Exploiting TIMSS & PIRLS combined data: multivariate multilevel modelling of student achievement *

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1 Abstract

We propose a multivariate multilevel model to analyse the Italian sample of the TIMSS&PIRLS 2011 Combined International Database on 4th grade students (Foy, 2013). The multivariate model jointly considers educational achievement on Reading, Mathematics and Science, thus allowing us to test for differential effects of the covariates on the three scores. This approach represents an advance with respect to official reports, where the three scores are analysed separately. Moreover, the multilevel approach allows us to disentangle student and contextual factors affecting achievement, also considering territorial differences in wealth.

In order to define the multivariate multilevel model, let Y_{mij} be the score on the m-th outcome for the i-th student of the j-th class, with $m = 1, 2, 3$ (1: Reading, 2: Math, 3: Science), $i = 1, \ldots, n_j$, $j = 1, \ldots, J$. The number of students of the j-th class is denoted with n_j, whereas the total number of students is denoted with $N = \sum_{j=1}^{J} n_j$. The Italian sample of the TIMSS&PIRLS 2011 Combined Dataset includes $N = 4,125$ students nested into $J = 239$ classes. We specify the following multivariate two-level model for outcome m of student i in class j:

$$Y_{mij} = \alpha_m + \boldsymbol{\beta}'_m \mathbf{x}_{mij} + \boldsymbol{\gamma}'_m \mathbf{w}_{mj} + u_{mj} + e_{mij} \tag{1}$$

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where $\mathbf{x}_{mij}$ is the vector of student-level covariates, $\mathbf{w}_{mj}$ is the vector of class-level covariates, also including covariates at higher level, e.g. school or province. Student-level errors $\mathbf{e}'_{ij} = (e_{1ij}, e_{2ij}, e_{3ij})$ are iid multivariate normal with zero means and unconstrained covariance matrix. Similarly, class-level errors $\mathbf{u}'_j = (u_{1j}, u_{2j}, u_{3j})$ are iid multivariate normal with zero means and unconstrained covariance matrix.

Model (1) is fitted by maximum likelihood. For each achievement scale, the TIMSS&PIRLS 2011 database provides five estimates of the student score, known as *plausible values*. In order to account for the variability induced by plausible values, estimation is performed separately for each of the five plausible values and then the results are combined by using multiple imputation formulas (Rubin, 1987). The analysis is carried out using the `mixed` and `mi` commands of Stata.

The main results are reported in (Grilli et al., 2014). The additional findings allowed by the multivariate approach are twofold. First, estimating the correlations among the three outcomes, we found that they are high at both student level and class level, even after adjusting for student and contextual factors. The correlations are extremely high at class level, so that in terms of school/class effectiveness the three outcomes are essentially indistinguishable. Second, testing for differential effects of the covariates on the outcomes, we found that females have a lower performance in Math and Science, but not in Reading, and student background covariates have similar effects on Reading and Science, as opposed to Math; on the other hand, contextual covariates have similar effects on the three outcomes.

A further peculiarity of our analysis lies in the use of the per-capita Gross Value Added at province level (GVA) as an external indicator accounting for territorial differences in wealth. The relationship between student achievement and GVA is well represented through a spline: it is found that student achievement is positively related to wealth for provinces below the national average, with no significant relationship for provinces above the national average.

The proportion of variability at class level is relevant even after controlling for the observed factors. The residual analysis allows us to locate classes with extremely high or low effectiveness.

References

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