Mathematical Methods for the Doubly-fed Induction Generator

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Abstract—

Motivation: A wind turbine powertrain includes a three-phase generator and switching electronics. The two most frequently installed types of generators are the synchronous induction machine and the doubly-fed induction machine (DFIG for short) [1]. Since the wind field is substantially unpredictable, whereas power has to be delivered to the network at (possibly) constant frequency and voltage amplitude, control of the powertrain plays an essential role in the design and operation of a wind power plant. This work focuses on the model of the DFIG for control purposes.

Co-energy and the Legendre Transform: The general process of power conversion from mechanical to electrical and the operation of a DFIG can be modeled by applying the mathematical methods of analytical mechanics and group theory. For example, let an electric machine have P poles and let its rotor form a mechanical angle $\theta_{r,mech}$ in the stator frame; then the electrical angle of the rotor θ_r with respect to the stator is given by $\theta_r = \frac{P}{2}\theta_{r,mech}$ (multiplier effect of P) and the resulting (motor) torque T can be shown to read [2, 3]

$$T = \frac{\partial W'_{fld}[\vec{i},\theta_r]}{\partial \theta_{r,mech}} = \frac{P}{2}\frac{\partial W'_{fld}[\vec{i},\theta_r]}{\partial \theta_r}$$

where $\vec{i}$ is the vector of electric currents and $W'_{fld}[\vec{i},\theta_r]$ is *co-energy*.

Standard textbooks do not generally introduce co-energy as the Legendre transform of the energy (stored in the magnetic field) with respect to magnetic flux linkage, $\vec{\lambda}$, nor point out the underlying differential geometric setting. Whereas $\vec{\lambda}$, as *energy* variables, are coordinates of the manifold $\mathcal{N}$, electric currents, $\vec{i}$, belong to the co-tangent bundle $T^*\mathcal{N}$ [4, 5]. The relation between co-energy and torque applies to any machine and can, in principle, deal with any functional dependence between $\vec{\lambda}$ and $\vec{i}$. Non-linear $\vec{\lambda}[\vec{i}]$ relations become of interest when saturation of the magnetic circuit has to be modeled.

Some examples and results in this direction will be provided.

The Blondel-Park Transformation and the Rotation Group: A linear DFIG is most effectively modeled in terms of stator and rotor flux linkages, $\vec{\lambda}_s$, $\vec{\lambda}_r$ (with obvious notation) and currents $\vec{i}_s$, $\vec{i}_r$, by means of inductance matrices. In the case of winding symmetry the equations read

$$\vec{\lambda}_s = \mathbf{L}_{ss}\cdot\vec{i}_s + L_x\mathbf{L}_{sr}[\theta_r]\cdot\vec{i}_r$$

and

$$\vec{\lambda}_r = L_x\mathbf{L}_{sr}^{\mathrm{Trs}}[\theta_r]\cdot\vec{i}_s + \mathbf{L}_{rr}\cdot\vec{i}_r.$$

The Blondel-Park transformation, which applies to electrical quantities of interest, is defined by the matrix

$$\mathbf{B}[\eta] := \sqrt{\frac{2}{3}}\begin{bmatrix} \cos[\eta] & \cos[\eta-\frac{2\pi}{3}] & \cos[\eta+\frac{2\pi}{3}] \\ -\sin[\eta] & -\sin[\eta-\frac{2\pi}{3}] & -\sin[\eta+\frac{2\pi}{3}] \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

where η stands for any of the electrical angles, usually denoted by θ, θ_r and $\beta(=\theta-\theta_r)$ respectively. As it is well known, the $\mathbf{B}[\eta]$'s form a one-parameter (η) group of unitary (power preserving) transformations.

The following properties can be shown to hold.

- Let $\mathbf{D}[\eta] := \mathbf{B}[0]^{-1}\cdot\mathbf{B}[\eta]$, then the exponential representation holds

$$\mathbf{D}[\eta] = e^{\eta\mathbf{F}},$$

where the matrix $\mathbf{F}$ is constant and is similar to the matrix $\mathbf{G}$, which row-wise reads $\mathbf{G} = [0\ 1\ 0| -1\ 0\ 0|0\ 0\ 0]$

$$\mathbf{F} = \mathbf{B}[0]^{-1} \cdot \mathbf{G} \cdot \mathbf{B}[0].$$

• By letting $\frac{\partial \mathbf{L}_{sr}[\theta]}{\partial \theta} := \mathbf{M}[\theta]$, where $\mathbf{L}_{sr}$ is the mutual, stator-to-rotor, inductance matrix, application of the $\mathbf{D}[.]$ transformation turns the electric torque into a bilinear, antisymmetric form for $\mathbf{G}$.

The extent to which the use of $\mathbf{D}[.]$ in the non-linear case (e.g., [6]) still simplifies results, and the class of tractable non-linearities, [7, 8] are currently being investigated.

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