Approximation of the Scattering Coefficients for a Non-RAYLEIGH Obstacle

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Let $\Omega \subset \Re^2$ be a star shaped obstacle with smooth boundary, $\mathit{\Gamma}$. Let u denote the incident scalar plane wave and v the scattered wave complying with $(u+v)|_{\mathit{\Gamma}} = 0$ and the SOMMERFELD radiation condition. Let λ denote a pair of indices and $\{u_\lambda\}$ be the family of real wave functions, which is linearly independent and complete in $L^2(\mathit{\Gamma})$, provided $k^2 \notin \sigma[-\Delta_D]$ (the wavenumber squared is not an eigenvalue of the interior DIRICHLET LAPLACE operator). The scattering coefficients are defined by $f_\lambda = -(i/4)\langle u_\lambda | \partial_N(u+v)\rangle$, where $\partial_N(.)$ is the outward normal derivative on $\mathit{\Gamma}$ and $\langle .| \ .\rangle$ denotes the inner product in $L^2(\mathit{\Gamma})$.

If L denotes the approximation order and $\Lambda[L]$ the related set of indices, approximate scattering coefficients $\{p_\lambda^{(L)}\}$ can be introduced

$$p_\lambda^{(L)} = -(i/4)\langle u_\lambda | \partial_N u + (\partial_N v)_2^{(L)}\rangle \tag{1}$$

with

$$(\partial_N v)_2^{(L)} = \sum_{\mu \in \Lambda[L]} c_\mu^{(L)} \partial_N v_\mu. \tag{2}$$

Here $F_2 = \{\partial_N v_\mu\}$ is the family of normal derivatives of outgoing waves $\{v_\mu\}$, which is unconditionally complete, and $\{c_\mu^{(L)}\}$, $\mu \in \Lambda[L]$, are suitable expansion coefficients. Let $W \equiv \{w_\mu\}$ denote a family of functions such that

$$w_\mu = (1/2)\partial_{N[\mathbf{r}]} v_\mu + (i/4) \int_\Gamma \partial_{N[\mathbf{r}]} H_0^{(1)}[kR] \partial_{N[\rho]} v_\mu d\Gamma[\rho], \tag{3}$$

where $R = |\mathbf{r} - \rho|$.

The following results can be shown to hold.

1) The family W is linearly independent and complete in $L^2(\Gamma)$ provided $k^2 \notin (\sigma[-\Delta_D] \cup \sigma[-\Delta_N])$ i.e., k^2 is neither an eigenvalue of the interior DIRICHLET nor of the interior NEUMANN LAPLACE operators.

2) The coefficients $\{c_\mu^{(L)}\}$, which form the vector $\mathbf{c}^{(L)}$ of card $[\Lambda[L]]$ components, solve the well-posed algebraic system
$$\mathbf{W}^{(L)} \cdot \mathbf{c}^{(L)} = \mathbf{g}^{(L)}, \tag{4}$$
where $\mathbf{W}^{(L)} = [\langle w_\lambda | w_\mu \rangle]$ is the GRAMian of $\{w_\mu\}$ and $\mathbf{g}^{(L)} = [\langle g | w_\mu \rangle]$ is a vector of known terms obtained from
$$g = (1/2)\partial_{N[\mathbf{r}]} u - (i/4) \int_\Gamma \partial_{N[\rho]} u \partial_{N[\mathbf{r}]} H_0^{(1)}[kR] d\Gamma[\rho]. \tag{5}$$

3) Finally, an error estimate for $\| \partial_N v - (\partial_N v)_2^{(L)} \|_{L^2(\Gamma)}^2$ can be provided in terms of the smallest eigenvalue of a double layer acoustic potential.

RAYLEIGH's hypothesis is nowhere required i.e., both F_2 and W only have to be linearly independent and complete.