Endogenous Market Structures and Corporate Finance

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by

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Abstract

We characterize the optimal financial structure as a strategic device to optimize the value of a firm competing in a market where entry is endogenous. Debt financing is always optimal under quantity competition and, contrary to the Brander-Lewis-Showalter results based on duopolies, we show the optimality of moderate debt financing also under price competition with cost uncertainty (but not with demand uncertainty). We derive the formulas for the optimal financial structure, which does not affect the strategies of the other firms but reduces their number.
1 Introduction

In this article we characterize the optimal financial structure as a strategic device to optimize the value of a firm competing in a market where entry is endogenous, and we show the general optimality of moderate debt financing under both quantity and price competition in the presence of cost uncertainty (but not with demand uncertainty).

The celebrated theorem by Modigliani and Miller (1958) states the irrelevance of the financial structure for a firm with access to perfectly competitive markets. A wide literature started with Brander and Lewis (1986), Showalter (1995) and others\(^1\) has shown that in the presence of strategic interactions and commitment power on the leverage choice, the Modigliani-Miller irrelevance breaks down and the optimal financial structure depends on the form of competition, in quantities or in prices.\(^2\) For instance, in case of a duopoly with uncertainty on the costs, it is optimal for a firm to issue debt under quantity competition with strategic substitutability, because this leads the equity holders to produce more (given their focus on low-cost scenarios) so as to reduce the production of the competitors and increase their expected profits (Brander and Lewis, 1986). However, debt financing is sub-optimal under price competition, with strategic complementarity, because it leads the equity holders to reduce their prices (again because of their focus on low-cost scenarios) inducing the competitors to do the same, which hurts expected profitability (Showalter, 1995).

A theoretical limit of the strategic approach to the choice of the optimal financial

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\(^2\)As well known, alternative theories based on agency costs (Jensen and Meckling, 1976; Myers, 1977; and Jensen, 1986), asymmetric information (Ross, 1977; Myers and Majluf, 1984) and incomplete contracts (Hart and Moore, 1995, 1998) have proved useful in explaining departures from the neutrality of the financial structure (see Harris and Raviv (1991) and Tirole (2006) for surveys), however it seems that strategic considerations related to product market competition can be seen as important complementary elements to these theories.
structure is that it focuses on the short run and neglects the endogeneity of entry of competitors in the market, which is crucial in the long run. When a firm commits to a certain financial structure as a strategic device, it appears reasonable that this firm is taking in consideration not only the impact on the competition with a rival, but the impact on the entire equilibrium market structure expected in the future, meaning the number of competitors and their price or quantity strategies. In particular, a financial structure inducing an aggressive behavior in the market will lead not only to aggressive replies of the rivals, but also to a lower number of entrants. The traditional analysis ignores this endogeneity of the market structure, whose importance for understanding strategic interactions is well established in the industrial organization literature (see Sutton, 2007 or Etro, 2007). The basic purpose of this article is to extend the models of Brander (1986) and Showalter (1995) to the long run case of an endogenous number of competitors in the market, showing that some of the traditional results are overturned.

On the empirical front, initial research provided limited support for the results by Brander and Lewis (1986) of a positive correlation between leverage and aggressive competition (Chevalier, 1995a; Phillips, 1995). Nevertheless, recent works have confirmed the presence of strategic effects of debt financing on product market competition especially in highly concentrated markets, showing that a moderate debt level or a recent LBO by a firm are associated with more aggressive strategies of the same firm, higher sale growth and lower prices (see Lyandres, 2006, and Campello, 2006). These effects tend to disappear for excessive debt levels (above industry average) and for the case of market leaders in an industry (Campello, 2006).

More importantly, the empirical research does not find out any differential impact of debt financing on product market strategies between industries characterized by strategic substitutability or complementarity. In particular, Lyandres (2006) examines this impact after classifying industries on the basis of the approach of Sundaram et al. (1996), that derives an empirical measure of the cross derivative of profits with respect to own and competitors’ sales: this derives from the coefficient of correlation between marginal profits and competitors’ sales, which should be negative under strategic substitutability and positive under strategic complementarity. The result of
Lyandres (2006) is that the relation between the extent of competitive interaction and firms’ leverage ratios is positive, statistically significant and economically important for both subsamples of firms competing in strategic substitutes and complements: this is in contrast with the results of Brander (1986) and Showalter (1995) which largely depend on the kind of strategic interaction.

Finally, some works have emphasized a relation between entry conditions and the impact of debt on pricing. Phillips (1995) analyzed four industries in which firms have sharply increased their financial leverage and noticed a positive association between debt and firm’s output only in an industry characterized by free entry and not in the other three industries which were characterized by high barriers to entry. Zingales (1998) analyzed the trucking industry after deregulation, finding a strong negative association between debt leverage and prices only during the liberalization process (when entry was endogenous). More recently, Khanna and Tice (2000) looked at the impact of entry by a market leader (Wal-Mart in supermarkets) and found that firms that undergo LBOs before entry occurs tend to be more aggressive when competing.\footnote{Also Chevalier (1995,b) examined changes in supermarket prices in local markets after LBOs and found out that prices decrease following LBOs in front of rival firms which are not highly leveraged, while they increase when the LBO firm’s rivals are also highly leveraged. In line with the implications of Brander and Lewis (1986) and Showalter (1995) for the case of price competition, she associated the former result with predatory strategies and the latter with a softening of price competition, but she did not control for the endogeneity of entry, which may vary across local markets with different financial features.} These results suggest that the endogeneity of entry may be important in understanding the relation between debt financing and product market competition.

We study the optimal financial structure for firms active in markets where entry is endogenous. This is the most natural situation for all the markets where there are not natural or legal barriers to entry, and where the market structure can be regarded as endogenous at least in the medium-long run. Endogenous entry does not change Brander and Lewis’ (1986) conclusion that debt has strategic value in case of quantity competition (regardless of whether there is demand or cost uncertainty), except for extending it beyond the case of strategic substitutability. However, the results are
radically different from the duopoly results of Showalter (1995) for the case of price competition: while in a short run (no-entry) setting debt has no value in case of price competition with uncertain costs (but may have value under demand uncertainty), in the long run (when entry is endogenized) debt has always value in the case of cost uncertainty (but not demand uncertainty).

To understand the rationale behind these results, consider our initial example of cost uncertainty: under endogenous entry the optimal financial structure remains characterized by moderate debt financing under both quantity and price competition, independently from whether strategic substitutability or complementarity holds. The intuition for this independence of the optimal financial structure from the form of competition is related to the one emerging in the literature on precommitments in the presence of endogenous market structures (see Etro, 2006, 2008). Under all forms of competition, debt induces the equity holders (or their managers) to choose their strategies focusing only on the positive (low cost) scenarios, and it is chosen taking in consideration the impact on the strategies of the competitors and on the entry process. Since any commitment to soften competition attracts entry until all the profitable opportunities are exploited, the only way to gain an advantage over the rivals is to commit to strengthen competition. Therefore, debt financing becomes useful whenever a positive shock increases the marginal profitability of a more aggressive strategy, independently from the form of competition.

Notice that debt leverage is only one mechanism to strengthen competition, which should be adopted for this purpose in alternative to other investments as, for instance, in production capacity or cost reductions (emphasized in Etro, 2006). These investments are likely to be more costly than debt leverage, at least when the credit market is relatively efficient, therefore we expect that the strategic use of debt can be a useful

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4 These results are reminiscent of the implications of entry for strategic trade policy. A well known literature started by Brander and Spencer (1985) has shown that the optimal policy for the domestic firms exporting to a foreign country in competition with a fixed number of competitors can be a subsidy or a tax depending on whether quantity or price competition occurs. Recently, Etro (2010a) has shown that whenever entry in the foreign market is endogenous, it is always optimal to adopt export subsidies to give a strategic advantage to the domestic firms.
commitment to expand output.\footnote{More generally, the availability of alternative commitment tools should be taken in consideration in the empirical investigations on the relation between leverage and competition.} Of course, market leaders (in the sense of Stackelberg) able to commit to a larger production would not need additional strategic tools as debt leverage for this purpose, and this is consistent with the empirical results that find no relation between leverage and aggressive behavior for market leaders in an industry but only for the other firms (Campello, 2006).

In conclusion, our theoretical results suggests that future empirical research should focus on the impact of entry conditions and on the source of uncertainty to verify the validity of the strategic approach to the financial structure: the prediction is that debt leverage should be more strongly associated with aggressive competition in markets characterized by free entry and relevant cost uncertainty.

The article is organized as follows. Section 2 describes the general model, Section 3 solves it for the cases of price and quantity competition and Section 4 concludes.

2 A general framework

In this section we present a general model of strategic interactions. Consider $n$ firms choosing a strategic variable $x_i > 0$ with $i = 1, 2, ..., n$. They all compete in Nash strategies, that is taking as given the strategies of each other. The profit functions of each firm $i$ are disturbed by a random shock $z_i \in [z, \bar{z}]$ independently and identically distributed according to the cumulative function $G(z)$ with density $g(z)$. This shock may affect the cost function, as in our main examples of the next section, or the demand function. Entry requires an initial fixed sunk cost $F$ paid by all the active firms, and net expected profits correspond to the expectation of the gross profits net of this fixed cost of entry.

The strategies chosen by the firms deliver the profit function $\Pi_i(x_i, \beta_i, z_i)$ for firm $i$, which is quasiconcave in the first argument and decreasing in the second one, which aggregates the strategies of the other firms in $\beta_i = \sum_{k=1, k \neq i}^n h(x_k)$ for some positive and increasing function $h(x)$. Without loss of generality, assume that the random variable is chosen so that a positive shock increases profits. Most of the commonly
used models of oligopolistic competition in quantities and in prices are nested in our general specification (in case of price competition, think of the strategy $x$ as the inverse of the price).

Each firm has enough cash to finance the costs of entry and production entirely without issuing debt. However, a firm may issue debt for purely strategic reasons if this affects product market competition as in Brander and Lewis (1986). Debt does not affect firm’s cash holdings, therefore the proceeds from its issue are paid out as dividends immediately. This assumption allows us to abstract from any other reason to have leverage in order to look at the strategic effect.\footnote{One could extend the model with cash constraints to characterize the optimal debt contract endogenously and to examine additional interactions between debt and strategic choices. See Khanna and Schroder (2009) for important progress in this direction.} The credit market is perfectly competitive, so that lenders break even and the firm issues debt at its market value. In such a context, the Modigliani-Miller neutrality result breaks down if and only if debt affects competition in the product market.

Following Brander and Lewis (1986), suppose that firm $i$ chooses a debt obligation which will be repaid out of gross profits if these are larger than $d_i$, which, from now, will be simply called the debt level. Otherwise the firm goes bankrupt and liquidated in favor of the lender. The value of equity, corresponding to the expected profits net of debt repayment can be written as:

$$E^i(x_i, \beta_i, d_i) = \int_{\hat{z}(d_i)}^{\hat{z}} [\Pi(x_i, \beta_i, z_i) - d_i] g(z) dz \quad (1)$$

where the lower bound $\hat{z}(d_i)$ is such that $\Pi(x_i, \beta_i, \hat{z}) = d_i$. Notice that $\hat{z}'(d) = 1/\Pi_z(x_i, \beta_i, \hat{z}) > 0$. The sign of the cross derivative $\Pi_{xz}(x_i, \beta_i, z)$ depends on the particular model adopted, but it can be determined unambiguously under reasonable conditions. Under price competition a positive cost shock tends to decrease the marginal profitability of a price increase ($\Pi_{xz}(x_i, \beta_i, z) > 0$), but the effect of a demand shock is the opposite under weak conditions (see Showalter, 1995). Under quantity competition, a positive shock tends to increase the marginal profitability of production ($\Pi_{xz}(x_i, \beta_i, z) > 0$). Notice that debt affects the expected marginal profits
and therefore the strategies of the firm.

For simplicity, we focus on the optimal strategic financial structure of a single firm, say firm $M$, assuming that the financial structure of the competitors implies no debt (however we will also discuss how to relax this hypothesis). Therefore, we consider the following game. First, firm $M$ pays the fixed cost of entry and adopts a financial structure by issuing debt $d$. Second, other firms decide whether to pay the fixed cost of entry and be active if this is expected to be profitable. Then, the equity holders of all the firms choose their market strategies. Finally, uncertainty is resolved and payoffs for equity holders and debt holders are assigned. The game is solved by backward induction. First, we characterize the endogenous market structure for a given level of debt $d$ for firm $M$, with strategies $x_M(d)$ for this firm and $x(d)$ for all the entrants, whose number is $n(d)$. Finally, we find out the optimal strategic debt for firm $M$.

The initial ownership of firm $M$ chooses debt to maximize the overall value of the firm, which can be seen as the sum of the equity value $E^M = E^M(x_M(d), \beta_M(d), d)$ and the market value of debt:

$$
D^M[x_M(d), \beta_M(d), d] = \int_{\hat{z}} \Pi[x_M(d), \beta_M(d), z] g(z) dz + d[1 - G(\hat{z}(d))] \tag{2}
$$

where the first term represents the expected repayment in the case of bankruptcy and the second one the expected repayment in case of successful outcome for the firm.\(^7\) Therefore, the optimal debt $d^*$, when positive, can be seen as maximizing the final expression for the total value of the firm:

$$
V(d) = E^M + D^M = \int_{\hat{z}} \Pi[x_M(d), \beta_M(d), z] g(z) dz \tag{3}
$$

so that $V'(d^*) = 0$. In the next section we will fully characterize the optimal strategic financial structure in specific models of competition in prices and quantities, while in the rest of this section we focus on the general principles.

\(^7\)The amount raised by the debt issue is $D^M$ under a competitive credit market, and we assume that this loan flows directly to shareholders, with the interest rate implicitly given by $r$ such that $(1 + r)D^M = d$. 9
Our game belongs to the general class of games of strategic commitment characterized by Fudenberg and Tirole (1984) for a duopoly and by Etro (2006) for the endogenous entry case. In the second case, the nature of the optimal strategic commitment depends only on the impact of the strategic tool on marginal profitability. Here, this corresponds to the impact of debt $d$ on the marginal effect that the strategic variable $x_M$ chosen by firm $M$ exerts on the value of equity $E_M$. As long as this impact is positive, there is always a strategic incentive to issue debt. To verify when this is the case in general, we start by deriving the marginal effect that the strategic variable chosen by the firm exerts on the value of equity:

$$E_x^M(x_M, \beta_M, d) = \int_{\tilde{z}(d)}^{\tilde{z}} \Pi_x(x_M, \beta_M, z)g(z)dz$$

The sign of the marginal profit at its bounds $\tilde{z}(d)$ and $\tilde{z}$ depends on the sign of $\Pi_{xz}(x_M, \beta_M, z)$. In particular $\Pi_x(x_M, \beta_M, \tilde{z}) \leq 0$ if $\Pi_{xz}(x_M, \beta_M, z) \geq 0$, which allows us to derive the sign of the cross effect:

$$E_{xd}^M(x_M, \beta_M, d) = -\Pi_x(x_M, \beta_M, \tilde{z})\tilde{z}'(d)$$

$$= \frac{-\Pi_x(x_M, \beta_M, \tilde{z})}{\Pi_{x}(x_M, \beta_M, \tilde{z})} \geq 0 \text{ if } \Pi_{xz}(x_M, \beta_M, z) \geq 0$$

This implies that, in the traditional case of a duopoly, under strategic substitutability there is a strategic incentive to issue debt when a positive shock increases marginal profits ($\Pi_{xz}(x_M, \beta_M, z) > 0$) and under strategic complementarity in the opposite case ($\Pi_{xz}(x_M, \beta_M, z) < 0$). For instance, under competition in quantities there is a strategic role for debt financing as long as strategic substitutability holds (Brander and Lewis, 1986), while under competition in prices there can not be a role for debt financing under cost uncertainty, but only under demand uncertainty (Showalter, 1995). However, things are different in the long run, that is when entry takes place endogenously until profitable opportunities are exhausted. In this case we can apply a result by Etro (2006, Prop. 1) and conclude with the following:

**Theorem.** The optimal strategic financial structure for a firm facing competition in a market with endogenous entry is characterized by positive debt whenever positive
shocks increase the marginal profitability of strategies that expand production or reduce prices, independently from the form of competition.

Endogenous entry does not change Brander and Lewis’ (1986) conclusion that debt has strategic value in case of quantity competition (regardless of the type of uncertainty), but extends it beyond the usual assumption of strategic substitutability. However, the results are strikingly different from the duopoly results of Showalter (1995) for the case of price competition. While in a duopoly setting debt has value in case of price competition with uncertain demand, the conclusion is reversed when entry is endogenized: now debt has value in the case of cost uncertainty, but not demand uncertainty.

To understand the intuition of this outcome, notice that debt induces the equity holders to choose their strategies focusing on the expected profits conditional on a positive scenario, because under negative scenarios they go bankrupt and their payoff is zero. The debt is chosen taking in consideration this and the impact on the strategies of the competitors and on the entry process. Any commitment to soften competition attracts entry until all the profitable opportunities are exploited, leaving the soft competitor with negative profits. The only way to gain an advantage over the rivals is to commit to strengthen competition and limit the number of entrants. Consequently, debt financing becomes useful whenever a positive shock increases the marginal profitability of a more aggressive strategy, independently form the kind of strategic interactions. This is always the case under quantity competition, because cost shocks induce a debt-constrained firm to expand production and demand shocks have the same effect. Moreover, this is also the case under price competition and cost uncertainty, because this leads a debt-constrained firm to reduce its prices.

From the general analysis we can derive further insights and verify how robust our results are. First of all, notice that the equilibrium is characterized by the optimality conditions for the market strategies $E^M(x_M, \beta_M, d^*) = 0$ and $E_x(x, \beta, 0) = 0$, by the endogenous entry condition $E(x, \beta, 0) = F$, and by the optimality condition for debt $V'(d^*) = 0$. In this equilibrium system the strategies of the entrants $x$ and their aggregators of the others’ strategies $\beta$ do not change with the debt level, while the strategy of the financially constrained firm $x_M$ is increasing in its debt, and the
endogenous number of firms satisfying $\beta = (n - 2)h(x) + h(x_M)$ is decreasing in it.
This implies that $\beta_M(d) = \beta + h(x) - h(x_M(d))$ with $\beta_M'(d) = -h'(x_M(d))x_M'(d)$ and
therefore the optimality condition for debt can be derived from the maximization of (3) as:
\[
\int_{\bar{z}}^{\tilde{z}} \Pi_x [x_M(d^*), \beta_M(d^*), z] g(z)dz = \int_{\bar{z}}^{\tilde{z}} h'(x_M(d^*)) \Pi_x [x_M(d^*), \beta_M(d^*), z] g(z)dz
\]
(6)

For future empirical analysis, one should keep in mind that a larger debt induces a
more aggressive strategy of the levered firm, but does not affect the strategies of the
other firms (contrary to the Brander-Lewis model) and it is associated with a smaller
number of them.

This optimal debt level $d^*$ induces the management to behave exactly as a Stackelberg leader (without debt) in front of endogenous entry (as in Etro, 2008). This follows from the fact that, in the absence of further costs of debt, the optimal financial structure replicates the best precommitment on the market strategy. A consequence of this is that market leaders cannot use debt financing in a strategic way: this cannot provide a better outcome than their pre-commitment strategy.

2.1 Bankruptcy costs

The model can be easily extended to take into account bankruptcy costs whose burden
is on the debtholders. Suppose that in case of bankruptcy the equity holders bear a
loss $C(d)$ that is increasing and convex in the debt leverage. Then, the optimal debt
maximizes total expected firm’s value $V(d) - G(\hat{z}(d))C(d)$. The optimal financial
structure satisfies $V'(d^*) = g(\hat{z}(d^*))C(d^*)\hat{z}'(d^*) + G(\hat{z}(d^*))C'(d^*)$ when there is an
interior solution. This condition requires a smaller debt level (since the right hand side

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8The best precommitment strategy solves the problem of the Stackelberg leader under the optimality and endogenous entry conditions for the followers:

\[
\max_{x_M} \int_{\tilde{z}}^{\bar{z}} \Pi_{x_M}(x_M, \beta_M, z) g(z)dz \quad \text{s.t.:} \quad E_x(x, \beta, 0) = E(x, \beta, 0) - F = 0, \beta_M = \beta + h(x) - h(x_M)
\]
is positive), and does not replicate the best precommitment equilibrium. Nevertheless, all the qualitative results on the incentives to issue debt go through.

### 2.2 Multiple levered firms

Finally, we can show that the optimal financial structure does not depend on whether other firms can adopt it as well (and is not jointly suboptimal as in the Brander-Lewis framework). Let us consider what would happen if other firms were choosing their financial structures strategically at the same time as firm $M$. In the duopolistic model of Brander and Lewis (1986) this was leading to higher debt leverage for both firms and lower profits compared to the zero debt case: in a sense, debt financing was jointly sub-optimal. This Prisoner’s Dilemma outcome disappears when the market structure is endogenous. To verify this, notice that, when there are $m$ strategic firms choosing their debt level, as long as there is also a fringe of unlevered entrants, the optimal financial structure for the other strategic firms remains exactly the same as the one characterized above. The reason is that the equilibrium optimality condition of the unlevered entrants, which is $E_x(x, \beta, 0) = 0$, and the endogenous entry condition, which is $E(x, \beta) = F$, remain unchanged, with $x$ and $\beta = (n - m - 1)h(x) + mh(x_M)$ independent from the debt decisions. Simultaneously, every levered firm $j$ with debt $d_j$ chooses its strategy $x_j$ according to the same condition as before, $E^j_x(x_j, \beta_j, d_j) = 0$ where $\beta_j = \beta + h(x) - h(x_j)$ as before. Since this last condition defines a unique increasing function $x_j(d)$, all the strategic firms choose their debt to maximize the total value $V(d_j) = \int_{\bar{z}}^{\bar{z}} [x_j(d_j), \beta + h(x) - h(x_j(d_j)), z]g(z)dz$, which provides the same equilibrium optimality condition as before: $V'(d^*) = 0$ for any $j$. All the strategic firms adopt the same debt level as above, but of course the total number of unlevered firms decreases when $m$ increases.

This simple extension suggests that our rules for the optimal strategic financial structure hold under a rather general set-up. However, it would important to extend our analysis to the case in which all the firms can simultaneously choose their financial structure and there are no unlevered firms.
3 Optimal financial structure

In this section, we fully develop two examples of optimal financial structure emerging under different forms of competition, in prices and in quantities. We focus on the case of cost uncertainty because it allows us to derive relatively straightforward results and to emphasize the differences between exogenous and endogenous market structures, that is short run and long run outcomes.

3.1 Optimal financial structure with Bertrand competition

Let us consider price competition with cost uncertainty. Denote with \( p_i \) the price of firm \( i \). Any model with direct demand \( D_i = D(p_i, P_{-i}) \), where \( D_1 < 0 \), \( P_{-i} = \sum_{j=1, j \neq i}^n b(p_j) \), \( D_2 < 0 \) and \( b'(p) < 0 \), is nested in our general framework after setting \( x_i \equiv 1/p_i \), \( \beta_i = P_{-i} \) and \( h(x_i) = b(1/x_i) \). Substitutability between goods is guaranteed by the fact that the cross derivative \( \frac{\partial D_i}{\partial p_j} \) is always positive:

\[ \Lambda_{ij} = D_2(p_i, P_{-i})b'(p_j) > 0 \text{ for any } i \text{ and } j. \]

Examples include common demand functions, as the isoelastic demand function à la Dixit-Stiglitz, any other demand derived from additively separable preferences, the Logit demand function and others.

Adopting a constant marginal cost \( c(z) \) with \( c'(z) < 0 \) and unconditional expectation \( c \), we obtain the gross profits for firm \( i \):

\[ \Pi(x_i, \beta_i, z_i) = [p_i - c(z_i)] \left\{ \frac{1}{x_i} - c(z_i) \right\} D(1/x_i, \beta_i) \tag{7} \]

which satisfies \( \Pi_{xz}(x, \beta, z) > 0 \). As usual for models of price competition, we assume that strategic complementarity holds, that is \( \Pi_{x\beta}(x_i, \beta_i, z_i) > 0 \). The value of equity of firm \( M \) can be written as:

\[ E^M = \int_{\hat{z}(d)}^{\hat{z}} \left\{ [p_M - c(z)] D(p_M, P_{-M}) - d \right\} g(z)dz \tag{8} \]

where the lower bound on the shock \( \hat{z}(d) \) is such that \( [p_M - c(\hat{z})] \cdot D(p_M, P_{-M}) = d. \)

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\(^9\)Nothing would change in case of increasing marginal costs, that is with a cost function \( c(x, z) \) that satisfies \( c_x(x, z) > 0 \), \( c_{xx}(x, z) \geq 0 \) and \( c_z(x, z) < 0 \), as long as a positive shock reduces the marginal cost \( (c_{xz}(x, z) < 0) \), because this implies \( \Pi_{xz}(x, \beta, z) > 0 \).
The expected profits of the other firms can be expressed as:

\[ E^i = \int_{z}^{\hat{z}} \{[p_i - c(z)]D(p_i, P_{-i})\} g(z)dz \]  

(9)

Brander and Lewis (1986) and Showalter (1995) have shown that in the short run, that is when the number of firms is exogenous (as in a duopoly), debt financing is counterproductive because it induces the firm and the rivals to reduce prices and profits. However, in the long run, that is when the number of firms is endogenous, the result breaks down and debt financing is always optimal.

Let us characterize the endogenous market structure given \( d \). The equilibrium first order condition of firm \( M \) is:

\[
[D(p_M, P_{-M}) + p_M D_1(p_M, P_{-M})][1 - G(\hat{z}(d))] = D_1(p_M, P_{-M}) \int_{\hat{z}(d)}^{\hat{z}} c(z) dg \]  

(10)

where the second order condition \( \Delta < 0 \) is assumed satisfied. The first order condition of the other firms and the free entry condition read as:

\[
\int_{z}^{\hat{z}} \{D(p, P) + [p - c(z)] D_1(p, P)\} g(z)dz = 0, \quad \int_{z}^{\hat{z}} [p - c(z)] D(p, P) g(z)dz = F \]  

(11)

This system defines the price of the other firms \( p \) and their price aggregator \( P \) independently from \( d \), according to the last two equations. Moreover, the first equation defines the price of the firm \( M \), \( p_M(d) \), as a function of \( \hat{z}(d) \) and therefore of \( d \). Total differentiation implies:

\[
p'_M(d) = \frac{D(p_M, P_{-M}) + p_M D_1(p_M, P_{-M}) - D_1(p_M, P_{-M}) c(\hat{z}(d))}{\Delta} g(\hat{z}(d)) \hat{z}'(d) \]  

\[ = - \frac{[c^e(z > \hat{z}(d)) - c(\hat{z}(d))] g(\hat{z}(d)) D_1(p_M, P_{-M})}{c_z(\hat{z}(d))D(p_M, P_{-M})} < 0 \]  

(12)

where we defined the expectation of the marginal cost conditional on \( z > \hat{z}(d) \) as follows

\[ c^e[z > \hat{z}(d)] = [1 - G(\hat{z}(d))]^{-1} \int_{\hat{z}(d)}^{\hat{z}} c(z)g(z)dz, \]  

which must be always smaller than the marginal cost at the cut-off \( \hat{z}(d) \).

This characterization of the endogenous market structure implies that the number of firms \( n \) satisfies \( P = (n - 2)b(p) + b(p_M(d)) \) and, therefore, it must be decreasing.
in the debt level:

\[ n(d) = 2 + \frac{P - b(p_M(d))}{b(p)} \quad \text{with} \quad n'(d) = -\frac{b'(p_M(d))p_M'(d)}{b(p)} < 0 \quad (13) \]

A larger debt of firm \( M \) is going to induce a more aggressive price strategy of this firm, which will not affect the price choice of the other firms, but will induce entry by fewer firms. Finally, the equilibrium market structure allows us to express the price aggregator of firm \( M \) as \( P_M = P + b(p) - b(p_M(d)) \).

The initial ownership chooses the debt contract to maximize the overall value of the firm:

\[
\max_d V(d) = \int_{\mathbb{Z}} \left\{ [p_M(d) - c(z)] D [p_M(d), P + b(p) - b(p_M(d))] \right\} g(z) dz
\]

subject to the equilibrium system for the definition of \( p_M(d) \). The optimality condition is:

\[
p_M'(d^*) \int_{\mathbb{Z}} \mathbb{E} \left\{ D(p_M(d^*), P_{-M}) + [p_M(d^*) - c(z)] [D_1(p_M(d^*), P_{-M}) - \Lambda] \right\} dg = 0
\]

where \( \Lambda = D_2(p_M(d^*), P_{-M})b'(p_M(d^*)) > 0 \) represents the indirect effect that an induced price change exerts on demand through the change in the endogenous number of entrants. Combining this with the first order equilibrium condition (10) and rearranging, one obtains an implicit expression for the optimal financial structure:

\[
c^e \left[ z > \hat{z}(d^*) \right] = c + \frac{(p_M(d^*) - c)\Lambda}{D_1(p_M(d^*), P_{-M})} \quad (14)
\]

The left hand side is the expectation of the marginal cost conditional on good realizations of the random variable (such that all debt is paid back and there is a positive return for the equity holders). This conditional expectation is the relevant reference for the equity holders (or the management appointed by them) in their pricing decisions, it is smaller than the unconditional expectation \( c \) and it is decreasing in the debt level. The right hand side is also smaller than \( c \), and decreases in the indirect effect \( \Lambda \) that an induced price change exerts on demand (through the change in the endogenous number of entrants). The optimal strategic financial structure requires a debt level high enough to equate the two sides.
The indirect effect of debt financing is to induce the equity holders to choose their price strategies with regard to the positive realizations of the random variable. These correspond to low cost realizations, which induce the choice of low prices. Such an aggressive strategy has a double impact on the competitors. On one side it leads them to reduce their prices as well (because of the strategic complementarity), and on the other side it reduces the incentives to enter in the market, which increases concentration and exerts a positive effect on the prices. The net consequence of these two effects is that the equilibrium price of the entrants remains unchanged, while their number is reduced. Therefore, in a long run perspective debt allows a firm to reduce its price below those of the rivals and to limit entry, which leads to unambiguously increase sales and to obtain a small profit margin which spreads over a large market share.

Finally, consider the same model with demand uncertainty. As long as a higher realization of the shock increases not only the direct demand but also its slope \( \frac{\partial^2 D_i}{\partial p_i \partial z_i} > 0 \), or does not decrease the latter too much, we have \( \Pi_{xz}(x, \beta, z) < 0 \). In such a case there is no value for debt: \( d^* = 0 \). As noticed above, this overturns the results for price duopolies in Showalter (1995).

### 3.2 Optimal financial structure with Cournot competition

In this section we describe the optimal financial structure in the case of competition in quantities with homogenous goods and cost uncertainty. Assume that the strategy \( x_i \) represents the quantity produced by firm \( i \) and \( X \) is total production corresponding to price \( p(X) \) decreasing in \( X \). The cost function \( c(x, z) \) is assumed to satisfy \( c_x(x, z) > 0 \), \( c_{xx}(x, z) \geq 0 \), \( c_z(x, z) < 0 \) and \( c_{xz}(x, z) < 0 \), so that a positive shock reduces the marginal cost. It follows that gross profits for firm \( i \) are:

\[
\Pi(x_i, \beta_i, z_i) = x_i p(x_i + \beta_i) - c(x_i, z_i)
\]

which is nested in our general model with \( \beta_i = \sum_{j=1,j\neq i}^n x_j \) and satisfies \( \Pi_{xz}(x, \beta, z) > 0 \). The cost function \( c(x, z) \) has an unconditional expectation \( c'(x) = \int_{\bar{z}}^{\bar{z}} c(x, z)g(z)dz \), with a conditional expectation for the marginal cost given by \( c'_x(x) = \int_{\bar{z}}^{\bar{z}} c_x(x, z)g(z)dz \).
The equity value for firm $M$ with debt $d$ is:

$$E^M = \int_{\bar{z}(d)}^{\bar{z}} [p(X)x_M - c(x_M, z) - d] g(z) dz$$

(16)

where the cut-off $\bar{z}(d)$ is defined by $p(X)x_M - c(x_M, \bar{z}) = d$. The expected profits of the other firms can be expressed as:

$$E^i = \int_{\bar{z}}^{\bar{z}} [p(X)x_i - c(x_i, z)] g(z) dz$$

(17)

Given the debt level $d$ of firm $M$, a symmetric equilibrium between the entrants is characterized by the first order and endogenous entry conditions:

$$p(X) + xp'(X) = c^e_x(x) \quad \text{and} \quad p(X)x - c^e(x) = F$$

(18)

which jointly define the production of each entrant $x$ and total production $X$. Contrary to what happens in the short run, as in a duopoly à la Brander and Lewis (1986) where entry is not possible, in the long run the financial structure does not affect the production of the other firms or the total production. The simultaneous first order equilibrium condition for firm $M$ is:

$$[p(X) + x_Mp'(X)] [1 - G(\bar{z}(d))] = \int_{\bar{z}(d)}^{\bar{z}} c_x(x_M, z) g(z) dz$$

(19)

and we assume that the second order condition $\Delta < 0$ is satisfied. This equilibrium condition establishes a positive relation between debt $d$ and production $x_M$:

$$x'_M(d) = \frac{\left[-[p(X) + x_Mp'(X)] + c_x(x_M, \bar{z}(d))\right] g(\bar{z}(d)) \bar{z}'(d)}{\Delta} = \frac{\left[c_x(x_M, \bar{z}(d)) - c_x^e[x_M, z > \bar{z}(d)]\right] g(\bar{z}(d)) \bar{z}'(d)}{\Delta} > 0$$

where we defined the expectation of the cost conditional on good realizations of the shock $z > \bar{z}(d)$ as follows $c^e_x[x, z > \bar{z}(d)] \equiv [1 - G(\bar{z}(d))]^{-1} \int_{\bar{z}(d)}^{\bar{z}} c_x(x, z) g(z) dz$. The sign of the derivative follows from the fact that this conditional expectation of the
marginal cost must be always smaller than the marginal cost evaluated at the cut-off \( \hat{z}(d) \). Finally, notice that the independence of total production \( X = (n-1)x + x_M(d) \) from the debt level, implies that the number of firms must be decreasing in the debt of firm \( M \):\(^{10}\)

\[
n(d) = 1 + \frac{X - x_M(d)}{x} \quad \text{with } n'(d) = -\frac{x'_M(d)}{x} < 0
\] (20)

A larger debt is going to induce a more aggressive strategy of firm \( M \), which does not affect the production choice of the other firms, but induces entry by fewer of them.

Let us move to the optimal choice of the debt level. We can express the problem of the optimal debt contract as:

\[
\max_d V(k) = \int_{z}^{\hat{z}} \left[ p(X) x_M(d) - c(x_M(d), z) \right] g(z) dz
\]

subject to the system for the definition of \( x_M(d) \). The optimality condition is:

\[
x_M'(d^*) \cdot \int_{z}^{\hat{z}} \left[ p(X) - c_x(x_M(d^*), z) \right] g(z) dz = 0
\] (21)

which requires the equality of the expected marginal cost of firm \( M \) with the price:

\[
c_x^e [x_M(d^*)] = p(X)
\] (22)

Combining this optimality condition with the first order equilibrium condition (19) and rearranging, one obtains another implicit expression for the optimal financial structure:

\[
c_x^e [x_M(d^*)] = c_x^e [x_M(d^*), z > \hat{z}(d^*)] - x_M(d^*) p'(X)
\] (23)

which clearly shows that the optimal debt must be positive: to verify this, notice that the left hand side is the expected marginal cost and the right hand side is

\(^{10}\)When the debt level arrives at a cut-off \( d \), the number of firms reaches \( n = 2 \) and entry deterrence occurs for any higher debt level. Here we focus on situations in which entry deterrence is not optimal. As shown in a related framework by Etro (2008), for this to be the case we need the marginal cost function to be increasing enough. Otherwise, the entry deterrence case can be easily characterized. However, notice that given the entry deterring debt \( d \), as long as there are no entrants it is \textit{ex post} optimal to adopt the monopolistic production (augmented for the debt bias).
the expectation of the same marginal cost conditional on good realizations of the
shock, augmented with a term reflecting the shape of the demand function. Since the
conditional expectation of the marginal cost is always smaller than the unconditional
expectation in case of positive debt, the optimal debt must be high enough to equate
the two sides.

From (22) we can easily verify the comparative statics of the optimal leverage
with respect to the fixed cost of entry. As long as an increase in the fixed cost
reduces total output and therefore increases the price (which is always the case under
strategic substitutability), the optimality condition requires a higher marginal cost,
and therefore a higher $x_M(d^*)$ which is only possible through a higher debt level.
Consequently, optimal debt leverage is increasing in the size of the fixed costs of
entry: $\partial d^*/\partial F > 0$. This should not be surprising since for $F$ going to zero we
revert to a perfectly competitive market with infinite firms where there is no strategic
value for debt (the Modigliani-Miller neutrality is back) and when $F$ increases the
number of firms goes down and strategic considerations become more important.
This suggests that in markets characterized by endogenous entry we should expect a
positive correlation between optimal debt leverage and market concentration.

Finally, one can examine demand uncertainty within the same framework. As
long as a higher realization of the shock increases not only the inverse demand but
also its slope ($\partial^2 p_i/\partial x_i \partial z_i > 0$), or does not decrease the latter too much, we have
$\Pi_{xz}(x, \beta, z) > 0$, which confirms the optimality of a positive debt leverage. For
instance this is the case with an additive shock on a linear demand function.

### 3.3 A closed form solution for the optimal debt

Consider an example with a linear demand:

$$p(X) = a - X$$ \quad (24)

and a quadratic cost function:

$$c(x, z) = (c - z)x + \frac{x^2}{2}$$ \quad (25)
which satisfies $c_x = c + x - z > 0$, $c_{xx} = 1 \geq 0$, $c_z = -x < 0$ and $c_{xz} = -1 < 0$.
Assume that $z$ is uniformly distributed in the interval $[-\sigma, +\sigma]$ with expectation
$E[z] = 0$ and variance $Var(z) = \sigma^2/3$.\footnote{Notice that we interpret the shock as a cost shock, but it may be reinterpreted as a demand shock affecting the intercept of the inverse demand. We assume that $\sigma$ and $F$ are in the relevant range to provide an interior solution for the optimal debt obtained in the text.}

Given any debt level $d$ of firm $M$, the endogenous market structure is characterized by an equilibrium price:

$$ p = c + 2\sqrt{2F/3} $$  \hspace{1cm} (26)

a production of each entrant $x = \sqrt{2F/3}$, and a production of the levered firm satisfying $p - x_M(d) = c_x^e [x_M(d), z > \hat{z}(d)]$. The cut-off $\hat{z}(d)$ is defined by $px_M(d) - (c - \hat{z})x_M(d) - x_M(d)^2/2 = d$ so that:

$$ \hat{z}(d) = \frac{d + x_M(d)^2/2 - (p - c)}{x_M(d)} = \frac{d}{x_M(d)} + \frac{x_M(d)}{2} - 2\sqrt{\frac{2F}{3}} $$  \hspace{1cm} (27)

and the expectation of the marginal cost conditional on higher realizations of $z$ is defined as:

$$ c_x^e [x, z > \hat{z}(d)] = c + x - \int_{\hat{z}(d)}^{\sigma} \frac{zdz}{\sigma - \hat{z}(d)} = c + x - \frac{\sigma^2 - \hat{z}(d)^2}{2[\sigma - \hat{z}(d)]} $$  \hspace{1cm} (28)

This allows us to solve for the equilibrium production of the levered firm $x_M(d) = p - c_x^e [x_M(d), z > \hat{z}(d)]$ as:

$$ x_M(d) = \sqrt{\frac{2F}{3}} + \frac{\sigma^2 - \hat{z}(d)^2}{4[\sigma - \hat{z}(d)]} $$  \hspace{1cm} (29)

Since the optimal debt must induce marginal cost pricing for the levered firm ($p = c + x_M(d^*)$), in equilibrium it must be that:

$$ x_M(d^*) = 2\sqrt{2F/3} $$  \hspace{1cm} (30)

which, using (27), implies:

$$ \hat{z}(d^*) = d^* \sqrt{3/8F} - \sqrt{2F/3} $$  \hspace{1cm} (31)
Substituting both expressions in (29) and solving for \( d^* \) we obtain the implied optimal debt leverage as:

\[
d^* = 4F - \sqrt{\frac{8F}{3} \left( \frac{8F}{3} + \sigma^2 - 2\sqrt{\frac{8F}{3}} \right)} \tag{32}
\]

This example provides an additional insight on the relationship between uncertainty and the financial structure: the optimal debt decreases with the variance of the shock because greater uncertainty strengthens the debt commitment on the equity holders. This result confirms the negative relation between volatility and debt leverage found by Franck and Le Pape (2008) and Haan and Toolsema (2008) in a duopoly setting (correcting a mistake by Wanzenried, 2003).

4 Conclusion

In this paper we have characterized the optimal financial structure as a strategic device to optimize the value of a firm competing in a market where entry is endogenous. The outcome is that, contrary to what happens in case of an exogenous market structure, the optimality of debt financing does not depend crucially on the form of competition, but only on the impact of uncertainty on the marginal profits. This allowed us to characterize the optimal debt leverage under both Cournot and Bertrand competition with endogenous entry.

Hopefully, this work can shed some light on the empirical debate. Initial empirical works (Chevalier, 1995a, Phillips, 1995) found that debt tends to soften competition, against the Brander and Lewis (1986) claim. However, the strategic approach could still be supported through the Showalter (1995) critique, according to which the form of competition or uncertainty may change the Brander-Lewis results; the same could be said for our results, with the addition that we show that debt increases the output of the levered firm but does not change that of the competitors (or even total output under quantity competition and homogenous goods). More recent empirical research has found some positive relation between leverage and aggressive competition but: not for excessive debt or for market leaders (Campello, 2006), independently from the form of strategic interaction in substitutes or complements (Lyandres, 2006), and
most of all only when entry is endogenous (Phillips, 1995; Zingales, 1998; Khanna and Tice, 2000). Our results are broadly in line with these facts, and suggest that future investigations should focus on the impact of entry conditions and on the source of uncertainty to verify the validity of the strategic approach to the financial structure.

One could extend the model with forms of heterogeneity between firms. Basic differences between the profit functions of the levered firm and of the entrants would not change the spirit of the results because debt would still serve the purpose of constraining entry. However, heterogeneity between firms that are cash constrained and bear a risk of bankruptcy and firms with “deep pockets” may substantially change the relation between debt and product market competition. A recent important work by Khanna and Schroder (2009) has characterized the optimal debt contract for a cash constrained firm with unobservable profits competing in prices against an unlevered firm (under demand uncertainty): this contract imposes the threat of nonrenewal à la Bolton and Scharfstein (1990) to induce truthful revelation of the profits, but it also affects product market competition in new ways. When nonrenewal attracts entry of a more efficient firm, the optimal contract induces the unlevered firm to soften price competition. It would be interesting to extend our model to the case of a cash constrained firm which is more likely to exit when competition is more intense (in which case the characteristics of the potential entrants would be crucial).

Our results can be seen as a step in the complex research program at the crossroads of the literatures on industrial organization and corporate finance. The former has been recently focused on the analysis of endogenous market structures most of the time taking as given the financial structure, and the latter has mainly analyzed the optimal financial structure taking as exogenous the market structure. Future research should explore in larger depth the joint characterization of endogenous financial and industrial structures, emphasizing the role of external variables (as technological conditions, the size of the market or the fixed costs of entry) on the endogenous financial and industrial decisions, and deriving the correlations between indicators of the industrial and financial structures. This may also lead to updates of the agenda for

\[12\] Related investigations on alternative contracts that can affect the endogenous market structures are analyzed in Etro (2010,b).
empirical research.

References


