Real Business Cycles with Cournot Competition and Endogenous Entry

Andrea Colciago, Federico Etro
No. 135 – February 2008
Real Business Cycles with Cournot Competition and Endogenous Entry *

by Andrea Colciago and Federico Etro
University of Milano, Bicocca, Department of Economics

Abstract

We introduce strategic interactions with quantity competition à la Cournot and endogenous entry in an RBC model with homogenous goods. In the long run, the steady state mark up is decreasing in the capital share, in the discount factor and in the level of technology, while it is increasing in the rate of bankruptcy and in the entry cost. In the short run, a competition effect amplifies the propagation of the shocks and generates procyclical profits and countercyclical mark ups. We extend the model to different forms of competition (as imperfect collusion and Stackelberg competition). The analysis of technology and preference shocks and of the second moments suggests that the model outperforms the RBC in terms of variability of output, labor and, of course, profits and mark ups.

JEL classification: L11, E32.
Keywords: Endogenous Market Structure, Firms’ Entry, Business Cycle.

*Correspondence: Andrea Colciago, University of Milano-Bicocca, Department of Economics, U6-360, Piazza dell’Ateneo nuovo 1, Milano 20126, Italy.
1 Introduction

Most of the business cycle literature is based on models with perfect competition between firms producing homogenous goods, as in the Real Business Cycles (RBC) tradition started by Kydland and Prescott (1982). This assumption leads to marginal cost pricing and to indeterminate market structures where strategic interactions are absent. The aim of this paper is to introduce imperfectly competitive markets characterized by strategic interactions between an endogenous number of firms in an otherwise standard DSGE model with capital accumulation, with the purpose of evaluating its business cycle properties relative to those of neoclassical models in the RBC tradition.\footnote{We also depart from the New Keynesian literature started by Blanchard and Kiyotaki (1987), which is based on monopolistic behavior of an exogenous number of firms. This leads to exogenous market structures where monopolistic prices are given by constant mark ups on the marginal cost and strategic interactions are, again, neglected.}

We build on the recent literature on endogenous entry in macroeconomics, in particular on the works by Ghironi and Melitz (2005), Bilbiie et al. (2007a,b) and the companion paper Etro and Colciago (2007),\footnote{The first papers to endogenize entry in a DSGE model are due to Chatterjee and Cooper (1993), Devereux et al. (1996) and Wu and Zhang (2001). Cooper (1999) surveys this early literature. Cook (2001), Devereux and Lee (2001) and Jaimovich and Floetotto (2007) have extended this class of models to strategic interactions but focusing on free entry in each period (which eliminates any dynamics for the profits). For an introduction to the competition effect emphasized here see Etro and Colciago (2007) and Etro (2007,b, 2009).} and we focus on Cournot competition between firms producing homogenous goods. Entry in the market requires a sunk investment, which is measured in effective units of labor, and it is determined endogenously in each period to equate the discounted value of profits to the entry cost. We set up a DSGE model where technology and preference shocks lead to a change in the supply or demand conditions and, through this, to changes in the market structure (i.e.: number of competitors, mark ups and individual production). These changes feed back in the aggregate economy affecting output and consumption.

The focus on homogeneous goods meets two desiderata, one on the theoretical side and the other on the empirical side. On the theoretical side we wish to compare our framework with the standard RBC model, which implicitly assumes homogenous goods (with perfect competition in the production of each good). On the empirical side, goods homogeneity allows us to identify the contribution of the intensive margin, as opposed to the extensive margin, to business cycle dynamics. This is reasonable since most of the available empirical evidence suggesting a significant correlation between GDP and measures of business creation, as that in Jaimovich and Floetotto (2007) and Lewis (2008), refers to the intensive margin. Moreover, many markets are characterized by a small (and endogenous) number of firms producing or distributing goods that are highly substitutable (think of wholesale and retail trade or hotels and restaurants at the local level) and/or for which strategic interactions are extremely important (as for many manufacturing or service activities in case of high fixed costs of advertising or R&D). The assumption of perfectly substitutable goods naturally
leads us to consider the standard forms of quantity competition starting from competition à la Cournot. In the companion paper Etro and Colciago (2007) we have analyzed the case of imperfect substitutable goods with firms competing in prices à la Bertrand.

In the short run, our endogenous market structures introduce a strong propagation mechanism of exogenous shocks. A temporary expansionary shock generates profit opportunities which attract entry of new competitors. This leads to a positive temporary competition effect on prices, in the form of a reduction in mark ups, which boosts consumption. As a result the effects of shocks are amplified. We analyze both technology shocks and preference shocks, and we obtain impulse response functions that are empirically plausible and reproduce procyclical profits and entry and countercyclical mark ups. The propagation mechanism we emphasize improves the explanation of second moments of variables such as output and hours with respect to a standard flexible prices model. While these results constitute a remarkable improvement with respect to the standard real business cycle model, it has to be said that the model substantially underestimates the volatility of aggregate profits and mark ups with respect to those in the data.

In the long run, the endogenous market structure is fully determined by structural parameters: in particular, we explicitly derive the steady state mark up and the number of firms. The mark up is decreasing in the elasticity of output to capital, which is proportional to the capital share (a higher elasticity reduces the wage and therefore the cost of entry, which promotes business creation and competition), in the discount factor (which tends to increase savings and investments in business creation) and in the labor productivity (which tends to increase output and demand and to create space for new business creation and stronger competition). Finally, the mark up is increasing in the rate of business destruction and in the cost of entry (two factors that reduce the incentives to enter and compete). The number of firms and the steady state capital/labor ratio follow the opposite comparative statics.

We show that the results discussed above are robust to other forms of quantity competition traditionally considered in the literature. Resorting to the conjectural variations approach, we show that competitive frameworks involving a degree of collusion stronger than that implied by the baseline Cournot model conform to the propagation mechanism described above and increase the variability of output and labor. Adopting the Stackelberg framework together with the assumption that a single leader is always active into the market we can, instead, address two pieces of empirical evidence. The first one, reported inter alia by Sutton (1997), is that the vast majority of entry and exit action in many U.S. manufacturing industries is due to small firms. The second one, emphasized by Vivek (2007), is that small and large firms are affected asymmetrically by shocks. Vivek (2007) emphasizes that this phenomenon is exacerbated by the presence of sunk entry costs. We show that production choices and profits of the leader can be markedly different from those of the followers in the aftermath of a technology shock. Hopefully, these applications will promote further investigations on the interaction between the microeconomic structure of the
markets and macroeconomic models.

Recent empirical works on the manufacturing sector by Broda and Weinstein (2007) and Bernard et al. (2008) have emphasized the importance of the extensive margin in the process of product creation or innovation. In the companion paper Etro and Colciago (2007) we account for the extensive margin by introducing a CES consumption index and deriving the equilibrium between firms producing differentiated goods. However, it should be emphasized that the entry process is mainly associated with new business creation, rather than product innovation, in most (non-high-tech) manufacturing and service industries, where competition is in the market rather than for the market. Even if they may be less innovative, these industries represent a large part of modern economies and therefore the impact of business creation in these sectors may have large aggregate effects. To give an example of our idea, in a boom, profitable opportunities may lead new “traditional” businesses to start, new restaurants to open, or new services to be provided, which will increase competition in the respective markets and reduce the mark ups. None of these businesses will affect the extensive margin as defined in the statistics in terms of new consumer products, but they will be relevant nevertheless. For this reason, the focus of this paper remains on markets with homogenous goods provided by a limited number of firms.\textsuperscript{3}

In what follows we will discuss the empirical motivations of our approach at the microeconomic and macroeconomic level. A wide industrial organization literature provides theoretical and empirical support for the relevance of the competition effect that we propose as a propagation mechanism. At the theoretical level, there is a crucial difference between models of monopolistic behavior \textit{à la} Dixit and Stiglitz (1977), recently employed by Bilbiie et al. (2007a,b) for business cycle analysis, and models with strategic interactions as ours. In the first class of models, the mark up and the production of each firm are constant, while the number of firms increases linearly with the size of the market. In the second class of models, positive shocks to the size of the market attract entry and strengthen competition in such a way that the mark up decreases, the production of each firm increases (to cover the fixed costs) and the number of firms increases less than proportionally. Early works of the New Empirical Industrial Organization literature starting with Bresnahan and Reiss (1987) and more recent works by Manuszak (2002), Campbell and Hopenhayn (2005), Manuszak and Moul (2008), Etro (2009) and others have provided convincing evidence in support of the second class of models and of the competition effect on mark ups, firms’ production and number of firms.


\footnotesize{\textsuperscript{3}See Etro (2009, b) for an empirical exercise based on our model which estimates the impact on business creation of a new technology able to reduce fixed entry costs for all firms (cloud computing).}
To provide further support to these empirical findings we constructed a labor share based measure of the price mark up for the U.S. along the lines suggested by Rotemberg and Woodford (1999) in their analysis of cyclical mark up behavior.\textsuperscript{4} Figure 1 plots the cross correlation between detrended GDP and the price mark up (panel a), net entry\textsuperscript{5} (panel b) and profits (panel c) at a quarterly frequency from 1948:1 to 1995:3.\textsuperscript{6} Cross correlations are provided for various leads and lags together with 95% confidence bands. Net entry and profits are strongly procyclical with a contemporaneous correlation with output equal to 0.45 and 0.71, respectively. The mark up is, instead, countercyclical and shows a contemporaneous correlation with output equal to -0.42. Notice also that the

\textsuperscript{4}The procedure used to derive our price mark up measure is described in Appendix B together with the data used in the remainder of the paper.

\textsuperscript{5}The measure of net entry adopted in the analysis is constituted by the index of net business formation supplied by the Brad&Broadstreet corporation. The index runs from 1948:1 to 1995:3 (1967=100).

\textsuperscript{6}Variables have been logged. We use a polynomial of time to detrend variables instead of the HP filter, as suggested by Galli et al. (2007). However detrending with the HP filter leads just to minor changes.
correlation between the price mark up and output is higher in absolute value when judged against lagged values of output. The dynamics delivered by the model in this paper, in response to both demand and supply shocks are consistent with this evidence.

A few other works have provided competing explanations for mark up countercyclicality. The closest to ours is the model by Rotemberg and Woodford (1992) in which a fixed number of firms collude to keep their prices above marginal cost. Collusion is supported by the threat to revert to lower prices in the future if a member of the cartel deviates from the announced price. However, when current demand is high with respect to future demand, there is an incentive to cut prices: to prevent the breakdown of the cartel the optimal collusive agreement implies that in these circumstances price mark up should be reduced. As a consequence entry does not play any role and its procyclicality cannot be explained within the model. Ravn et al. (2006) consider a situation where habits are formed at the level of individual goods, as opposed to at the level of aggregate consumption goods. In this case producers face dynamic demand functions which are characterized by a price-inelastic component. This term causes the price elasticity of demand to be increasing in current aggregate demand. Since in their setting mark ups are inversely related to the price elasticity of demand it follows that periods of higher than average aggregate demand are characterized by lower mark ups. However, this demand-based explanation of countercyclical mark ups has no implications for changes in the market structures over the business cycle. Finally, sticky prices models such as, inter alia, Kimball (1995) and Galì et al. (2007), can also address the countercyclicality of mark ups by assuming that an exogenous fraction of firms can optimally adjust their prices just occasionally. Notice, however, that since mark ups variations originating from nominal rigidities are undesired, sticky prices model can hardly match mark ups countercyclicality without implying profits countercyclicality as well.

The remainder of the paper is organized as follows. In Section 2 we lay out the model. We initially consider the case of Cournot competition where labor is the only factor of production. This helps understanding the strategic behavior of producers and the entry and exit process. Next, we introduce capital accumulation and consider alternative forms of competition. Section 3 studies the impulse response functions to technology and preference shocks. Section 4 studies the second moments of the model and compares them to those delivered by a standard RBC model. Section 5 concludes. Further details are left in the Appendix.

2 The Model

We develop a simple dynamic stochastic general equilibrium model with imperfect competition and endogenous entry. For simplicity, we first develop a model in which firms compete à la Cournot and produce homogenous goods employing labor as the only input. Prior to entry into the market firms are subject to a
sunk entry cost in units of effective labor. Then we augment this model with accumulation of physical capital. Finally we extend the model to alternative forms of quantity competition as imperfect collusion and Stackelberg competition.

2.1 Strategic interactions

Let us consider a representative sector characterized by $N_t$ firms in each period $t$, all producing the same consumption good. We can think of this as a representative sector of the economy. All our results would go through in the presence of multiple sectors producing differentiated goods and each one with the structure of our representative sector.\(^7\)

Each firm $i = 1, 2, ..., N_t$ produces $y_t(i)$ according to a linear production function:

$$y_t(i) = A_t L_t^c(i) \quad \text{(1)}$$

where $A_t$ is exogenous TFP common to all firms, and $L_t^c(i)$ is the labor input used by firm $i$ for the production of the final good. Given the nominal wage $W_t$, the constant nominal marginal cost of production is $MC_t = W_t / A_t$. Total expenditure in the sector is:

$$EXP_t = p_t C_t = p_t \sum_{j=1}^{N_t} y_t(j)$$

where $p_t$ is the price of the homogenous good and $C_t$ its consumption in period $t$.

Assuming that all firms take total expenditure as given in each period, their perceived inverse demand function must be $p_t = EXP_t / \sum_{j=1}^{N_t} y_t(j)$. Accordingly, the nominal profits of firm $i$ are:

$$\Pi_t(i) = (p_t - MC_t) y_t(i) = \frac{y_t(i) EXP_t}{N_t} - MC_t y_t(i) \quad \text{(2)}$$

Assume that in each period, the $N_t$ firms compete in quantities, choosing their individual production $y_t(i)$ to maximize profits taking as given the production of all the other firms. The Cournot equilibrium generates the symmetric individual output:

$$y_t = \frac{(N_t - 1) EXP_t}{N_t^2 MC_t} \quad \text{(3)}$$

Substituting in the inverse demand, one obtains the equilibrium price:

$$p_t = \frac{N_t}{N_t - 1} MC_t \quad \text{(4)}$$

\(^7\)In the companion paper Etro and Colciago (2010), we provide a model with a continuum of sectors producing differentiated goods and competing in prices.
which is associated with the mark up \( \mu_t = N_t/(N_t - 1) \). This equilibrium generates individual profits \( \Pi_t(N_t) = EXP_t/N_t^2 \) in nominal terms. Taking the consumption good as the *numeraire*, we have \( EXP_t = C_t \) and we can express the real profits as:

\[
\pi_t(N_t) = \frac{C_t}{N_t^2}
\]

and the real wage as follows from the equilibrium pricing relation:

\[
w_t = \frac{N_t - 1}{N_t} A_t
\]

When the number of firms increases, the equilibrium price goes down and the wage goes up, with the former approaching the marginal cost and the latter approaching the marginal productivity of labor only for \( N_t \to \infty \). However, the number of firms in the market is constrained by the presence of fixed costs of entry that endogenously rule the entry of new firms in the goods market.

In every period \( N_{t+1}^* \) new firms enter in the market, and, following Ghironi and Melitz (2005), we assume that a fraction \( \delta \in (0,1) \) of the (old and new) firms exits the market for exogenous reasons. Therefore, the number of firms follows the equation of motion:

\[
N_{t+1} = (1 - \delta)(N_t + N_{t+1}^*)
\]

The real value of a firm \( V_t \) is the present discounted value of its future expected profits, or in recursive form:

\[
V_t = (1 - \delta)E_t \left( (1 + r_{t+1})^{-1} \left( V_{t+1} + \frac{C_{t+1}}{N_{t+1}^2} \right) \right)
\]

where \( r_t \) is the real interest rate at time \( t \). The number of firms is endogenous in the sense that in each period entry occurs until the real value of the representative firm equates the fixed cost of entry. The latter is equal to \( \eta/A_t \) units of labor, where \( \eta > 0 \) parametrizes the entry cost. Thus, the endogenous entry condition amounts to:

\[
V_t = \frac{\eta w_t}{A_t}
\]

which can also be interpreted as a no-arbitrage condition of a model of occupational choice for the labor market (in the simpler case where \( \eta/A_t = 1 \), the labor force is allocated so as to equalize the wage of a worker to the value of firms created by an entrepreneur). Using the equilibrium expression for the wage we have:

\[
V_t = \frac{\eta(N_t - 1)}{N_t}
\]
2.2 Consumer choices

The expected lifetime utility of the representative agent reads as:

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log (C_t - \Theta_t) - \nu \frac{L_t^{1+1/\varphi}}{1 + 1/\varphi} \right] \tag{11} \]

where \( E_0 \) denotes the mathematical expectations operator conditional on information available at time 0, \( \beta \in (0, 1) \) is the discount factor while \( \varphi \) is the Frish elasticity of labor supply. The variable \( C_t \) represents consumption of the homogenous good, \( L_t \) denotes hours worked and \( \Theta_t \) represents a consumption demand shock. A positive variation in \( t \) leads to an urgency to consume by temporarily increasing marginal utility of consumption. The time \( t \) flow budget constraint in real terms is:

\[ B_{t+1} + V_t (N_t + N_t^e) x_{t+1} + C_t = w_t L_t + (1 + r_t) B_t + [\pi_t (N_t) + V_t] N_t x_t \tag{12} \]

where \( B_t \) is net bond holdings and \( x_t \) is the share of the stock market value of the firms that are owned by the agent. The agent faces the, usual, problem of maximizing expected lifetime utility subject to (12), and chooses consumption, hours and how much to invest in risk free bonds and risky stocks out of labor and profit income. The first order conditions with respect to \( x_{t+1}, L_t \) and \( B_{t+1} \) are respectively:

\[ V_t (N_t + N_t^e) (C_t - \Theta_t)^{-1} = \beta E_t \left\{ [\pi_{t+1} (N_{t+1}) + V_{t+1}] N_{t+1} (C_{t+1} - \Theta_{t+1})^{-1} \right\} \tag{13} \]

\[ L_t = \left( \frac{1}{\nu} w_t (C_t - \Theta_t)^{-1} \right)^{\varphi} \tag{14} \]

\[ (C_t - \Theta_t)^{-1} = \beta (1 + r_{t+1}) E_t (C_{t+1} - \Theta_{t+1})^{-1} \tag{15} \]

where \((C_t - \Theta_t)^{-1}\) is the time \( t \) marginal utility of consumption. Equation (13) is an asset pricing equation for stocks, while equation (15) is a standard Euler equation for bonds. Equation (14) can, instead, be interpreted as a labor supply schedule. Rearranging equation (13) and considering the equation of motion for the number of firms (7), it follows that the value of a firm can be expressed as:

\[ V_t = \beta E_t \left\{ \left( \frac{C_{t+1} - \Theta_{t+1}}{C_t - \Theta_t} \right)^{-1} [\pi_{t+1} (N_{t+1}) + V_{t+1}] \frac{N_{t+1}}{N_t + N_t^e} \right\} = \tag{16} \]

\[ = \beta (1 - \delta) E_t \left\{ \left( \frac{C_{t+1} - \Theta_{t+1}}{C_t - \Theta_t} \right)^{-1} [\pi_{t+1} (N_{t+1}) + V_{t+1}] \right\} \]

which corresponds to the present discounted sum of future profits.
2.3 Endogenous market structures in the short run

Using the value of the firms (7) and the definition of profits (5), we can rewrite the endogenous entry condition (10) as:

$$\frac{\eta(N_t - 1)}{N_t} = \beta(1 - \delta)E_t \left[ \left( \frac{C_{t+1} - \Theta_{t+1}}{C_t - \Theta_t} \right)^{-1} \left( \frac{\eta(N_{t+1} - 1)}{N_{t+1}} + \frac{C_{t+1}}{N_{t+1}} \right) \right]$$  (17)

where the flow budget constraint under the equilibrium conditions $B_t = 1 - x_t = 0$ for any $t$, leads to the aggregate resource constraint of the economy:

$$C_t + V_t N_t^e = w_t L_t + \pi_t (N_t) N_t$$  (18)

which states that the sum of consumption and investment must be equal to total income from labor and profits. Substituting the labor supply schedule and the free entry condition into the aggregate resource constraint leads to the number of new entrants in each period:

$$N_t^e = \frac{1}{\eta} \left[ A_t \left( \frac{A_t}{C_t - \Theta_t} \frac{(N_t - 1)}{v N_t} \right)^{v} - C_t \right]$$

The latter together with the dynamic equation describing the evolution of the number of firms yields:

$$N_{t+1} = (1 - \delta)N_t + \frac{1 - \delta}{\eta} \left[ A_t \left( \frac{A_t}{C_t - \Theta_t} \frac{(N_t - 1)}{v N_t} \right)^{v} - C_t \right]$$  (19)

Given an exogenous processes for $A_t$ and $\Theta_t$, the system composed by equations (17) and (19) fully characterizes the dynamic behavior of $C_t$ and $N_t$, and thus of all the other variables. In particular, it provides the short run characterization of the endogenous market structures in terms of number of firms, mark ups and production per firm.

2.4 Endogenous market structures in the long run

The long run market structure is endogenously identified by the solution to the following system derived from (17) and (19) in steady state:

$$NN : \quad C^* = A - \frac{\eta \delta N^*}{1 - \delta}$$  (20)

$$CC : \quad C^* = \eta \left[ \frac{1}{(1 - \delta)\beta} - 1 \right] (N^* - 1) N^*$$  (21)

where a starred variable denotes the steady state value. The first steady state relation (CC) is a negative relation between consumption and the number of firms (more consumption implies a lower investment to replace firms), the second one (NN) is a positive and convex relation (more firms produce more and increase
Figure 2: **Phase Diagram for the Cournot Model.** Point SS₁: steady state consumption (C⁺) and number of producers (N⁺) associated with the initial level of technology. In the face of a permanent technology increase, curve NN shifts to NN’. The steady state associated to the higher level of technology is represented by point SS₂.

Substituting CC into NN, one can derive that the steady state endogenous market structure is characterized by following the number of firms:

\[ N^* = \frac{\eta(1 - \beta)}{2\eta(1 - \beta)(1 - \delta)} \Phi(\beta, \eta, A, \delta) \]  

where:

\[ \Phi(\beta, \eta, A, \delta) = \sqrt{\eta^2(1 - \beta)^2 + 4\beta\eta(1 - \delta)[1 - \beta(1 - \delta)]A} \]

Without loss of generality we normalized steady state hours worked to unity. For a given Θ, the value of ν can be selected accordingly. Alternatively, one can see this as the equilibrium system for the case of exogenous unitary labor supply.
and by the equilibrium mark up:

$$\mu^* = \frac{\Phi(\beta, \eta, A, \delta) + \eta(1 - \beta)}{\Phi(\beta, \eta, A, \delta) - \eta(1 - \beta + 2\beta\delta)}$$

(23)

The mark up is decreasing in the discount factor $\beta$, in total factor productivity $A$ and increasing in the exit rate $\delta$, and in the entry cost parameter $\eta$. The intuitions for the impact of the four structural parameters determining the endogenous market structure in the long run are the following. First of all, higher productivity increases labor income and demand, which attracts entry and strengthens competition. As shown in Figure 2, a permanent increase in productivity shifts the locus NN to NN': the new steady state is reached in point $SS_2$, which is characterized by a higher number of producers and thus by lower mark ups. The second determinant is the size of the costs of entry: when these are high, profitability is low and the long run equilibrium is characterized by high concentration and high mark ups. To the extent that the costs of entry are artificial, in the sense that there are barriers to entry due to product market regulations, reforms leading to deregulation reduce concentration and mark ups in the long run. The third factor is the way people discount future utility: when agents are more patient, the interest rate is lower and the discounted sum of future profits is higher, which attracts more entry, strengthens competition and ultimately reduces the mark ups - therefore more patient agents lead to a higher number of firms in the steady state. The last element is the rate of business destruction due to exogenous reasons: when the risk of bankruptcy is high, there are only few firms in the long run (but with a high rate of turnover), and they apply a high mark up to their goods.

Summing up, the equilibrium endogenously generates imperfect competition between a positive but limited number of firms producing the homogenous good. Notice that dynamic inefficiency holds, since a better allocation of resources could be achieved reducing the number of firms and the waste in fixed costs of production of a homogenous good.

2.5 Capital Accumulation

In this section we augment the baseline model with endogenous investment in physical capital. Differently from Bilbiie et al. (2007,b) we assume that capital is used solely in the production of final goods. The resulting framework nests,

---

9 The positive effect of a permanent technology shift on the number of firms would disappear if the cost of entry parameter $\eta$ was proportional to the size of the economy. However, notice that the fact that mark ups are decreasing in the number of firms implies that scale effects become negligible when TFP increases indefinitely.


11 The afore mentioned authors introduce capital in the business creation sector as well. We could adopt the same approach and confirm their result for which the equilibrium exhibits a cycling path, which is converging for high depreciation and diverging for low depreciation. The simpler approach adopted here avoids these phenomena.
as discussed below, both the standard RBC model and our model with Cournot competition. Let us assume that final goods are produced with a standard Cobb-Douglas production function of the form:

\[ y_t(i) = A_t K_t^\alpha L_t^\nu (i)^{1-\alpha} \]  

(24)

where \( \alpha \in [0, 1) \) is the elasticity of output to capital, which will be proportional (but not equal to) the capital share. New firms are created with the same technology as before. The stock of capital evolves according to:

\[ K_{t+1} = (1 - \delta^k) K_t + I_t^k \]  

(25)

where \( I_t^k \) is time-\( t \) investment in physical and \( \delta^k \) represents the rate of physical depreciation. The representative household holds the stock of capital and rents it to the producers of the final good. In this case total investment amounts to the sum of investment in physical capital and in new firms, thus \( I_t = I_t^k + V_t N_t^r \). Moreover the household has a further intertemporal optimality condition with respect to the earlier model, which translates into the following Euler equation:

\[ (C_t - \Theta_t)^{-1} = \beta E_t (C_{t+1} - \Theta_{t+1})^{-1} \left( 1 + r_{t+1}^k - \delta^k \right) \]  

(26)

where \( r_t^k \) is the rental rate of capital.

The symmetric Cournot equilibrium in the market for final homogenous goods leads again to the price (4), where the marginal cost derives from the standard problem of cost minimization under Cobb-Douglas production function as \( MC_t = \left( r_t^k \right)^\alpha (w_t)^{1-\alpha} / \alpha^\alpha (1 - \alpha)^{1-\alpha} A_t \). The demand of inputs in the good producing sector requires:

\[ w_t = (1 - \alpha) A_t \left( \frac{K_t}{L_t} \right)^\alpha \left( \frac{N_t - 1}{N_t} \right) \]  

(27)

and:

\[ r_t^k = \alpha A_t \left( \frac{K_t}{L_t} \right)^{\alpha-1} \left( \frac{N_t - 1}{N_t} \right) \]  

(28)

Profits are given by:

\[ \pi_t(N_t) = \frac{C_t + I_t^k}{N_t^2} \]  

(29)

Using the fact that \( C_t + I_t^k = A_t K_t^\alpha L_t^\nu (i)^{1-\alpha} \) to solve for the labor employed in the production of final goods, we can obtain the following new equation governing the dynamic of the number of firms:

\[ N_{t+1} = (1 - \delta) N_t + (1 - \delta) \frac{A_t}{\eta} L_t - \left( \frac{C_t + I_t^k}{A_t K_t^\alpha} \right)^{\frac{1}{1-\alpha}} \]  

(30)

while equation (16) remains unchanged.
The steady state endogenous market structures can be characterized as before, with the following number of firms in the long run:

\[ N^* = \frac{\eta[(1 - \alpha)(1 - \beta) + \alpha \beta \delta] + \Phi(\beta, \eta, A, \delta, \alpha)}{2\eta(1 - \alpha)[1 - \beta(1 - \delta)]} \quad (31) \]

where:

\[ \Phi(\beta, \eta, A, \delta, \alpha) = \sqrt{\eta^2[(1 - \alpha)(1 - \beta) + \alpha \beta \delta]^2 + 4\beta \eta(1 - \alpha)(1 - \delta)[1 - \beta(1 - \delta)]}A \]

with \( \Phi(\beta, \eta, A, \delta, 0) \equiv \Phi(\beta, \eta, A, \delta) \). The long run mark up converges to:

\[ \mu^* = \frac{\Phi(\beta, \eta, A, \delta, \alpha) + \eta[(1 - \alpha)(1 - \beta) + \alpha \beta \delta]}{\Phi(\beta, \eta, A, \delta, \alpha) - \eta(1 - \alpha)(1 - \beta + 2\beta \delta)} \quad (32) \]

and of course both these expressions tend to those in (22) and (23) when \( \alpha \to 0 \).

An increase of \( \alpha \) increases the steady state number of firms (and therefore reduces the mark up): a higher elasticity of output to capital reduces the wage and therefore the cost of entry, which promotes business creation and competition while reducing the mark ups. The steady state level of the capital/labor ratio in the final goods’ sector \( k_t \equiv K_t/L_t \) can be derived as:

\[ k^* = \left\{ \frac{\alpha \beta A[\Phi(\beta, \eta, A, \delta, \alpha) - \eta(1 - \alpha)(1 - \beta + 2\beta \delta)]}{1 - \beta(1 - \delta^k)} \left[ \frac{\Phi(\beta, \eta, A, \delta, \alpha) + \eta[(1 - \alpha)(1 - \beta) + \alpha \beta \delta]}{(1 - \beta + 2\beta \delta)} \right] \right\}^{1/\gamma} \quad (33) \]

which is always increasing in \( \alpha, \beta, A \), and decreasing in the rate of capital depreciation \( \delta^k \), but is now also decreasing in the rate of business destruction \( \delta \) and in the fixed entry cost \( \eta \). The two processes of accumulation of capital and business creation are complementary: more firms facilitate capital accumulation and more capital accumulation promotes entry.

Notice that this model nests the model of the previous paragraph for \( \alpha = 0 \) and the standard RBC model when \( \eta \to 0 \), and will constitute the workhorse for the analysis in the remainder of the paper.

### 2.6 Other forms of Quantity Competition

Until now we have focused our analysis of the endogenous market structures on Cournot competition. One of the main aim of this paper is to emphasize the need of a deeper investigation of the industrial organization of the markets in macroeconomic models. Therefore, in this section we extend the model to other forms of competition with endogenous entry: one is based on conjectural variations (which is traditionally applied to homogenous goods models), and another is based on Stackelberg competition (for which we have a closed form solution only in the case of homogenous goods).\(^{12}\) The first extension provides a reduced form for a more general model explaining collusion, the second allows

\(^{12}\)See Etro (2007,a) on the industrial organization of endogenous market structures.
us to take into account asymmetries between small and large firms. In Section 3, beside considering the baseline case of pure Cournot competition, we study also the dynamics delivered by the Conjectural Variations and by the Stackelberg approach. In Section 4 we take the Cournot model as a benchmark, and assess whether these extensions of the baseline framework contribute to the explanation of the second moments of the U.S. business cycle in the face of a technology shock.

2.6.1 Conjectural Variations Approach

A simple extension of the Cournot model of competition can be obtained assuming general conjectural variations of the firms.\footnote{See Bresnahan (1981) for an early discussion of the conjectural variations model.} This methodology provides a simple theoretical rationale for degrees of market power above those of Cournot competition and associated with forms of imperfect collusion, which may emerge especially in dynamic contexts.

Assuming that each firm takes as given the differential impact of its output choice on the output choice of the other firms $\lambda \equiv \partial x_t(j)/\partial x_t(i)$, the equilibrium price can be obtained as:

$$p_t = \frac{N_t}{(N_t - 1)(1 - \lambda)} MC_t$$  \hspace{1cm} (34)

which nests the case of Cournot competition in quantities for $\lambda = 0$ and tends to the (indeterminate) case of perfect collusion for $\lambda \to 1$. More importantly, intermediate situations with $\lambda \in (0, 1)$ describe cases of imperfect collusion between the firms which achieve mark ups above the Cournot level but below the perfect collusion level. While we will experiment the performance of our model for values of $\lambda$ different from the Cournot case of zero, it would be interesting to derive empirical measure of this parameter in the calibration exercise.

Finally, under general conjectural variations, the real profits become:

$$\pi_t(N_t) = \frac{[(N_t - 1)\lambda + 1](C_t + I_k^l)}{N_t^2}$$  \hspace{1cm} (35)

and the remaining equations are unchanged with respect to those of the previous sections.

2.6.2 Stackelberg Competition

Another relevant case is based on the theory of Stackelberg competition. For simplicity, we assume that the leader cannot commit to future strategies and therefore adopts the optimal static strategy in each period.\footnote{The assumption that also the leader cannot commit to a sequence of strategies is crucial here. If the leader could commit, it would engage in aggressive strategies aimed at reducing or deterring entry in the long run. See Etro (2008, b) for a recent analysis of Stackelberg competition with commitment and endogenous entry.}
that a single leader is always active in the representative sector. In this case the variable \( N_t \) defines the number of followers active in period \( t \).

This modelling choice is supported by the findings in Dunne et al. (1988), Audretsch (1995) and Sutton (1997) who report that in many U.S. industries the vast majority of entry and exit action is due to small firms and that larger firms have higher survival rates. This pervasive asymmetry suggests that the Stackelberg model could provide a more realistic picture of the structure of many markets.

The Stackelberg equilibrium is characterized by the following equilibrium price (see Etro, 2007a, Ch. 3):

\[
p(N_t) = \frac{N_t}{(N_t - 1/2)} MC_t
\]

which is lower than under pure Cournot competition in quantities. The output levels of the leader and of the representative follower (which must be compared to (3) under Cournot competition) are respectively:

\[
y_t^L = \frac{(2N_t - 1)EXP_t}{4N_t MC_t} \quad y_t^F = \frac{(2N_t - 1)EXP_t}{4N_t^2 MC_t}
\]

and the corresponding real profits are respectively larger and smaller than the profits under Cournot competition:

\[
\pi_t^L(N_t) = \frac{C_t + I_t^k}{4N_t} \quad \pi_t^F(N_t) = \frac{C_t + I_t^k}{4N_t^2}
\]

Entry and exit concerns only the followers, whose value is always pinned down by the endogenous entry condition \( V_t^F = \eta w_t/A_t \). In this case total investment is given by \( I_t = I_t^k + V_t^F N_t^e \). The value of the leading firm must be calculated separately as the expected discounted value of its future profits, \( V_t^L = \beta E_t \left[ (1 + r_{t+1})^{-1} \left( V_{t+1}^L + \pi_t^L(N_{t+1}) \right) \right] \).

3 **Impulse Response Analysis**

As customary, we solve the model by log-linearization around the deterministic steady state. In the next subsections we consider the behavior of the model in response to both technology and demand shocks.

The calibration is conducted on a quarterly basis. The discount factor, \( \beta \), is set to 0.99. As mentioned above we set \( \psi \) such that steady state hours worked equal unity. The Frisch elasticity of labor supply is \( \varphi \), to which we assign a value of 4 corresponding to the elasticity implicit in King and Rebelo (2000). As in Bilbiie et al. (2007a,b) the rate of business destruction, \( \delta \), equals 0.025 implying an annual rate equal to 10 percent. We follow King and Rebelo (2000) calibrating \( \alpha = 1/3 \) and \( \delta^k = 0.025 \). Empirical evidence cannot say much about the steady state ratio \( \Theta/C \), for this reason we simply follow Wen (2006) and set it equal to 0.1. The steady state mark up depends on the ratio \( A/\eta \) through the
impact on the steady state number of firms. To obtain mark ups in the empirically relevant range we set this ratio equal to 1.\textsuperscript{15} The baseline parameterization delivers an endogenous steady state mark up equal to 18 percent. This value stays in the mid of the empirically relevant range. Oliveira Martins and Scarpetta (1999) consider U.S. manufacturing industries over the period 1970-1992 and report estimates of mark ups in value added data which range from 20 percent to 40 percent, while those in gross output vary between 5 percent and 15 percent.

3.1 Technology shocks

Let us consider a temporary increase in technology.\textsuperscript{16} As usual we assume that the technology shock evolves according to an autoregressive process of the form:

\begin{equation}
\log(A_t/A) = \rho_A \log(A_{t-1}/A) + \varepsilon_t^A
\end{equation}

where $\varepsilon_t^A$ is an independently and identically distributed normal random variable with variance $\sigma^2_{\varepsilon}$. Figure 3 depicts the impulse response functions (IRFs) of the main macroeconomic variables to a one percent temporary shock to $A_t$ assuming $\rho_A = 0.9$. On the vertical axis we report percentage deviations of variables from the steady state. Time on the horizontal axis is in quarters. Solid lines denote IRFs under Cournot competition, while dashed lines represent the IRFs of a standard RBC model.\textsuperscript{17}

As in the standard RBC model the temporary technology shock increases consumption. However, differently from what happens in the afore mentioned framework, the impact of the shock on consumption is enhanced by deeper competition. The amplification mechanism works as follows. The number of firms is a state variable and thus it remains unchanged on impact as well as the price mark up. A higher demand for the final good together with a muted mark up determine the formation of profits which boosts investment in new firms and thus the number of new entrants. Entry of new firms fosters competition leading to a temporary reduction in mark ups. This provides an extra boost to the intertemporal substitution of future for present consumption with respect to the case in which mark ups are constant or absent. Increased private demand of the final good fuels profits which, despite lower mark ups, stay above the baseline for several quarters. Also, the interaction between sunk entry costs and oligopolistic behavior delivers a stronger propagation mechanism on hours and output with respect to the RBC model. The qualitative dynamics described in the case of Cournot Competition are preserved also under alternative market arrangements. Figure 4 provides IRFs dynamics for the Conjectural Variations

\textsuperscript{15} Notice that none of our results is qualitatively affected by alternative parameterization of the ratio $A/\eta$.

\textsuperscript{16} The working paper version of this article also reports transitional dynamics in the face of a permanent technology shock.

\textsuperscript{17} The benchmark RBC model is that in King and Rebelo (2000). Needless to say we adopt the same parametrization for both models. In particular the Frish elasticity of labor supply is $\varphi = 4$ in both cases.
Figure 3: **Impulse Response Functions to a Temporary Technology Shock.** IRFs under Cournot competition (solid lines) and the standard RBC model (dashed lines). Percentage changes from the steady state on the vertical axis, quarters on the horizontal one.

The steady state mark up in the conjectural variation case amounts to 30% while it is 12% under Stackelberg competition.
Figure 4: Impulse Response Functions to a Temporary Technology Shock. IRFs under Conjectural Variations (solid lines) and Stackelberg competition (dashed lines). Percentage changes from the steady state on the vertical axis, quarters on the horizontal one.
cality of mark ups and procyclicality of aggregate profits.

3.2 Preference shocks

In this section we consider the impact of changes in demand determined by preference shocks. Our focus will be on temporary shocks, assuming that the preference shifter $\Theta_t$ follows the first order autoregressive process:

$$\log (\Theta_t / \Theta) = \rho_\Theta \log (\Theta_{t-1} / \Theta) + \varepsilon_{t}^{\Theta}$$

where $\varepsilon_{t}^{\Theta}$ is an independently and identically distributed random variable. Figure 5 depicts the response of key variables to a one percent increase in $\Theta_t$, assuming $\rho_\Theta = 0.9$. As in the previous section, solid lines denote IRFs under Cournot competition, while dashed lines represent the IRFs of a standard RBC model. The taste shock leads to an increase in the demand for the final good. As it is, by now, understood this is the channel which leads to entry of firms into the market, to an increase in the overall number of producers and finally to a countercyclical mark up.

As emphasized by Baxter and King (1992) and Wen (2006), taste shocks in standard general equilibrium models generate countercyclical investment dynamics due to the crowding out effect. This is also the case in our model. Nevertheless, it should be noted that households reduce the stock of physical capital in order to satisfy their desire to consume, but profit opportunities warrant an increase in investment in new firms. Although this effect does not overturn the negative response of aggregate investment, it is, in our view, supportive of the relevance of the mechanism we put forward.

Clearly the volatility of variables in response to the shock is low. However our intent is that of proving the capability of the model to deliver mark up countercyclicality together with profits procyclicality in the face of a demand shock and not that of explaining business cycle variability resorting solely on demand shocks.

4 Business Cycle Moments

To further assess the implications of endogenous market structures for the business cycle, we compute second moments of the key macroeconomic variables. In this exercise we follow the RBC literature by assuming that the only source of random fluctuations are technology shocks. We take the calibration of the parameters characterizing the technology process from King and Rebelo (2000),

19 As mentioned above, empirical evidence does not help addressing the steady state ratio $\Theta/C$. As the value of the ratio increases the impact response of variables, as well as their variability, in the face of a demand shock are amplified.

20 Investment increases on impact in the baseline RBC model if one assumes that the taste shock has a near random walk behavior. Clearly, however, this ad hoc solution does not settle the issue.
namely we set the shock persistence to $\rho_A = 0.979$ and its standard deviation to $\sigma_{\tau A} = 0.0072$. We use the same process as in King and Rebelo (2000) for comparison purposes with the RBC literature. Jaimovich and Floetotto (2008) have prepared a measure of the TFP based on U.S. data taking into account the mark up variability and then fitting an AR (1) process to the constructed series. While they do not find significant differences from King and Rebelo (2000) in the estimated autoregressive coefficient, they estimate a lower standard deviation of TFP innovations. Although our results are marginally affected by a change in $\sigma_{\tau A}$, the main message of the analysis is not altered.

Table 1 (Panel A) reports in the left column the statistics on US data (1947–1/2007-3) for output $Y$, consumption $C$, investment $I$, labor force $L$, aggregate profits $\Pi$ and the mark up $\mu$\textsuperscript{21}. The middle column displays the theoretical moments produced by our baseline model with capital accumulation. The right column provides theoretical moments delivered by the benchmark RBC model under the same calibration adopted for the Cournot model.

Endogenous mark up fluctuations together with endogenous entry deliver a

\textsuperscript{21}Variables have been logged. We report theoretical moments of HP filtered variables with a smoothing parameter equal to 1600. Profits include both the remuneration of capital and the extra-profits due to market power: while we could not distinguish between the two, future research may try to do it.
allows both models to outperform the RBC model in terms of variability of Conjectural Variations approach with the model with Stackelberg competition while the right column refers to the is higher in absolute value when judged against lagged values of output. more in future periods. This means that the correlation of mark ups and output on impact however, since the number of …rms increases slowly, the mark up falls similar to that in Figure 1. In response to the technology shock output increases the correlation between the mark up and the cycle is non-linear and strikingly GDP at various lags and leads. The time proﬁle of these cross-correlations is in cross-correlations between the mark up, entry and aggregate real proﬁts with substantially higher output volatility with respect to the standard RBC model, basically matching the one emerging from U.S. data. A strong improvement with respect to the RBC also emerges for what concerns variability of hours worked. Also, the Cournot framework with homogeneous goods performs quite well in matching the contemporaneous correlation with output of mark ups and profits, on which the neoclassical model is completely silent. The model delivers a variability of consumption identical to that of the baseline RBC model, for this reason the relative (with respect to output) variability of consumption is rather low. Unfortunately, the volatility of real profits and that of the mark up are relevantly underestimated. However, to further document the ability of the model at reproducing business cycle facts Figure 6 plots model-generated cross-correlations between the mark up, entry and aggregate real profits with GDP at various lags and leads. The time profile of these cross-correlations is in line with that in the data, reported in Figure 1. In particular, the time-profile of the correlation between the mark up and the cycle is non-linear and strikingly similar to that in Figure 1. In response to the technology shock output increases on impact however, since the number of ﬁrms increases slowly, the mark up falls more in future periods. This means that the correlation of mark ups and output is higher in absolute value when judged against lagged values of output.

Finally Table 1 (Panel B) displays the second moments for our two extensions of the Cournot model. The left column displays the statistics delivered by the model with Stackelberg competition while the right column refers to the Conjectural Variations approach with $\lambda = 0.15$. Again, mark up variability allows both models to outperform the RBC model in terms of variability of

<table>
<thead>
<tr>
<th></th>
<th>$\sigma (X)$</th>
<th>$\sigma (X)/\sigma (Y)$</th>
<th>$E(X_t, X_{t-1})$</th>
<th>Corr$(X,Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Data, Cournot, RBC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>1.66, 1.69, 1.39</td>
<td>1</td>
<td>0.84, 0.69, 0.72</td>
<td>1</td>
</tr>
<tr>
<td>$C$</td>
<td>1.24, 0.60, 0.60</td>
<td>0.75, 0.34, 0.44</td>
<td>0.80, 0.77, 0.78</td>
<td>0.76, 0.90, 0.94</td>
</tr>
<tr>
<td>$I$</td>
<td>5.00, 4.58, 4.68</td>
<td>3.01, 2.71, 3.38</td>
<td>0.88, 0.68, 0.70</td>
<td>0.79, 0.99, 0.97</td>
</tr>
<tr>
<td>$L$</td>
<td>1.82, 1.30, 0.67</td>
<td>1.10, 0.77, 0.49</td>
<td>0.90, 0.66, 0.70</td>
<td>0.88, 0.96, 0.97</td>
</tr>
<tr>
<td>$\mu$</td>
<td>8.08, 1.57, n.a.</td>
<td>4.87, 0.93, n.a.</td>
<td>0.76, 0.61, n.a.</td>
<td>0.67, 0.95, n.a.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.87, 0.11, n.a.</td>
<td>1.13, 0.06, n.a.</td>
<td>0.85, 0.93, n.a.</td>
<td>-0.27, -0.37, n.a.</td>
</tr>
<tr>
<td>Panel B: Stackelberg, Conjectural Variations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>1.62, 1.76</td>
<td>1</td>
<td>0.66, 0.68</td>
<td>1</td>
</tr>
<tr>
<td>$C$</td>
<td>0.58, 0.57</td>
<td>0.35, 0.33</td>
<td>0.77, 0.77</td>
<td>0.91, 0.91</td>
</tr>
<tr>
<td>$I$</td>
<td>4.67, 4.67</td>
<td>2.87, 2.65</td>
<td>0.67, 0.69</td>
<td>0.99, 0.99</td>
</tr>
<tr>
<td>$L$</td>
<td>1.14, 1.49</td>
<td>0.71, 0.85</td>
<td>0.64, 0.68</td>
<td>0.96, 0.99</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.98, 0.94</td>
<td>0.60, 0.54</td>
<td>0.62, 0.73</td>
<td>0.98, 0.98</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.06, 0.06</td>
<td>0.04, 0.03</td>
<td>0.93, 0.94</td>
<td>-0.43, -0.24</td>
</tr>
</tbody>
</table>

Table 1: Second moments of the main macroeconomic variables in the aftermath of a temporary technology shock. Panel A: Data (left), Cournot (center), RBC (right). Panel B: Stackelberg (Left), Conjectural Variations (Right)
Figure 6: Model-Based Cross Correlations under Cournot Competition. Cross correlation on the vertical axis, leads and lags on the horizontal one. Cross correlation between real GDP at time $t+k$ and the Markup at time $t$. Panel b): cross correlation between real GDP at time $t+k$ and net business formation at time $t$. Panel c): cross correlation between real GDP at time $t+k$ and real profits at time $t$. 
output and hours. Low variability of profits and price mark up remains instead an issue that future research should try to address.

The table shows that variables’ autocorrelation is lower with respect to that which shows up in the data. This could be accommodated, as suggested by Bilbiie et al. (2007a,b), by imposing a longer time to build up a new firm.

5 Conclusions

In this article we have studied a dynamic model with flexible prices where the structure of the markets is endogenous and accounts for strategic interactions of different kinds.

Our approach belongs to the emerging literature on endogenous entry in macroeconomics (Devereux and Lee, 2001; Bilbiie et al., 2007a,b; Jaimovich and Floetotto, 2008; Etro, 2007b, 2009; Etro and Colciago, 2007) and, as others, it provides some improvements in the explanation of the business cycle compared to the standard real flexible prices framework.

In particular, when tested against a temporary technology shock our basic model delivers second moments of macroeconomic variables which are in line with those provided by the standard RBC model. Further, it adds to the latter framework an endogenous characterization of the market structure which allows to explain the procyclical variability of profits together with the countercyclical variability of mark ups found in the data.

The model could be easily extended in various directions. For example in Etro and Colciago (2007), we extend a related model to a multisector economy with imperfect substitutability and Bertrand competition and we evaluate the welfare properties of the equilibrium, while in the working paper version Etro and Colciago (2007) we study government spending shocks. An analogous model could be employed for the analysis of open economy issues to study international business cycles in an environment where the gains from trade derive from more competition and lower markups (due to larger integrated markets producing homogenous goods) rather than from more varieties or from selection effects as in the model of Ghironi and Melitz (2005). Moreover, the simple form of Cournot competition allows one to easily incorporate technological differences between countries and to verify the mechanisms of propagation of the shocks across borders. Finally, one could extend the model with frictions in the labor market to study the dynamics of unemployment in a context where the process of entry and exit is endogenous and hiring and firing of workers can be associated with it, or with frictions in the credit market (see Etro, 2009, for a wide discussion).

Acknowledgements.

We are grateful to seminar participants at the University of Saint Andrews (Scotland), IMT (Lucca), the University of Rome, the University of Milan, Bicocca, the Catholic University of Leuven, Osaka University, the University of Tokyo, State University of Milan, CERGE-EI (Czech Republic) and the Central Bank of Hungary for important suggestions. Guido Ascarì, Florin Bilbiie, Lilia
Cavallari, Monica Corti, Alex Cuckierman, Alberto Dalmazzo, Paolo Garella, Mark Gertler, Fabio Ghironi, Stefano Gnocchi, Giammario Impullitti, Michihiro Kandori, Toshihiro Matsumura, Alessandro Missale, Pietro Peretto, Tiziano Ropele and Patrizio Tirelli provided insightful discussions on this topic. Further details are available at www.intertic.org.

References


Vivek, Gh., 2007. Small is Beautiful but Size Matters: the Asymmetric Impact of Uncertainty and Sunk Costs on Small and Large Businesses, MPRA wp 5461.

Appendix A: Technical details

This appendix reports the log-linearized equilibrium conditions and the steady state of the model with endogenous capital accumulation. We show that the basic model displays, for a given number of firms, constant steady state output shares of consumption, investment, profits and labor income. The mark up function is left implicit so that the analysis applies to all the forms of competition considered.

Log-Linear Equilibrium Conditions

In what follows we provide the main log-linear equilibrium conditions for the model with capital. Hatted variables and variables without time subscript denote log-deviations from the steady state and the steady state value of the corresponding variables, respectively.

\[
\text{Labor supply: } \frac{1}{\varphi} \delta \tilde{L}_t = \tilde{w}_t - \frac{1}{1 - \frac{\varphi}{c}} \tilde{C}_t + \frac{1}{(\frac{\varphi}{c})^{-1} - 1} \tilde{\Theta}_t
\]

\[
\text{Capital accumulation: } \tilde{K}_{t+1} = \left(1 - \delta^k\right) \tilde{K}_t + \delta^k \tilde{I}_t
\]

\[
\text{Number of firms } \tilde{N}_t^c = \frac{1}{\delta} \tilde{N}_{t+1} - \tilde{N}_t
\]

\[
\text{Factors’ compensation: } \tilde{w}_t = \tilde{Y}_t^c - \tilde{\mu}_t - \tilde{L}_t^c; \quad \tilde{R}_t^k = \left(1 - \frac{1}{\beta + \delta^k}\right) \left(\tilde{Y}_t^c - \tilde{\mu}_t - \tilde{K}_t\right)
\]

\[
\text{Consumption output: } \tilde{Y}_t^c = \frac{C}{C + I^k} \tilde{C}_t + \frac{I^k}{C + I^k} \tilde{R}_t^k
\]

\[
\text{Stock market value: } \tilde{V}_t = \tilde{w}_t - \tilde{A}_t
\]

\[
\text{Euler equation for shares: } \tilde{V}_t = \frac{1}{(\frac{\varphi}{c})^{-1} - 1} \Delta \tilde{\Theta}_{t+1} + \frac{1}{1 - \frac{\varphi}{c}} \Delta \tilde{C}_{t+1} + \frac{r + \delta}{1 + r} E_t \tilde{\pi}_{t+1} + \frac{1 - \delta}{1 + r} E_t \tilde{V}_{t+1}
\]

\[
\text{Euler equation for capital: } \left(1 + \beta \delta^k\right) \tilde{R}_{t+1}^k + \frac{1}{(\frac{\varphi}{c})^{-1} - 1} \Delta \tilde{\Theta}_{t+1} - \frac{1}{1 - \frac{\varphi}{c}} \Delta \tilde{C}_{t+1} = 0
\]

\[
\text{Labor used for production of new firms: } \tilde{L}_t^c = \tilde{N}_t^c - \tilde{A}_t
\]

\[
\text{Profits } \tilde{\pi}_t = \frac{1}{\mu - 1} \tilde{\mu}_t + \tilde{Y}_t^c - \tilde{N}_t
\]

\[
\text{mark up (Cournot) } \tilde{\mu}_t = -\frac{1}{N - 1} \tilde{N}_t
\]
where \( R^k_t = 1 + r^k_t \), and thus \( \tilde{R}_t^k = \log \frac{1 + r^k_t}{\pi_\delta^k} \). Also we defined \( Y_t^c = C_t + R_t^k \). Notice that Steady state values involved are defined below. The parameter \( \varphi > 0 \) represents the Frish elasticity of labor supply which we set equal to 4 as in the RBC model of King and Rebelo (2000). The latter is taken as the benchmark model for most of the analysis.

### Steady State

Evaluating at the steady state the Euler equation for shares we get

\[
\frac{\eta}{A} \frac{w}{1 + (r + \delta)} = \frac{(1 - \delta)}{(1 - \mu)} \frac{Y^c}{N}
\]

Using the wage relation \( w = (1 - \alpha) Y^C/\mu L^c \), we have:

\[
\frac{\eta}{A} \frac{(1 - \alpha)}{\mu} = \frac{(1 - \delta)}{(r + \delta)} \frac{(1 - \delta)}{(1 - \mu)} \frac{L^c}{N}
\]

Labor market clearing requires \( L^c = L - N^e \eta/A = L - \frac{\delta}{1 - \sigma} \frac{\eta}{\pi} N \). This leads to steady state number of firms as the unique positive solution to:

\[
N = \frac{(1 - \delta)A}{\eta \left( \frac{(1 - \alpha)(r + \delta)}{\mu - 1} + \delta \right)} L
\]

whose right hand side is increasing in \( \alpha \) and decreasing in \( N \) as long as the markup \( \mu \) is inversely related with the number of firms: this implies that the steady state number of firms must be always increasing with \( \alpha \).

Under Cournot competition the mark up is \( \mu = N/(N - 1) \), therefore we can solve for the steady state number of firms as:

\[
N = \frac{\eta \left[ (1 - \alpha)(r + \delta) - \delta \right] + \Phi}{2 \eta (1 - \alpha)(r + \delta)}
\]

where \( \Phi = \sqrt{\eta^2 \left[ (1 - \alpha)(r + \delta) - \delta \right]^2 + 4(1 - \delta)(1 - \alpha)(r + \delta)\eta AL} \). When \( L = 1 \) this expression can be rewritten as (31) in the text using \( (1 + r)\beta = 1 \). An identical procedure can be adopted to obtain the number of producers under the Conjectural Variations approach and the number of followers under Stackelberg competition.

Considering the expression for the rental rate of capital delivers:

\[
K = \left[ \frac{\alpha A}{r + \delta^k \mu_t} \right]^{\frac{1}{\pi}} \left( L - \frac{\delta}{1 - \delta} \frac{\eta}{A} N \right)
\]

The steady state counterpart of the aggregate resource constraint is \( Y = Y^c + N^e V \). The latter can equivalently be written as \( 1 = Y^c/Y + N^e V/Y \). Evaluating at the steady state the Euler equation for bonds yields \( \beta (1 + r) = 1 \), where \( r \) is the steady state interest rate. The steady state counterpart of the Euler equation for stock holdings yields the steady state value of a firm as:

\[
V = \frac{1 - \delta}{r + \delta}\pi
\]

28
where steady state profits are given by \( \pi = (1 - 1/\mu) Y^c / N \). To obtain the share of investment in new firms over consumption output notice that:

\[
N^c V = \frac{1 - \delta}{r + \delta} \pi Y = \frac{\delta}{r + \delta} \pi N
\]

which, since \( \pi N / Y^c = (1 - 1/\mu) \), leads to:

\[
\frac{N^c V}{Y^c} = \frac{\pi N}{r + \delta Y^c} = \frac{\delta}{r + \delta} \left( 1 - \frac{1}{\mu} \right)
\]

To compute shares over aggregate output recall that:

\[
1 = \frac{Y^c}{Y} + \frac{N^c V Y^c}{Y^c} = \frac{Y^c}{Y} \left[ 1 + \frac{\delta}{r + \delta} \left( 1 - \frac{1}{\mu} \right) \right]
\]

thus we have:

\[
\frac{Y^c}{Y} = \left[ 1 + \frac{\delta}{r + \delta} \left( 1 - \frac{1}{\mu} \right) \right]^{-1}
\]

We next compute the share of factor compensation over consumption output \( Y^c \). Recall that \( w = (1 - \alpha) \frac{Y^c}{\mu} \), which implies:

\[
wL^c / Y^c = (1 - \alpha) / \mu \tag{44}
\]

Similarly, it can be shown that \( \frac{r^k K}{Y^c} = \frac{\alpha}{\mu} \). Since \( L^c = L - \frac{\delta}{r + \delta} \frac{\alpha}{\mu} N \), it follows that:

\[
\frac{L^c}{L} = \frac{(1 - \alpha) (r + \delta)}{(1 - \alpha) (r + \delta) + \delta (\mu - 1)}
\]

Therefore, we have:

\[
\frac{wL^c}{Y^c} \frac{L^c}{L} = \frac{wL}{Y^c} \frac{1}{\mu} \left[ (1 - \alpha) + \frac{\delta (\mu - 1)}{(r + \delta)} \right] \tag{45}
\]

Recalling that \( r^k = r + \delta^k \) we obtain:

\[
\frac{K}{Y} = \frac{\alpha}{(r + \delta^k) \mu} \frac{Y^c}{Y} \quad \text{and} \quad \frac{r^k}{Y} = \frac{\delta^k}{r + \delta^k} \frac{1 - \alpha}{\mu} \frac{Y^c}{Y} \tag{46}
\]

Total investment is composed by investment in physical capital and investment in new firms, which can be computed as

\[
\frac{I + V N^c}{Y} = \frac{I^k}{Y} + \frac{V N^c Y^c}{Y^c Y}
\]

Notice also that:

\[
\frac{Y^c}{Y} = C \frac{1}{Y} + \frac{I^k}{Y} = C \frac{1}{Y} + \delta \frac{K}{Y}
\]
where $Y^c = \left[1 + \frac{\delta}{r+\delta} \left(1 - \frac{1}{\rho} \right) \right]^{-1}$. Thus we can write:

$$C \frac{Y^c}{Y} = Y^c - \delta K \frac{Y^c}{Y} - \delta k \frac{(1 - \alpha)}{r + \delta} \frac{Y^c}{Y} = \left(1 - \frac{\delta k}{r + \delta} \frac{\alpha}{C} \right) \frac{Y^c}{Y}$$

Given

$$L = \left( \frac{wL}{C - \Theta} \right)$$

we can determine the value of $\theta$ such that $L = 1$ as $\theta = \frac{wL}{C - \Theta}$. In the model we set $\Theta/C = 0.1$.

**Appendix B: Data and Mark up measure**

Data on consumption, GDP, labor share, Investment, labor productivity, GDP deflator and profits derive from FRED, the Federal Reserve Economic Database of the Federal Reserve Bank of St. Louis. Below, we report in brackets the mnemonics of each series.

- Proprietors’ Income with inventory valuation adjustment (IVA) and capital consumption adjustment (PROPINC): Billions of Dollars, Quarterly, saar, 1947-01-01 2007-07-01.
- Corporate Profits with inventory Valuation Adjustment (IVA) and Capital Consumption Adjustment (CPROFIT): Billions of Dollars, Quarterly, saar, 1947-01-01 2007-04-01.

The index of net business formation is supplied by the Brad&Broadstreet corporation. The net business formation index runs from 1948:1 to 1995:3 (1967=100), for this reason we restrict our empirical analysis to this period. We thank Vivien Lewis for providing us the series on firms’ data.

To derive the empirical measure of the markup we proceed as follows Rotemberg and Woodford (1999) assume the existence of overhead labor so that effective labor
for the production of consumption goods is \( L_t^c = \hat{L}_t - L_t^\sigma \). Assuming a linear in (effective) labor production function, the price markup can be written as:

\[
\mu_t = \frac{Y_t}{w_t L_t} \frac{L_t}{\hat{L}_t - \hat{L}_t^\sigma}
\]

whose log-linearization is:

\[
\tilde{\mu}_t = -\frac{l^o}{1 - l^o} \tilde{L}_t - \tilde{s}_t
\]

where \( l^o \equiv L_t^o/L_t \) represents the average share of overhead labor over total labor input (assumed to be equal to 0.2), \( s_t \equiv w_t L_t/Y_t \) is the labor share of income, and hatted variables indicate percentage deviations from its long run trend.