Models of Partnerships*

Paolo Balduzzi†
University of Milan-Bicocca
Edinburgh School of Economics
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Abstract

Beside the traditional public-private dichotomy for the provision of public services, an increasing attention has been devoted to the use of public-private partnerships (PPPs). This paper compares relative inefficiencies of public provision, traditional private provision and PPPs. It also analyses the effect of workers’ efforts and incentives on the success of this new device.

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†Contacts: paolo.balduzzi@unimib.it; +39.02.6448.6551.
1 Introduction

The debate about public and private provision of public goods and services has always been lively, both in the political and in the academic arena. Recently, the application of the “incomplete contracts” framework has enriched this debate (Hart, 2003; Bennet and Iossa, 2004). The importance of contractual relationships is now even more emphasized by the increasing relevance assumed by partnerships between public (central or local) authorities and private firms. In this paper, we refer in particular to Public-Private Partnerships (henceforth, PPPs). The aim of this work is to present the main theoretical contributions to this debate, to discuss some experiences across UK\(^1\), to introduce our models of partnerships and workers’ contribution to performance and, finally, to provide policy suggestions.

The introduction of workers’ incentives is our main contribution. We believe that political debate has said probably too much and conversely academia too little about this topic. As a matter of fact, most of the political debate is nowadays focusing exactly on this point: who should employ whom. For example, recent guidances for new partnership contracts in Scotland\(^2\) explicitly underline that workers and unions should be informed about the privatization process, that workers’ conditions are essential for a satisfactory provision of the service and that two tier workforce should be limited. By two tier workforce, we refer to the coexistence of workers who are employed under different conditions. This is indeed a very common situation in partnerships, where some of the staff is usually transferred from a public to a private employer.

The paper is organized as follows. In section two, we define partnership contracts, discuss some of the main UK experiences of partnerships, and highlight advantages and drawbacks of these choices. In section three, we present a review of the main theoretical literature. In section four we introduce our main contribution: a model allowing for public provision, beside private provision and PPP, and the presence of workers. We divide the section in two parts, in order to have simplified versions of our model and better focus our analysis. Finally, in section five we draw our general conclusions and provide policy suggestions implied by the model.

2 Partnerships in UK

Partnerships are not a completely new device for the delivery of public services. For instance, Scott Fosler and Berger (1982) witness the presence of partnerships in seven U.S. municipalities during the ‘70s. More recently, Rossenau (2000) reports evidences from successful American and British experiences. Nevertheless, the history of partnerships is not always a list of successes: some of the first partnership contracts failed to be satisfied and still nowadays criticism arises when a new partnership is proposed\(^3\).

\(^1\)An excellent review about some Italian cases can be found in Ambrosanio, Bordignon and Etro (2004).
\(^3\)See, for example, www.unison.co.uk for the Scottish case.
The rest of the section is organized as follows: first we draw a brief historical background of the policies which brought to the introduction and development of PPP contracts in UK; then, we analyze different types of partnerships and try to understand the main elements of these contracts. Finally, we present a taste of UK experience and, in particular, we focus on health, education and prisons.

2.1 From traditional private provision to PPP

The main arguments supporting the shift from public to private provision are usually connected to the necessity of solving budget problems, without raising extra revenues, and the desire to obtain efficiency gains.

The first wave of privatization in the UK spread with PM Margaret Thatcher, during the eighties. Despite some expected benefits, new problems emerged: the government sold most of its assets at an excessively low price, competition was not always possible, and benefits were unevenly distributed among management and employees (HM Treasury, 2000).

The first half of the last decade opened with the introduction of Private Finance Initiative (henceforth, PFI) contracts: in this way, the government wished to keep the level of public investments high, to provide further incentives for private capital and, at the same time, to retain an overall public control on the projects. Pollit (2000) classifies three kinds of PFI projects:

1. the public sector buys the service (e.g.: roads, prisons) from the private sector, which is responsible for the capital investments;

2. the private sector designs, builds, finances and operates an asset; fees are paid by the public authority over the life of the contract, providing the required standards are met (Bennett and Iossa, 2004, and Hart, 2003, are based on these particular contracts, henceforth referred as DBFO); according to the nature of the asset, the private sector might in some cases directly charge the users (e.g., a bridge);

3. joint ventures between private and public sector.

Research, debate and propaganda mainly focused on the following most relevant elements characterizing PFI contracts:

- the possibility of delaying payments as long as the contract would last;

- the transfer of risks from the public to the private sector;

- the ability to provide “value for money”: a PFI should be set up only if the output could be provided at a cheaper cost than by a different type of provision (on a cost-benefits analysis).
As it will become clearer later, we believe in Hart’s view, that is that PFIs find their peculiarity more in the length of the contract involved rather than the listed points. We add to his theory the role of workers’ incentives which, at least initially, was not even considered by policy makers.

These main characteristics (especially risk transfer and value-for-money) were also the limits of this wave of PFI contracts. Even if the point is still debated (for example, see Grout, 1997), it is not clear why the cost of the private project should be lower, as borrowing for the private is more expensive than for the Government. Moreover, following Clark and Root (1999, p. 352), “risk was assumed to involve: design and construction risk (...); commissioning and operating risk (...); demand risk (...); residual value risk (...); technology and obsolescence risk”. It is straightforward to understand that most of these risks could not be specified a priori for a lot of projects. Even lack of experience and of project management skills in the public sector were such that progress in PFI projects in the early years was very slow.

Despite all these limitations, first failures could provide lessons for future agreements. It is on the basis of these experiences that PPP contracts have been launched in Britain since 1997.

2.2 The elements of a PPP

Although they are often used as synonyms, PPPs encompass a wider variety of relationships than PFIs. According to Broadbent and Laughlin (2003), the main aspect differentiating PPPs from traditional private provision is the presence of some control from the public authority over the nature and pricing of the service offered. This control is possible through the exploitation of ownership rights. Now, as they recognize, this point is highly debatable. Even if assets are not always technically owned by the public, we believe there is still scope for economic (or political) ownership. In particular, it is important to understand who is legally entitled to residual rights of control, which define what we call “economic ownership”. We will soon come back to this important point when discussing the flexibility of contracts.

A first synthetic definition of partnership, obtained by collecting from different sources (see, for example, HM Treasury, 2000, p. 10, and IPPR, 2001, p. 40), is “a long term risk sharing relationship between the public and the private sector to realize a mutual benefit”. This definition is quite comprehensive of the main elements of a partnership, which are listed and more extensively commented on below.

- Private and public sectors. As regards the subjects involved, the definition above refers to private firms (for profit ones, voluntary sector and charities) and public authorities, being them central or local (but also, for example, hospitals trusts and central government departments). The main responsibilities for the latter subjects are political ones, that is: deciding and defining the objectives, fix the standards (above all, quality
and quantity levels), monitoring the performances and ensuring that public interest is safeguarded. In some sectors, the public authority also provides part of the staff. The contribution of the private sector is supposedly based on its better management and business skills and efficiency driven culture. More precisely, public and private sectors are only words, which nest a lot of different stakeholders: customers, employees, private sector investors, banks and taxpayers. Most of the existing theoretical models often fail to consider the issues brought by each of these different agents. In this sense, our model acts as a first step, through the concern given to public and private sector employees. Taking into account that benefits must be shared among all the stakeholders is becoming only nowadays a fixed point in the political design of PPPs.

- **Risk sharing.** Within a partnership, risks should be borne by the party who can best manage them. Normally, political risk, plus a share of market risk, is retained by the public. It is this subject who is responsible for satisfying social needs. In addition, charges might be still due to the private firm even if the demand for the service drops (e.g., a demographic change leads to less pupils in a school). On the contrary, construction, design, standard satisfaction, operating costs and delivery risks are transferred to the private sector (and then reallocated within the consortium, when existing). There is also a final risk associated to the residual value of the facility, which is borne by the party who owns it.

- **Long term.** The duration of a contract may vary from five to seven years for local authorities outsourcing, and up to twenty-thirty years for schools. This “long-term” characteristic of the contracts and the size of the projects give rise to different problems. Pollit (2000) spots some particular difficulties. First, very often inputs and outputs cannot be specified in contracts, especially when they deal with quality issues. The less these items are sharply defined, the less the incentives for the private investors to respect the agreement. As already noticed, many risks are involved and transferred from the public authority to the private agent. In order to avoid problems (trials, delays, legal costs), they should be listed and specified. It is now very easy to understand the necessity to assume “incompleteness” in models dealing with this topic. Long-term contracts are also very hardly modifiable if unexpected contingencies realize during the provision period. This lack of flexibility is the source of well known hold up problems. In theoretical models, a no cost renegotiation could partially solve them. In reality, renegotiation is not always possible without breaking down the existing contract. With PFI contracts, key flexibility rights are given to the public sector (HM Treasury, 2003). In particular, provided an agreement on costs variation is found, the public sector has the right to change any aspect of the building or service provision. As anticipated above, this looks like a residual right of control over the asset, even if this is not necessarily owned by the public subject\textsuperscript{5}.

\textsuperscript{5}In other terms, if the public authority bears the cost of the proposed change (i.e., it undertakes the investment), then it does not need to share the additional benefit with the private firm.
• Relationship. Contracts are usually tailored on the specific case following standardized schemes, in order to reduce writing and legal costs. Contracts specify outputs related to the service required by the public sector, rather than inputs specification and asset characteristics; the basis for payment is also an element of the contract. Outputs are typically designed in consultation with public sector workers (e.g., doctors and teachers). The public sector evaluates bids received from the private firms and selects an option. It is quite natural to expect a PPP to be signed when a public provision would turn out to be more expensive. Unfortunately, it is not easy to compare costs and benefits of two different providers. A PFI is likely to be chosen when it offers greater value for money; the investment horizon is sufficiently long (no less than five years) and outcomes can be well specified. Greater value for money means that the overall cost of the PFI satisfies a public sector comparator (PSC) criterion. This criterion involves comparisons about interest rates (the cost of borrowing), which are lower for a public subject, and tax on private profits (which can be transferred on the final purchaser of the good/service). The evaluation of the overall costs should be made also with regard to other not economic aspects, such as employment condition. A further element of the contract is the performance evaluation and the system of deductions and penalties following poor standards levels. A constant monitoring of the performance of PFI projects is therefore required.

• Mutual benefit. Private and public sectors typically have different objectives. Private firms usually seek to maximize their profits whereas public authorities, at least in principle, wish to grant the highest benefit to the society. The difference might be less sharp when not-for-profit organizations are concerned but, again, they do not necessarily share the same utility function of society as a whole (on this point, see Dixit, 2002b).

Finally, following IPPR (2001, pp. 40-41) and HM Treasury (2000, pp. 46-48), we can list, and very briefly comment on, some possible types of PPPs:

1. PFI, as explained above, which constitutes the dominant form of partnership in UK. Variation of DBFO contracts are also possible, such as DBO or DBF agreements.

2. Wider markets: partnerships where private skills and finance should better exploit public assets or human resources.

3. Long term service provision contracts: these are agreements where no building stage is required but only management of existing assets.

4. Strategic (or policy) partnerships: agreements where the private sector is involved in the development and implementation of public services.

5. Sales of businesses: they involve the sale of shares of state-owned businesses, with the hope that the presence of private investments and market discipline would release the full potential of these firms.
6. Joint ventures: partnerships with a pooling of public and private assets, finance and workers under a common management.

2.3 Some UK experiences

PFI in England plays a still limited but increasingly relevant role in public sector capital investment: 11% of total investment in public services in 2003-2004 is estimated to be due to PFI. These investments have now delivered more than 600 new public facilities, including 34 hospitals, 119 other health schemes and 239 schools. PFI is used following a particular criterion, that is, it must offer value for money and efficiency gains must not be made at the cost of the workers’ conditions. First evaluations seem encouraging: of 61 chosen projects, 89% were delivered on time and 77% of public sector managers were happy with the delivery. We now want to illustrate some particular experiences and cases of partnerships. There are many examples we could choose from, but we shall focus on schools, hospitals and prisons, as we find them particularly relevant and inspiring for the theoretical model we have in mind. We also concentrate on the employment aspects of these agreements. Other cases we do not have the space to discuss here, like London Underground, Post Office, National Air Traffic Service and British Nuclear Fuels, are exposed in Balduzzi (2000).

2.3.1 Health service and education

A common problem of these sectors (but probably of every PFI) was initially the difficulty to develop and write contracts. After the very first experiences, there is now a tendency to use standardized (but flexible) partnership contracts.

In the education sector, a wide typology of contracts is possible. For example (IPPR, 2001, p. 164):

- Design, build, finance and operate: this is the typical PFI contract, carried out by a consortium of private firms; this consortium owns the school over the entire period of the contract.
- Education Business Partnership: the private sector participates as future employer and community stakeholder.
- Dual use Facilities: the private sector can recover part of its costs by exploiting the facility for its own business.
- LEA (Local Education Authority) management and provision of services: the private sector provides only strategic services, such as management.

In any of the possible schemes (which can be mixed as well), the Head and the Governing Body of the school continue to be responsible for teaching, while cleaning and catering are

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6If not differently and explicitly stated, the information in this subsection is based on HM Treasury (2000, 2003).
provided by external staff. The idea is that this choice allows schools to focus on their main business (i.e.: education) and to raise their standards. Typically, some employees need to be transferred to the private contractor (e.g.: the school Caretaker, IT technicians) whereas the teaching staff will be unaffected. In this case, the transferred staff is protected by the TUPE (Transfer of Undertakings Protection of Employment) Regulation of 1981. Moreover, unions should be consulted as early as possible and service provider and Head teacher should develop close relationships between them.

In the health sector\(^7\), PPPs always take the form of PFI contracts (in particular, DBFO). The consortium is usually required to build and maintain the facilities for the contract period. Employment design is similar to the one in education. The private sector provides ancillary services such as catering, cleaning, laundry security and portering. The public subject is responsible for the employment of clinical staff: it is believed that, in this way, the quality of the service would be better guaranteed. In addition, the contracts normally specify the range of services to be delivered, the performance standards required and the price of the provision, which is not due until services are provided to the agreed standard. The NHS has recently developed an original system called retention of employment (henceforth, RoE) in new PFI hospitals\(^8\). Under RoE, some categories of the ancillary staff are employed and retained as NHS employees but seconded to work for the consortium. The objective is to avoid a two-tier workforce but, in practice, the system is very complex and still highly debatable.

2.3.2 Prisons

Prisons constitute a really mixed sector. In England and Wales, custodial services are provided in 137 prisons (NAO, 2003), belonging to the public sector, the (traditional) private one or to PFIs. Among them, there are some interesting cases. Two prisons that were built and financed conventionally by the public sector are now run by private companies under management-only contracts. Three other prisons, two of which had previously been operated by the private sector, are now run by local management teams following successful in-house bids. Finally, since 1995, the Prison Service has signed nine Private Finance Initiative (PFI) contracts for new prisons, seven of which are already operational.

The most interesting aspect of this sector, at least according to the direction of our research, concerns employment choices. The full range of services, from management to staff, is here provided by the PFI. And it is a relevant chapter because staff costs account for nearly 80% of the running costs of a prison. It is natural to think that efficiency gains in the private sector might start exactly from the use of employees. And indeed the workforce has been subject to some reduction. Two main problems, related to staff conditions, are that staffing level in some prisons (e.g., HMP&YOI Ashfield; NAO, 2003) is failing to meet the original agreement. Furthermore, there is a high degree of turnover and a consequent lack of experience of constantly new employees. While most of the staff is recruited in the market or developed internally, senior managers positions are not. Directors have been recruited

\(^7\)We focus here only on secondary care even if PPPs exist in primary and intermediate care as well.

\(^8\)See http://www.ippr.org.
from the ranks of previous Prison Service Governors (public employees).

3 Partnerships, Contracts and Workers: a Review

Several contributions have recently discussed privatization and partnerships in an incomplete contracts framework. We also choose to develop our model in the well known “incomplete contracts” framework introduced in subsequent papers by Grossman and Hart (1986), Hart and Moore (1990) and finally Hart (1995). From an empirical point of view, it should be clear by now why partnership contracts would probably fail to be fully detailed. From a theoretical point of view, two main similarities about our models are worth of analysis. Among the other contributions they provide, these papers study the rationale for merging (or not) between private firms, whose objective is solely profit, and the optimal employment contract in these firms.

Hart, Shleifer and Vishny (1997) have already enlarged these models in order to consider a private firm and a government, whose objective function deals with quality rather than profit. This work provides criteria for the choice about contracting-out or not the provision of a public good, but it is not specific for PPPs. In a following paper (Hart, 2003), partnerships are explicitly considered. The author offers a new insight into the microeconomic principles that PPPs are (or should have been) based on. His preliminary conclusions are that conventional private provision (unbundling the construction of the facility and operation of the public service) is good if the quality of the building can be well specified in an initial contract, whereas the quality of the service cannot be. A series of short-term contracts (i.e.: more competition) will provide less distorted levels of investments. In contrast, PPP (bundling construction and operation) is good if the quality of the service can be well specified in a single long-term contract. Despite the advantages of his model, his analysis is still preliminary and fails to consider the effect of these two different provision devices on workers’ efforts and the role of ownership.

First steps in the latter direction have been made by Besley and Ghatak (2001) and Bennett and Iossa (2004)9. The former contribution highlights how, when a public good is involved, ownership should be based on valuations and not on investments or technology. The authors argue that sometime it is socially better for private firms to own public assets (for instance, a school) and focus their attention on not-for-profit organizations, as they are more easily driven towards public objectives. On the drawbacks of this form of provision, we must remember Dixit (2002b). He shows that these firms might in fact have also additional objectives than the government’s ones. The externality they produce by carrying on a public service needs to be taken into account.

The latter contribution (Bennett and Iossa, 2004) is very similar to Hart (2003) in considering building and management as the two main stages of the partnership contract. But PPP is now defined as an ownership structure rather than simple “bundling” of these stages.

9See also Shleifer (1998) on the importance of innovation in ownership choice.
In addition, they generalize the effects of the builder’s investments on the running of the project, in the sense that investments in the first stage have either positive or negative effects on the costs in the second stage. They consider the problem of ownership and closely link it with whether the externality is positive or negative. Given the usual hold up problem causing underinvestment, not internalizing a negative externality (unbundling the stages) may indeed reduce the related inefficiency. With a positive externality, on the contrary, PPP (ownership by a consortium) or public ownership must be preferred.

Several papers consider employment issues\textsuperscript{10}. We choose to focus in particular on two of them. Francois (2000) introduces “public service motivation” as a possible incentive for workers employed in the public sector. The author develops a more formal approach, based on economic rationality, rather than relying on psychological considerations. In his model, the presence of market incentives may in fact diminish employees’ effort. This happens because a profit oriented provider acts as a residual claimant and has an incentive in adjusting inputs to fulfill the contract whenever a worker is underperforming. Even under public service motivation, the worker prefers to shirk as the outcome will still be guaranteed. Relevant elements of the model are the adjustment costs, the degree of substitutability of inputs and the ability to write complete contracts about the outcome. Our model is different, as he deals with employers’ decisions about the production quantity rather than with investment choices.

A peculiar approach is developed by Besley and Ghatak (2004). In their model, productivity is increased by a correct match between the mission of an organization and the motivation of its employees. In particular, motivation acts as a substitute of pecuniary incentives and contracting on the mission can provide a firm with more productive and cheaper workers. We decided to develop a more explicit model, without using types as in Besley and Ghatak (2004), but where objective functions of the players may have common elements.

<table>
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<tr>
<th>Topic</th>
<th>Author</th>
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<td><strong>The incomplete contracts framework</strong></td>
<td>Grossman and Hart (1986)</td>
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<td>Hart and Moore (1990)</td>
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<td>Hart (1995)</td>
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<td><strong>Public versus private ownership</strong></td>
<td>Hart, Shleifer and Vishny (1997)</td>
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<td>Bennett and Iossa (2004)</td>
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<td><strong>Partnerships</strong></td>
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<td>Bentz, Grout and Halonen (2001)</td>
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<td><strong>Workers</strong></td>
<td>Francois (2000)</td>
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<td>Besley and Ghatak (2004)</td>
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\textsuperscript{10}See, for instance, Corneo and Rob (2003) or Delfgaauw and Dur (2004). Prendergast (1999) and Dixit (2002a) provide excellent reviews for incentives respectively in private and public organizations.
Finally, we must acknowledge the existence of other contributions about privatization and partnerships. These models are less relevant to our purpose as they are either developed in a different framework or focus on different aspects of partnerships (i.e., risk transfer). In our opinion, the most relevant ones are by Grout (1997, 2003) and Bentz, Grout and Halonen (2001). All the contributions are summarized in Table 4.1.

Grout’s papers deals with financing and risk issues. In Grout (1997), the author challenges the common opinion that public sector provision necessarily implies lower risk than private provision. The risk is the same even if the government can in fact borrow at a lower rate. This lower rate is not an indication of lower risk, as this is transferred on the general public in form of higher taxes in the future. Then, a standard criterion for implementing PPP projects is that it should be cheaper for the Government to provide a service with a PPP than with public provision. But this criterion raises questions about both the choice of discount factors, in the private and in the public sector, and the reasons for their difference. Starting from a pure finance test, Grout (2003) argues that discount rates used for the public sector should be lower than for the private one. This result is not based on risk arguments, but on the different nature of costs considered for public or partnership provision. It would disappear if the public authority chose to assess partnerships by their construction and maintenance costs rather than by the contracting costs involved.

Finally, Bentz, Grout and Halonen (2001) adopt a complete contracts approach and focus on the design of incentives from the public to the private sector. The distinction between PPP and public provision is made on the basis of the choice, by the government, respectively to purchase a service or a facility. When investments to make this facility efficient for the service are expensive, than public provision must be preferred. However, when these investments are cheap and delivery costs are low, then PPP is a good choice.

In the next section, we finally present our original models of partnerships.

4 Models of PPP

The model we present builds on some previous papers, in particular by Hart (2003), Besley and Ghatak (2004) and Francois (2000). Our contribution is related to the following themes:

- we allow for public provision; to do this, we assume the existence of two stages as in Hart (2003), and that the government does not have the ability to build the facility. There is still a private builder in the first stage but there is room for more possibilities in the second stage. If the provider is public, then we have public provision; if it is a different private firm then we have traditional private provision; finally, if we have a consortium of the builder and the provider, we have a partnership;

- we allow for one additional investment in the second stage; we also allow for the presence of workers in this stage: they can undertake some effort influencing the level of social benefit and are consumers of the good they produce;
• we try to motivate an explicit economic problem and not simply a problem based on “types”: motivation refers to how the provision of the service influences the worker’s utility.

We divide the problem in two stages: first we allow for workers and public provision; then we introduce some social costs related to the investment in the second stage.

4.1 Putting the workers into the picture

Hart (2003) does not consider workers. Besley and Ghatak (2004) do and focus their model on the matching between employers and employees; yet, the relation between workers and employers is based on types and not on explicit utility functions, as in our model. Finally, Francois (2000) is closer to our contribution in making his “public service motivation” an economical problem. Nevertheless, his model deals with employers’ decisions about the production quantity rather than with investments choices.

Our aim is to introduce two corrections in Hart’s model. The first one is about the presence of workers, as we believe their contribution is not irrelevant for the success of the good or service under provision. The second one is about the possibility for the provider to undertake some investments. In order to understand the relevance of these two new elements, an example might be useful. Let’s consider a radiologist in a hospital. His ability to diagnose an illness may depend on the accuracy of some exam: without, for instance, an X-ray machine, he is not able to understand the problem of his patient. The choice of buying or not this kind of machine is an investment bearing on the provider’s side. Finally, costlier machines provide more accurate results, which are easier to interpret for the radiologist. In other words, there is some complementarity between the investment of the employer and the effort of the worker. Both of them can influence the quality of the service which is provided (in this case, health services).

We also want to differentiate our model from Besley and Ghatak (2004) by making incentives for workers more explicit, without relying on matching of exogenous types or psychological motivations. The worker can find some satisfaction in his job, which is higher the greater his contribution to the final result. He may also enjoy the service himself as a consumer. For instance, a teacher, whose children go to the school where he works, might have an incentive to improve the level of teaching.

We now present a first version of the model, with a single worker. First we work out the efficient levels of investments, as a benchmark case. Then, we recall Hart’s results (Hart, 2003) which we are going to use. Finally, we discuss our original contribution to his model.

4.1.1 The setting

There are four subjects: a builder $B$, a private provider $P$, the government $G$, and a worker $L$. The government $G$ is not a social welfare maximizer agent strictu sensu\textsuperscript{11}, but only cares

\textsuperscript{11}As it will become clear below (see Lemma 3 and Lemma 8), as long as we do not consider any social cost caused by the provider’s investment, $G$ eventually acts as a social welfare maximizer even if it is not
about the social effect of the service. This is measured by a social benefit function $SB$ rather than by the sum of the agents’ utility and profit functions (worker, government itself and firms). $G$ could also be a public agency. In reality, this is the most natural interpretation. We therefore need to assume that the objective functions of the government and of its agency are the same. The production of the service requires two stages: a building one and an operating one. In the former stage a facility or a capital asset is built; this is used in the latter stage to actually provide the good or service. For instance, the facility can be a school, a hospital or a prison. The building stage is technologically impossible for $G$, so it must contract it out to $B$. Before this contract is signed, $G$ must also decide which kind of provider will operate the facility. $G$ has three choices: it can operate the facility itself (public provision), it can contract out the provision to an independent private provider $P$ (traditional private provision), or it can contract it out to the builder itself. The builder can then subcontract the operational stage to $P$ or form with him a consortium. We refer to the latter as a PPP and we assume, for simplicity and without loss of generality, that a partnership is precisely formed by a consortium of $B$ and $P$. The worker $L$ is employed in the second stage: therefore he is a public employee under public provision and a private employee under private provision or PPP.

All the subjects can undertake some non contractible effort or investments: exactly as in Hart (2003), the builder can decide whether to invest in $i$, a productive investment increasing $SB$ and decreasing operational costs $OC$ for the provider (e.g., a nicer building which is also easier to operate), and in $e$, an unproductive investment, decreasing both $SB$ and $OC$ (a facility may be easier to operate but less safe to its users). We add two more investments. First, the provider can invest in $a$, a productive investment with a positive effect on $SB$ (for instance, a better or more powerful machine in a hospital or teaching instruments). Finally, $L$ can undertake some effort on his job. This effort $d$ increases the quality, or level, of the service which is provided. Recalling the example about the radiologist, it is clear that the returns from these two investments, $a$ and $d$, are mutually dependent: the result of a medical test needs an interpretation to become a diagnosis; likewise, the radiologist’s opinion needs to be funded on solid bases.

The utility functions of the agents under different provision forms are the following:

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<tr>
<th>Private provision</th>
<th>Public provision</th>
<th>Partnership</th>
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<tr>
<td>$U^G_G = SB - \lambda_B - \lambda_P$</td>
<td>$U^G_G = SB - \lambda_B - OC$</td>
<td>$U^{PPP}<em>G = SB - \lambda</em>{PPP}$</td>
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<tr>
<td>$\Pi_B = \lambda_B - i - e$</td>
<td>$\Pi_B = \lambda_B - i - e$</td>
<td>$\Pi_{PPP} = \lambda_{PPP} - i - e - OC$</td>
</tr>
<tr>
<td>$\Pi_P = \lambda_P - OC$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and

$$U_L = w + \delta f(a, d) - d$$

with

$$SB = B_0 + b(i) - \beta(e) + f(a, d)$$

$$OC = C_0 - c(i) - \gamma(e) + w + a$$

defined as such.
The utility of $G$ ($U_G$) is given by $SB$ minus the price (or prices) $\lambda$, paid to the private firms. Under traditional private provision, $G$ pays $\lambda_B$ to the builder and $\lambda_P$ to the provider; under partnership, $G$ pays $\lambda_{PPP}$ to the consortium. Finally, under public provision, $G$ pays $\lambda_B$ to the builder and directly suffers the operational costs $OC$. We assume a fully competitive market for the firms. Therefore, they are paid competitive prices by $G$ and their profits are simply driven to the market ones, which we assume equal to 0. More specifically, the profit of the builder, $\Pi_B$, is given by the competitive price paid by $G$ for its service, $\lambda_B$, minus the level of investments he decides to undertake in equilibrium. The provider $P$ is paid a competitive price $\lambda_P$ by $G$ and incur in some operational costs, $OC$. If a partnership is formed, then it is paid a price $\lambda_{PPP}$ by $G$ and incurs in all the costs of the service, that is $i$, $e$ and $OC$. Finally, the worker’s utility ($U_L$) depends on his wage $w$, the level of his effort $d$, and on the quality, or level, of the service which is provided. This is particularly true when the good is a public or a collective one, as it is in our model. The parameter $\delta$, with $\delta \in (0, 1)$, may measure the sensitivity of the worker to the service. For instance, local workers might be more affected if they use the service they provide. We can interpret $\delta$ also as a characteristic of the good itself, that is, the degree of its public dimension. Finally, $\delta$ can reflect the worker’s commitment to the public service. The worker’s effort $d$ directly increases $SB$ through $f(a,d)$ and indirectly increases the worker’s own utility. The marginal contribution of $d$ is increasing in $a$, that is, $\frac{\partial^2 f(a,d)}{\partial d \partial d} > 0$, and, in particular, without any investment in $a$, $d$ has no effect: $f(0,d) = 0 \ \forall d$. Recalling the previous example, a radiologist might find it impossible to interpret some analysis if the machine he uses is not good enough. Finally, the worker has a reservation utility from not working equal to $\overline{U}$.

The functions $\beta(e)$, $c(i)$, $\gamma(e)$, $b(i)$ and $f(a,d)$ embody the effects of the various investments on $SB$, $OC$ and $U_L$. The social benefit function, that we defined as the social effect of the service provided, is positively affected both by the productive investment $i$, undertaken by the builder, and jointly by $a$ and $d$. The unproductive investment $e$ has a negative effect on the social benefit. As regards the operational costs $OC$, they decrease with both the builder’s investments, $i$ and $e$. They also include the provider’s investment, $a$, and the wage $w$ of the worker, as he is employed by the provider.

The function $f(a,d)$ appears twice in the payoffs functions. The first time as a positive contribution to $SB$. Both $a$ and $d$ positively affect the quality of the service. The second time, $f(a,d)$ appears as a positive effect on the worker’s utility. We can interpret this effect in different ways. The most straightforward one is the presence of satisfaction from the final result of the job. A better result, which depends both on $a$ and $d$, can make the worker happier about himself. A second interpretation is that the worker can be a consumer himself, therefore enjoying a share of the public or collective good that he helps to provide.

For technical reasons, we assume $f(a,d)$ to be a Cobb-Douglas production function with decreasing return of scale\textsuperscript{12}; that is:

$$f(a,d) = a^\alpha d^\beta$$

\textsuperscript{12}In this way, we ensure the existence of an internal equilibrium and we are able to provide explicit comparisons among solutions.
with:

\[ \alpha, \beta \in (0, 1) \]
\[ \alpha + \beta < 1 \]

If not strictly necessary to our computations, through the paper we will refer to \( f(a, d) \) in its generic expression. Finally, further technical assumptions about the functions above apply. All the functions are non negative and increasing in their arguments. The function \( \beta(e) \), which decreases the social benefit, is strictly convex, whereas functions increasing profits and utilities \( (c(i); \gamma(e); b(i); f(a, d)) \) are strictly concave in their arguments. We further assume that \( f(a, d) \) satisfies Inada conditions.

4.1.2 The benchmark case

Efficiency requires the maximization of the social welfare function \((SWF)\), which we define as the sum of all the agents’ payoffs. Taking into account all the cross payments, this is equal to:

\[ SWF = B_0 + b(i) - \beta(e) + (1 + \delta)f(a, d) - C_0 + c(i) + \gamma(e) - i - e - a - d, \]

(1)

hence the social welfare maximizer’s problem:

\[ \max_{i, e, a, d} SWF \]

with non-negativity constraints:

\[ i, e, a, d \geq 0 \]

The first order conditions are:

\[
\begin{align*}
[b_i(i^*) + c_i(i^*) - 1] i^* &= 0 \\
[-\beta_e(e^*) + \gamma_e(e^*) - 1] e^* &= 0 \\
[(1 + \delta)f_a(a^*, d^*) - 1] a^* &= 0 \\
[(1 + \delta)f_d(a^*, d^*) - 1] d^* &= 0 
\end{align*}
\]

(2)

where \( h_j(j) = \frac{\partial h_j(j)}{\partial j} \) and \( j^* \) is the efficient level of investment \( j \).

We are keeping Hart’s assumptions about the solutions for \( i \) and \( e \). These assumptions refer to the existence of an interior solution for \( i \) and a corner one for \( e \) (that is, \( \gamma_e(0) - \beta_e(0) - 1 \leq 0 \)). In addition, we assume that both \( a \) and \( d \) have an internal solutions as well:

\[
\begin{align*}
b_i(i^*) + c_i(i^*) - 1 &= 0 \\
e^* &= 0 \\
(1 + \delta)f_a(a^*, d^*) - 1 &= 0 \\
(1 + \delta)f_d(a^*, d^*) - 1 &= 0 
\end{align*}
\]

Before analyzing the worker’s contribution, we recall Hart’s results (Hart, 2003) and highlight the relationships between our models.
4.1.3 Hart (2003)

In this model, the government can choose only between traditional private provision and PPPs. In the former case, the builder has no incentive in investing either in $i$ or $e$, as he does not consider the effects of these investments on the provider’s operational costs $OC$ and on the social benefit $SB$. With a partnership between the builder and the provider, the external effects of $e$ and $i$ on $OC$ are taken into account, but not the effect on $SB$. The result is that a partnership will provide a more efficient level of investment $i$ but a less efficient level of investment $e$. As far as investments in $i$ and $e$ are concerned, no difference emerges between our two models. The worker worries only about the level of $a$ and $d$, which are not present in Hart (2003). In this way, comparisons between the models are easier. In addition, the level of investments $i$ and $e$ reached under traditional private provision are the same that we reach under public provision.

We can formalize this discussion, in order to better understand his results. At this point, we focus only on investments in the first stage: $i$ and $e$. We label the investments under traditional private provision, under partnerships and under public provision respectively as $i_p$, $i_{ppp}$, $i_g$ and $e_p$, $e_{ppp}$, $e_g$.

**Investments under private provision** Under private provision, the government signs two short-term contracts with two different private firms: $B$ and $P$. The problem of the builder is therefore the following:

$$\max_{i,e} \Pi_B = \lambda_B - i - e$$

In the second period, the problem of the provider is:

$$\max_a \Pi_P = \lambda_P - OC$$

With the resulting investments:

$$i_p = e_p = 0$$

None of the firms has an incentive in investing: the builder considers $i$ and $e$ simply as a loss, as they do not influence its costs or productivity.

**Investments under partnership** When a partnership between $B$ and $P$ is formed, the consortium takes into account the effects of $i$ and $e$ on the provider’s costs. The governments signs one long term contract with this consortium, whose problems is:

$$\max_{i,e,a} \Pi_{PPP} = \lambda_{PPP} - OC - i - e$$

With the resulting investments:

$$\left\{ \begin{array}{l} c_i(i_{ppp}) = 1, \\
\gamma_e(e_{ppp}) = 1, \end{array} \right.$$
In this case the consortium takes into account the effect of \( i \) and \( e \) on the provider’s costs. As regards \( i \), this is still insufficient, as no weight is given to the social benefit. As for \( e \), the lack of consideration for its negative effect on social benefits leads to an overinvestment.

**Investments under public provision**  Finally, with public provision the government signs only one short term contract with the builder and then provides the service or good in-house. The problem of the builder is the same as with private provision:

\[
\max_{i,e} \Pi_B = \lambda_B - i - e
\]

In the second period, the public provider solves the following problem:

\[
\max_a U_G = SB - \lambda_B - OC
\]

With the resulting investments:

\[
i_g = e_g = 0
\]

The builder does not make any difference about the identity of the provider, as long as it is not part of a consortium. Therefore no investment in \( i \) and \( e \) will B’s choice, as the investment in \( i \) is not contractible.

Proposition 1 is based on these results, which will be restated in Proposition 2 in the light of our original contribution.

**Proposition 1 (Hart, 2003 revisited)**  We can order the levels of investments reached under different provision mechanisms. We have that:

\[
i_p = i_g < i_{ppp} < i^*
i^* = e_p = e_g < e_{ppp}
\]

Traditional private provision and public provision (as defined above) are better when the quality of the building can be easily detailed but the quality of the service cannot. In other words, when it is relatively easier to write the contract about the quality of the building, private provision or public provision minimize the inefficiency. The underinvestment in “\( i \)” is not a serious issue and investment in “\( e \)” is even efficient. Mutatis mutandis, partnerships should be preferred when it is easier to measure or specify the quality of the service in the contract.

**Proof.**  See Hart (2003) and the discussion above. ■

We now present a model where the additional investments (by the provider and the worker) can lead to different choices. We first introduce a setting with general functions, and then we provide a more specific example.
4.1.4 Private versus public employment

We can now compare public and private employment in a second best setting. To do this, it is important to clarify a time-line of decisions in this economy (Graph 4.1).

At $t = 1$, $G$ decides what kind of provision it wants (public, private or a partnership). Consequently, $G$ also decides the identity of $L$’s employer and signs a particular contract: with public or private provision this would be a short term contract with the builder $B$ about the quality of the building; with a partnership, this would be a long term contract with the consortium of $B$ and $P$ about the quality of the service. At $t = 2$, the builder decides the level of investments $i$ and $e$. At $t = 3$, the provider decides the level of investment $a$ and the level of the wage $w$; in case of private provision, $G$ previously signs a short term contract with $P$ about the quality of the service. Finally, at $t = 4$ the worker observes $a$ and $w$. Then, he decides whether to accept the job or not and, in the positive case, what level of effort $d$ to undertake.

All these investments decisions are \textit{ex post} observable, but are not verifiable. So, it is not possible to write complete contracts and set up a system of payments and transfers such that the efficient investment levels are realized.

Having already shown the results for $i$ and $e$, we can now focus only on the provider’s problem with respect to $w$ and $a$. The model is solved as a typical principal-agent one and by backwards induction, starting from $t = 4$. It is important to stress that the worker’s problem is independent of the identity of his employer (whereas its choice is not, as we will show later). We also notice that no difference arises between private provision and a partnership. Therefore we analyze both the cases together.

At $t = 4$, $L$ solves the following problem:

$$\max_d w + \delta f(a, d) - d$$

s.t. $d \geq 0$

Assuming $f(a, d) = (a^\alpha d^\beta)$, we obtain the following first order condition:

$$\delta \beta a^\alpha d^{\beta - 1} = 1$$

(3)

We can rearrange the terms so that we have:

$$d = g(a) = (\delta a^\alpha \beta)^{\frac{1}{1-\beta}}$$

(4)
where $g(a)$ denotes the reaction function for the worker’s effort. The worker’s choice is therefore between working, and reacting to $a$ according to (4), and not working, if the incentive provided by the employer is not high enough:

$$\max \left\{ w + \delta f(a, g(a)) - g(a), \bar{U} \right\}$$

In other words, the worker will accept a contract only if:

$$w + \delta f(a, g(a)) - g(a) \geq \bar{U}$$

The relationship between $a$ and $d$ is used by the principal at $t = 3$. We now analyze the employer’s problem by discriminating between private and public provision. From the first order conditions of his problem, we already know that, for fixed values of $a$ and $\delta$, $d$ is inefficiently low, as the worker takes into account only his private benefit and not the entire social one. With an abuse of notation, we call the worker’s socially optimal reaction function $g^*(a)$. So we can state that:

$$d = g(a) < d^* = g^*(a)$$

and:

$$f(a, d) < f(a, d^*)$$

given our assumption on $f(\bullet)$ that $f_{ad}(a, d) > 0$ and $f_{ae}(a, d) < 0$.

**Private provision.** At $t = 3$, the private provider $P$ (the consortium would face a similar situation: see proof of Lemma 5 below) must solve the following problem: the firm wants to minimize its costs but has to consider the worker’s constraints. The individual rationality (or participation) constraint $IR$ states that the worker will accept the job only if it provides him at least his reservation utility $\bar{U}$. The incentive compatibility constraint ($IC$) states that the action the worker will take on the job must be optimal from his point of view, and directly follows from the first order conditions of his maximization problem:

$$\min_{a,w} OC = C_0 - c(i) - \gamma(e) + w + a$$

subject to:

$$w + \delta(a^\alpha d^\beta) - d \geq \bar{U} \quad IR$$

$$d = g(a) = (\delta a^\alpha \beta)^{1/\gamma} \quad IC$$

where $i$ and $e$ were chosen at $t = 2$, and therefore $c(i)$ and $\gamma(e)$ are treated as constant terms.

The worker’s participation constraint is binding in equilibrium, as otherwise the principal would have an incentive to decrease $w$. The provider has indeed two possibilities to attract the worker, and can choose the cheapest one: it can either invest in some minimum level of $a$ or it can raise the wage, such that the reservation utility is equalized. We can first substitute $IC$ into $IR$, and then, knowing that the latter is binding, we can directly substitute for $w$
in the objective function. The principal’s problem (in a simplified version) is then only one dimensional:

$$\min_a \left\{ \bar{U} - \delta a^\alpha \left[ (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} \right]^\beta + a + (\delta a^\alpha \beta)^{\frac{1}{\beta-\alpha}} \right\}$$

The first order condition of the problem is:

$$1 - \beta + a^{-1} \alpha (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} - \alpha \delta a^\alpha - 1 \left[ (\delta a^\alpha \beta)^{\frac{1}{\beta-\alpha}} \right]^\beta = 0 \quad (7)$$

The principal determines the value $a_p$ such that:

$$a_p \in \arg \min OC$$

In this case, we have that:

$$a_p = \left[ \delta \alpha^{1-\beta} \beta^\beta \right]^{\frac{1}{1-\beta-\alpha}} \quad (9)$$

Finally, the wage in the private sector ($w_p = w_{ppp}$) is easily determined from the worker’s $IC$:

$$w_p = \bar{U} - \delta f(a_p, d_p) + d_p$$

**Public provision.** With respect to the worker, the government must solve the following (simplified) problem:

$$\max_{a,w} U_G = (a^\alpha d^\beta) - a - w \quad \text{s.t.} \quad w + \delta(a^\alpha d^\beta) - d \geq \bar{U} \quad IR$$

$$d = g(a) = (\delta a^\alpha \beta)^{\frac{1}{\beta-\alpha}} \quad IC$$

Applying the same logic as above, we reduce the government’s problem to the following:

$$\max_a \left\{ (1 + \delta) a^\alpha \left[ (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} \right]^\beta - a - (\delta a^\alpha \beta)^{\frac{1}{\beta-\alpha}} - \bar{U} \right\}$$

whose first order condition is:

$$1 - \beta + \alpha a^{-1} (\delta a^\alpha \beta)^{\frac{1}{1-\beta}} - \alpha (1 + \delta) a^\alpha - 1 \left[ (\delta a^\alpha \beta)^{\frac{1}{\beta-\alpha}} \right]^\beta = 0 \quad (10)$$

Finally, the government determines the value $a_g$ such that:

$$a_g \in \arg \max U_G$$

In this case, we have that:

$$a_g = \left[ \delta^\beta \alpha^{1-\beta} \beta^\beta \right]^{\frac{1}{1-\beta-\alpha}} \left[ \frac{1 - \beta}{(1 + \delta - \delta \beta)} \right]^{\frac{\beta-1}{\beta-\alpha}} \quad (11)$$

Finally, the wage in the public sector ($w_g$) is easily determined from the worker’s $IR$:

$$w_g = \bar{U} - \delta f(a_g, d_g) + d_g$$

---

\(^{13}\)We are ignoring the terms $c(i)$ and $\gamma(e)$ as the government has no control over them.

---

20
Comments, comparison and an example. These results allow some initial comment. Using Hart’s approach (Hart, 2003), we can state that if the characteristics of the facility are easier to specify, the government should provide the service. If, on the contrary, the quality of the service is easier to measure, then the choice is between public provision and partnerships. Public provision is preferred when worker’s effort is very relevant for the success of the service.

We now formalize our findings with Proposition 2, which updates Proposition 1, and then comment on it.

Proposition 2 We can compare the levels of investments provided under different provision choices. We have that:

\[
\begin{align*}
  i_p &= i_g < i_{ppp} < i^* \\
  e^* &= e_p = e_g < e_{ppp} \\
  a_{ppp} &= a_p < a_g < a^* \\
  d_{ppp} &= d_p < d_g < d^*
\end{align*}
\]  

When we allow for public provision, for workers’ efforts and investments in the second stage, private provision is always dominated by public provision. Investments in $i$ and $e$ are the same, but investments in $a$ and $d$ are closer to the efficient level.

Proof. The first two rows in (12) are part of Proposition 1 and have been proven above.

As regards, $a$ and $d$, it follows from direct comparison of $a_{ppp} = a_p$ in (9) and $a_g$ in (11) that $a_{ppp} = a_p < a_g$. The efficient value of $a$, $a^*$, can be worked out by substituting $f(a, d) = a^\alpha d^\beta$ in the social planner’s problem. We obtain that:

\[
a^* = \left[ (1 + \delta) \alpha^{1-\beta} \beta^3 \right]^{\frac{1}{1-\alpha-\beta}}
\]

By comparing $a^*$ and $a_g$, we obtain the following condition:

\[
\delta^\beta \left( \frac{1 + \delta(1 - \beta)}{1 - \beta} \right)^{1-\beta} < 1 + \delta
\]

For the range of values of the parameters under consideration\(^{14}\), this condition is always satisfied\(^{15}\) (equality holds for $\beta = 0$, but we do not consider this case). So we can state that:

\[
a_{ppp} = a_p < a_g < a^*
\]

Given (14), it must follow that:

\[
d_{ppp} = d_p < d_g < d^*
\]

\(^{14}\)That is, $\beta \in (0, 1)$ and $\delta \in (0, 1)$

\(^{15}\)The condition in (13) is formally proven in Appendix.
The reason for the result in (15) is quite intuitive. In second best (public and private provision), the worker’s reaction function \( g(a) \) is the same; therefore, if \( a_p < a_g \), then it is straightforward that \( d_p < d_g \). Moreover, from the social planner’s problem, we can obtain the efficient reaction function \( g^*(a) \):

\[
g^*(a) = [(1 + \delta) a^{1-\beta} \beta]^{\frac{1}{1-\beta}}
\]

This is such that:

\[
g(a) < g^*(a) \quad \forall a > 0,
\]

and so it is a fortiori true that, if \( a_g < a^* \), then \( d_g < d^* \).

As regards the result in (14), the intuition is that the public provider correctly takes into account the full benefit from \( a \) and \( d \). Nevertheless, this is not enough to produce an efficient level of \( a \), as \( g(a) < g^*(a) \). In other words, for any unit of investment \( a \) which is undertaken by \( G \), the return (social benefit) is smaller than in the efficiency case, as the worker’s contribution \( (g(a)) \) is inefficiently low.

In Graph 4.2, we show the ranking of the investments in \( a \) under different provision mechanisms.

The government takes correctly into account both the social benefit and the worker’s private benefit from the good. Nevertheless, we know from (6) that the benefits are definitely smaller than in the first best: both \( a_g \) and \( d_g \) are set at an inefficient level.

The best choice is the one maximizing the SWF, as defined in (1). In Table 4.2 we compare the values taken by the SWF under different provision schemes. The first best is never reached; from comparisons in (12) and in Table 4.2, and recalling Proposition 1, Proposition 2 directly follows.

<table>
<thead>
<tr>
<th>Table 4.2: The optimal choice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Private provision:</strong> SWF(_P): ( B_0 + (1 + \delta) f(a_p, d_p) - C_0 - a_p - d_p )</td>
</tr>
<tr>
<td><strong>Public provision:</strong> SWF(_G): ( B_0 + (1 + \delta) f(a_g, d_g) - C_0 - a_g - d_g )</td>
</tr>
<tr>
<td><strong>Partnership</strong> SWF(<em>{PPP}): ( B_0 + b(i</em>{PPP}) - \beta(e_{PPP}) + (1 + \delta) f(a_{PPP}, d_{PPP}) - C_0 + c(i_{PPP}) + \gamma(e_{PPP}) - i_{PPP} - e_{PPP} - a_p - d_p )</td>
</tr>
</tbody>
</table>

22
We want to focus our comments in particular on $a$ and $d$. As long as no social cost is associated to the investment in the second stage ($a$), public provision is always the best provision choice. The government is interested in maximizing the social benefit, net of its provision costs, and, even if it does not take into account all the benefits from $a$, its choice is the closest to the first-best one. But this also means that public provision is better as long as a provider can influence the quality of the service. The importance of the investment is summarized by the shape of the function $f(\cdot)$.

If we think about health services, for instance, investments in the provision stage could be expensive and technological machinery. Even if a private provider could afford such an investment, it is not straightforward that he would undertake it. In education, these kinds of expensive investments seem less frequent. Most of the issues usually regard the quality of the building, something that the government cannot really influence. Our model would therefore suggest that the health service should be publicly provided whereas education could be left to the private sector.

Nevertheless, if we consider also the role of $i$ and $e$, partnership may be better than public provision. This is particularly true when the reduction in operational costs associated to these investments is higher than the corresponding reduction in the social benefit. Hart (2003) reckons that health service should be provided under a PPP. Our model suggests that the reduction in operational costs should offset the reduction in social benefit following not only higher investments in $i$ and $e$, but also lower investments in $a$ and $d$. Recalling how we stress the importance of complementarity between $a$ and $d$, our model may actually support the opinion that some of the staff should be privately occupied and part of the staff publicly employed. In particular, the public sector should retain its control over workers whose contribution is very high to the level of the service and leave the others to the private sector.

The government is facing two kinds of choices: an investment choice (i.e.: the level of $a$) and a provision choice (i.e.: the identity of the provider). The former choice induce distorted level of investments, as we showed above. What about the latter? The maximization of the SWF gives us a normative criterion to decide which provider is better. Nevertheless, in reality governments are not social welfare maximizers but have their own utility functions. In our model, we allow the government to have social concerns by considering $SB$ as part of its maximization problem. Nevertheless, $U_G$ does not consider all the utilities (or profit functions) of every agent in the economy. This may lead to a clash between what is best for the society (i.e., a provision choice according to the SWF) and what is actually implemented (i.e.: a provision choice according to $U_G$, the utility function of the decision maker).

In other words, as the government is maximizing $U_G$ and not SWF, it would choose according to the resulting investments maximizing its utility function rather than the social welfare function. Lemma 3 solves this doubt.

**Lemma 3** There is no distortion in the second best “provision” choice by the government.

**Proof.** We can compare the government’s actual choice, based on $U_G$, evaluated for different
provision choices, and the optimal one, based on SWF, evaluated in the same values. We summarize this problem in Table 4.3.

<table>
<thead>
<tr>
<th>Private provision:</th>
<th>$U_G^P : SB - \lambda_B - \lambda_P = B_0 + (1 + \delta)f(a_p, d_p) - C_0 - a_p - d_p - U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public provision:</td>
<td>$U_G^P : SB - \lambda_B - OC = B_0 + (1 + \delta)f(a_g, d_g) - C_0 - a_g - d_g - U$</td>
</tr>
<tr>
<td>Partnership</td>
<td>$U_G^{PPP} : SB - \lambda = B_0 + b(i_{ppp}) - \beta(e_{ppp}) + (1 + \delta)f(a_p, d_p) - C_0 + c(i_{ppp})$</td>
</tr>
<tr>
<td></td>
<td>$+ \gamma(e_{ppp}) - i_{ppp} - e_{ppp} - a_p - d_p - U$</td>
</tr>
</tbody>
</table>

As $\bar{U}$ is simply a constant, the government faces the same problem as a hypothetical social welfare maximizer. Therefore its provision choice is not distorted. When a partnership is better than public provision according to a SWF criterion, then it is better also according to $G$’s maximization problem. And so it is for the other way round.

The government is forced to consider the other agents’ profit and utility functions, as he is the ultimate payer for the service. Under public provision, its choice of $a$ is not optimal just because the worker’s reaction function is distorted.

As regards the role of workers, the fact that they are consumers makes them indirectly sympathetic with the government’s preferences. Public provision is therefore better as workers’ and government’s efforts are complementary to the success of the service. Again, a similar caveat regarding the shape of $d(\bullet)$ is worth of mention. Public employment is better as long as the workers’ contribution to the quality of the service is relevant. As suggested above, this result might also justify the case for mixed employment schemes. For instance, in the case of public transport, our model may suggest that buses designers should be public employees whereas it doesn’t really matter who hires, for instance, the administrative staff.

Additional comments follow the example below.

**Example 4** In this example, we only focus on the worker’s problem about “$d$” and the principal’s choice about “$a$”. In other words, we completely ignore issues about the remaining investments “$i$” and “$e$”. We assume that $\alpha = \beta = \frac{1}{3}$:

$$f(a, d) = (ad)^{\frac{2}{3}}$$

that is, the contribution of the two investments is symmetric. The FOCs in the first best case are:

$$\begin{align*}
\frac{\partial SWF}{\partial a} & : a = \left(1 + \delta\right)^{\frac{2}{3}} \sqrt[3]{d} \\
\frac{\partial SWF}{\partial d} & : d = \left(1 + \delta\right)^{\frac{2}{3}} \sqrt[3]{a}
\end{align*}$$

with interior solutions:

$$a^* = d^* = \left(\frac{1 + \delta}{3}\right)^3$$
The worker’s reaction function \( g(a) \) is:
\[
d = \left( \frac{\delta}{3} \right)^2 \sqrt{a} < \left( \frac{1 + \delta}{3} \right)^2 \sqrt{a} \text{ for any } a > 0
\]

When we solve for the private firm’s and the government’s optimal \( a \), we obtain respectively:
\[
\begin{align*}
a_p &= \left( \frac{\delta}{3} \right)^3 < \left( \frac{1 + \delta}{3} \right)^3 \\
a_g &= \frac{\delta}{3} \left( \frac{3 + 2\delta}{6} \right)^2 < \left( \frac{1 + \delta}{3} \right)^3
\end{align*}
\]
with:
\[a^* > a_g > a_p\]

The corresponding worker’s effort are respectively:
\[
\begin{align*}
d_p &= \left( \frac{\delta}{3} \right)^3 < \left( \frac{1 + \delta}{3} \right)^3 \\
d_g &= \frac{\delta}{3} \left( \frac{3 + 2\delta}{6} \right)^2 < \left( \frac{1 + \delta}{3} \right)^3
\end{align*}
\]
with:
\[d^* > d_g > d_p\]

Finally, we can determine wages; recalling that in equilibrium:
\[
w = \bar{U} - \delta f(a, d) + d
\]
we have:
\[
\begin{align*}
w_p &= \bar{U} - 2\left( \frac{\delta}{3} \right)^3 \\
w_g &= \bar{U} - 2\left( \frac{\delta}{3} \right)^2 \left( \frac{3 + 2\delta}{6} \right)
\end{align*}
\]
so that:
\[w_g \leq w_p\]

Example 4 gives us the opportunity to draw some additional considerations about wages in the private and the public sector, but has no direct relevance for the PPP debate.

From (16), we know that if \( \delta = 0 \), there is no reason why we should observe wage differentials in the private and in the public sector. We introduced \( \delta \) as a characteristic of the worker, who is also the consumer of the good he produces. We also offered a different interpretation of \( \delta \), that is, the degree of public dimension of the good itself. In public and private firms producing the same kind of pure private good, we expect workers to demand the same wage. If they do not, it is probably because of other dimensions of the problem (e.g.: job security, retirement plans). What our model explains is why people accept similar jobs in the public sector for lower monetary incentives (see also, for example, Dixit, 2002a and 2002b). They do so as long as they believe the can cooperate with their employer to satisfy their preferences.

The general result is stated and proven in Lemma 5.
Lemma 5 The wage of the worker in the private sector is never lower than the wage in the public sector:

\[ w_g \leq w_p = w_{ppp} \quad (17) \]

Proof. First of all, we show that \( w_p = w_{ppp} \). As explained above, at \( t = 3 \) a private provider \( P \) solves the following problem:

\[
\min_a \left\{ \bar{U} - \delta f(a, g(a)) + a + g(a) + C_0 \right\}
\]

whereas a consortium has a slightly different cost function:

\[
\min_a \left\{ \bar{U} - \delta f(a, g(a)) + a + g(a) + C_0 - c(i_{ppp}) - \gamma(e_{ppp}) + i_{ppp} + e_{ppp} \right\}
\]

The difference emerges as, with traditional private provision, \( P \) cannot control \( i \) and \( e \). But, as at \( t = 3 \) both the private provider \( P \) and the consortium only choose \( a \), their solution is the same, and so it is the wage they pay to \( L \).

Then, we know that:

\[
w_g = \bar{U} - \delta f(a_g, d_g) + d_g
\]

\[
w_p = \bar{U} - \delta f(a_p, d_p) + d_p
\]

and that:

\[ a_p < a_g; d_p < d_g \]

so

\[ f(a_g, d_g) > f(a_p, d_p) \]

The function \( f(a, d) \) is increasing in \( a \) by assumption. This means that:

\[ f(a_g, d_g) > f(a_p, d_p) \]

The utility \( \delta f(a, d) - d \) that a public worker obtains from observing \( a_g \) and producing \( d_g \) must be bigger than the his utility from observing \( a_g \) and producing \( d_p \). Otherwise, he would be better off by decreasing his effort level to \( d_p \). Therefore:

\[ \delta f(a_g, d_g) - d_g > \delta f(a_p, d_p) - d_p \]

and (17) follows.

Under a different point of view, the model can explain why people are happy to work for free in charities like Oxfam but they require a wage from, say, Blackwell’s. Though the economic activity is the same (selling books), people working in Oxfam probably have some preferences for the social activity of the charity.

This conclusion is extremely similar to the one in Besley and Ghatak (2004), except for the fact that we do not model our workers using “types” but explicit utility functions. \( G \) and \( L \) have, at least partially, the same objective, which is the quality of the service. This argument can be expressed in Besley and Ghatak’s terms as: “\( G \) and \( L \) are of the same type”.

26
4.1.5 The choice under budget constraint

A final remark about the (second) best provision choice is worthy of mention. So far we have implicitly assumed that $G$ could spend any amount of money, that is, its choice was unconstrained. It can be interesting to compare the three provision choices purely in terms of their cost to the government, that is, without considering the social benefit they imply.

Let’s define $TC$ as the total cost function for the government. Under different provision mechanisms, we have three possible cases:

$$TC^p = \lambda_B + \lambda_P$$

$$TC^{ppp} = \lambda_{PPP}$$

$$TC^g = \lambda_B + OC$$

where:

$$\lambda_B = i_p + e_p$$

$$\lambda_P = C_0 - c(i_p) - \gamma(e_p) + a_p + w$$

$$\lambda_{PPP} = C_0 - c(i_{ppp}) - \gamma(e_{ppp}) + a_{ppp} + w$$

and:

$$w = \bar{U} - \delta f(a, d) + d$$

Under private and public provision, $\lambda_B = 0$, as $i_p = e_p = 0$. Following the analysis in the previous sections, and recalling that $a_p = a_{ppp}$ and $d_p = d_{ppp}$, we can conclude that:

$$TC^p = C_0 + a_p + \bar{U} - \delta f(a_p, d_p) + d_p \quad (18)$$

$$TC^{ppp} = C_0 + i_{ppp} + e_{ppp} - c(i_{ppp}) - \gamma(e_{ppp}) + a_p + \bar{U} - \delta f(a_p, d_p) + d_p \quad (19)$$

$$TC^g = C_0 + a_g + \bar{U} - \delta f(a_g, d_g) + d_g \quad (20)$$

Lemma 6 directly follows.

**Lemma 6** Partnerships are the cheapest provision mechanism: $TC^{ppp} < TC^p < TC^g$
Proof. It is straightforward to see that $T_{C^{ppp}} < T_{C^{P}}$ by direct comparison of (18) and (19). We have that:

$$T_{C^{ppp}} = T_{C^{P}} + c_{ppp} + e_{ppp} - c(i_{ppp}) - \gamma(e_{ppp})$$

The quantity $c(i_{ppp}) + \gamma(e_{ppp}) - i_{ppp} - e_{ppp}$ must be positive for internal solutions of $i$ and $e$, as it is maximized by the partnership. Therefore $T_{C^{ppp}} < T_{C^{P}}$.

As regards $T_{C^{P}} < T_{C^{G}}$, the comparison reduces to the following:

$$a_{p} - \delta f(a_{p}, d_{p}) + d_{p} \leq a_{g} - \delta f(a_{g}, d_{g}) + d_{g}$$

The left hand side of the inequality corresponds to the private provider’s minimization problem, as expressed in (7). Therefore, it is at its minimum exactly when $a = a_{p}$. So the right hand side must be bigger. \[\blacksquare\]

When $G$ hires a partnership, it incurs the lowest possible price $\lambda_{PPP}$ for the service. A partnership obtains operational costs savings which neither a public nor a private provider can realize. The private sector pays a higher wage, but this wage is not the only labour cost. Both the public and the private sector must give their worker a level of utility equal to $\bar{U}$. What the government saves in $w$ must be paid through a higher investment in $a$. Our model suggests that private provision (traditional or partnership) is a good way of realizing savings for the government and confirms the common opinion that private provision is cheaper. Whether it is also preferable from a social welfare point of view is not always clear, as stated in Proposition 2.

Partnerships are chosen because they provide “value for money”; this criterion is always realized when the government does not have a budget constraint. If $G$ is bounded to spend no more than some level $T_{C} < T_{C^{g}}$, then privatization might be an inefficient solution, though the only available one.

### 4.2 A wider choice

In this subsection, we want to show that the scope for public provision is not limited by the presence of workers. Efficiency or inefficiency of public provision can have also a different source. In order to do this, we introduce some social costs associated to the investment in the second stage ($a$), with the aim of compensating the previous bias against traditional private provision. As anticipated, we completely ignore the role of workers. The rest of the setting is the same as in the previous section. The main difference concerns the government’s utility function, which is now equal to:

$$U_{G} = \theta SB + (1 - \theta) PB$$

where:

$$SB = B_{0} + b(i) - \beta(e) + f(a)$$
is the social benefit given by the investments in the production (building and running) process; and

\[ PB = n(a) \]

is the private benefit for the public provider. \( PB \) can have the following interpretation: \( n(a) \) is the probability of being re-elected due to the votes-catching investment \( a \); \( a \) has a positive effect on the collective benefit, but might be used also to obtain electoral consensus (e.g., through excessive employment).

The parameter \((1 - \theta)\), with \( \theta \in (0, 1) \), can be interpreted as the degree of corruptibility of the government. In the previous subsection, we simply assumed \( \theta = 1 \).

We further assume that the private benefit \( PB \) is a complete waste for the society; that is, an investment of \( a \) in the second stage creates an increase in the social benefit equal to \( f(a) \) but also a social cost (excessive staff, propaganda) equal to \( n(a) \). Therefore:

\[ SC = n(a) \]

The private firms’ objective functions are unchanged:

\[
\begin{align*}
\Pi_B &= \lambda_B - i - e \\
\Pi_P &= \lambda_P - OC \\
\Pi_{PPP} &= \lambda_{PPP} - i - e - OC
\end{align*}
\]

with

\[ OC = C_0 - c(i) - \gamma(e) + a \]

As before, the profit of the builder, \( \Pi_B \), is given by the competitive price paid by \( G \) for its service, \( \lambda_B \), minus the level of investments he decides to undertake in equilibrium. The provider \( P \) is paid a competitive price \( \lambda_P \) by \( G \) and incur in some operational costs, \( OC \). If a partnership is formed, then it is paid a price \( \lambda_{PPP} \) by \( G \) and incurs in all the costs of the service, that is \( i, e \) and \( OC \).

All the previous assumptions about the functions above still apply. In particular, we recall that \( \beta(e) \) is strictly convex whereas \( c(i), \gamma(e), b(i), f(a) \) and also \( n(a) \) are strictly concave.

Efficiency requires the maximization of the following social welfare function, which is the algebraic sum of objective functions of the subjects in this economy and of the social costs function. Once taken into account all the cross payments, this is equal to:

\begin{equation}
SWF = U_G + \Pi_B + \Pi_P - SC = \theta[B_0 + b(i) - \beta(e) + f(a)] - \theta n(a) - C_0 + c(i) + \gamma(e) - i - e - a
\end{equation}

The problem is solved as above:

\[
\max_{i,e,a,d} SWF
\]
with non-negativity constraints:

\[ i, e, a, d \geq 0 \]

The first order conditions are:

\[
\begin{align*}
[\theta b_i(i^*) + c_i(i^*) - 1] i^* &= 0 \\
[-\theta \beta e(e^*) + \gamma e(e^*) - 1] e^* &= 0 \\
[\theta f_a(a^*) - \theta n_a(a^*) - 1] a^* &= 0
\end{align*}
\]

where \( h_j(j) = \frac{\partial h_j(j)}{\partial j} \) and \( j^* \) is the efficient level of investment \( j \).

We are still keeping Hart’s assumptions about the solutions for \( i \) and \( e \). In addition, we assume that \( a \) still has an internal solution as well:

\[
\begin{align*}
\theta b_i(i^*) + c_i(i^*) &= 1 \\
e^* &= 0 \\
f_a(a^*) - n_a(a^*) &= \frac{1}{\theta}
\end{align*}
\]  

(22)

We are now ready to compare the relative inefficiencies of the three forms of provision.

### 4.2.1 Investments under private provision

Under private provision, the government signs two short-term contracts with two different private firms: \( B \) and \( P \). The problem of the builder is therefore the following:

\[
\max_{i, e} \Pi_B = \lambda_B - i - e
\]

In the second period, the problem of the provider is:

\[
\max_a \Pi_P = \lambda_P - OC
\]

With the resulting investments:

\[ i_p = e_p = a_p = 0 \]

None of the firms has an incentive in investing; the builder considers \( i \) and \( e \) simply as a loss, as they do not influence its costs or productivity. The same holds for the provider and \( a \).

### 4.2.2 Investments under partnership

When a partnership between \( B \) and \( P \) is formed, the consortium takes into account the effects of \( i \) and \( e \) on the provider’s costs. The governments signs one long term contract with this consortium, whose problems is:

\[
\max_{i, e, a} \Pi_{PPP} = \lambda_{PPP} - OC - i - e
\]
With the resulting investments:
\[
\begin{aligned}
&c_i(i_{ppp}) = 1, \\
&\gamma_e(e_{ppp}) = 1, \\
&a_{ppp} = 0
\end{aligned}
\]

In this case the consortium takes into account the effect of \(i\) and \(e\) on the provider’s costs. As regards \(i\), this is still insufficient, as no weight is given to the social benefit. As for \(e\), the lack of consideration for the its negative effect on social benefits leads to an overinvestment. The consortium still does not undertake any investment in \(a\).

### 4.2.3 Investments under public provision

Finally, with public provision the government signs only one short term contract with the builder and then provides the service or good in-house. The problem of the builder is the same as with private provision:

\[
\max_{i,e} \Pi_B = \lambda_B - i - e
\]

In the second period, the public provider partially takes into account the effects of the investment \(a\) on the collective benefit (but also on its private one). The problem for the government is:

\[
\max_a U_G = \theta CB + (1 - \theta) PB
\]

With the resulting investments:

\[
\begin{aligned}
&i_g = 0, \\
&e_g = 0, \\
&f_a(a_g) + (\frac{1-e_g}{\theta})n_a(a_g) = \frac{1}{\theta}
\end{aligned}
\]

The builder does not make any difference about the identity of the provider, as long as it is not part of a consortium. Therefore no investment in \(i\) and \(e\) will be undertaken. The government is investing in \(a\), as it correctly takes into account its effect on the social benefit. Nevertheless, this provision is still inefficient, as \(G\) does not consider the existence of social costs.

### 4.2.4 Comparison and comments

Hart’s suggestions still hold but we now have to take into account a third option. Given our discussion, we can state that public provision is good when traditional privatization is better then partnership and social costs are low. Following Hart (2003), traditional private provision is better than partnership when the characteristics of the facility are easier to specify. When social costs are high, traditional private provision should be preferred. If the quality of the service is easier to measure, then the choice is between public provision and partnerships. With low social costs, public provision should be preferred. Proposition 7 formalizes this result.
Proposition 7 When the government has a private benefit from the investment “a”, the levels of investments provided under different provision choices are as follows:

\[
\begin{align*}
\begin{cases}
i_p &= i_g < i_{ppp} < i^* \\
e^* &= e_p = e_g < e_{ppp} \\
a_{ppp} &= a_p = 0 < a^* < a_g
\end{cases}
\end{align*}
\]  

(24)

Proof. The first two rows are part of Proposition 1 and have been proven above.

As regards a, from (22) we know that the efficient level of a, \(a^*\), is positive: so it is always higher than \(a_{ppp} = a_p = 0\).

Furthermore, from direct comparison of (22) and (23) and given our concavity assumptions about the functions, we have that:

\[a^* < a_g\]

as

\[
\frac{1 - \theta}{\theta} > -1, \ \forall \theta \in (0, 1)
\]

Graph 4.3 shows the relations among \(i, e\) and \(a\) under different provision choices. The best choice is the one maximizing the SWF, as defined in (21). In Table 4.4 we compare the values taken by the SWF under different provision schemes. The first best is never reached; from comparisons in (24) and in Table 4.4, and recalling Proposition 1, Proposition 7 directly follows.

<table>
<thead>
<tr>
<th>Table 4.4: The optimal choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private provision:</td>
</tr>
<tr>
<td>(SWF_p) : (\theta B_0 - C_0)</td>
</tr>
<tr>
<td>Public provision:</td>
</tr>
<tr>
<td>(SWF_g) : (\theta [B_0 + f(a_g) - n(a_g)] - C_0 - a_g)</td>
</tr>
<tr>
<td>Partnership</td>
</tr>
<tr>
<td>(SWF_{ppp}) : (\theta [B_0 + b(i_{ppp}) - \beta(e_{ppp})] - C_0 + c(i_{ppp}) + \gamma(e_{ppp}))</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>- (i_{ppp} - e_{ppp})</td>
</tr>
</tbody>
</table>

Graph 4.3: The choice of the provider

Graph 4.4: The choice of the provider
Public provision is unambiguously bad when the government can easily use the facility (or the service itself) to maximize its private benefit. That is, when the social cost is very relevant or even bigger than the social benefit (in which case, \( a \) should probably be equal to 0). For instance, in Italy, schools are typically seen as places where hiring teachers is a fast (but very costly) way of increasing consensus. The model suggests that education could be efficiently left to private providers to avoid overstaffing. Prisons, on the contrary, appear to have a very low “electoral” characteristic (overstaffing, for instance, is never an issue), therefore public provision could be better.

The fact that, in reality, the provision of a lot of the collective goods is not (fully) privatized (e.g.: education) does not weaken our conclusions. In fact, our model provides a rationale for inefficient government’s choices: if the private benefit associated to the provision of a good is big enough, then the government, who actually decides about the identity of the provider, does not want to lose it. We state this result more formally in Lemma 8.

**Lemma 8** The choice of the government is inefficiently biased towards public provision.

**Proof.** We can compare the government’s actual choice, based on \( U_G \), evaluated for different provision choices, and the optimal one, based on \( SWF \), evaluated in the same values. We summarize this problem in Table 4.5.

<table>
<thead>
<tr>
<th>Choice</th>
<th>( U_G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private provision: ( U^p_G )</td>
<td>( SB - \lambda_B - \lambda_P = \theta B_0 - C_0 )</td>
</tr>
<tr>
<td>Public provision: ( U^g_G )</td>
<td>( SB - \lambda_B - OC = \theta</td>
</tr>
<tr>
<td>Partnership ( U^ppp_G )</td>
<td>( SB - \lambda = \theta</td>
</tr>
</tbody>
</table>

As \((1 - \theta)n(a) > -\theta n(a)\) for any positive value of the social cost \( n(a) \), the government’s utility is distorted towards public provision, whereas the choice between partnership and traditional privatization is still (constrained) optimal. ■

It is worth noting that, in the analysis above, we could substitute the government with a not-for-profit institution, if we think that \( G \) is always incapable of providing the service
in-house. In the case of a not-for-profit provider, our model recalls the one in Dixit (2002b). Beside the interest for the social benefit, these organizations might have also private benefits (e.g.: the spread of their religious beliefs) which are not necessarily compatible with public interests.

5 Conclusions

The aim of this paper was to introduce new and more complete models of partnerships. In particular, we distinguish from the existing literature in our effort to introduce the government as an alternative provider and in studying the more efficient employment choices.

There is a trade off given by the choice of a public provider: the government $G$ is the only employer which is able to provide a more efficient incentive to the worker. Nevertheless, if $G$ sees the opportunity of exploiting these investments for a private benefit (for instance, for higher probability of election), then the scope for public provision is dramatically reduced. Public provision, as opposed to partnerships, is also particularly good when employer’s and employee’s effort are complementary and relevant. Our models seem to suggest that health service, usually requiring very expensive investments by the provider, should be kept under the public sector. On the contrary, education can be contract out to the private sector (in particular, to partnerships). This is especially true in countries where employing teachers is considered a way of buying consensus.

Our model also explains wage differentials in the private and the public sector: public workers might accept lower wages, due to the ability of the public provider to increase the worker’s utility through the investment in $a$.

Finally, our model stresses that often the government choice is financially constrained. In this case, the solution is biased towards the private sector, and in particular partnerships, which are the cheapest alternative.

Further research should be aimed at developing at least the following problem: what would happen if there were different types of workers or different types of workers’ efforts (i.e.: laziness)? We may expect some workers to be more efficiently hired and managed by a private provider, so that a further trade off emerges. The model above is indeed very incomplete and too counter-intuitive: it might further explain “public service motivation” as in Francois (2000) and the presence of higher wages in the public sector: they are paid to compensate the lower satisfaction of the workers. Nevertheless, in reality private workers are often seen as very productive whereas in our model they appear to be lazier.

6 Appendix: Proof of the condition in (13)

We now want to show that the condition:

$$\delta \beta \left( \frac{1 + \delta (1 - \beta)}{1 - \beta} \right)^{1-\beta} < 1 + \delta$$
is always satisfied for any $\beta, \delta \in (0, 1)$.

First of all, we apply a logarithmic transformation (which is positive monotonic) to both sides of the inequality:

$$\log \left[ \delta^\beta \left( \frac{1 + \delta(1 - \beta)}{1 - \beta} \right)^{1-\beta} \right] < \log [1 + \delta]$$

Therefore:

$$\beta \log \delta + (1 - \beta) \log [1 + \delta - \delta \beta] - (1 - \beta) \log [1 - \beta] - \log [1 + \delta] < 0$$

We evaluate the function on the left hand side in $\beta = 0$ and obtain:

$$\log [1 + \delta] - \log [1] - \log [1 + \delta] = 0$$

As $\beta \in (0, 1)$, it is now sufficient to show that the first derivative with respect to $\beta$ of the function on the left hand side of the inequality is always negative:

$$\frac{\partial}{\partial \beta} \beta \log \delta + (1 - \beta) \log [1 + \delta - \delta \beta] - (1 - \beta) \log [1 - \beta] - \log [1 + \delta] =$$

$$\log \delta - \frac{1 - \beta}{1 + \delta - \delta \beta} - \log [1 + \delta - \delta \beta] + \frac{1 - \beta}{1 - \beta} + \log [1 - \beta] < 0$$

which reduces to:

$$1 < \log \left[ \frac{1 + \delta - \delta \beta}{\delta(1 - \beta)} \right]^{1+\delta-\delta \beta}$$

This condition is satisfied when:

$$\left( \frac{1 + \delta - \delta \beta}{\delta(1 - \beta)} \right)^{1+\delta-\delta \beta} > e$$

where $e$ is the Napier’s constant\textsuperscript{16}. It is easier to solve this inequality in (25) if we reduce it to a one variable problem; we set:

$$1 + \delta - \delta \beta = x$$

and it can be easily checked that, for $\beta \in (0, 1)$ and $\delta \in (0, 1)$, we have:

$$1 \leq x \leq 2$$

After the necessary transformations, (25) becomes:

$$\left( \frac{x}{x - 1} \right)^x > e$$

\textsuperscript{16}The constant $e$ is occasionally called Euler’s number after the Swiss mathematician Leonhard Euler, or Napier’s constant in honour of the Scottish mathematician John Napier who introduced logarithms. In honour of the University where I studied and of the Country I’ve been living in for the last four years, I think Napier’s constant is more appropriate.
The function \( \frac{x}{x-1} \) is monotonically decreasing (its slope is always negative); so must be \( (\frac{x}{x-1})^x \), as it is obtained as a positive monotonic transformation of the former function. The function \( (\frac{x}{x-1})^x \) reaches its minimum when \( x \) equals its maximum possible value (= 2). If (26) is satisfied for this \( x = 2 \), then it is \textit{a fortiori} true for smaller values of \( x \):

\[
\left(\frac{2}{1}\right)^2 = 4 > e
\]

\textit{Q.E.D.}

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