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Filippo Pavesi, Massimo Scotti
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Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
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Filippo Pavesi† and Massimo Scotti‡

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Abstract

This paper studies the impact of reputation on the reporting strategy of experts that face conflicts of interest. The framework we propose applies to different settings involving decision makers that rely on experts for making informed decisions, such as financial analysts and government agencies. We show that reputation has a non-monotonic effect on the degree of information revelation. In general, truthful revelation is more likely to occur when there is more uncertainty on an expert's ability. Furthermore, above a certain threshold, an increase in reputation always makes truthful revelation more difficult to achieve. Our results shed light on the relationship between the institutional features of the reporting environment and informational efficiency.

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†Department of Economics, University of Milan-Bicocca, Piazza dell’Ateneo 1, 20126, Milan, Italy and Department of Public Policy, Central European University (CEU), Nador Utca 11, Budapest, Hungary. Email: filippo.pavesi@unimib.it

‡School of Finance and Economics, University of Technology, Sydney, PO Box 123 Broadway NSW 2007 Australia. Email: massimo.scotti@uts.edu.au
I Introduction

Individuals frequently rely on the information provided by experts when making economic decisions. This information can take either the form of a direct recommendation to follow a specific course of action or the form of a forecast that individuals use to inform their decisions. In all cases, the value of the expert’s information relies on at least two components. First, the presumed ability of the expert to recover accurate information about an unobserved state of the world upon which the success of a specific action depends. Second, the presumption that the expert truthfully reports his information.

However, experts often face incentives that are not fully compatible with truthful revelation of their information. In particular, there are situations where experts have a clear bias in favor of reporting over-optimistically (or over-pessimistically) on some unknown state of the world upon which receivers must base their decisions. In all these cases, the expert faces a conflict of interest with the party that eventually uses his information. Reputation acquisition is typically regarded as a mechanism that is able to offset this bias and to mitigate the negative effects associated with conflicts of interest.

The main contribution of this paper is to propose a theoretical framework that captures these essential features of experts’ incentives. Since these characteristics are common to several economic and political settings where decision makers rely on experts for making informed decisions, the model is well suited for analyzing different contexts that share these features. Financial analysts, for instance, may have incentives to provide biased reports, and thus may face a conflict of interest with investors.\textsuperscript{1} On the other hand, analysts are also concerned about their reputation as valuable information providers, since this influences their future payoffs. An analyst who provides biased reports will be identified by the market as a bad information provider, thereby reducing his future wage and possibly jeopardizing

\textsuperscript{1}There is a large body of literature showing evidence that affiliated analysts have an optimism bias resulting from their involvement in the investment banking activity of their brokerage house (Michaeli and Womack (1999), Barber et al. (2006, 2007)).
his career.\textsuperscript{2}

In the political sphere, some government agencies are responsible for providing macro-
economic or fiscal forecasts for the purpose of efficiently allocating scarce public resources
and effective public and private sector planning. In this case, the conflict of interest stems
form the fact that government agencies face incentives to bias their forecasts away from
objective reports, towards those that favor politicians.\textsuperscript{3} Also in this case, reputation costs
can constrain such biased behavior in several ways.\textsuperscript{4}

We model a reporting environment where an expert is concerned about his reputation as
an accurate provider of information, but at the same time receives some form of compensation
whenever he manages to induce the receivers to believe that the world is in one specific state.
We analyze the effect of these contrasting objectives on the expert’s incentives to truthfully
reveal his information, focusing on the effectiveness of the reputational mechanism to act as
a disciplining device that positively affects the incentives for truthful information revelation.

The nature of the bias we consider is such that regardless of the initial beliefs of decision-
makers, an expert always has an incentive to induce them to attribute greater probability
to a particular state of the world. Even if public information regarding a particular state
of the world (such as economic growth prospects) happens to be pessimistic, we assume the
expert always benefits from convincing those who rely on his advice that things are not as

\textsuperscript{2}Stickel (1992), Mikhail, Walther, and Willis (1999), Hong and Kubik (2003), Fang and Yasuda (2009)
all document that reputation has a disciplining effect on analyst behavior.

\textsuperscript{3}The political science literature documents that incumbent governments generally prefer agencies that
are more inclined to provide optimistic forecasts as a way to signal to the electorate that the politician is a
interest originates from the fact that the executive branch has the power to sanction agencies that fail to act
in their interest by proposing budget cuts, disposing of political executives or even advocating termination
of the agency.

\textsuperscript{4}For instance if the electorate is to view the incumbent executive as a competent public manager the
agencies issuing reports must be considered reliable sources of information (Heclo (1975), Rourke (1992),
Carpenter (2001)). Government economists also value the esteem of their peers and act in order to maintain
their professional reputation for career concerns (Wilson 1989). Finally, loss of reputation may also result
in auditory sanctions that may pose a serious threat to the agency’s existence (Bendor, Taylor, and Van
Gaalen (1985), Banks and Weingast (1992)).
bad as they think.

As a first result we show that, despite the bias, reputation is still effective in reducing the incentives to misreport. Moreover, the nature of the most informative equilibrium in our setting is qualitatively similar to that of a reputational cheap talk model a la Ottaviani and Sorensen (2006), where conflicts of interest are not present. As in Ottaviani and Sorensen (2006), reputational concerns fail to be an effective disciplining device only when public information is characterized by little uncertainty. In these cases, experts disregard their private information and conform to public information fearing that any contrarian signal they receive is probably incorrect.

Our main result is that we find that improvements in the quality of information may have negative effects on information revelation. In particular, a variation in the share of experts with high quality information (i.e. higher initial reputation), has a non-monotonic effect on the incentives to truthfully reveal information, and therefore on the level informational efficiency. An increase in this share leads to less misreporting as long as the initial fraction of better-informed experts is not too high. Beyond a certain threshold, however, any increase in initial reputation results in a decrease in informational efficiency. Intuitively, when their reputation is high to start with, experts have less scope for reputation acquisition, but at the same time face greater incentives to be over-optimistic, since decision makers attribute more weight to the advice of well established experts. Analogously, an improvement in the accuracy of information of less talented experts has a negative effect on informational efficiency, when the difference between the quality of information of good and bad types becomes thinner. Also in this case, as the abilities of experts converge, the reputational gain of being recognized as a good expert tends to fade, reducing the disciplining role of reputation. Meanwhile, the improved quality of information generated by an increase in the accuracy of less talented experts, enhances the credibility of advice, increasing the returns from biased reports.

The remainder of the paper is organized as follows. In Section I, we review the relevant
literature. Section II introduces the general setup of the model. In Section III we characterize
the most informative equilibrium and analyze the conditions under which truthtelling is
possible, highlighting the incentives that lead experts to deviate from truthtelling. Section
IV examines how informational efficiency is affected by variations in the institutional features
that characterize the reporting environment. Section V concludes.

II Literature Review

Our paper is closely related to two main strands of the literature on sender-receiver models of
information transmission. The first deals with experts that are exclusively concerned about
their reputation for being good information providers (Ottaviani and Sorensen (2001, 2006)
and Trueman (1994)). The second strand instead considers information providers that care
only about the credibility of their messages, as this affects their payoff through the impact of
recommendations on the actions of receivers (Benabou and Laroque (1992), Brandenburger
and Polak (1997) and Morgan and Stocken (2003)). By combining these approaches in a
unique setup, we show that the interaction between reputational concerns and conflicts of
interest, plays a crucial role in determining the level of information revelation.

Ottaviani and Sorensen (2006) study information transmission by privately informed
experts concerned about being perceived to have accurate information. They characterize
experts’ incentives to deviate from truthtelling, by analyzing different information struc-
tures. In particular, they consider information providers with known or unknown ability,
and different signal structures, discrete versus continuous, in a setup in which the experts
are solely concerned about the receivers’ perceptions of their forecasting ability.

Trueman (1994) considers a model where analysts with different forecasting abilities
are concerned about building a good reputation for their forecasting accuracy. He finds
that analysts display herding behavior, whereby they disregard their private information
and release forecasts similar to those previously announced by other analysts, in order to
maximize their expected reputation. Trueman’s findings are in line with Scharfstein and Stein (1990), where managers exhibit herd behavior in a framework in which the expert has to make an investment decision as opposed to reporting his private information to a third party. In both these papers, experts choose their actions sequentially, and as in Ottaviani and Sorensen (2006) are solely concerned about their reputation.

In Benabou and Laroque (1992), insiders perform the joint actions of speculating and spreading information at no intrinsic cost, managing to manipulate prices repeatedly without being fully discovered. Insiders do not differ in their forecasting abilities (i.e., they all receive an equally informative signal), and are exclusively concerned about their credibility for reporting private information, as this affects the impact of their report on prices. In particular, some types of insiders are constrained to provide truthful reports, while others are allowed to act strategically. In our model, experts are characterized by different forecasting abilities, and the reporting strategies of all types of experts are determined endogenously.

In Brandenburger and Polak (1996), managers that are more informed with respect to the market on the true state of the world, must take an action whose effect on expected profits is conditional on the state of the world. The price of the firm, determined by public beliefs about the true state of the world, is updated based on the decision of the manager. They find that managers will tend to take an action that goes in the direction of prior market beliefs, in order to maximize the firm’s share price. This bias does not disappear, even when the payoff function of managers is a convex combination of the short term objective of maximizing current share price, and the long term objective of maximizing future profits. As in our model, biased actions are driven by the incentives to influence the beliefs of receivers (prices) before the true state of the world is revealed. However, unlike our model there is no scope for reputation acquisition, since managers do not differ in terms of the quality of their private information.

Morgan and Stocken (2003) present a theoretical model that analyzes the informational content of stock reports, when investors are uncertain about the analyst’s incentives. These
incentives may either be aligned or misaligned with those of investors. They find that
any investor uncertainty about incentives makes full revelation of information impossible.
Under certain conditions, analysts with aligned incentives can credibly convey unfavorable
information, but can never credibly convey favorable information. In their model, analysts
do not differ in the degree of informativeness of their signals (as they do in our work), but
in the degree of divergence of their preferences with respect to those of investors. As in
Benabou and Laroque (1992), analysts are not concerned about being perceived as having
accurate information, but about being perceived as credible.

III The Model

An expert is called upon to provide information to a pool of individuals who have to make
a forecast about the state of world. The state of the world \( w \) is either high or low, i.e. \( w \in \{h, l\} \), and all players hold the same prior belief \( \theta \) that the state is \( h \). At the beginning of
the game, the expert observes a private and non-verifiable signal \( s_i \in \{s_h, s_l\} \) about the true
state, whose accuracy depends on the expert’s ability \( t \). We assume that the expert is either
good or bad, i.e. \( t \in \{g, b\} \), and that ability affects the accuracy of the signal as follows:

\[
\Pr(s_h | t = g, \omega = h) = \Pr(s_l | t = g, \omega = l) = p, \ p \in (1/2, 1) \tag{1}
\]
\[
\Pr(s_h | t = b, \omega = h) = \Pr(s_l | t = b, \omega = l) = z, \ z \in (1/2, p] \tag{2}
\]

Therefore, both types of experts can count on an informative (yet imperfect) signal, with
the good type having a more accurate signal than a bad type. We assume that neither the
expert nor the receivers know the expert’s type, and all players hold the same prior belief \( \alpha \)
that the expert is good.\(^5\) We interpret \( \alpha \) as the prior reputation of the expert.

\(^5\)This assumption is without loss of generality as far as the key results of paper are concerned, and makes
the analysis more tractable. Assuming that the expert knows his own type does not affect the nature of the
results.
After observing the signal, the expert chooses a report that is publicly released in the form of a costless binary message $m_j \in \{m_h, m_l\}$. Receivers observe message $m_j$ and revise their beliefs about the true state of the world. We denote with $\tilde{\theta}_{\alpha, m_j} \equiv \Pr(\omega = h|m_j)$, the receivers’ posterior belief that the state of the world is $h$, given that message $m_j$ was sent by an expert with prior reputation $\alpha$. As we will see, in an equilibrium where some information is transmitted, the higher the reputation of the expert, the more the receivers trust the message sent. The subscript $\alpha$ highlights this relationship.

At the end of the game, the true state of the world is revealed and together with the message of the expert is used by the receivers to revise their beliefs about the expert’s ability. We denote with $\tilde{\alpha}_{\omega, m_j} \equiv \Pr(t = g|w, m_j)$, the receivers’ posterior belief that the expert is good upon observing state $w$ and message $m_j$. We interpret $\tilde{\alpha}_{\omega, m_j}$ as the new level of reputation acquired by the expert at the end of the game.

To model the expert’s concern about establishing a reputation for being a valuable provider of information and the contemporaneous existence of conflicts of interest, we construct a psychological game where the payoff of the expert depends positively on the receivers’ posterior beliefs $\tilde{\theta}_{\alpha, m_j}$ and $\tilde{\alpha}_{\omega, m_j}$, as follows:

$$\pi(m_j) = k\tilde{\theta}_{\alpha, m_j} + (1 - k)\tilde{\alpha}_{\omega, m_j}, \; k \in [0, 1]$$

The component $\tilde{\alpha}_{\omega, m_j}$ captures the reputational concerns of the expert. The component $\tilde{\theta}_{\alpha, m_j}$ gives the expert an incentive to inflate the receivers’ belief that the state is $h$, and thus creates a conflict of interest with the receivers, since the expert now has a bias in

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6In fact, in our model the receivers perform the task of forecasting the state of the world and the expert’s ability. Notice that we do not explicitly model the payoff of the receivers. Instead, we follow the approach of Ottaviani and Sorensen (2006) and implicitly assume that receivers are rewarded for accurately forecasting both the state of the world and the ability of the expert.

7This reduced form to account for reputational concerns is widely adopted in studies that model the reputation of experts and managers (see for example Sharfstein and Stein (1990), Ottaviani and Sorensen (2006) and Gentzkow and Shapiro (2006)).
favor of information that increases the receivers’ perception that the state is $h$. Finally, the parameter $k \in [0, 1]$ weighs these two components and can be seen as a measure of the severity of conflicts of interest. The structure and the parameters of the game (with the sole exception of the expert’s signal) are common knowledge.\footnote{See Battigalli and Dufwenberg (2009) for an analysis of extensive-form psychological games.}

Notice that interpreting $h$ and $l$ respectively as favorable and unfavorable states for the receivers, the model represents the over-optimism bias that has been discussed both in the finance literature on sell side analysts and in the political science literature on government agencies’ forecasts.\footnote{It is worth noticing that since also $k$ is common knowledge, we do not address the case when receivers are uncertain about the incentives of the expert (see Sobel (1985), Benabou and Laroque (1992), Morgan and Stocken (2003) for a formal analysis of the case when there is uncertainty about the expert’s incentives).} For the sake of exposition, in the remainder of the paper we will adopt this interpretation and refer to the expert’s bias as to the over-optimism bias.

\section*{IV Equilibrium Analysis}

In this section, we analyze the incentives of an expert to truthfully report his information and characterize the most informative equilibrium.\footnote{Assuming that the expert has an interest in inflating the receivers’ belief about the state being $h$, is without loss of generality. Our setup is well suited for analyzing a more general setting, where the expert has an incentive to manipulate the receivers’ beliefs in a desired direction.}

At the moment of sending message $m_j$, the true state of the world is unknown to the expert. The expert uses his signal $s_i$ to compute the expected impact of message $m_j$ on his reputation, as follows:

$$E(\hat{\alpha}_{\omega,m_j}|s_i) = \Pr(\omega = h|s_i)\hat{\alpha}_{h,m_j} + \Pr(\omega = l|s_i)\hat{\alpha}_{l,m_j}$$

\footnote{Our model presents the well-known problem of equilibrium multiplicity that is common to any cheap-talk game. A babbling equilibrium where all messages are taken to be meaningless and ignored always exists.}
Therefore, the expected payoff of the expert from sending message \( m_j \) reads:

\[
E(\pi(m_j)|s_i) = k\hat{\theta}_{\alpha,m_j} + (1 - k)E(\tilde{\alpha}_{\omega,m_j}|s_i)
\]

Before analyzing the incentives of an expert to truthfully report his information, it is convenient to gain an intuition of the tensions involved in the reporting decision of the expert. In any equilibrium where some information is transmitted we have that \( \hat{\theta}_{\alpha,m_h} > \hat{\theta}_{\alpha,m_l} \).

This introduces an incentive to report message \( m_h \) and represents a threat to truthtelling whenever signal \( s_l \) is received. In fact, the presence of reputational concerns counterbalances this over-optimism bias. As long as \( k \in (0, 1) \), the expert has to trade off the temptation of sending \( m_h \), with the negative effects that this message might have on his reputation, in case the message turns out to be incorrect.

The equilibrium concept we use is that of Perfect Bayesian Equilibrium (PBE). The expert will truthtfully report signal \( s_i \), if and only if, the expected payoff of truthtelling is greater than the payoff of reporting a message that is different from the signal received. Thus, a truthtelling equilibrium exists, if and only if, for every \( i, j \in \{h, l\} \), \( E(\pi(m_i)|s_i) \geq E(\pi(m_j)|s_j) \), or equivalently:

\[
\begin{align*}
  k\hat{\theta}_{\alpha,m_l} + (1 - k)E(\tilde{\alpha}_{\omega,m_l}|s_l) & \geq k\hat{\theta}_{\alpha,m_h} + (1 - k)E(\tilde{\alpha}_{\omega,m_h}|s_l) \quad (4) \\
  k\hat{\theta}_{\alpha,m_h} + (1 - k)E(\tilde{\alpha}_{\omega,m_h}|s_h) & \geq k\hat{\theta}_{\alpha,m_l} + (1 - k)E(\tilde{\alpha}_{\omega,m_l}|s_h) \quad (5)
\end{align*}
\]

In a truthtelling equilibrium, posterior reputation takes on only two possible values, which we denote with \( \alpha \) and \( \bar{\alpha} \), where:

\[
\begin{align*}
\alpha & \equiv \hat{\alpha}_{l,m_h} = \hat{\alpha}_{h,m_l} \\
\bar{\alpha} & \equiv \hat{\alpha}_{h,m_h} = \hat{\alpha}_{l,m_l}
\end{align*}
\]

\[\text{12Since the expert’s signals are informative, in any equilibrium where signals are truthfully reported with some positive probability, the messages of the expert contain some information.}\]
with $\bar{\alpha} > \alpha > \underline{\alpha}$. Making a correct evaluation increases the expert’s reputation from its initial level $\alpha$ to the higher level $\bar{\alpha}$. Making a wrong evaluation decreases the expert’s reputation from $\alpha$ to the lower level $\underline{\alpha}$. In the rest of the paper we denote $(\bar{\alpha} - \alpha)$ as the reputational reward of being recognized as a good expert. This allows us to write conditions (4) and (5) in the following way:

\begin{align}
    k \left( \hat{\theta}_{\alpha,mh} - \hat{\theta}_{\alpha,ml} \right) & \leq (1 - k)(\bar{\alpha} - \alpha)(1 - 2 \Pr(\omega = h|s_l)) \\
    k \left( \hat{\theta}_{\alpha,mh} - \hat{\theta}_{\alpha,ml} \right) & \geq (1 - k)(\bar{\alpha} - \alpha)(1 - 2 \Pr(\omega = h|s_h))
\end{align}

For each of the above conditions, we refer to the left hand side as the benefit of providing a high message, and to the right hand side as the expected reputational gain of sending a low message. Notice that the right hand side of (6) represents the expected reputational gain of truthtelling when receiving a low signal, while the right hand side of (7) represents the expected reputational gain of misreporting when receiving a high signal.

**Lemma 1** In a truthtelling equilibrium, the benefit of sending a high message, $k \left( \hat{\theta}_{\alpha,mh} - \hat{\theta}_{\alpha,ml} \right)$ satisfies the following properties: a) it is strictly positive positive for $\theta \in (0, 1)$ and equal to zero for $\theta = 0, 1$; b) it is strictly concave in $\theta$ with a maximum at $\theta = \frac{1}{2}$.

*(Proof: see Appendix)*

The benefit of sending a high report, is therefore increasing up until a threshold value of the prior on the state of the world, and decreasing from that point onwards. Notice also, that when there is little uncertainty on the state of the world (i.e. when $\theta$ is close to 0 or 1), this benefit tends to zero.

The previous lemma immediately implies that in the limit case, when reputation does not play any role (i.e. when $k = 1$), condition (6) is never satisfied and a truthtelling

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\footnote{We show this result in the Appendix.}
equilibrium never exists. \footnote{This case resembles Branderburger and Polak (1996), the only difference being that the absence of an over optimism bias in their model, allows for the existence of partially informative mixed strategy equilibria.} In this case, the incentive of the expert to report \( m_h \) destroys any putative equilibrium where some information is transmitted, and the expert plays no role in reducing information asymmetries. This provides some justification as to why making experts’ compensation depend on reputation, is a necessary condition for them to be credible information providers.

**Lemma 2** The expected reputational gain of sending the low message, \((1-k)(\alpha-\alpha) (1 - 2 \Pr(w = h|s_i))\) satisfies the following properties: a) it is positive at \( \theta = 0 \) and negative at \( \theta = 1 \) for \( i = h, l \); b) it is strictly decreasing in \( \theta \) for \( i = h, l \); it is strictly concave in \( \theta \) for \( i = l \) and strictly convex in \( \theta \) for \( i = h \).

It is important to notice, that the reputational reward of being recognized as a good expert \((\bar{\alpha})\), is not affected by variations in the prior on the state of the world. Variations in \( \theta \) simply affect the expected reputational gains.

We now establish that when experts have reputational concerns some information can be transmitted. The most informative equilibrium is reminiscent of Ottaviani and Sorensen (2001, 2006) as described in the following proposition:

**Proposition 1** For \( k \in [0, 1) \), the most informative equilibrium is separating (i.e. fully revealing) for \( \theta \in [\bar{\theta}, \overline{\theta}] \) and pooling (i.e uninformative) for \( \theta \not\in [\bar{\theta}, \overline{\theta}] \).

(Proof: see Appendix)

For an intuition of Proposition 1, first notice that Lemma 1 implies that when \( \theta \) is very low (high), receivers expect the economy to be in state \( l \) (\( h \)) regardless of the message sent by the expert. As a result, the net gain from inflating the beliefs of the receivers by sending \( m_h \) instead of a \( m_l \), is very small and the choice of the expert is mainly driven by reputational concerns. However, reputational concerns make truth-telling impossible when
the prior is relatively extreme. In these cases, the expert may believe that any contrarian
signal he receives is probably incorrect. Being worried about the adverse impact of ex-post
incorrect messages on his reputation, he disregards his private information and reports the
signal that is more likely to be correct ex-post. This is illustrated in Lemma 2, that shows
how as the ex-ante probability that the true state is \( h \) increases, the expected reputational
gain of reporting the low message decreases independently from the signal received. This
conservative behavior on the part of the expert exists as long as the expert has some concerns
about his reputation (i.e. for \( k < 1 \)).

On the other hand, Proposition 1 also highlights how truthful revelation occurs for interior
values of \( \theta \). As illustrated in Lemma 1, in these cases conflicts of interest play a greater role
with respect to the limit cases when \( \theta \) approaches 0 or 1. Therefore, reputational concerns are
still somewhat effective in inducing truth-telling behavior, even in the presence of conflicts of
interest. Indeed, Proposition 1 suggests that the nature of the most informative equilibrium
in the presence of over-optimism bias (\( k \in (0, 1) \)), is not qualitatively different from the case
when conflicts of interest are absent and the expert is solely concerned about his reputation
(\( k = 0 \)).

V Comparative Statics

In this section, we examine how variations in the severity of conflicts of interest, in reputation,
and in the difference between the signal informativeness of good and bad types affect the
most informative equilibrium of Proposition 1. In particular, we analyze how changes in the
parameters \( k, \alpha \) and \( (p - z) \) affect the truth-telling region \([\bar{\theta}, \tilde{\theta}]\), as measured by the difference
\( \bar{\theta} - \tilde{\theta} \). With a slight abuse of terminology, we refer to any increase (decrease) in \( \bar{\theta} - \tilde{\theta} \) as to
an increase (decrease) in informational efficiency. To gather further insight on our findings,
we carry out numerical analysis which we refer to in presenting the results.

The key finding is that significantly different results arise when conflicts of interest are
present \((k \in (0, 1))\), as opposed to the case when conflicts of interest are absent \((k = 0)\). For the sake of exposition, it is convenient to define some properties of the truth-telling equilibrium in the case when \(k = 0\):

**Remark 1** Let \(\underline{\theta}^*\) and \(\overline{\theta}^*\) denote the threshold values for an expert with no conflicts of interest (i.e. \(k = 0\)). Then, \(\underline{\theta}^* = 1 - \alpha p + (1 - \alpha)z\) and \(\overline{\theta}^* = \alpha p + (1 - \alpha)z\).

(Proof: see Appendix)

The previous remark suggests that in the absence of conflicts of interest, the truth-telling region is symmetrically centered around \(\theta = \frac{1}{2}\), and expands as \(\alpha, p\) and \(z\) increase. In particular, \(\underline{\theta}^* (\overline{\theta}^*)\) is decreasing (increasing) in \(\alpha, p\) and \(z\).

### A Variations in the Severity of Conflicts of Interest

We start by analyzing how variations in \(k\) affect the truth-telling thresholds \(\underline{\theta}\) and \(\overline{\theta}\) as described by the following proposition:

**Proposition 2** Both \(\underline{\theta}\) and \(\overline{\theta}\) are decreasing in \(k\).

(Proof: see Appendix)

In the case of no conflicts of interest \((k = 0)\), the truth-telling region is centered around \(\theta = \frac{1}{2}\). Proposition 2 suggests that as conflicts of interest become more severe, the truth-telling region progressively shifts toward values of the prior on the state of the world that are closer to zero. Indeed, as \(k\) increases the bias in favor of the high message increases. As a consequence, the expert is willing to send the high message for lower values of the prior \(\theta\), and truthful revelation becomes possible, only when public information is rather contrary to the state the expert wishes public opinion to be swayed towards (i.e. state \(h\)).

As conflicts of interest become fiercer, not only does the bias to report \(m_h\) becomes stronger, but informational efficiency progressively declines. This occurs because as \(k\) increases, the expert’s interest to sway the beliefs of decision makers in favor of state \(h\),
progressively dominates the expert’s concern for his reputation (i.e., we approach the limit case when $k = 1$). The following proposition summarizes this result:

**Proposition 3** There always exists a level of $k$ above which informational efficiency (i.e. $(\bar{\theta} - \theta)$) is decreasing in $k$.

*Proof: see Appendix*

As shown in figure 1, numerical analysis suggests that informational efficiency is decreasing in $k$. Furthermore, the decline in efficiency is quite sharp for relatively low values of $k$.

**B Variations in prior reputation ($\alpha$)**

We next analyze how variations in prior reputation affect informational efficiency. As a first step, we focus on the relationship between $\alpha$ and the different payoff components of the expert, as described in the following remark:

**Remark 2** (i) The benefit of sending a high report, $(\hat{\theta}_{\alpha,m} - \hat{\theta}_{\alpha,m_1})$ is increasing in initial reputation $\alpha$; (ii) The reputational reward of being recognized as a good expert, $\bar{\alpha} - \underline{\alpha}$ is strictly concave in $\alpha$, with $(\bar{\alpha} - \underline{\alpha}) = 0$ for $\alpha = 0, 1$.

*Proofs: see Appendix*

The benefit of sending a high report increases with the level of reputation. An expert with higher reputation receives a more accurate signal, and his message therefore has a greater impact on the beliefs of decision makers. The way $(\bar{\alpha} - \underline{\alpha})$ changes in response to variations in the initial level of reputation, reflects the common idea that individuals sluggishly change their mind in response to new evidence, when they already hold a strong prior belief about something or somebody. On the contrary, new information typically leads to larger swings in beliefs when the level of uncertainty is high.
The previous remark suggests that above a certain level of $\alpha$, the reputational reward of being recognized as a good expert, becomes negligible with respect to the benefit of sending a high report (indeed, the difference between these two components grows larger as $\alpha$ increases). As a result, above a threshold level of $\alpha$ the expert’s bias in favor of the high message becomes stronger and actually increases with $\alpha$. This makes both truthtelling thresholds $\bar{\theta}$ and $\theta$ decrease with $\alpha$, reflecting the idea that as $\alpha$ grows larger, the expert has a stronger incentive to report a high message for any level of $\theta$. A similar argument reveals that an increase in $\alpha$, when $\alpha$ is below a certain threshold, determines an increment in $\theta$ and $\bar{\theta}$.

This leads us to the following proposition:

**Proposition 4** There always exist: (i) a level of initial reputation $\alpha$ above which an increase in $\alpha$ reduces $\bar{\theta}$ and $\theta$; (ii) a level of initial reputation $\alpha$ below which an increase in $\alpha$ increases $\bar{\theta}$ and $\theta$.

*(Proof: see Appendix)*

Remark 2 bears a deeper consequence as far as the impact of reputation on informational efficiency in concerned. As $\alpha$ increases above a certain threshold, the difference between $(\bar{\alpha} - \alpha)$ and $(\hat{\theta}_{\alpha,m_h} - \hat{\theta}_{\alpha,m_l})$ grows larger (with the former in fact progressively shrinking to zero), meaning that the reporting incentives of the expert are increasingly dominated by his interest to sway the beliefs of decision makers in favor of state $h$. As a result, for relatively large values of $\alpha$, the benefit of sending the high message, irrespectively of the signal observed, dominates the expected reputational gain of making a correct evaluation.

\footnote{At $\theta = \hat{\theta}$ an expert that has received a high signal is indifferent between reporting a high message and reporting a low message. Ceteris paribus, an increase in $\alpha$ breaks this indifference in favour of the high message, which in fact implies that at $\theta = \hat{\theta}$ the expert is now truthfully reporting the high signal (i.e. the new truthtelling threshold, say $\hat{\theta}$, is lower than the initial one, $\hat{\theta}$). On the other hand, at $\theta = \bar{\theta}$ an expert that has received a low signal is indifferent between reporting a high message and reporting a low message. Again, ceteris paribus, an increase in $\alpha$ breaks this indifference in favour of the high message, implying that at $\theta = \bar{\theta}$ the expert is now pooling on the high signal (i.e. the new truthtelling threshold, say $\bar{\theta}$, is lower than the initial one, $\bar{\theta}$).}
thus reducing informational efficiency. This effect clearly intensifies as alfa approaches to 1, where the truthtelling region becomes an empty set.

A similar reasoning applied to the case when initial reputation is below a certain threshold, suggests that an increase in $\alpha$ leads to an expansion of the truthtelling region, when $\alpha$ is indeed below a certain threshold. The following proposition summarizes the previous reasoning:

**Proposition 5** There always exist: (i) a level of initial reputation $\alpha$ above which an increase in $\alpha$ reduces informational efficiency (i.e. $(\overline{\theta} - \theta)$); (ii) a level of initial reputation $\alpha$ below which an increase in $\alpha$ increases informational efficiency (i.e. $(\overline{\theta} - \theta)$ increases).

*(Proof: see Appendix)*

The result in Proposition 5, contrasts with the case of no conflicts of interest ($k = 0$), where an increase in reputation always translates into an improvement of informational efficiency.\(^{16}\) Now, a further increase in prior reputation above a certain threshold (i.e. a reduction of uncertainty on expert ability), makes the truthtelling space shrink.

Numerical analysis illustrates how both $\overline{\theta}$ and $\underline{\theta}$ are hump-shaped in $\alpha$ (figure 2). Furthermore, the threshold level of $\alpha$ above which an increase in prior reputation leads to a stronger bias towards $h$, is a relatively intermediate value (i.e. close to $1/2$). Thus this effect cannot be considered as a limit case that sets in only for extreme values of initial reputation. Prior reputation therefore has a non-monotonic effect on informational efficiency when conflicts of interest are present. Notice that for extreme values of $\alpha$ informational efficiency tends to zero. In other words, a very high level of reputation, is as bad as a very low level of initial reputation as far as informational efficiency is concerned.

\(^{16}\)Notice from Remark (When $k = 0$ the truthtelling region monotonically expands from $2z - 1$ (when $\alpha \rightarrow 0$) to $2p - 1$ (when $\alpha \rightarrow 1$) and the greatest amount of information is transmitted when $\alpha \rightarrow 1$.)
C Variations in Signals’ Informativeness

In analyzing variations in the quality of information, we examine the impact of variations in the gap between expert abilities, by fixing $p$ and letting $z$ vary. The following proposition summarizes the main findings:

**Proposition 6** Holding $p$ fixed, there always exists a level of $z$ above which an increase in $z$ reduces informational efficiency (i.e. $(\bar{\theta} - \theta)$ decreases).

*(Proofs: see Appendix)*

The intuition for this result, is that as the evaluation technology (or ability) of the worst expert improves, the spread $(\bar{\theta}_{a,m_h} - \bar{\theta}_{a,m_l})$ increases since in an equilibrium with some information revelation, expert reports are more informative. On the other hand, as $z$ approaches $p$, the reputational reward of being recognized as a good expert decreases, since the difference between good and bad experts shrinks (figure 3). Thus, when conflicts of interest are present, as the abilities of experts converge the information revealed tends to zero. This result is quite striking, as it paradoxically implies that the coexistence of experts of different abilities, guarantees a higher level of informational efficiency. $^{17}$

VI Conclusion

Conflicts of interest are relevant in many economic settings where experts with privileged information are called upon to provide information to uninformed receivers. In particular, in this paper we have focused on the trade-off that experts typically face, between the short term benefit of providing biased reports, versus the long term reward of acquiring a reputation for being valuable information providers.

$^{17}$In the absence of conflicts of interest ($k = 0$), an increase in $z$ has an unambiguously positive effect on informational efficiency resulting in maximum efficiency when $z \to p$. 

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We find that reputation plays an important role in shaping the incentives of experts that face conflicts of interest driven by an over-optimism bias. The main result of our model is that reputation has a non-monotonic effect on information transmission, and greater uncertainty on expert ability is associated with more information revelation. In other words, those experts that have established a reputation for providing valuable information, may have strong incentives to release biased reports, much like those that have a stable record of incorrect evaluations. It is precisely the uncertainty on ability, that creates greater incentives for experts to truthfully reveal their information, in order to distinguish themselves from the poorly informed and acquire a higher reputation. Once this standing has been attained, the over-optimism bias tends to prevail over the reputational losses that experts may incur, by erroneously forecasting a future state of the world.

These results suggest an empirical implication for the case of sell-side financial analysts. In a situation where the market for analysts is populated by a large share of well established analysts, less information will be contained in financial reports. If investors are rational, this should on average lead stock prices to exhibit a milder reaction to analyst reports, with respect to other market scenarios characterized by more uncertainty on analyst ability. Testing this empirical implication represents a step for future research.

Another suggested direction for future research, is to gather a better understanding of the link between informational efficiency and the institutional framework in which experts operate. In particular, the characteristics of the market and institutions that govern the expert environment, may affect the degree of uncertainty on ability (or reputation) in different ways. Capturing how these institutional settings may influence the degree of informational efficiency, through the reputational channel, represents an open question.
VII  Appendix

Expert’s Posterior Beliefs.

\[
\Pr(\omega = h | s_h) = \frac{\theta(\alpha p + (1 - \alpha)z)}{\theta(\alpha p + (1 - \alpha)z) + (1 - \theta)(\alpha(1 - p) + (1 - \alpha)(1 - z))} \\
\Pr(\omega = l | s_h) = 1 - \Pr(\omega = h | s_h) \\
\Pr(\omega = h | s_l) = \frac{\theta(\alpha(1 - p) + (1 - \alpha)(1 - z))}{\theta(\alpha(1 - p) + (1 - \alpha)(1 - z)) + (1 - \theta)(\alpha p + (1 - \alpha)z)} \\
\Pr(\omega = l | s_l) = 1 - \Pr(\omega = h | s_l)
\]

Posterior Reputations under Truthtelling. In a truthtelling equilibrium the expert reports the signal he has observed. Therefore:

\[
\hat{\alpha}_{\omega,m_j} \equiv \Pr(t = g|\omega, m_j) = \begin{cases} 
\frac{\alpha p}{\alpha p + (1 - \alpha)z} & \text{for } (w = h, j = h), (w = l, j = l) \\
\frac{\alpha(1 - p)}{\alpha(1 - p) + (1 - \alpha)(1 - z)} & \text{for } (w = h, j = l), (w = l, j = h)
\end{cases}
\]

Let \( \overline{\alpha} \equiv \frac{\alpha p}{\alpha p + (1 - \alpha)z} \) and \( \underline{\alpha} \equiv \frac{\alpha(1 - p)}{\alpha(1 - p) + (1 - \alpha)(1 - z)} \). Then for \( \alpha \in (0, 1) \), \( p \in (\frac{1}{2}, 1) \) and \( z \in [\frac{1}{2}, p) \):

\[
\overline{\alpha} - \underline{\alpha} = \frac{\alpha p}{\alpha p + (1 - \alpha)z} - \frac{\alpha(1 - p)}{\alpha(1 - p) + (1 - \alpha)(1 - z)} = \frac{\alpha(1 - \alpha)(p - z)}{(1 - \alpha)(p - z) - z} > 0
\]

Proof of Lemma 1. Since \( k \in [0, 1] \), we can analyze \( f(\theta) \equiv \hat{\theta}_{\alpha,m_h} - \hat{\theta}_{\alpha,m_l} \). In a truthtelling equilibrium the expert reports the signal he has observed. Therefore:

\[
\hat{\theta}_{\alpha,m_j} \equiv \Pr(\omega = h | m_j) = \Pr(\omega = h | s_j) = \begin{cases} 
\frac{\theta(\alpha p + (1 - \alpha)z)}{\theta(\alpha p + (1 - \alpha)z) + (1 - \theta)(\alpha(1 - p) + (1 - \alpha)(1 - z))} & \text{for } j = h \\
\frac{\theta(\alpha(1 - p) + (1 - \alpha)(1 - z))}{\theta(\alpha(1 - p) + (1 - \alpha)(1 - z)) + (1 - \theta)(\alpha p + (1 - \alpha)z)} & \text{for } j = l
\end{cases}
\]
With a bit of algebra we obtain:

\[
\begin{align*}
  f(\theta) & \equiv \tilde{\theta}_{\alpha,m_h} - \tilde{\theta}_{\alpha,m_i} = \\
  & = \frac{\theta(1 + \theta)(-1 + 2(\alpha(p - z) + z))}{\theta(2(\alpha(p - z) + z) - 1)} - \frac{\theta(1 + \theta)(2(\alpha(p - z) + z) - 1)) - (\alpha(p - z) + z)}{(1 + \theta)(2(\alpha(p - z) + z) - 1)) - (\alpha(p - z) + z)} \\
  \end{align*}
\]

Let \( q \equiv \alpha(p - z) + z \). Then,

\[
\begin{align*}
  f(\theta) &= -\frac{\theta(1-\theta)(2q-1)}{(2q\theta-\theta-q)(1+2q\theta-\theta-q)}. \\
  \end{align*}
\]

Notice that for \( \alpha \in (0,1), p \in (\frac{1}{2}, 1) \) and \( z \in [\frac{1}{2}, p] \), we have that \( \frac{1}{2} < q < 1 \). Then:

\[
\begin{align*}
  f(\theta) > 0 & \text{ for } 0 < \theta < 1 \\
  f(\theta) &= 0 \text{ for } \theta = 0, 1 \\
  \frac{\partial f(\theta)}{\partial \theta} &= -\frac{q(1-q)(2q-1)(2\theta-1)}{(2q\theta-\theta-q)^2(1+2q\theta-\theta-q)^2} \\
  & \begin{cases} > 0 & \text{ for } 0 < \theta < \frac{1}{2} \\
  = 0 & \text{ for } \theta = \frac{1}{2} \\
  < 0 & \text{ for } \frac{1}{2} < \theta < 1 \end{cases} \\
  \frac{\partial^2 f(\theta)}{\partial \theta^2} &= 2q(1-q)(2q-1)\left(\frac{1}{(2q\theta-\theta-q)^3} - \frac{1}{(1+2q\theta-\theta-q)^3}\right) < 0 \text{ for } 0 < \theta < 1
\end{align*}
\]

\[\Box\]

**Proof of Lemma 2.** Let \( g(\theta) \equiv (1 - k)(\overline{\alpha} - \alpha)1 - 2 \Pr(\omega = h|s_i) \) and \( v(\theta) \equiv (1 - k)(\overline{\alpha} - \alpha)(1 - 2 \Pr(\omega = h|s_h)) \). Using the values of \( \overline{\alpha}, \alpha, \Pr(\omega = h|s_i) \) and \( \Pr(\omega = h|s_h) \) we obtain:

\[
\begin{align*}
  g(\theta) &= \frac{(1-k)(1-\alpha)(p-z)(-\theta + \alpha(p-z) + z)}{(-1 + \alpha(p-z) + z)(\alpha(p-z) + z)(\alpha(-1 + 2\theta)(p-z) - z + \theta(-1 + 2z))} \quad (\text{RHS of (6)}) \\
  v(\theta) &= \frac{(1-k)\alpha(1-\alpha)(p-z)(-\theta + \alpha(p-z) + z)}{(-1 + \alpha(p-z) + z)(\alpha(p-z) + z)(1 + \alpha(-1 + 2\theta)(p-z) - z + \theta(-1 + 2z))} \quad (\text{RHS of (7)}) \\
  \end{align*}
\]

Let \( q \equiv \alpha(p - z) + z \). Then,

\[
\begin{align*}
  g(\theta) &= \frac{\alpha(p-q)(\theta-q)}{q(1-q)(2q\theta-\theta-q)} \quad \text{and} \quad v(\theta) = \frac{\alpha(p-q)(\theta-q)}{q(1-q)(2q\theta-\theta-q+1)}. \\
  \end{align*}
\]

Notice that
for $\alpha \in (0, 1)$, $p \in (\frac{1}{2}, 1)$ and $z \in [\frac{1}{2}, p)$, we have that $\frac{1}{2} < z < q < p < 1$. Then:

$$g(\theta) = \begin{cases} > 0 & \text{for } 0 < \theta < q \\ = 0 & \text{for } \theta = q \\ < 0 & \text{for } q < \theta < 1 \end{cases}$$

$$g(0) = \frac{\alpha(p - q)}{q(1 - q)} > 0, \quad g(1) = -\frac{\alpha(p - q)}{q(1 - q)} < 0$$

$$\frac{\partial g(\theta)}{\partial \theta} = -\frac{2\alpha(p - q)}{(q + \theta - 2q\theta)^2} < 0 \quad \text{for } 0 < \theta < 1$$

$$\frac{\partial^2 g(\theta)}{\partial \theta^2} = -\frac{4\alpha(p - q)(2q - 1)}{(q + \theta - 2q\theta)^3} < 0 \quad \text{for } 0 < \theta < 1$$

$$v(\theta) = \begin{cases} > 0 & \text{for } 0 < \theta < 1 - q \\ = 0 & \text{for } \theta = 1 - q \\ < 0 & \text{for } 1 - q < \theta < 1 \end{cases}$$

$$v(0) = \frac{\alpha(p - q)}{q(1 - q)} > 0, \quad v(1) = -\frac{\alpha(p - q)}{q(1 - q)} < 0$$

$$\frac{\partial v(\theta)}{\partial \theta} = -\frac{2\alpha(p - q)}{(-1 + q + \theta - 2q\theta)^2} < 0 \quad \text{for } 0 < \theta < 1$$

$$\frac{\partial^2 v(\theta)}{\partial \theta^2} = \frac{4\alpha(p - q)(2q - 1)}{(1 - q - \theta + 2q\theta)^3} > 0 \quad \text{for } 0 < \theta < 1$$

$$g(\theta) - v(\theta) = \frac{2\alpha(p - q)(2q - 1)(1 - \theta)(1 - \theta)\theta}{q(1 - q)(1 - q - \theta + 2q\theta)(q + \theta - 2q\theta))} > 0 \quad \text{for } 0 < \theta < 1$$

**Proof of Proposition 1.** Consider the two conditions for truthtelling:

$$k[\tilde{\theta}_{a,m_h} - \hat{\theta}_{a,m_l}] \leq (1 - k)(\overline{\alpha} - \alpha)[1 - 2 \Pr(\omega = h|s_l)] \quad (8)$$

$$k[\tilde{\theta}_{a,m_h} - \hat{\theta}_{a,m_l}] \geq (1 - k)(\overline{\alpha} - \alpha)[1 - 2 \Pr(\omega = h|s_h)] \quad (9)$$
We first prove that for every value of $\alpha \in (0,1)$, $k \in [0,1)$, $p \in \left( \frac{1}{2}, 1 \right)$ and $z \in \left[ \frac{1}{2}, p \right)$, there exist $\theta \in [0,1]$ and $\overline{\theta} \in [0,1]$ such that for $\theta \in [\overline{\theta}, \theta]$ conditions (8) and (9) are satisfied simultaneously. Consider condition (8) first. Using lemmas 1 and 2, (8) can be written as follows:

$$
\frac{k\theta(1-\theta)(2q-1)}{(2q\theta-\theta-q)(1+2q\theta-\theta-q)} \leq \frac{(1-k)\alpha(p-q)(\theta-q)}{(1-q)q(2q\theta-\theta-q)}
$$

Notice that $\frac{1}{2} \leq z < q < p < 1$. Thus, for $\theta \in (0,1)$, $2q\theta - \theta - q < 0$ and (8) is equivalent to:

$$
\frac{k\theta(1-\theta)(2q-1)}{1+2q\theta-\theta-q} \leq -\frac{(1-k)\alpha(p-q)(\theta-q)}{(1-q)q}
$$

Finally, let $h(\theta) = -\frac{k\theta(1-\theta)(2q-1)}{2q\theta-\theta-q}$ and $r(\theta) = \frac{(1-k)\alpha(p-q)(\theta-q)}{(1-q)q}$, and notice that:

a) $r(0) > h(0) = 0$, $r(1) < h(1) = 0$

b) $r(\theta)$ is a negatively sloped straight line.

c) $h(\theta)$ is non-negative, continuous, and strictly concave for $\theta \in (0,1)$.

Properties a), b) and c) imply that there exists a unique $\overline{\theta} \in (0,1)$ such that for any $\theta < \overline{\theta}$ (10) (and therefore (8)) are satisfied.

Focusing on condition (9) and following the same steps above, we can prove the existence and uniqueness of a $\underline{\theta} \in (0,1)$ such that, for any $\theta > \underline{\theta}$, (9) is satisfied. From lemma 2 we know that for $\theta \in (0,1)$ the RHS of condition (8) is strictly greater than the RHS of condition (9). This result, together with the uniqueness of $\underline{\theta}$ and $\overline{\theta}$ implies that $\overline{\theta} > \underline{\theta}$. Therefore, (8) and (9) are simultaneously satisfied for $\theta \in [\underline{\theta}, \overline{\theta}]$.

Finally, notice that a babbling equilibrium where the expert sends $m_h$ with probability $\pi$ and $m_l$ with probability $1 - \pi$ irrespectively of the signal observed always exists. In this case all messages are taken to be meaningless and ignored: $\tilde{\theta}_{\alpha,m_j} = \theta$ for any $i = h,l$, and $\tilde{\alpha}_{\omega,m_j} = \alpha$ for any $\omega = h,l$ and $j = h,l$, making the expert indifferent between the two messages. •

**Corollary 1** For condition (8), $\frac{\partial \text{RHS}}{\partial \theta} \bigg|_{\theta = \overline{\theta}} > \frac{\partial \text{LHS}}{\partial \theta} \bigg|_{\theta = \overline{\theta}}$. For condition (9), $\frac{\partial \text{RHS}}{\partial \theta} \bigg|_{\theta = \underline{\theta}} > \frac{\partial \text{LHS}}{\partial \theta} \bigg|_{\theta = \underline{\theta}}$. 

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\[ \partial \text{LHS} \big|_{\theta = \theta^*} \]

**Proof of Corollary 1.** The result in Corollary 1 is an immediate consequence of
uniqueness of \( \overline{\theta} \) and \( \theta^* \), together with the properties in lemma 1 and lemma 2. In words, the
RHS of (8) always intersects the LHS from above. The same is true for condition (9). ■

**Proof of Remark 1.** When \( k = 0 \), condition (10) boils down to \( 0 \leq \alpha(p - q)(\theta - q) \).
The associated equation has solution \( \theta = q = \alpha p + (1 - \alpha) z \equiv \overline{\theta}^* \). The value of \( \theta^* \) is obtained
in the same way from condition (9) ■

**Proof of Proposition 2.** Consider condition (8). We know from lemma 1 that the
LHS is strictly positive for any \( \theta \in (0, 1) \). This implies that at \( \theta = \overline{\theta} \), \( \text{RHS} = \text{LHS} > 0 \). We
know from Lemma 2 that: \( \text{RHS} \) is strictly decreasing in \( \theta \) for any \( \theta \in (0, 1) \), being equal to
zero at \( \theta = q = \overline{\theta}^* \). Therefore, it must be that \( \overline{\theta} < \overline{\theta}^* \). Having established this result, notice
that for any \( \theta \in (0, \overline{\theta}^*) : \frac{\partial \text{LHS}}{\partial k} = -\frac{\theta(1-\theta)(2q-1)}{(2q-\theta-q)(1+2q-\theta-q)} > 0 \) and \( \frac{\partial \text{RHS}}{\partial k} = -\frac{\alpha(p-q)(\theta-q)}{(1-q)(2q-\theta-q)} < 0 \).
This, together with the result from Corollary 1 implies that \( \overline{\theta} \) is decreasing in \( k \). The same
reasoning applies to condition (9) to show that \( \theta \) is decreasing in \( k \). ■

**Proof of Proposition 3.** Consider condition (8). Notice that for \( k \to 1 \), \( \text{LHS}_1 \to -\frac{\theta(1-\theta)(2q-1)}{(2q-\theta-q)(1+2q-\theta-q)} \) and \( \text{RHS}_1 \to 0 \). Thus, for \( k \to 1 \): \( \overline{\theta} \to 0 \). For \( k = 0 \), \( \text{LHS}_1 = 0 \) and
\( \text{RHS}_1 = -\frac{\alpha(p-q)(\theta-q)}{(1-q)(2q-\theta-q)} \). Thus for \( k = 0 \), \( \overline{\theta} = \overline{\theta}^* \).

Consider condition (9). Notice that for \( k \to 1 \), \( \text{LHS}_2 \to -\frac{\theta(1-\theta)(2q-1)}{(2q-\theta-q)(1+2q-\theta-q)} \) and
\( \text{RHS}_2 \to 0 \). Thus, for \( k \to 1 \): \( \theta \to 0 \). For \( k = 0 \), \( \text{LHS}_2 = 0 \) and \( \text{RHS}_2 = \frac{\alpha(p-q)(1-\theta-q)}{q(1-q)(2q-\theta-q+1)} \).
Thus for \( k = 0 \), \( \theta = \theta^* \).

The results above imply that: for \( k = 0 \), \( \overline{\theta} - \theta = \overline{\theta}^* - \theta^* > 0 \); for \( k \to 1 \), \( \overline{\theta} - \theta = 0 \). By
continuity of \( \overline{\theta} \) and \( \theta \), there must exists a \( k' \in [0, 1) \) such that for \( k > k' \), \( \frac{\partial (\overline{\theta} - \theta)}{\partial k} < 0 \). ■

**Proof of Remark 2.** Let \( q = \alpha(p - z) + z \), where \( z < q < p \). Notice that:

(i) \( \frac{\partial(\theta_0, m_0 - \theta_0, m_0)}{\partial z} = \theta(1 - \theta)(p - z) \left( \frac{1}{q + \theta(1-2q)} + \frac{1}{(1-q-\theta(1-2q))} \right) > 0 \) for any \( \alpha \in (0, 1) \).

(ii) \( \frac{\partial (\pi/\alpha)}{\partial \beta} = \frac{(p-z)(a^2(m-p)+(m-1)^2z-(m-1)^2z^2)}{(a-1)^2q^2} \). Notice that: \( \frac{\partial (\pi/\alpha)}{\partial \alpha} = 0 \iff \alpha_0 = \frac{z - z^2 - \sqrt{p^2 - p^2 z^2 - p z^2 + p^2 z^2}}{p^2 - p z - z^2} \), \( \alpha_1 = \frac{z - z^2 + \sqrt{p^2 - p^2 z^2 - p z^2 + p^2 z^2}}{p^2 - p z - z^2} \), where \( \alpha_1 < 0 < \alpha_0 < 1 \).
$$\frac{\partial^2 (\pi - q)}{\partial z^2} = 2 (p - z) \left( - \frac{(1-p)(1-z)}{(1-q)^3} - \frac{p^2}{q^3} \right) < 0$$ for $\alpha \in (0, 1)$. Therefore, for $\alpha \in (0, 1)$, $\bar{\alpha} - \alpha$ is strictly concave with a maximum at $\alpha = \alpha_0$. ■

Proof of Proposition 4. Consider condition (8) and notice that: (i) For $\alpha \to 0$, $LHS_1 \to \frac{k\theta(2z-1)(1-\theta)}{(2z\theta - \theta - z)(2z\theta - \theta - z + 1)}$ and $RHS_1 \to 0$; thus, for $\alpha \to 0$, $\bar{\theta} \to 0$; (ii) For $\alpha \to 1$, $LHS_1 \to \frac{k\theta(2p-1)(1-\theta)}{(2p\theta - \theta - p)(2p\theta - \theta - p + 1)}$ and $RHS_1 \to 0$; thus, for $\alpha \to 1$ : $\bar{\theta} \to 0$.

Now notice that $\bar{\theta}$ is positive and continuous for $\alpha \in (0, 1)$. This, together with (i), (ii) imply that: There exist an $\alpha' \in (0, 1)$ such that for $\alpha \in (0, \alpha')$, $\frac{\partial \bar{\theta}}{\partial \alpha} > 0$; There exist an $\alpha'' \in (0, 1)$ such that for $\alpha \in \alpha', 1$, $\frac{\partial \bar{\theta}}{\partial \alpha} < 0$.

A similar argument applies to condition (9) to show that: (iii) For $\alpha \to 0$, $\bar{\theta} \to 0$; (iv) For $\alpha \to 1$, $\bar{\theta} \to 0$. Again, continuity and the fact that $\bar{\theta}$ is positive for any $\alpha \in (0, 1)$ imply that: There exist an $\alpha^+ \in (0, 1)$ such that for $\alpha \in (0, \alpha^+)$, $\frac{\partial \bar{\theta}}{\partial \alpha} > 0$; There exist an $\alpha^{++} \in (0, 1)$ such that for $\alpha \in (\alpha^+, 1)$, $\frac{\partial \bar{\theta}}{\partial \alpha} < 0$. ■

Proof of Proposition 5. From the results in the proof of proposition 4 we have that: (i) For $\alpha \to 0$, $\bar{\theta} - \theta \to 0$; (ii) For $\alpha \to 1$, $\bar{\theta} - \theta \to 0$. Since $\bar{\theta} - \theta$ is positive for any value of $\alpha \in (0, 1)$, by continuity there exist a value of $\alpha \in (0, 1)$ below which $\bar{\theta} - \theta$ is increasing in $\alpha$, and a value of $\alpha \in (0, 1)$ above which $\bar{\theta} - \theta$ is decreasing in $\alpha$. ■

Proof of Proposition 6. Consider conditions (8) and (9).Notice that for $z \to p$: (i) $LHS_1 \to \frac{k\theta(1-\theta)(2p-1)}{(2p\theta - \theta - p)(2p\theta - \theta - p + 1)}$ and $RHS_1 \to 0$, which implies that $\bar{\theta} \to 0$; (ii) $LHS_2 \to \frac{k\theta(1-\theta)(2p-1)}{(2p\theta - \theta - p)(2p\theta - \theta - p + 1)}$ and $RHS_2 \to 0$, which implies that $\bar{\theta} \to 0$. From (i) and (ii) it follows that for $z \to p$, $\bar{\theta} - \theta \to 0$. Since $\bar{\theta} - \theta$ is positive for any value of $z \in (0, p)$, by continuity there exist a value of $z \in (0, 1)$ above which $\bar{\theta} - \theta$ is decreasing in $z$. ■
References


RAND Journal of Economics 34, 183-203.


Figure 1: Truth-telling thresholds as functions of $k$.

Figure 2: Truth-telling thresholds as functions of $\alpha$. 
Figure 3: Truth telling thresholds as functions of $z$