Optimal Simple Monetary and Fiscal Rules under Limited Asset Market Participation

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Optimal Simple Monetary and Fiscal Rules under Limited Asset Market Participation *

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Abstract

When the central bank is the sole policymaker, the combination of limited asset market participation and consumption habits can have dramatic implications for the optimal monetary policy rule and for stability properties of a business cycle model characterized by price and nominal wage rigidities. In this framework, a simple countercyclical fiscal rule plays a twofold role. On the one hand it ensures uniqueness of the rational expectations equilibrium when monetary policy follows a standard Taylor rule. On the other hand it brings aggregate dynamics substantially closer to their socially efficient levels.

JEL classification: E52.

Keywords: Rule of Thumb Consumers, DSGE, Determinacy, Limited Asset Market Participation, Taylor Principle, Optimal Simple Rule

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1 Introduction

The standard New-Keynesian framework is characterized by optimizing agents and by a number of nominal and real frictions in goods, labor and financial markets. Following a seminal contribution by Mankiw (2000), a second strand of New Keynesian literature emphasizes the role of rule-of-thumb consumers (RT consumers henceforth) who do not participate to financial markets and therefore cannot save or borrow. Erceg, Guerrieri and Gust (2006) in their Sigma model calibrate the share of RT consumers at 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Gali et al. (2007) and Furlanetto and Seneca (2009) show that the RT consumers can rationalize the empirically observed response of aggregate consumption to public spending shocks. In Furlanetto and Seneca (2011) the RT hypothesis helps in accounting for recent empirical evidence on productivity shocks. Andres et al. (2008) show that nominal rigidities and RT consumers can rationalize the empirically observed negative correlation between government size and consumption volatility in OECD countries. In Boscà et al. (2009) the combination of RT consumers and consumption habits significantly improves the ability of an otherwise standard search model in reproducing some of the stylized facts characterizing the US labour market. Airaudo (2008) and De Graeve et al. (2010) exploit the RT consumers assumption to model asset prices.

Empirical research cannot reject the RT consumers hypothesis. Estimates of structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989). More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain values around 35%. The findings in Johnson et al. (2006), Shapiro and Slemrod (2009) and Parker et al. (2011) are also consistent with the RT assumption. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets (Bilbiie, Meier and Müller, 2008). In fact, increasing regulation in the aftermath of the 2008 crisis (OECD 2009) is likely to raise the share of liquidity-constrained households in the near future.

The RT consumers hypothesis has triggered a controversy about the properties that simple and implementable interest rate rules must fulfill in order to guarantee the uniqueness of the rational expectations equilibrium and to maximize the social welfare. Earlier contributions, based on the representative, optimizing agent framework emphasize the importance of satisfying the Taylor principle (Woodford, 2003; Schmitt-Grohé and Uribe 2004, 2007). By contrast Bilbiie (2008) shows that, in a world of flexible nominal wages, a low elasticity of labor supply combined with a sufficiently large share of constrained agents leads to an equilibrium where an interest rate policy based on the Taylor principle cannot ensure model determinacy. Colciago (2007) downplays this conclusion: he finds that even a mild degree of wage stickiness dampens the sensitivity
of RT consumption to shocks and restores the standard Taylor Principle even for a very large share of RT consumers. In addition, Ascari et al. (2010) show that the optimal monetary policy, i.e. the optimal inflation coefficient in the interest rate rule, is almost unaffected by the presence of RT consumers when nominal wages are sticky.

In this paper we show that - just like wage stickiness undermines the RT consumers effect outlined in Bilbiie - other frictions may weaken the sensitivity of optimizing agents’ consumption decisions to the interest rate rule. In fact, this happens when consumption habits enter the utility function. In modern New Keynesian business cycle models consumption habits are relied on to explain movements in aggregate consumption data and to generate the “hump-shaped” impulse responses widely recognized to characterize the responses of output and consumption to demand and supply shocks (see Dennis 2009 and references therein). In our model the combination of consumption habits and RT consumers has dramatic implications for model determinacy, resurrecting Bilbiie’s inverted Taylor principle for empirically plausible values of the RT consumers share. In addition, the central bank optimal reaction to inflation is far stronger than in Ascari et al. (2010).

Having confirmed that RT consumers potentially have important policy implications when the central bank is the sole policymaker, we then consider the impact of fiscal policies on the necessary conditions for equilibrium determinacy and on the features of the optimal monetary policy rule. Research on simple optimal monetary policy rules under limited asset market participation has ignored the simultaneous role played by fiscal policy. In models based on full asset market participation this may be justified by the findings in Schmitt-Grohé and Uribe (2007) who show that optimal fiscal rules should simply ensure debt solvency. In our context the limited asset market participation hypothesis paves the way for an additional role of fiscal policy, because tax reactions to temporary shocks may stabilize consumption decisions of RT households. The fiscal rules we consider are akin to the so-called automatic stabilizers, i.e. those elements of fiscal policy which react to the business cycle without requiring discretionary fiscal policy action. Automatic stabilizers characterize modern market economies and their working is typically associated to a reduction in the volatility of output and consumption (Fatas and Mihov, 2001, 2010; Dolls et al. 2010; Debrun et al. 2008; Debrun and Kapoor, 2010). Our contribution provides a theoretical background to the policy-oriented literature that sees automatic stabilizers as an important component of the future macroeconomic policy framework (Baunsgaard and Symansky, 2009, Blanchard et al. 2010).

We find that our fiscal rules have strong implications for dynamic stability: the Taylor principle is restored even for economies characterized by a relatively large amount of RT consumers. By contrast, the introduction of the fiscal rules does not change our former result that RT consumers require a much stronger sensitivity of the interest rate rule to inflation. Finally, all the fiscal rules considered here have a welfare-improving effect.
The remainder of the paper is organized as follows. In section 2 we describe the model. In section 3 we analyze determinacy and the robustness of the results under alternative calibrations. In section 4 we investigate the performance on simple optimal monetary and fiscal rules. Section 5 concludes.

2 The Model

We consider a cashless DSGE model where nominal rigidities are characterized by the Calvo formalism. Following Gaì et al (2004 and 2007), households are characterized by the same utility function, but we draw a distinction between the fraction $\theta$ of RT consumers and the $(1 - \theta)$ Ricardian agents who have unrestricted access to financial markets and . The key difference between the two groups concerns intertemporal consumption optimization, which is precluded to households who have no access to financial markets.

2.1 Households preferences

Preferences are defined as follows.

$$U^i_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left( C^i_t - bC^i_{t-1} \right) - \frac{\psi^i_t}{1 + \phi^i_t} (h^i_t)^{1 + \phi^i_t} \right\}$$

where $i : o, rt$ stands for the household type (Ricardian and RT consumers respectively), $\beta$ is the discount factor, $C^i_t$ represents total individual consumption, $b$ denotes internal habits and $h^i_t = \left( \int_0^1 \left( h^i_{t:j} \right)^{\omega - 1} \omega d\omega \right)^{\omega - 1}$ denotes individual supply of the labour bundle. $C^i_t$ is a standard consumption bundle

$$C^i_t = \left[ \int_0^1 \left( c^i_t (z) \right)^{\frac{\omega - 1}{\eta}} dz \right]^{\frac{\eta}{\omega - 1}} ; z \in [0, 1]$$

The aggregate consumption price index is

$$P_t = \left[ \int_0^1 p^i_t (z)^{1-\eta} dz \right]^{\frac{1}{1-\eta}}$$

and demand for good $z$ is

$$c^i_t(z) = C^i_t \left( \frac{P_t(z)}{P_t} \right)^{-\eta}$$

Finally, we define $\lambda^i_t$ as the marginal utility of consumption

$$\lambda^i_t = \frac{1}{C_t^i - bC_{t-1}^i} - \frac{\beta b}{E_i C_{i+1}^i - bC_t^i}$$
2.1.1 Ricardian Households

Ricardian households maximize (1) subject to the following period budget constraint:

\[ P_t C_t^o + E_t \Lambda_{t,t+1} (B_t + Q_{t+1}) = B_{t-1} + Q_t + P_t (D_t - S_t) + W_t h_t^o \]  \hspace{1cm} (4)

In each time period \( t \) Ricardian agents can purchase any desired state-contingent nominal payment \( Q_{t+1} \) in period \( t+1 \) at the dollar cost \( E_t \Lambda_{t,t+1} Q_{t+1} \). The variable \( \Lambda_{t,t+1} \) denotes the stochastic discount factor between period \( t \) and \( t+1 \). Real dividends are denoted by \( D_t \), while \( B_t \) is the quantity of nominally riskless bonds purchased in period \( t \) at price \( R_t^{-1} \) and paying one unit of the consumption numeraire at period \( t+1 \). \( P_t S_t \) represents nominal lump sum taxes.

The solution for the optimizing household problem is standard. The Euler equation is

\[ \lambda_t^o = \beta E_t \left( \lambda_{t+1}^o \frac{R_t}{\pi_{t+1}} \right) \]  \hspace{1cm} (5)

The stochastic discount factor is defined as \( E_t \Lambda_{t,t+1} = \beta E_t \frac{\lambda_{t+1}^o}{\pi_{t+1}} \frac{R_t}{\lambda_t^o} \) and absence of arbitrage profits in the asset markets implies that \( E_t \Lambda_{t,t+1} = R_t^{-1} \).

2.1.2 Rule-of-Thumb Households

As pointed out above, RT consumers neither save or borrow, in each period they entirely consume their labor income net of taxes.

\[ P_t C_t^{rt} = W_t h_t^{rt} - S_t \]  \hspace{1cm} (6)

2.2 Firms

Good \( z \) is produced in monopolistically competitive markets with the following technology:

\[ y_t (z) = a_t h_t (z) \]

Where

\[ \ln a_t = \rho_a \ln a_{t-1} + \varepsilon_t \]

defines a technology process. For any given level of its labor demand \( h_t (z) \), the optimal allocation of labor inputs implies

\[ h_t^* (z) = \left( \frac{W_t^2}{W_t} \right)^{-\alpha_w} h_t (z) \]  \hspace{1cm} (7)
where \( W_t = \left( \int_0^1 \left( W_t^j \right)^{1-\alpha_w} \, dj \right)^{1/(1-\alpha_w)} \) is the standard wage index. Firm \( z \)'s real marginal costs are:

\[
mc_t = (1 - \rho) \frac{w_t}{a_t}
\]  

where \( w_t = \frac{W_t}{P_t} \) is the real wage and \( \rho \) is a production subsidy.

\[\text{2.2.1 Sticky Prices}\]

In each period firm \( z \) faces a probability \( 1 - \lambda_p \) of being able to reoptimize its price. All the \( 1 - \lambda_p \) firms which reoptimize at time \( t \) will face symmetrical conditions and set the same \( \tilde{P}_t \), chosen to maximize a discounted sum of expected future profits:

\[
E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \Lambda_{t+s} \left( \tilde{P}_t - P_{t+s} mc_{t+s} \right) y_{t+s} (z)
\]

subject to:

\[
y_{t+s} (z) = Y_{t+s}^d \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\eta}
\]

where \( Y_{t+s}^d \) is aggregate demand.

\( \tilde{P}_t \) solves the following first order condition

\[
E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \Lambda_{t+s} (P_{t+s})^\eta Y_{t+s} \left[ \tilde{P}_t + \mu_p P_{t+s} mc_{t+s} \right] = 0
\]

where \( \mu_p = \frac{\eta}{(\eta-1)} \) defines the markup chosen under flexible prices.

\[\text{2.3 Labor market}\]

Each labor market \( j \) is monopolistically competitive and there is a union \( j \) which sets the nominal wage, \( W_t^j \), subject to (7). Each household \( i \) supplies all labour types at the given wage rates \(^1\) and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

\[
h_i^t = \int_0^1 h_t^{i,j} \, dj = \int_0^1 \left( \frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t \, dj
\]

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and

\(^1\)Under the assumption that wages always remain above all households’ marginal rate of substitution, households are willing to meet firms’ labour demand.
non-Ricardian households therefore work for the same amount of time, \( h_t \). We posit that the representative union objective function is a weighted average \((1 - \theta, \theta)\) of the utility functions of the two households types. This, in turn, implies that with flexible wages

\[
\hat{w}_t = \frac{W_t}{\hat{P}_t} = \mu_w \frac{\psi_t h_t^{\hat{\phi}_t}}{[(1 - \theta) \lambda_t^o + \theta \lambda_t^{rt}]} \tag{12}
\]

where \( \mu_w = \frac{\alpha_w}{(\alpha_w - 1)} \) represents the wage markup.

### 2.3.1 Sticky wages

In each period a union faces a constant probability \((1 - \lambda_w)\) of being able to reoptimize the nominal wage. Unions that cannot reoptimize simply set their wages equal to the one in the previous period.

Following Colciago (2006), the representative union objective function is defined as

\[
L^u = E_t \sum_{s=0}^{\infty} \left( \beta \lambda_w^s \right) \left\{ \left[(1 - \theta) \ln \left(C_{t+s}^o - bC_{t+s-1}^o\right) + \theta \ln \left(C_{t+s}^{rt} - bC_{t+s-1}^{rt}\right)\right] - \frac{\psi_t}{1 + \phi_t} (h_{t+s})^{1+\phi_t} \right\} \tag{13}
\]

The relevant constraints are (4), (6),(11).

The nominal wage \( \hat{W}_t \) solves the following first order condition

\[
\sum_{s=0}^{\infty} \left( \beta \lambda_w^s \right) \left[(1 - \theta) \lambda_t^{o} + \theta \lambda_t^{rt}\right] h_{t+s} (W_{t+s})^{\alpha_w} \left[ \frac{\hat{W}_t}{\hat{P}_{t+s}} - \frac{\psi_t h_t^{\hat{\phi}_t}}{[(1 - \theta) \lambda_t^{o} + \theta \lambda_t^{rt}]} \right] = 0
\]

### 2.4 Government sector

We make the assumption of an efficient steady state in order to study the welfare properties of the economy without resorting to a second-order approximation to the equations of the model. We therefore posit that the production subsidy \( \rho \) brings production at the competitive level. The real wage in the zero-inflation steady state equilibrium is

\[
\hat{w} = \frac{1}{(1 - \rho) \bar{MPL}} \mu_w \frac{\psi_t h_t^{\hat{\phi}_t}}{[(1 - \theta) \lambda_t^{o} + \theta \lambda_t^{rt}]} \]

Since \( \bar{MPL} = 1 \) must hold at the efficient steady state, the optimal subsidy is

\[
\rho^* = 1 - \frac{1}{\bar{\rho}_p \mu_w}
\]

\(^2\)It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.
Following Ascari et al. (2010) we assume that in each period the subsidy is entirely financed by lump-sum taxes, \(T\), levied on firms. This in turn implies that steady-state firms profits are \(D = Y - (1 - \tau) h \frac{W}{P} - T = 0\), and both consumption and the marginal rate of substitution are identical for the two consumer groups.

As pointed out in the introduction, the RT consumers assumption paves the way for fiscal stabilization policies. To keep complications at a minimum, we neglect government supply of public goods and assume that fiscal policy is based on a lump-sum tax, \(S_t\), which is levied on households. \(^3\) The government’s flow budget constraint is then given by

\[
R_t B_{t-1} - P_t S_t = B_t
\]  

(14)

The features of \(S_t\) are discussed in section (3.1) below. It suffices here to state that in good times the fiscal policymaker builds up a "rainy days" fund to be used in the face of adverse shocks. \(^4\)

### 2.5 Aggregate resource constraint

The aggregate resource constraint is

\[
C_t = \int_0^\theta C_t^o(j) \, dj + \int_1^1 C_t^p(j) \, dj = \theta C_t^o + (1 - \theta) C_t^o = Y_t
\]  

(15)

#### 2.6 The model in log-linear form

Hatted letters denote log deviations from the steady state.

Household \(i\) marginal utility of consumption

\[
\lambda_t^i = \frac{\beta b}{(1 - \beta b) (1 - b)} E_t \hat{c}_{t+1}^i - \frac{(1 + \beta b^2)}{(1 - \beta b) (1 - b)} \hat{c}_t^i + \frac{b}{(1 - \beta b) (1 - b)} \hat{c}_{t-1}^i
\]  

(16)

Euler equation

\[
\hat{c}_t^o = \left\{ \frac{b}{1+\beta+\beta b^2} \hat{c}_{t-1}^o - \frac{\beta b}{1+\beta+\beta b^2} E_t \hat{c}_{t+2}^o + \left( \frac{1+\beta b+\beta b^2}{1+\beta+\beta b^2} \right) E_t \hat{c}_{t+1}^o - \frac{(1-\beta b)(1-b)}{1+\beta+\beta b^2} \left( R_t - E_t \hat{r}_{t+1} \right) \right\}
\]  

(17)

RT consumption

\[
\hat{c}_t^r = \hat{w}_t + \hat{h}_t - \hat{s}_t
\]  

(18)

Aggregate consumption

\[
\hat{c}_t = (1 - \theta) \hat{c}_t^o + \theta \hat{c}_t^r
\]  

(19)

\(^3\)We have also controlled for supply side effects of the rule by modelling \(S\) as a labor income tax. Our conclusions are fully confirmed (results available upon request).

\(^4\)Therefore it would be straightforward to show that optimizing consumers would not react to cyclical variations of taxes.
Production function

\[ \dot{y}_t = \dot{h}_t + \dot{a}_t \]  

(20)

Aggregate resource constraint

\[ \dot{y} = \dot{c}_t \]  

(21)

Marginal costs

\[ \dot{m}c_t = \dot{w}_t - \dot{a}_t \]  

(22)

Phillips Curve

\[ \dot{\pi}_t = \frac{(1 - \lambda_p)(1 - \beta \lambda_p)}{\lambda_p} \dot{m}c_t + \beta E_t \dot{\pi}_{t+1} \]  

(23)

Wage-setting condition

\[
\begin{bmatrix}
\left(1 + \frac{\lambda_w^2}{1 - \lambda_w}ight) \dot{w}_t - \beta \lambda_w E_t \dot{w}_{t+1} - \beta \frac{\lambda_w}{1 - \lambda_w} E_t \dot{\pi}_{t+1} + \\
+ \frac{\lambda_w}{1 - \lambda_w} \dot{\pi}_t - \frac{\lambda_w}{1 - \lambda_w} \dot{w}_{t-1}
\end{bmatrix}
= \begin{bmatrix}
(1 - \beta \lambda_w) \varphi \dot{h}_t + \\
- (1 - \beta \lambda_w) \left[ \theta \dot{\lambda}_t + (1 - \theta) \dot{\lambda}_t^o \right]
\end{bmatrix}
\]  

(24)

Public debt dynamics

\[ \dot{b}_t = \frac{1}{\beta} \dot{b}_{t-1} - s_t \]

2.7 Policy rules

Monetary policy follows a standard Taylor Rule \(^5\)

\[ \hat{R}_t = \phi_n \hat{\pi}_t \]  

(25)

Following Schmitt-Grohe and Uribe (2007), we neglect cyclical adjustments of public consumption and define our fiscal rule as a feedback that reacts to aggregate nominal income growth:

\[ s_t = \tau_s \left( \dot{y}_t - \dot{y}_{t-1} + \frac{1}{Y^t} \dot{\pi}_t \right) + \tau^b \dot{b}_{t-1} \]  

(26)

\(^5\)We also experimented with several alternative specifications, such as a forward-looking rule, and a rule including a feedback on the output gap, but result were basically unaffected (results available upon request).
Condition (26)\textsuperscript{6} is consistent with empirical evidence suggesting that the primary balance in the OECD economies is more sensitive to output growth than to the output gap (Auerbach and Feenberg, 2000; Auerbach, 2009; Dolls et al., 2009, Fatas and Mihov 2010) and that the real progression of tax rates may be affected by inflation (Immervoll, 2003; Tanzi 1980). We shall also consider restricted versions of (26), where $\hat{s}$ reacts to either real income growth or to the output gap. To ensure stability of the debt accumulation process the fiscal rule includes a feedback on past debt accumulation.\textsuperscript{7}

2.8 Calibration

Parameters are calibrated following Christiano et al. (2005), the technology process is modeled as in Schmitt-Grohe, Uribe (2007).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.7</td>
<td>degree of habit persistence</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>subjective discount factor</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>0.6</td>
<td>price stickiness</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.64</td>
<td>wage stickiness</td>
</tr>
<tr>
<td>$\psi_l$</td>
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<td>preference parameter</td>
</tr>
<tr>
<td>$\phi_l$</td>
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<td>inverse of Frisch elasticity</td>
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<tr>
<td>$\mu_p$</td>
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<td>price mark-up</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>1.05</td>
<td>wage mark-up</td>
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<tr>
<td>$\rho_a$</td>
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<tr>
<td>$\sigma_a$</td>
<td>(0.0064)$^2$</td>
<td>shock std. deviation</td>
</tr>
</tbody>
</table>

3 Determinacy

Bilbiie (2008) has shown that, in a world of flexible nominal wages and sticky prices, a low elasticity of labor supply combined with a sufficiently large share of constrained agents leads to an equilibrium where an interest rate policy based on the Taylor principle cannot ensure model determinacy. The intuition behind this result is as follows. Suppose that firms form an arbitrary expectation of future price increases and therefore choose to raise the current price. The simultaneous (real) interest rate response induces a substitution effect in the consumption decisions of Ricardian households: $\hat{c}_t$ is such that $E_t \{ \Delta \hat{c}_{t+1} \} > 0$ (see (17) for $b = 0$). If all consumers were Ricardians, such a policy would allow a unique $\hat{c}_t < 0$ consistent with convergence to steady state, thus generating in $t$ a negative output gap sufficient to rule out the initial price increase as a possible equilibrium. By contrast, in this class of models Ricardian agents can react to the real interest rate surge by choosing $\hat{c}_t > 0$ because RT consumers induce a "Keynesian multiplier" effect that raises profits which

\textsuperscript{6}Both $\hat{s}$ and $\hat{b}$ are defined as percentages of steady-state output.

\textsuperscript{7}In our simulations coefficient $\psi^b$ takes the value 0.02.
are entirely appropriated by Ricardian agents. If this wealth effect is sufficiently strong, i.e. the share of RT consumers is sufficiently large, the choice of $\tilde{c}_t^o > 0$ such that $E_t \{ \Delta \tilde{c}_{t+1}^o \} > 0$, may be consistent with the rational expectation of future return to steady state. $\tilde{c}_t^o$ therefore confirms the increases in current and expected inflation. Colciago (2006) downplays this conclusion: he finds that a mild degree of wage stickiness is sufficient to dampen the Keynesian multiplier effect generated by RT consumers, restoring the standard Taylor Principle even for a very large share of RT consumers.

Without consumption habits, i.e. when we set $b = 0$, our model replicates this latter result even if fiscal policy is switched off. By contrast, under our calibration of $b$ determinacy requires an inversion of the Taylor principle when $\theta > 0.42$ when fiscal policy is inactive (Figure 1). The rationale for this result is straightforward: from (17) it is easy to see that consumption habits reduce the sensitivity of Ricardian consumers to real interest rate changes, weakening the substitution effect that is crucial to obtain determinacy under the Taylor principle.

![Figure 1: Determinacy region in the benchmark model.](image)

3.0.1 Sensitivity analysis

The threshold value $\theta^*$ that triggers an inversion of the Taylor principle is crucially affected by three key parameters, i.e. the degrees of price and nominal wage stickiness, respectively $\lambda_p$ and $\lambda_w$, and consumption habits $b$. Empirical DSGE models yield quite different estimates for these parameters. Fernandez-Villaverde et al. (2008) estimates a relatively large habit parameter ($b = 0.88$) and find that prices and nominal wages are re-optimized every 11 and 1.8 quarters respectively. With these parameter values a very low share of RT consumers ($\theta = 0.04$) is sufficient to require an inverted the Taylor principle. An almost identical result
obtains if we follow Guerron-Quintana (2010) who estimate an even stronger degree of habit formation \( b = 0.91 \), but find that prices and nominal wages are re-optimized every 5.5 and 2.6 quarters respectively. By contrast in Smets and Wouters (2007) \( b = 0.71 \) and prices and wages are reoptimized every 3 and 3.3 quarters respectively. In this case the inversion of the Taylor principle obtains at \( \theta = 0.5 \).

Given our benchmark calibration of \( b \), the beneficial effects of nominal wage stickiness emphasized in Colciago (2006) obtain only in extreme cases, i.e. when either prices are close to be fully flexible or when nominal wages contracts last at least one year (Figure 2). Taking as given our benchmark calibration for price stickiness, figure 3 shows that wage stickiness overturns the consumption habits effects when either \( b \) is far below existing estimates or when nominal wages are implausibly rigid.

\[ \begin{align*}
\lambda_w & \lambda \pi \\
0 & 0
\end{align*} \]

Figure 2

---

8 These estimated models allow for price and wage indexation to past inflation. Introducing inflation indexation in our model has no effect on the threshold for the value of \( \theta \) that causes inversion of the Taylor principle. Results available upon request.
3.1 The fiscal rule and the taylor principle

The fiscal rule has substantial implications for dynamic stability (Figure 4).

For instance, the Taylor principle holds irrespective of the size of $\theta$ when $\tau_s = 0.55$. The figure also shows that the fiscal policy effects would be weaker if, instead of reacting to nominal income growth, the tax feedback targeted either real output growth or the current output gap. To understand our results one should bear in mind that in this class of models the Taylor principle fails to achieve determinacy when it cannot prevent a self-fulfilling expectation in the growth of profits (see our discussion above). Thus fiscal rules should be
effective to the extent they indirectly stabilize profits by limiting the "Keynesian multiplier effect" of RT consumers. The effect of the fiscal rules is clearly shown in figure 5, where we plot the dynamics of profits in response to a productivity shock when $\theta = 0.4$ and $\theta = 0.45$.

![Figure 5: Impulse responses to a positive productivity shock.](image)

It remains to be explained why the fiscal rules have different effects on profits and, consequently, on determinacy. To grasp intuition, compare the working mechanism of a feedback on the current output gap with that of the rule targeting real output growth. As pointed out above, indeterminacy occurs when RT consumers induce on Ricardian consumers a wealth effect which dominates the substitution effect caused by the Taylor principle. Intuitively, the tax feedback on the current output gap stabilizes RT consumption and limits such wealth effect in each period. The rule controlling real output growth exploits a different mechanism, based on the complementarity with monetary policy. In fact the stronger $\phi_x$, the larger $E_t \{ \Delta \hat{c}^o_{t+1} \}$ and the expected tax on RT consumers in $t+1$. The tax, in turn, depresses the multiplier effect of RT consumers, bringing down output, profits and $E_t \{ \hat{c}^o_t \}$. As a result, $\hat{c}^o_t$ must fall with the fiscal feedback in order to satisfy the Euler equation (17). Fiscal reaction to output growth therefore strengthens the grip of the monetary policy rule on $\hat{c}^o_t$. A fortiori, the stronger complementarity with the monetary rule explains why the fiscal feedback on nominal income growth is more effective than the rule targeting real output growth.

---

9 In the simulations we set $\tau_x = 0.4$, $\phi_y = 1.5$. In the right-hand-side panel we do not plot the IRF for the case in which fiscal policy is switched off, because for $\theta > 0.42$ monetary policy alone is not sufficient to guarantee determinacy under the Taylor principle.
4 Optimal simple monetary and fiscal policy rules

We now turn to the analysis of the optimal simple policy rules, subject to the determinacy constraints of the model. The first step in our analysis is the identification of the solution to the social planner problem.

4.1 The social planner problem

The social planner problem can be characterized as:

$$\max_{c_t^o, c_t^r, h_t^o, h_t^r} \sum_{t=0}^{\infty} \beta^t E_t \left[ \theta \left( \log (C_t^r - bC_{t-1}^r) - \frac{\psi_t}{1+\phi_t} (h_t^r)^{1+\phi_t} \right) + (1-\theta) \left( \log (C_t^o - bC_{t-1}^o) - \frac{\psi_t}{1+\phi_t} (h_t^o)^{1+\phi_t} \right) \right]$$

subject to:

$$\theta C_t^r + (1-\theta) C_t^o = C_t$$

$$\theta h_t^r + (1-\theta) h_t^o = h_t$$

$$Y_t = C_t$$

$$Y_t = a_t h_t$$

By assumption, the two household groups have symmetrical preferences, but RT consumers have no access to financial markets. As a result, from the social planner perspective both consumption and worked hours should be identical for the two groups. In addition, the social planner faces an intertemporal problem due to internal habit formation, which affects socially optimal dynamics in response to shocks. It is easy to demonstrate that the log-linear solutions to the social planner problem are \((\hat{c}_t^r)^* = (\hat{c}_t^o)^* = (\hat{c}_t)^* = (\hat{y}_t)^*, (\hat{h}_t^r)^* = (\hat{h}_t^o)^* = (\hat{h}_t)^*, \hat{a}_t^* = \hat{a}_t, and$$

$$\left( \phi_t + \frac{1+\beta b^2}{1-\beta b (1-b)} \right) \hat{y}_t^* = \frac{\beta b}{1-\beta b (1-b)} \hat{y}_{t+1}^* + \frac{b}{1-\beta b (1-b)} \hat{y}_{t-1}^* + (\phi_t + 1) \hat{a}_t$$

Both technology and consumption habits drive output dynamics (Figure 6) \(^{10}\)

\(^{10}\)The "hump-shaped" response is due to the habit formation in households’ utility function.
Figure 6: Social Planner Response to a Technology Shock

4.2 The policymaker’s welfare function

Following Bilbiie (2008) and Ascari et al. (2010) the policymaker’s period objective function assigns weights \( \theta, (1 - \theta) \) to utilities of the two households groups:

\[
W_t = \left\{ (1 - \theta) \ln (C_t^o - bC_t^{o t}) + \theta \ln (C_t^r - bC_t^{r t}) \right\} - \frac{\psi t}{1 + \phi t} (h_t)^{1 + \phi t}
\]  

(27)

We derive the second order approximation to (27) around the efficient steady state, and then re-express it as deviations from the solutions to the social planner problem outlined above, obtaining the discounted value of the Central Bank loss function. \(^{11}\)

\[
L_t \approx -\frac{1}{2} \frac{(1 - \beta b)}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(1-b)}{(1-\beta b)} \left[ \theta \left( \dot{x}_{t+s} - \dot{x}_{t+s}^* \right)^2 + (1 - \theta) \left( \ddot{x}_{t+s} - \ddot{x}_{t+s}^* \right)^2 \right] + \right. \\
+ \phi \left( \dot{y}_{t+s} - \dot{y}_{t+s}^* \right)^2 + \frac{\alpha_w}{\kappa_w} \left( \ddot{x}_{t+s} \right)^2 + \frac{\eta}{\kappa_p} \left( \ddot{x}_{t+s} \right)^2 \right\} + t + O \left( ||\xi||^3 \right)
\]  

(28)

where \( \dot{x}_{t+s}^* = \frac{1}{1-b} \ddot{y}_{t+s}^* - \frac{b}{1-b} \ddot{y}_{t+s-1}^* \), \( \ddot{x}_{t+s}^* = \frac{1}{1-b} \ddot{c}_{t+s}^* - \frac{b}{1-b} \ddot{c}_{t+s-1}^* \), \( \kappa_p = \frac{(1-\lambda_p)(1-\beta \lambda_p)}{\lambda_p} \) and \( \kappa_w = \frac{(1-\lambda_w)(1-\beta \lambda_w)}{\lambda_w} \). Straightforward manipulations of (28) show that

\[
\theta \left( \dot{x}_{t+s} - \dot{x}_{t+s}^* \right)^2 + (1 - \theta) \left( \ddot{x}_{t+s} - \ddot{x}_{t+s}^* \right)^2 = \left( \ddot{x}_{t+s} - \ddot{x}_{t+s}^* \right)^2 + \theta \left[ \left( \ddot{x}_{t+s} - \ddot{x}_{t+s}^* \right)^2 + \left( \ddot{x}_{t+s} - \ddot{x}_{t+s}^* \right) \right]
\]  

(29)

\(^{11}\)Proof available in appendix.
The policymaker is therefore concerned with the differences in consumption utility, \( \hat{x}_{t+s}^r - \hat{x}_{t+s}^o \), between the two consumers groups. These are determined by firm profits and may arise only to the extent that marginal costs gaps are tolerated.

4.3 Optimal interest rate rule without fiscal stabilization

The optimization problem consists in deriving the strength of the policy parameter \( \phi_r \), which minimizes (28) subject to the behavior of households, wage setters and firms in response to a technology shock. In analogy with Schmitt-Grohe and Uribe (2004,2007), we restrict the admissible range of \( \phi_r \) values to the interval \([-10,10]\). \(^{12}\)

The optimal inflation coefficient \( \phi_r^* \) in (25) rapidly grows with \( \theta \) (Figure 7). \(^{13}\) For instance, \( \frac{\phi_r^*(\theta=20\%)}{\phi_r^*(\theta=0)} \approx 1.5, \frac{\phi_r^*(\theta=25\%)}{\phi_r^*(\theta=0)} \approx 2.4 \). To support intuition note that when wages are flexible and consumption frictions are absent, monetary policy is very aggressive, \( \phi_r^* (\theta = b = \lambda_w = 0) = 10 \). In the case of sticky wages and no consumption frictions, the strength of the inflation coefficient in (25) falls dramatically, \( \phi_r^* (\theta = b = 0) = 2.4 \).

This happens because optimal policy now tolerates inflation in order to limit wage adjustment costs (as in Erceg et al., 2006). At this stage, introducing RT consumers has a minor impact on the optimal rule: \( \phi_r^* (0 < \theta < 0.42, b = 0) = 2.8 \). Similarly, the policy rule does not change much when only habit frictions are introduced but RT consumers are absent, \( \phi_r^* (\theta = 0, b = 0.7) = 3.9 \). \(^{14}\) Thus, the combination of consumption habits and RT consumers is necessary to obtain \( \phi_r^* \). From (29) we know that the policymaker is concerned with consumption utility gaps. Without habits, differences in consumption utility among the two groups are relatively unimportant. With habits, incomplete stabilization of profit gaps generates stronger and more persistent differences in consumption utilities. As a result, wage adjustment costs become relatively less important and the anti-inflation policy becomes more aggressive. In addition, from (5), (3), (15) it is clear that the combination of RT consumers and consumption habits strongly reduces the aggregate demand elasticity to interest rate changes.

\(^{12}\)This choice is justified by the idea that rules characterized by stronger interest rate reaction to changes in inflation are difficult to communicate to the public and therefore unlikely to be implemented in practice (Schmitt-Grohe and Uribe, 2007).

\(^{13}\)As soon as \( \theta \) reaches the 0.42 threshold, coefficient \( \phi_r^* \) switches to the lower bound.

\(^{14}\)The increase in \( \phi_r^* \) from 2.4 to 3.9 is caused by the habit-induced reduction in the sensitivity of consumption to the real interest rate (see (17) above).
Figures 8a and 8b display the impulse response functions (IRFs henceforth) to a positive technology shock, where variables are respectively defined as deviations from steady state and gaps from the social planner’s efficient response. Consider IRFs under the optimal policy rules that obtain when $\theta = 0$ ($\phi^*_\pi = 3.9$, blue lines) and $\theta = 0.3$ ($\phi^*_\pi = 10$, green dotted lines). In both cases output increases but worked hours fall on impact, in line with the findings of Galli (1999), Basu et al. (2006), Canova et al (2010). The different strength of coefficient $\phi_{\pi}$ allows to obtain similar responses for inflation and the real interest rate. In line with the findings in Basu et al. (2006), under both policy experiments the real wage increase is driven by the inflation fall, whereas the nominal wage adjustment is almost nil. The more tenuous response of aggregate demand to the real interest rate fall explains why the increase in output and worked hours is much weaker in presence of RT consumers. Monetary policy cannot avoid substantial consumption differences between the two consumer groups when $\theta = 0.3$. Optimizing consumers’ demand for goods increases because expected future consumption grows and because the real interest rate falls. By contrast, RT consumers demand is constrained by current labor income. However, for both groups actual consumption is equal to current income because there is no capital in the model. The surge in profits therefore explains why optimizing households can increase their consumption.

When looking at differences relative to the social planner’s response (figure 8b) we find that monetary policy can close gaps in total consumption (output) and worked hours when all agents optimize. By contrast, negative gaps in output and worked hours cannot be avoided when $\theta = 0.3$. 

![Figure 7](image-url)
4.4 Optimal fiscal and monetary rules

In figure 9a we plot the IRFs under different the fiscal rules. Coefficients $\phi_s$ and $\tau_s$ are chosen to minimize (28) conditional to the fact that the fiscal rule alternatively controls nominal income growth, real income growth and the current output gap (Rules 1,2,3 respectively in Table 2). All the fiscal rules reduce welfare
Table 2:

<table>
<thead>
<tr>
<th>Rule</th>
<th>$\tau_s^*$</th>
<th>$\phi_\pi^*$</th>
<th>$G^*$</th>
<th>$\tau_s^*$</th>
<th>$\phi_\pi^*$</th>
<th>$G^*$</th>
<th>$\tau_s^*$</th>
<th>$\phi_\pi^*$</th>
<th>$G^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1</td>
<td>0.45</td>
<td>5.7</td>
<td>0.1618</td>
<td>0.57</td>
<td>6</td>
<td>0.4</td>
<td>0.74</td>
<td>6.7</td>
<td>0.9743</td>
</tr>
<tr>
<td>Rule 2</td>
<td>0.99</td>
<td>10</td>
<td>0.0956</td>
<td>0.99</td>
<td>10</td>
<td>0.205</td>
<td>0.99</td>
<td>10</td>
<td>0.9394</td>
</tr>
<tr>
<td>Rule 3</td>
<td>0.99</td>
<td>8.8</td>
<td>0.1029</td>
<td>0.99</td>
<td>10</td>
<td>0.295</td>
<td>0.99</td>
<td>10</td>
<td>0.9454</td>
</tr>
</tbody>
</table>

$G^*$ represents the percentage welfare gain with respect to the case of no fiscal stabilization.

losses relative to the benchmark regime where only monetary policy is activated.\textsuperscript{15} It is interesting to note that controlling nominal income growth requires less activist monetary and fiscal policies. The IRFs describing the dynamic patterns of taxes help to understand the performance of the economy under the different fiscal rules. Only control of nominal income growth allows to obtain the tax reduction necessary to raise consumption of RT agents whereas the other two rules cannot avoid an increase in taxation (Figure 9a). In this regard, the rule based on a feedback on the current output gap is particularly penalizing for RT consumers. In figure 9b we plot the IRFs of deviations from the social planner’s response. It is easy to see that, relative to a situation where fiscal policy is switched off, all the fiscal rules dampen the volatility of gaps in worked hours, output and aggregate consumption. However, the reduction in aggregate consumption volatility is due to a composition effect unless the rule targets nominal income growth. This latter rule generates a reduction in gaps volatility for all the variables that enter the objective function (28).

\textsuperscript{15} We take $\theta = 0.4$ as an upper bound because beyond this level an inversion of the Taylor principle is necessary at for the benchmark case.
5 Conclusion

The key message of the paper is simple. Limited asset market participation has potentially strong policy implications when the central bank is the sole policymaker, but a well-crafted system of automatic fiscal stabilizers dampens the undesirable effects of limited asset market participation. Proper design of a fiscal feedback on nominal income growth brings the optimal interest rate rule relatively close to the one obtained when all agents have full access to financial markets.

The paper has normative implications for the design of automatic stabilizers. An important result concerns the desirability of linking taxes to inflation and to income growth in order to restore the Taylor principle and to stabilize consumption gaps. This conclusion indirectly supports Auerbach’s (2009) statement that the US policy of introducing full inflation indexation of the individual income tax - as well as the reduction in marginal tax rates - is likely to have complicated the task of monetary policy in the aftermath of the 2008 financial crisis, eventually forcing the government to implement a strongly discretionary fiscal action. Given the widespread consensus about the inefficient use of discretionary fiscal policies in a large number of countries (Alesina et al., 2008), it might be sound to preserve the role of automatic stabilizers.

Our results also sound a note of caution, suggesting that fiscal rules should be carefully designed. Further research should therefore investigate the optimal design of automatic stabilizers in medium-scale models, accounting for both a richer set of tax instruments and countercyclical public consumption expenditure.
References


Appendix A

Welfare-based Loss Function

The derivation of (28) strictly follows Ascari et al. (2010) and Leith et al. (2009).

Given the efficiency of the steady state in our model, we have that:

\[
\frac{U_i^h}{U_i^C} = \frac{U_o^h}{U_o^C} = \frac{Y}{h} = 1
\]

where \(U_i^h\) and \(U_i^C\) are the steady state derivatives of the agent \(i\) utility function with respect to hours, \(h\) or consumption, \(C\) respectively.

As in Bilbiie (2008) we assume that the Central Bank maximizes a convex combination of the utilities of two types of households, weighted by their mass in the economy, i.e.:

\[
W_t = \theta \left[ U \left( X_i^{rt} \right) - U \left( h_i^{rt} \right) \right] + (1 - \theta) \left[ U \left( X_o^o \right) - U \left( h_o^o \right) \right]
\]

where

\[
X_i^t = C_i^t - bC_{i-1}^t
\]

We start by assuming a generic CRRA utility for agent \(i\) at period \(t\) function as

\[
U_i^t = \frac{(X_i^t)^{1-\sigma}}{1-\sigma} - \frac{\psi_t}{1+\phi_t} (h_i^t)^{1+\phi_t}
\]

since \(h_i^o = h_i^{rt} = h_t\), because of the unionized labour market,

\[
W_t = \theta U \left( X_i^{rt} \right) + (1 - \theta) U \left( X_o^o \right) - U \left( h_t \right) = U \left( X_t \right) - U \left( h_t \right)
\]

A second order approximation of \(\theta U \left( X_i^{rt} \right)\) around the efficient steady state delivers

\[
\theta U \left( X_i^{rt} \right) \simeq \theta U_{X_i} X_i^{rt} \left( \hat{x}_i^{rt} + \frac{1-\sigma}{2} (\hat{x}_i^{rt})^2 \right)
\]

and for Ricardians

\[
(1 - \theta) U \left( X_o^o \right) \simeq (1 - \theta) U_{X_o} X_o^o \left( \hat{x}_o^o + \frac{1-\sigma}{2} (\hat{x}_o^o)^2 \right)
\]
where $U_X^i$ is the derivative of agent $i$ steady state utility function with respect to $X^i$.

Log-linear approximation of $X_t^i$ yields

$$\hat{X}_t^i = \frac{1}{1-b} \hat{c}_t^i - \frac{b}{1-b} \hat{c}_{t-1}^i = \frac{1}{1-b} (\hat{c}_t^i - b\hat{c}_{t-1}^i)$$

and therefore

$$(\hat{X}_t^i)^2 = \frac{1}{(1-b)^2} (\hat{c}_t^i - b\hat{c}_{t-1}^i)^2$$

$X_t^i$ can be approximated to second order by

$$\frac{X_t^i - X^i}{X^i} = \hat{X}_t^i + \frac{1}{2} (\hat{X}_t^i)^2$$

so that

$$\hat{X}_t^i + \frac{1}{2} (\hat{X}_t^i)^2 = \frac{1}{1-b} \left( \hat{c}_t^i + \frac{1}{2} (\hat{c}_t^i)^2 \right) - \frac{b}{1-b} \left( \hat{c}_{t-1}^i + \frac{1}{2} (\hat{c}_{t-1}^i)^2 \right)$$

and

$$\hat{X}_t^i = \frac{1}{1-b} \left( \hat{c}_t^i + \frac{1}{2} (\hat{c}_t^i)^2 \right) - \frac{b}{1-b} \left( \hat{c}_{t-1}^i + \frac{1}{2} (\hat{c}_{t-1}^i)^2 \right) - \frac{1}{2} (\hat{X}_t^i)^2$$

The following equations

$$(1 - \theta) \left[ U(X_t^o) - U(X^o) \right] \simeq (1 - \theta) U_X^o X^o \left( \hat{c}_t^o + \frac{1 - \sigma}{2} (\hat{c}_t^o)^2 \right)$$

$$\theta \left[ U(X_t^r) - U(X^r) \right] \simeq \theta U_X^r X^r \left( \hat{c}_t^r + \frac{1 - \sigma}{2} (\hat{c}_t^r)^2 \right)$$

become

$$(1 - \theta) \left[ U(X_t^o) - U(X^o) \right] \simeq (1 - \theta) U_X^o X^o \left( \frac{1}{1-b} \left( \hat{c}_t^o + \frac{1}{2} (\hat{c}_t^o)^2 \right) - \frac{b}{1-b} \left( \hat{c}_{t-1}^o + \frac{1}{2} (\hat{c}_{t-1}^o)^2 \right) - \frac{1}{2} (\hat{X}_t^o)^2 \right)$$

$$\theta \left[ U(X_t^r) - U(X^r) \right] \simeq \theta U_X^r X^r \left( \frac{1}{1-b} \left( \hat{c}_t^r + \frac{1}{2} (\hat{c}_t^r)^2 \right) - \frac{b}{1-b} \left( \hat{c}_{t-1}^r + \frac{1}{2} (\hat{c}_{t-1}^r)^2 \right) - \frac{1}{2} (\hat{X}_t^r)^2 \right)$$

Approximation of $U(h_t) = \frac{\psi_t}{1+\psi_t} (h_t)^{1+\phi_t}$ delivers

$$U(h_t) - U(h) = U_h h \left( h_t + \frac{1+\phi_t}{2} \hat{h}_t^2 \right)$$

Summing all the terms, given that steady state consumption and hours worked level are identical for the
two groups of agents, \( U_t^X X^{\tau t} = U_t^X X^o = U_X X = U h = U_C C \)

\[
W_t - W = \left[ (1 - \theta) U_X X \left( \frac{1}{1-b} \left( c_t^\theta + \frac{1}{2} (c_t^\theta)^2 \right) - \frac{b}{1-b} \left( c_{t-1}^\theta + \frac{1}{2} (c_{t-1}^\theta)^2 \right) - \frac{\sigma}{2} (x_t^\theta)^2 \right) + \\
+ \theta U_X X \left( \frac{1}{1-b} \left( c_t^{\tau t} + \frac{1}{2} (c_t^{\tau t})^2 \right) - \frac{b}{1-b} \left( c_{t-1}^{\tau t} + \frac{1}{2} (c_{t-1}^{\tau t})^2 \right) - \frac{\sigma}{2} (x_t^{\tau t})^2 \right) - U h (\hat{h}_t + \frac{1+\phi_1}{2} \hat{d}_t^2) \right]
\]

From the economy production function we know that

\[
\hat{h}_t = \hat{y}_t + \hat{d}_{w,t} + \hat{d}_{p,t} - \hat{a}_t
\]

where \( \hat{d}_{w,t} = \log \int_0^1 \left( \frac{W_j^t}{W_j^{\tau t}} \right)^{-\alpha_w} d j \), i.e. the log of the wage dispersion and \( \hat{d}_{p,t} = \log \int_0^1 \left( \frac{P_i^t}{P_i^{\tau t}} \right)^{-\eta} d i \), i.e. the log of the price dispersion are of second order and therefore cannot be neglected in our approximation. Since

\[
\hat{h}_t^2 = (\hat{y}_t + \hat{d}_{w,t} + \hat{d}_{p,t} - \hat{a}_t)^2 = \hat{y}_t^2 + \hat{a}_t^2 - 2\hat{y}_t\hat{a}_t
\]

thus

\[
W_t - W = (1 - \theta) U_X X \left( \frac{1}{1-b} \left( c_t^\theta + \frac{1}{2} (c_t^\theta)^2 \right) - \frac{b}{1-b} \left( c_{t-1}^\theta + \frac{1}{2} (c_{t-1}^\theta)^2 \right) - \frac{\sigma}{2} (x_t^\theta)^2 \right) + \\
+ \theta U_X X \left( \frac{1}{1-b} \left( c_t^{\tau t} + \frac{1}{2} (c_t^{\tau t})^2 \right) - \frac{b}{1-b} \left( c_{t-1}^{\tau t} + \frac{1}{2} (c_{t-1}^{\tau t})^2 \right) - \frac{\sigma}{2} (x_t^{\tau t})^2 \right) + \\
- U h (\hat{y}_t + \hat{d}_{w,t} + \hat{d}_{p,t} - \hat{a}_t + \frac{1+\phi_1}{2} (\hat{y}_t^2 + \hat{a}_t^2 - 2\hat{y}_t\hat{a}_t) + \text{tip.} + O (||\xi||^3)
\]

or, summing over the future

\[
W_t - W = U_X X \theta \sum_{s=0}^{\infty} \beta^s \left[ \left( \frac{1 - \beta b}{1-b} \left( c_{t+s}^\theta + \frac{1}{2} (c_{t+s}^\theta)^2 \right) - \frac{\sigma}{2} (x_{t+s}^\theta)^2 \right) \right] + \\
+ U_X X (1 - \theta) \sum_{s=0}^{\infty} \beta^s \left[ \left( \frac{1 - \beta b}{1-b} \left( c_{t+s}^{\tau t} + \frac{1}{2} (c_{t+s}^{\tau t})^2 \right) - \frac{\sigma}{2} (x_{t+s}^{\tau t})^2 \right) \right] + \\
- U h \sum_{s=0}^{\infty} \beta^s \left[ (\hat{y}_{t+s} + \hat{d}_{w,t+s} + \hat{d}_{p,t+s} + \hat{a}_t + \frac{1+\phi_1}{2} \hat{y}_{t+s}^2 - (1 + \phi) \hat{y}_{t+s} \hat{a}_{t+s}) \right] + \text{tip.} + O (||\xi||^3)
\]

After tedious but straightforward manipulation we obtain

\[
\frac{W_t - W}{U_C C} = - \frac{(1 - \beta b)}{(1-b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{\phi}{2} \hat{y}_{t+s}^2 + \theta \frac{(1 - b)}{1 - \beta b} \frac{\sigma}{2} (x_{t+s}^\theta)^2 + (1 - \theta) \frac{(1 - b)}{1 - \beta b} \frac{\sigma}{2} (x_{t+s}^{\tau t})^2 + \\
+ (\hat{d}_{w,t+s} + \hat{d}_{p,t+s}) - (1 + \phi) \hat{y}_{t+s} \hat{a}_{t+s} \right] + \text{tip.} + O (||\xi||^3)
\]
-Following Woodford (2003) and recalling that in our model utility is logarithmic in consumption, i.e. \( \sigma = 1 \)

\[
\frac{W_t - W}{U_C} = - \frac{(1 - \beta) b}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{(1-b)}{(1-\beta b)} \left( \theta \left( \hat{x}_{t+s}^r \right)^2 + (1 - \theta) \left( \hat{x}_{t+s}^p \right)^2 \right) + \frac{\alpha_w}{\alpha_w} \left( \hat{\pi}_{t+s}^w \right)^2 + \frac{\alpha_p}{\alpha_p} \left( \hat{\pi}_{t+s}^p \right)^2 + \frac{\alpha_l}{\alpha_l} \left( \hat{y}_{t+s} - \frac{(1+\beta_2)}{\phi_l} \hat{\pi}_{t+s} \right)^2 + \text{tip} + O \left( ||\xi||^3 \right) \right] + O \left( ||\xi||^3 \right)
\]

where \( \alpha_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p} \) and \( \alpha_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w} \).

From the social planner problem we have that:

\[
(\phi_t + 1) \hat{\alpha}_t = \frac{1}{(1 - \beta b)} \hat{\alpha}_t \hat{\lambda}_t - \frac{\beta b}{(1 - \beta b)} \hat{\alpha}_{t+1} \phi_t \hat{y}_t^* + \text{tip} + O \left( ||\xi||^3 \right)
\]

therefore

\[
\frac{W_t - W}{U_C} = - \frac{(1 - \beta) b}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{(1-b)}{(1-\beta b)} \left( \theta \left( \hat{x}_{t+s}^r \right)^2 + (1 - \theta) \left( \hat{x}_{t+s}^p \right)^2 \right) + \frac{\alpha_w}{\alpha_w} \left( \hat{\pi}_{t+s}^w \right)^2 + \frac{\alpha_p}{\alpha_p} \left( \hat{\pi}_{t+s}^p \right)^2 + \frac{\alpha_l}{\alpha_l} \left( \hat{y}_{t+s} - \frac{(1+\beta_2)}{\phi_l} \hat{\pi}_{t+s} \right)^2 + \text{tip} + O \left( ||\xi||^3 \right) \right] + \text{tip} + O \left( ||\xi||^3 \right)
\]

Further manipulating yields

\[
\frac{W_t - W}{U_C} = - \frac{(1 - \beta) b}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{(1-b)}{(1-\beta b)} \left( \theta \left( \hat{x}_{t+s}^r \right)^2 + (1 - \theta) \left( \hat{x}_{t+s}^p \right)^2 \right) + \frac{\alpha_w}{\alpha_w} \left( \hat{\pi}_{t+s}^w \right)^2 + \frac{\alpha_p}{\alpha_p} \left( \hat{\pi}_{t+s}^p \right)^2 + \frac{\alpha_l}{\alpha_l} \left( \hat{y}_{t+s} - \frac{(1+\beta_2)}{\phi_l} \hat{\pi}_{t+s} \right)^2 + \text{tip} + O \left( ||\xi||^3 \right) \right] + \text{tip} + O \left( ||\xi||^3 \right)
\]

Considering that

\[
\sum_{s=0}^{\infty} \beta^s \frac{1}{1 - \beta b} \hat{y}_{t+s} (\hat{x}_{t+s}^r - \frac{\beta b \hat{x}_{t+s+1}^r}{1 - \beta b}) = \sum_{s=0}^{\infty} \beta^s \frac{1}{1 - \beta b} (\hat{y}_{t+s} - \frac{\beta b \hat{y}_{t+s-1}}{1 - \beta b}) (\hat{x}_{t+s}^r) = \sum_{s=0}^{\infty} \beta^s \frac{(1 - b)}{1 - \beta b} (\hat{x}_{t+s}^r) \hat{x}_{t+s}^r
\]

and that \( \hat{x}_{t+s}^r = (\hat{x}_{t+s}^r)^* = (\hat{x}_{t+s}^o)^* \) the welfare function becomes

\[
\frac{W_t - W}{U_C} = - \frac{(1 - \beta) b}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{(1-b)}{(1-\beta b)} \left( \theta \left( \hat{x}_{t+s}^r \right)^2 + (1 - \theta) \left( \hat{x}_{t+s}^p \right)^2 \right) + \frac{\alpha_w}{\alpha_w} \left( \hat{\pi}_{t+s}^w \right)^2 + \frac{\alpha_p}{\alpha_p} \left( \hat{\pi}_{t+s}^p \right)^2 + \frac{\alpha_l}{\alpha_l} \left( \hat{y}_{t+s} - \frac{(1+\beta_2)}{\phi_l} \hat{\pi}_{t+s} \right)^2 + \text{tip} + O \left( ||\xi||^3 \right) \right] + \text{tip} + O \left( ||\xi||^3 \right)
\]
which can be easily rewritten as (28)

\[
L \approx - \frac{1}{2} \frac{(1 - \beta b)}{(1 - b)} \sum_{s=0}^{\infty} \beta^s \left[ \frac{(1 - \beta b)}{(1 - b)} \left[ \theta \left( \hat{x}_{t+s}^e - \hat{x}^*_{t+s} \right)^2 + (1 - \theta) \left( \hat{x}_{t+s}^o - \hat{x}^*_{t+s} \right)^2 \right] + \phi_1 \left( \hat{y}_{t+s} - \hat{y}^*_{t+s} \right)^2 + \frac{n_w}{\alpha_w} \left( \hat{\pi}_{t+s}^w \right)^2 + \frac{n_p}{\alpha_p} \left( \hat{\pi}_{t+s} \right)^2 \right] + \text{tip} + O \left( ||\xi||^3 \right)
\]