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Do Campaign Finance Policies Really Improve Voters' Welfare?*

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Abstract

In an electoral race, interest groups will be willing to finance political candidates' campaigns in return for favors that are costly to voters. Starting from the empirical observation of split contributions, we develop a theoretical model of directly informative campaign advertising with rational voters. In this setting, interest groups that demand more favors are less likely to finance candidates to enhance their electoral prospects. We find that the only feasible Pareto improving policy involves providing specific limits and subsidies to each candidate. Unfortunately, this policy is very demanding in terms of information for the policy maker and always involves candidates providing favors to interest groups. We argue that bans on contributions without public subsidies may not be welfare improving, since they negatively affect the informational value of advertisements.

Keywords: Campaign Finance, Interest Groups, Elections, Welfare

JEL Classification: D72, H40

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1 Introduction

There is no reason to cast doubts on the importance of money in elections. In the 2008 U.S. presidential race, under the limits to contributions of the McCain-Feingold Act of 2002, Barack Obama raised 745 million dollars, John McCain raised 370 and overall more than 1 billion dollars were spent on campaigns. The McCain-Feingold Act, on the one hand, limited contributions from individuals to candidates and political committees, and on the other hand banned direct contributions from corporations, unions and firms to candidates. In January 2010 the U.S. Supreme Court decided that the law regulating political campaign finance was in violation of corporations' and unions' First Amendment rights, abolishing all limits to contributions. This recent ruling has not been regarded favorably by public opinion and the issue remains highly debated.[1]

We propose a framework that aims to shed light on the welfare economics of campaign finance policy. More specifically, we analyze the contribution strategies of interest groups and the effects of such contributions on the electoral race and voters' welfare from a theoretical point of view. The empirical evidence shows that interest groups typically contribute to more than one candidate in the same electoral race financing both friendly and non-friendly politicians (see, for example, Austen-Smith and Wright 1994 and Hojnacki and Kimball 1998).[2] More recent papers have also highlighted that, in order to reach its own goal, the issue sponsored by the interest group needs to be non-partisan and non-ideological, thus finding evidence against the idea that contributions are driven by ideology (see, for example, Hall and Deardorff 2006). Based on the empirical evidence, we consider a pairwise electoral competition under majority rule where a non-ideological interest group may finance both candidates' political campaigns. Assuming that the interest group is non-ideological allows for our model to incorporate split contributions, since if the interest group were ideologically oriented, it would typically contribute only to the candidate closer to its own ideology.

We assume that voters are rational and derive positive utility from electing qualified politicians, but are also concerned about the ideology of candidates. Candidates differ in ideology and run political campaigns in order to inform the electorate of their ability. Since candidates have no funds of their own, campaign advertisements are entirely financed by contributions from an interest group. These ads convey hard information that voters use to update their beliefs on the ability of candidates, before casting their vote. We assume that

[1] A poll by Washington Post-ABC News in February 2010 (one month after the U.S. Supreme Court decision) showed that 80% of Americans and in particular 85% of democrats, 76% of republicans and 81% of independents were opposed to the Supreme Court's ruling.

[2] As documented the the non-partisan organization OpenSecrets (www.opensecrets.org), splitting contributions seems to be the rule rather than the exception. Considering contributions to politicians by sector and focusing on the top contributors by sector, the U.S. data suggests that contributions are typically between 40/60 and 60/40 to each party. The sector with the highest total contributions is that related to investment banks and financial firms, where only two out of the top ten contributors gave more that 60% to one party from 1990 to 2010.

contributions are provided in exchange for favors that candidates commit to provide (to the interest group) if elected. The cost of favors is paid by the electorate after the elections take place, based on the favors promised by the winning candidate. Electoral campaigns are assumed to be directly informative in the sense that candidates can only advertise their own characteristics (their qualification) and ads have to be truth-telling, hence only qualified candidates can raise funds and advertise.[3] Throughout the analysis we consider a model with a single interest group. In the discussion we show how the results can be extended to a common agency framework (Bernheim and Whinston 1986).

The effectiveness of advertisements measures the impact that campaigns have in inducing voters to switch their vote in favor of the candidate whose add they observed. This effectiveness depends on the amount of favors that each candidate that receives funding promises to provide to the interest group, in return for contributions. A feature of our model is that advertising must always be effective in order for qualified candidates to be weakly better off from accepting the interest group's offer. In other words, the benefit for voters of electing a qualified candidate that advertises is always greater than the cost in terms of favors that the candidate is expected to concede.

The interest group finances candidates in order to obtain favors in exchange (influence motives) but contributions can also affect the electoral outcome (electoral motives). We find that there is a trade-off between the two motives for contributing: in particular, the electoral motive tends to be less relevant when an interest group requires higher levels of favors. Moreover, electoral motives appear to be relatively weak confirming the empirical findings that cast doubts on the possibility of political campaigns to significantly affect electoral outcomes (see, for example, Levitt 1994).[4]

The intuition for this result comes from the following observation. A marginal increase in campaign advertising has both a positive and a negative effect on the electoral probability of a qualified candidate. More specifically, for a given level of favors an increase in contributions augments the effectiveness of *observing* an advertisement and reduces the effectiveness of *not observing* an ad. The positive effect originates from the fact that spending more on campaigns allows a greater share of voters to be reached by ads, and therefore to infer that a candidate is qualified. The negative effect is less apparent, and is based on the fact that if a rational voter does not observe an ad from a given candidate, this is a more informative signal on the candidate's lack of ability, the greater is the equilibrium level of contributions that he receives. The magnitude of the positive effect is always larger than the negative

[3] In our paper "qualification" could measure any valence characteristic of the candidate: managerial competence, policy, creativity, track record but also image, look and charisma. The significance of the valence characteristic for political candidates has already been extensively discussed in the literature (see, for example, Aragones and Palfrey 2002).

[4] The results of more recent empirical papers like Stratmann (2009) and Gerber (1998) are in line with those of Levitt (1994). Evans (2007) finds empirical evidence of an inverse proportionality between total contributions and electoral effects, confirming our theoretical finding.

one, which implies that spending more on campaigns always enhances the electoral odds of a qualified candidate. However, when the advertising technology is concave, (i.e. when the additional share of voters that is reached by an increment in adds is decreasing), the negative effect tends to dampen the positive one. The marginal effect of contributions is in fact decreasing and tends to zero. Interest groups that demand more favors will necessarily need to provide more contributions just to induce politicians to accept without increasing their electoral odds. For a "favor hungry" group, any additional funding beyond what is strictly necessary to obtain the desired favors has a smaller impact on the electoral outcome, with respect to that of a group that demands less favors.

We also investigate the effects of campaign finance policies on voters' welfare. Any limit or ban on contributions has the positive effect of reducing the level of favors that a given candidate will promise to special interests in equilibrium. However, when contributions are limited voters rationally expect qualified candidates to spend less on advertising. Therefore, the event of not observing an add is a less informative signal on a candidate's lack of ability, with respect to the setting in which contributions are unrestricted. Introducing limits to contributions thus may have a negative effect on the expected utility of voters, since it reduces the informational conveyed by advertisements, increasing the chances of electing a bad candidate when no advertisements are observed.

First, we focus on policies that involve limits and corresponding public subsidies. Public funding serves the purpose of increasing the effectiveness of campaigns by substituting tainted private funds with clean public financing, and is financed by taxes. Although it is possible to design a Pareto improving policy, it may be difficult to implement because very demanding in terms of information requirements. It requires the policy maker to fine tune the limits and corresponding subsidies to each candidate, on the basis of the specific unrestricted equilibrium. More specifically, the policy must guarantee that the effectiveness of both observing and not observing advertisements remains unchanged, in order to leave the electoral probabilities unvaried. A policy that alters the electoral probabilities would in fact increase the utility of one partisan group of voters at the expense of reducing that of its counterpart.

Moreover, a Pareto improving policy always imposes a strictly positive cost on voters in terms of both monetary favors and taxes. On the one hand, Pareto improving policies necessarily require public subsidies, since pure limit policies always alter the effectiveness of advertisements. On the other hand however, public money can never completely substitute private contributions when a given candidate has an electoral advantage. This occurs because the interest group will always find it optimal to offer greater contributions (and request more favors) to a qualified candidate with an electoral advantage, since he has greater chances of winning. A policy that symmetrically drives favors to zero for both candidates would therefore enhance the electoral probability of the advantaged candidate, by increasing the effectiveness of his advertisements. Therefore, a positive level of favors must always be tolerated, in order to dampen the effectiveness of the advantaged candidate's ads, allowing for the welfare of all voters to (weakly) increase, including those that share the ideology of the less advantaged candidate.

We also focus on policies that simply ban contributions for at least one candidate. Such policies are less demanding in informational terms for the policy maker; that is, they are the only ones available when the policy maker does not have information about the interest group's preferences. Bans on contributions eliminate favors but also suppress the informational value of advertising, altering the electoral probability of candidates with respect to the unrestricted political equilibrium. Therefore, bans are detrimental to voters in the sense of Pareto, whereas their net effect on voters' aggregate welfare crucially depends on the unrestricted political equilibrium.

Thus, overall our model suggests that there may be limited scope for campaign finance policy. When policy makers have limited information on the preferences of interest groups, it may not be possible for campaign finance regulation to produce welfare improvements for voters. In any case, public funds can never completely substitute private contributions, and voters must always tolerate a positive level of costly favors.

The remainder of the paper is organized as follows. Section 2 discusses the related literature and our contribution. Sections 3 and 4 present the model and the relevant details, and Sections 5 analyses the equilibrium in the absence of campaign finance policy considering the role of the influence and electoral motives in determining the contributions of interest groups. Section 6 studies campaign finance policy, and Section 7 provides a discussion of the assumptions. Section 8 concludes.

2 The Literature and our Contribution

Analyzing the welfare implications of campaign finance policy is an interesting yet challenging endeavour. As mentioned by Coate (2004a), "it is necessary to take a particular stand on how and why campaign spending impacts voter behavior and why contributors give to candidates-issues on which the empirical literature offers no clear guidance". We argue that the specific modeling assumptions we make are particularly appropriate for assessing the welfare effects of policies.

Baron (1994) and Grossman and Helpman (1996), first addressed the issue of campaign financing by assuming that a share of voters are uninformed and can mechanically be swayed by campaign advertising. This approach trivially implies that banning contributions is welfare improving since these irrational agents are incapable of inferring that candidates distort their policy in order to obtain funds to finance campaigns. More recent literature that seeks to introduce campaign financing and elections in a rational voter framework can be divided in two categories. The first is based on the idea that political advertising may be considered indirectly informative (Potters et al. 1997, and Prat 2002a, b), meaning that ads do not convey hard information. Viewers are influenced by ads not because of their message, but because of the signalling role of burning money. The second strand, which is the one adopted in this paper, relies on the fact that advertisement is directly informative (Coate 2004a,b, Ashworth 2006, Schultz 2007). In other words, political campaigns provide voters with verifiable information on candidates' qualification or ability.

A distinctive feature of our model is that unlike the previous models of directly informative advertising, our setup allows us to consider split contributions in a rational voter framework. In Prat (2002a) split contributions are also considered, but the welfare implications are very different. Since advertising is indirectly informative, both candidates advertise only in an uninformative pooling equilibrium in which bad candidates burn money in the attempt of mimicking qualified candidates. Therefore, banning split contributions naturally produces a welfare improvement. In our model on the contrary, campaigns are directly informative. Political ads help voters to update their beliefs on candidates' qualification, and qualification is a necessary condition for candidates to raise funds. Thus, also split contributions are informative, and policies that limit or ban campaign financing are not necessarily welfare improving, since they alter the informational content of advertisements.

In Coate (2004a) each interest group shares the same ideology with one of the two candidates. As in our model interest groups finance candidates in exchange for favors that are costly for voters. Candidates are concerned about being elected but are also motivated by policy. Unlike our model however, candidates are the principals in the agency relationship with their partisan interest group so they have all the bargaining power, but can only request funds from their like-minded interest group. In this setting, Coate finds that it is possible to design a Pareto Improving policy for all agents (including interest groups) that eliminates favors.[5] This result is driven by the assumptions that candidates have all the bargaining power and that split contributions are ruled out since interest groups are ideological. These two assumptions imply that interest groups are always willing to accept a take-it-or-leave-it offer that involves positive contributions without providing favors, from their like-minded candidate. Thus, there always exists a policy that limits contributions that can eliminate favors. In our model instead, interest groups are non-ideological, which besides allowing for split contributions, also implies that independently of their bargaining power with respect to candidates, they would never accept to finance a campaign without being promised favors, as this would give them a negative expected utility.

Ashworth (2006) also obtains that a campaign finance policy that uses public funds to finance the same set of candidates that would receive contributions in the absence of regulation, can be welfare improving. He assumes that voters face uncertainty regarding candidates' ideologies and advertising serves the purpose of resolving this uncertainty. Advertising is financed by interest groups, that, like in our paper, are not concerned about

[5] A critical aspect of Coate (2004a) is that interest group members may actually be worse off with respect to non interest group members of the same partisan cohort. This raises the issue of why rational individuals should actually choose to join an interest group, that author justifies by assuming that there is an exogenous benefit of collective action that offsets this loss.

ideology but seek to obtain favors. However, in much the same way as for Coate (2004a), candidates have all the bargaining power and split contributions are not considered.

Another distinctive feature of our model, which is also present in Ashworth (2006) and Grossman and Helpman (1996), is that we consider asymmetric candidates. Both these papers assume that the incumbent has an ex-ante advantage in fund-raising and in terms of visibility among the electorate. In Ashworth, advertising is a discrete choice since there is a fixed cost of informing the electorate, which is represented by a single voter. We instead assume that contributions are a continuous choice. The combination of these features, as well as the fact that voters are rational and advertising is directly informative, allows for equally qualified candidates to receive different levels of contributions in equilibrium. This allows for the informational content of the signals (observing or not observing advertisements) to differ between candidates.

3 The Model - Overview

The model describes an electoral competition under majority rule in a jurisdiction with two candidates, a special interest group and a continuum of voters normalized to 1.

Voter i is described by his fixed policy (ideology) $p_i \in [0,1]$. The voting population is made up of three groups of citizens: leftists, rightists and swing voters. Leftist citizens have ideology 0 and rightists have ideology 1. Swing voters' ideologies instead are distributed uniformly on $[m-\tau, m+\tau]$. The ideology of the median swing voter is distributed uniformly on $[1/2-\theta-e, 1/2-\theta+e]$, where θ represents the bias or proclivity the median swing voter has in favor of one party or the other. If $\theta > 0$ (< 0) the leftist (rightist) candidate has an ex-ante electoral advantage; without loss of generality, we assume θ is positive.

Voters cannot abstain and are called on to choose between two ideological candidates indexed by $j \in \{L, R\}$ selected by parties, whose ideologies are 0 for party L, and 1 for party R. Candidates may differ also in their qualification for office denoted by $q_j \in \{0,1\}$: they are either qualified ($q_j = 1$) or unqualified ($q_j = 0$). The probability of find a qualified candidate of each party is denoted by $\sigma = \Pr(q_j = 1) \in (0,1)$, which is common knowledge. All voters derive positive utility from electing a qualified candidate and derive a negative utility from monetary favors awarded by candidates to the interest group in case of election, denoted by f_j, where $f_j \in \mathbb{R}^+$. The utility of voter i is therefore:

$$U_i^V = \begin{cases} \delta q_e - \beta\,|p_i - 0| - f_e & \text{if } e = L \\ \delta q_e - \beta\,|p_i - 1| - f_e & \text{if } e = R \end{cases},$$

where: $e \in \{L, R\}$ denotes the candidate who wins the election, δ measures the benefit of having a qualified candidate in power, and β measures the loss of electing a candidate with a different ideology.

We assume that there is an equal number of partisan voters for each candidate. Moreover, we assume that partisan voters always vote for their party's candidate irrespective of favors or qualification, implying that $\delta < \beta$. Behavior of swing voters is therefore crucial in determining the electoral outcomes.

Candidates are assumed to be opportunistic, their only objective is that of winning the election. Voters are not informed about candidate qualification. Candidates can convey information regarding qualification to voters through advertising, but have no funds of their own. By accepting contributions from the interest group, qualified politicians can enhance voters' perception of their qualification. Only qualified candidates can obtain funding for campaign advertising and they can only truthfully advertise their own characteristics. Campaign advertising is therefore assumed to be directly informative.[6] We assume that all contributions received from the interest group must be spent on campaign advertising.

The advertising function determines the share of the population that is reached by the candidate's message. A qualified candidate that raises (and spends) an amount of contributions C manages to inform a fraction $\Psi(C)$ of the population that he is qualified. Voters cannot ignore political ads because they are bundled with media broadcasting. The advertising technology is such that if a candidate spends and amount C his message reaches a fraction $\Psi(C) \in (0, 1)$ of the population. We assume that $\Psi(C)$ is non-negative and increasing in C, with $\Psi(0) = 0$. In common with Coate (2004a), in order to obtain tractable closed form solutions, we adopt the following specific functional form for the advertising technology: $\Psi(C) = C/(C + \alpha)$ where $\alpha > 0$.

The interest group is not interested in fixed policies (ideology) or qualification and is concerned only about favors from candidates.[7] The special interest group therefore has the following objective function:

$$U^{IG} = \pi_L b(f_L) + \pi_R b(f_R) - C_L(f_L) - C_R(f_R),$$

where π_j is the election probability of candidate j, $b(f)$ is a function that represents the utility that the interest group obtains for each level of favors, and $C_j(f_j)$ are the contribution schedules offered to candidate j. The interest group proposes a contribution schedule to each qualified candidate in exchange for favors. We assume that only qualified candidates can receive contributions and therefore $C_j(f_j) = 0$ for every f_j when $q_j = 0$. Schedules must be continuous, differentiable and non-negative.[8] When the interest group obtains favors

[6] We rule out negative advertising in the sense that each qualified candidate can try to inform the electorate about his own charachteristics and cannot convey any information about his opponent. Existing models of negative advertising include Skaperdas and Grofman (1995) and Polborn and Yi (2006).

[7] We consider the case where the interest groups are, for example, international corporations or firms. The interest groups are utility maximizers and non-ideological, whereas each voter has his own ideological position.

[8] Schedules are assumed to be common knowledge and each party can observe the schedule offered to the other. This can be justified by the fact that schedules are not explicit contracts but are conveyed by

from the elected candidate it enjoys a benefit $b(f)$ at a uniform cost of f to each citizen. The function $b(f)$ is increasing, strictly concave and satisfies $b(0) = 0$. We assume that the interest group does not have privileged information on the qualification of candidates and designs its schedules before knowing if each candidate is qualified or not.[9]

Candidates cannot choose the fixed policy (ideology) p_j which is predetermined by their party but can choose the level of favors, f_j. Candidates have no bargaining power since we assume that the interest group proposes the contribution schedules. A qualified candidate that is presented with a contribution schedule will rationally choose to provide a level of favors that corresponds to the amount of contributions that maximizes his chances of being elected. In other words, he will commit to carry out the agreed favors if elected and receive the corresponding level of contributions according to the schedule proposed by the interest group.

The timing of the game is the following:

1. Two candidates L and R are selected by each party.
2. The interest group designs its contribution schedules for each candidate (conditional on the candidate being qualified), before knowing if each candidate is qualified or not.
3. Qualified candidates receive the schedules and all candidates, whether qualified or not, choose the level of favors.
4. Contributions are set and campaigns are waged.
5. Elections take place.
6. Policies are implemented.
7. Payoffs are realized.

4 Model - Details

In what follows we make use of the following assumptions on the parameters and functions of the model

the public behavior of interest groups. Given our distributional assumptions the equilibria we obtain are independent of the observability of schedules. As observed by Grossman and Helpman (1996), even with unobservable schedules, the equilibrium obtained under observable schedules can be considered focal for more general distributional assumptions.

[9] This assumption is made to simplify the analysis. We could alternatively assume that the interest group announces its schedules after having observed if each candidate is qualified or not. This would naturally lead the interest group to offer different schedules based on whether one or both candidates are qualified.

Assumption 1 $\tau \geq \frac{\delta}{2\beta} + \theta + \varepsilon$.

Assumption 2 $\varepsilon \geq \frac{\delta}{2\beta} + \theta$.

Assumption 3 $b'(0) > \frac{2\varepsilon\alpha\sigma}{(\varepsilon-\theta)(1-\sigma)^2\delta}$.

The role of each of these will be highlighted within the analysis.

4.1 Swing Voter Behavior

Proceeding by backward induction we first consider the behavior of swing voters. Considering swing voter i when he is called upon to vote, he may have seen ads from both, one or neither candidate. Let (I_L, I_R) denote the voter's information set, where $I_j = 1$ when he observes an ad from candidate j and $I_j = \varnothing$ when he does not. Let $\rho_j(I_L, I_R)$ denote his belief that party $j's$ candidate is qualified conditional on his information set (I_L, I_R). Since only qualified candidates advertise both $\rho_L(1, I_R)$ and $\rho_R(I_L, 1)$ must equal 1, while beliefs $\rho_L(\varnothing, I_R)$ and $\rho_R(I_L, \varnothing)$ will be derived as part of the equilibrium. The voter will also have beliefs about the amount of favors that each candidate will offer to the interest group. We denote the amount of favors that a voter with information (I_L, I_R) believes candidate j will implement, if qualified, with $\widetilde{f_j}(I_L, I_R)$. It follows that a voter with ideology p_i and information set (I_L, I_R) prefers candidate L whenever:

$$\rho_L(I_L, I_R)(\delta - \widetilde{f_L}(I_L, I_R)) - \beta\,|p_i - 0| > \rho_R(I_L, I_R)(\delta - \widetilde{f_R}(I_L, I_R)) - \beta\,|p_i - 1|\,,$$

and the ideology of the indifferent swing voter, $\overline{p}(I_L, I_R)$ is therefore:

$$\overline{p}(I_L, I_R) = \frac{1}{2} + \frac{\rho_L(I_L, I_R)(\delta - \widetilde{f_L}(I_L, I_R)) - \rho_R(I_L, I_R)(\delta - \widetilde{f_R}(I_L, I_R))}{2\beta}. \tag{1}$$

4.2 Election Probabilities

If $p_i(I_L, I_R) < \overline{p}(I_L, I_R)$, where $p_i(I_L, I_R)$ is the fixed policy preference of swing voter i, he will vote for party L, otherwise he will vote for R. Given the ideology of the median swing voter m, the fraction of swing voters voting for candidate L is therefore:

$$\frac{1}{2} + \frac{\overline{p}(I_L, I_R) - m}{2\tau}.$$

Assumption (1) ensures that $\overline{p}(I_L, I_R)$ is always included between $m - \tau$ and $m + \tau$ for all m, which implies that there is always some uncertainty on the behavior of swing voters.[10]

[10]Notice that when $\overline{p}(I_L, I_R) < m - \tau$, all swing voters vote for candidate R and when $\overline{p}(I_L, I_R) > m + \tau$, all swing voters vote for candidate L.

Assuming that both candidates are qualified and that they receive contributions C_L and C_R, candidate L will win if he gets at least half the votes; that is, he will win if $m < \overline{m}(C_L, C_R)$, where

$$\begin{aligned}\overline{m}(C_L, C_R) &= \overline{p}(1,1)\Psi(C_L)\Psi(C_R) + \overline{p}(1,\varnothing)\Psi(C_L)(1-\Psi(C_R)) \\ &\quad + \overline{p}(\varnothing,1)(1-\Psi(C_L))(\Psi(C_R) + \overline{p}(\varnothing,\varnothing)(1-\Psi(C_L))(1-(\Psi(C_R)).\end{aligned}$$

Given the distribution of m, it follows that the probability of winning function of candidate L is:

$$\pi_L(C_L, C_R) = \begin{cases} 0 \text{ if } \overline{m}(C_L, C_R) < 1/2 - \theta - \varepsilon \\ \frac{1}{2} + \frac{\theta}{2\varepsilon} + \frac{\overline{m}(C_L,C_R)-1/2}{2\varepsilon} \text{ otherwise} \\ 1 \text{ if } \overline{m}(C_L, C_R) > 1/2 - \theta + \varepsilon \end{cases}.$$

If only candidate L receives contributions in equilibrium, he will win the election with probability $\pi_L(C_L, 0)$, whereas if only candidate R receives contributions candidate L will win with probability $\pi_L(0, C_R)$. If instead neither candidate receives financing from the interest group, candidate L's probability of winning is $\pi_L(0, 0)$.

4.3 Campaign Contributions

When designing its contribution schedules to qualified candidates, an interest group must decide whether to contribute only to obtain policy favors (*influence motive*) or also to enhance the electoral prospects of a particular candidate (*electoral motive*). A qualified candidate will observe the contribution schedule proposed by the interest group and choose an amount of favors that he will commit to provide if elected. If a candidate does not receive an offer because he turns out to be unqualified or if he refuses to accept contributions, he will choose a level of favors that maximizes his chances of being elected, namely $f_j = 0$. A qualified candidate will refuse contributions only if he receives a contribution schedule for which there does not exist a pair (C_j, f_j) that weakly increases his probability of being elected with respect to refusing.

5 Equilibrium

A political equilibrium consists of:

- a pair of feasible policy favors (f_L^*, f_R^*) and a pair of contribution schedules $C_L^*(f_L)$ and $C_R^*(f_R)$.
- Voter belief functions $\rho_j(I_L, I_R)$ and $\widetilde{f}_j(I_L, I_R)$ respectively describing: voters' beliefs concerning the likelihood that each candidate j is qualified, and the level of favors the winning candidate will implement, in each information set (I_L, I_R).

- Cut-points for swing voters $\overline{p}(I_L, I_R)$ describing their behavior as a function of the information they have received during the political campaign.

Interest group and candidate strategies must be mutual best responses given voter behavior. Voter beliefs must be consistent with interest group and candidate strategies, and voter behavior must be consistent with their beliefs.

5.1 Voter Beliefs

Given the equilibrium schedules, the candidates' equilibrium choice of favors, f_L^* and f_R^* gives rise to the following contributions: $C_L^* = C_L^*(f_L^*)$ and $C_R^* = C_R^*(f_R^*)$. Bayes' rule implies that voters beliefs about unadvertised candidates' must satisfy:

$$\rho_L(\varnothing, \cdot) = \frac{\sigma(1 - \Psi(C_L^*))}{\sigma(1 - \Psi(C_L^*)) + (1 - \sigma)}, \tag{2}$$

$$\rho_R(\cdot, \varnothing) = \frac{\sigma(1 - \Psi(C_R^*))}{\sigma(1 - \Psi(C_R^*)) + (1 - \sigma)}, \tag{3}$$

where $\rho_L(\varnothing, \cdot)$ and $\rho_R(\cdot, \varnothing)$ represent the probabilities that voters assign to unadvertised candidates being qualified. Notice that $\partial \rho_j / C_j < 0$, so that if voters do not observe an ad from a given candidate, his likelihood of being qualified decreases as campaign contributions increase. Voters' beliefs regarding the level of favors that qualified candidates will offer must satisfy:

$$\widetilde{f}_j(I_L, I_R) = f_j^* \text{ for } \forall j \in \{L, R\}.$$

If a candidate for which no ads are observed does not advertise, so that $C_j^* = f_j^* = 0$, Bayes' rule implies that equilibrium beliefs must be:

$$\rho_L(\varnothing, I_R) = \sigma \text{ and } \widetilde{f}_L(\varnothing, I_R) = 0 \text{ whenever } (C_L^*, f_L^*) = (0, 0),$$

$$\rho_R(I_L, \varnothing) = \sigma \text{ and } \widetilde{f}_R(I_L, \varnothing) = 0 \text{ whenever } (C_R^*, f_R^*) = (0, 0).$$

However, since the event of observing an ad when a candidate does not advertise does not arise along the equilibrium path, Bayes' rule does not apply for

$$\rho_L(1, I_R) \text{ and } \widetilde{f}_L(1, I_R) \text{ whenever } (C_L^*, f_L^*) = (0, 0)$$

and

$$\rho_R(I_L, 1) \text{ and } \widetilde{f}_R(I_L, 1) \text{ whenever } (C_R^*, f_R^*) = (0, 0).$$

In these cases, we will focus on out-of-equilibrium beliefs that are such that $\rho_L(1, I_R) = \sigma$ and $\widetilde{f}_L(1, I_R) = 0$ if $(C_L^*, f_L^*) = (0, 0)$, and $\rho_R(I_L, 1) = \sigma$ and $\widetilde{f}_R(I_L, 1) = 0$ if $(C_R^*, f_R^*) = (0, 0)$. This assumption rules out equilibria with no advertising supported by the out-of-equilibrium beliefs that any candidate who advertises must have been offered a contribution schedule with favors in excess of δ. Moreover, any equilibria in which the above out-of-equilibrium beliefs do not hold are not sequential equilibria.

Thus, voters' beliefs regarding the likelihood that a candidate for which no advertisement was observed is qualified are entirely determined by (2) and (3), which we denote as ρ_R and ρ_L respectively. Moving to voter behavior, using (1) we can compute the cut-points for asymmetrically informed (i.e. $(I_L, I_R) \in \{(\varnothing, 1), (1, \varnothing)\}$) and symmetrically informed (i.e. $(I_L, I_R) \in \{(\varnothing, \varnothing), (1, 1)\}$) voters, and we can therefore rewrite $\overline{m}(C_L^*, C_R^*)$ as:

$$
\begin{aligned}
\overline{m}(C_L^*, C_R^*) &= \frac{1}{2} + \Psi(C_L^*)\left[\tfrac{(1-\rho_L)(\delta - f_L^*)}{2\beta}\right] + \\
&\quad -\Psi(C_R^*)\left[\tfrac{(1-\rho_R)(\delta - f_R^*)}{2\beta}\right] + \left[\tfrac{\rho_L(\delta - f_L^*) - \rho_R(\delta - f_R^*)}{2\beta}\right].
\end{aligned}
$$

Assumption (2) ensures that $\overline{m}(C_L^*, C_R^*) \in [1/2 - \theta - \varepsilon, 1/2 - \theta + \varepsilon]$ and therefore the probability of electing candidate L can be written as:

$$
\begin{aligned}
\pi_L(C_L^*, C_R^*) &= \frac{1}{2} + \frac{\theta}{2\varepsilon} + \Psi(C_L^*)\left[\tfrac{(1-\rho_L)(\delta - f_L^*)}{4\beta\varepsilon}\right] + \\
&\quad -\Psi(C_R^*)\left[\tfrac{(1-\rho_R)(\delta - f_R^*)}{4\beta\varepsilon}\right] + \left[\tfrac{\rho_L(\delta - f_L^*) - \rho_R(\delta - f_R^*)}{4\beta\varepsilon}\right].
\end{aligned} \tag{4}
$$

Notice that the election probabilities of a candidate endogenously turn out to be separable in the variables describing his own level of policy favors and contributions, and those of his opponent. Therefore each candidate chooses the level of favors he will enact and the corresponding contributions, independently of his knowledge regarding the contribution schedule and decisions of the other candidate.

Each term in expression (4) has the following interpretation:

- $\frac{\theta}{2\varepsilon} > 0$ represents the ex ante electoral advantage of candidate L.

- $\Psi(C_j)\left[\frac{(1-\rho_j)(\delta - f_j)}{4\beta\varepsilon}\right]$ represents the change in j's electoral probability when voters observe an ad from j, which we denote as the *effectiveness of observing* an ad. Each of the two terms $\Psi(C_L)\left[\frac{(1-\rho_L)(\delta - f_L)}{4\beta\varepsilon}\right]$ and $\Psi(C_R)\left[\frac{(1-\rho_R)(\delta - f_R)}{4\beta\varepsilon}\right]$ measures the effectiveness in inducing swing voters to switch from their natural allegiances when they observe an add from a given candidate (switching from candidate R to candidate L and from L to R respectively).

- $\left[\frac{\rho_j(\delta - f_j)}{4\beta\varepsilon}\right]$ represents the change in j's electoral probability in the event that voters do not observe ads from j, which we denote as the *effectiveness of not observing* an ad. Each of the two terms $\left[\frac{\rho_L(\delta - f_L)}{4\beta\varepsilon}\right]$ and $\left[\frac{\rho_R(\delta - f_R)}{4\beta\varepsilon}\right]$ measures the effectiveness of campaign ads in inducing swing voters to switch from their natural allegiances when they do not observe an add from a given candidate (switching from candidate R to candidate L and from L to R respectively).

Notice that a necessary condition for both measures of effectiveness to be positive is that $\delta > f_j$. While the effectiveness of observing an ad is positive only when a candidate receives positive contributions, the effectiveness of not observing an ad is positive even when a given candidate does not receive contributions. This is because the event of not observing an ad, may arise with positive probability independently of whether a candidate advertises or not. Notice also that if candidates' equilibrium contributions and favors differ, the effectivenesses of observing and not observing advertisements also differ for each candidate.

5.2 Contributions and Favors

Every qualified candidate accepts money from the interest group in exchange for favors if such a contribution weakly increases his electoral probability. Notice that a candidate that receives a contribution schedule, that is positive for at least one value of f_j, is certain of being qualified. Formally, equilibrium schedules must satisfy:

$$\pi_j(C_j, C_{-j}) \geq \pi_j(0, C_{-j}) \text{ for } \forall j \in \{L, R\}.$$

We compute the probabilities of electing each candidate by making use of (4) and the corresponding voter beliefs. Simplifying we obtain the following expression:

$$\left(\frac{[\Psi(C_j)(1-2\sigma)+\sigma]}{1-\sigma\Psi(C_j)}\right)(\delta - f_j) \geq \sigma\delta. \tag{5}$$

When only the influence motive applies the above expression is satisfied with equality and is non binding when the electoral motive applies.[11] First notice that the first term on the right hand side of (5) is always positive and increasing in $\Psi(C_j)$. This implies that it must always be that $f_j < \delta$, in order for the candidate to accept the offer. This is because if f_j were greater δ, voters would rationally expect a negative utility from electing a qualified candidate, inducing them to vote for an unqualified candidate. Therefore, each candidate would never commit to offering a level of favors greater than δ.[12]

Remark 1 *When advertising is directly informative and the advertising technology is non-negative, the effectiveness (of both observing and not observing an advertisement) is always*

[11] The separability of the electoral probabilities in the variables describing each candidate's level of contributions and favors and those of his opponent implies that the choice of contributions and favors that satisfy the participation constraint of each candidate are independent of those of the other.

[12] If the electoral motive applies, $\pi_j(C_j, C_{-j}) - \pi_j(0, C_{-j}) > 0$. Its partial derivative with respect to $\Psi(C_j)$ equals $(\delta - f_j)(1-\sigma)^2/(1-\sigma\Lambda(C_j))^2$ which is negative if $f_j > \delta$.

positive for every $C_j > 0$, since for every level of positive contributions C_j it must be that $f_j < \delta$ in order for a candidate to accept the interest group's offer.

Advertising is always effective because a positive share of swing voters that observe an add from a given candidate, will be induced to switch their vote in favor of the candidate whose advertisement they observed. This is because in equilibrium voters always benefit from electing a qualified candidate that advertises, despite the fact that the interest group has all the bargaining power in setting contribution schedules. Notice also that since $\Psi(C_j)$ is increasing in contributions, (5) implies that C_j is increasing in f_j. Intuitively, holding C_j constant, when f_j increases this reduces voters' benefits of electing a qualified candidate reducing the effectiveness of observing ads. Contributions must therefore increase, in order to continue to satisfy a given candidate's participation constraint.

5.3 Influence Motive Only

In this section, we assume that contributions from the interest group just affect the policy choice (*influence motive*) without affecting the electoral probabilities (*electoral motive*). Thus, (5) holds with equality.

In order to obtain an explicit function for the influence motivated contributions for every level of favors, as well as a specific upper bound on favors, we substitute the functional form for the advertising technology that we introduced in Section 2, $\Psi(C_j) = C/(C + \alpha)$ in (5) obtaining:

$$\gamma(f_j) = \frac{\alpha\sigma f_j}{(1-\sigma)\left[(1-\sigma)\delta - f_j\right]}, \tag{6}$$

where $\gamma(f_j)$ is a function that denotes the minimum contributions for every level of favors, which is the same for both candidates. It is immediate to notice that $0 \leq f_j < \overline{f}_j$ where $\overline{f}_j = \delta(1-\sigma)$ and that $\gamma(f_j)$ is a convex function ($\gamma'(f_j) > 0$ and $\gamma''(f_j) > 0$).

When only the influence motive applies, this is equivalent to choosing f_j for every $j \in \{L, R\}$ that maximizes expected utility, conditional on contributions satisfying the candidates' participation constraints:

$$\begin{aligned} U^{IG} &= b(f_L)[\sigma^2\pi_L(C_L, C_R) + \sigma(1-\sigma)\pi_L(C_L, 0)] + \\ &\quad + b(f_R)[\sigma^2\pi_R(C_L, C_R) + \sigma(1-\sigma)\pi_R(0, C_R)] \\ &\quad - \gamma(f_L)[\sigma^2 + \sigma(1-\sigma)] \\ &\quad - \gamma(f_R)[\sigma^2 + \sigma(1-\sigma)]. \end{aligned}$$

Notice that since the participation constraint is always satisfied with equality, the election probabilities are not affected by contributions, which implies that $\pi_L(\cdot,\cdot) \equiv \pi_L$. Taking the first order conditions with respect to f_L and f_R we therefore obtain:

$$\begin{aligned} b'(f_L^*) &\leq \frac{\gamma'(f_L^*)}{\pi_L}, \\ b'(f_R^*) &\leq \frac{\gamma'(f_R^*)}{(1-\pi_L)}. \end{aligned}$$

Given the properties of $\gamma(f_j)$ and $b(f_j)$, we have that the equilibrium pairs of contributions and policy favors for every $j \in \{L, R\}$, are such that:

$$f_L^* \geq f_R^* \text{ and } C_L^* \geq C_R^*,$$

thus we can state the following proposition:

Proposition 1 *When contributions are exclusively influence motivated, the candidate with an ex-ante electoral advantage receives at least as many funds from the Interest Group and concedes at least as many favors, with respect to his opponent.*

Assumption (3) allows us to focus on the interesting case in which both candidates receive positive contributions if they are qualified.[13]

5.4 Influence and Electoral Motives

Up to now we have assumed that the interest group will offer each party only what is strictly necessary to win its support for the desired platform. Now we set out to show when this is not the case.

The first order conditions for the maximization of the interest group utility function U^{IG} with respect to C_L and C_R subject to the participation constraint $C_j \geq \gamma(f_j^*)$ for every j, imply that:

$$\phi_L^*[b(f_L^*) - b(f_R^*)] = 1 - \lambda_L, \tag{7}$$

$$\phi_R^*[b(f_R^*) - b(f_L^*)] = 1 - \lambda_R, \tag{8}$$

where ϕ_j^* denotes the marginal return of contributions for candidate j in terms of the variation in the probability of winning the election, and λ_j are the Lagrangian multipliers applicable to the participation constraints of each candidate j. When $\lambda_j = 0$, this implies that the participation constraint is not binding for candidate j, and that the interest group contributes with the intent of influencing the electoral outcome.

[13] Note that when $\theta > 0$ if we assume that $b'(0) > \gamma'(0)/(1-\pi_L)$ by continuity it must be that $f_L^* > f_R^* > 0$ and $C_L^* > C_R^* > 0$ and we have an interior solution. By substituting $\pi_L = 1/2+\theta/2\varepsilon$ and using the expression for $\gamma(f_j)$ derived in the paper, we obtain $b'(0) > \frac{2\varepsilon\alpha\sigma}{(\varepsilon-\theta)(1-\sigma)^2\delta}$.

In order to gather further insight on when the electoral motive kicks in, it useful to derive an expression for the marginal benefit of contributions, denoted by ϕ_j^*. Considering the probability of electing candidate j given by (4), it is sufficient to take the derivative of this probability with respect to C_j when $C_j = \gamma(f_j^*)$:

$$\phi_j^* = \frac{\partial \pi_j(C_j, \cdot)}{\partial C_{j|C_j=\gamma(f_j^*)}} = \frac{(\delta - f_j^*)}{4\beta\varepsilon} \left\{ \Psi'(C_j) \left[(1-\rho_j^*) \right] + \rho_j'^* [1 - \Psi(C_j)] \right\}, \tag{9}$$

where ρ_j^* represents the probability that a candidate is qualified given that an ad was not observed and contributions are such that the candidate's participation constraint is satisfied with equality, at the equilibrium level of favors f_j^*. The two terms in curly brackets in the above expression for ϕ_j^*, can be interpreted in the following way:

- $\Psi'(C_j) \left[(1-\rho_j^*) \right] > 0$ represents the marginal increase in the *effectiveness of observing an ad* given an increase in contributions.
- $\rho_j'^* [1 - \Psi(C_j)] < 0$ represents the marginal decrease in the *effectiveness of not observing an ad.* It denotes the fact that for higher levels of contributions, the event of not observing an add from a given candidate implies that there are less chances of him being qualified.

It turns out that the first effect prevails and that ϕ_j^* is always strictly positive. This implies that it will never occur that both candidates receive more contributions than are necessary to induce each of them to adopt the desired policy:

Proposition 2 *The only candidate that may receive contributions for electoral motives from the interest group is the ex-ante more advantaged candidate (proof in appendix).*

From the previous section we know that, if $\theta > 0$, we have $b(f_L^*) > b(f_R^*)$. Since $\phi_j^* > 0$, if $\lambda_j = 0$, it must be that $\lambda_{-j} > 0$ because the right hand sides of equations (7) and (8) have opposite signs. In addition, from equation (7) it follows that when $\lambda_L = 0$ it must be that $f_L^* > f_R^*$, implying that $C_L^* > C_R^*$; therefore, the party that sees its chances of being elected enhanced provides more policy favors and receives greater contributions.[14].

The only party that may receive additional campaign support is therefore the ex-ante more advantaged one. To see this, assume that the ex-ante less advantaged candidate ($j = R$) were receiving the larger contribution. Suppose now that the interest group were to invert the offers made to each candidate, offering to candidate L what it was offering to candidate R and vice versa. At this point, the interest group could reduce the offer made to candidate

[14] This propostion is in the spirit of proposition 4 of Grossman and Helpman (1996). While their result applies to a setting with irrational voters, our proposition extends this result to a setting with rational voters and directly informative ads.

L obtaining the original probability distribution over policy outcomes. This represents a profitable deviation because it allows the interest group to obtain the same probability distribution over policy outcomes but at a lower cost.

5.5 Determinants of the Electoral Motive

When the interest group provides more funding to the advantaged candidate than what is strictly necessary to guarantee that its preferred level of favors will be adopted, it does so to enhance the candidate's electoral prospects. The interest group will do this if the expected marginal benefit from the first additional unit of contributions:

$$\phi_L^*[b(f_L^*) - b(f_R^*)]$$

is greater than the marginal cost, which is equal to 1. In the above expression, ϕ_L^* represents the marginal impact of contributions on vote shares, when contributions and policies are such that candidate L has a binding participation constraint. Therefore, contributions are defined by $\gamma(f_L^*)$, and the policies of candidate L and R are respectively f_L^* and f_R^*.

It is apparent that the IG will have a reason to apply the electoral motive in addition to the influence motive, the greater is the impact of contributions on vote shares which is represented by the term ϕ_L^*. Bearing in mind that $f_L^* > f_R^*$, this condition is more easily satisfied the greater is the distance between these optimal policies. This distance depends on two factors: 1) positively on the ex-ante electoral advantage of one candidate over the other and 2) negatively on the concavity of the benefit function. The concavity of $b(\cdot)$ is related to the pliable policy preferences of the interest group. It is reasonable to assume that if the interest group's marginal utility of favors decreases at a slower rate, it has more extreme preferences. In practice, the interaction between the impact of an additional unit of contributions on popularity, and the distance between the optimal policies determines whether the electoral motive applies or not.

From proposition 2 we know that ϕ_j^* is always positive. However, we can also show that ϕ_j^* is decreasing in favors and tends to zero if favors are sufficiently high:

Proposition 3 *The marginal effect of contributions on vote share ϕ_j^* is decreasing in the level of favors promised, f_j^* and tends to zero as f_j^* tends to the upper bound on favors $\overline{f}_j$ (proof in appendix).*

Hence, even if "favor hungry" candidates will tend to have a less concave benefit function, leading to a greater distance between $b(f_L^*)$ and $b(f_R^*)$, the marginal impact of contributions on vote share, ϕ_L^* dampens this effect since it is decreasing in favors. Moreover, numerical simulations (figure 1) indicate that the magnitude of ϕ_j^* is significantly smaller than 1, suggesting that electoral motives are relatively weak. This result is consistent with the empirical evidence that highlights how campaign advertising rarely affects electoral outcomes.[15]

6 Campaign Finance Policy

We now consider campaign finance policies that may potentially improve voters' welfare. We look for the possibility of both welfare and Pareto improvements with respect to the unrestricted equilibrium. Notice that the interest group is always worst off under any policy setup, since in the absence of policies it has all the bargaining power with respect to candidates in setting contribution schedules, and benefits from favors.

We consider both pure limit policies as well as policies that impose a limit l on contributions, but provide a public subsidy of rate s: a candidate that raises C in private funds therefore receives a corresponding subsidy of sC. This policy scheme ensures that only qualified candidates receive public funding, namely those who manage to raise private financing, avoiding wasteful use of public resources. Public subsidies are levied by a head tax T on all voters. The policy maker may set different limits and subsidies for each candidate. Therefore, the complete set of available policy instruments is: (l_L, s_L) and (l_R, s_R).

In terms of timing, the campaign finance policy is introduced in the first stage of the game before candidates are selected. The interest group therefore announces its schedules conditional on the policy. We denote $\widehat{C}_j$ and f_j^P respectively as the equilibrium contributions and favors under the campaign finance policy, and the corresponding expected head tax is $T = \sigma(s_L\widehat{C}_L + s_R\widehat{C}_R)$. We define the total level of contributions (public and private) received by a given candidate as a consequence of the policy as $C_j^P = \widehat{C}_j(1 + s_j)$, and the resulting electoral probabilities as $\pi_j^P(\cdot, \cdot)$. Due to the properties of $b(f)$ and Assumption (3) it follows that whenever $0 < l_j < C_j^*$, the interest group will always design the contribution schedules in order to obtain a level of contributions that is exactly equal to the limit, so that $\widehat{C}_j = l_j$. A policy (l_j, s_j) is a pure limit policy if $s_j = 0$, while it involves public subsidies when $s_j > 0$. In addition, a policy that targets only one candidate j, implies that $l_k = \infty$ and $s_k = 0$ for $k \neq j$.

We begin by considering policies that target only one candidate, therefore addressing the issue of split contributions, and then move on to policies involving both candidates. In each of these cases we analyze pure limit policies as well as those that entail public financing.

We assume $\theta > 0$, but notice that whenever $\theta \neq 0$, that is, whenever one candidate has an ex-ante electoral advantage over the other, each partisan's expected utility depends positively on the probability that his party's candidate is elected independently of whether he is qualified or not. Therefore, even if candidates are "equally qualified" (i.e.(C_L, C_R)

[15] Levitt (1994) provides evidence of how campaign advertising does not significantly effect electoral outcomes.

and $(0,0)$), partisans' utilities will vary in opposite directions if the policy affects these probabilities. In our model there is a natural antagonism between partisan voters, and in order for a policy to be Pareto improving it must leave electoral probabilities unaltered in all states (i.e.(C_L, C_R), $(C_L, 0)$, $(0, C_R)$, $(0,0)$), with respect to the unrestricted case.[16]

Lemma 1 *A necessary condition for a policy to be Pareto improving is that it does not alter election probabilities with respect to the unrestricted case and reduces favors (proof in appendix).*

We begin by showing that no pure limit policy satisfies this lemma. For example, when an interest group finances candidates for electoral motives, by Proposition 2 we know that this occurs only for the ex-ante more advantaged candidate. At a first glance we might be led to think that a policy that limits or bans contributions to the candidate receiving less funds, could be Pareto improving. However, this is not the case because such a policy alters the electoral probabilities, even if the limit or ban is imposed on the candidate that was financed exclusively for influence motives. This occurs because when contributions are limited, a qualified candidate will offer less contributions (and favors) and the event of not observing ad induces voters to believe that the candidate is qualified with a greater probability, with respect to the unrestricted case. In other words the effectiveness of not observing an ad from the candidate targeted by the policy increases. If candidate L is qualified and his opponent is not, any limit for R would reduce the election probability of L :

$$\pi_L^P(C_{L,}^* 0) - \pi_L(C_{L,}^* 0) = \frac{-\rho_R^P(\delta - f_R^P) + \rho_R^*(\delta - f_R^*)}{4\beta\varepsilon} < 0,$$

where superscript P stands for "under policy".

As a next step we therefore move on to analyze policies that affect both candidates. We obtain that banning or limiting contributions (to both candidates) is never Pareto improving for all voters since this alters at least one electoral probability in a given state. Intuitively, a reasoning similar to the case where limits are imposed on a single candidate applies. For instance, banning contributions increases the leftist candidate's probability of being elected when neither candidate is qualified. This is because the ban has a greater impact on the effectiveness of not observing an ad for the ex-ante advantaged candidate, since in the unrestricted case he would receive greater contributions with respect to an equally qualified but disadvantaged contender. Intuitively, in the absence of restrictions, not observing an ad from an advantaged candidate is more informative on his lack of ability in relation to that of his opponent, then when contributions are limited.

[16] In the knife edge case where $\theta = 0$ this antagonism between partisan voters breaks down. In this case when candidates are "equally qualified", voters' utility is not affected by the probability of electing one candidate or the other. The utility of all partisan voters is instead increasing in the probability of electing a qualified candidate when the other candidate is not qualified, independently of whether the qualified candidate belongs to their partisan cohort or not.

Proposition 4 *It is* never *possible to design a pure limit policy that limits (or bans) contributions for at least one candidate, that is Pareto improving (proof in appendix).*

The next question we address is whether it is possible to design a Pareto improving policy targeted at one candidate only, that set limits to contributions while offering corresponding subsidies financed through taxation.

Disregarding the tax cost of subsidies, even if contributions to one candidate are partially substituted by public funding, it is not possible to generate a Pareto improvement for voters. If for example the policy targets candidate R, in order to compensate leftist voters for the loss of utility due to the increased chances of electing a rightist candidate, in the state where only the leftist candidate is qualified (which implies setting $\pi_L^P(C_{L,}^* 0) = \pi_L(C_{L,}^* 0)$), the policy maker must provide a subsidy to increase total contributions to R, so that $C_R^P > C_R^*$ for any $l_R < C_R^*$. However, if both candidates turn out to be qualified, the subsidy increases the probability of electing the rightist candidate (i.e. $\pi_L^P(C_{L,}^* C_R^P) - \pi_L(C_{L,}^* C_R^*) < 0$) making the leftist voters worst off. Thus, a policy that affects only one candidate can never leave electoral probabilities unaltered in all states, independently from the tax cost of subsidies.

Proposition 5 *It is* never *possible to design a Pareto improving policy that imposes limits and subsequent subsidies on one candidate only (proof in appendix).*

We now ask whether it is possible to design a Pareto improving policy that makes use of all the available policy instruments, potentially setting different limits and subsidies for each candidate. Although such a policy always exists, it is never possible to design a Pareto improving policy that eliminates all private contributions and therefore all favors:

Proposition 6 *It is* always *possible to design a Pareto improving policy that imposes a limit on contributions and offers corresponding subsidies for each candidate. This policy involves a level of favors, f_L^P for the ex-ante advantaged candidate that is significantly greater than zero, such that $f_L^* - f_L^P = f_R^* - f_R^P$ (proof in appendix).*

The best the policy maker can do is to set the limit l_R such that f_R^P is equal to its lower bound ξ, where ξ is strictly positive but arbitrarily small, and to set l_L so that $f_L^* - f_L^P = f_R^* - \xi$.[17] Consequently subsidies s_L and s_R must be set so that the effectiveness, of both observing and not observing an ad from a given candidate, is unaltered with respect to the unrestricted case. Besides being very demanding in terms of information requirements, since the policy maker must have perfect information on the preferences of the interest group, this policy involves a level of favors that is significantly greater than zero for the ex-ante advantaged

[17] Setting a strictly postive limit (i.e $f_R^P = \xi > 0$) is a necessary requirement, since it allows policy makers to identify qualified candidates by observing positive ads, thus providing matching funds only in these cases and avoiding wasteful spending.

candidate. Moreover, the greater is the ex-ante electoral advantage, the higher is $f_L^* - f_R^*$ and the higher the level of favors f_L^P that must be tolerated, in order to weakly improve the utility of all voters.

The Pareto improving policy must satisfy the necessary condition of leaving electoral probabilities unaltered while reducing favors (Lemma 1), but must also balance the social benefits (in terms of reduced favors) versus the costs (in terms of the taxes needed to finance the subsidies). There may be some cases where it can be optimal to set the lower bound on favors f_R^P to be greater than ξ, so that both candidates must offer a level of favors that is significantly greater than zero. This may occur because the tax cost of public contributions may be decreasing in limits when limits are low. More specifically, while setting higher limits increases the favors provided by a given candidate, it also reduces the share of contributions that must be financed through subsidies. Thus, raising limits (and favors) above the lower bound may in some cases lead to a tax reduction that offsets the additional cost of favors.

Let's focus now on welfare improvements. We begin by considering policies that leave the level of contributions unaltered by substituting private financing with public money such that $C_j^* = l_j(1 + s_j)$ for every $j \in \{L, R\}$, thus lowering the level of favors. Notice that these policies are as demanding as the Pareto improving policy in terms of informational requirements, since they require the policy maker to have perfect information on the preferences of the interest group. Therefore, we focus on policies that are less demanding in terms of information for the policy maker, investigating whether it is possible to design a welfare improving policy that simply bans favors, with $l_j = 0$ and $s_j = 0$ for at least one candidate. Although it may be possible to find such a policy that is welfare improving for a subset of political equilibria, we obtain that:

Proposition 7 *The only policy that does not require the policy the maker to have knowledge of the interest group's preferences is a pure ban on contributions for at least one candidate. Such a policy does not increase the expected aggregate utility of voters in all possible political equilibria (proof in appendix).*

The ban on contributions has both a positive and a negative effect on voters' welfare. In other words, the benefit from eliminating favors comes at the cost of increasing the chances of electing a bad candidate. This cost is a consequence of the fact that if contributions to a given candidate are banned, the event of not observing an advertisement no longer conveys information on qualification, driving the effectiveness of not observing an ad to zero. This results in a loss of valuable information that may be greater than the benefit that comes from eliminating favors. Therefore, the net effect on welfare crucially depends on the specific contributions and favors resulting from the unrestricted political equilibrium. Although for some unrestricted equilibria banning contributions for at least one candidate may be welfare improving, this is not always the case.

7 Discussion

In this section we discuss some of the key assumptions of our model highlighting how relaxing them would affect our results. The first assumption we consider is that related to the existence of a unique interest group. If we introduce multiple interest groups competing for favors, there continues to be a natural limit to the total amount of favors that interest groups can request, and the favors obtained by each lobby are decreasing in the number of contributing interest groups.[18] Increasing the number of interest groups would not necessarily generate a Pareto or welfare improvement in the unrestricted case. This is because the total amount of policy favors may be higher or lower in the multiple lobby case depending on the distribution of preferences of the different contributors. In any case the only Pareto improving policy continues to be the one we mentioned, that involves at least one candidate providing a level of favors that is significantly greater than zero.

We also assumed that policy favors exclusively benefit interest group members. We could more realistically assume that these favors may have positive effects on a wider share of the population. For example increasing import tariffs may also produce benefits for firms that are not part of the interest group and did not make contributions. In this case favors may generate a utility for some voters therefore reducing the scope for campaign finance policy to be welfare or Pareto improving. The non-contributing voters that benefit from the activities of the interest group, may in fact be harmed by a policy that reduces favors.

Another assumption regards the fact that the interest group is not concerned about ideology or qualification, but is simply motivated by favors. This allows us to abstract from

[18] Assuming that there are K interest groups, in a common agency framework (Bernheim and Whinston (1986)) where $k \in \{1, .., K\}$, and the advertising function is:

$$\Lambda(C_j) = \frac{\sum\limits_k C_j^k}{\sum\limits_k C_j^K + \alpha}$$

where C_j^k denotes the contribution of interest group k to candidate j, $C_j = \sum\limits_k C_j^k$, and $C_j^{-k} = \sum\limits_{h \neq k} C_j^h$. We obtain that the minimum level of contributions, for each level of favors f_j^k of each interest group, must satisfy the participation constraint of candidate j with equality:

$$\pi_j(C_j, C_{-j}) = \pi_j(C_j^{-k}, C_{-j}),$$

which implies that the following condition must be satisfied in order for f_j^k to be a feasible equilibrium level of favors :

$$f_j^k < \frac{\alpha(1-\sigma)[\delta - f_j^{-k}]}{\left[\alpha + C_L^{-k}(1-\sigma)\right]}.$$

When $C_j^{-k} = 0$ and $f_j^{-K} = 0$ this reduces to the single IG case: $f_j^* < (1-\sigma)\delta$. The limit on the level of favors that can be obtained for each IG is therefore decreasing in the number of IGs that contribute.

the fact that interest group members might also be voters. If they were ideological, it seems plausible to assume that interest group members would all be part of the same partisan group as in Coate (2004a). However, this would immediately rule out split contributions, which is one of the key features captured by our model. If interest group members were also concerned about the qualification of candidates, this would attenuate their willingness to demand favors as this would represent a cost for them as voters, reducing the scope for regulation. When there is less of a conflict, between the preferences of the interest group and those of voters, there is also less need for regulation.

8 Conclusion

Interest groups make campaign contributions both for influence and electoral motives. Electoral motives are shown to be relatively weak since a marginal increase in contributions for campaign advertising has both a positive effect and a negative effect, on the probability of being elected of a qualified candidate. The negative effect can be summarized as follows: rational voters that do not observe an advertisement from a given candidate infer that there are less chances that he is qualified, the greater is the equilibrium level of contributions that a qualified candidate receives. The positive effect originates from the fact that increasing contributions allows a greater share of voters to be reached by ads, and therefore to infer that a candidate is qualified. When the advertising technology is concave (i.e. when the additional share of voters that is reached by an increment in adds is decreasing), the negative effect tends to offset the positive one.

Thus, limiting or banning contributions affects the informational content of both observing and not observing ads, possibly altering electoral probabilities. Whenever one candidate has an ex-ante advantage over the other and ads convey hard information, policies imposing limits (or bans) without provision of public subsidies cannot be Pareto improving. Moreover, although in some cases a welfare improving policy of this type may exist, this is not necessarily the case for all political equilibria.

There are also policies that impose limits and subsidies. A "fully informed" policy maker could Pareto improve the welfare of voters by setting specific limits and corresponding subsidies (financed through taxation) for each candidate, thus leaving the electoral probabilities unaltered with respect to the unrestricted political equilibrium. Notice that this requires the policy maker to have complete information about interest group's contribution schedules, which may be unrealistic. In any case, candidates must continue to receive private financing from interest groups, implying that voters must always pay a strictly positive cost of monetary favors in exchange for valuable electoral information.

Thus when policy makers have limited information on the preferences of interest groups, it may not be possible for campaign finance regulation to produce welfare improvements for voters. In any case, public funds can never completely substitute private contributions, and voters must always tolerate a positive level of costly favors.

Appendix

Proof of Proposition 2. Given the argument of Section 5.4, it is sufficient to prove that ϕ_j^* is always greater than zero to prove the proposition. Substituting ρ_j^* and $\partial\rho_j^*/\partial C_j$ in equation (9) we obtain that whenever $f_j^* > 0$:

$$\tfrac{(\delta - f_j^*)}{4\beta\varepsilon}\left\{\tfrac{\Psi'(C_j)(1-\sigma)^2}{(1-\sigma\Psi(C_j))^2}\right\} > 0,$$

since by assumption $\Psi'(C_j) > 0$ and by Remark 1 $f_j^* < \delta$. ■
Proof of Proposition 3. If we substitute $C_j = \gamma(f_j^*)$ in (9) we obtain ϕ_j^* as a function of f_j^* which simplifies to:

$$\phi_j^*(f_j^*) = -\tfrac{(f_j^* + \delta(\sigma-1))^2}{4\alpha\beta\varepsilon(f_j^* - \delta)}.$$

Taking the derivative with respect to f_j^* we obtain:

$$\frac{\partial\phi_j^*(f_j^*)}{\partial f_j^*} = -\frac{(f_j^*)^2 + 2f_j^*\delta + \delta^2(\sigma^2-1)}{4\alpha\beta\varepsilon(f_j^* - \delta)^2}.$$

Since the denominator is always positive, this expression is negative whenever the numerator is negative. The numerator is negative if:

$$f_j^* < \delta(1-\sigma) = \overline{f},$$

and we know by Remark 1 that this is always satisfied. Having proved that $\frac{\partial\phi_j^*(f_j^*)}{\partial f_j^*} < 0$ since $\frac{\partial\phi_j^*(f_j^*)}{\partial f_j^*} = 0$ when $f_j^* = \overline{f}$, it follows that $\lim\limits_{f_j^* \to \overline{f}} \frac{\partial\phi_j^*(f_j^*)}{\partial f_j^*} = 0$. ■

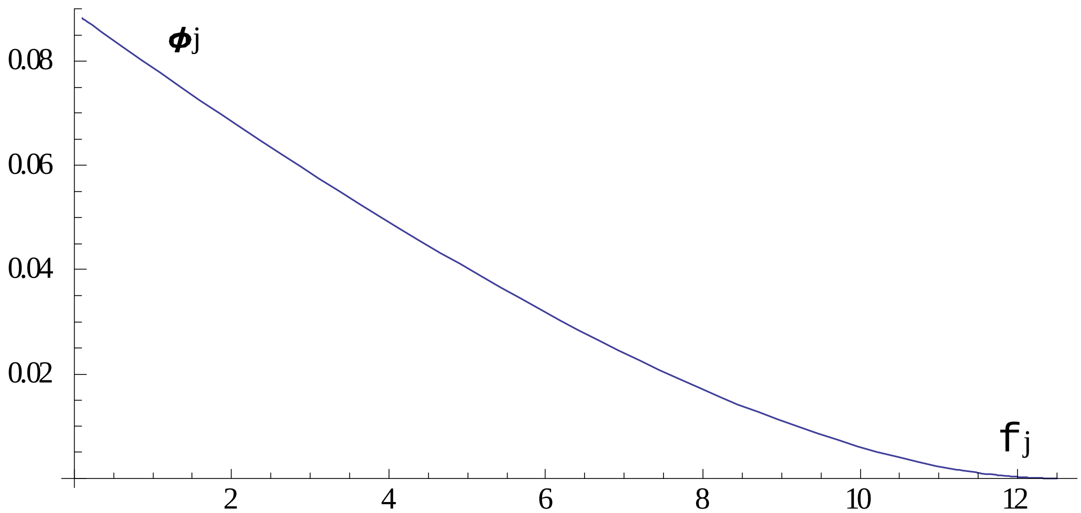

Figure 1: Relationship between ϕ_j^* and f_j^*
($\beta = 35, \delta = 25, \alpha = 10, \varepsilon = 0.05, \sigma = 0.5$).

Campaign Finance Policy

We first write the expected utilities of the different types of voters for equilibrium contributions and favors (C_j^P, f_j^P) and a given campaign finance policy (l_j, s_j) for every $j \in \{L, R\}$. These expressions naturally apply also in the unrestricted regime.

The expected utility of a *leftist partisan* is:

$$\begin{aligned}
&\sigma^2[\pi_L^P(C_L^P, C_R^P)(\delta - f_L^P) + \pi_R^P(\widehat{C}_L,\widehat{C}_R)(\delta - f_R^P)]+ \\
&+\sigma(1-\sigma)[\pi_L^P(C_L^P,0)(\delta - f_L^P) + \pi_R^P(0, C_R^P)(\delta - f_R^P)]+ \\
&-\beta\left\{\sigma^2\pi_R^P(C_L^P, C_R^P) + (1-\sigma)^2\pi_R^P(0,0) + \sigma(1-\sigma)\left[\pi_R^P(C_L^P,0) + \pi_R^P(0, C_R^P)\right]\right\} \\
&-\sigma(s_L\widehat{C}_L + s_R\widehat{C}_R).
\end{aligned} \tag{10}$$

The expected utility of a *rightist partisan* is:

$$\begin{aligned}
&\sigma^2[\pi_L^P(C_L^P, C_R^P)(\delta - f_L^P) + \pi_R^P(C_L^P, C_R^P)(\delta - f_R^P)]+ \\
&+\sigma(1-\sigma)[\pi_L^P(C_L^P,0)(\delta - f_L^P) + \pi_R^P(0, C_R^P)(\delta - f_R^P)]+ \\
&-\beta\left\{\sigma^2\pi_L^P(C_L^P, C_R^P) + (1-\sigma)^2\pi_L^P(0,0) + \sigma(1-\sigma)\left[\pi_L^P(C_L^P,0) + \pi_L^P(0, C_R^P)\right]\right\} + \\
&-\sigma(s_L\widehat{C}_L + s_R\widehat{C}_R).
\end{aligned} \tag{11}$$

We treat swing voters as ex-ante identical, so that for a given draw of m each one is equally likely to have any fixed policy preference on $[m-\tau, m+\tau]$. With this assumption the utility of each swing voter is equal to that of the average swing voter. When computing this payoff we must also take into account of the correlation between the candidate that wins the election and the ideology of the average swing voter. As a result of these considerations the expected utility of a *swing voter* is:

$$\begin{aligned}
&\sigma^2[\pi_L^P(C_L^P, C_R^P)(\delta - f_L^P) + \pi_R^P(C_L^P, C_R^P)(\delta - f_R^P)]+ \\
&+\sigma(1-\sigma)[\pi_L^P(C_L^P, 0)(\delta - f_L^P) + \pi_R^P(0, C_R^P)(\delta - f_R^P)]+ \\
&+\beta\sigma^2\left[2\pi_L(C_L^P, C_R^P)(\varepsilon + \theta - \pi_L(C_L^P, C_R^P)\varepsilon)\right]+ \\
&+\beta\sigma(1-\sigma)\left[2\pi_L(C_L^P, 0)(\varepsilon + \theta - \pi_L(C_L^P, 0)\varepsilon)\right]+ \\
&+\beta\sigma(1-\sigma)\left[2\pi_L^P(0, C_R^P)(\varepsilon + \theta - \pi_L^P(0, C_R^P)\varepsilon)\right]+ \\
&+\beta(1-\sigma)^2\left[2\pi_L^P(0,0)(\varepsilon - \theta - \pi_L^P(0,0)\varepsilon)\right]+ \\
&+\beta\left(\theta - \frac{1}{2}\right) - \sigma(s_L\widehat{C}_L + s_R\widehat{C}_R).
\end{aligned} \tag{12}$$

Proof of Lemma 1. Adding up the expected utilities of the different types of voters (10), (11) and (12) we obtain an expression for voter welfare. We can write this as a function of f_L^P and f_R^P:

$$\begin{aligned}
W[f_L^P, f_R^P] \;=\; &\sigma^2[\pi_L^P(C_L^P, C_R^P)(\delta - f_L^P) + \left(1 - \pi_L^P(C_L^P, C_R^P)\right)(\delta - f_R^P)]+ \\
&\sigma(1-\sigma)[\pi_L^P(C_L^P, 0)(\delta - f_L^P)] + (1-\sigma)\sigma[\pi_R^P(0, C_R^P)(\delta - f_R^P)]+ \\
&+\eta\beta\sigma^2\left[2\pi_L^P(C_L^P, C_R^P)(\varepsilon + \theta - \pi_L^P(C_L^P, C_R^P)\varepsilon)\right]+ \\
&+\eta\beta\sigma(1-\sigma)[2\pi_L^P(C_L^P, 0)(\varepsilon + \theta - \pi_L^P(C_L^P, 0)\varepsilon)]+ \\
&+\eta\beta\sigma(1-\sigma)\left[2\pi_L^P(0, C_R^P)(\varepsilon + \theta - \pi_L^P(0, C_R^P)\varepsilon)\right]+ \\
&+\eta\beta(1-\sigma)^2\left[2\pi_L^P(0,0)(\varepsilon + \theta - \pi_L^P(0,0)\varepsilon)\right]+ \\
&+\beta\left(\theta - \frac{1}{2}\right) - \sigma(s_L l_L(f_L^P) + s_R l_R(f_R^P)).
\end{aligned} \tag{13}$$

From this expression it is immediate to see that if a policy leaves electoral probabilities unaltered and reduces favors it can be Pareto improving. It is necessary but not sufficient since if a policy requires limits and corresponding subsidies in order to maintain electoral probabilities unaltered, the tax cost of subsidies may offset the benefit from reducing favors. ■

Policies and Variations in Electoral Probabilities

Using (4) we can easily calculate the variations in the electoral probability of candidate L in all states for policies targeting one or both candidates, namely $\pi_L^P(\cdot,\cdot) - \pi_L(\cdot,\cdot)$

- *Policies Targeting one candidate only*

We have $\pi_L^P(C_L^*, C_R^P) - \pi_L(C_L^*, C_R^*) = \pi_L^P(0, C_R^P) - \pi_L(0, C_R^*) = \Delta\pi_L^1(C_R)$, where:

$$\Delta\pi_L^1(C_R) = \Psi(C_R^*)\left[\tfrac{(1-\rho_R^*)(\delta-f_R^*)}{4\beta\varepsilon}\right] - \Psi(C_R^P)\left[\tfrac{(1-\rho_R^P)(\delta-f_R^P)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right],$$

and $\pi_L^P(C_L^*, 0) - \pi_L(C_L^*, 0) = \pi_L^P(0, 0) - \pi_L(0, 0) = \Delta\pi_L^1(C_L)$, where:

$$\Delta\pi_L^1(C_L) = -\left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right].$$

Thus, a policy targeting one candidate only leaves electoral probabilities unaltered with respect to the unrestricted case, only if the following two conditions are satisfied:

$$\begin{aligned} \tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon} &= 0, && (14)\\ \Psi(C_R)\left[\tfrac{(1-\rho_R^*)(\delta-f_R^*)}{4\beta\varepsilon}\right] - \Psi(C_R^P)\left[\tfrac{(1-\rho_R^P)(\delta-f_R^P)}{4\beta\varepsilon}\right] &= 0. && (15) \end{aligned}$$

- *Policies targeting both candidates*

  We have:

$$\begin{aligned} \pi_L^P(C_L^P, C_R^P) - \pi_L(C_L^*, C_R^*) &= \Psi(C_L^P)\left[\tfrac{(1-\rho_L^P)(\delta-f_L^P)}{4\beta\varepsilon}\right] - \Psi(C_L^*)\left[\tfrac{(1-\rho_L^*)(\delta-f_L^*)}{4\beta\varepsilon}\right] + \\ &\quad + \Psi(C_R^*)\left[\tfrac{(1-\rho_R^*)(\delta-f_R^*)}{4\beta\varepsilon}\right] - \Psi(C_R^P)\left[\tfrac{(1-\rho_R^P)(\delta-f_R^P)}{4\beta\varepsilon}\right] + \\ &\quad + \left[\tfrac{\rho_L^P(\delta-f_L^P)-\rho_L^*(\delta-f_L^*)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right], \end{aligned}$$

$$\begin{aligned} \pi_L^P(C_L^P, 0) - \pi_L(C_L^*, 0) &= \Psi(C_L^P)\left[\tfrac{(1-\rho_L^P)(\delta-f_L^P)}{4\beta\varepsilon}\right] - \Psi(C_L^*)\left[\tfrac{(1-\rho_L^*)(\delta-f_L^*)}{4\beta\varepsilon}\right] + \\ &\quad + \left[\tfrac{\rho_L^P(\delta-f_L^P)-\rho_L^*(\delta-f_L^*)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right], \end{aligned}$$

$$\begin{aligned} \pi_L^P(0, C_R^P) - \pi_L(0, C_R^*) &= \Psi(C_R^*)\left[\tfrac{(1-\rho_R^*)(\delta-f_R^*)}{4\beta\varepsilon}\right] - \Psi(C_R^P)\left[\tfrac{(1-\rho_R^P)(\delta-f_R^P)}{4\beta\varepsilon}\right] + \\ &\quad + \left[\tfrac{\rho_L^P(\delta-f_L^P)-\rho_L^*(\delta-f_L^*)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right], \end{aligned}$$

$$\pi_L^P(0,0) - \pi_L(0,0) = \left[\tfrac{\rho_L^P(\delta-f_L^P)-\rho_L^*(\delta-f_L^*)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right].$$

Thus, a policy targeting both the two candidates leaves electoral probabilities unaltered with respect to the unrestricted case only if the following three conditions are satisfied:

$$\begin{aligned} \left[\tfrac{\rho_L^P(\delta-f_L^P)-\rho_L^*(\delta-f_L^*)}{4\beta\varepsilon}\right] - \left[\tfrac{\rho_R^P(\delta-f_R^P)-\rho_R^*(\delta-f_R^*)}{4\beta\varepsilon}\right] &= 0, && (16)\\ \Psi(C_L^P)\left[\tfrac{(1-\rho_L^P)(\delta-f_L^P)}{4\beta\varepsilon}\right] - \Psi(C_L^*)\left[\tfrac{(1-\rho_L^*)(\delta-f_L^*)}{4\beta\varepsilon}\right] &= 0, && (17)\\ \Psi(C_R^*)\left[\tfrac{(1-\rho_R^*)(\delta-f_R^*)}{4\beta\varepsilon}\right] - \Psi(C_R^P)\left[\tfrac{(1-\rho_R^P)(\delta-f_R^P)}{4\beta\varepsilon}\right] &= 0. && (18) \end{aligned}$$

Proof of Proposition 4.

- *Pure limit /ban one candidate* ($l_j < C_j^*$ for at least one $j; s_j = 0$ for every j)

Notice that conditions (14) and (15) apply only for the candidate that is being targeted, so they are independent of the other. For the sake of exposition we use candidate R. Notice that (14) can never be satisfied by a pure limit policy of $l_R < C_R^*$

- *Pure ban for both candidates* ($l_j = 0$ and $s_j = 0$ for every j)

Note that a complete ban implies that (16) becomes:

$$\left[\frac{\rho_R^*(\delta - f_R^*) - \rho_L^*(\delta - f_L^*)}{4\beta\varepsilon}\right] = 0,$$

which can never be satisfied whenever one candidate has an electoral advantage (i.e. $\theta \neq 0$). If for example$\theta > 0$, so that the leftist candidate has an ex-ante electoral advantage, it follows that $\rho_R^* > \rho_L^*$ and $(\delta - f_R^*) > (\delta - f_L^*)$ implying that the above equation is positive.

- *Pure limit policy for both candidates* ($l_j < C_j^*$ and $s_j = 0$ for every j)

To prove this it is sufficient to consider one candidate. We prove this for the leftist candidate, a symmetric argument holds for the rightist candidate. The necessary conditions for a policy to be Pareto improving are (16) and (17) (alternatively (18) for the rightist candidate). Taking into account (17), substituting ρ_j and simplifying, we obtain:

$$(\delta - f_L^P)\frac{\Psi(C_L^P)}{1-\sigma\Psi(C_L^P)} = (\delta - f_L^*)\frac{\Psi(C_L^*)}{1-\sigma\Psi(C_L^*)}. \tag{19}$$

Let us define $g(f, C) = (\delta - f)\frac{\Psi(C_L)}{1-\sigma\Psi(C_L)}$.

If only the influence motive applies we can express g as a function of f by substituting $\gamma(f)$ in g:

$$g(f) = \frac{\alpha\sigma(f\delta - f^2)}{f\alpha[\sigma(2-\sigma)-1]+\alpha\delta(1+\sigma^2)}.$$

Since we know that in the unrestricted equilibrium $f_j^* < \delta$ for every $j \in \{L, R\}$, $g(f)$ is a strictly increasing function if $f < \delta$.[19] Therefore, $g(f_L^P) \neq g(f_L^*)$ for every $f_j^P < f_j^*$, and so condition (19) can never be satisfied. Thus no pure limit Pareto improving policy exists in this case.

[19] This implies that the IG never raises favors beyond the level where the effectiveness of observing ads decreases.

If the electoral motive applies, for every policy that reduces contributions without reducing favors, $C_L^P < C_L^*$ and $f_L^P = f_L^*$ and we can write $g(\cdot)$ as $g(\overline{f}, C)$. Since $\frac{\partial g(\overline{f},C)/}{\partial C} > 0$, condition (19) can never be satisfied by pure limit policies that do not affect favors.

More generally, if the electoral motive holds we have that $g(f_L^*, C_L^*) > g(f_L^*)$ since $C_L^* > \gamma(f_L^*)$ and $\frac{\partial g(\overline{f},C)}{\partial C} > 0$. It follows that (19) can never be satisfied even when the limit on contributions reduces favors.

The other necessary condition that must be satisfied is (16) that can be written as:

$$\begin{aligned} & \tfrac{(\delta - f_L^P)}{1-\sigma\Psi(C_L^P)} - \tfrac{\Psi(C_L^P)(\delta - f_L^P)}{1-\sigma\Psi(C_L^P)} - \tfrac{(\delta - f_L^*)}{1-\sigma\Psi(^*C_L)} - \tfrac{\Psi(C_L)(\delta - f_L^*)}{1-\sigma\Psi(C_L^*)} \\ = \; & \tfrac{(\delta - f_R^P)}{1-\sigma\Psi(C_R^P)} - \tfrac{\Psi(C_R^P)(\delta - f_R^P)}{1-\sigma\Psi(C_R^P)} - \tfrac{(\delta - f_R^*)}{1-\sigma\Psi(C_R^*)} - \tfrac{\Psi(C_R)(\delta - f_R^*)}{1-\sigma\Psi(C_R^*)}. \end{aligned}$$

Using (19) we get:

$$\tfrac{(\delta - f_L^P)}{1-\sigma\Psi(C_L^P)} - \tfrac{(\delta - f_L^*)}{1-\sigma\Psi(C_L^*)} = \tfrac{(\delta - f_R^P)}{1-\sigma\Psi(C_R^P)} - \tfrac{(\delta - f_R^*)}{1-\sigma\Psi(C_R^*)}, \tag{20}$$

and we can express (19) as:

$$\Psi(C_j^P) = \frac{(\delta - f_j^*)\Psi(C_j^*)}{(\delta - f_j^P) + \sigma\Psi(C_j^P)(f_j^P - f_j^*)}, \tag{21}$$

and substituting in (20) we obtain:

$$f_L^* - f_L^P = f_R^* - f_R^P, \tag{22}$$

which implies that $f_L^P > f_R^P$ since $f_L^* > f_R^*$. ■

Proof of Proposition 5. $(l_j < C_j^*, s_j > 0\,; l_k = \infty, s_k = 0$ for $j \neq k)$

For the sake of exposition we use candidate R. If the electoral motive applies, a policy that limits contributions without reducing favors requires that $C_R^P = C_R^*$, so that $l_R(1+s_R) = C_R^*$. This guarantees that both (14) and (15) are satisfied, but it is never Pareto improving since it implies greater costs on citizens (in terms of taxes), without reducing favors. If instead the policy reduces equilibrium favors (whether the electoral motive applies or not), then it must be that $C_R^P > C_R^*$ for (14) to be satisfied, and so $l_R(1+s_R) > C_R^*$ implying that $(1-\rho_R^P)(\delta - f_R^P) > (1-\rho_R^*)(\delta - f_R^*)$. Notice, that in order for (15) to be satisfied, it must be that $\Psi(C_R^P) < \Psi(C_R^*)$ which implies that $C_R^P < C_R^*$. This is clearly a contradiction. ■

Proof of Proposition 6. $(l_j < C_j^*, s_j > 0$ for every $j)$

We define $\widehat{l}_L(>0)$ and $\widehat{l}_R(>0)$ as limits with associated favors f_L^P and f_R^P such that (22) is satisfied. For all limits $\widehat{l}_L$, and $\widehat{l}_R$, contribution levels $C_L^P(\widehat{l}_L)$ and $C_R^P(\widehat{l}_R)$ such that (19) holds, always exist. Therefore any limit $\widehat{l}_L$ and $\widehat{l}_R$ that induces the favors f_L^P and f_R^P, can be matched by corresponding subsidies s_L and s_R such that $\widehat{l}_L(1+s_L) = C_L^P(\widehat{l}_L)$ and

$\widehat{l}_R(1+s_R) = C_R^P(\widehat{l}_R)$. Thus, for all limits $\widehat{l}_j$, subsidies s_j are defined endogenously by the following relation:

$$s_j(\widehat{l}_j) = \frac{C_j^P(\widehat{l}_j)}{\widehat{l}_j} - 1.$$

Therefore, Pareto improving policies are completely defined by $\widehat{l}_L$ and $\widehat{l}_R$. ■

Designing a Pareto Improving Policy

We now address the issue of analyzing the trade-off involved in designing a Pareto improving policy.

The first thing to observe is that the greater is the distance $f_L^* - f_R^*$ the closer is f_L^P to the lower bound on favors given by f_R^P. In other words, the smaller is the ex-ante advantage of one candidate over the other, the more successful is campaign finance policy in reducing favors and therefore in increasing voter welfare.

The second thing is that reducing favors by setting a lower limit $\widehat{l}_j$ reduces the private cost of campaigns per candidate C_j^P, as can be shown from (21).

We can rewrite the expression for voter welfare (13) as function of $\widehat{l}_L$ and $\widehat{l}_R$:

$$\begin{aligned} W[\widehat{l}_L,\widehat{l}_R] \;=\; & \sigma^2[\pi_L(C_L,C_R)(\delta - f_L^P(\widehat{l}_L)) + (1-\pi_L(C_L,C_R)\,(\delta - f_R^P(\widehat{l}_R))] + \\ & +(1-\sigma)[\pi_L(C_L,0)(\delta - f_L^P(\widehat{l}_L))] + (1-\sigma)\sigma[\pi_R(0,C_R)(\delta - f_R^P(\widehat{l}_R))] + \\ & +\eta\beta\sigma^2\left[2\pi_L(C_L,C_R)(\varepsilon+\theta-\pi_L(C_L,C_R)\varepsilon)\right] + \\ & +\eta\beta\sigma(1-\sigma)[2\pi_L(C_L,0)(\varepsilon+\theta-\pi_L(C_L,0)\varepsilon)] + \\ & +\eta\beta\sigma(1-\sigma)\left[2\pi_L(0,C_R)(\varepsilon+\theta-\pi_L(0,C_R)\varepsilon)\right] + \\ & +\eta\beta(1-\sigma)^2\left[2\pi_L(0,0)(\varepsilon+\theta-\pi_L(0,0)\varepsilon)\right] + \\ & +\beta\left(\theta-\frac{1}{2}\right) - \sigma(C_L^P(\widehat{l}_R)) - \widehat{l}_L + C_R^P(\widehat{l}_R) - \widehat{l}_R), \end{aligned}$$

where $\pi_L(\cdot,\cdot)$ are the election probabilities of the unrestricted case, since any Pareto improving policy must maintain these unaltered. The tax cost for voters of subsidies to each candidate j is $s_j\widehat{l}_j = C_j^P(\widehat{l}_j) - \widehat{l}_j$. We know that for $\widehat{l}_j < C_j^*$, $f_j^P(\widehat{l}_j)$ is an increasing function of $\widehat{l}_j$, and $C_j^P(\widehat{l}_j)$ is an increasing function of f_j^P, therefore $\frac{\partial C_j^P(\widehat{l}_j)}{\partial \widehat{l}_j} > 0$.

Analyzing the first order conditions of $W[\widehat{l}_L,\widehat{l}_R]$ with respect to $\widehat{l}_j$ we obtain:

$$\frac{\partial W[\widehat{l}_L,\widehat{l}_R]}{\partial \widehat{l}_j} = -\frac{\partial f_j^P(\widehat{l}_j)}{\partial \widehat{l}_j}[\sigma^2\pi_j(C_j,C_{-j}) + \sigma(1-\sigma)\pi_j(C_j,0)] - \sigma[\frac{\partial C_j^P(\widehat{l}_j)}{\partial \widehat{l}_j} - 1] \leq 0.$$

The first term in the above expression represents the marginal loss of an increase in favors by candidate j as a consequence of increasing the limit on contributions, l_j. The second term instead represents the variation in the marginal cost in terms of taxes. Whenever increasing limits increases the total cost of contributions more than it decreases the cost in terms of taxes; that is, whenever $\partial C_j^P(\widehat{l}_j)/\partial \widehat{l}_j > 1$, an increase in limits generates an increase in taxes.

Analyzing (21) we obtain that $\partial C_j^P/\partial^2 f_j^P > 0$ which implies that $\partial C_R^P(\widehat{l}_R)/\partial \widehat{l}_R < \partial C_L^P(\widehat{l}_L)/\partial \widehat{l}_L$ since $\widehat{l}_R < \widehat{l}_L$ and $\partial f_j^P(\widehat{l}_j)/\partial \widehat{l}_j > 0$. This means that it is more likely that $\partial C_j^P(\widehat{l}_j)/\partial \widehat{l}_j < 1$ for small values of f_R^P, implying that raising the limit on contributions such that $f_R^P > \xi$,where ξ is positive but close to 0, can reduce the tax cost. However we know that the minimum level of interest group contributions for every level of favors are given by $\gamma(f_j^P)$. Since $\gamma(f_j^P)$ is a convex function we know that its inverse function in concave, and so $\partial f_j^P(\widehat{l}_j)/\partial^2 \widehat{l}_j < 0.$ This implies that the marginal cost in terms of favors is greater for lower levels of $\widehat{l}_j$. So this could offset the tax reduction effect of raising limits above the lower bound, and setting $f_R^P = \xi$ may turn out to be the optimal level of favors that satisfies (22).
Proof of Proposition 7. *($l_j = 0$ and $s_j = 0$ for at least one j)*

We analyze the welfare effects of a policy that requires little information on behalf of the policy maker. Therefore, we consider a policy that bans favors for at least one candidate without providing subsidies. Such a policy is welfare improving for any political equilibrium, if for any unrestricted equilibrium $(C_j^*, f_j^*)_{j\in\{L,K\}}$ where $f_L^* > 0$ and $f_R^* > 0$, either dW/df_L^* or dW/df_R^* are always negative. We analyze dW/df_L^* (the same reasoning applies for dW/df_R^*). Using the expression for voter welfare (13), we have:

$$\begin{aligned} dW/df_L \quad = \quad & \sigma^2\pi'(C_L^*)[f_R^* - f_L^*) + 2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(C_L^*, C_R^*))] \\ & \sigma(1-\sigma)\pi'(C_L^*)[(\delta - f_L^*) + 2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(C_L^*, 0))] + \\ & \sigma(1-\sigma)\pi'(0)[(f_R^* - \delta) + 2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(0, C_R^*))] + \\ & (1-\sigma)^2\pi'(0)[2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(0, C_R^*))] \\ & -\sigma^2\pi_L(C_L^*, C_R^*) - \sigma(1-\sigma)\pi_L(C_L^*, 0), \end{aligned}$$

where $\pi'(C_L) = \frac{d\pi(C_L,C_R)}{df_L} = \frac{d\pi(C_L,0)}{df_L}$ and $\pi'(0) = \frac{d\pi_L(0,0)}{df_L} = \frac{d\pi_L(0,C_R)}{df_L} < 0$. Notice that $\pi'(0) < 0$, since reducing favors and therefore contributions for a given candidate makes the event of not observing and ad from that candidate, a less informative signal on his lack of ability. Notice that $\pi'(C_L^*) = 0$ if the electoral motive does not apply. If the electoral motive applies reducing favors may attenuate or eliminate the incentive of the interest group to contribute to affect electoral probabilities. Thus, reducing favors for the ex-ante advantaged candidate may lead to a further reduction in welfare caused by a reduction in the effectiveness of observing ads from the electorally motivated interest group. For this reason it is sufficient to consider the terms that are different from zero when the electoral motive does not apply. Notice that:

$$\begin{aligned} \sigma(1-\sigma)\pi'(0)[(f_R^* - \delta) + 2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(0, C_R^*))] \quad &> \quad 0, \\ (1-\sigma)^2\pi'(0)[2\eta\beta(\varepsilon + \theta - 2\varepsilon\pi_L(0, C_R^*))] \quad &> \quad 0, \\ -\sigma^2\pi_L(C_L, C_R) - \sigma(1-\sigma)\pi_L(C_L, 0) \quad &< \quad 0. \end{aligned}$$

Therefore, the sign of dW/df_L can be either positive or negative depending on the equilibrium favors and contributions. Thus, such policies are not necessarily welfare improving. $\blacksquare$

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