The Comeback of Inflation as an Optimal Public Finance Tool

Giovanni Di Bartolomeo, Patrizio Tirelli, and Nicola Acocella

Department of Economics and Law, Sapienza University of Rome
DEMS, University of Milan Bicocca
MEMOTEF, Sapienza University of Rome

We challenge the widely held belief that New Keynesian models cannot predict optimal positive inflation rates. In fact, interest rates are justified by the Phelps argument that monetary financing can alleviate the burden of distortionary taxation. We obtain this result because, in contrast with previous contributions, our model accounts for public transfers as a component of fiscal outlays. We also contradict the view that the Ramsey policy should minimize inflation volatility and induce near-random-walk dynamics of public debt in the long run. In our model it should instead stabilize debt-to-GDP ratios in order to mitigate steady-state distortions. Our results thus provide theoretical support to policy-oriented analyses which call for a reversal of debt accumulated in the aftermath of the 2008 financial crisis.

JEL Codes: E52, E58, J51, E24.

1. Introduction

Optimal monetary policy analyses (Khan, King, and Wolman 2003; Schmitt-Grohé and Uribe, SGU henceforth, 2004a) identify two key
frictions driving the optimal level of long-run (or trend) inflation. The first one is the adjustment cost of goods prices, which invariably drives the optimal inflation rate to zero. The second one is monetary transaction costs that arise unless the central bank implements the Friedman rule, i.e., a zero nominal inflation rate in steady state. In their survey of the literature, SGU (2011) argue that the optimality of zero inflation is robust to other frictions, such as nominal wage adjustment costs, downward wage rigidity, hedonic prices, the existence of an untaxed informal sector, and the zero bound on the nominal interest rate. This latter result is broadly confirmed by Coibion, Gorodnichenko, and Wieland (2012), who find that the optimal inflation rate is low, typically less than 2 percent, even when the economy is hit by costly but infrequent episodes at the zero lower bound. A consensus therefore seems to exist that monetary transaction costs are relatively small at zero inflation, and that implementing low and stable inflation is the proper policy.

This theoretical result is in sharp contrast with empirical evidence. For instance, both in the United States and in the euro area, average inflation rates over the 1970–99 period have been close to 5 percent. Even the widespread central bank practice of adopting inflation targets between 2 percent and 4 percent is apparently at odds with theories of the optimal inflation rate (SGU 2011).

Furthermore, following the buildup of large stocks of debt in the aftermath of the 2007–8 financial crisis, some economists have argued that the public debt surge should be reversed and that a temporary increase in inflation might be necessary to achieve this goal. For instance, Rogoff (2010) suggests that “two or three years of slightly elevated inflation strikes me as the best of many very bad options.” Blanchard, Dell’Ariccia, and Mauro (2010) point at the potential role of the inflation tax as one among several distortionary taxes which are available to policymakers. Aizenman and Marion (2011) predict that a 6 percent inflation rate would reduce the debt-to-GDP ratio by 20 percent within four years. These contributions are in line with the well-known Phelps (1973) argument that to alleviate the burden of distortionary taxation, it might be optimal for governments to resort to monetary financing, driving a wedge between the private and the social cost of money.

The Phelps argument has been widely investigated in the framework of general equilibrium models, and never found sufficient to
warrant the optimality of a significantly positive inflation rate. Two main results have been established. The first one is that distortionary taxation does not warrant deviations from the Friedman rule unless factor incomes are sub-optimally taxed (see SGU 2011 and references cited therein). The underlying intuition is that since all resources are eventually used for consumption, then the inflation tax, which affects consumption transaction costs, is desirable only to the extent that other taxes have a sub-optimal effect on consumption. The second main result is that when the goods market characterization is modified to account for (sub-optimally taxed) monopolistic distortions, numerical simulations suggest that the optimal inflation rate is negative and very close to zero, even accounting for the Phelps effect (SGU 2004a). This conclusion carries over to the optimality of near-zero volatility of inflation and near-random-walk behavior in government debt and tax rates in response to shocks, implying that the recent increase of public debt in developed economies should be regarded as a tax-smoothing device in response to the financial crisis.

Our paper reconsiders the importance of the Phelps effect and obtains results that challenge the optimality of near-zero inflation rates when the tax system is incomplete. We show that a non-negligible inflation rate might indeed be optimal and that inflation (and tax rates) volatility should be exploited in order to stabilize debt-to-GDP ratios in the long run.

The starting point in our analysis is that the optimal zero-inflation result obtained in dynamic stochastic general equilibrium (DSGE) models with incomplete tax systems is the consequence of unrealistic assumptions about the size and composition of public expenditure. In the literature, standard calibrations of public expenditures focus on public-consumption-to-GDP ratios, typically set at 20 percent (SGU 2004a; Aruoba and Schorfeide 2011). This follows a long-standing tradition in business-cycle models, where only public consumption decisions have real effects. In our framework this choice is not correct, because the focus here is on distortionary financing of public expenditures in steady state, where also other components of public expenditure matter. To the best of our knowledge, the only exception is SGU (2006), who determine the optimal inflation rate in a medium-scale model where public consumption and transfers respectively amount to about 20 percent and 9 percent of GDP.
They find that the optimal inflation rate is positive but very small, half a percentage point, and that the inclusion of public transfers accounts for a 0.7 percent increase in the optimal inflation rate. Their intuition for the inflationary effect of public transfers is that only transfers are pure rents to households and inflation is an indirect tax on those pure rents.

As a matter of fact, public consumption accounts for a limited component of the overall public expenditures in OECD countries, and transfers are relatively large (table 1).

We show that just allowing for a plausible parameterization of public consumption and transfers in the SGU (2004a) model reverses the standard conclusion about the optimal inflation rate, which now monotonically increases from 2 percent to 12 percent as the transfers-to-GDP ratio goes from 10 percent to 20 percent. Further, our calculations contradict the claim that public transfers per se require an inflation tax (SGU 2006). In fact, we also find that, absent public transfers, very large public-consumption-to-GDP ratios are also associated with a positive inflation rate. For instance, we obtain that the optimal inflation rate monotonically increases from 2 percent to 12 percent as the public-consumption-to-GDP ratio grows from 40 percent to 47 percent. Given the historically observed public consumption ratios, these latter results are not empirically relevant, but they challenge received wisdom about the reasons why level and composition of public expenditures should matter for the identification of the optimal inflation rate.

By working with a simplified version of our model, we are able to show that changes in public consumption and public transfers would generate identical variations in the optimal inflation rate if public consumption did not affect the aggregate resource constraint. The limited incentive to inflate that we observe in response to a public consumption variation is due to its contemporaneous effects that operate through the aggregate resource constraint and impact on (i) inflation- and labor-tax revenues, and (ii) the planner’s desired marginal rate of substitution between consumption and leisure.

We also investigate the optimal fiscal and monetary policy responses to shocks. The issue is admittedly not new, but we are able to provide new contributions to the literature. When prices are
Table 1. Government Expenditures and Revenues, 1998–2008 (average ratios to GDP)

<table>
<thead>
<tr>
<th></th>
<th>Public Consumption</th>
<th>Other Public Expenditures</th>
<th>Total Revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>18.00</td>
<td>16.97</td>
<td>36.26</td>
</tr>
<tr>
<td>Austria</td>
<td>19.10</td>
<td>32.29</td>
<td>49.71</td>
</tr>
<tr>
<td>Belgium</td>
<td>22.13</td>
<td>27.82</td>
<td>49.93</td>
</tr>
<tr>
<td>Canada</td>
<td>19.49</td>
<td>21.56</td>
<td>42.08</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>21.24</td>
<td>22.81</td>
<td>40.12</td>
</tr>
<tr>
<td>Denmark</td>
<td>25.84</td>
<td>27.88</td>
<td>55.96</td>
</tr>
<tr>
<td>Finland</td>
<td>21.75</td>
<td>27.74</td>
<td>53.12</td>
</tr>
<tr>
<td>France</td>
<td>23.39</td>
<td>29.21</td>
<td>52.60</td>
</tr>
<tr>
<td>Germany</td>
<td>18.96</td>
<td>27.58</td>
<td>46.11</td>
</tr>
<tr>
<td>Greece</td>
<td>16.52</td>
<td>28.32</td>
<td>44.19</td>
</tr>
<tr>
<td>Hungary</td>
<td>21.98</td>
<td>27.42</td>
<td>43.40</td>
</tr>
<tr>
<td>Ireland</td>
<td>15.11</td>
<td>29.84</td>
<td>45.25</td>
</tr>
<tr>
<td>Italy</td>
<td>19.10</td>
<td>21.28</td>
<td>40.38</td>
</tr>
<tr>
<td>Japan</td>
<td>17.07</td>
<td>21.28</td>
<td>38.35</td>
</tr>
<tr>
<td>Netherlands</td>
<td>23.57</td>
<td>22.19</td>
<td>45.34</td>
</tr>
<tr>
<td>New Zealand</td>
<td>17.97</td>
<td>20.89</td>
<td>42.01</td>
</tr>
<tr>
<td>Norway</td>
<td>20.76</td>
<td>23.54</td>
<td>55.51</td>
</tr>
<tr>
<td>Poland</td>
<td>17.95</td>
<td>25.34</td>
<td>43.29</td>
</tr>
<tr>
<td>Portugal</td>
<td>19.57</td>
<td>25.48</td>
<td>44.25</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>20.24</td>
<td>21.35</td>
<td>41.59</td>
</tr>
<tr>
<td>Spain</td>
<td>17.75</td>
<td>21.52</td>
<td>36.55</td>
</tr>
<tr>
<td>Sweden</td>
<td>26.67</td>
<td>29.03</td>
<td>55.70</td>
</tr>
<tr>
<td>Switzerland</td>
<td>11.4</td>
<td>23.48</td>
<td>34.80</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>19.83</td>
<td>22.28</td>
<td>40.38</td>
</tr>
<tr>
<td>United States</td>
<td>15.26</td>
<td>20.51</td>
<td>33.77</td>
</tr>
<tr>
<td>Euro Area</td>
<td>20.17</td>
<td>27.11</td>
<td>45.39</td>
</tr>
</tbody>
</table>

Source: OECD.

flexible and governments issue non-contingent nominal debt (Chari, Christiano, and Kehoe 1991), it is optimal to use inflation as a lump-sum tax on nominal wealth, and the highly volatile inflation rate allows to smooth taxes over the business cycle. This result is intuitive insofar as taxes are distortionary whereas inflation volatility is costless. SGU (2004a) show that when price adjustment is costly,
optimal inflation volatility is in fact minimal and long-run debt adjustment allows to obtain tax smoothing over the business cycle. In our paper, the SGU result is reversed, even when the amount of public transfers is relatively small (12 percent of GDP). In this case, tax and inflation volatility are exploited to limit debt adjustment in the long run.

The interpretation of our result is simple. As discussed above, public transfers increase the tax burden in steady state. In this case, the accumulation of debt in the face of an adverse shock—which would work as a tax-smoothing device in SGU (2004a)—is less desirable, because it would further increase long-run distortions. To avoid such distortions, the policymaker is induced to front-load fiscal adjustment and to inflate away part of the real value of outstanding nominal debt. Consumption smoothing is therefore reduced relative to SGU (2004a).

To the best of our knowledge, this is the first study of the optimal interaction between inflation and tax policies when transfers account for the relatively large proportion of public expenditures that is documented in the data. A number of recent papers have analyzed the macroeconomic implications of public transfer schemes, but their focus is different from ours. Alonso-Ortiz and Rogerson (2010) investigate the labor supply response and the welfare implications of an optimal public transfer scheme in the context of a model with idiosyncratic productivity shocks, incomplete financial markets, and flexible prices. Oh and Reis (2011) analyze the role of transfers for consumption stabilization in the context of heterogeneous agents, incomplete markets, and sticky prices—when taxes are lump sum, no public debt accumulation is allowed and the central bank is constrained to implement a zero-inflation policy. Angelopoulos, Philippopoulos, and Vassilatos (2009) maintain the representative-agent hypothesis and incorporate an uncoordinated redistributive struggle for transfers into an otherwise standard DSGE model. Zubairy (2014) investigates the consequences of temporary public transfer shocks in an estimated representative-agent DSGE model.

The remainder of the paper is organized as follows. The next section describes the model. Section 3 introduces the Ramsey policy and illustrates our main results. Section 4 discusses optimal monetary and fiscal stabilization policies. Section 5 concludes.
2. The Model

We consider a simple infinite-horizon production economy populated by a continuum of households and firms whose total measures are normalized to one. Monopolistic competition and nominal rigidities characterize product markets. The labor market is competitive. A demand for money is motivated by assuming that money facilitates transactions. The government finances an exogenous stream of expenditures by levying distortionary labor income taxes and by printing money. Optimal policy is set according to a Ramsey plan.

As discussed by SGU (2011), positive inflation may be a desirable instrument if some part of income is sub-optimally taxed. In the narrow framework of our model, the choice of inflating the economy depends on untaxed monopolistic profits in the goods market, and the introduction of a uniform income tax would reduce the incentive to inflate. However, the tax system might be incomplete or sub-optimal for other reasons. For instance, one might take into account the existence of an informal sector of the economy, or introduce monopolistic competition in the labor market. Here the model is deliberately simple to highlight the theoretical challenge to the claim that price stability is indeed optimal even when the tax system is incomplete. Providing a complete quantitative analysis of the optimal inflation rate is beyond the scope of the paper.

2.1 Households

The representative household \((i)\) maximizes the following utility function:

\[
U = E_{t=0}^{\infty} \beta^t u(c_{t,i}, l_{t,i}) ;
\]

\[
u(c_{t,i}, l_{t,i}) = \ln c_{t,i} + \eta \ln (1 - l_{t,i}) ,
\]

where \(\beta \in (0, 1)\) is the intertemporal discount rate, \(c_{t,i} = \left( \int_0^1 c_{t,i}(j)^\rho \, dj \right)^{\frac{1}{\rho}}\) is a consumption bundle, and \(l_{t,i}\) denotes the individual labor supply. The consumption price index is \(P_t = \left( \int_0^1 p_t(j)^\varphi \, dj \right)^{\frac{\varphi - 1}{\varphi}}\).
The flow budget constraint in period $t$ is given by

$$c_t,i (1 + s(v_{t,i})) + \frac{M_{t,i} + B_{t,i}}{P_t} = (1 - \tau_t) w_{t,i} t_{t,i} + \frac{M_{t-1,i}}{P_t}$$

$$+ \theta_t + \frac{R_{t-1} B_{t-1,i}}{P_t} + t_t,$$  \hspace{1cm} (2)

where $w_{t,i}$ is the real wage; $\tau_t$ is the labor income tax rate; $t_t$ denotes real fiscal transfers; $\theta_t$ is firms' profits; $R_t$ is the gross nominal interest rate; and $B_{t,i}$ is a nominally riskless bond that pays one unit of currency in period $t+1$. $M_{t,i}$ defines nominal money holdings to be used in period $t+1$ in order to facilitate consumption purchases.

Consumption purchases are subject to a transaction cost

$$s(v_{t,i}), \quad s'(v_{t,i}) > 0 \text{ for } v_{t,i} > v^*,$$  \hspace{1cm} (3)

where $v_{t,i} = \frac{P_{t,i} c_{t,i}}{M_{t,i}}$ is the household’s consumption-based money velocity. The features of $s(v_{t,i})$ are such that a satiation level of money velocity ($v^* > 0$) exists where the transaction cost vanishes and, simultaneously, a finite demand for money is associated with a zero nominal interest rate. Following SGU (2004a) the transaction cost is parameterized as

$$s(v_{t,i}) = Av_{t,i} + \frac{B}{v_{t,i}} - 2\sqrt{AB}.$$  \hspace{1cm} (4)

The first-order conditions of the household’s maximization problem are

$$c_t(j) = c_t \left( \frac{p_t(j)}{P_t} \right)^{1-\rho}$$  \hspace{1cm} (5)

---

2. Our results are robust to the alternative specification for the transaction cost used by Brock (1989) and Kimbrough (2006), which implies a Cagan (1956) money demand function. A proof is available upon request. The model is also compatible with Baumol (1952) demand for money (see SGU 2004a).
3. When solving its optimization problem, the household takes as given goods and bond prices. As usual, we also assume that the household is subject to a solvency constraint that prevents it from engaging in Ponzi schemes.
\begin{align*}
\lambda_t &= \frac{u_c(c_t, l_t)}{1 + s(v_t) + v_t s'(v_t)} \\
\lambda_t &= \beta E_t \left( \frac{\lambda_{t+1} R_t}{\pi_{t+1}} \right) \\
\lambda_t (1 - \tau_t) w_t &= -u_t (c_t, l_t) \\
\frac{R_t - 1}{R_t} &= s'(v_t) v_t^2.
\end{align*}

Equation (5) is the demand for the good \( j \). As in SGU (2004a), condition (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth that vanishes only if \( v = v^* \). Equation (7) is a standard Euler condition where \( \pi_{t+1} = P_{t+1}/P_t \) denotes the gross inflation rate. Equation (8) defines the individual labor supply condition. Finally, equation (9) implicitly defines the money demand function, such that

\begin{equation}
\frac{M_t}{P_t} = \left( \frac{R_t - 1}{R_t A} + \frac{B}{A} \right)^{-\frac{1}{2}} c_t.
\end{equation}

\subsection*{2.2 Firms’ Pricing Decisions}

Each firm \((j)\) produces a differentiated good \(^4\)

\[ y_t(j) = z_t l_{t,j}, \]

where \( z_t \) denotes a productivity shock \(^5\).

We assume a sticky-price specification based on Rotemberg (1982) quadratic cost of nominal price adjustment:

\begin{equation}
\frac{\xi}{2} y_t (\pi_t - 1)^2,
\end{equation}

\(^4\)We abstract from capital accumulation and assume constant returns to scale of employed labor. The consequences of these two assumptions are discussed in SGU (2006) and SGU (2011), respectively. Our results are not affected by the introduction of diminishing returns to scale for labor (simulation results available upon request).

\(^5\)We assume that \( \ln z_t \) follows an AR(1) process.
where $\xi_p > 0$ is a measure of price stickiness. In line with Ascari, Castelnuovo, and Rossi (2011), we assume that the reoptimization cost is proportional to output.\(^6\)

In a symmetrical equilibrium, the price adjustment rule satisfies

$$
\frac{z_t (\rho - mc_t)}{1 - \rho} + \xi_p \pi_t (\pi_t - 1) = E_t \beta \frac{y_{t+1} \lambda_{t+1}}{y_t \lambda_t} \xi_p [\pi_{t+1} (\pi_{t+1} - 1)],
$$

where

$$
mc_t = \frac{1}{z_t} w_t. \tag{14}
$$

From (5) it would be straightforward to show that $\frac{1}{\rho} = \mu^p$ defines the price markup that obtains under flexible prices.

\subsection*{2.3 Government Budget and Aggregate Resource Constraints}

The government supplies an exogenous, stochastic, and unproductive amount of public good $g_t$ and implements exogenous transfers $t_t$. Government financing is obtained through a labor income tax, money creation, and issuance of one-period, nominally risk-free bonds. The government’s flow budget constraint is then given by.\(^7\)

$$
R_{t-1} b_{t-1} + g_t + t_t = \tau_t w_t l_t + \frac{M_t - M_{t-1}}{P_t} + b_t, \tag{15}
$$

where $b_t = \frac{B_t}{P_t}$ defines real debt.

The aggregate resource constraint closes the model:

$$
y_t = c_t (1 + s(v_t)) + g_t + \frac{\xi_p}{2} y_t (\pi_t - 1)^2. \tag{16}
$$

\footnote{Our results are independent of this assumption. A proof is available upon request.}

\footnote{As in SGU (2004a), $\ln(g_t/y_t)$, is assumed to evolve exogenously following an independent $AR(1)$ process. We assume instead that the level of the real transfer ($t_t/y_t$) is non-stochastic.}
3. Ramsey Policy

3.1 Optimal Fiscal and Monetary Policy

The Ramsey policy is a set of plans \( \{c_t, l_t, \lambda_t, mc_t, \pi_t, v_t, R_t, \tau_t, b_t\}_{t=0}^{+\infty} \) that maximizes the expected value of (1) subject to the competitive equilibrium conditions (6), (7), (8), (9), (11), (13), (14), (15), and (16), and to the exogenous fiscal and technology shocks. Given (6), (8), and (14), labor-tax revenues may be written as

\[
\tau_t w_l = \left( z_t mc_t + \frac{u_l(c_t, l_t) (1 + s(v_t) + v_t s'(v_t))}{u_c(c_t, l_t)} \right) l_t.
\]  

Condition (17) simply states that government fiscal revenues are equivalent to the wedge between the firm’s wage cost and the household’s desired wage rate.

The Lagrangian of the Ramsey planner problem can be written as follows:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, l_t) + \lambda_t^{AR} \left[ z_t l_t - c_t (1 + s_t) - g_t - \frac{\xi_p z_t l_t (\pi_t - 1)^2}{2} \right] 
+ \lambda_t^B \left[ \lambda_t - \beta \frac{\lambda_{t+1} R_t}{\pi_{t+1}} \right] 
+ \lambda_t^{GBC} \left[ \frac{c_t}{v_t} + \frac{B_t}{P_t} + \left( z_t m c_t + \frac{u_l(c_t, l_t) [1 + s(v_t) + v_t s'(v_t)]}{u_c(c_t, l_t)} \right) l_t 
- R_{t-1} \frac{B_{t-1}}{P_{t-1}} - \frac{c_{t-1}}{\pi_t v_{t-1}} - g_{t-1} - t_{t-1} \right] 
+ \lambda_t^{Ph} \left[ \frac{\beta y_{t+1} \lambda_{t+1} \xi_p \pi_{t+1} (\pi_{t+1} - 1)}{y_t \lambda_t} - \frac{z_t (\rho - m c_t)}{1 - \rho} \right] 
- \frac{\xi_p \pi_t (\pi_t - 1)}{y_t \lambda_t} \right\} + \lambda_t^{MUC} \left[ \frac{u_c(c_t, l_t)}{1 + s(v_t) + v_t s'(v_t)} - \lambda_t \right],
\]

where \( R \) and \( s(v) \) are defined in (4) and (9), respectively.

The solution requires numerical simulations. For the sake of comparison, we calibrate our model as SGU (2004a). The time unit

\textsuperscript{8} These are obtained implementing SGU (2004b) second-order approximation routines.
is meant to be a year; the subjective discount rate $\beta = 0.96$ is consistent with a steady-state real rate of return of 4 percent per year. Transaction cost parameters $A$ and $B$ are set at 0.011 and 0.075, the debt-to-GDP ratio is set at 0.44 percent, the benchmark level for the public-consumption-to-GDP ratio is 0.20, the gross price markup is 1.2, and the annualized Rotemberg price adjustment cost is 4.375 (this implies that firms change their price on average every nine months; see SGU 2004a, p. 210). The preference parameter $\eta$ is set so that in the flexible-price steady state households allocate 20 percent of their time to work when public transfers are nil.

In figure 1 we describe the steady-state optimal inflation response to the transfer increase and to a corresponding variation in public consumption in addition to the benchmark 20 percent value. Both public consumption and transfers are defined as GDP ratios: $g_{PC} = g_t/y_t$ and $g_{PT} = t_t/y_t$. Simulations show that steady-state inflation rapidly increases when $g_{PT}$ grows beyond the 8 percent threshold. For instance, the optimal inflation rate is close to 3 percent when $g_{PT}$ is 10 percent, and exceeds 12 percent when the transfer ratio is 20 percent. When public expenditure is confined to public consumption, a 40 percent public-consumption-to-GDP ratio is associated with a 2 percent optimal inflation rate, and optimal inflation monotonically grows up to 12 percent as the public consumption share reaches 47 percent.$^9$

Note that when different public consumption and transfers levels induce the Ramsey planner to choose identical inflation rates, we also obtain identical consumption, labor market, and inflation wedges, $s(v), \frac{1+s(v)+vs'(v)}{(1-\tau)},$ and $\frac{\xi}{2} l (\pi - 1)^2$, respectively (figure 2). It is also interesting to note that when either $g_{PT}$ or $g_{PC}$ reach the levels which trigger the optimality of positive inflation, the optimal policy generates an almost identical consumption pattern. In both cases, abandoning price stability allows to stabilize consumption in spite of the increasing burden of fiscal revenues.

It is interesting to compare our results with the interpretation of the inflationary outcome generated by the need to finance transfers offered by SGU (2006, p. 385). They claim that when the private

---

$^9$As pointed out above, in our model untaxed monopolistic profits are necessary to generate the planner’s incentive to inflate. For instance, if one sets $\mu^p = 1.1$, the optimal inflation rate remains close to zero for $g_{PT} \leq 15$ percent.
sector must receive an exogenous amount of after-tax transfers, it is optimal to exploit the inflation tax on money balances in order to impose an indirect levy on the (transfers-determined) source of household income. Given our finding that relatively large levels of public consumption exist such that the planner chooses identical inflation rates, this intuition must be incorrect.

One mechanism driving the choice of the optimal policy mix might be related to the distortionary taxation necessary to finance the additional transfers, which adversely affects the labor supply and reduces the tax base, whereas the increase in public consumption generates a negative wealth effect that triggers a positive labor supply response and expands the tax base. In this case, the incentive to increase inflation should be much reduced. To check the importance of the wealth effect of \( g_{PC} \) on the labor supply, we solved the Ramsey problem under a different specification of the utility function, such as the Greenwood-Hercowitz-Huffman (1988; GHH henceforth) preferences,

\[
GHHu(C_{t,i}, l_{t,i}) = \frac{\left(C_{t,i} - \eta l_{t,i}^{1+\phi}\right)^{1-\sigma}}{1 - \sigma}.
\] 

(18)
Notes: The wedges are computed as follows: Price adjustment cost is (12). The consumption wedge is (4). The labor wedge is divided by $\lambda$. 

Figure 2. Policy Wedges and Consumption 

[Graphs showing the relationships between public consumption, public transfers, and adjustment cost related to policy wedges.]
Under (18) the marginal rate of substitution \( -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \eta l^{\phi}_{t,i} \) is independent of consumption, i.e., there is no wealth effect on the labor supply, and the labor market equilibrium condition in steady state is \( \eta l^{\phi} = w \). Simulations contradict our conjecture. In fact, under GHH preferences we obtain almost identical inflation rates in response to the variations in \( g_{PC} \) and \( g_{PT} \) considered in figure 1. In particular, an increase in public consumption is met by an expansion in the labor tax, whereas inflation remains very close to zero unless public consumption is relatively large.\(^{10}\)

3.2 The Ramsey Solution in a Simplified Model

Further insights can be obtained by imposing restrictions on some parameter values, which allow to simplify the Ramsey solution in the steady state.\(^{11}\) To begin with, we set \( \beta = 1, \xi_p = 0 \). In this case, the Friedman rule is satisfied for \( \pi = 1 \) and price adjustment frictions do not matter, restricting the policymaker’s trade-off to two dimensions: the Friedman rule calls for complete price stability, whereas the public finance motive calls for positive inflation because the tax system is incomplete. From (13) it is easy to see that in steady state \( w = \rho \) irrespective of the inflation rate. We also assume that steady-state debt is nil, and set \( B = 0 \) in (4). Under this latter assumption,\(^{12}\) we obtain \( s(v) = Av \) and, since \( R = \frac{\pi}{\beta}, v = \left( \frac{R-1}{AR} \right)^{\frac{1}{2}} = \left( \frac{\pi-1}{A\pi} \right)^{\frac{1}{2}} \).\(^{13}\)

By setting \( g_{PC} = 0.2 \) and \( g_{PT} = 0 \), the optimal inflation rate in the simplified model is \( \pi = 1.3 \) percent, whereas for the full model we obtained \( \pi = -0.16 \) percent. This result is obviously due to the assumed reduction in inflation costs, but our focus here is on obtaining a better understanding of the reason why similar levels of public transfers and public consumption are associated with different optimal inflation rates in steady state. In this regard, note that a 5 percent increase in \( g_{PC} \) now is matched by \( \pi = 1.8 \) percent, whereas an identical variation in \( g_{PT} \) is associated with \( \pi = 2.6 \) percent.

\(^{10}\) Results are available upon request.  
\(^{11}\) See the appendix for a derivation of our results.  
\(^{12}\) With \( B = 0 \), the model is characterized by a standard Tobin money demand.  
\(^{13}\) The result described in this section can also be obtained by removing the assumptions \( \xi_p = B = 0 \), but in this case the algebra is rather cumbersome and it is more difficult to support the intuition.
In this simplified model, the steady-state Ramsey solution is characterized by $\lambda^{P_h} = \lambda^{MUC} = 0$ and $\lambda^B = -\lambda^{GBC} \frac{c}{v^2 \pi}$. This latter condition implies that the marginal effect of inflation on the Euler equation constraint must equal the marginal effect of $\pi$ on the government budget constraint. Further, we obtain that the marginal effect of money velocity on the aggregate resource constraint must equal its marginal effect on the government budget constraint, i.e.,

$$\lambda^{AR} c_s'(v) = \lambda^{GBC} \left[ \frac{R'(v)}{v^2 \pi^2} - \frac{\pi - 1}{v^2 \pi} - \frac{2\delta Al}{1 - l} \right] c. \tag{19}$$

Finally, the solution for $\lambda^{GBC}$ is

$$\lambda^{GBC} = u_c(c, l) \left[ \left( \frac{R'(v)}{v^2 \pi^2} - A - \frac{2\delta Al}{1 - l} \right) \frac{1 + s(v)}{s'(v)} + \frac{1}{v} \frac{\pi - 1}{\pi} - \frac{\delta l \gamma(v)}{1 - l} \right]^{-1}. \tag{20}$$

The Ramsey planner’s choice of $c$ takes into account effects on the marginal utility of consumption; on the aggregate resource constraint, $\lambda^{AR} \left[ 1 + s(v) \right]$; and on the government budget constraint, where $\frac{1}{v} \frac{\pi - 1}{\pi}$ and $-\frac{\delta l \gamma(v)}{1 - l}$ define consumption effects on revenues from inflation and from labor taxes, respectively. In a sense, just like monetary transaction costs drive a wedge between the marginal utility of consumption and the marginal utility of wealth in the representative-household first-order condition (6), here monetary transaction costs and the need to enforce distortionary taxation drive a wedge between the Ramsey planner’s marginal utility of consumption and the marginal utility of revenues.

The optimal labor supply condition is

$$-u_h(c, l) = \lambda^{AR} + \lambda^{GBC} \left( \rho - \frac{\delta c \gamma(v)}{(1 - l)^2} \right) \tag{21}$$

$$= \left( \frac{R'(v)}{Av \pi^2} - 1 - \frac{2\delta l}{1 - l} + \rho - \frac{\delta c \gamma(v)}{(1 - l)^2} \right) \lambda^{GBC}. \tag{22}$$

The right-hand side of (21) accounts for the marginal effects of $l$ on the aggregate resource constraint (23), which is proportional to
the multiplier $\lambda^{AR}$, and on the government budget constraint (24), which is determined by the multiplier $\lambda^{GBC}$ and by the marginal effect of $l$ on tax revenues, $\left(\rho - \frac{\delta c\gamma(v)}{(1-l)^2}\right)$. This is the Ramsey planner’s equivalent of the representative-agent first-order condition (8).

Using (19), (20), and the explicit functional forms for $v$, $s(v)$, and $s'(v)$, the Ramsey planner’s problem collapses to the following conditions:

\[
c = \frac{1 - g_{PC}}{1 + \left(A\frac{\pi-1}{\pi}\right)^{1/2}}l
\]

\[
\frac{c\pi - 1}{lv\pi} + \left\{\rho - \frac{\delta c}{1-l}\left[1 + 2\left(A\frac{\pi-1}{\pi}\right)^{1/2}\right]\right\} = g_{PC} + g_{PT}
\]

\[
\frac{\delta c}{1-l} = \frac{\frac{1-\pi^2}{\pi^4} - \frac{\delta l}{1-l} + \rho - \delta c\left(\frac{1}{2} + \left(A\frac{\pi-1}{\pi}\right)^{1/2}\right)}{\left(\frac{1-\pi^2}{\pi^4} - \frac{\delta l}{1-l}\right)[1 + \left(A\frac{\pi-1}{\pi}\right)^{1/2}] + \frac{1}{2}\left(A\frac{\pi-1}{\pi}\right)^{1/2} - \frac{\delta l\left(\frac{1}{2} + \left(A\frac{\pi-1}{\pi}\right)^{1/2}\right)}{1-l}}
\]

Conditions (23) and (24), respectively, are the aggregate resource and government balance constraints that the Ramsey planner solution must satisfy. Condition (25) simply rearranges (21). It is easy to see that changes in $g_{PC}$ and $g_{PT}$ would generate identical variations in consumption hours and inflation if public consumption did not enter the aggregate resource constraint (23). Therefore, the smaller incentive to inflate that we observed in response to a public consumption variation is due to its contemporaneous effects that operate through the aggregate resource constraint and impact on (i) inflation- and labor-tax revenues, i.e., on the left-hand side of (24), and (ii) the planner’s desired marginal rate of substitution in (25).

In figure 3 we present a graphical solution. Substituting for $c$ from (23) into (24), we obtain the $GBC$ schedule which defines combinations of $l$ and $\pi$ that are consistent with a balanced government budget constraint for given values of $\rho$, $g_{PC}$, and $g_{PT}$. It is upward sloping\(^{14}\) because an increase in employment can be obtained through a labor-tax reduction. This, in turn, requires an increase in

\(^{14}\)It is very steep for the small inflation range (1.00–1.10) used in figure 3.
inflation in order to compensate for the revenue loss. By substituting (23) into (25), we obtain the MRS schedule which defines combinations of $l$ and $\pi$ such that the Ramsey planner’s desired marginal rate of substitution obtains for given values of $\rho$ and $g_{PC}$. It is downward sloping because an increase in $l$ brings the consumption-to-labor ratio below its desired level. A fall in inflation is therefore necessary to raise desired relative consumption. The MRS and GBC schedules are plotted by assuming $g_{PC} = 0.2$ and $g_{PT} = 0$. The corresponding Ramsey equilibrium is then point A.

Figure 4 describes the effects on the Ramsey equilibrium of a 5 percent increase in public expenditure. Specifically, panel A

\footnote{Due to its complex functional form, the slope of (25) is not trivial. Note that, in addition to $g_{PC}$, (25) includes only two calibrated parameters, $A$ and $\delta$, which take values 0.011 and 2.9 as in SGü (2004a). We experimented for values of $A$ and $\delta$ in the ranges $10^{-4}$–10 and 0.5–8, respectively. In all cases, we obtained a downward-sloping MRS schedule around realistic values for $l$ and $\pi$ (including the Ramsey equilibrium). Note that the different values for $\delta$ imply an equilibrium for $l$ between about 0.1 and 0.9. Therefore, we explored the range of all possible values.}
Figure 4. The Effects of a Change in Government Expenditures

![Graph Illustrating Effects](image)

A. Public transfers variations.  
B. Public consumption variation.

illustrates the effects of an increase in public transfers, whereas panel B shows the effects of an equivalent increase in public consumption. The common initial equilibrium in the two panels is described by point A, where \( g_{PC} = 0.2 \) and \( g_{PT} = 0 \).

Starting from point A in panel A, the 5 percent increase in \( g_{PT} \) shifts the \( GBC \) locus to the left to \( GBC' \) because, holding inflation constant, the increase in the tax rate necessary to balance the budget inevitably reduces employment. As pointed out above, the \( MRS \) schedule is not affected by \( g_{PT} \) and the new equilibrium \( A' \) is characterized by a relatively large increase in inflation.

The effects of a 5 percent increase in \( g_{PC} \), panel B, are more complex. Consider the \( GBC \) locus; in this case, for any given value of \( l \), private consumption must fall, causing a twofold effect on government revenues. On the one hand, the reduction in real money holdings lowers inflation-tax proceedings. On the other hand, from (17) we know that for any given value of \( l \), the lower private consumption is associated with larger fiscal revenues. This latter effect unambiguously dominates, limiting the leftward shift of \( GBC' \).

\[16\] This latter effect also helps to explain why incentives to inflate remain limited when the utility function is characterized by GHH preferences.
Turning to the $MRS$ locus, we find that an increase in $g_{PC}$ now also causes a rightward shift to $MRS'$. This happens because for any given level of inflation, the planner seeks a reduction in leisure to partly offset the reduction in consumption determined by the increase in $g_{PC}$. Thus, relative to the increase in $g_{PT}$, the shift in $MRS$ would cause larger inflation, but this is dominated by the corresponding shift in $GBC$.

4. Optimal Monetary and Fiscal Stabilization Policies

In this section we investigate whether our characterization of steady-state public expenditures also bears implications for the conduct of macroeconomic policies over the business cycle. SGU (2004a) show that, when public transfers are nil, costly price adjustment induces the Ramsey planner to choose a minimal amount of inflation volatility and to select a permanent public debt response to shocks in order to smooth taxes over the business cycle. Benigno and Woodford (2004), who emphasize the complementarity between fiscal and monetary policies, substantially confirm the optimality of near-zero inflation volatility for a plausible degree of nominal price stickiness.

We discuss how the optimal fiscal and monetary stabilization policies change when, in steady state, $g_{PT}$ is 0.1 instead of zero, whereas other fiscal figures are assumed to be 0.44 for debt-to-GDP ratio and $g_{PC} = 0.2$. In table 2 we show that the volatility of both taxes and inflation dramatically increases, whereas the strong persistence of taxes vanishes. Thus, even if we still obtain a unit root in the dynamic process for debt accumulation, a more realistic calibration of fiscal outlays has important implications for the dynamic pattern of fiscal and monetary stabilization policies. To grasp this intuition, consider the impulse response functions to a temporary increase in government purchases (figure 5).

---

17 We consider a productivity shock and a public consumption shock. Properties of stochastic processes are described in table 2. We compute the second-order approximation using SGU (2004b) routines (see also SGU 2004a, section 7).

18 To sharpen the analysis, we assume the shock is not serially correlated.

19 Additional experiments are reported in the working paper version of the paper (downloadable from Ideas). Specifically, there we report the impulse response functions for different composition of the public expenditure and levels of government debt.
Table 2. Dynamic Properties of the Ramsey Allocation (second or approx.)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Auto. Corr.</th>
<th>corr(x,y)</th>
<th>corr(x,g)</th>
<th>corr(x,z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{PT} = 0, g_{PC} = 0.2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>25.19</td>
<td>1.062</td>
<td>0.759</td>
<td>-0.305</td>
<td>0.436</td>
<td>-0.236</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-0.16</td>
<td>0.177</td>
<td>0.034</td>
<td>-0.108</td>
<td>0.374</td>
<td>-0.275</td>
</tr>
<tr>
<td>$R$</td>
<td>3.82</td>
<td>0.566</td>
<td>0.863</td>
<td>-0.942</td>
<td>-0.044</td>
<td>-0.962</td>
</tr>
<tr>
<td>$y$</td>
<td>0.21</td>
<td>0.007</td>
<td>0.820</td>
<td>1.000</td>
<td>0.204</td>
<td>0.938</td>
</tr>
<tr>
<td>$l$</td>
<td>0.21</td>
<td>0.003</td>
<td>0.823</td>
<td>-0.085</td>
<td>0.590</td>
<td>-0.402</td>
</tr>
<tr>
<td>$c$</td>
<td>0.17</td>
<td>0.007</td>
<td>0.824</td>
<td>0.940</td>
<td>-0.123</td>
<td>0.954</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Auto. Corr.</th>
<th>corr(x,y)</th>
<th>corr(x,g)</th>
<th>corr(x,z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{PT} = 0.1, g_{PC} = 0.2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>42.69</td>
<td>2.860</td>
<td>-0.053</td>
<td>-0.110</td>
<td>0.284</td>
<td>-0.356</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.46</td>
<td>0.962</td>
<td>-0.054</td>
<td>-0.062</td>
<td>0.304</td>
<td>-0.309</td>
</tr>
<tr>
<td>$R$</td>
<td>5.50</td>
<td>0.489</td>
<td>0.775</td>
<td>-0.790</td>
<td>0.142</td>
<td>-0.926</td>
</tr>
<tr>
<td>$y$</td>
<td>0.17</td>
<td>0.005</td>
<td>0.823</td>
<td>1.000</td>
<td>0.408</td>
<td>0.884</td>
</tr>
<tr>
<td>$l$</td>
<td>0.17</td>
<td>0.003</td>
<td>0.714</td>
<td>-0.237</td>
<td>0.699</td>
<td>-0.651</td>
</tr>
<tr>
<td>$c$</td>
<td>0.13</td>
<td>0.005</td>
<td>0.783</td>
<td>0.851</td>
<td>-0.091</td>
<td>0.985</td>
</tr>
</tbody>
</table>

Notes: In the table, $\tau$, $\pi$, $R$, $y$, $l$, and $c$ stand for the tax rate, inflation rate, nominal interest rate, output, hours, and consumption, respectively.
Figure 5. Fiscal Shock Impulse Response Functions under Different Levels of Public Transfers

Notes: The solid line shows no transfers and the dashed line shows 10 percent of the transfers-to-GDP ratio. The figure shows impulse responses to an i.i.d. government purchases shock. The size of the innovation in government purchases is one standard deviation (a 3 percent increase in $g$). The shock takes place in period 1. Public debt, consumption, and output are measured in percent deviations from their pre-shock levels. The tax rate, the nominal interest rate, and the inflation rate are measured in percentage points.

Under both scenarios, the permanent debt adjustment allows to smooth tax distortions. However, the different magnitudes of the permanent debt and tax adjustments associated with the two cases ($g_{PT} = 0$ and $g_{PT} = 0.1$) are also evident. When $g_{PT} = 0.1$, the long-run debt adjustment is reduced by 70 percent. In this case, long-run tax and inflation distortions are already relatively large, and the steady-state accumulation of debt in the face of an adverse shock becomes less desirable. Instead, the planner finds it optimal to front-load tax adjustment and to inflate away part of the real value of outstanding nominal debt. In addition, the increase in inflation has
a positive impact on seigniorage revenues. This explains the surge in inflation volatility reported in table 2. Our model is also able to match the positive empirical correlation between average inflation and inflation variability. For the sake of fairness, it is worth noticing that inflation volatility still appears to be substantially limited relative to the case of flexible prices, which is the main point of SGU (2004a). Our contribution here is that a substantial complementarity exists between inflation and taxes in response to the public consumption shock.

5. Conclusions

Incompleteness of the tax system is a necessary condition for the existence of a public finance justification for inflation. The strong point of SGU (2004a, 2011) was to argue that irrespective of the incompleteness of the tax system, optimal inflation should be between zero and the Friedman rule.

The point of this paper is that for the same incompleteness of the tax system, a non-negligible inflation rate in steady state is indeed optimal if one adopts a realistic calibration for fiscal outlays, including public transfers. Differently from SGU (2011), who argue that central bank inflation targets are too high, our contribution shows that a 2 percent target might indeed be too low.

However, to obtain an empirically relevant assessment of the optimal inflation rate, the model should be extended to account for a number of country-specific factors, such as governments’ ability to optimally tax factor incomes, composition of public expenditures, monetary transaction costs, other frictions such as nominal wage stickiness, and the existence of an informal sector. All this should be done bearing in mind that the tax system incompleteness probably is an inherent feature of modern economies. Similar considerations can be made concerning inflation costs. For instance, Calvo pricing, which implies price dispersion, might generate higher inflation costs than Rotemberg pricing, but one should also take into account inflation indexation and its correlation with the underlying

\[^{20}\text{See, e.g., Friedman (1977), Ball and Cecchetti (1990), Caporale and McKier-}
\]

\[^{20}\text{nan (1997).}\]
inflationary regime, as shown in Fernández-Villaverde and Rubio-Ramírez (2008). All this is left for future research.

Further, our analysis of the optimal fiscal and monetary stabilization policies strengthens the Benigno and Woodford (2004) argument that the two policy tools should be seen as complements and that the monetary authority should consider the consequences of their actions for the government budget. In this regard, we show that a substantial amount of inflation volatility is indeed desirable to deflate nominal debt and to limit the accumulation of real debt in the long run. Our results thus provide theoretical support to policy-oriented analyses which call for a reversal of debt accumulated in the aftermath of the 2008 financial crisis and for a reconsideration of the role of inflation in facilitating debt reductions.

Appendix

The steady-state solution of the Ramsey problem defined in section 3 is characterized by the following set of first-order conditions:

\[ l = [1 + s(v)] c + g_c l + \frac{\xi_p}{2} l(\pi - 1)^2 \quad (26) \]

\[ 1 = \beta r(v) \frac{1}{\pi} \quad (27) \]

\[ \frac{c}{v} + b + [mc + Z\gamma(v)] l = \frac{r(v)b}{\pi} + \frac{c}{v\pi} + (g_{PC} + g_{PT}) l \quad (28) \]

\[ \xi_p(1 - \beta)\pi(\pi - 1) = \frac{mc - \rho}{1 - \rho} \quad (29) \]

\[ u_c(c, l) = \lambda \gamma(v) \quad (30) \]

\[ u_c(c, l) - \lambda^{AR}[1 + s(v)] + \left[ \frac{1}{v} \left( 1 - \frac{\beta}{\pi} \right) - \frac{\delta}{1 - l} \gamma(v) \right] \lambda^{GBC} + \lambda^{MUC} u_{cc} = 0 \quad (31) \]

\[ u_l(c, l) + \lambda^{AR} \left( 1 - \frac{\xi_p}{2} (\pi - 1)^2 \right) \]

\[ + \lambda^{GBC} \left[ mc - \left( \frac{\delta c}{1 - l} + \frac{\delta c}{(1 - l)^2} l \right) \gamma(v) \right] = 0 \quad (32) \]
\[
\left(1 - \frac{r(v)}{\pi}\right) \lambda^B + (1 - \beta) \frac{\lambda^{Ph}}{\lambda} \pi (\pi - 1) - \lambda^{MUC} \gamma(v) = 0 \tag{33}
\]

\[
- \lambda^{AR} s'(v)c - \lambda^B \beta r'(v) \frac{\lambda}{\pi}
\]

\[
- \lambda^{GBC} \left[ \left(1 - \frac{\beta}{\pi}\right) \frac{c}{v^2} + \frac{\delta c}{1 - l} \gamma'(v) + \frac{\beta br'(v)}{\pi}\right]
\]

\[
- \lambda^{MUC} \lambda \gamma'(v) = 0 \tag{34}
\]

\[
- \lambda^{AR} \xi_p (\pi - 1) l + \frac{1}{\pi^2} \left[ \lambda^B r(v) \lambda + \lambda^{GBC} \left( r(v) b + \frac{c}{v} \right) \right] = 0 \tag{35}
\]

\[
\pi = \beta r(v) \tag{36}
\]

\[
\xi_p \lambda^{GBC} l = - \frac{\lambda^{Ph}}{1 - \rho}, \tag{37}
\]

where we have expressed public consumption and transfers as GDP ratio (i.e., \(g = g_{PC} l\) and \(t = g_{PT} l\), recall that \(y = l\)) and \(r(v) = \frac{1}{1 - s'(v) v^2}\) from (9).

As said in the main text, we impose \(\beta = 1, b = 0, \xi_p = 0,\) and \(B = 0\). In that case, \(v = \left(\frac{R - 1}{AR}\right)^{\frac{1}{2}} = \left(\frac{\pi - 1}{A\pi}\right)^{\frac{1}{2}}, s(v) = Av, s'(v) = A,\) and \(\gamma(v) = 1 + s(v) + vs'(v) = 1 + 2A^{\frac{1}{2}} \left(\frac{\pi - 1}{\pi}\right)^{\frac{1}{2}}.\)

From (37) we get \(\lambda^{Ph} = 0.\) Then from equation (33) we get \(\lambda^{MUC} = 0.\) Equation (29) implies \(mc = \rho.\) From (35) we obtain

\[
\lambda^B \frac{\lambda}{\pi} = - \left[ \lambda^{GBC} \frac{c}{v \pi^2} \right] \tag{21}
\]

Then substitute \(\lambda^B \frac{\lambda}{\pi} = - \lambda^{GBC} \frac{c}{v \pi^2}\) into (34) to obtain

\[
\lambda^{AR} = \frac{\lambda^{GBC}}{s'(v)} \left[ \frac{\rho'(v)}{v \pi^2} - A - \frac{2 \delta A l}{1 - l} \right]. \tag{38}
\]

Substituting for \(\lambda^{AR}\) in (31), we obtain

\[
\lambda^{AR} = \frac{U_c}{s'(v)} \left[ \frac{\rho'(v)}{v \pi^2} - A - \frac{2 \delta A l}{1 - l} \right] \tag{38}
\]

\[
\lambda^s = u_c(c, l) \left\{ \left[ \frac{\rho'(v)}{v \pi^2} - A - \frac{2 \delta A l}{1 - l} \right] \frac{1 + s(v)}{s'(v)} + \frac{1}{v} \left( \frac{\pi - 1}{\pi} \right) - \frac{\delta l}{1 - l} \gamma(v) \right\}^{-1} \tag{39}
\]

\[21\]This latter condition implies that the marginal effect of \(\pi\) on savings must equal the marginal effect of \(\pi\) on the government budget constraint.
Then substituting for $\lambda^{GBC}$, $\lambda^{AR}$ into (32), we get

$$\left[ \frac{2}{\pi^2} \frac{1}{\pi^2} - 1 - \frac{2\delta l}{1-l} \right] + \left\{ \rho - \frac{\delta c\gamma(v)}{1-l} - \frac{\delta c\gamma(v)l}{(1-l)^2} \right\} = \frac{c}{1-l}. \quad (40)$$

The model is then solved using (26), (28), and (40).

References


