From a correlation (covariance) matrix, the determination of significant signals results in a functional connectivity network. Then, the pattern of functional connectivity is summarized into a map of the brain. As the correlation coefficient increases, the number of links included in the brain network decreases.

Bayes False Discovery Rate (FDR) and Bayes Power (BP): The corresponding theoretical test statistic $t$ follows a Student's $t$ distribution ($n - 1$ degrees of freedom) with $p = 2 - 2F_{2}\left(\gamma\right)$, where $F_{2}$ is the Student's $t$ distribution function.

Empirical Bayesian Estimates: The cumulative distribution function of the $p$-value is estimated as $F_{2}\left(\gamma\right) = \left(1 - F_{2}\left(\gamma\right)\right)^{2}$, where $F_{2}$ represents the empirical cumulative distribution function.

Balancing FDR and BP: To identify the optimal values of the tuning parameter $\lambda_{1}$ and $\lambda_{2}$ we had to balance the Bootstrap version of $F_{2}\left(\gamma\right)$ and $|\lambda_{2}|\alpha$ over a range of $\lambda$ values.

Data acquisition and preprocessing: We acquired MRI scans of two healthy women, a 33 and a 67 years old, both with a normal cognitive profile and no evidence of medical disorders.

We selected $\tau = 0.2$ for the elderly participant and $\tau = 0.25$ for the young one as the values which ensure the best balance between the Bayes FDR and the BP. In fact, the trade-off between the Bayes FDR and the BP requires that the test statistic $t$ be greater than the maximum value of $\gamma$ such that the Bayes FDR is smaller than the maximum. A smaller $\gamma$ means that we may detect a smaller number of links.

Data analysis: Graphical representation in stereotactic coordinates of the brain networks of the healthy participants (woman, 33 years old (left panel) and woman, 67 years old (right panel)). Each grey line represents the significant links.

Simulation Study with Independent $p$-values: We simulated $n = 1000$ independent and normally distributed random variables $Z_{i} \sim N(0,1)$ under the null hypothesis $\mu = 0$. We computed the associated $p$-value $p_{i} = 2 \Phi\left(-\frac{|\lambda_{2}|\alpha}{\tau}\right)$, where $\Phi\left(x\right)$ is the cumulative distribution function of the standard normal variable. To provide Monte Carlo estimates we repeated our simulation $N = 10000$ times for each parameter $\tau = 0.0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5$.

Simulation Studies with Dependent $p$-values: We assumed the set of random variables $Z_{i} \sim N(0,1)$ to have a multivariate normal distribution. Each marginal distribution $Z_{i} \sim N(0,1)$ has the same variance and mean equal to 0 under the null hypothesis while 1 under the alternative hypothesis. We explored three different patterns of dependency among variables:

1. autoregressive pattern of dependency: each correlation among pairs of random variables equals to $Corr\left(Z_{i}, Z_{i+p}\right) = \rho^{p}$ and we fixed $\rho = 0.6$.
2. unstructured pattern of dependency: given a covariance matrix made up as block matrix with $100 \times 100$ matrices of size 10 x 10 its diagonal, we fixed such correlation within clusters equal to the others. We simulated the covariance pattern with respect to three different constant covariances $c = 0.0, 0.15, 0.3$.

References:


A Bayesian approach to False Discovery Rate and Power in Multiple Testing

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