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Abstract

Falsification in Financial Advice

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This work gives another fundamental explanation for the stylized fact of active management underperformance. I develop a model of delegated portfolio management with adverse selection, where the preferences of investor and advisor are misaligned. The information structure I focus on is intermediate between the cases of private and public information: the advisor can distort the observed Sharpe ratio at some cost. I show that investor may strictly prefer the contract that induces falsification as it helps him to manipulate the information rents of the agent. Within the model I show that investor prefers to deal with the advisors that have higher partisan objective as it leads to higher expected utility. I study how optimal contract and welfare changes when the information structure changes from private to public. The relation between the social welfare and the degree of information publicness is non-monotonic: although social welfare is maximum under pure public information, increasing the publicness of information may decrease the welfare.

Contents

Introduction

The role of financial advisor is crucial nowadays. During the last decades the demand for financial advice dramatically increased. In modern times the major part of financial wealth is not managed by the savers, but through intermediaries. According to a 2015 report by Investment Adviser Association there is a steady growth not only by the number of advisers (from 2012 up 5.3 percent) but also by the amount of assets managed by SEC-registered investment advisers (up 8.1 percent).

More empirical studies confirm the importance of financial advisors. The study of 2007 by Bergstresser, Charlmers and Tufano (2008) states that about 80% of mutual funds claimed they received financial advice. Other survey presented by Hung et al. (2008) reveals that 73% of all US investors were seeking for financial advice. Also in the online survey among the buyers of investment product conducted by Chater et al. (2010) 80% of respondents answered that they were buying through the intermediary, as 58% of them revealed that their choice was influenced by advisor.

However, while these studies demonstrate a trend of growing demand for financial advice, there exist the facts of a poor performance of active management compared to the passive one. For example, Gruber (1996) demonstrated average mutual fund underperformance of 65 basis points per year. Later French (2008) updated this result to 67 basis points per year. In their next survey of mutual funds returns Fama and French (2010) obtain the simulation results that are different. Now there is slightly more evidence of performance, which leads to more shaded conclusions. For example, before this study, the simulation tests on net returns produced no evidence of returns that are able cover costs. The new results say that a small fraction of managers covers costs. However, the active management still underperform. Bergstresser et al. (2007) look at the performance of mutual funds comparing the returns when the investment was made directly and through the brokers. And they find that the returns in the first case are lower. Chen et al. (2004) reports that the mutual funds that are ruled externally underperform compared to those that are managed internally.

There may be several reasons for this evidence of underperformance in the face of steady demand. First one lies in the institutional nature of Investor-advisor relationship: advisor possess an information which is hidden from the investor so that the standard agency problem arises. The delegated portfolio management literature shows that in certain frameworks it is more difficult for principal to sign an incentive compatible contract.

The second reason lie in misalignment of the interests: the financial advisors have the objectives that are not in line with the ones of investor, which creates the possibility for the opportunistic behavior. A number of authors study the conflict of interests in the settings of strategic communication, when the commissions, so called "kickbacks", influence the behavior of an agent. The majority of works, starting from the pioneer article by Crawfrod and Sobel (1982), assume that the conflict already exists and focus on its origins. Inderst and Ottaviani (2009), however, look at this conflict endogenously and show how it may affect the size of "misselling".

The third reason of the active management underperformance could be financial illiteracy. It seems very trustworthy that a financially educated investor has a better understanding of financial markets and would be keen on making the decision on his own, without intermediaries. As demonstrates van Rooij et al. (2011) financial literacy is a significant factor for the financial market participation among Dutch households. One can assume that if financial literacy increases the probability of buying a financial product, it is also increasing the probability of doing so without an advisor. However, the empirical literature on this topic demonstrates that advice is demanded by knowledgeable investors. From the paper by Lusardi and Mitchell (2006) it is clear that less literate investors tend to seek the advice from the informal sources like relatives, colleagues, neighbors while more literate ones use the formal channels. These results were confirmed also with analysis on Netherlands data pool by van Rooij et al. (2011).

In my study I propose another possible explanation of active management underperformance.

I assume that the problem may lay not only in misalignment of preferences of investor and agent but also in the fact that falsification is a part of an optimal contract for investor. There is a broad evidence that relationships between principal and agent are characterized by systematic distortion of the information. According to Coalition against Insurance Fraud, falsification takes about 10% of property-casualty insurance losses per year. Nearly one-third of insurers say fraud was as high as 20% of the costs. Many empirical papers, like, e.g. Artis et al. (1999) try to measure the actual amount or fraction of fraudulent claims in different insurance markets. For example, Caron and Dionne (1997) find that about 10% of all claims in the Quebec automobile insurance market can be attributed to fraudulent behavior.

I claim that investor may strictly prefer the contract that induces falsification as it helps him to reduce the information rents and get an additional surplus from extracting a part of agent kickback commission. To analyze the role of information distortion I use the delegated portfolio management framework. An investor has a desire to invest his money into a risky asset but he doesn't have the knowledge to do that. So he hires a financial advisor to provide him with the information based on which investment decision has to be made. I assume that the preferences of the investor and advisor are not aligned as advisor has a conflict of interests. But unlike the majority of studies, see for example Piccolo, Puopolo and Vasconcelos (2013), I model not only bias to partisan objective but also the cost of information distortion. The agent has the possibility to distort the signal about the state of nature at a cost. When this cost is close to zero the model goes to the version of private information and when it is prohibitively costly for agent to lie the public information case arises.

In the second chapter I develop a simple model of fully private information. Agent faces a conflict of interest as he has professional objective of reporting the truth and a partisan objective, - a commission paid by the third party. This conflict is modeled in such a way that the problem of countervailing incentives arises so the agent has a motive not just to overstate the return of the risky asset but understate it as well. The optimal behavior of the agent compared to the first-best benchmark includes a constant understatement of the possible asset return. The degree of distortion depends on the size of partisan objective: the higher the partisan objective is the less understatement arises. Welfare analysis shows that social welfare, computed as a sum of surpluses of investor and agent, depends positively on the amount of commission paid to advisor. Moreover, for a relatively high partisan objective, investor is better off from the higher partisan objective: he gets an additional surplus from "partisan rents" of advisor.

In the third chapter I add the costs of information distortion to the model. Now it is costly for an advisor to distort the signal, this expense can be interpreted for example as time needed to make the fake report or efforts to manipulate the facts. The optimal contract still includes some amount of falsification which is less though than in the purely private information model. As the informational structure is now only partly private it becomes more difficult for advisor to extract her informational rents, and, in opposite, it is easier now for investor to manipulate with the incomes of the agent. The investor's unconditional expected utility is now higher the higher partisan objective is as there are more possibilities for him to extract the "partisan rents"

Welfare analysis gives an interesting results. Unlike one may assume that the social welfare depends positively on magnitude of distortion costs the model shows that relation between these two parameters is non-monotonic: although social welfare is maximum under the pure public information, increasing the publicness of information may decrease the welfare. This result is accordant with some second-best theorems. For example, increasing the number of markets can be socially harmful when the social optimum is achieved at perfect competition state. Another implication can be the following: if investor wants to invest in financial regulations that will make the process of lying more costly for advisor, the social value of this investment my fall short of the principal's private value and thus the investment will exceed the socially optimal level.

The work is organized as following: Chapter 1 provides the review of literature connected with the present research. Chapter 2 describes the framework and environment, a simple model of financial advice. In Chapter 3 a model with costs of falsification is presented.

Chapter 1

Literature review

The present work contributes to different spheres of research, namely: delegated portfolio management, financial advice, costly information distortion and countervailing incentives literature. This chapter provides an overview of the studies connected with our work.

1.1 Delegated portfolio management

One of the biggest survey on the delegated portfolio management was made by Stracca (2005) where he highlighted the importance of investor-advisor relationship within principal-agent framework and gives brief reviews of the most significant works. He makes an accent on the differences of delegated portfolio management models from standard agency problem and brilliantly sums up the existent challenges of the sphere. Existing works highlight a lot of important aspects of delegated portfolio management problem and further we will discuss the most important ones for our research.

We start our review with the works of Bhattacharya and Pfleiderer (1985) (further BH) and Allen (1985) who were the first ones who presented the models where the investor wants to get the truthful information about the financial asset: rate of return and/or the riskiness, from a better informed agent. Their models are focused on the understanding of the information market and the deal more with the case of hidden information. "In addition to being important in its own right, a full understanding of the market for information is therefore necessary for explaining the existence and operation of many of the intermediaries that are observed".[1]

In the model of BP, investors that formed a mutual fund want to hire a manager to do a research for them. The problem that a mutual fund faces is to screen a big number of agents that are different in their forecasting abilities, better forecasters have higher opportunity costs. The utility functions of principal and the agents are observable. Investor's problem is to design a contract that insures that an advisor has a better forecasting abilities than their own and obtain at least a utility as high as their reserved one. Once an agent is hired he observes a private signal, and the second problem of the principal is that of eliciting the agent's private signal in order to make the right portfolio decision. Bhattacharya and Pfleiderer show the existence of an optimal payment schedule in which agents truthfully report their forecasting ability and their private signal. They show that the quadratic scoring rule can elicit the mean of a random variable if the distribution is symmetric and the agent is a (weakly) risk-averse expected utility maximizer.

Allen is also focused on the adverse selection problem and, however, the structure of his model is very similar to the one of BP in terms of nature of the information, instead of looking at the problem as investors hiring advisors, he prefers to consider an information owner as a monopolist over the information. In this case the possessor of the information receives the surplus from the information unlike in BP's model

[1] Allen "Information-Contracting in Financial Markets," in Financial Markets and Incomplete Information-Frontiers of Modern Financial Theory, Volume 2, edited by S. Bhattacharya and G. M. Constantinides, 1989, 188-197, Rowman and Littlefield.

where the investor obtain it. Allen also assumes that the alternative opportunity of the advisor is endogenous and his utility function is unobservable. The timing of the game in his model is the following:

- (t=0) A seller announces a set of payment schedules to potential buyers, one schedule for each signal
- (t=1) The nature draws the return and advisor receive the signal
- (t=2) The contracts are signed
- (t=3) Seller observes the signal and announces the corresponding payment to the buyer
- (t=4) Markets meet and the risky asset's payoff is realized.

The key problem for the seller is to make his actions credible: the buyers need to be sure that he will observe the signal after signing the contract. Although the set of constraints is complicated the tractable version of solution can be found for each possible signal. As in BP model, it involves a quadratic function of the returns, but due to the difference in the assumptions they are not entirely the same.

Our work is similar to the one of BP and Allen as it has the same focus on a hidden information, but the framework structure is closer to BP's one. The main difference lies in the defining of manager's preferences: we generate the conflict of interest between investor and advisor on providing the truthful advice (countervailing incentives) and make the falsification possible in the equilibrium outcome.

Another work we share an approach with is the one by Piccolo, Puopolo and Vasconcelos (2013) where the authors study the interaction between small investors and their financial advisor when the last one has private information about the asset and the objective of the investor and advisor are not identical. This interaction takes place in a particular environment where the money transfer between principal and the agent does not exist and the relationships between them are not exclusive.

This is a model of non-exclusive financial advice where two investors have a desire to invest but do not have the enough knowledge or skills and have to get the help of a professional advisor. The formal analysis is developed under the hypothesis that the investors choose their equilibrium portfolio allocations by committing to the direct mechanisms, however, later the authors show that the same result can be achieved with a certain delegation rule and that this rule is robust to the renegotiation. As our work, this model follows the adverse-selection approach, the interests of the investor and the advisor are misaligned, but we focus on the exclusive advice and the welfare applications.

1.2 Financial Advice

There is a vast quantity of literature on financial advice. We would like to highlight a few directions of research that are of particular interest of ours. A particularly relevant strand of the literature is the one that focuses on advisors' conflict of interests whose preferences are biased towards a partisan objectives. The less informed investor seeks the advice from a better informed advisor in the strategic communication framework. The most famous model of this kind was proposed by Ottaviani (2000) where he was following classic cheap-talk tradition (Crawford and Sobel (1982)). Investors in his model differ in the level of sophistication and can choose between the full delegation of their decision and the possibility of being engaged in a cheap talk. The nature of communication game changes dramatically as the level of investors' sophistication goes down, the equilibrium is totally revealing and includes distortions. However, if the preferences of the investor and advisor are not too different, some information can be successfully communicated in the equilibrium.

This result is generalized by Dessein (2002) who shows that less informed principals tend to delegate their decisions rather than interact with the agent. He suggests that there exist some complementarity between information and the demand for professional advice.

Relying on these studies we model the conflict of the interest of advisor in the same manner but using the agency problem framework.

Hackethal et al. (2012) study the question of delegation from an empirical perspective using German data and show that richer and experienced investors with potentially higher financial literacy tend to delegate more often to financial advisors. The explanation they provide for this is related to the higher opportunity cost of time of richer investors.

Calgano and Monticone (2015) show that there are cases when it is more profitable for the advisors not to reveal the information he possesses and that this happen when the investors are less knowledgeable.

The other direction of research is connected with the reasons of this conflict of interest. In many works mentioned above this bias is simply assumed, while in paper by Inderst and Ottaviani (2009) it is endogenous and takes the origins from the agency relation between the advisor and the firm which defines the optimal compensation in a way it induces the agent to sell and to missell. In their latter work Inderst and Ottaviani (2012) overview the pros and cons of various market interventions as "conflict of interest [...] can turn advice into a curse rather than blessing for consumers".

Our paper contributes to this strand of literature with another way of modelling the conflict of interest and also adds the cost of information distortion as another possible origin of this conflict

1.3 Costly Information distortion

As we have already mentioned in the Introduction, falsifications are the often characteristics of the agency relationship. The standard principal-agent models (see, for example, Laffont and Matrimort (2001)) are unable to explain the fact why the principal may strictly prefer the contract that induces falsification. Two big strands of the literature were developed as an explanation. The first one, Costly State Falsification, was inspired by the pioneer work by Lacker and Weinberg (1989). There are two main assumptions within it: the monitoring of the agent's action is not feasible and the information distortion is costly. In a model of costly state falsification, Lacker and Weinberg (1989) study the optimal contract in which a risk-averse agent observes a state of the nature after signing the contract and can falsify with a cost this state. They have shown that optimal contracts may involve paying the manager a fixed amount in the lowest states and sharing profits between manager and investor in higher states. Their model explains this result through the agent's risk aversion. Another work where the costly information distortions are represented through costs of the agent is the paper by Maggi and Rodriguez-Claire (1995). They present a model where falsification is a part of optimal contract, they study how the equilibrium payoffs change with a change of informational structure from private to public. They conclude that there exist a strong relationship between the level of publicness of the information and the strength of the countervailing incentives.

With these two works we share the way of distortion costs representation that can be interpreted as the communication costs, when the agents' abilities to misrepresent the information are limited, Deneckere and Severinov (2001), or as the costs of manipulating or falsifying information, for example the time needed to "cook the numbers" or produce a falsifying report, Lacker and Weinberg (1989), Maggi and

Rodriguez-Claire (1995).

The work of Lacker and Weinberg was applied to an insurance settings by Crocker and Morgan (1997). In their model it is assumed that policyholders may be involved in costly falsification process in order to magnitude of their losses in an environment where verification of claims is not possible. Here also the coverage schedule affects the incentives to claim falsification, but the cost of generating insurance claims through falsification differs among policyholders according to their true level of loss. These differential costs make it possible to implement loss-contingent insurance payments with some degree of claims falsification at equilibrium.

The lying costs also have recently been introduced into the framework of strategic communication games by Kartik (2009). Lying costs transform the standard Crawford and Sobel (1982) cheap-talk model into one of costly signaling, although the induced structure is different from traditional "monotonic" signaling games. The analysis shows how lying costs can significantly affect the outcome of strategic information transmission. The focus of these last two models is quite different from ours as well as the environment, as we are looking at the problem of falsification in delegated portfolio management framework.

The second direction of research looking into the falsifications as a part of an optimal contract is "Costly State Verification". This paradigm is attributed to the work of Townsend (1979). Here the principal is exposed to the costs in order to observe a signal of a true state, and the agent cannot manipulate the information. The focus of these models as well as their structure is different from our model in the sense that the agent is not involved in wasteful activities.

1.4 Countervailing incentives

Most of the existing models in the principal-agent framework with the adverse selection deal with a constant incentive of the agent to overstate the private information or to understate it. As a result principal deviates from the first-best contract either below or above the efficient levels for all the continuum of the agents in order to reduce the information rents. But when the efficient's type reserved utility becomes high enough, the principal stops decreasing the information rent of the agent as it is bounded by this outside opportunity. When the inefficient type incentive compatibility constraint becomes binding the countervailing incentives take place.

First the term "countervailing incentives" was proposed by Lewis and Sappington (1989) who reconsidered Baron-Myerson model with a firm having the fixed cost negatively correlated with its marginal costs. They studied a model with the continuum of types and emphasized the pooling region they obtain in the transition from upward to downward binding incentives constraint.

Maggi and Rodriguez-Claire (1994) have shown that pooling is not a general consequences of the countervailing incentives, it depends crucially on the assumption of concavity of the status quo utility in the private parameter. If the function is convex, countervailing incentives are compatible with the separating contracts. In their latter work of 1995 they presented a model where principal can observe on top of the output a noisy signal on the agent's marginal costs. The agent manipulates this observable by playing at this noise with a cost. Countervailing incentives there may arise because the Spence-Mirrlees condition may not be longer satisfied.

Our work is another example of the model where separating equilibrium exists together with countervailing incentives.

Chapter 2

Fully Private Information Model

2.1 Introduction

We begin our analysis with the case in which the advisor (she) provides the financial advice to a risk-averse investor (he). This chapter will provide the benchmark against which we will compare the results obtained in chapter 3. Here I study the case where an advisor has private information about the return on the risky asset but his objectives are not identical to the ones of the investor. The agent needs to find a trade-off between his professional and partisan objectives and the goal of the principal to sign a most profitable contract. As a result I show that the higher partisan objective of the advisor is more preferable for investor as they are able to share the "partisan rent" while signing the contract that induces falsification.

In a perfect world investor and advisor have the preferences that are totally aligned and there is a full information. In this case advisor reports the truth and investor invests this exact amount of money. If we change the assumptions a bit and will add the partisan objective to the full information, situation changes. Now, investor puts more money in the market for the same Sharpe ratio. The indirect utility of the investor increases in partisan objective when it is smaller a certain threshold: investor extracts all the kickback commission of the advisor through the tariff. But with the increase of partisan objective it becomes too costly for investor to manipulate the information rents of advisor.

When the players deal in the situation of asymmetrical information with preferences that are not aligned the optimal contract induces falsification – the optimal investment is not equal to the first best one, no matter that the contract with first best outcome was available to investor. Two scenarios are possible for the different level of partisan objective. When it is relatively small, the unconditional expected utility of investor depends negatively on partisan objective. But social welfare depends positively on it, which means that society is better off when falsification is a part of a contract. When the partisan objective is high, the expected utility of investor depends positively on it as well as social welfare. And here I give the same explanation as above: investor and advisor share with each other the kickbacks rent and are both better off from the falsification in the contract. If we make the partisan objective equal to zero than the optimal contract will coincide with the one from the "perfect world".

2.2 The model

There are two individuals: an investor (principal) and a advisor (agent). Due to the lack of proper financial education, the investor must rely on a financial advisor to make his investment choice. The advisor is better informed than the investors about the return structure of the risky asset.

Consider an investor with initial wealth normalized to 1. There is only one risky investment opportunity (e.g., equities or bonds) and the riskless asset. The stochastic return of the risky asset $\tilde{r}$ is normally distributed with mean $E(r)$ and variance σ^2. The riskless asset pays the riskfree rate $r_f < E(r)$, $r_f \leq 1$. For mathematical simplicity assume that the agent is privately informed about the Sharpe ratio $\theta = \frac{E(r)-r_f}{\sigma^2}$ which is uniformly distributed on $\left[-\frac{\Delta}{2}; \frac{\Delta}{2}\right] \equiv \Theta$,with $\Delta \in [0,1]$. In order to obtain a one dimensional problem, further we contemplate cases where only mean is unknown and $\sigma^2 = 1$.

Let α be the fraction of wealth that investor allocates to the risky asset, or his risk exposure. We consider a situation where a principal contracts with an agent to provide a financial advise on investment in a risky asset and compensates the agent with a monetary transfer, t. The investor's preferences are described with utility function CARA — i.e., for any level of wealth $w = \alpha\tilde{r} + (1-\alpha) r_f - t$

$$u(w) = 1 - e^{-\gamma w} \tag{2.1}$$

where $\gamma > 0$ measures his risk attitude. Assuming that $w\tilde{\ }N(E(w); var(w))$ the investors' expected utility can be described by mean-variance utility function. $u(\alpha,\theta) = \alpha\theta + r_f - t - \dfrac{\gamma}{2}\alpha^2$

To achieve interpretable results in this chapter and chapter 3 we take the following values for the parameters that are lying out of the focus of interest: $r_f = 1$, $\gamma = 1$. So the preferences of investor are represented as following

$$u(\alpha,\theta) = \alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \tag{2.2}$$

Consider the preferences of an advisor who has both a professional and partisan objective. On one hand advisor has an incentive to provide the best advice for investor as the divergence of the amount invested and the Sharpe ratio enters the utility function of an agent with a weight $0 < \phi \leq 1$. On the other, she cares about the amount invested and parameter λ measures the advisor's intrinsic bias relative to the investor's ideal portfolio choice. When $\lambda > 0$ the partisan objective arise. Ottaviani (2000) gave a interpretation to this term as a commission from sales that manager received as a bonus. Consequently, manager has an incentive to maximize his compensation.

$$v(\alpha, \theta) = \lambda\alpha + t - \frac{\phi}{2}(\alpha - \theta)^2 \tag{2.3}$$

Again, to focus on the most important implications of the model and avoid cumbersome expressions without losing generality we take $\phi = 1$[1]. The principal's uncertainty about $\theta = \mu - r_f$ is represented by a (common knowledge) probability distribution $F(\theta)$ with associated density function $f(\theta)$. As we are assuming that θ is uniformly distributed on Θ, the standard assumption that $\frac{F(\theta)}{f(\theta)}$ *and* $\frac{F(\theta)-1}{f(\theta)}$ are increasing functions of θ is fulfilled. Investor chooses a direct mechanism $\mathcal{M} \equiv \{\alpha(\theta_l), t(\theta_l)\}_{\theta_l \in \Theta}$, with $\alpha(.) : \Theta \to \Re$, which specifies a portfolio allocation $\alpha(\theta_l)$ for any (private) report $l \in \Theta$ made by the advisor to investor about the sate of the world θ. As standard in this literature, mechanisms are restricted to be piecewise differentiable and continuous. The expert cannot refuse advice to his clients: an intrinsic common agency game.

Timing. The timing of the game is as follows:

- ($t = 0$) Nature draws θ and only the advisor observes its realization.
- ($t = 1$) Investor announces (and commits to) a mechanism $\mathcal{M}$.
- ($t = 2$) The advisor (privately) reports θ_l to investor. Investment choices are made according to the mechanisms chosen at $t = 1$.

[1] You may find the calculations for an optimal contract with ϕ in the Appendix. Section 2.3.2 - for the First Best outcome, section 2.3.4 - for the assymetric information case.

- ($t = 3$) Asset returns materialize.

The commitment assumption is standard in the mechanism design literature that studies delegation in the absence of monetary incentives. In particular, it allows to avoid the typical selection issue of cheap talk games (e.g., Crawford and Sobel, 1982, among others), and is often motivated with a reputation argument. That is, the relationship between an investor and his financial advisor is usually long-lasting (due to switching costs).

2.2.1 The complete information benchmark

When the investor observes the realization of the asset's return the problem of the information asymmetry doesn't exist. We are looking at the complete information outcome to understand the "perfect" investment realization to have a benchmark to compare with. In this case the marginal utility of the principal supposed to be equal to the marginal utility of the agent. The principal solves the following optimization problem

$$\begin{aligned} \max_{a(\theta)} U &= \alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \\ & s.t \\ v(\alpha, \theta) &= \lambda\alpha + t - \frac{1}{2}(\alpha - \theta)^2 \end{aligned} \tag{2.4}$$

Proposition 1 *The allocation $\alpha^{FB} = \theta + \frac{\lambda}{2}$, $t^{FB} = \frac{5\lambda^2}{8} - \lambda\theta$ is the optimal for investor and advisor when the problem of asymmetric information is absent. The indirect utility of investor is $U = \frac{1}{2}\theta^2 + \theta\lambda - \frac{3}{4}\lambda^2 + 1$.*

This Proposition characterizes optimal contract between investor and advisor when there preferences are misaligned in the situation of full information. If we substitute $\lambda = 0$ in these results we will get the "perfect world" outcome when the preferences of investor and advisor are perfectly aligned. It is easy to notice that compared to the situation of full alignment in our case investor is willing to invest more in the market for a given Sharpe ratio. Moreover the partisan objective enters the utility function of investor not necessarily with the minus: when the Sharpe ratio is relatively high $\left(\theta > \frac{3}{4}\lambda\right)$ the investor's utility is higher then the one in case of full alignment. As the information is totally public investor has an opportunity to extract not only all information rents from advisor, but also the "partisan rent" - the surplus of the agent received from the partisan objective, kickback commissions.

2.2.2 The asymmetric information case

We assume that the agent has a constant reservation utility that is normalized to zero. With asymmetric information by revelation principal we can derive optimal allocations $\{\alpha(\theta), t(\theta)\}$ as a solution to the following maximization problem:

$$
\begin{aligned}
& \max_{\alpha(\theta), t(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(\alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \right) df\left(\theta\right) \qquad (2.5) \\
& s.t. \\
IC \quad : \quad & \theta \in \arg\max_{\Theta_l} \left\{ v\left(\alpha, \theta_l\right) = \lambda\alpha + t - \frac{1}{2}\left(\alpha - \theta_l\right)^2 \right\} \\
PC \quad : \quad & v\left(\alpha, \theta\right) \equiv \lambda\alpha + t - \frac{1}{2}\left(\alpha - \theta\right)^2 \geq 0
\end{aligned}
$$

To solve 2.5 we need to use the standard scheme for the continuum of types case, i.e. we replace IC condition with necessary and sufficient conditions: a local first-order condition:

$$ICFOC: \frac{\partial v(\cdot)}{\partial \theta_l} = 0 \tag{2.6}$$

and a monotonicity conditions $\alpha'(\theta) \leq 0$. Differentiating 2.6 with respect to θ gives us equivalent representation of ICFOC if the monotonicity conditions are hold.

$$\dot{v} = \alpha - \theta \tag{2.7}$$

This condition is central in understanding the nature of countervailing incentives of the model because it shows how the information rent of the agent must change so the contract remains incentive compatible. Overstating the Sharpe ratio means that the agent reports θ higher than the true one. The gains from overstating increase with the level of optimal investment and it is easy to explain with the partisan objective of the advisor. Understating the Sharpe ratio means that the advisor reports θ lower than the true one and this can be explained with the second term on RHS of 2.7. The existence of the information rents leads the principal to distort the level of investment from the first-best level, which was computed in the previous section.

The usual procedure for solving this problem is the following: replace the IC constraint in the 2.5 with 2.7 , ignore the monotonicity conditions (MC), conclude that PC is binding on one of the extremes and find the solution for this "relaxed" problem; then check if this solution satisfies PC and MC. However, this method is based on the assumption that $\dot{v}(\theta)$ doesn't change the sign, which might not be true in our case due to existence of the countervailing incentives. So within the present mechanism it must be taken into consideration that beside the standard mimicking effect of overstating the realization of θ, the one of understating θ may present. 2.7

represents how the information rents changes with θ for the contract to be incentive compatible. Therefore, the participation constraint can be violated for some θ. So we introduce PC explicitly in the problem. Rewriting Principal's utility function as $u = 1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2$ will let us reformulate the 2.5 as following

$$\begin{aligned}
& \max_{\alpha(\theta), v(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2\right) f(\theta)\, d\theta \qquad (2.8) \\
& s.t. \\
\dot{v} &= \alpha - \theta \\
v(\alpha, \theta) &\geq 0
\end{aligned}$$

The Hamiltonian associated with a problem is

$$H = \left(1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2\right)\frac{1}{\Delta} + \mu(\alpha - \theta) \qquad (2.9)$$

Hence the Lagrange function to be maximized is

$$L = H + \tau v$$

where the control variable is α, the state variable is v and the costate variable is μ. Due to the fact that the problem 2.8 contains pure state constraints the solution can be find with a set of sufficient conditions that allow a jump in the costate variable. The strategy will be to look for a solution with a continuous costate variable. If none exists, we will allow jumps in the costate variable. The first-order conditions for the maximization of Hamiltonian with respect to α are:

$$\begin{aligned} \frac{\partial H}{\partial \alpha} &= 0 \rightarrow (\lambda - (\alpha - \theta) + \theta - \alpha) \frac{1}{\Delta} + \mu = 0 \\ \alpha^* &= \frac{1}{2} (\theta + \lambda + \theta + \Delta \mu) \end{aligned} \tag{2.10}$$

Since H is concave in α condition 2.10 is sufficient for the maximization of Hamiltonian.

The other sufficient conditions are

$$\dot{\mu} = -\frac{dH}{dv} = f(\theta) - \tau(\theta) \tag{2.11}$$

$$\dot{v} = (\alpha - \theta) \tag{2.12}$$

$$\tau(\theta) v(\theta) = 0; \quad \tau(\theta) \geq 0; \quad v(\theta) \geq 0 \tag{2.13}$$

$$\mu(\underline{\theta}) v(\underline{\theta}) = 0; \; \mu(\overline{\theta}) v(\overline{\theta}) = 0; \quad \mu(\underline{\theta}) \leq 0; \quad \mu(\overline{\theta}) \geq 0 \tag{2.14}$$

Let $\hat{\mu}(\theta)$ denote the solution in μ when the agent's utility is maximum and constant: $\dot{v}(\alpha^*) = 0$. The slope of $\hat{\mu}(\theta)$ is crucial in determining the optimal contract. If PC is binding on a nondegenerate interval then $\mu(\theta)$ supposed to be equal to $\hat{\mu}(\theta)$ on this interval.

$$\hat{\mu}(\theta) = -\frac{1}{\Delta}\lambda \tag{2.15}$$

Our strategy to solve the problem is to conjecture a solution and verify that is satisfies the sufficient conditions. The structure of the optimal contract depends crucially on whether or not $\hat{\mu}(\theta)$ crosses the distribution function $F(\theta)$ and $F(\theta) - 1$. It is easy to see that $F(\theta) > \hat{\mu}(\theta)$ on all the interval which means that the optimal

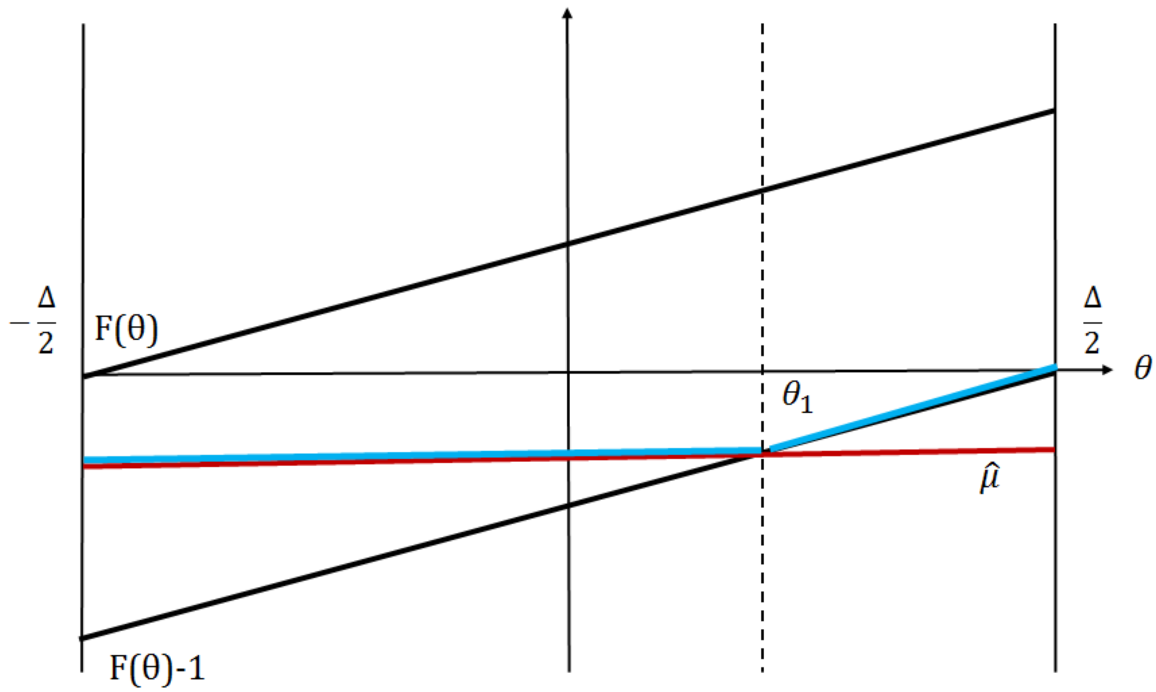


Figure 2.1: The schedual for optimal μ^*

costate variable cannot be equal to $F(\theta)$ for all θ: if we set $\mu^*(\theta) = F(\theta)$, rents would be increasing $\dot{v} > 0$ and since, from the transversality condition, $v(\theta_1) = 0$, the participation constraint will be violated.

$$\mu^*(\theta) = \begin{cases} \hat{\mu}(\theta) & \hat{\mu}(\theta) \in [F(\theta) - 1, 0] \\ F(\theta) - 1 & \hat{\mu}(\theta) < F(\theta) - 1 \end{cases} \tag{2.16}$$

Using the functions' specifications we have:

$$\mu^*(\theta) = \begin{cases} -\frac{1}{\Delta}\lambda & \theta \in \left[-\frac{\Delta}{2}, \tilde{\theta}_1\right) \\ \frac{\theta - \frac{\Delta}{2}}{\Delta} & \tilde{\theta}_1 \leq \theta \leq \frac{\Delta}{2} \end{cases} \tag{2.17}$$

At the graphical representation above the blue line shows μ^*. The point of interception $\theta_1 = \frac{\Delta}{2} - \lambda$ takes it's value depending on the parameters of the model and can appear both positive or negative. However we are interested if θ_1 can be bigger or smaller than the endpoints. In this framework it is possible that $\theta_1 < -\frac{\Delta}{2}$ which means that the other type of solution is possible where μ^* can be described with the single pattern. Condition $\theta_1 < -\frac{\Delta}{2}$ corresponds to the one $\lambda > \Delta$. Which means that there exists a critical level of partisan objective after which the advisor changes his behavior and reports to the principal according to one scheme only.

Proposition 2 *For the case when $\lambda < \Delta$, the optimal mechanism $\mathcal{M} \equiv \{\alpha(l), t(l)\}_{l\in\Sigma}$ satisfies the following properties*

The investment decision and the optimal tariff are given by:

$$\alpha^*(\theta) = \begin{cases} \theta & \theta < \theta_1 \\ \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda & \theta \geq \theta_1 \end{cases} \tag{2.18}$$

$$t^*(\theta) = \begin{cases} -\theta\lambda & \theta < \theta_1 \\ \frac{1}{32}\left(4\theta^2 + \Delta^2 - 4\lambda^2 - 4\theta\Delta - 40\theta\lambda + 4\Delta\lambda\right) & \theta \geq \theta_1 \end{cases} \tag{2.19}$$

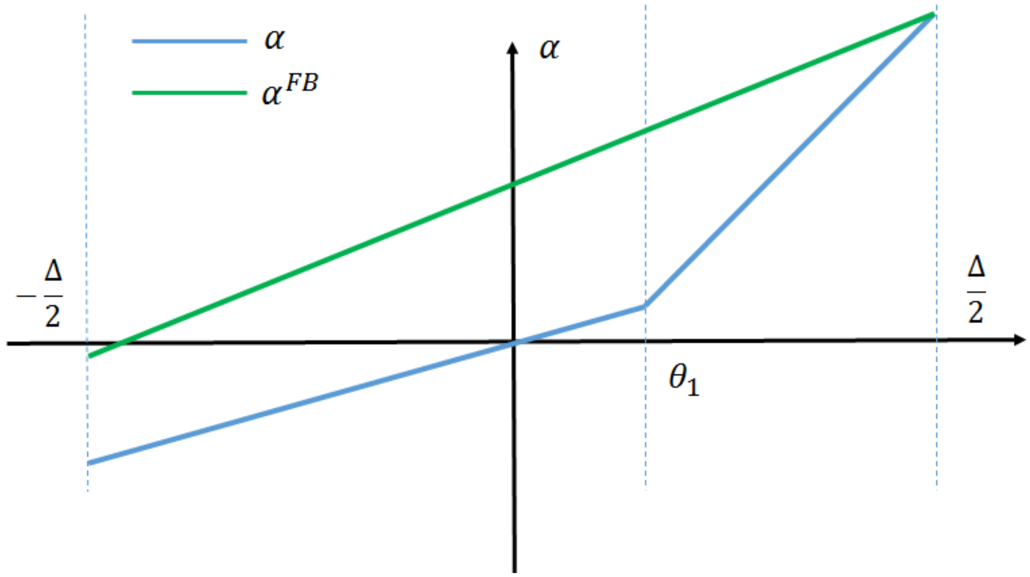
The expected utility of the principal and the rent of the advisor are the following (respectively):

$$EU = \frac{1}{24}\frac{24\Delta + \Delta^3 - 2\lambda^3}{\Delta} \tag{2.20}$$

$$EV = \frac{1}{4\Delta}\lambda^3 \tag{2.21}$$

And the social welfare is

$$SW = \frac{1}{24}\frac{24\Delta + \Delta^3 + 4\lambda^3}{\Delta} \tag{2.22}$$



Optimal investment level α^* compared with the FB benchmark

For all meanings of θ on the interval the principal underivests compared to the First Best outcome (FB) which means that the agent reports the state of the nature lower than the true one[2]. To understand why it is happening we need again to have a closer look at 2.7 which is the key for understanding the countervailing incentives of the agent. To induce truthtelling the principal has to compensate the incentives to misreport for the agent and 2.7 shows how the information rent of the agent has to change with θ for contract to be incentive compatible. Information rents will be increasing or decreasing on θ depending on which of the incentives to overstate or understate dominates. Understating the Sharpe ratio advisor reports θ lower then the actual one and the gain of it can be explained with the second term on RHS of 2.7.

This solution is valid for the relatively small partisan objective, λ, and the compensation the agent receives is positive when Sharpe ratio is negative. Up till θ_1

[2] I note again that this result is independent of our parametrisation. The solution for optimal investment with ϕ can be seen at the Appendix.

the graphs of the FB investment and the optimal investment are parallel and the difference between them is $\frac{\lambda}{2}$, so the degree of the understatement depends on the partisan objectives of the advisor: the higher λ the deeper understatement is. However, the break point when the agent starts the movement towards the FB level, θ_1, depends negatively on λ. The more agent cares about personal objectives the faster she switches to the second scheme. At the point $\theta = \frac{\Delta}{2}$ the strength of countervailing incentives is highest and the incentives to overstate and understate are perfectly balanced and the agent has no incentives to lie.

In other words it means that the agent gives up the possible commission to receive a higher tariff: if the Sharpe ratio is high, the investor would like to invest more and will pay a smaller tariff because she understands that the agent receives a higher "partisan rent", but the agent prefers a tariff to the kickback commission as in this case the commission is bounded on top ($\lambda < \Delta$). So the "partisan rent" in this particular case is not high enough to influence the incentive of principal to share it with agent.

Corollary 3 *The expected utility of the principal, EU, depends negatively on the meaning of partisan objective of the agent, λ and positively on Δ.*

The expected utility of the agent, EV, depends positively on the meaning of partisan objective of the agent, λ and negatively on Δ.

The social welfare, SW, depends positively on λ and on Δ positively if $\lambda \leq \frac{\Delta}{\sqrt[3]{2}}$ and negatively if $\lambda > \frac{\Delta}{\sqrt[3]{2}}$[3]

The logic behind the outcome that utility of investor depends negatively on the partisan objective is straightforward: the more the advisor cares about his own

[3] The calculations of comparative statics can be found in the Appendix.

benefit the more she distort the information the less the expected utility of investor is. However, the positive dependence on Δ is less obvious. We may say that Δ shows the dispersion of θ or degree of uncertainty. Principal benefits from the higher uncertainty because it makes an interval for λ bigger: the agent continues preferring the tariff over "partisan rent". Unlike First Best outcome, when there is no information asymmetry and investor extracts all the kickback commission of the agent, here to manipulate the agent's rents is too costly for the principal and she prefers the contract that induces falsification.

Agent prefers a higher partisan objective which lays on the line. But the situation with uncertainty level is again not usual: in the standard case the agent usually is better from higher asymmetry of information as it helps to gain the information rents, however here it is on contrary. The explanation I offer is the following: a higher volatility serves better to the party that has an advantage over the other one. So here the principal has a higher weight and more possibility to manipulate the rents of the agent.

As for social welfare, it depends positively on λ because the growth of EV prevails the decrease of EU. The society turns out to be better off from the higher partisan objective.

Proposition 4 *For the case when $\lambda \geq \Delta$, the optimal mechanism $\mathcal{M} \equiv \{\alpha(l), t(l)\}_{l \in \Sigma}$ satisfies the following*

The investment decision is given by:

$$\alpha^* (\theta) = \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda \tag{2.23}$$

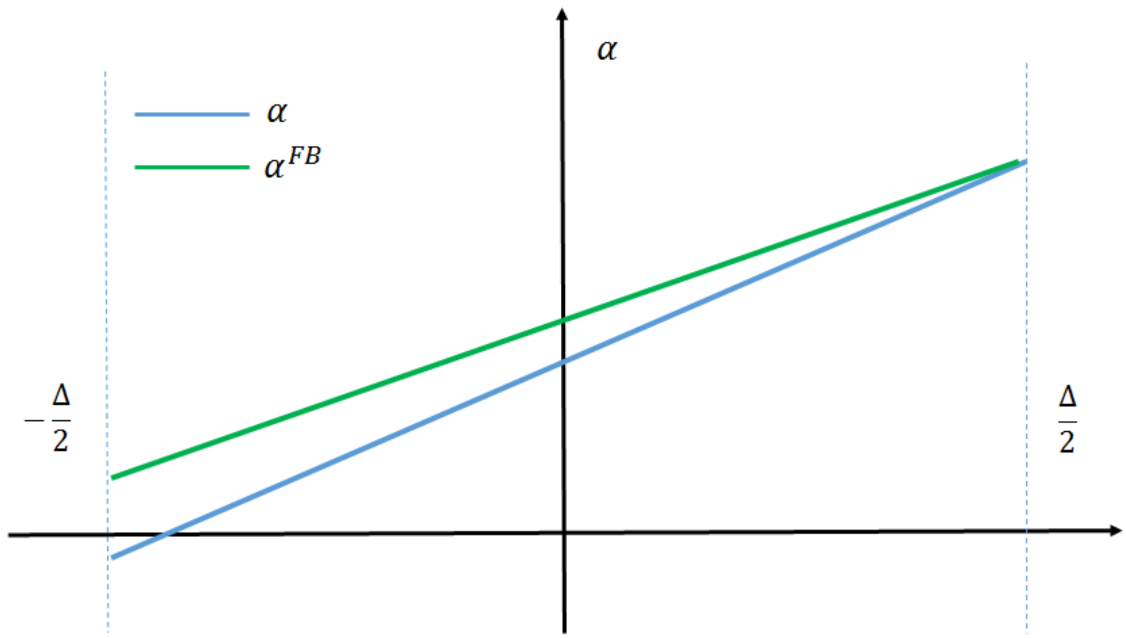


Figure 2.2: Optimal investment level compared with FB benchmark

the optimal tariff is

$$t^*(\theta) = \frac{1}{32}\left(4\theta^2 + \Delta^2 - 4\lambda^2 - 4\theta\Delta - 40\theta\lambda + 4\Delta\lambda\right)$$

The expected utility of the principal and the rent of the advisor are the following (respectively):

$$EU = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 24\right)$$

$$EV = -\frac{1}{4}\Delta\left(\Delta - 2\lambda\right)$$

And the social welfare is

$$SW = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 - 24\right)$$

In the case when the partisan objective is high enough the agent follows the only behavioral scheme: she understates the true state of the nature however the degree of understatement is getting smaller with the growth of θ. Again, at $\theta = \frac{\Delta}{2}$ the countervailing incentives are at maximum so the advisor reports the true θ. But because the partisan objective is high now the understatement for the lower θ is deeper compared to the previous case.

Corollary 5 *The expected utility of the principal, EU, depends positively on the meaning of partisan objective of the agent,* λ *and negatively on* Δ*. The expected utility of the agent, EV, depends positively on the both meanings of partisan objective of the agent,* λ *and on* Δ*. The social welfare, SW,depends positively on* λ *and on negatively on* Δ*.*

This case is particularly different form the one with a relatively law partisan objective. Here the principal is better off from increase in level of partisan objective as now the kickback commission is high enough to extract a part of it and manipulate the information rent of advisor. The comparative statics of the agent becomes standard as his utility now depends positively on the level of uncertainty. The major conclusion we have here that both principal and agent prefer the higher partisan objective as they share the "partisan rent" and "agree" on a certain level of falsification.

2.2.3 Concluding remarks

This chapter was devoted to a simple model of financial advice when the information is fully private and the preferences of investor and advisor are not aligned. It is the first phase of modelling principal-agent relationship within financial advice framework. In spite of its siplicity we gain help in better understanding of players' behavior in particular environment, their incentives and the mechanisms behind the countervailing incentives. Next step we make is to introduce the costly signal into the model to incorporate falsification.

In the next chapter I enlarge the current model to see how the situation changes when the information is not fully private to see if the results change under different information structure.

2.3 Appendix

2.3.1 Proof of Proposition 1

For the optimization problem

$$\begin{aligned} \max_{a(\theta)} U &= \alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \\ & s.t \\ v(\alpha,\theta) &= \lambda\alpha + t - \frac{1}{2}(\alpha-\theta)^2 \end{aligned} \tag{2.24}$$

the following first order condition can be obtained

$$FOC : \theta - \alpha - (\alpha - \theta) + \lambda = 0$$

Out of which the derivation of optimal level of investment is straightforward.

$$\alpha^{FB} = \theta + \frac{\lambda}{2}$$

To find the optimal transfer we need to use the fact that in this case there is no hidden information so the information rents is equal to zero and so is his surplus.

$\lambda\alpha^{FB} + t - \frac{1}{2}\left(\alpha^{FB} - \theta\right)^2 = 0$

$$t^{FB} = \frac{5\lambda^2}{8} - \lambda\theta$$

To find the indirect utility function of investor we need to substitute the obtained meanings of α^{FB} and t^{FB} into maximized function: $U = \frac{1}{2}\theta^2 - \frac{1}{8}\lambda^2 - \left(\frac{5\lambda^2}{8} - \lambda\theta\right) + 1 = \frac{1}{2}\theta^2 + \theta\lambda - \frac{3}{4}\lambda^2 + 1$

2.3.2 Complete information framework with ϕ

$$\begin{aligned} \max_{a(\theta)} U &= \alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \qquad (2.25) \\ & s.t \\ v(\alpha, \theta) &= \lambda\alpha + t - \frac{\phi}{2}(\alpha - \theta)^2 \end{aligned}$$

We proceed with the solution as in the previous section and the first order condition will have the following form:

$$FOC : \theta - \alpha - \phi(\alpha - \theta) + \lambda = 0$$

Out of which we obtain the firs-best level of investment:

$$\alpha^{FB} = \theta + \frac{\lambda}{1 + \phi} \qquad (2.26)$$

2.3.3 Proof of proposition 2

To compute the optimal level of investment we used the FOC 2.10 and the schedule for μ^* 2.17.

In the first interval, $\theta \in \left[-\frac{\Delta}{2}, \tilde{\theta}_1\right)$, which we will notate with a lower index $_1$, $\mu_1 = -\frac{1}{\Delta}\lambda$. We substitute μ with this expression in the formula for an optimal investment $\alpha^* = \frac{1}{2}(\theta + \lambda + \theta + \Delta\mu)$ to get the optimal level of investment for all the intervals:

$$\alpha^*(\theta) = \begin{cases} \theta & \theta < \theta_1 \\ \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda & \theta \geq \theta_1 \end{cases} \tag{2.27}$$

In this part we will use the notations with lower indexes which correspond to the intervals $i = 1, 2$: $1 : \theta \in \left[-\frac{\Delta}{2}; \theta_1\right]$, $2 : \theta \in (\theta_1; \frac{\Delta}{2}]$. In order to find the schedule for the optimal tariff we need to compute the unconditional utility of the agent, which we can find integrating 2.7 by parts $V = \int_{-\frac{\Delta}{2}}^{\theta_1} v_1 d\theta + \int_{\theta_1}^{\frac{\Delta}{2}} v_2 d\theta$, where $v_i = \alpha_i - \theta$.

$v_1 = \int_{-\frac{\Delta}{2}}^{\theta_1} (\alpha_1 - \theta) d\theta = 0$, the rent on the first interval $\theta \in \left[-\frac{\Delta}{2}; \theta_1\right]$

$v_2 = \int_{\theta_1}^{\frac{\Delta}{2}} (\alpha_2 - \theta) d\theta = \frac{1}{4}\lambda^2$, the utility on the second interval $\theta \in (\theta_1; \frac{\Delta}{2}]$

From 2.3 we can express $t = v - \lambda\alpha + \frac{1}{2}(\alpha - \theta)^2$. To write the optimal schedule for the tariff we need to substitute the relevant meanings of v, α^* into this formula:

$t_1 = v_1 - \lambda\alpha_1 + \frac{1}{2}(\alpha_1 - \theta)^2 = -\theta\lambda$

$t_2 = v_2 - \lambda\alpha_2 + \frac{1}{2}(\alpha_2 - \theta)^2 = \frac{1}{32}\left(4\theta^2 + \Delta^2 - 4\lambda^2 - 4\theta\Delta - 40\theta\lambda + 4\Delta\lambda\right)$

In order to compute the unconditional tariff we need to integrate t_1 and t_2 on the corresponding intervals

$T = \int_{-\frac{\Delta}{2}}^{\theta_1} t_1 d\theta + \int_{\theta_1}^{\frac{\Delta}{2}} t_2 d\theta = \frac{1}{24}\lambda^3$

Expected utility, agent's expected rent and social welfare can be found through integrating the conditional expressions on the relevant intervals

$$EU = \int_{-\frac{\Delta}{2}}^{\theta_1} \left(1 - t_1 + \alpha_1\theta - \tfrac{1}{2}\alpha_1^2\right) \tfrac{d\theta}{\Delta} + \int_{\theta_1}^{\frac{\Delta}{2}} \left(1 - t_2 + \alpha_2\theta - \tfrac{1}{2}\alpha_2^2\right) \tfrac{d\theta}{\Delta} = \tfrac{1}{24} \tfrac{24\Delta + \Delta^3 - 2\lambda^3}{\Delta}$$

expected utility of investor

$EV = \int_{-\frac{\Delta}{2}}^{\theta_1} V_1 \frac{d\theta}{\Delta} + \int_{\theta_1}^{\frac{\Delta}{2}} V_2 \frac{d\theta}{\Delta} = \frac{1}{4\Delta}\lambda^3$ expected utility of advisor

Social welfare we define as the sum of unconditional utilities of principal and agent.

$SW = EV + EU = \frac{1}{24} \frac{24\Delta + \Delta^3 + 4\lambda^3}{\Delta}$

2.3.4 Optimal investment with ϕ

We solve the problem 2.8 without substitution ϕ with 1. So the Hamiltonian 2.9 takes form:

$$H = \left(1 - v + \lambda\alpha - \frac{\phi}{2}(\alpha - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2\right)\frac{1}{\Delta} + \mu\phi(\alpha - \theta)$$

The first order condition for the optimal investment is

$$\frac{\partial H}{\partial \alpha} = \frac{1}{\Delta}(\theta - \alpha + \lambda + \theta\phi - \alpha\phi + \Delta\mu\phi) = 0$$

Which gives us $\alpha = \frac{1}{\phi+1}(\theta + \lambda + \theta\phi + \Delta\mu\phi)$

The derivation of μ doesn't change as the sufficient conditions 2.11, 2.13, 2.14 remain the same. So we substitute the schedule 2.16 for μ in the expression for α to get the optimal pattern for investment:

$$\alpha^*(\theta) = \begin{cases} \frac{1}{\phi+1}(\theta + \lambda + \theta\phi - \lambda\phi) & \theta < \theta_1 \\ \frac{1}{2\phi+2}(2\theta + 2\lambda + 4\theta\phi - \Delta\phi) & \theta \geq \theta_1 \end{cases}$$

One can easily check that there are no interceptions of optimal level of investment and the first-best level (2.26) for $\theta \in \left[-\frac{\Delta}{2}; \frac{\Delta}{2}\right]$ for any possible ϕ. Which means that for any meaning of ϕ there is a underinvestment compared to FB-level, except for $\theta = \frac{\Delta}{2}$ when the advisor reports the truth and the optimal amount of investment is equal to FB level.

2.3.5 Comparative statics computations for Corollary 3

Case $\lambda < \Delta$

Principal's expected utility derivative with respect to λ:

$\frac{\partial EU}{\partial \lambda} = -\frac{1}{4\Delta}\lambda^2$ which is negative because $\lambda > 0$, $\Delta \in [0; 1]$

Expected Utility derivative with respect to Δ:

$\frac{\partial EU}{\partial \Delta} = \frac{1}{12\Delta^2}\left(\Delta^3 + \lambda^3\right) > 0$

The derivatives of expected utility of the agent with respect to λ and Δ correspondingly

$\frac{\partial EV}{\partial \lambda} = \frac{3}{4\Delta}\lambda^2 > 0$

$\frac{\partial EV}{\partial \Delta} = -\frac{1}{4\Delta^2}\lambda^3 < 0$

The derivatives of social welfare with respect to λ and Δ correspondingly

$\frac{\partial SW}{\partial \lambda} = \frac{1}{2\Delta}\lambda^2 > 0$

$\frac{\partial SW}{\partial \Delta} = \frac{1}{12\Delta^2}\left(\Delta^3 - 2\lambda^3\right)$

2.3.6 Proof of Proposition 4

When $\lambda \geq \Delta$, there is only one scheme for $\mu = \frac{\theta - \frac{\Delta}{2}}{\Delta}$. We substitute this expression in the first order conditions 2.10 and obtain $\alpha^*(\theta) = \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda$

As in before we proceed our computation with the expressing the tariff from the utility function of the agent $t = v - \lambda\alpha + \frac{1}{2}(\alpha - \theta)^2$ and substitute in the corresponding parameters where v is an integrated 3.4: $v = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} (\alpha - \theta) d\theta = -\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda\right)$. So after all the substitutions we get:

$t = \frac{1}{32}\left(4\theta^2 + \Delta^2 - 4\lambda^2 - 4\theta\Delta - 40\theta\lambda + 4\Delta\lambda\right)$

The unconditional utility of investor is an integrated indirect utility on the compact Θ:

$U = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \left(1 - t + \alpha\theta - \frac{1}{2}\alpha^2\right)\frac{d\theta}{\Delta} = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 24\right)$

The unconditional utility of agent is an integrated indirect utility on the compact Θ:

$V = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \left(-\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda + \frac{1}{2}\Delta\beta\right)\right)\frac{d\theta}{\Delta} = -\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda\right)$

As before we compute the social welfare as a sum of expected utilities of principal and agent:

$SW = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 - 24\right)$

2.3.7 Comparative statics for Corollary 6

Case $\lambda \geq \Delta$

$$t^*(\theta) = \frac{1}{32}\left(4\theta^2 + \Delta^2 - 4\lambda^2 - 4\theta\Delta - 40\theta\lambda + 4\Delta\lambda\right)$$

$$EU = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 24\right)$$

$$EV = -\frac{1}{4}\Delta(\Delta - 2\lambda)$$

$$SW = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 - 24\right)$$

Principal's expected utility derivative with respect to λ:

$\frac{\partial(EU)}{\partial\lambda} = \frac{\partial\left(\frac{1}{24}\left(5\Delta^2+6\lambda^2-12\Delta\lambda+24\right)\right)}{\partial\lambda} = \frac{1}{2}\lambda - \frac{1}{2}\Delta$ which is positive because $\lambda > \Delta$, $\Delta \in [0;1]$

Expected Utility derivative with respect to Δ:

$\frac{\partial EU}{\partial\Delta} = \frac{\partial\left(\frac{1}{24}\left(5\Delta^2+6\lambda^2-12\Delta\lambda+24\right)\right)}{\partial\Delta} = \frac{5}{12}\Delta - \frac{1}{2}\lambda$ which is negative because $\lambda > \Delta$

The derivatives of expected utility of the agent with respect to λ and Δ correspondingly

$\frac{\partial EV}{\partial\lambda} = \frac{\partial\left(-\frac{1}{4}\Delta(\Delta-2\lambda)\right)}{\partial\lambda} = \frac{1}{2}\Delta > 0$

$\frac{\partial EV}{\partial\Delta} = \frac{\partial\left(-\frac{1}{4}\Delta(\Delta-2\lambda)\right)}{\partial\Delta} = \frac{1}{2}\lambda - \frac{1}{2}\Delta > 0$

The derivatives of social welfare with respect to λ and Δ correspondingly

$$\frac{\partial\left(-\frac{1}{24}\left(\Delta^2-6\lambda^2-24\right)\right)}{\partial\lambda} = \frac{1}{2}\lambda > 0$$

$$\frac{\partial\left(-\frac{1}{24}\left(\Delta^2-6\lambda^2-24\right)\right)}{\partial\Delta} = -\frac{1}{12}\Delta < 0$$

Chapter 3

Partly Private Information Model

3.1 Introduction

In this chapter I continue to study an optimal contract between investor and advisor but the information structure changes. I make an assumption that there exist costs for an agent to distort the information. An interpretation for this parameter can be such that an advisor may need some additional time to falsify the report or resources to produce the fake evidence. I introduce a parameter β that defines the magnitude of this costs and can be interpreted as a degree of the publicness of the information: the higher β is the bigger the costs are and distortion of the signal becomes more complicated for the agent, so the information becomes more public. So when β is equal to zero we have a fully private information and that is exactly the case we analyzed in Chapter 2. When β is close to infinity the information becomes public and we get closer to the First Best outcome. In this Chapter I analyze the situation when information is not purely public or private, so this case lies in between the model with fully public information (first best) and the model with purely private information (model of Chapter 2).

The changing in the information structure leads to the certain changing in results. First of all, it becomes more difficult for advisor to manipulate the information and, on contrary, it is easier for investor to extract the "partisan rent": he let the advisor falsify information and get the kickback commission but then extracts it through the tariff.

The optimal contract again, as in Chapter 2, is characterized with a certain amount of falsification however it is closer to the first best level. But in welfare analysis there are differences. First, the unconditional expected utility of investor now depends always positively on the partisan objective of the agent. The explanation I offer is the following: as the information now is not fully private it is easier for investor to manipulate the information rents and it is now possible to extract the "partisan" rent of advisor even when the market is low, unlike in Chapter 2, it was possible only for the high Sharpe ratio. Second, welfare analysis of the publicness information level gives an interesting result: however the maximum of social welfare corresponds to the highest possible β, the relation between these two variables is not monotonic and increasing the publicness of information may lead to decrease in welfare.

3.2 The Model

Like in the previous chapter the principal contracts with the agent to receive an advice about the return of the risky asset. The risk-averse investor with the initial wealth normalized to 1 has a risky opportunity and the riskless asset. The Sharpe ratio $\theta = \frac{E(r)-r_f}{\sigma^2}$ is still uniformly distributed on $\Theta \equiv \left[-\frac{\Delta}{2}; \frac{\Delta}{2}\right]$, with $\Delta \in [0;1]$ and corresponding cumulative and distribution functions $F(\theta)$ and $f(\theta)$.

$$u(\alpha, \theta) = \alpha\theta + 1 - t - \frac{1}{2}\alpha^2 \tag{3.1}$$

The informational structure of the model is different: the principal observes the signal $s = \theta + e$, where $e \in (-\infty; +\infty)$ is unobservable action, that the agent can take to distort the signal, in either direction. There are the costs that are associated with this action - which we call the costs of falsification. The agent's utility function includes the costs of falsification $\frac{\beta}{2}(s-\theta)^2$which captures the publicness of information: if this cost is zero, the model coincides with the standard private-information model that we saw in chapter 2; if it is infinite the model coincides with public-information model. As before advisor's preferences also include the partisan objective λ. This time the preferences of the advisor are represented the following way.

$$v(\alpha, \theta) = \lambda\alpha + t - \frac{1}{2}(\alpha - \theta)^2 - \frac{\beta}{2}(s-\theta)^2 \tag{3.2}$$

Investor chooses a direct mechanism $\mathcal{M} \equiv \{\alpha(\theta_l), t(\theta_l), s(\theta_l)\}_{\theta_l \in \Theta}$, with $\alpha(.) : \Theta \to \Re$, which specifies a portfolio allocation $\alpha(l)$ for any (private) report $\theta_l \in \Theta$ made by the advisor to investor about the sate of the world θ. As standard in this literature, mechanisms are restricted to be piecewise differentiable and continuous. The expert cannot refuse advice to his clients: an intrinsic common agency game.

The use of direct mechanisms allows us to rely on intuitive and easy to characterize incentive constraints.

Timing. The timing of the game is the following

- $(t = 0)$ Nature draws θ and only the advisor observes its realization.

- $(t = 1)$ Investor announces (and commits to) a mechanism $\mathcal{M}$.

- ($t = 2$) The advisor (privately) reports s to investor. Investment choices are made according to the mechanisms chosen at $t = 1$.

- ($t = 3$) Asset returns materialize.

The commitment assumption is standard in the mechanism design literature that studies delegation in the absence of monetary incentives. In particular, it allows to avoid the typical selection issue of cheap talk games (e.g., Crawford and Sobel, 1982, among others), and is often motivated with a reputation argument. That is, the relationship between an investor and his financial advisor is usually long-lasting (due to switching costs).

With asymmetric information by revelation principal we can derive optimal allocations $\{\alpha(\theta), s(\theta), t(\theta)\}$ as a solution to the following problem:

$$\begin{aligned} & \max_{\alpha(\theta),s(\theta),t(\theta)} E\left(U^p\left(\alpha(\theta), s\left(\theta\right), t(\theta)\right)\right) \\ & s.t. \\ IC \quad : \quad & \theta \in \arg\max_{\Theta_l} \left\{ v\left(\alpha, \theta_l\right) = \lambda\alpha + t - \frac{1}{2}\left(\alpha - \theta_l\right)^2 - \frac{\beta}{2}\left(s - \theta_l\right)^2 \right\} \\ PC \quad : \quad & v\left(\alpha, \theta\right) \equiv \lambda\alpha + t - \frac{1}{2}\left(\alpha - \theta\right)^2 - \frac{\beta}{2}\left(s - \theta_l\right) \geq 0 \end{aligned} \tag{3.3}$$

As before in order to solve this problem we differentiate the incentive compatibility constraint (IC) with respect to θ and assuming that the monotonicity conditions hold $\alpha'(\theta) \leq 0$ and $s'(\theta) \geq 0$:

$$\dot{v} = (\alpha - \theta) + \beta\left(s - \theta\right) \tag{3.4}$$

As in previous chapter this equation is the key one for understanding the countervailing incentives, however now it is expanded with β and s. β here makes both incentives (to overstate or to understate) stronger: the higher it is the bigger the second term will be with deviation s from θ in any direction.

3.4 allows us to rewrite the Principal's utility function as $u = 1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 - \frac{\beta}{2}(s - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2$ and to reformulate the 3.3 as following

$$\begin{aligned} & \max_{\alpha(\theta), s(\theta), v(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left(1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 - \frac{\beta}{2}(s - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2\right) f(\theta) d\theta \\ & s.t. \\ \dot{v} &= (\alpha - \theta) + \beta(s - \theta) \\ v(\alpha, \theta) &\geq 0 \end{aligned} \tag{3.5}$$

As we consider α and s are the control variables and v is a state variable then the Hamiltonian and Lagrangian associated with the problem 3.5 are the following:

$$\begin{aligned} H &= \left(1 - v + \lambda\alpha - \frac{1}{2}(\alpha - \theta)^2 - \frac{\beta}{2}(s - \theta)^2 + \alpha\theta - \frac{1}{2}\alpha^2\right)\frac{1}{\Delta} + \mu((\alpha - \theta) + \beta(s - \theta)) \\ L &= H + \tau v \end{aligned}$$

The first-order conditions for the maximization of the Hamiltonian with respect to α is:

$$\begin{aligned} \frac{\partial H}{\partial \alpha} &= 0 \rightarrow (\lambda - \alpha + \theta + \theta - \alpha)\frac{1}{\Delta} + \mu = 0 \\ \alpha^* &= \frac{\lambda + 2\theta + \Delta\mu}{2} \end{aligned} \tag{3.6}$$

And with respect to s:

$$\begin{aligned}\frac{\partial H}{\partial s} &= 0 \rightarrow -\beta\left(s-\theta\right)\frac{1}{\Delta}+\mu\beta=0 \\ s^{*} &= \mu\Delta+\theta\end{aligned} \tag{3.7}$$

Since H is concave in α and s condition 3.6 and 3.7 are sufficient for the maximization of Hamiltonian.

The other sufficient conditions are

$$\dot{\mu} = -\frac{dH}{dv}=\frac{1}{\Delta}-\tau\left(\theta\right) \tag{3.8}$$

$$\dot{v} = \left(\alpha-\theta\right)+\beta\left(s-\theta\right) \tag{3.9}$$

$$\tau\left(\theta\right)v\left(\theta\right) = 0;\quad \tau\left(\theta\right)\geq 0;\quad v\left(\theta\right)\geq 0 \tag{3.10}$$

$$\mu\left(\underline{\theta}\right)v\left(\underline{\theta}\right) = 0;\ \mu\left(\overline{\theta}\right)v\left(\overline{\theta}\right)=0;\quad \mu\left(\underline{\theta}\right)\leq 0;\quad \mu\left(\overline{\theta}\right)\geq 0 \tag{3.11}$$

Let $\hat{\mu}\left(\theta\right)$ denote the solution in μ when the agent's utility is maximum and constant: $\dot{v}\left(\alpha^{*};s^{*}\right)=0$. The slope of $\hat{\mu}\left(\theta\right)$ is crucial in determining the optimal contract. If PC is binding on a nondegenerate interval then $\mu\left(\theta\right)$ supposed to be equal to $\hat{\mu}\left(\theta\right)$ on this interval.

$$\hat{\mu}\left(\theta\right)=-\frac{\lambda}{\Delta+2\Delta\beta} \tag{3.12}$$

Our strategy is the same as in the previous chapter: to conjecture a solution and verify that it satisfies the sufficient conditions. The structure of the optimal contract depends crucially on whether or not $\hat{\mu}\left(\theta\right)$ crosses the distribution function $F\left(\theta\right)$ and

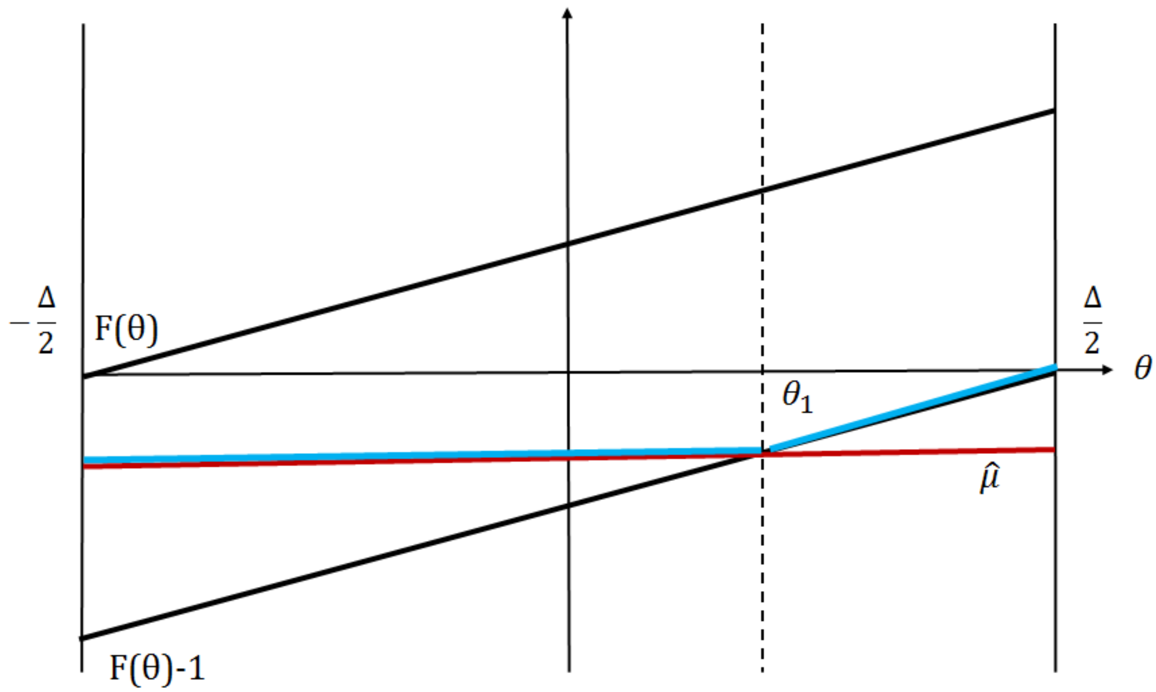


Figure 3.1: Optimal scheme for μ

$F(\theta) - 1$. It is easy to see that $F(\theta) > \hat{\mu}(\theta)$ on all the interval which means that the optimal costate variable cannot be equal to $F(\theta)$ for all θ: if we set $\mu^*(\theta) = F(\theta)$, rents would be increasing $\dot{v} > 0$ and since, from the transversality condition, $v(\theta_1) = 0$, the participation constraint will be violated.

$$\mu^*(\theta) = \begin{cases} \hat{\mu}(\theta) & \hat{\mu}(\theta) \in [F(\theta) - 1, 0] \\ F(\theta) - 1 & \hat{\mu}(\theta) < F(\theta) - 1 \end{cases} \tag{3.13}$$

Using the functions' specifications we have

$$\mu^*(\theta) = \begin{cases} -\frac{\lambda}{\Delta + 2\Delta\beta} & \theta \in \left[-\frac{\Delta}{2}, \tilde{\theta}_1\right) \\ \frac{\theta - \frac{\Delta}{2}}{\Delta} & \tilde{\theta}_1 \leq \theta \leq \frac{\Delta}{2} \end{cases} \tag{3.14}$$

Where $\theta_1 = \frac{\Delta}{2} - \frac{\lambda}{1+2\beta}$. 3.1 below provides a graphical representation of the μ^*:

The point of interception θ_1 takes it's value depending on the parameters of the model[1] and can appear both positive or negative. However we are interested if θ_1 can be bigger or smaller than the endpoints. In this framework it is only possible that $\theta_1 < -\frac{\Delta}{2}$ which means that the other type of solution is possible where μ^* can be described with the single pattern. Condition $\theta_1 < -\frac{\Delta}{2}$ corresponds to the one $\lambda \geq \Delta + 2\Delta\beta$ or $\beta \leq \frac{\lambda-\Delta}{2\Delta}$. We also can interpret $\hat{\mu}(\theta)$ as a measure for the size of distortion needed to neutralize information rents: when $\hat{\mu}$ is small, a small distortion is enough to neutralize the information rents.

Proposition 6 *For the case when* $\lambda < \Delta + 2\Delta\beta$ $\left(\beta > \frac{\lambda-\Delta}{2\Delta}\right)$, *the optimal mechanism* $\mathcal{M} \equiv \{\alpha(l), t(l), s(l)\}_{l\in\Sigma}$ *satisfies the following*

The investment decision is given by:

$$\alpha^*(\theta) = \begin{cases} \theta + \frac{\beta\lambda}{2\beta+1} & \theta < \theta_1 \\ \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda & \theta \geq \theta_1 \end{cases}$$

The signal is given by:

$$s^*(\theta) = \begin{cases} \theta - \frac{\lambda}{2\beta+1} & \theta < \theta_1 \\ 2\theta - \frac{1}{2}\Delta & \theta \geq \theta_1 \end{cases}$$

the optimal tariff is

$$t^*(\theta) = \begin{cases} -\frac{1}{2}\frac{\lambda}{(2\beta+1)^2}\left(2\theta + 8\theta\beta + \beta\lambda + 8\theta\beta^2 + 3\beta^2\lambda\right) & \theta < \theta_1 \\ \frac{1}{64\beta+32}\left(\begin{array}{c} 32\theta^2\beta^2 + 24\theta^2\beta + 4\theta^2 - 32\theta\Delta\beta^2 - 24\theta\Delta\beta - 4\theta\Delta - 80\theta\beta\lambda \\ -40\theta\lambda + 8\Delta^2\beta^2 + 6\Delta^2\beta + \Delta^2 + 8\Delta\beta\lambda + 4\Delta\lambda - 24\beta\lambda^2 - 4\lambda^2 \end{array}\right) & \theta \geq \theta_1 \end{cases}$$

The expected utility of the principal and the rent of the advisor are the following

[1] The one can easily compute that $\theta_1 = -\Delta\left(\frac{\lambda}{\Delta+2\Delta\beta} - \frac{1}{2}\right)$

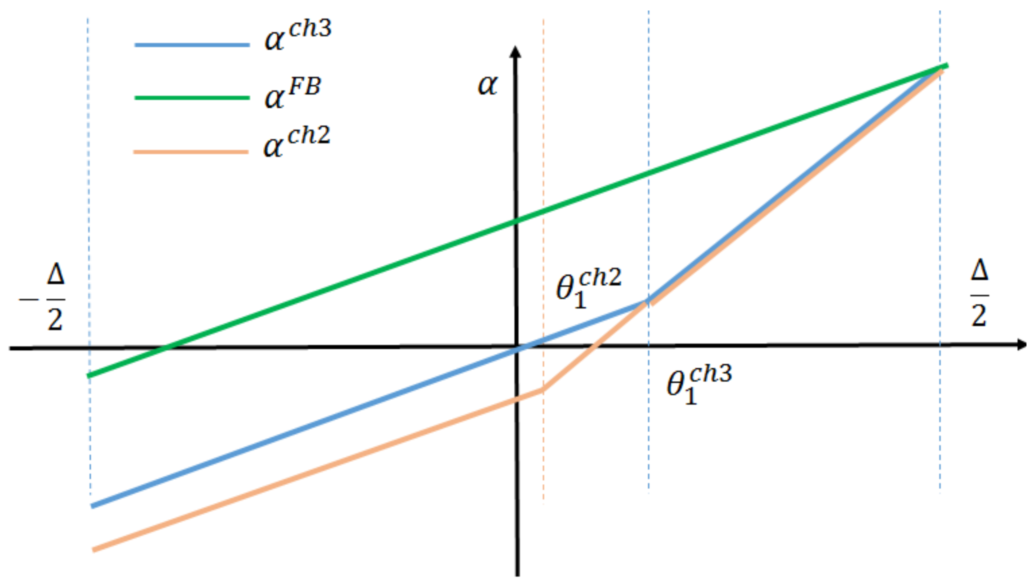
(respectively):

$$EU = \frac{1}{24\Delta\left(2\beta+1\right)^2}\left(4\Delta^3\beta^2 + 4\Delta^3\beta + \Delta^3 + 24\Delta\beta^2\lambda^2 + 96\Delta\beta^2 + 12\Delta\beta\lambda^2 + 96\Delta\beta + 24\Delta - 2\lambda^3\right)$$

$$EV = \frac{1}{4\Delta}\frac{\lambda^3}{\left(2\beta+1\right)^2}$$

And the social welfare is

$$SW = \frac{1}{24}\frac{24\Delta + \Delta^3 + 4\lambda^3 + 4\Delta^3\beta^2 + 96\Delta\beta + 96\Delta\beta^2 + 4\Delta^3\beta + 12\Delta\beta\lambda^2 + 24\Delta\beta^2\lambda^2}{\Delta\left(2\beta+1\right)^2}$$



Optimal investment compared to the FB benchmark and Chapter 2 case

From the graphical representations it can easily be seen that despite of the fact that the optimal investment is still lower then the first best level, the introduction of costly information distortion moves the outcome closer to the first-best. The gap between the investment levels of Ch 2 and Ch 3 (the gap between the blue and orange lines) is equal to $\frac{\beta\lambda}{2\beta+1}$, so the higher β is the more blue line moves towards the

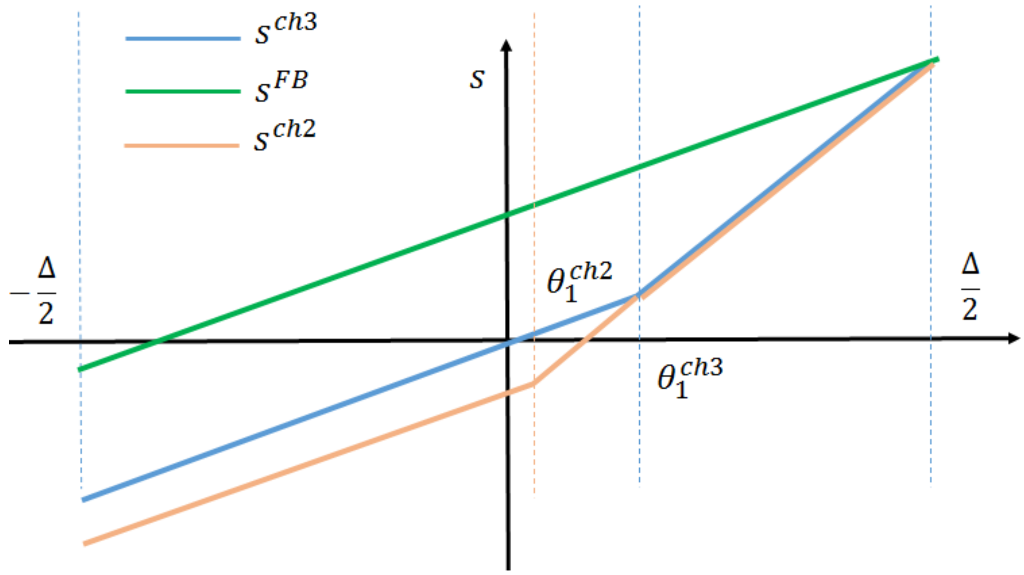


Figure 3.2: Optimal signal compared to FB and Ch2

green one (FB level) and this is pretty tractable: the higher the costs of information distortion are the closer the investment level to the first-best level. The difference between α^{ch3} and α^{FB} is equal to $\left(\frac{\lambda}{2} + \frac{\lambda}{2\beta+1}\right)$ and it demonstrates that the increase in partisan objective moves the outcome away from the FB level while the raise in β moves the solution towards it.

As it can also be seen from the graph θ_1 lies closer to the right end when there exist the costs of information distortion. As we discussed in the Chapter 2 the agent switches to the second scheme for higher θ faster if his personal objective (λ) is higher because for higher θ the incentive to overstate becomes more attractive. In expanded model though this mechanism is slower due to existence of β which holds the desire to overstate by making it costly and that is why $\theta_1^{ch3} > \theta_1^{ch2}$.

To plot the signal of Chapter 2 model we took the solution for the signal and made $\beta = 0$ which is the case when the models coincide. The graphical representation of the optimal signal confirms our conclusions from above: the agent understates the true state of the nature compared to FB outcome. This result can also be interpreted with $\hat{\mu}$ behavior. As it was mentioned earlier $\hat{\mu}$ can be interpreted as a size of distortion needed to neutralize the information rents and, as following, the strength of the countervailing incentives, which in our case is negative and constant.

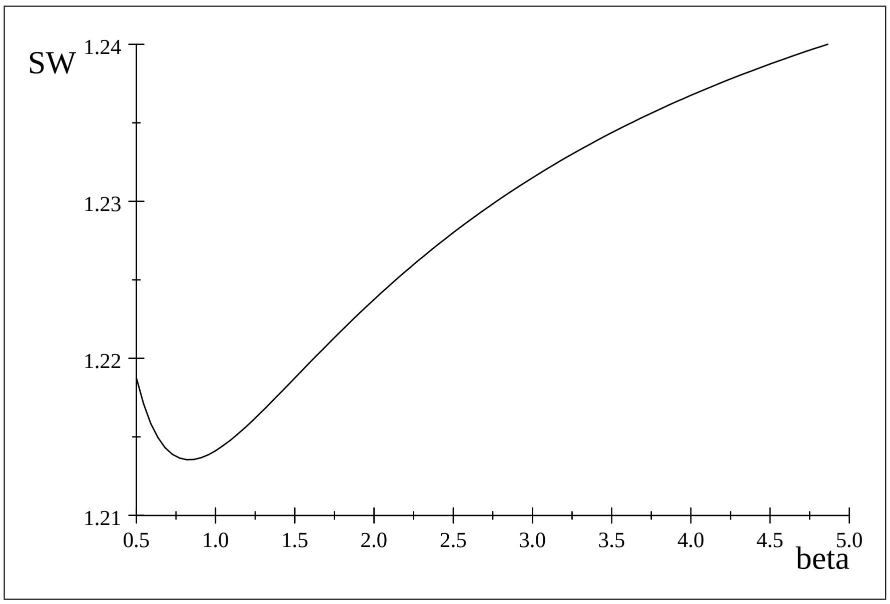
The question is, again, why principal prefers the contract that induces falsification when the First Best one is feasible. To understand the logic we must look into the comparative statics.

Corollary 7 *The expected utility of the principal, EU, depends positively on the meaning of partisan objective of the agent, λ, Δ and the level of publicness of information, β. The expected utility of the agent, EV, depends positively on the meaning of partisan objective of the agent, λ but negatively on Δ and the level of publicness of information, β. The social welfare, SW,depends positively on λ and Δ; but for β the derivative changes it's sign: positive if $\beta \geq 2 - \frac{2\lambda}{3\Delta}$ and negative otherwise.*

In this case the principal has a better off from a higher partisan objective, i.e. the higher commission receives the advisor the higher becomes the welfare of investor. And here I propose the same logic of a shared "partisan" rent. As the information now is not purely private it becomes easier for investor to manipulate the rents of advisor and he takes a part of her "partisan" profit. This contract and comparative statics are true when $\beta > \frac{\lambda - \Delta}{2\Delta}$, i.e. the level of publicness information is relatively high. As it is more costly for the agent to lie, the investor has an advantage and puts all the risks of higher uncertainty on the shoulders of the agent through decreasing the tariff with raise of Δ[2]. And so the investor's utility is growing in Δ, while the utility of advisor decreases in Δ.

[2]Please see appendix for the corresponding derivative

The result that EU depends positively on β is very much expected as it becomes almost impossible for agent to lie and investor extracts a higher surplus. As for the agent, her utility decreasing with β as the information rent is decreasing with growth in level of publicness of information. The impact of β on the total social welfare is different depending on the value of β. As we can see from the graph, welfare reaches the maximum when the information gets public ($\beta \to \infty$), but increasing β may lead to fall in social welfare.



Social welfare

This result corresponds to well-known second-best theorems. For example, increasing the number of markets can be socially harmful when the social optimum is achieved at perfect competition state. Another implication can be the following: if investor wants to invest in financial regulations that will make the process of lying more costly for advisor, the social value of this investment my fall short of the principal's private value and thus the investment will exceed the socially optimal level.

Proposition 8 *For the case when* $\lambda \geq \Delta + 2\Delta\beta$ $\left(\beta \leq \frac{\lambda - \Delta}{2\Delta}\right)$, *the optimal mechanism*

$\mathcal{M} \equiv \{\alpha(l), t\,(l)\,, s\,(l)\}_{l \in \Sigma}$ *satisfies the following*

The investment decision is given by:

$$\alpha^*\,(\theta) = \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda \tag{3.15}$$

The signal is given by:

$$s^*\,(\theta) = 2\theta - \frac{1}{2}\Delta \tag{3.16}$$

the optimal tariff is

$$t^*\,(\theta) = \frac{1}{32}\left(4\theta^2 - 7\Delta^2 - 12\lambda^2 - 4\theta\Delta - 40\theta\lambda + 20\Delta\lambda + 16\theta^2\beta - 12\Delta^2\beta - 16\theta\Delta\beta\right)$$

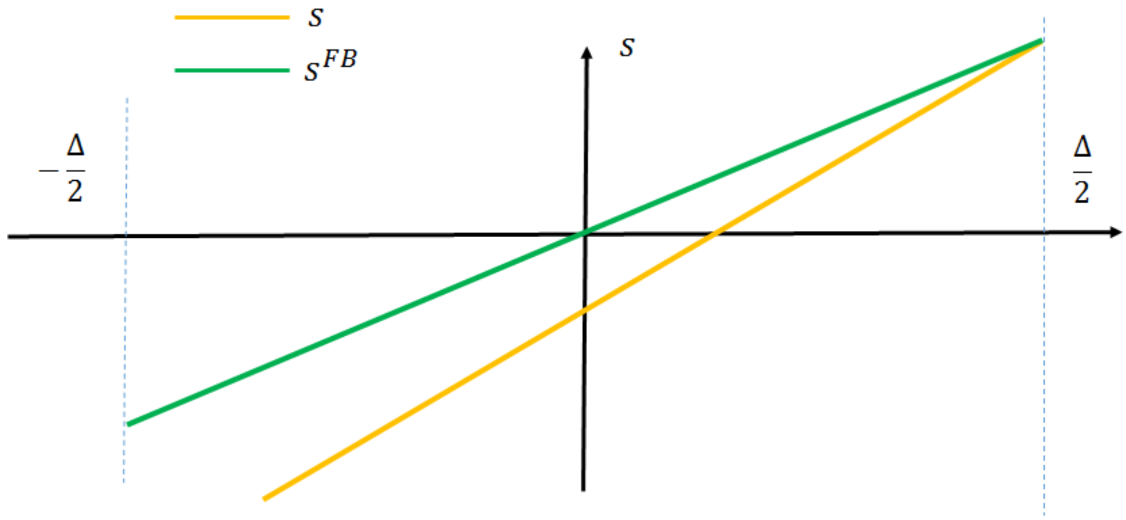
The expected utility of the principal and the rent of the advisor are the following (respectively):

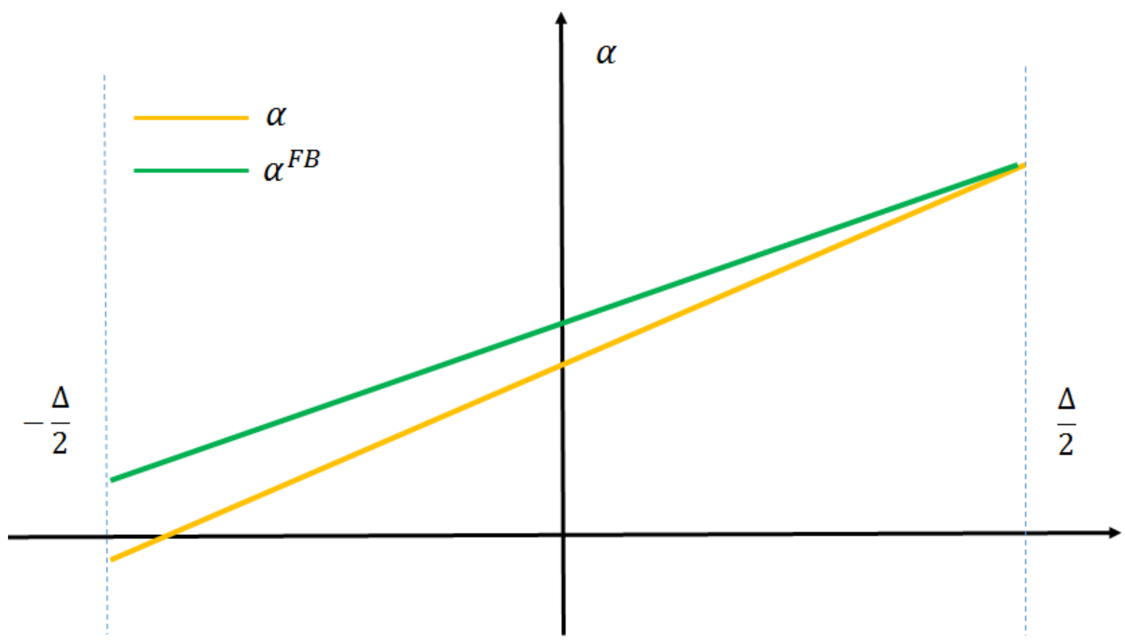
$$EU = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 8\Delta^2\beta + 24\right)$$

$$EV = -\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda + \frac{1}{2}\Delta\beta\right)$$

And the social welfare is

$$SW = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 + 4\Delta^2\beta - 24\right)$$

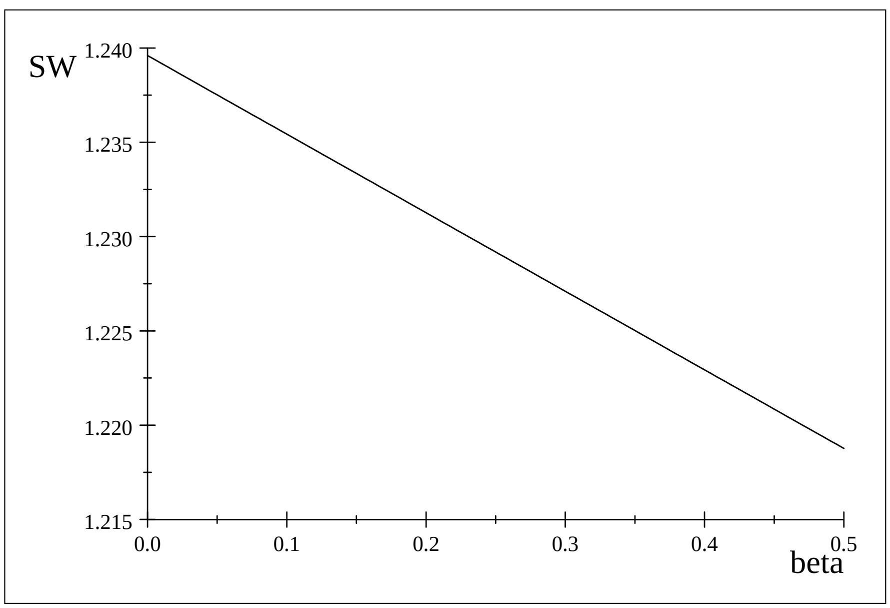




This contract is optimal when the level of publicness of information is relatively low and the market state (high-low Sharpe ration) doesn't influence the variables and the solution for optimal investment level and the signal transmitted coincide with the one of Chapter 2, that is why on presented graphs there is only one line beside the first-best outcome.

Corollary 9 *The expected utility of the principal, EU, depends positively on the meaning of partisan objective of the agent,* λ *and the level of publicness of information,* β*, but negatively on* Δ *. The expected utility of the agent,* EV*, depends positively on the meaning of partisan objective of the agent,* λ*, and* Δ*, but negatively on the level of publicness of information,* β*. The social welfare, SW,depends positively on* λ*, but negatively on* Δ *and* β *.*

Investor is still better off from the higher partisan objective of the advisor, but now he cannot anymore put the burden of uncertainty on the agent as the last one has more opportunity to misrepresent the state of the nature compared to the previous case. So now the investor's utility is decreasing with Δ, when the advisor's utility increases in Δ.



Social welfare

In this case social welfare depends negatively on the degree of the publicness of information. Explanation here is simple: we are looking at the case where $\beta \leq \frac{\lambda - \Delta}{2\Delta}$ which means that β lies in the interval where it is still inefficient to increase the publicness information (on very left side if look at the previous image of social welfare).

3.3 Concluding remarks

If we think about the real world we understand that it consists of shades: there is very little pure black or pure white, very little examples of extreme cases. The same we can relate to information structure. It is almost impossible to find an example in financial advising sector with purely private or purely public information. That is why the presented model of partly private information adds a value to the discussion.

This model is the first step towards general explanation of steadily growing demand for financial advice despite the significant underperformance of financial managers. The main substantive question we have tackled is whether the falsification in the optimal contract benefits to both investor and advisor. As it was demonstrated under certain conditions investor, as well as society, benefits from higher kickback commission of advisor. Presented model gives an understanding of possible transmission mechanisms behind the falsification as an optimum. And this exactly the point where this work contributes to the literature: a new explanation of the existent underperformance and reasoning how investor benefits from advisor who transmits "wrong" state of nature. In our model investor chooses the contract with falsification and receives the bonus from advisor's kickback commission. That means that investor agrees to have a bad advice in exchange for extra income, which we can interpret as some kind of services and discounts provided by advising company. That may be one of explanations of the existent puzzle with underperformance.

Another interesting implication we found is that the relation between social welfare and the level of information publicness is non-monotonic: increasing it may lead be ineffective from the point of welfare.

3.4 Appendix

3.4.1 Proof of proposition 6

We follow the same scheme as in the previous chapter to compute the optimal α, s and t.

To compute the optimal levels of α and s we substitute the expression for μ for each region of θ from 3.14 in the first order conditions 3.6 and 3.7. One can easy obtain our result

$$\alpha^*(\theta) = \begin{cases} \theta + \frac{\beta\lambda}{2\beta+1} & \theta < \theta_1 \\ \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda & \theta \geq \theta_1 \end{cases}$$

$$s^*(\theta) = \begin{cases} \theta - \frac{\lambda}{2\beta+1} & \theta < \theta_1 \\ 2\theta - \frac{1}{2}\Delta & \theta \geq \theta_1 \end{cases}$$

Then we proceed with the computation of t: first we calculate the unconditional level of the agent's utility for each region of θ, and then substitute it in the expression for t. Here we use the same notation as in Section 2.3.2.

$v_1 = \int\limits_{-\frac{\Delta}{2}}^{\theta_1} ((\alpha_1 - \theta) + \beta(s_1 - \theta))\, d\theta = 0$ unconditional level of information rents for the interval $\theta \in \left[-\frac{\Delta}{2}; \theta_1\right]$

$v_2 = \int\limits_{\theta_1}^{\frac{\Delta}{2}} ((\alpha_2 - \theta) + \beta(s_2 - \theta))\, d\theta = \frac{\lambda^2}{8\beta+4}$, the utility of the agent on the second interval $\theta \in (\theta_1; \frac{\Delta}{2}]$

Now we compute the optimal schedule of the tariff for all possible meanings of θ:

$$t_1 = v_1 - \lambda\alpha_1 + \frac{1}{2}(\alpha_1 - \theta)^2 + \frac{\beta}{2}(s_1 - \theta)^2 = -\frac{1}{2}\frac{\lambda}{(2\beta+1)^2}\left(2\theta + 8\theta\beta + \beta\lambda + 8\theta\beta^2 + 3\beta^2\lambda\right)$$

receives the agent for $\theta \in \left[-\frac{\Delta}{2}; \theta_1\right]$

$$t_2 = v_2 - \lambda\alpha_2 + \frac{1}{2}\left(\alpha_2 - \theta\right)^2 + \frac{\beta}{2}\left(s_2 - \theta\right)^2 = \frac{1}{64\beta+32}\left(32\theta^2\beta^2 + 24\theta^2\beta + 4\theta^2 - 32\theta\Delta\beta^2 - 24\theta\Delta\beta - 4\theta\right.$$

he receives on $\theta \in (\theta_1; \frac{\Delta}{2}]$

Which gives us the expression for the unconditional level of tariff.

$$T = \int\limits_{-\frac{\Delta}{2}}^{\theta_1} t_1 d\theta + \int\limits_{\theta_1}^{\frac{\Delta}{2}} t_2 d\theta = -\frac{1}{24}\lambda^2 \frac{-\lambda + 12\Delta\beta + 2\beta\lambda + 60\Delta\beta^2 + 72\Delta\beta^3}{(2\beta+1)^3}$$

Level of investor's expected utility we compute while substituting the relevant parameters and integrating on all possible θ :

$$U = \int\limits_{-\frac{\Delta}{2}}^{\theta_1} \left(1 - t_1 + \alpha_1\theta - \frac{1}{2}\alpha_1^2\right)\frac{d\theta}{\Delta} + \int\limits_{\theta_1}^{\frac{\Delta}{2}} \left(1 - t_2 + \alpha_2\theta - \frac{1}{2}\alpha_2^2\right)\frac{d\theta}{\Delta} = \frac{1}{24\Delta(2\beta+1)^2}\left(4\Delta^3\beta^2 + 4\Delta^3\beta + \Delta^3 + \right.$$

Level of expected utility of advisor:

$$V = \int\limits_{-\frac{\Delta}{2}}^{\theta_1} V_1 \frac{d\theta}{\Delta} + \int\limits_{\theta_1}^{\frac{\Delta}{2}} V_2 \frac{d\theta}{\Delta} = \frac{1}{4\Delta}\frac{\lambda^3}{(2\beta+1)^2}$$

Level of social welfare we compute as a sum of investor's unconditional utility and information rents of the advisor/

$$SW = \frac{1}{24}\frac{24\Delta + \Delta^3 + 4\lambda^3 + 4\Delta^3\beta^2 + 96\Delta\beta + 96\Delta\beta^2 + 4\Delta^3\beta + 12\Delta\beta\lambda^2 + 24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2}$$

3.4.2 Comparative statics for Corollary 7

We compute the derivatives for the following expressions with respect to the key parameters:

$$U = \frac{1}{24\Delta(2\beta+1)^2}\left(4\Delta^3\beta^2 + 4\Delta^3\beta + \Delta^3 + 24\Delta\beta^2\lambda^2 + 96\Delta\beta^2 + 12\Delta\beta\lambda^2 + 96\Delta\beta + 24\Delta - 2\lambda^3\right)$$

unconditional utility of investor

$V = \frac{1}{4\Delta}\frac{\lambda^3}{(2\beta+1)^2}$ unconditional utility of advisor

$S_w = \frac{1}{24}\frac{24\Delta+\Delta^3+4\lambda^3+4\Delta^3\beta^2+96\Delta\beta+96\Delta\beta^2+4\Delta^3\beta+12\Delta\beta\lambda^2+24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2}$ social welfare computed as a sum of expected utilities of principal and agent

Investor

Derivative with respect to partisan objective:

$\frac{\partial U}{\partial \lambda} = \frac{1}{4}\lambda\frac{-\lambda+4\Delta\beta+8\Delta\beta^2}{\Delta(2\beta+1)^2} > 0$, as $\lambda < \Delta + 2\Delta\beta$ the numerator of the fraction is strictly positive and the whole expression is positive

Derivative with respect to asset dispersion Δ:

$\frac{\partial U}{\partial \Delta} = \frac{1}{12\Delta^2(2\beta+1)^2}\left(4\Delta^3\beta^2 + 4\Delta^3\beta + \Delta^3 + \lambda^3\right) > 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial U}{\partial \beta} = \frac{1}{6\Delta}\frac{\lambda^2}{(2\beta+1)^3}\left(3\Delta + 2\lambda + 6\Delta\beta\right) > 0$

Advisor

Derivative with respect to partisan objective:

$\frac{\partial V}{\partial \lambda} = \frac{3}{4\Delta}\frac{\lambda^2}{(2\beta+1)^2} > 0$

Derivative with respect to asset dispersion Δ:

$\frac{\partial V}{\partial \Delta} = -\frac{1}{4\Delta^2}\frac{\lambda^3}{(2\beta+1)^2} < 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial V}{\partial \beta} = -\frac{1}{\Delta}\frac{\lambda^3}{(2\beta+1)^3} < 0$

Social Welfare

Derivative with respect to partisan objective:

$\frac{\partial S_w}{\partial \lambda} = \frac{1}{24\Delta(2\beta+1)^2}\left(48\Delta\beta^2\lambda + 24\Delta\beta\lambda + 12\lambda^2\right) > 0$

Derivative with respect to asset dispersion Δ:

$\frac{\partial S_w}{\partial \Delta} : \frac{1}{12\Delta^2(2\beta+1)^2}\left(4\Delta^3\beta^2 + 4\Delta^3\beta + \Delta^3 - 2\lambda^3\right) > 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial S_w}{\partial \beta} = \frac{1}{6\Delta}\frac{\lambda^2}{(2\beta+1)^3}\left(3\Delta - 4\lambda + 6\Delta\beta\right)$, can be both signs as the expression in the brackets can be rewritten in the following way $3\Delta - 4\lambda + 6\Delta\beta = 3\left(\Delta + 2\Delta\beta\right) - 4\lambda$, it is more clear that the sigh depends on the ratio of parameters. So after a certain threshold $\beta \geq \frac{4\lambda-3\Delta}{6\Delta}$ the social welfare increases with β.

Tariff

Derivative of tariff with respect to level of uncertainty

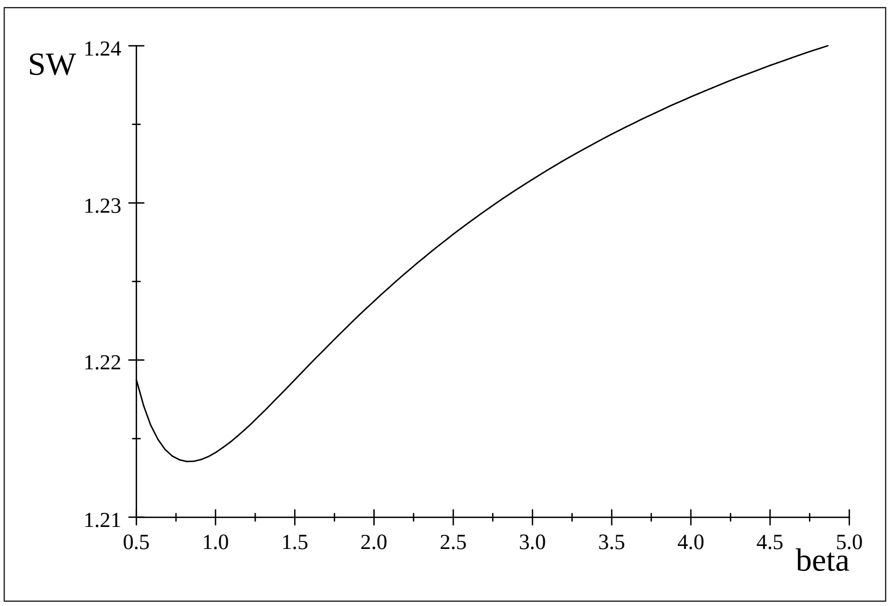
$$\frac{\partial T}{\partial \Delta} = -\frac{1}{2}\beta\frac{\lambda^2}{(2\beta+1)^2}(3\beta+1) < 0$$

3.4.3 Simulation for social welfare when $\lambda < \Delta + 2\Delta\beta$

In this section we show how the graph of social welfare changes.

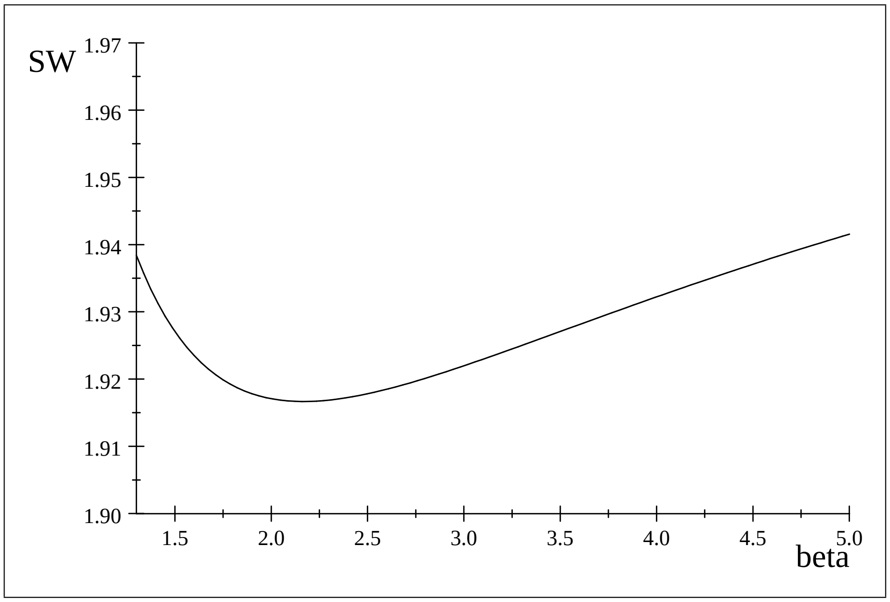
We give the following meanings to the parameters to built the figure for social welfare, the meaning are taken from the possible intervals: $\Delta = 0.5, \lambda = 1$

$$S_w = \frac{1}{24}\frac{24\Delta+\Delta^3+4\lambda^3+4\Delta^3\beta^2+96\Delta\beta+96\Delta\beta^2+4\Delta^3\beta+12\Delta\beta\lambda^2+24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2} = \frac{8.3333\times10^{-2}}{(2\beta+1)^2}\left(60.5\beta^2 + 54.5\beta + 16.12\right.$$



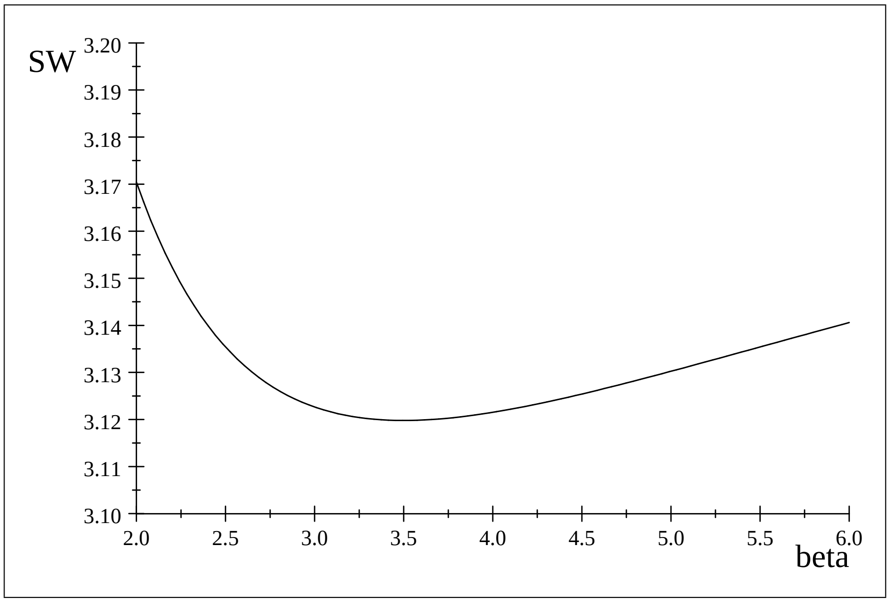
If we change the parameter of partisan objective to $\lambda = 2$ the shape of the curve remains the same, however it moves up-right.

$S_w = \frac{1}{24}\frac{24\Delta+\Delta^3+4\lambda^3+4\Delta^3\beta^2+96\Delta\beta+96\Delta\beta^2+4\Delta^3\beta+12\Delta\beta\lambda^2+24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2} = \frac{8.3333\times 10^{-2}}{(2\beta+1)^2}\left(96.5\beta^2 + 72.5\beta + 44.12\right.$



If we change the parameter of partisan objective to $\lambda = 3$

$S_w = \frac{1}{24}\frac{24\Delta+\Delta^3+4\lambda^3+4\Delta^3\beta^2+96\Delta\beta+96\Delta\beta^2+4\Delta^3\beta+12\Delta\beta\lambda^2+24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2} = \frac{8.3333\times 10^{-2}}{(2\beta+1)^2}\left(156.5\beta^2 + 102.5\beta + 120\right.$

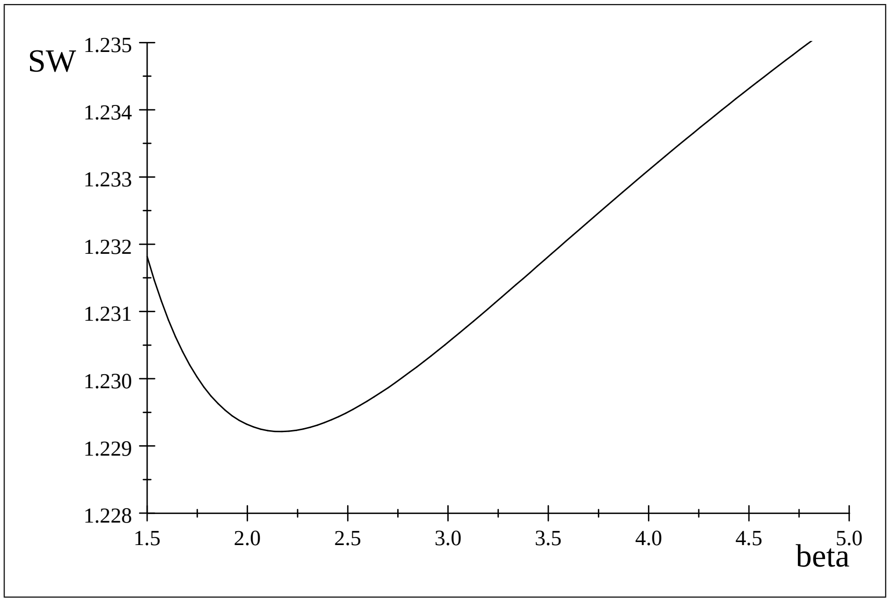


As we change the dispersion of the return Δ the graph moves also up-right, however more slowly than in case of changing λ

$\Delta = 0.25$

$\lambda = 1$

$S_w = \frac{1}{24} \frac{24\Delta+\Delta^3+4\lambda^3+4\Delta^3\beta^2+96\Delta\beta+96\Delta\beta^2+4\Delta^3\beta+12\Delta\beta\lambda^2+24\Delta\beta^2\lambda^2}{\Delta(2\beta+1)^2} = \frac{0.166\,67}{(2\beta+1)^2} \left(30.\,063\beta^2 + 27.\,063\beta + 10.\,0\right.$



3.4.4 Proof of Proposition 3.15

Computation of consumer's surplus and social welfare

When $\lambda \geq \Delta + 2\Delta\beta$, there is only one scheme for $\mu = \frac{\theta - \frac{\Delta}{2}}{\Delta}$. We substitute this expression in the first order conditions 3.6 and 3.7 and obtain $\alpha^*(\theta) = \frac{3}{2}\theta - \frac{1}{4}\Delta + \frac{1}{2}\lambda$ and $s^*(\theta) = 2\theta - \frac{1}{2}\Delta$

As in section 3.3.1 we proceed our computation with the expressing the tariff from the utility function of the agent $t = v - \lambda\alpha + \frac{1}{2}(\alpha - \theta)^2 + \frac{\beta}{2}(s - \theta)^2$ and substitute in the corresponding parameters where v is an integrated 3.4: $v = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} ((\alpha - \theta) + \beta(s - \theta))\, d\theta = -\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda + \frac{1}{2}\Delta\beta\right)$. So after all the substitutions we get:

$t = \frac{1}{32}\left(4\theta^2 - 7\Delta^2 - 12\lambda^2 - 4\theta\Delta - 40\theta\lambda + 20\Delta\lambda + 16\theta^2\beta - 12\Delta^2\beta - 16\theta\Delta\beta\right)$

To compute the unconditional tariff we need to integrate t on all the interval for

θ :

$$T = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} t d\theta = -\tfrac{1}{24}\Delta\left(5\Delta^2 + 9\lambda^2 - 15\Delta\lambda + 8\Delta^2\beta\right)$$

The unconditional utility of investor is an integrated indirect utility on the compact Θ:

$$U = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \left(1 - t + \alpha\theta - \tfrac{1}{2}\alpha^2\right)\tfrac{d\theta}{\Delta} = \tfrac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 8\Delta^2\beta + 24\right)$$

The unconditional utility of agent is an integrated indirect utility on the compact

Θ:

$$V = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \left(-\Delta\left(\tfrac{1}{4}\Delta - \tfrac{1}{2}\lambda + \tfrac{1}{2}\Delta\beta\right)\right)\tfrac{d\theta}{\Delta} = -\Delta\left(\tfrac{1}{4}\Delta - \tfrac{1}{2}\lambda + \tfrac{1}{2}\Delta\beta\right)$$

As before we compute the social welfare as a sum of expected utilities of principal and agent:

$$SW = -\tfrac{1}{24}\left(\Delta^2 - 6\lambda^2 + 4\Delta^2\beta - 24\right)$$

3.4.5 Comparative Statics for Corollary 9

We compute the derivatives for the following expressions with respect to the key parameters:

$U = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 8\Delta^2\beta + 24\right)$ unconditional utility of investor

$V = -\Delta\left(\frac{1}{4}\Delta - \frac{1}{2}\lambda + \frac{1}{2}\Delta\beta\right)$ unconditional utility of advisor

$S_w = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 + 4\Delta^2\beta - 24\right)$ social welfare computed as a sum of expected utilities of principal and agent

Investor

Derivative with respect to partisan objective:

$\frac{\partial U}{\partial \lambda} = \frac{1}{2}\lambda - \frac{1}{2}\Delta > 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial U}{\partial \beta} = \frac{1}{3}\Delta^2 \geq 0$

Derivative with respect to asset dispersion Δ:

$\frac{\partial U}{\partial \Delta} = \frac{5}{12}\Delta - \frac{1}{2}\lambda + \frac{2}{3}\Delta\beta$

Advisor

Derivative with respect to partisan objective:

$\frac{\partial V}{\partial \lambda} = \frac{1}{2}\Delta \geq 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial V}{\partial \beta} = -\frac{1}{2}\Delta^2 < 0$

Derivative with respect to asset dispersion Δ:

$\frac{\partial V}{\partial \Delta} = \frac{1}{2}\lambda - \frac{1}{2}\Delta - \Delta\beta \geq 0$ we can rewrite this expression as $\frac{1}{2}\left(\lambda - (\Delta + 2\Delta\beta)\right)$, as we are considering the case when $\lambda \geq \Delta + 2\Delta\beta$, it is obvious this derivative is positive.

Social Welfare

Derivative with respect to partisan objective:

$\frac{\partial S_w}{\partial \lambda} = \frac{1}{2}\lambda > 0$

Derivative with respect to degree of publicness of information:

$\frac{\partial S_w}{\partial \beta} = -\frac{1}{6}\Delta^2 < 0$

Derivative with respect to asset dispersion Δ:

$\frac{\partial S_w}{\partial \Delta} = -\frac{1}{12}\Delta - \frac{1}{3}\Delta\beta < 0$

3.4.6

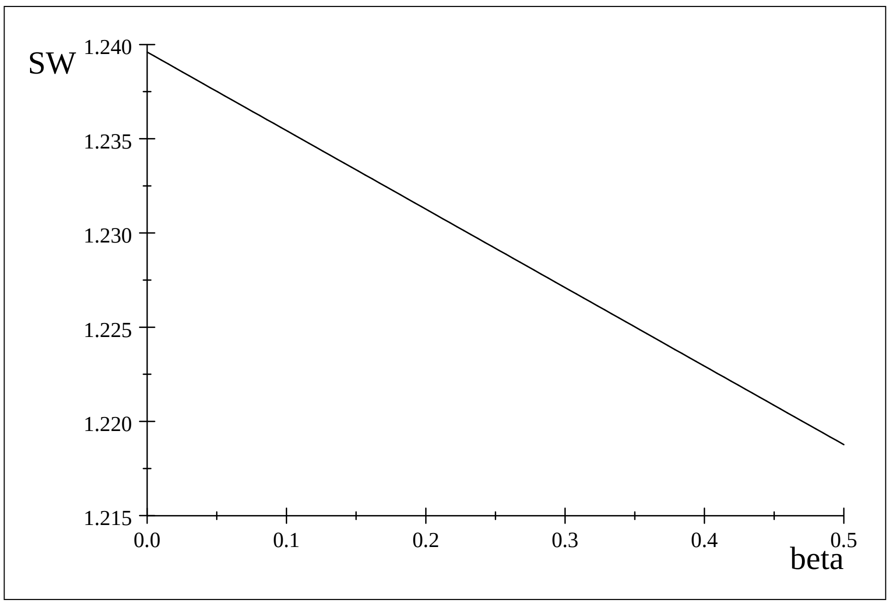
3.4.7 Simulations for social welfare when $\lambda \geq \Delta + 2\Delta\beta$

$\lambda = 1$

$\Delta = 0.5$

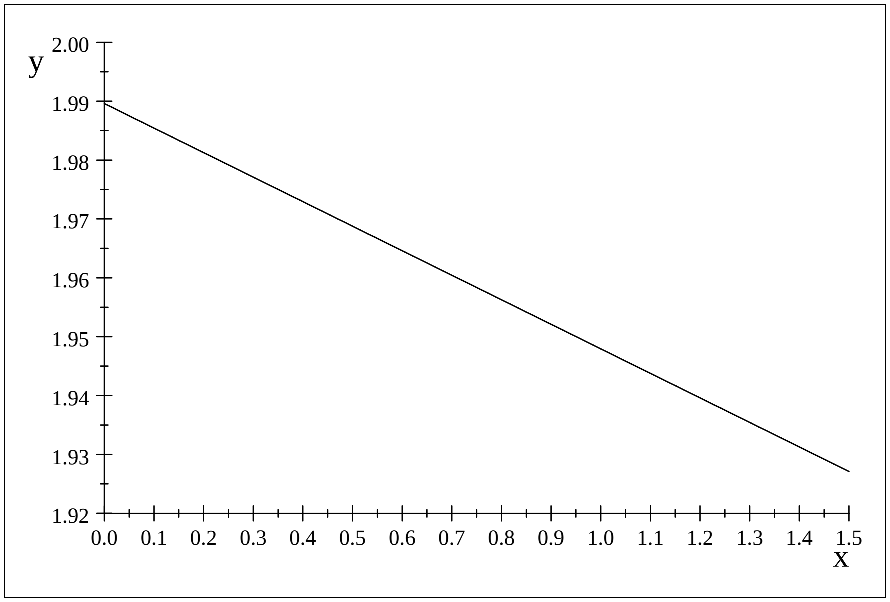
$U = \frac{1}{24}\left(5\Delta^2 + 6\lambda^2 - 12\Delta\lambda + 8\Delta^2\beta + 24\right)$

$S_w = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 + 4\Delta^2\beta - 24\right) = 1.2396 - 4.1667 \times 10^{-2}\beta$



$\lambda = 2$

$S_w = -\frac{1}{24}\left(\Delta^2 - 6\lambda^2 + 4\Delta^2\beta - 24\right) = 1.9896 - 4.1667 \times 10^{-2}\beta$


y
2.00
1.99
1.98
1.97
1.96
1.95
1.94
1.93
1.92
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
x

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