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Some Uniqueness Conditions and Stability Estimates for the Identification of Conductivity in a Parabolic PDE

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UNIQUENESS CONDITIONS: *Cauchy datum*

Hp.1 data pair $\{u, f\}, \{v, f\} \cdot\ni\cdot\ u_t, v_t, f \in C^0(\bar{T}; H^{-1}(D))$

and $u_x, v_x \in C^0(\bar{T}; C^0(\bar{D}))$; $v_x \neq 0, \forall (x, t) \in \bar{Q}$

Def.1 $w := v - u$

Hp.2 $\exists\ \hat{a}(u, f), b(v, f) \in A_{ad}$

Def.2 $g := b - \hat{a}$

Hp.3 $g \in B_{ad} := \{ g \mid g \in L^{\infty}(D); \exists \lim_{x \to x_0^+} g \text{ and } \lim_{x \to x_0^+} g = 0 \}$

Def.3 the defect $r := -w_t + (a w_x)_x$; $R := \frac{r}{v_x}$

Hp.4 properties of antiderivatives of distributions

$\{R^{[-1]}\} \subset C^0(\bar{T}; L^{\infty}(D))$; $\exists \lim_{x \to x_0^+} R^{[-1]}\ \forall\ t \in \bar{T}$

Thm. *IF* $\exists\ s \in \bar{T} \cdot\ni\cdot\ w(s) = w_t(s) = 0\ \forall x \in D$ *THEN* $g =_{a.e.} 0$ in D.

UNIQUENESS CONDITIONS:

Kitamura & Nakagiri extended*

Def. (set of points where u stationary)

$$E_u := \{y \mid y \in \bar{D},\ y(.) \in C^0(\bar{T}),\ \text{either } u_x(y,t)=0 \text{ or } \lim_{x \to y^{\pm}} u_x(y,t)=0\}$$

$f \in L^2(T, H^{-1}(D))$; $u \in \mathcal{U}_{ad}$

Hp. $\exists\, \tilde{a} \in A_{ad} \ni u(\hat{a}, f) =_{\text{a.e.}} u(\tilde{a}, f)$ in Q.

Thm.

IF

either $\exists\, s \in T \ni \{E_u(s) \neq \emptyset;\ \text{meas}[E_u(s)] = 0\}$ (KN1)

or $\forall\, t \in T\, \{E_u(t) \neq \emptyset;\ \text{meas}[\bigcap_t E_u(t)] = 0\}$ (KN2)

or $\exists\, E_u$ indep. of $t \ni \{E_u \neq \emptyset;\ \text{meas}[E_u] = 0\}$ (KN2.1)

or $\forall\, t \in T,\ \{E_u(t) \neq \emptyset;\ \text{meas}[E_u(t)] = 0\}$ (KN2.2)

THEN $\tilde{a} =_{\text{a.e.}} \hat{a}$.

(*) SICON, (1977) 15, 785-802

STABILITY ESTIMATE
in L^∞ (Cauchy-unique solution)

Hp 1) (reference and second potentials)

$u, v \in \mathcal{X} := \mathcal{C}^0(\bar{T}, H^2(D)) \cap \mathcal{C}^1(\bar{T}, L^2(D))$

2) (available data)

u, u_t, v, v_t @ $t = \tau$

3) $\left|\frac{1}{v_x}\right|(\tau) \leq c_v$; $\|v_{xx}\|_{0,2}(\tau) \leq c_2$

4) (reference solution)

$\hat{a}(u,f) \in \mathcal{C}^1(\bar{D} \setminus \{y_i\})$; $y_i \neq x_0, x_1, \forall i$

Rem. 1) $\hat{a}_x = \Sigma_i c_i \delta(x - y_i) + \Sigma_i \psi_{i,x} \chi_{(y_i, y_{i+1})}$

Hp. 5) $\Sigma_i |c_i| < \infty \Rightarrow \{|a_x|^{[-1]}\} \subset L^\infty(D)$

6) $\exists\, b(v,f) \in \mathcal{A}_{ad}$; $b - \hat{a} := g \in \mathcal{B}_{ad}$

Rem. 2) (defect sc.)

$\{R^{[-1]}\}(\tau) \in L^\infty(D)$; $\exists \lim_{x \to x_0^+} R^{[-1]}(\tau)$

Th. $\|b - \hat{a}\|_{0,\infty} \leq [1 + \|\hat{a}\|_{0,\infty} + \||\hat{a}_x|^{[-1]}\|_{0,\infty}] \cdot c_v \cdot$

$\cdot \|v - u\|_{\mathcal{X}(\tau)} \cdot \exp[c_v c_2 \sqrt{\mathrm{meas}[D]}]$

$$\|w\|_{\mathcal{X}(\tau)} := \max_{\bar{D}} (|w| + |w_x|)(\tau) + \sqrt{\mathrm{meas}[D]}\,[\|w_t\|_{0,2} + \|w_{xx}\|_{0,2}](\tau)$$

STABILITY ESTIMATE
L^1 (KN2 - unique solution)

Hp. 1) $u \in \mathcal{X}$ (as above)

2) (special case) $E_u(t) = [\zeta_u(t), \xi_u(t)]$

Def. 1) $I_u(t) := [x_0, \zeta_u(t))$; $J_u := (\xi_u(t), x_1]$

Rem. 1) $\overline{D} \setminus E_u(t) = I_u(t) \cup J_u(t)$

Hp. 3) $\frac{1}{u_x} \in \mathcal{C}^0(\overline{T}, L^1(D \setminus E_u(t)))$ i.e. $\left(\frac{1}{u_x}\right)(t) \in L^1(D \setminus E_u(t)), \forall t$

Def. 2) $I_a := [\min_{\overline{T}} \zeta_u(t) - \varepsilon, \max_{\overline{T}} \xi_u(t) + \varepsilon] \cap \overline{D}$; $\varepsilon > 0$

Hp. 4) $\hat{a} \in \mathcal{C}^0(I_a) \cap A_{ad}$

5) $v \in \mathcal{X}$, $E_v(t) \neq \emptyset$, $\bigcup_t E_v(t) \subseteq I_a$

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6) $\text{meas}[\bigcap_t E_v(t)] = 0$ $\quad\quad$ $\text{meas}[\bigcap_t E_v(t)] > 0$

7) $\left(\frac{1}{v_x}\right)(t) \in L^1(D \setminus E_v(t))$, $\forall t \in \overline{T}$

8) $\max_{\overline{T}} \left\| \frac{1}{v_x} \right\|_{L^1(D \setminus E_v(t))} (t) \leq c_v$

Notation $\quad \mathcal{Y} := L^1(D \setminus \bigcap_t E_v(t))$

Th. $\|b - \hat{a}\|_{L^1(D)} \leq \quad \|b - \hat{a}\|_{\mathcal{Y}} \leq$

$$\leq 2 c_v (1 + 2 \|\hat{a}\|_{0,\infty}) \|v - u\|_{\mathcal{X}}$$