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Reframing coverage estimation under line-strip sampling in the Monte Carlo integration framework

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ABSTRACT

For the first time in ecological applications, the coverage of an attribute is estimated by line-strip sampling in which several strips of fixed width, running across the whole study area, are selected on a baseline and the coverage within these strips is recorded. Under line-strip sampling, the coverage can be expressed as the integral of the partial coverages within the strips, thus enabling its estimation through Monte Carlo integration methods, in which strips are randomly placed on the baseline according to uniform random sampling, tessellation stratified sampling, and systematic grid sampling. A simulation study based on real habitat maps of three coastal dune systems in the United Kingdom is conducted to assess the performance of these three integration strategies. Simulation results suggest tessellation stratified sampling to be the most suitable scheme to locate strips. Moreover, a case study on alien species coverage in a Mediterranean dune ecosystem in Italy is examined. Finally, the advantages of using line-strip sampling with respect to the use of familiar schemes as point sampling and line-intercept sampling are discussed.

1. Introduction

In environmental and ecological studies, strip sampling, which involves the complete survey of several “strips” located on the study region in accordance with some probabilistic schemes, has a long-standing tradition for estimating totals or densities of interest attributes in ecological and forest communities (e.g., Chapman, 1924). When strips are of prefixed length and width, they can be regarded as long, narrow plots, and standard results from plot sampling and related edge effects directly apply (e.g., Gregoire and Valentine, 2008, Chapter 7). However, strip sampling can alternatively be implemented by defining the baseline as the projection of the study area onto a line of arbitrary direction and by selecting points along it, from which perpendicular strips of fixed width and random length, running across the whole study area, are established. In early ecological applications, this protocol was usually referred to as Line-Strip Sampling (LSS) and was often preferred to the use of long strips of prefixed length and width, as it avoids the location and delineation of strips and can be performed by simply following the randomly placed lines, starting from the selected points and perpendicular to the baseline, and extending the survey half a width to each side (see, e.g., Weaver and Clements, 1929; Bauer, 1943; Borman, 1953). In particular, the importance of LSS in phytosociological studies was emphasized in two pioneering papers by Woodin and Lindsey (1954) and Lindsey (1955). Several years later, Eberhardt et al. (1979) raised the issue of whether it was preferable to adopt relatively narrow line-strips to ensure that all objects were detected within them, or to indefinitely enlarge

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the width, giving rise to incomplete counts that could be corrected by auxiliary information provided by the distances of the detected objects from the lines, in accordance with the newly developed line-transect sampling theory (e.g., Eberhardt, 1968; Gates et al., 1968; Eberhardt, 1978). The issue was subsequently investigated by Burnham and Anderson (1984) and Burnham et al. (1985), who delineated pros and cons of the two sampling approaches. Since then, in distance sampling literature, LSS is viewed as a special case of line-transect sampling and it is usually referred to as strip-transect sampling (Buckland et al., 2001).

In environmental surveys, the estimation of the size of the portion of the study area covered by an attribute of interest, usually referred to as coverage, plays a central role. Applications include assessing forest canopy extent (e.g., Korhonen et al., 2006), wetland extent (e.g., Davidson and Finlayson, 2018), quantifying impervious surface area (e.g., Wang et al., 2018), monitoring snow cover (e.g., Bormann et al., 2018), detecting invasive species spread (e.g., Tobin et al., 2007), and delineating zones of soil contamination (e.g., Largueche, 2006). Traditionally, as early as the 1950s, coverage estimation was performed via Point Sampling (PS), based on the presence/absence of the interest attribute at sample points (e.g., Goodall, 1952; Evans and Love, 1957). Later, after the seminal paper by Lucas and Seber (1977), a sampling protocol referred to as Line-Intercept Sampling (LIS) became increasingly popular to estimate coverage. In the LIS protocol, lines are randomly located on the baseline and the lengths of their intersections with the attribute of interest are recorded. Through the homogeneous linear estimation criterion (e.g., Hedayat and Sinha, 1991, Section 2.2), for any coverage defined as the total size of particles of arbitrary shape and spatial location, the authors provided an unbiased estimator based on a linear combination of the intersection lengths. Elzinga et al. (2001, Chapter 12) provided a thorough discussion of the practical advantages and disadvantages of PS vs. LIS.

The purpose of this paper is to exploit LSS for estimating coverage. Since LSS records the size of the intersection between the area covered by the attribute of interest and the strips, rather than presence/absence data or intersection lengths, the information acquired from LSS is far greater than that obtained from PS or LIS. Therefore, LSS would obviously be more efficient than these two schemes in coverage estimation, albeit at the cost of a considerable increase in field work, since the on-field measurement of areas is more cumbersome than recording presence/absence or lengths. Despite the presumable superiority of LSS over PS and LIS, apart from a stereological application in microscopy analysis (Baddeley and Jensen, 2004, Section 7.1.3), to our knowledge this proposal constitutes the first attempt to adopt LSS for coverage estimation in ecological and environmental studies while ensuring statistical soundness of the resulting estimator.

In particular, this paper originates from the need to estimate the vegetation coverage of a Natura 2000 coastal dune site located along the northern Adriatic coast of Italy. The estimation of vegetation coverage is indeed a key step for monitoring biodiversity and ecological integrity, evaluating the conservation status of native plant communities in the face of alien species invasion, and assessing the success of ecological restoration efforts (e.g., Park et al., 2024). This is especially relevant in coastal dune ecosystems, which are fragile and dynamic environments supporting numerous species of high conservation priority (Beccari et al., 2024). These systems are typically characterized by steep environmental gradients, leading to pronounced vegetation zonation from the shoreline to the inland (Acosta et al., 2007), a feature that makes LSS suitable from a practical point of view (see, e.g., Tordoni et al., 2018; Attorre et al., 2013).

The paper is organized as follows. In Section 2, coverage is alternatively expressed by three different integrals in accordance with the sampling scheme adopted, i.e., PS, LIS, and LSS. In particular, the novel formulation of coverage, expressed as the integral of the function that, for any point of the baseline – suitably enlarged to avoid edge effects – gives the coverage within the strip of prefixed width centred at that point, allows for viewing coverage estimation based on LSS as Monte Carlo integration over the enlarged baseline. Section 3 then introduces design-based coverage estimation using LSS through three Monte Carlo integration strategies to select starting points on the baseline: uniform random sampling, tessellation stratified sampling, and systematic grid sampling (see, e.g., Gregoire and Valentine, 2008, Chapter 10; Kiêu et al., 1999). Section 4 provides an extensive simulation study to empirically assess the performance of these strategies, Section 5 presents the case study, and finally Section 6 offers concluding remarks.

2. Notation and setting

Denote by $A \subset \mathbb{R}^2$ the study area and let $C \subset A$ be the region covered by the interest attribute. The coverage, that is the size of C, can be expressed as the integral over A of a function identifying the presence or absence of the attribute defined at any point of the study area, specifically

$$|C| = \int_A I_C(x,y)\,dx\,dy \tag{1}$$

where I_C is the indicator function such that $I_C(x,y)=1$ when $(x,y)\in C$ and 0 otherwise.

To express $|C|$ as an integral in a one-dimensional interval, there is the need to define the baseline, that is the projection of A onto a line of arbitrary direction (e.g., Thompson, 2002, Section 17.7). Denoting by B the interval constituting the baseline, coverage can be expressed as the following integral over B

$$|C| = \int_B c(x)\,dx \tag{2}$$

where for any $x \in B$

$$c(x) = \int_{A_x} I_C(x,y)\,dy$$

is the coverage in the one-dimensional region $A_x = \{y : (x, y) \in A\}$ having fixed direction and random length. Usually, the orientation and placement of the baseline is determined by the geometry of the study area and by the presence of spatial and environmental gradients, thereby ensuring that the baseline effectively captures heterogeneity across the landscape.

Eqs. (1) and (2) play a key role in coverage estimation. In particular, (1) allows the coverage estimation based on the proportion of sample points falling within the area covered by the interest attribute to be viewed as a Monte Carlo integration over A, in which the dichotomous presence/absence I_C of the attribute is recorded at each sample point selected in A. Similarly, Eq. (2) allows the coverage estimation based on LIS to be viewed as a Monte Carlo integration over B, in which the intersection length c of the attribute is recorded for each sample line whose starting point is selected on B. While sampling by points or lines does not involve any edge effects, since sample points and lines lie entirely within A, when sampling by strips of fixed width w edge effects arise because strips starting near the ends of the baseline may extend partially outside the study region A. To handle edge effects, for each $x \in B$ consider the indicator function $I_{[x-\frac{w}{2},x+\frac{w}{2}]}$, that is $I_{[x-\frac{w}{2},x+\frac{w}{2}]}(u) = 1$ iff $|u - x| \leq w/2$, and enlarge B to achieve an interval B^* such that

$$\int_{B^*} I_{[x-\frac{w}{2},x+\frac{w}{2}]}(u)du = w. \tag{3}$$

Thanks to the enlargement, the projection of the strip centred at any point on the baseline completely lies within the enlarged baseline B^*. Then, for each $x \in B^*$ consider the function

$$h(x) = \int_B c(u)I_{[x-\frac{w}{2},x+\frac{w}{2}]}(u)du \tag{4}$$

that gives the coverage in the strip of width w centred at x. Therefore, owing to (2), (3) and (4), it holds that

$$\begin{aligned}
\int_{B^*} h(x)dx &= \int_B \left\{ \int_{B^*} c(u)I_{[x-\frac{w}{2},x+\frac{w}{2}]}(u)dx \right\} du \\
&= \int_B c(u) \left\{ \int_{B^*} I_{[x-\frac{w}{2},x+\frac{w}{2}]}(u)dx \right\} du \\
&= \int_B c(u) \left\{ \int_{B^*} I_{[u-\frac{w}{2},u+\frac{w}{2}]}(x)dx \right\} du \\
&= w \int_B c(u)du \\
&= w|C|
\end{aligned}$$

so that

$$|C| = \frac{1}{w} \int_{B^*} h(x)dx. \tag{5}$$

Eq. (5) offers an alternative formulation in terms of the integral extended to the enlarged baseline B^* of the function h. This naturally leads to viewing coverage estimation based on LSS as a Monte Carlo integration over B^*, in which the coverage h of the interest attribute is recorded within any sample strip whose central line has its starting point selected on B^*.

As to the choice of w, it is based on considerations that account for the characteristics of the interest attribute, the features of the study region, and the ability to quantify the coverage within the strip. When w is very small compared to the baseline length (e.g., few metres against some kilometres), edge effects are negligible, and strips can be selected directly on B.

Finally, even if in this section the LSS theory has been approached referring to strips centred at the sampled lines, alternatively, strip sampling can be implemented considering strips located to the left or to the right of the lines. Also in these cases, the integral representation of coverage of type (5) remains valid, provided that appropriate edge corrections are applied.

3. Coverage estimation by LSS

Interestingly, familiar Monte Carlo integration techniques involve selection schemes analogous to sampling schemes widely employed in environmental surveys (Barabesi, 2003; Gregoire and Valentine, 2008). In the case of LSS, n strips are placed across the study area centred on n lines with as many starting points selected onto the enlarged baseline B^*, from now denoted by $[0, b^*]$.

The more straightforward scheme for locating strips, which corresponds to the basic Monte Carlo procedure, is Uniform Random Sampling (URS) which consists in randomly and independently selecting n starting points onto B^*. Denoting by $X_1, \ldots, X_n$ the i.i.d. random variables uniformly distributed on $[0, b^*]$ representing the starting points, the Monte Carlo integration gives rise to the estimator

$$\widehat{|C|}_{n,\mathrm{URS}} = \frac{b^*}{nw} \sum_{i=1}^{n} h(X_i) \tag{6}$$

which is unbiased with variance

$$V(\widehat{|C|}_{n,\mathrm{URS}}) = \frac{b^*}{nw^2} \int_0^{b^*} h(x)^2 dx - \frac{|C|^2}{n}$$

which can be unbiasedly estimated by

$$\hat{V}_1(\widehat{|C|}_{n,\mathrm{URS}}) = \frac{(b^*)^2}{w^2 n(n-1)} \sum_{i=1}^{n} (h(X_i) - \overline{h})^2 \tag{7}$$

where $\overline{h} = \frac{1}{n}\sum_{i=1}^{n} h(X_i)$.

Despite its simplicity, URS may lead to uneven surveying of the study area, an issue which can be straightforwardly solved by using Tessellation Stratified Sampling (TSS), which is the analogue of the modified Monte Carlo procedure. Under LSS, TSS consists in partitioning the baseline into n segments of equal length and in randomly and independently selecting one starting point in each segment. Thus, the random variables $X_1, \ldots, X_n$ are now independent and uniformly distributed on $[\frac{i-1}{n}b^*, \frac{i}{n}b^*]$ for $i = 1, \ldots, n$. The Monte Carlo integration gives rise to the estimator $\widehat{|C|}_{n,\mathrm{TSS}}$ whose analytical formulation is identical to (6), it is unbiased with variance

$$V(\widehat{|C|}_{n,\mathrm{TSS}}) = \frac{b^*}{nw^2} \int_0^{b^*} h(x)^2 dx - \sum_{i=1}^{n} |C_i|^2$$

where $|C_i| = \frac{1}{w}\int_{\frac{i-1}{n}b^*}^{\frac{i}{n}b^*} h(x)dx$. Barabesi et al. (2012) have pointed out that, since $\sum_{i=1}^{n}|C_i|^2 \geq \frac{1}{n}(\sum_{i=1}^{n}|C_i|)^2$, then $V(\widehat{|C|}_{n,\mathrm{TSS}}) \leq V(\widehat{|C|}_{n,\mathrm{URS}})$, i.e., TSS is always preferable to URS. In addition, they proved that, under some smoothness conditions on h, TSS gives rise to a "superefficient" estimator of $|C|$ with respect to that obtained under URS, since $V(\widehat{|C|}_{n,\mathrm{TSS}})$ is of order $n^{-\gamma}$, with $\gamma \in (1, 2]$ while $V(\widehat{|C|}_{n,\mathrm{URS}})$ is of order n^{-1}.

The estimation of $V(\widehat{|C|}_{n,\mathrm{TSS}})$ is not trivial as $\widehat{|C|}_{n,\mathrm{TSS}}$ is a linear combination of non-identically distributed random variables and a single observation from each variable is available. A straightforward conservative estimator of $V(\widehat{|C|}_{n,\mathrm{TSS}})$, say $\hat{V}_1(\widehat{|C|}_{n,\mathrm{TSS}})$ is given by (7), as if points were selected by URS. Barabesi et al. (2012) alternatively proposed the following more refined conservative estimator

$$\hat{V}_2(\widehat{|C|}_{n,\mathrm{TSS}}) = \frac{(b^*)^2}{2w^2 n^2} [h(X_1)^2 + \sum_{i=1}^{n-1} (h(X_i) - h(X_{i+1}))^2 + h(X_n)^2] \tag{8}$$

A further sampling scheme ensuring an even coverage of the study region is Systematic Grid Sampling (SGS) which, under LSS, consists of partitioning B^* into n segments of equal length, randomly selecting a starting point in the first segment and systematically repeating the selection in the remaining segments. By this scheme, the random variables $X_1, \ldots, X_n$ are dependent and uniformly distributed on $[\frac{i-1}{n}b^*, \frac{i}{n}b^*]$ for $i = 1, \ldots, n$. The Monte Carlo integration gives rise to the estimator $\widehat{|C|}_{n,\mathrm{SGS}}$ whose analytical formulation is identical to (6), it is unbiased with variance

$$V(\widehat{|C|}_{n,\mathrm{SGS}}) = V(\widehat{|C|}_{n,\mathrm{TSS}}) + 2\frac{(b^*)^2}{w^2 n^2} \sum_{h>i=1}^{n} Cov(h(X_i), h(X_h)).$$

The variance expression clearly indicates that, in presence of negative correlations of within-strip coverages, SGS outperforms TSS. However, in the presence of periodicities of coverages that possibly align with the segmentation of the baseline, its performance can heavily deteriorate, potentially yielding higher variance than that obtained under URS. The dependence among $X_1, \ldots, X_n$ makes variance estimation an extremely challenging task. Once again, a first rough estimator $\hat{V}_1(\widehat{|C|}_{n,\mathrm{SGS}})$ is simply given by (7), as if points were selected by URS. Alternatively, Kiêu et al. (1999) prove that, under some regularity conditions on h, a consistent estimator (n large) is given by

$$\hat{V}_2(\widehat{|C|}_{n,\mathrm{SGS}}) = \frac{(b^*)^2}{12w^2 n^2}[3\sum_{i=1}^{n} h(X_i)^2 - 4\sum_{i=1}^{n-1} h(X_i)h(X_{i+1}) + \sum_{i=1}^{n-2} h(X_i)h(X_{i+2})].$$

The three Monte Carlo estimators, together with their corresponding variance estimators, are straightforward to compute. In particular, in both the simulation study and the empirical case study presented below, they were implemented in R (R Core Team, 2023) and executed on a standard workstation (AMD Ryzen 5 PRO 5650G with 6 cores, 12 logical processors, and 32 GB of RAM). R codes are available on request from the authors.

4. Simulation study

The performance of the three Monte Carlo estimation strategies is assessed through a simulation study. Specifically, three coastal dune systems located within distinct protected areas in the United Kingdom are considered: Holkham Dunes (North Norfolk), Sandwich Dunes (Kent), and North Walney Dunes (Walney Island, Cumbria). The habitats within these three dune systems were completely mapped through the analysis of LiDAR and Compact Airborne Spectrographic Imager data using Object-Based Image Analysis (Brownett and Mills, 2017). Habitat maps were developed as a part of a collaborative project between the Environment Agency and Natural England, integrating advanced remote sensing techniques with ecological expertise. The resulting maps are publicly accessible and can be downloaded in shapefile format under the Open Government Licence via Data.gov.uk (Defra, 2016).

The Holkham Dunes cover 162 hectares and support 25 distinct habitats. Nearly half of this area is dominated by Bare Sand (48%), followed by Floodplain and Coastal Grazing Marsh (13%), Fixed Dune Grassland (13%), and Marram-Dominated Dune (12%). The Sandwich Dunes span 151 hectares and feature 16 habitats. Fixed Dune Grassland occupies 36% of the area, with Wetlands

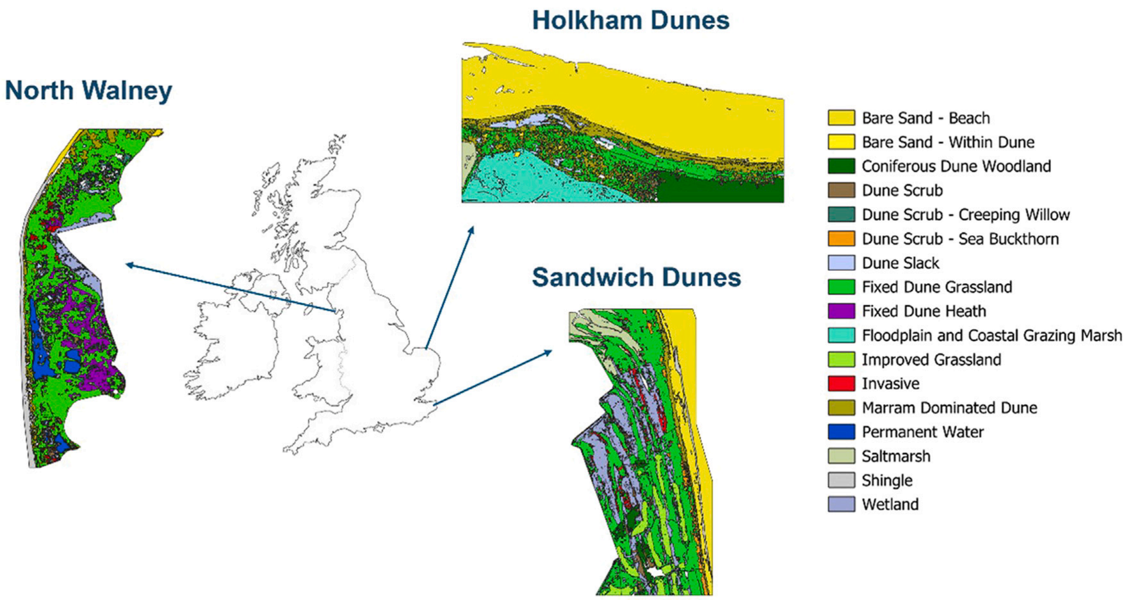

Fig. 1. Habitat map of North Walney, Holkham, and Sandwich dunes.

(19%) and Bare Sand (16%) being the next most prevalent habitat types. Finally, the North Walney Dunes cover 84 hectares and host 15 habitats. Fixed Dune Grassland constitutes 51% of the area, while the remainder consists mainly of Fixed Dune Heath (13%) and Wetlands (9%). Habitat maps of the three dune systems are presented in Fig. 1 and constitute the ground truth from which the simulation study is performed to estimate the coverage of each habitat.

For each of the three dune systems, LSS is simulated by artificially selecting $n = 25, 50$ strips of width $w = 4$ m along an enlarged baseline of 2.004 km according to URS, TSS, and SGS and the habitat coverages are recorded within the n strips. For each combination of dune system, sampling scheme and sample size, $M = 10\,000$ samples are independently selected by means of URS, TSS and SGS and the coverage estimates are obtained for each habitat by means of (6). In addition, standard error estimates are computed by the square root of $\hat{V}_1(\widehat{|C|}_{n,\mathrm{URS}})$ in the case of URS, the square root of $\hat{V}_1(\widehat{|C|}_{n,\mathrm{TSS}})$ and $\hat{V}_2(\widehat{|C|}_{n,\mathrm{TSS}})$ in the case of TSS, and the square root of $\hat{V}_1(\widehat{|C|}_{n,\mathrm{SGS}})$ and $\hat{V}_2(\widehat{|C|}_{n,\mathrm{SGS}})$ in the case of SGS. From the empirical distribution of each estimator, its Empirical Standard Error (ESE) is computed as

$$ESE(\widehat{|C|}_{n,S}) = \left[\frac{1}{M}\sum_{m=1}^{M}(\widehat{|C|}_{n,m,S} - |C|)^2\right]^{1/2}$$

where $\widehat{|C|}_{n,m,S}$ is the estimate achieved in the mth sample of size n selected using the sampling scheme S=URS, TSS, SGS and $|C|$ is the habitat coverage under estimation. Then, the corresponding Relative Standard Error (RSE) is obtained as the ratio between ESE and $|C|$. Moreover, the ratio between the empirical expectation of the standard error estimator to the ESE is computed and denoted by R_1 when the variance is estimated using estimators of type (7) and by R_2 when the variance is estimated using $\hat{V}_2(\widehat{|C|}_{n,\mathrm{TSS}})$ or $\hat{V}_2(\widehat{|C|}_{n,\mathrm{SGS}})$. The values of RSEs and ratios are reported in Tables 1–3 for the three coastal dunes systems and for each habitat whose coverage is at least 1% of the extent of the study area.

As expected, the precision of the coverage estimators heavily depends on the characteristics of the habitats both in terms of the extent of the area covered by the attribute and in terms of its spatial distribution. However, in general, both TSS and SGS outperform URS for any dune system and sample size, with SGS showing a slightly better performance. Moreover, the increase in precision tends to be more marked when the shape of the area covered by the habitat tends to be elongated, such as Dune Scrub and Marram Dominated Dune in Holkham Dunes or Fixed Dune Heath and Shingle in North Walney Dunes. This is due to the ability of TSS and SGS to provide spatial balance, which in this framework means that the selected points are well spread onto the baseline.

As to the variance estimation, the values of R_1 under URS are obviously near to 1, since the variance estimator is unbiased, while they are always larger than 1 under TSS, confirming the theoretical results on the conservativeness of $\hat{V}_1(\widehat{|C|}_{n,\mathrm{TSS}})$, and also under SGS. Under TSS, the values of R_2 remain above 1, confirming the conservative nature of $\hat{V}_2(\widehat{|C|}_{n,\mathrm{TSS}})$, but are always lower than the corresponding values of R_1, and in some cases show a substantial reduction for both sample sizes, such as in the estimation of Floodplain and Coastal Grazing Marsh coverage in Sandwich Dunes or Saltmarsh coverage in Holkham Dunes, thus highlighting the good performance of this estimation criterion. On the other hand, under SGS, the values of R_2 are always smaller than those of R_1, even though they can be also noticeably smaller than 1, such as when estimating Marram Dominated Dune in Holkham Dunes for both sample sizes. This highlights the risk of over evaluating the precision of $\widehat{|C|}_{n,\mathrm{SGS}}$ if $\hat{V}_2(\widehat{|C|}_{n,\mathrm{SGS}})$ is adopted.

Table 1
For Sandwich Dunes, habitat coverage is reported along with RSE and R_1 and R_2 values under URS, TSS and SGS schemes for sample sizes $n = 25, 50$.

Habitat	Coverage (%)	n	URS		TSS			SGS		
			RSE	R_1	RSE	R_1	R_2	RSE	R_1	R_2
Bare Sand - Beach	783 561.00 (48.3%)	25	0.02	1.00	0.01	2.46	4.70	0.01	2.20	1.65
Bare Sand - Within Dune	23 375.85 (1.44%)		0.23	0.98	0.14	1.63	1.41	0.11	2.14	0.93
Coniferous Dune Woodland	114 429.91 (7.05%)		0.25	0.99	0.04	5.89	2.56	0.04	6.91	1.25
Dune Scrub	26 093.26 (1.61%)		0.26	1.00	0.13	2.04	1.73	0.10	2.59	1.03
Dune Slack	36 342.92 (2.24%)		0.26	1.01	0.11	2.55	1.45	0.06	4.46	1.31
Fixed Dune Grassland	202 994.77 (12.51%)		0.11	1.00	0.04	3.15	1.73	0.04	3.17	0.81
Floodplain & Coastal Grazing Marsh	208 186.53 (12.83%)		0.23	1.00	0.03	8.14	2.24	0.01	26.74	1.78
Marram Dominated Dune	186 920.20 (11.52%)		0.09	1.00	0.04	2.57	1.51	0.03	3.19	0.90
Saltmarsh	16 868.76 (1.04%)		0.97	1.00	0.29	3.39	3.30	0.31	3.20	1.54
Bare Sand - Beach	783 561.10 (48.3%)	50	0.02	1.00	0.01	2.80	3.70	0.00	3.56	1.87
Bare Sand - Within Dune	23 375.85 (1.44%)		0.16	1.00	0.09	1.86	1.39	0.08	2.02	0.71
Coniferous Dune Woodland	114 429.91 (7.05%)		0.18	0.99	0.02	7.42	2.22	0.02	9.11	1.03
Dune Scrub	26 093.26 (1.61%)		0.18	0.99	0.08	2.39	1.51	0.08	2.32	0.62
Dune Slack	36 342.92 (2.24%)		0.19	0.99	0.06	3.48	1.66	0.02	9.16	2.08
Fixed Dune Grassland	202 994.77 (12.51%)		0.08	0.99	0.02	3.65	1.59	0.02	4.60	0.89
Floodplain & Coastal Grazing Marsh	208 186.53 (12.83%)		0.16	1.00	0.01	14.39	2.23	0.01	30.86	1.10
Marram Dominated Dune	186 920.20 (11.52%)		0.07	1.00	0.02	3.08	1.51	0.02	3.86	0.87
Saltmarsh	16 868.76 (1.04%)		0.68	1.01	0.16	4.33	3.15	0.17	4.18	1.28

Table 2
For Holkham Dunes, habitat coverage is reported along with RSE and R_1 and R_2 values under URS, TSS and SGS schemes for sample sizes $n = 25, 50$.

Habitat	Coverage (%)	n	URS		TSS			SGS		
			RSE	R_1	RSE	R_1	R_2	RSE	R_1	R_2
Bare Sand - Beach	249 241.90 (16.48%)	25	0.09	0.98	0.01	6.52	4.29	0.02	4.39	1.18
Coniferous Dune Woodland	28 137.25 (1.86%)		0.34	1.00	0.16	2.16	1.42	0.13	2.67	0.88
Dune Scrub	40 805.04 (2.7%)		0.18	0.99	0.09	2.03	1.47	0.06	3.04	1.07
Dune Scrub - Sea Buckthorn	29 529.23 (1.95%)		0.17	1.00	0.12	1.51	1.32	0.11	1.66	0.66
Fixed Dune Grassland	551 083.22 (36.45%)		0.05	1.00	0.03	2.12	2.38	0.01	5.10	2.53
Improved Grassland	111 935.44 (7.4%)		0.26	0.99	0.08	3.47	2.41	0.07	3.84	1.38
Invasive	37 491.08 (2.48%)		0.21	1.00	0.10	2.09	1.32	0.07	3.21	0.84
Marram Dominated Dune	41 856.22 (2.77%)		0.09	0.99	0.05	1.94	1.35	0.05	1.78	0.59
Saltmarsh	78 151.74 (5.17%)		0.43	1.00	0.10	4.52	2.21	0.08	5.51	0.94
Shingle	23 484.78 (1.55%)		0.12	1.00	0.07	1.81	1.40	0.05	2.35	0.82
Wetland	292 380.40 (19.34%)		0.14	1.00	0.03	4.77	1.70	0.02	7.18	1.14
Bare Sand - Beach	249 241.90 (16.48%)	50	0.06	1.01	0.01	7.59	3.53	0.01	4.86	0.91
Coniferous Dune Woodland	28 137.25 (1.86%)		0.24	1.02	0.09	2.80	1.56	0.07	3.62	0.96
Dune Scrub	40 805.04 (2.7%)		0.13	1.02	0.05	2.69	1.53	0.03	4.56	1.07
Dune Scrub - Sea Buckthorn	29 529.23 (1.95%)		0.12	1.01	0.07	1.81	1.44	0.08	1.47	0.59
Fixed Dune Grassland	551 083.22 (36.45%)		0.04	0.99	0.01	2.85	2.21	0.01	4.84	1.46
Improved Grassland	111 935.44 (7.4%)		0.18	1.01	0.04	4.66	2.65	0.04	5.09	1.31
Invasive	37 491.08 (2.48%)		0.15	1.00	0.05	2.85	1.50	0.02	7.52	1.90
Marram Dominated Dune	41 856.22 (2.77%)		0.06	1.01	0.03	2.18	1.29	0.04	1.51	0.41
Saltmarsh	78 151.74 (5.17%)		0.30	1.00	0.04	7.04	2.23	0.04	7.13	0.83
Shingle	23 484.78 (1.55%)		0.08	0.99	0.03	2.62	1.59	0.03	2.52	0.67
Wetland	292 380.40 (19.34%)		0.10	1.00	0.02	6.72	1.73	0.01	11.81	1.16

In practice, the simulation results clearly suggest the precautionary use of TSS, which invariably outperforms URS, joined with the use of (8) for the standard error estimation, which turns out to be moderately conservative. On the other hand, the use of SGS may provide unreliable coverage estimates in presence of periodicities and with uncomfortable difficulties for the standard error estimation, as both extremely conservative estimates and underestimates are possible.

5. Coverage estimation in the coastal dunes of Valle Vecchia: a case study

Coastal sand dunes represent one of the most fragile ecosystems in the Mediterranean basin and they often host species of high conservation value. Therefore, there is a need to constantly assess or update their conservation status to promote appropriate management strategies and preserve these peculiar environments. Along the northern Adriatic coastline, in particular, dune ecosystems have been severely impacted by sustained increases in tourism and urban development (Tordoni et al. 2018), resulting in the few natural remnants present along the coastline, now mostly enclosed within protected areas. The site of Valle Vecchia beach, located along the northern Adriatic coast and belonging to *Valle Vecchia, Zumelle and Valli di Bibione* (Italy) Natura 2000 Site

Table 3
For North Walney Dunes, habitat coverage is reported along with RSE and R_1 and R_2 values under URS, TSS and SGS schemes for sample sizes $n = 25, 50$.

Habitat	Coverage (%)	n	URS		TSS			SGS		
			RSE	R_1	RSE	R_1	R_2	RSE	R_1	R_2
Bare Sand - Beach	9977.94 (1.19%)	25	0.53	0.99	0.18	3.06	2.09	0.19	2.79	0.89
Dune Scrub	13 739.58 (1.63%)		0.29	1.01	0.20	1.48	1.29	0.23	1.29	0.57
Dune Scrub - Creeping Willow	20 843.83 (2.48%)		0.34	1.00	0.17	2.08	1.49	0.14	2.53	0.80
Dune Slack	10 878.23 (1.29%)		0.46	1.01	0.30	1.58	1.48	0.19	2.45	1.15
Fixed Dune Grassland	427 781.08 (50.82%)		0.08	1.00	0.03	2.50	2.01	0.02	4.35	1.51
Fixed Dune Heath	108 637.84 (12.91%)		0.23	1.01	0.09	2.65	1.42	0.08	2.79	0.69
Invasive	17 049.16 (2.03%)		0.30	1.01	0.19	1.55	1.20	0.11	2.64	1.08
Marram Dominated Dune	49 993.88 (5.94%)		0.29	1.00	0.11	2.84	2.57	0.11	2.79	1.23
Permanent Water	40 178.56 (4.77%)		0.42	1.00	0.16	2.69	1.86	0.13	3.35	1.01
Shingle	44 197.65 (5.25%)		0.10	0.99	0.02	4.50	2.66	0.02	4.05	1.03
Wetland	78 540.24 (9.33%)		0.18	1.00	0.07	2.60	1.54	0.07	2.86	0.81
Bare Sand - Beach	9977.94 (1.19%)	50	0.38	0.99	0.07	5.20	3.09	0.05	8.40	2.39
Dune Scrub	13 739.58 (1.63%)		0.20	1.01	0.12	1.81	1.40	0.06	3.76	1.44
Dune Scrub - Creeping Willow	20 843.83 (2.48%)		0.24	1.00	0.10	2.45	1.38	0.07	3.42	0.89
Dune Slack	10 878.23 (1.29%)		0.33	1.00	0.20	1.67	1.29	0.06	5.59	1.91
Fixed Dune Grassland	427 781.08 (50.82%)		0.06	1.01	0.02	3.19	1.77	0.01	7.35	1.54
Fixed Dune Heath	108 637.84 (12.91%)		0.17	1.00	0.05	3.55	1.58	0.05	3.51	0.70
Invasive	17 049.16 (2.03%)		0.21	1.00	0.12	1.70	1.21	0.08	2.77	0.92
Marram Dominated Dune	49 993.88 (5.94%)		0.21	0.99	0.06	3.75	2.66	0.03	7.72	2.44
Permanent Water	40 178.56 (4.77%)		0.30	0.99	0.08	3.62	1.75	0.02	13.43	2.69
Shingle	44 197.65 (5.25%)		0.07	0.99	0.01	5.66	2.38	0.01	6.86	1.16
Wetland	78 540.24 (9.33%)		0.13	1.00	0.03	3.85	1.71	0.02	7.24	1.34

of Community Importance, constitutes one of the few remnants of the predominantly sedimentary coastline where extensive sand dune vegetation persists.

Assessing the conservation status of native vegetation in relation to alien species presence is essential, and to this purpose the estimation of the coverage of alien species is of interest. Specifically, data on sand dune vegetation (available at https://sites.google.com/view/svebio-project) were collected in 2022 using a sampling protocol that can be viewed as a LSS where strips are determined by placing points onto a baseline by means of TSS. Indeed, Tordoni et al. (2018) overlaid a rectangle onto the study area, subsequently partitioned into a grid of 6 rectangular cells with a base of 500 m each, and in each cell, they selected a random transect following a sea-inland gradient with variable length due to dune width and coast morphology. This procedure corresponds to the implementation of TSS where the baseline of 3 000 m is partitioned into 6 segments of 500 m and a point is randomly and independently selected in each segment. From each point, strips of 4 m width were considered and, to assess the species coverage in each strip, the strips were partitioned in a set of contiguous square plots of 4 m × 4 m where the percentage visual cover of each vascular plant species was assessed, from which the coverage of the species in the strip can be straightforwardly obtained. The sampling scheme was implemented without the enlargement of the baseline. However, as already pointed out in Section 2, neglecting edge corrections is expected to have negligible effects on the estimates, as the baseline length of 3 000 m is notably larger than the strip width. Data are reported only for plots where vascular plants are present, while in the remaining plots the coverage is zero for all species. The dataset available online comprises 82 species, but the focus lies on three alien species: *Ambrosia psilostachya*, *Oenothera biennis* and *Xanthium strumarium subsp. Strumarium*.

Coverage and variance estimation are performed by means of (6) and (8), respectively. As to *Ambrosia psilostachya*, the coverage is estimated equal to 5 414 m^2 with a value of the relative standard error of 18.14%, while for *Oenothera biennis* and *Xanthium strumarium subsp. Strumarium* the coverage estimate is 1 708.6 m^2 and 1 355.4 m^2, respectively, with corresponding relative standard error of 46.71% and 43.45%. The high values of the relative standard errors may partly be attributable to the use of a slight conservative variance estimator, but especially to the limited number of transects and the narrow strip width, which was necessary to enable coverage assessment through visual inspection. Moreover, the large values of the relative standard error estimates for the last two species may also be influenced by their small extents, with coverages that are estimated to be less than one-third of that of *Ambrosia psilostachya*. It is indeed a well-known result of spatial sampling that, irrespective of the sampling scheme adopted, the relative precision of any coverage estimator decreases with the coverage extent.

6. Concluding remarks

Coverage estimation of ecological attributes is approached for the first time by LSS in a design-based framework relying on Monte Carlo integration methods. The theoretical considerations of Section 3 and the empirical results of the simulation study suggest locating strips on the enlarged baseline in accordance with TSS. Owing to the relevant gain in information achieved by LSS with respect to PS and LIS, the proposal has the potential to greatly improve precision relative to familiar estimation methods based on points or lines. Notwithstanding this, in the paper we have deliberately avoided comparing LSS-based estimators of coverage with estimators based on PS or LIS. Indeed, in this scenario any comparison would be unfair, and results would be obvious, given the formidable increase in information gained using LSS, even if achieved at the cost of a massive increase in field work. However, on this issue, it is worth noting that recent advancements in the unmanned aerial vehicle industry, commonly known as drones, along with the improvement and miniaturization of camera technology (e.g., Witczuk and Pagacz, 2021; Nuijten et al., 2021), have opened up new possibilities for reducing the sampling effort involved in LSS. This fact is likely to render more feasible the future use of this scheme and the related estimation.

Although the paper focuses on the estimation of coverage, the achieved results are by far more general. Indeed, firstly they are valid whenever the parameter under estimation can be expressed as the integral over the study area of an arbitrary function. Moreover, these results can be straightforwardly applied to the case of populations of units scattered over the study area, with a value of an interest attribute associated to each unit, when the parameter of interest is the total of the variable.

As to the case study, LSS has been applied to estimate the coverage of three alien species along a northern Adriatic coastline in Italy. Coastal dune monitoring is indeed an issue of great concern in Italy, a country with about 8 000 km of coasts against a relatively small territorial extension of about 300 000 km^2. On this issue, since the beginning of this century, the Institute of Bioeconomy of the National Research Council (CNR-IBE) has organized a biennial symposium on "Monitoring of Mediterranean Coastal Areas: Problems and Measurement Techniques". Our proposal of estimating coverage by LSS will be presented at the 11th edition, to be held in Livorno in June 2026, with the hope of giving rise to new possibilities of application.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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