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Two Essays on Endogenous Distributions in Macroeconomics

Surname: CANNIOTO Name: FABRIZIO

Registration number 833160

Supervisor: Domenico Delli Gatti

Coordinator: Matteo Manera

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Two Essays on Endogenous Distributions in Macroeconomics

Fabrizio Cannioto

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"I've still got to raise four dollars! That ain't much when you've got it, but an awful lot when you ain't"

Donald Duck, 1952

Declaration of Authorship

I, FABRIZIO CANNIOTO, declare under my responsibility that this thesis titled “Two Essays on Endogenous Distributions in Macroeconomics” is my own work and that:

- This work was done entirely while in candidature for a research degree at Bicocca University.
- This thesis was never submitted in part or in full for a degree at any other University.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from works of others, the source is always given. Apart from such quotations, the work is my own.
- I have acknowledged all the main sources of help.
- The thesis does not contain any work that has been done jointly with others.

Ringraziamenti

I miei ringraziamenti vanno prima di tutto a chi mi ha trasmesso amore. L'amore di mio papà Marco e mia mamma Cettina per il proprio figlio. L'amore di mio nonno Vincenzo e di mia nonna Rosa, l'uno per l'altro, assieme per 60 e oltre anni di matrimonio. L'amore della mia compagna Gaia in ogni mio momento di difficoltà di questo percorso. L'amore dei miei amici tutte le volte che non mi restano vicini quando cerco di sparire. E' stato un percorso duro, ed è merito di ognuna di queste persone se arriverò al traguardo.

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'Cause love's such an old fashioned word
And love dares you to care for
The people on the edge of the night
And love dares you to change our way of
Caring about ourselves
This is our last dance
This is our last dance
This is ourselves under pressure
Under pressure
Pressure

Queen and David Bowie, 1995.

General Abstract

This thesis consists of two papers that investigate the aggregate and distributional implications of an expansionary monetary policy shock and the introduction of a minimum wage, respectively. In particular, in this thesis I place emphasis on the importance of featuring endogenous wealth distributions in macroeconomic models. Despite I tackle two different research questions in the two papers, a common theme emerges: distributional and aggregate effects of a policy are closely linked, such that its cross-sectional implications cannot be neglected by the policymakers.

In the first paper, I extend the literature that has investigated the effects of monetary policy shock on aggregate consumption and the net worth distribution using heterogenous agent in continuous time models (see, e.g., the seminal Kaplan et al., 2018). Namely, I address an important limitation of the existing literature, that is, it features an exogenous borrowing constraint on the monetary balance. This setup makes debt opportunities independent of net worth (i.e. liquid plus illiquid assets) with debatable implications concerning the effects of an expansionary monetary policy shock on marginal propensities to consume and private debt. For these reasons, in this paper I assume a two-asset structure with a collateral-based borrowing constraint, which allows us to reproduce the non-monotonicity of marginal propensities to consume uncovered by the empirical literature (e.g. Crawley and Kuckler, 2020) and the rise in private debt that was experienced during the global financial crisis. The transition dynamics following an unexpected cut in policy rates show a boost in aggregate consumption backed by an increase in private debt and an increase in aggregate durable assets, which sum up into a reduction in the net worth inequality (as captured by the reduction in the Gini Index of net worth distribution). Notably, the presence

of an endogenous borrowing constraint has crucial implications for monetary policy. In fact, compared to a framework where the borrowing limit is exogenous, the model presented in the first paper features a lower increase in the aggregate consumption, a lower decrease in inequality, as measured by the Gini coefficient and a higher increase in private debt following a temporary expansionary monetary policy shock. In addition, I show that, while decreasing inequality because of a left shift in the net worth distribution, a permanent expansionary monetary shock is unable to boost consumption; on the contrary, it pushes the economy towards a lower aggregate consumption equilibrium. Despite mostly qualitative in their nature, the results in this paper should caution policymakers against the consequences of neglecting the role played by a realistic, endogenous, collateral-based borrowing limit in the transmission of monetary policy.

In the second paper, I adopt an heterogenous agent in continuous time model to discuss the distributional and cross-sectional implications of a minimum wage policy. Because setting the minimum wage involves a trade-off between reducing inequality and destroying unskilled jobs, the model considers two different types of labour input (skilled and unskilled). Similar to Galor and Zeira (1993), households may become skilled through an indivisible human capital investment. As a borrowing constrain exists, the households become skilled when the investment in human capital is both profitable and affordable. In this way, the investment choice (and, therefore, individual skills) depends on individual wealth. The model in this paper considers a minimum wage setting similar to Dehez and Fitoussi (1996); in particular, wage setting is strictly interconnected with fiscal policy because the Public Sector subsidizes the representative firm in order to avoid the dismissal of unskilled workers. A minimum wage policy succeeds in boosting aggregate consumption and reducing income inequality; however, the size of the unskilled group increases and wealth inequality also increases because this policy represents a disincentive to the human capital investment and thus it fosters polarization among households. I show that, if a policymaker is interested in reducing wealth inequality, it is more effective to redistribute wealth towards the unskilled individuals, because this will help their dynasties to invest in human capital in the long run.

Overall, an important implication emerges from the joint reading of these papers: policymakers need to consider the distributional effects of their measures since these actively determine the aggregate effects: endogenous distributions need to receive more attention in theoretical models.

Contents

Chapter 1

General Introduction

This thesis extends the Aiyagari (1994) model in two directions with the goal of answering two relevant macroeconomic research questions. In the first Chapter, I address some limitations of investigating monetary policy implications with a model featuring an exogenous borrowing limit by proposing instead a collateral-based borrowing constraint. In the second Chapter, I introduce heterogeneity in the labour input (i.e. individual skills) to investigate the implications of setting a minimum wage on both inequality and macroeconomic aggregates. In this introductory Chapter I review the Aiyagari model, which remains a fundamental benchmark for both Chapters. Then, I introduce the fundamental concepts of heterogeneous agent models in continuous time. The continuous time framework, which represents our modelling choice in both the Chapters, entails several computational advantages, as I shall detail later in this Introduction. In this respect, I will also describe the solution method in Achdou et al. (2021), which represents the computational framework of reference.

1.1 The Aiyagari Model

Many interesting questions in economics concern distributions and addressing such issues requires the introduction of heterogeneous agent models. In particular, a benchmark model for distributional macroeconomics is the Aiyagari heterogeneous agent model (1994), which I summarize below. Time is discrete ($t = 0, 1, ...$). There exists a continuum of infinitely lived households with expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0, 1)$ is the utility discount factor common across the agents, c_t is the consumption of the final good, and σ is the coefficient of relative risk aversion. The households are endowed with an initial wealth a_0 and inelastically supply one unit of work, since leisure does not generate utility. The wealth yields a return r_t in period t. The households earn $w_t l_t$ from working, where w_t is the wage rate and l_t is labour productivity state, which follows a Markov chain process such that each agent's productivity is independent of others'. There exists a continuum of firms which is characterized by the following production technology:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

where K_t and L_t are capital and labour inputs, respectively. Because Y is a constant return to scale (CRS) function, we can assume a single representative firm without loss of generality. Instead of solving the general problem, Aiyagari focuses on the steady-state equilibrium, where the distributions of agents and prices are stationary. It is worth emphasizing that "steady-state" assumes a special meaning in this context: even though the distribution, the aggregates and the equilibrium prices are constant over time, the individual agents are hit by idiosyncratic productivity shocks and shift from the right to the left of the distribution and viceversa. There are two equilibrium prices in the model: the interest and wage rates. These can be obtained as functions of the capital stock K, given that the aggregate labour supply is constant over time (the total number of agents can be normalized to one). Therefore, it is only needed to find one among K, r or w to compute the steady state. The recursive formulation of the agents' problem is

$$V(a_t, l_t) = max_{c,a_{t+1}}\{u(c) + \beta \sum_{l_{t+1}} \pi_{l_t,l_{t+1}} V(a_{t+1}, l_{t+1})\}$$

$$c_t + a_{t+1} = wl_t + (1 + r)a_t$$

$$c_t \geq 0, a_t \geq -b$$

where $\pi_{l_t,l_{t+1}}$ is the probability of switching the productivity state and b is an exogenous borrowing limit. This can be fixed to the natural limit $\frac{wl_t}{r}$ or to some smaller ad hoc value that gives rise to a tighter borrowing constraint. Solving the optimization problem gives the optimal decision rules $c_t = g_c(a_t, l_t)$ and $a_{t+1} = g_a(a_t, l_t)$, which are shared by all households (ideally with different a_t and l_t). The steady state consists of the prices, the value function, the optimal decision rules, the distribution of capital and the aggregate inputs such that agents and firms are optimizing, and the markets are cleared.

1.2 The Solution Method

In this section, in order to illustrate the solution method in Achdou et al. (2021), I recast Aiyagari's model in continuous time and show how the computation applies. In fact, Achdou et al. (2021) developed a simple, efficient and sufficiently portable algorithm to solve numerically heterogeneous agent models that feature endogenous distributions, such as the Aiyagari model. They exploit the result that, in a continuous time formulation, every heterogeneous agent model reduces to a system of two coupled partial differential equations (PDEs). The first is a Hamilton-Jacobi-Bellman (HJB) equation that describes the optimal choices of an atomistic individual who takes the evolution of the distribution (and hence prices) as given. The second is a Kolmogorov Forward (KF) equation, which characterizes the evolution of the distribution given individuals' optimal choices. These two equations are coupled since optimal choices depend on prices, and prices are determined in equilibrium by the distribution. Assume that labour productivity l_t follows a two-state Poisson process $l \in \{l_1, l_2\}, l_2 > l_1$, with intensities λ_1 and λ_2. In this case, the steady state of Aiyagari's model is characterized by the following system of equations

$$\rho v_j(a) = \max_c u(c) + v_j'(a)(wl_j - c + (1+r)a) + \lambda(v_{-j}(a) - v_j)$$

$$0 = -\frac{d}{da}(s_j(a)g_j(a)) - \lambda_j g_j(a) + \lambda_{-j}g_{-j}(a)$$

$$K = \int_{-b}^{\infty}(ag_1(a) + ag_2(a))da$$

where $g_j(a)$ is the wealth distribution of agents of type j (with $j = 1, 2$), $-j$ denotes a state other than j, and $c_j(a) = (U')^{-1}(v_j'(a))$ and $s_j(a) = wl_j - c_j(a) + (1-r)a$ are the optimal consumption and saving policy functions. The prices (the interest and wage rates) are equal the marginal productivities of capital and labour, respectively.

1.2.1 Derivation of the HJB Equation

Consider the discrete time general problem of maximizing consumption c_t subject to the evolution of wealth a_t. The length of the periods is Δ, the households have discount factor $\beta = e^{-\rho\Delta}$, and the households of type j maintain the same productivity with probability $p_j = e^{-\lambda_j \Delta}$ and switch to state $-j$ with probability $(1 - p_j) = 1 - e^{-\lambda_j \Delta}$. The Bellman equation is

$$v_j(a_t) = \max_c u(c)\Delta + \beta[p_j v_j(a_{t+\Delta}) + (1 - p_j)v_{-j}(a_{t+\Delta})]$$

when $\Delta \to 0$, the following approximations hold:

$$\beta = e^{-\rho\Delta} \approx 1 - \rho\Delta$$

$$p_j = e^{-\lambda_j \Delta} \approx 1 - \Delta\lambda_j$$

Therefore the Bellman equation becomes:

$$v_j(a_t) = \max_c u(c)\Delta + (1 - \rho\Delta)[(1 - \Delta\lambda_j)v_j(a_{t+\Delta}) + \Delta\lambda_j v_{-j}(a_{t+\Delta})]$$

Subtracting $(1 - \rho\Delta)v_j(a)$ from both sides this can be rewritten as:

$$v_j(a_t) = \max_c u(c)\Delta + (1 - \rho\Delta)[v_j(a_{t+\Delta}) - v_j(a) + \Delta\lambda_j(v_{-j}(a_{t+\Delta}) - v_j(a_t))]$$

Exploiting the fact that

$$\lim_{\Delta \to 0} \frac{v_j(a_{t+\Delta}) - v_j(a_t)}{\Delta} = \lim_{\Delta \to 0} \frac{v_j(\Delta(wl_j - c + (1 + r)a_t) - v_j(a_t)}{\Delta} =$$
$$= v_j'(wl_j - c + (1 + r)a_t)$$

The HJB equation can be restated as

$$\rho v_j(a) = \max_c u(c) + v_j'(a)(wl_j - c + (1 + r)a) + \lambda(v_{-j}(a) - v_j(a))$$

1.2.2 Derivation of the KF Equation

The evolution of wealth is characterized by

$$d\tilde{a}_t = s_j(\tilde{a}_t, t)dt$$

where the tilde denotes stochastic variables. Consider the discrete time analogue. Individuals make their saving optimal decisions according to

$$\tilde{a}_t = \tilde{a}_{t+\Delta} + \Delta s_j(\tilde{a}_{t+\Delta})$$

After optimal decisions are made, the next period's productivity is realized. The fraction of people with productivity l_j and wealth a is

$$G_j(a,t) = P(\tilde{a}_t \leq a, \tilde{l}_t = l_j)$$

where G is the cumulative density function (CDF). The density g_j satisfies

$$g_j(a,t) = \partial G_j(a,t)$$

In order to derive a law of motion for G, it is necessary to determine recursively what level of capital a_t a type j household had, given that she has capital $a_{t+\Delta}$ at time $t+\Delta$. Consider the fraction of households with wealth below a at $t+\Delta$ and let us temporarily ignore productivity switches and assume that individuals do not accumulate capital, i.e., $s_j(a) \leq 0$ (the opposite case is symmetric). Then, the probability of being characterized by a wealth below a at $t+\Delta$ is the sum of the probability of already being below a at time t and the probability of falling below a in the period Δ

$$P(\tilde{a}_{t+\Delta} \leq a) = P(\tilde{a}_t \leq a) + P(\tilde{a}_t \leq a - \Delta s_j(a)) = P(\tilde{a}_t \leq a - \Delta s_j(a))$$

Introducing probability switches, this becomes

$$\begin{aligned} &P(\tilde{a}_{t+\Delta} \leq a, \tilde{l}_{t+\Delta} = l_j) = \\ &= (1-\Delta\lambda_j)P(\tilde{a}_t \leq a - \Delta s_j(a), \tilde{l}_t = l_j) + \Delta\lambda_{-j}P(\tilde{a}_t \leq a - \Delta s_{-j}(a), \tilde{l}_t = l_{-j}) \end{aligned}$$

Therefore, the CDF is

$$G(a, t+\Delta) = (1-\Delta\lambda_j)G_j(a - \Delta s_j(a), t) + \Delta\lambda_{-j}G_{-j}(n - \Delta s_{-j}(a), t)$$

Subtracting $G_j(a,t)$ from both sides and dividing by Δ, I obtain

$$\frac{G_j(a,t+\Delta)-G_j(a,t)}{\Delta}=$$
$$\frac{G_j(a-\Delta s_j(a,t))-G_j(a,t)}{\Delta}+\lambda_j G_j(a-\Delta s_j(a),t)-\lambda_{-j}G_{-j}(a-\Delta s_{-j}(a),t)$$

which becomes

$$\partial G_j(a,t)=-s_j(a)\partial_a G_j(a,t)-\lambda_j G_j(a,t)+\lambda_{-j}G_{-j}(a,t)$$

when $\Delta\to 0$. Differentiating with respect to wealth and given that $g_j(a,t)=\partial G_j(a,t)$, I obtain the KF equation:

$$0=-\frac{d}{da}(s_j(a)g_j(a))-\lambda_j g_j(a)+\lambda_{-j}g_{-j}(a)$$

1.2.3 Solution to the HJB Equation

In order to solve the HJB equation, the value function and the distribution need to be approximated. The approach is to use a discretization method that transforms these two PDEs into a system of matrix equations. The HJB equation is solved through a finite difference method. First, the value function is approximated at J discrete points in the space (capital) dimension, $a_i=1,...,I$. Grids are equispaced and I denote by Δa the distance between grid points.

The derivative $v'(a)$ is approximated with either a backward or a forward approximation:

$$v'(a_i)\approx\frac{v(a_{i+1})-v(a_i)}{\Delta a}$$
$$v'(a_i)\approx\frac{v(a_i)-v(a_{i-1})}{\Delta a}$$

The choice between backward and forward approximation is governed by an *upwind* scheme (similar to Candler, 1999), which approximates the derivative in the direction of the movement of the state (i.e., the forward approximation is used when saving is positive and the backward approximation when saving is negative). Second, once the value function is approximated, the HJB equation must be solved using an iterative (implicit) scheme because of its non-linearity. One starts with an initial guess $v_j^0 = (v_{i,j}^0, ..., v_{I,j}^0), j = 1, 2$ and then updates v_a^0, $a = 1$, according to:

$$\frac{v_{i,j}^{n+1} - v_{i,j}^n}{\Delta} + \rho v_{i,j}^{n+1} = u(c_{i,j}^n) + (v_{i,j}^{n+1})'(\dot{a}) + \lambda_j(v_{i,-j}^{n+1} - v_{i,j}^{n+1})$$

where Δ is the step size (which can be chosen to be arbitrarily large in an implicit scheme). Achdou et al. (2021) show that, combined with the upwind scheme, this system of equations can be written in matrix notation as:

$$\rho v^{n+1} = u^n + A^n v^{n+1}$$

where A is a lower tridiagonal matrix. Moreover, in order to be solved with matrix routines, this system can be written as:

$$B^n v^{n+1} = b^n$$

$$B^n = \rho I - A^n$$

$$b^n = u^n$$

1.2.4 Solution to the KF Equation

In contrast to the HJB equation, which is nonlinear in the value function, the KF equation is linear in the density. Achdou et al. (2021) show that the finite difference approximation of the KF equation takes the form

$$\underline{0} = \underline{A}(\underline{v}, \underline{p})^T \underline{g}$$

The intuition for this matrix equation is that the KF equation is the "transpose" problem of the HJB equation, i.e. the transition matrix in the discretized KF equation is the adjoint of the transition matrix in the discretized HJB equation. Hence, the product of this discretization is the following system of matrix equations:

$$\underline{\rho}\underline{v} = u(\underline{v}) + \underline{A}(\underline{v}, \underline{p})\underline{v}$$

The interesting and convenient feature of this system is that the discretized KF equation is the "transpose problem" of the discretized HJB equation. Then, the KF equation can be solved by inverting the A matrix.

1.2.5 The Algorithm for the Stationary Equilibrium

The stationary equilibrium is obtained by iterating on the equilibrium system; namely, I use a bisectional algorithm on the stationary aggregate capital. The iteration starts with an initial guess for K. Then, for $i = 0, 1, ...$, the algorithm works as follows:

1. Given the initial guess, compute prices (r and w) from their equilibrium equations;
2. Solve the HJB equation using a finite difference method and derive the saving policy function $s_{j,i}(a)$;
3. Given the saving policy function, solve the KF equation for the density $g_{j,i}(a)$ using a finite difference method;
4. Given the density, compute aggregate capital. Update K accordingly to the sign in the difference between its value and the guess;
5. Stop when the updated aggregate capital is "close enough" to the guess.

1.2.6 Computational Advantages of Continuous Time

A continuous time framework yields several advantages compared to a discrete time one. In what follows I discuss the most important advantage when solving the Aiyagari model (see Achdou et al. 2017 for a detailed discussion of the other advantages), which concerns the handling of state constraints such as $a_t \geq -b$. Namely, Achdou et al. (2021) show that, in continuous time formulations, the state constraint never binds in the interior of the state space. Hence, the state constraint does not appear in the HJB equation and gives rise to a state constraint boundary condition

$$v_j'(-b) \geq u'(wl_j - c - b(1+r))$$

Moreover, this boundary condition is easily imposed through the upwind scheme used in the discretization of the HJB equation, i.e. the special structure of the upwind scheme itself will ensure that the state constraint is never violated.

1.3 Results

In this section, I show the main features of Aiyagari's model steady state when solved using the computational framework in Achdou et al. (2021). In steady state, the households consume and save depending on their rank in the joint distribution of capital and labour productivity. Figures 1.1 and 1.2 show optimal consumption and saving.

The high productivity households choose a higher level of consumption and savings with respect to the low productivity households; the high productivity households' savings are positive regardless of their wealth, while the low productivity households' savings are negative regardless of their wealth. Optimal consumption (saving) increases (decreases) monotonically with wealth

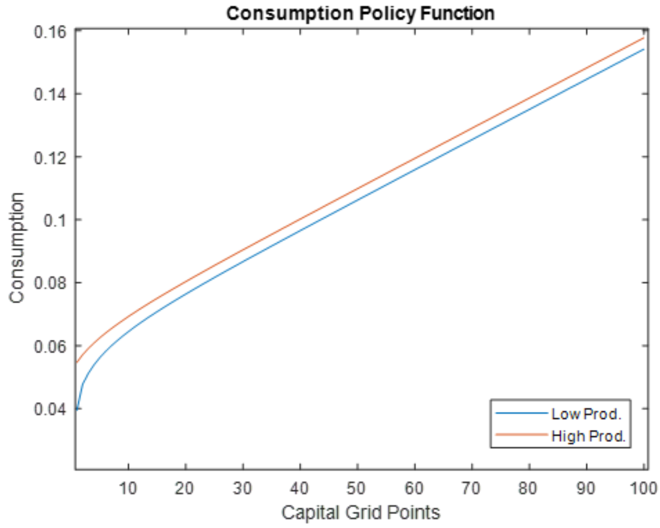

Figure 1.1: Aiyagari Model: Consumption Policy Function

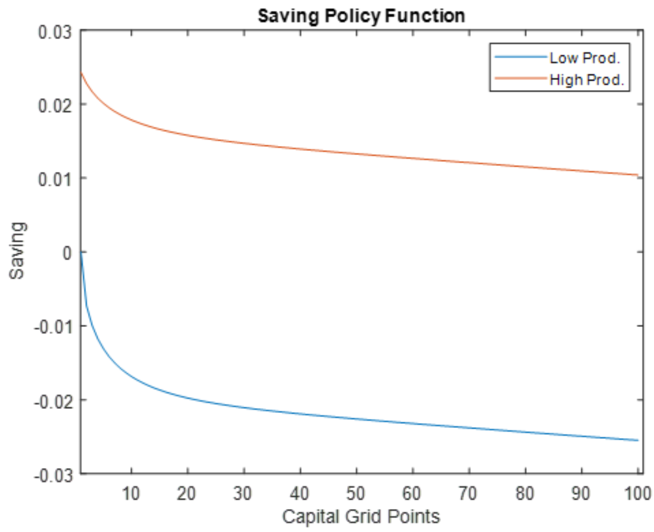

Figure 1.2: Aiyagari Model: Saving Policy Function

for both households' types. None of these features can be captured by a representative agent model. Most importantly, the model generates heterogeneity in the marginal propensity to consume (as shown in Figure 1.3), which is crucial in many macroeconomic applications.

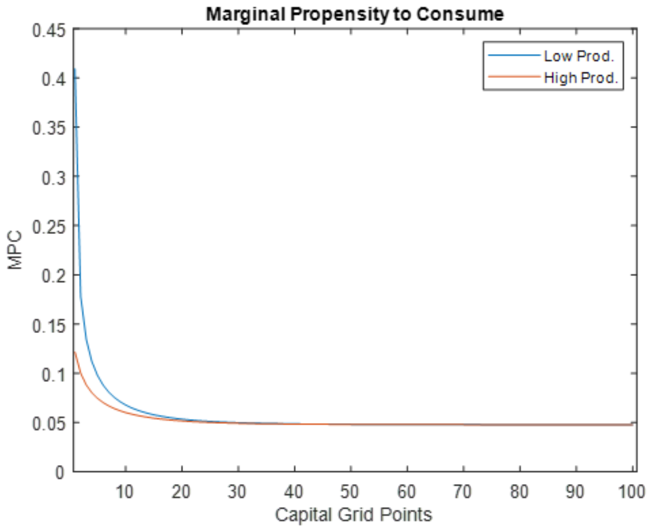

Figure 1.3: Aiyagari Model: Marginal Propensity to Consume

The households' optimal consumption and saving policy functions produce, in equilibrium, the wealth distribution depicted in Figure 1.4.This functional form features a log-normal distribution. The low productivity households' distribution features a Dirac point mass at the exogenous borrowing constraint.

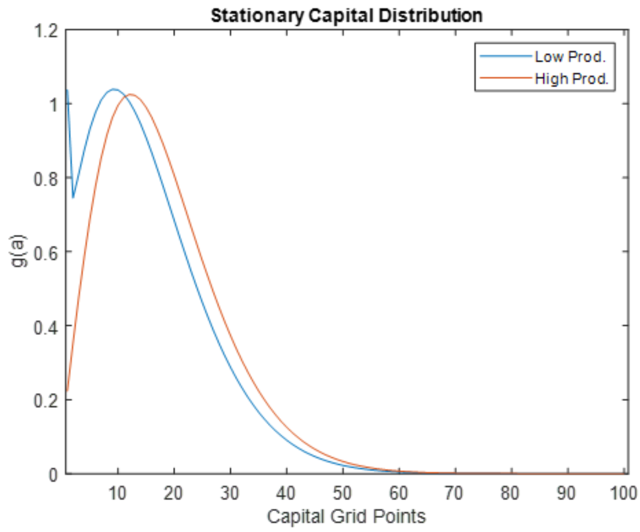

Figure 1.4: Aiyagari Model: Wealth Distribution

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Chapter 2

Endogenous Net Worth Distribution and Monetary Policy: a Heterogeneous Agent in Continuous Time Model

Abstract

This paper aims at assessing the aggregate and distributional effects of an expansionary monetary policy shock when the households are heterogenous in wealth and labour productivity and face a collateral-based borrowing limit. The households derive utility from consumption and durable assets. The presence of a collateral-based borrowing constraint represents a key novelty in the literature studying the effects of monetary policy under a heterogenous agent framework. Notably, it not only adds reality to the model (consistently with the role played by collateral during the financial crisis of 2007 – 2008), but it also carries crucial implications for monetary policy. In fact, compared to a framework where the borrowing limit is exogenous, the model presented in this paper features a lower increase in the aggregate consumption, a lower decrease in inequality, and a higher increase in private debt following a temporary expansionary monetary policy shock.

2.1 Introduction

The Great Financial Crisis (henceforth, GFC) challenged the standard beliefs held by macroeconomists and central bankers concerning the implications of monetary policy shocks and their channels of transmission to the real economy. In fact, the sharp and prolonged decrease in the Effective Federal Funds rate from 6.5% at the end of 2000 to 1% five years later did not have the desired stimulating effect on the U.S. macroeconomy, as shown by a vast empirical literature (see, e.g., Fitoussi and Saraceno, 2010; Rajan, 2010; Fazzari and Cynamon, 2013; Mian and Sufi, 2014). For instance, Fitoussi and Saraceno (2010) report that the lax monetary policy that preceded the GFC barely succeed in sustaining the aggregate demand. On the contrary, it led to a large increase in private debt with the household debt as a share of disposable income rising from around 90% in the early 2000s to a peak of more than 120% in 2007. The strong trend in the aggregate household leverage between the mid-1980s and 2007 ultimately undermined the overall stability of the financial system, as emphasized by Fazzari and Cynamon (2013). Moreover, the borrowing attitude of households had crucial implications in terms of net worth inequality. For instance, Casiraghi et al. (2017) and Ampudia et al. (2018) show that an expansionary monetary policy compresses income and wealth inequality, in line with standard theoretical predictions, but the effects that they report are overall very modest.

In the attempt of reproducing those empirical facts, macroeconomists have turned to models that feature the presence of households' heterogeneity (with two or more agents). These models offer strikingly different results with respect to the representative agent models in terms of their monetary policy implications. In fact, monetary policy transmission depends on the marginal propensities to consume (MPCs) and the households' response to changes in monetary conditions is heterogeneous with respect to income and net worth (see, e.g., Voinea et al., 2017; Mian and Sufi, 2014). In particular, Kaplan et al. (2018) showed that in the heterogeneous agent (HA) models the indirect effects of an unexpected cut in policy rates, which work through general equilibrium mechanisms, outweigh the direct effects such as intertemporal substitution; therefore, the effects of an expansionary monetary policy shock on consumption are considerably smaller than usually predicted when representative agent models are employed. My paper builds on this seminal work to assess the aggregate and distributional effects of an expansionary monetary policy shock when the agents face a collateral-based borrowing constraint.

Notably, while Bilbiie (2018) argued that a qualitatively similar aggregate consumption effects can be obtained using a two-agent (TA) model, in this paper, I place emphasis on modelling the entire agents' distribution. In fact, despite acknowledging the importance of agents' heterogeneity, TA literature has generally focused on the effects of a monetary policy shock on the aggregate demand (and, more precisely, on its major component, the aggregate consumption). In contrast, this framework is not adequate to investigate the other two important implications discussed above, namely, the effects in terms of private debt and inequality, which are crucial for many macroeconomic policies. In this respect, in contrast to the TA framework, the HA models have the potential to allow for a joint discussion of the aggregate and cross-sectional implications of monetary policy. In fact, the HA framework assumes a full distribution of agents, thus allowing for a discussion of inequality in terms of both concentration indexes and quantile ratios; in addition, it enables a proper characterization of the MPCs that is crucial for the description of the households' borrowing attitude. As (qualitatively) replicating the huge increase in private debt due to the changes in the borrowing attitude of the households that follows an expansionary monetary policy shock and investigating the related implications in terms of net worth inequality constitute the two central goals of this paper, HA models represent the framework of election.

In this respect, my main contribution is to overcome the limitations of investigating monetary policy implications with a model that features an exogenous borrowing limit (which makes debt opportunities independent of net worth), as it is common in the HA literature starting from Aiyagari (1994), by introducing a borrowing constraint that is based on collateral assets. In fact, the role played by collateral in the GFC was crucial because, following the cut in policy rates, many households did not increase their consumption but decided to collateralize their durable assets to increase their borrowing (see, e.g., Mian and Sufi 2010). Therefore, to reproduce the large increase in private debt that has been documented by the empirical literature and derive its implication on inequality, I extend the Aiyagari model in two directions: on the one hand, as discussed above, I replace the exogenous borrowing limit with a collateral-based borrowing constraint; on the other hand, I assume a two-asset structure where unproductive durable assets (which can be interpreted as houses) are included in the households' utility function, as it is common in macroeconomic models featuring collateral assets, such as Iacoviello (2005).

The model is cast in continuous time to exploit the computationally convenient results in Achdou et al. (2021), who describe a simple and yet efficient algorithm to solve numerically the HA models in the presence of borrowing constraints. The economy in the model is populated by a continuum of households, who are heterogeneous in net worth and labour productivity.

The net worth is composed by monetary balance and durable assets and the latter may be collateralized for borrowing purposes up to a certain fraction. Households have preferences over consumption and durable assets and maximize their flow of utilities subject to the law of motion of net worth and the collateral-based borrowing constraint. The firms' sector is kept purposefully simple and replicates the structure of the real business cycle (RBC) models. Namely, there is a representative firm who produces using capital and labour as inputs. The Monetary Authority controls the monetary balance rate and ensures the equilibrium in the market by introducing (absorbing) the missing (excess) liquidity. This mechanism replicates the one in Kaplan et al. (2018) and assumes that monetary and fiscal policy are strictly interconnected. Although this is a simplifying assumption, it is less unrealistic than one may think, given the crucial role played by quantitative easing after the GFC.

It is crucial to emphasize that an exogenous borrowing constraint is not consistent with reality, since borrowing opportunities actually depends on the individual illiquid assets holdings. Moreover, an unexpected shock to policy rates would not affect an exogenous borrowing limit, while in the presence of a collateral-based borrowing constraint the agents (may) face an increase (decrease) in the borrowing opportunities. In addition, the collateral-based borrowing constraint, as well as the presence of the durable assets in the households' utility function, which represent the key innovations of my paper, allow us to produce MPCs that are non-monotonic with respect to net worth, as in Crawley and Kuchler (2020). When the durable assets are included in the borrowing constraint as collateral, the model implies the existence of a fraction of households who decides to increase borrowing to purchase durable assets as a response to an expansionary monetary policy (subtracting resources to the adjustment in consumption).

Needless to say, this model carries relevant implications in terms of monetary policy. In fact, in this setting, the transition dynamics following an expansionary monetary policy shock feature a "hump-shaped" positive response in the aggregate consumption that is backed by an increase in private debt and aggregate capital. Overall, net worth inequality is reduced. The importance of a collateral-based borrowing constraint is assessed by a comparison with a simplified version of the model that relies on a simple (exogenous) borrowing limit. In particular, the same expansionary monetary policy shock leads to a smaller increase in aggregate consumption, a higher increase in private debt and a smaller decrease in net worth inequality when the model features a collateral-based borrowing constraint. This is crucial for a monetary authority, which needs to acknowledge that relying on a model based on an exogenous constraint will lead to overestimating the effects of an expansionary shock on aggregate consumption; in addition, the presence of a collateral-based borrowing attitude amplifies the instability of the financial system through an increase in the private debt, and mitigate the reduction of net worth inequality. Overall, neglecting the presence of the dynamics generated by a realistic collateral-based borrowing constraint would lead policy-makers to overstating the effects of easing the stance of the monetary policy both in terms of stimulating aggregate consumption and reducing inequality; in addition, it would also lead to underestimating the perverse effects of expansionary monetary policy on private debt.

In addition to considering the effects of a transitory monetary policy shock, in section 2.3.7, I also discuss the results of a permanent change (decrease) in the rate set by the monetary authority. While the overall inequality, as measured by the Gini index, will be lower in the new steady state than before the permanent monetary shock, the aggregate consumption will be permanently reduced. This is due to the shift of the net worth distribution to the left and represents an important warning for the monetary authority: a permanent easing of the stance of the monetary policy may lead to a permanent increase of private debt, pushing the economy towards a low-consumption long run equilibrium. In other words, monetary policy cannot produce a long run increase in the aggregate consumption.

The rest of this paper proceeds as follows. Section 2.2 describes the model. Section 2.3 describes the results, both for the steady-state of the model and the transition dynamics following an unexpected cut in the policy rate. Section 2.4 concludes.

2.2 The Model

2.2.1 Households

The economy is populated by a continuum of infinitely lived households of unit mass, indexed by their net worth n and their idiosyncratic labor productivity z. Labour productivity follows a two-states Poisson process $z \in \{z_1, z_2\}$, with $z_2 > z_1$. The process jumps from state 1 to state 2 with intensity λ_1 and from state 2 to state 1 with intensity λ_2. The net worth is composed by monetary balance b and durable assets a. Asset of type a are illiquid, meaning that the households have to pay a cost to deposit into (withdrawing from) their illiquid account. The household's deposit rate is denoted with d_t ($d_t < 0$ corresponds to withdrawals); this ensures that r^a (the return on durable assets) is higher than r^b (the return on the monetary balance) in equilibrium.

Durable assets are composed by a productive and an unproductive fraction with returns r^a and r^h, respectively. The unproductive fraction, denoted by ω, can be used as collateral for borrowing. I assume that the consumption good and durable assets can be exchanged at a constant and unitary rate. In the same spirit of Kaplan et al. (2018), the unproductive durable assets can be interpreted as housing services and r^h as the service flow from owner-occupied houses. Namely, rather than being an interest rate determined in equilibrium, r^h should be interpreted as a convenience yield arising from holding a property. The monetary balance yields a return r^b and is negative when the household is a net borrower (in which case r^b represents a cost).

The households have the following preferences:

$$E_0 \int_0^{\infty} e^{-\rho t} u(c_t, \omega a_t) dt$$

where c_t is consumption, $\rho \geq 0$ is the discount rate and the expectation is taken over the realizations of the idiosyncratic productivity shocks. Because ω is a constant fraction of durable assets, in the rest of the paper, we assume without consequences $u(c_t, a_t)$. Individual assets evolve according to

$$\dot{b}_t = w_t z_t - c_t + \tau_t + r_t^b b_t - d_t \tag{2.1}$$

$$\dot{a}_t = (r_t^a(1-\omega) + r_h \omega) a_t + d_t \tag{2.2}$$

where w_t is the wage rate and τ_t is an individual net transfer from the public sector. The net worth's evolution law, which represents the individual's budget constraint, can be obtained from (2.1) and (2.2) as follows:

$$\dot{n}_t = w_t z_t - c_t + \tau_t + r_t^b n_t + (r_t^m - r_t^b) a_t$$

where

$$r_t^m = (r_t^a(1-\omega) + r^h \omega)$$

is the weighted average return of durable assets.

2.2.2 Borrowing Constraint

A household can borrow up to the unproductive fraction of the owned durable assets plus a constant, H:

$$-b_t \leq \omega a_t + H$$

which may be rearranged as

$$a_t \leq \phi(n_t + H) \tag{2.3}$$

with

$$\phi = \frac{1}{1-\omega}$$

The set of admissible choices for durable assets is given by the borrowing constraint (2.3) and a no short-selling condition $a \geq 0$

$$A(n) = \{a : 0 \leq a \leq \phi(n + H)\}$$

which implicitly defines the state constraint $n \geq n_{min}$, with $n_{min} = -H$. This maintains the model coherent with the mathematical and computational framework of Achdou et al. (2021) ensuring that their results concerning the existence and uniqueness of the steady state still hold, despite the intuition on the borrowing constraint is different. In fact, differently from the case of the exogenous borrowing constraint, in this formulation of the model, the borrowing limit depends on illiquid asset holdings, such that individuals with a larger net worth can borrow up to a larger amount, as depicted in Figure 2.1.

2.2.3 Firms

The firms' sector is kept purposefully simple and follow the RBC structure. Namely, firms use capital and labour with the following Cobb-Douglas con-

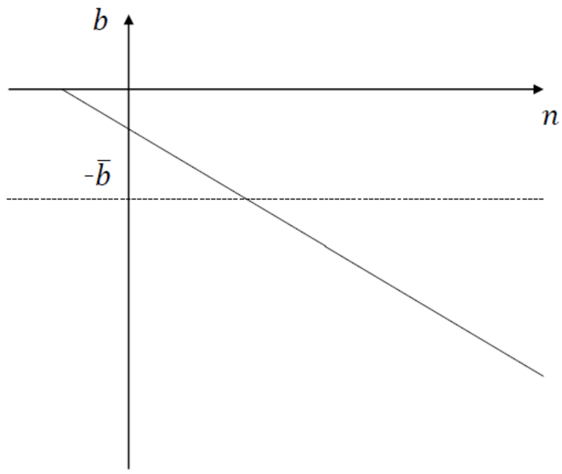

Figure 2.1: Collateral based Borrowing Constraint

stant return to scale technology:

$$Y_t = AK_t^{\alpha} L_t^{(1-\alpha)} \tilde{z}^{(1-\alpha)}$$

where A is the technology parameter, K_t is the aggregate capital, L_t is the aggregate labour; $\tilde{z}$ is the average households' productivity, which implies that there is no aggregate uncertainty in the output. In this model, households do not get disutility from labour, hence labour supply is anelastic and fixed at 1. Therefore, firms' technology reduces to:

$$Y_t = AK_t^{\alpha} \tilde{z}^{(1-\alpha)}$$

and capital and labour prices will be respectively:

$$r_t^a = \alpha AK_t^{(\alpha-1)} \tilde{z}^{(1-\alpha)} - \delta$$

$$w_t = (1-\alpha) AK_t^{\alpha} \tilde{z}^{(1-\alpha)}$$

where δ is the depreciation rate.

2.2.4 Monetary Policy and Public Sector

The monetary authority controls the monetary balances rate r^b. In this model, there is a unique value of r^b which ensures endogenously the equilibrium in the monetary balance market; therefore, if the monetary authority fixes a value for r^b different from this equilibrium value, it will be required to ensure the equilibrium by introducing (absorbing) the missing (excess) liquidity. Following a common modelling choice in the literature (see, e.g., Kaplan et al. 2018), this model assumes that monetary and fiscal policies are strictly interconnected; namely, the public sector will subsidize/tax households to ensure the monetary balance market equilibrium, which will take the following form:

$$S = \int_{n_{min}}^{\infty} (b_1(n)g_1(n) + b_2(n)g_2(n))dn$$

where S represents the aggregate net transfer from the public sector to the private sector, and $g_j(n)$ is the net worth distribution of productivity type $j = 1, 2$. Therefore, a value of S higher (lower) than zero represents a subsidy (a tax). In the baseline model, I assume this transfer to be equally distributed among households. In section 2.3.6, I will also show the results when the transfer is distributed among some specific categories of households.

2.2.5 Stationary Equilibrium

A stationary equilibrium is fully characterized by the following system of equations. The first and the second are the HJB and KF equations; the third and the fourth equations show how the aggregate capital and monetary balance are determined in steady-state:

$$\rho v_j(n) = \max_{c,a \in A(n)} u(c,a) + v_j'(n)(wz_j - c + \tau + r^b n + (r^m - r^b)a) + \lambda(v_{-j}(n) - v_j(n))$$

$$0 = -\frac{d}{dn}(s_j(n)g_j(n)) - \lambda_j g_j(n) + \lambda_{-j}g_{-j}(n)$$

$$\left(\frac{r^a+\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}}\tilde{z} = (1-\omega)\int_{n_{min}}^{\infty}(a_1(n)g_1(n)+a_2(n)g_2(n))dn$$

$$S = \int_{n_{min}}^{\infty}(b_1(n)g_1(n)+b_2(n)g_2(n))dn$$

where $c_j(n) = (U')^{-1}(v_j'(n))$, $s_j(n) = wz_j - c_j(n) + \tau + r^b n + (r^m - r^b)a_j(n)$, $a_j(n)$ and $b_j(n) = n - a_j(n)$ are the optimal consumption, saving, durable assets holding and monetary balance policy functions, $g_j(n)$ is the net worth distribution of type j, and $-j$ is the opposite type of j. Assuming quasi-linear utility $u(c,a) = \tilde{u}(c + f(a))$ and defining $x = c + f(a)$, the HJB equation becomes

$$\rho v_j(n) = \max_x \tilde{u}(x) + v_j'(n)(wz_j - x + r^b n + f(n)) + \lambda(v_{-j}(n) - v_j(n))$$

$$f(n) = \max_{a\in A(n)} f(a) + (r^m - r^b)a$$

where $f(n)$ is the pecuniary equivalent of the utility benefit of durable assets net of return and cost. The quasi-linearity of $u(c,a)$ is a simplifying assumption that allows me to exploit the results of Achdou et al. (2021) about the existence and uniqueness of the stationary equilibrium; in particular, Propositions 1 to 4 in their Appendix apply to the model in this paper with no exceptions.

2.2.6 Optimal Durable Assets Choice

We choose the functional form of $f(a)$ as:

$$f(a) = \frac{\gamma a^{1-\beta}}{1-\beta}$$

where β is the coefficient of relative risk aversion and γ is a scale factor. Then the optimal durable asset choice becomes

$$f(n) = \max_{a\in A(n)}\left(\frac{\gamma a^{1-\beta}}{1-\beta} + (r^m - r^b)a\right)$$

Two cases must be distinguished. First, if $r^m \geq r^b$, the household will maximize $f(n)$ choosing the highest possible value for a, which is defined by the borrowing constraint (i.e., by the specific net worth level of the household). Second, if $r^m < r^b$ the optimal durable assets choice is derived from the following first order condition:

$$\gamma a^{-\beta} + r^m - r^b = 0$$

which gives

$$a^* = \left(\frac{r^b - r^m}{\gamma}\right)^{-\frac{1}{\beta}}$$

Note that if $a^* > \phi(n+H)$ the households with $n < n^*$ will be borrowing constrained, with:

$$n^* = \frac{a^*}{\phi} - H$$

2.2.7 Transition Dynamics

The transition dynamics following an expansionary monetary policy shock is characterized by the following system of equations:

$$\rho v_j(n,t) = \max_{c,a \in A(n)} u(c,a) + \partial_a v_j'(n,t)(wz_j - c(n,t) + r^b n + (r^m(t) - r^b(t))a(n,t))$$
$$+ \lambda(v_{-j}(n,t) - v_j(n,t)) + \partial_t v_j(n,t)$$

$$\partial_a g_j(n,t) = -\partial_a(s_j(n,t)g_j(n,t)) - \lambda_j g_j(n,t) + \lambda_{-j} g_{-j}(n,t)$$

$$\left(\frac{r^a(t)+\delta}{\alpha A}\right)^{\frac{1}{\alpha-1}} \tilde{z} = (1-\omega)\int_{n_{min}}^{\infty} (a_1(n,t)g_1(n,t) + a_2(n,t)g_2(n,t))dn$$

$$S(t) = \int_{n_{min}}^{\infty} (b_1(n,t)g_1(n,t) + b_2(n,t)g_2(n,t))dn$$

where $c_j(n,t) = (U')^{-1)}(\partial_a v_j(n,t)), a_j(n,t), b_j(n,t) = n - a_j(n,t)$ and $s_j(n,t) = wz_j - c_j(n,t) + \tau + r^b(t)n + (r^m(t) - r^b(t))a_j(n,t)$ are the optimal consumption, durable asset holdings, monetary balance and saving policy functions.

The density g_j satisfies the initial condition:

$$g_j(n,0) = g_{j,0}(n)$$

and the value function satisfies the terminal condition:

$$v_j(n,T) = v_{j,\infty}(n)$$

for T sufficiently large.

2.2.8 Parametrization

Table 2.1 reports the parameters' value, which are mostly taken from Achdou et al. (2021). The parameters affecting preferences and production take standard values. The coefficient of relative risk aversion is set to $\beta = 0.5$, while the scalar factor is $\gamma = 0.02$. The representative firm's total factor productivity is $A = 0.1$ and the capital elasticity is set to $\alpha = 0.33$ (labour elasticity is equal to $1 - \alpha = 0.67$). The Poisson process for labour productivity is such that $z \in \{z_1 = 1, z_2 = 2\}$ with switching probabilities $\lambda_1 = \lambda_2 = 0.1$. Therefore, the average households' productivity is equal to $\tilde{z} = 3$. The depreciation rate is $\delta = 0.05$. The monetary balance and housing returns, r^b and r^h, are set equal to 0.045 and 0.015 respectively. The parameters affecting the borrowing constraint are the unproductive fraction of capital that is set to $\omega = 0.25$ and the net worth limit that is $n_{min} = -H = -0.75$. These are the parameters that may influence the most the key results in this paper. Therefore in section 2.3.4, I will provide some robustness checks on their values.

Table 2.1: Parametrization

Parameter	Value
z_1	2
z_2	4
λ_1	0.1
λ_2	0.1
r_h	0.015
ω	0.25
r^b	0.045
H	0.75
α	0.33
δ	0.05
A	0.1
$\tilde{z}$	3
μ	0.2
β	0.5
γ	0.02

2.3 Results

2.3.1 Steady-State

In the steady state, households take optimal decisions depending on their ranking in the joint distribution of net worth and productivity. Figure 2.2 and 2.3 represent consumption and saving optimal policy functions. The high productivity households (i.e, those facing the high productivity shock, z_2) choose a higher level of consumption and savings with respect to the

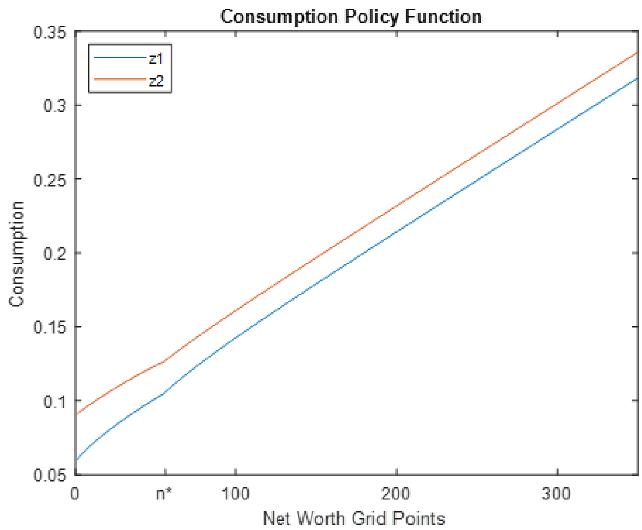

Figure 2.2: Consumption Policy Function

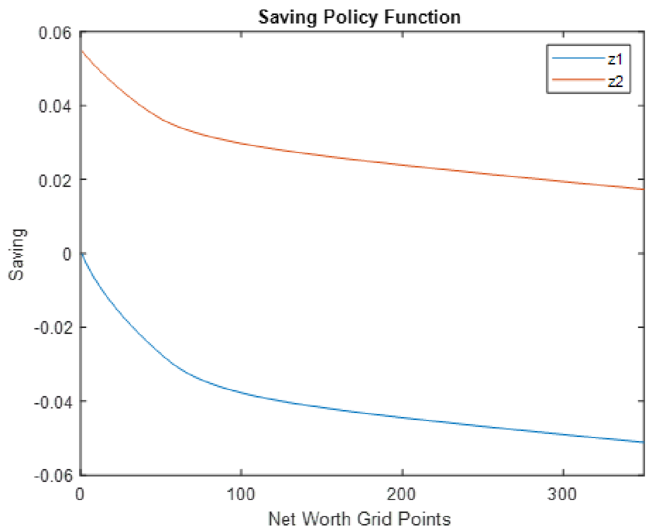

Figure 2.3: Saving Policy Function

low productivity households (i.e, those facing the low productivity shock z_1); the high productivity households' savings are positive regardless of their net worth, while low productivity households' savings are negative regardless of their net worth. The optimal consumption (savings) is increasing (are decreasing) in the net worth for both households' types and is (are) characterized by a kink in n^*, i.e., the net worth level below which households are borrowing constrained. The model allows replicating the high heterogeneity in marginal propensities to consume. I define the (instantaneous) MPC as the derivative of the optimal consumption function with respect to the net worth. Figure 2.4 and 2.5 show the instantaneous MPC as a function of net worth and for quintiles of the net worth distribution, respectively.

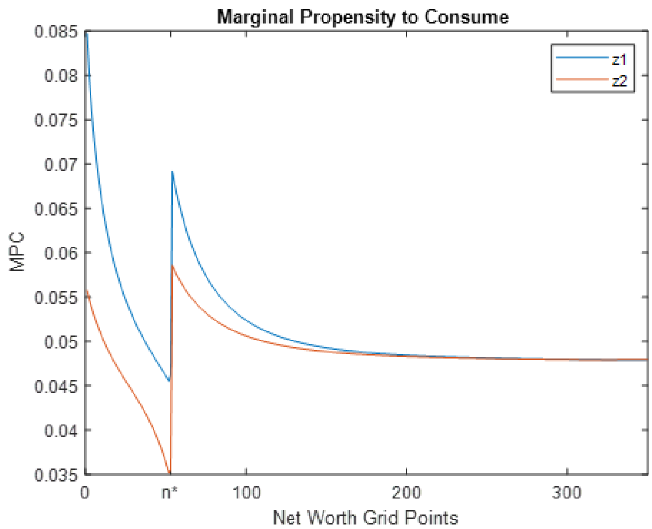

Figure 2.4: Marginal Propensity to Consume

Figures 2.4 and 2.5 highlight a non-monotonic relationship and feature a peak in n^*; in particular, the low productivity households MPC is characterized by an absolute maximum at the net worth's lower bound and a local maximum at n^*, while the high productivity households' MPC is maximized at n^*. Hence, the model allows replicating qualitatively the most recent literature regarding consumption heterogeneity (see Crawley and Kuchler, 2020). This result is driven by the introduction of a collateral-based borrowing constraint that improves the theoretical description of the borrowing attitude,

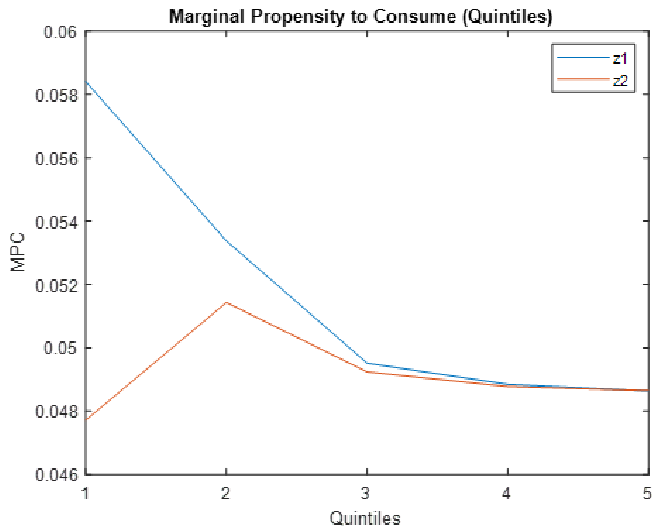

Figure 2.5: Marginal Propensity to Consume, Quintiles

and implies that a fraction of households increases its borrowing to purchase durable assets, thus subtracting resources to consumption.

Because of the assumption that durable assets are (partly) collateralizable, the model returns the monetary balance policy function depicted in Figure 2.6. Visibly, the model features a non-monotonic relation also between the monetary balance and the net worth, with the monetary balance policy function displaying an absolute minimum at n^*. This entails that the household with the lowest monetary balance is ranked in the middle of the distribution and not at the bottom, as it would be the case with an exogenous borrowing limit. The figure shows that households with the same net worth are characterized by the same choices in terms of monetary balance notwithstanding their level of productivity; namely, the households' borrowing attitude does not depend on their idiosyncratic productivity. In equilibrium, the households' optimal consumption, saving and monetary balance policy functions originate the net worth distribution plotted in Figure 2.7. Figure

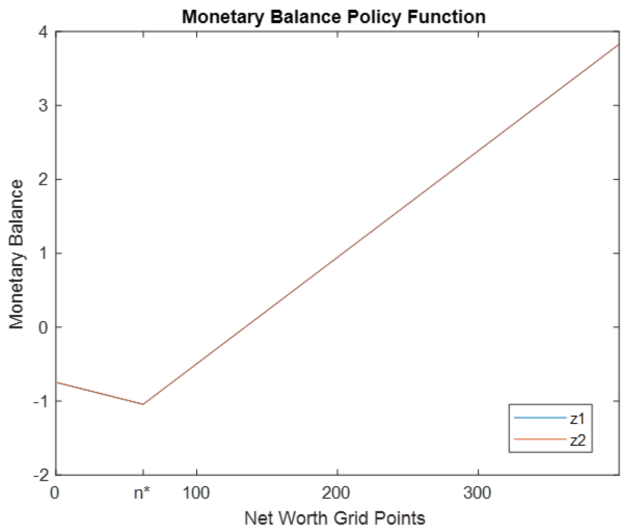

Figure 2.6: Monetary Balance Policy Function

2.8 distinguishes among the high and low productivity households. Their

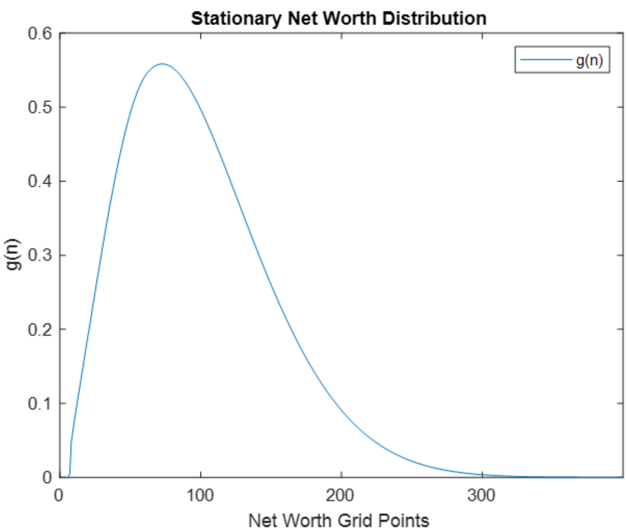

Figure 2.7: Stationary Net Worth Distribution

functional form reproduces a log-normal distribution.

Table 2.2 summarizes the main cross-sectional results of this paper. The 20.03% of the households have a negative net worth ($n < 0$) and, intuitively,

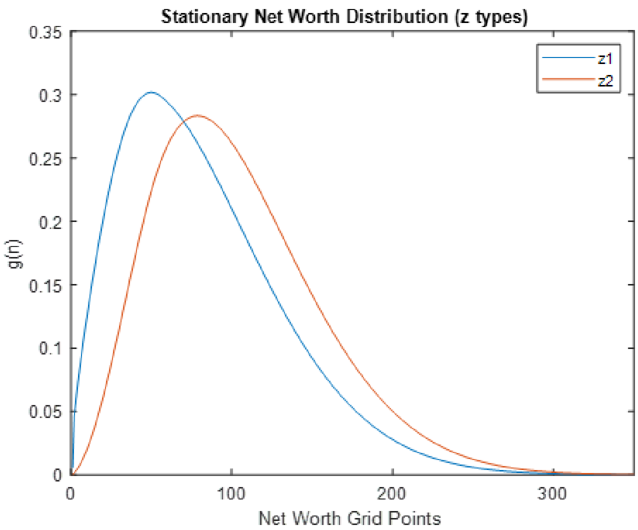

Figure 2.8: Stationary Net Worth Distribution, Low and High Productivity Households

Table 2.2: Stationary Net Worth Distribution

	$n < 0$	$0 < n < n^*$	Gini Index
Total	20.03%	6.91%	0.293
Low Prod.	13.95%	4.86%	0.317
High Prod.	6.08%	2.05%	0.269

the majority of these (13.95%) belongs to the low productivity group. Moreover, the model implies a relatively small fraction of borrowing constrained households with a positive net worth (6.91%). Importantly, it must be noted that these results may be obtained only within a framework that models the full distribution, while the high majority of TA models (for instance the rule-of-thumb consumer framework, see e.g., Galì et al., 2004) consider only exogenous fractions of agents. The Gini index of the net worth distribution is equal to 0.293; in particular, net worth is highly concentrated among low productivity households (the Gini Index of the low productivity households' distribution is equal to 0.317, while the Gini Index of high productivity households' distribution is equal to 0.269).

2.3.2 Transition Dynamics

The main goal of my application is to investigate the aggregate and distributional effects of an expansionary monetary policy shock. In what follows, I consider a negative shock to the monetary balance rate r^b. The shock is modeled as the deterministic version of the Ornstein-Uhlenbeck process (the continuous time analogue of an AR(1) process).

2.3.3 IRF - Expansionary Monetary Policy

In this section, I consider a 10%, 20% and 30% reduction in the monetary balance rate r^b. Following an expansionary monetary policy shock (a cut in the policy rate) the aggregate consumption jumps above its steady-state value. The impulse response functions of aggregate consumption are plotted in Figure 2.9.

The figure clearly shows that the IRFs are "hump-shaped", i.e. the highest response is reached only a few periods after the intervention, and then the consumption reverts to the steady-state level showing some persistence. The consumption response is supported by a decrease in the aggregate monetary balance (i.e., an increase in private debt), as it can be easily seen from Figure 2.10.

Transition dynamics may be summarized as follows. An unexpected cut in r^b causes a disequilibrium in the monetary balance market. This must be rebalanced through additional liquidity and therefore the net transfer from

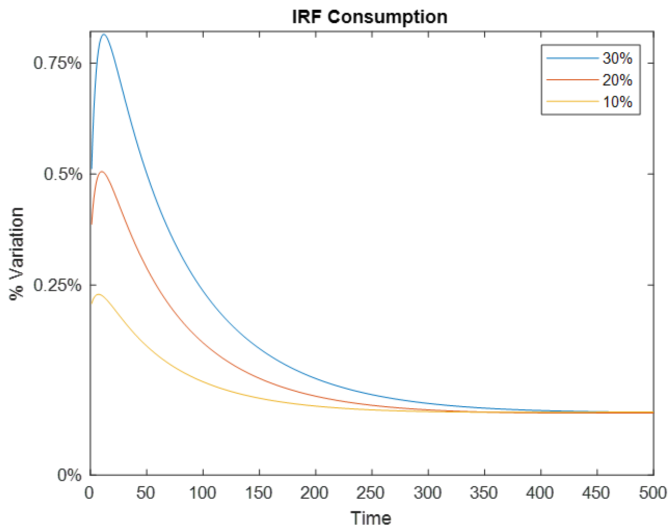

Figure 2.9: IRFs Aggregate Consumption

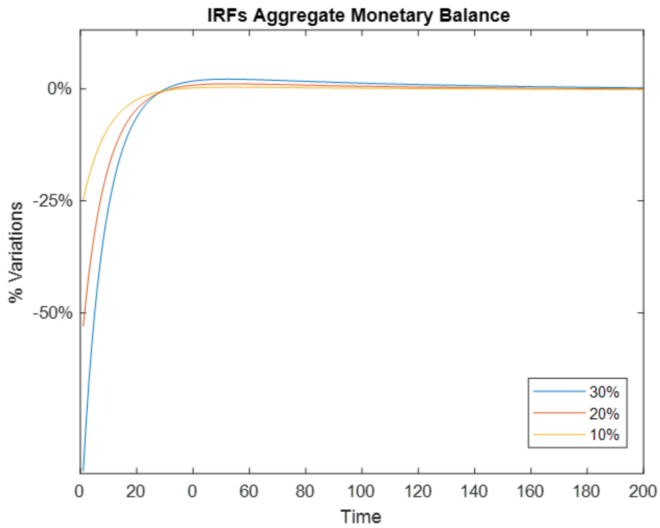

Figure 2.10: IRFs Aggregate Monetary Balance

the public sector needs to increase. This improves the households' credit conditions and, consequently, relax their borrowing constraint so that they can obtain new loans and increase their consumption levels. Therefore, the monetary policy shock has the ultimate effect of easing the credit conditions of the households. Moreover, these loans allow the households to purchase durable assets and, as a result, the aggregate capital increases as displayed in Figure 2.11. This increase in aggregate capital is also due to firms' production

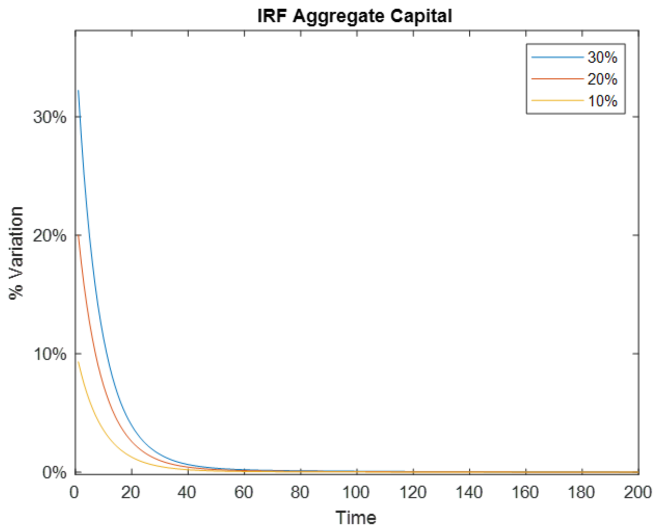

Figure 2.11: IRFs Aggregate Capital

choices. The increase in the demand of durable assets from the households, in fact, puts upward pressure on their price; however, this model does not feature an endogenous price for durable assets, so that this is reflected in a downward pressure on the durable assets return, r^a, as showed by Figure 2.12. Figure 2.13 shows that, as r^a is a cost for firms, they will substitute labour with capital, reducing labour demand and putting upward pressure on the wage rate since labour supply is anelastic. The model, therefore, succeeds in reproducing the collateral-based dynamics that characterized the GFC, when an expansionary monetary policy led to a modest increase in consumption but a huge expansion of private debt because households contracted new loans to buy durable assets (see, e.g., Mian and Sufi 2010).

Finally, as previously discussed, the characteristics of this model allow

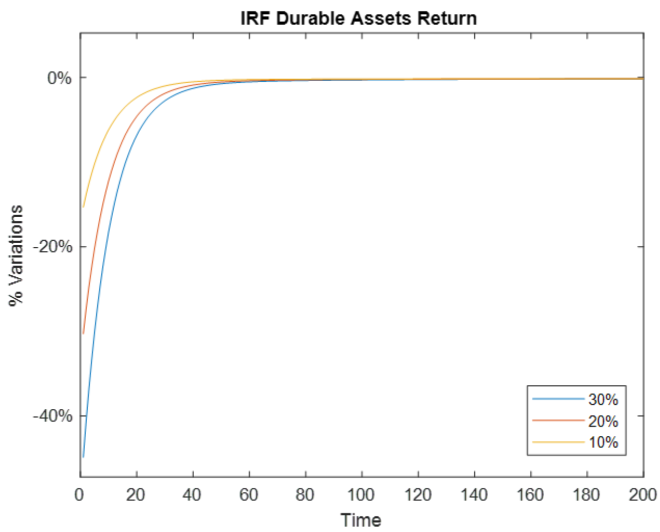

Figure 2.12: IRFs Durable Assets Return

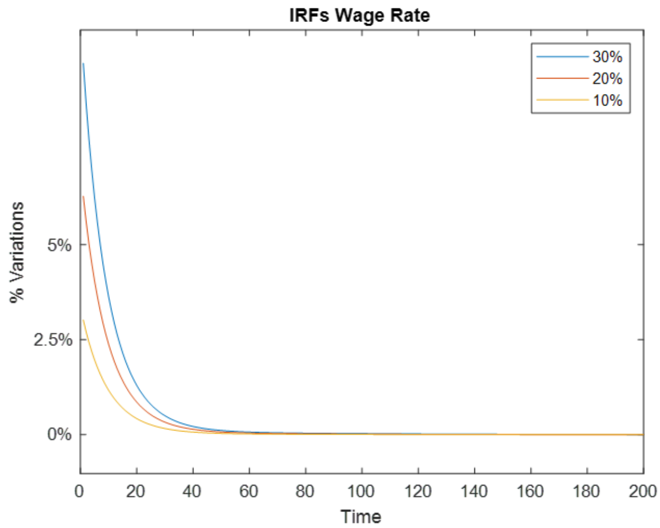

Figure 2.13: IRFs Wage Rate

us to corroborate the aggregate analysis with the discussion of the cross-sectional results, such that we can consider the effects of the shock on net worth inequality. Figure 2.14 shows the impulse-response function of the Gini Index of net worth distribution An expansionary monetary policy shock

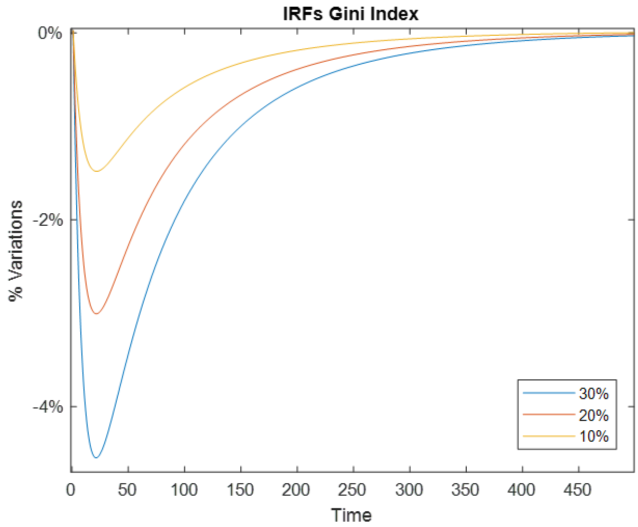

Figure 2.14: IRFs Gini Index

reduces the Gini Index, showing some persistence. This is similar to the pattern showed by the aggregate consumption given that households' optimal choices depend on their specific net worth level. On the one side, the households at the bottom of the distribution benefit from an improvement in their financial conditions (a reduction in the monetary balance rate, which is negative in their case) and borrowing opportunities so that they increase consumption and durable assets purchases. On the other side, the households at the top of the distribution are worse off because of the reduction in both asset types' returns. Overall, the net worth inequality is temporarily reduced.

2.3.4 Robustness Checks

This section presents some robustness checks about the implications of a 20% reduction in the monetary balance rate r^b under the assumption of different values of ω and H, that are the collateralizable fraction of the durable assets and the net worth limit. I only report the IRFs concerning the key results of the paper, which I show to be robust to different parametrization choices, as their functional forms do not vary when I change ω and H. Figure 2.15 and 2.16 show the IRFs of the aggregate consumption and the Gini Index of the wealth distributions, respectively, for $\omega \in \{0.20, 0.25, 0.3\}$.

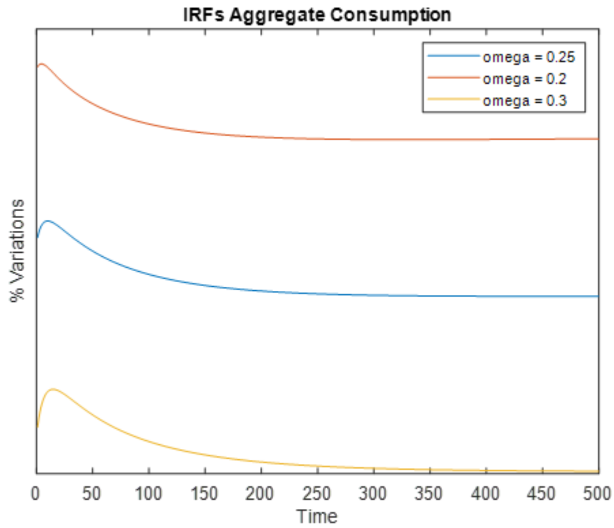

Figure 2.15: Robustness Check ω, IRF Aggregate Consumption

Intuitively, different values of ω affect the stationary values of all the variables of interest; in particular, a higher value of ω implies a lower stationary aggregate consumption and a higher value for the Gini Index of the stationary net worth distribution. The implications of a reduction in the monetary balance rate r^b are qualitatively unchanged: the aggregate consumption is boosted and the highest response is only reached a few periods after the intervention, then the consumption reverts to the steady-state level showing some persistence. The consumption response is still accompanied by a decrease in the aggregate monetary balance and an increase in the aggregate capital. Wealth inequality is mitigated. Then, the results in this paper are

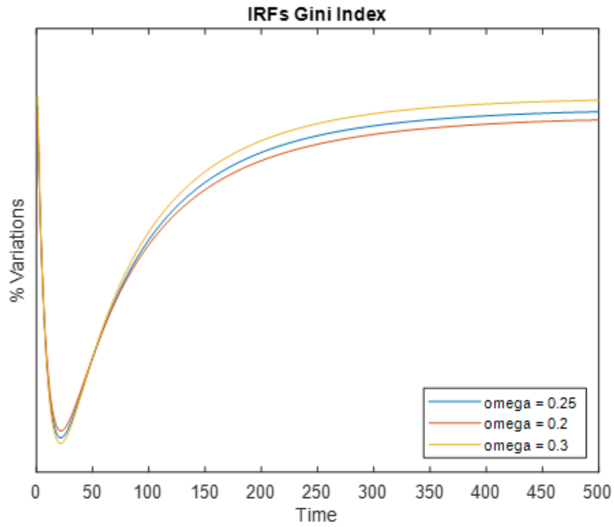

Figure 2.16: Robustness Check ω, IRF Gini Index

robust to different parametrization choices for ω. Figure 2.17 and 2.18 show the IRFs of the aggregate consumption and the Gini Index of the wealth distributions, respectively, for $H \in \{0.7, 0.75, 0.8\}$:

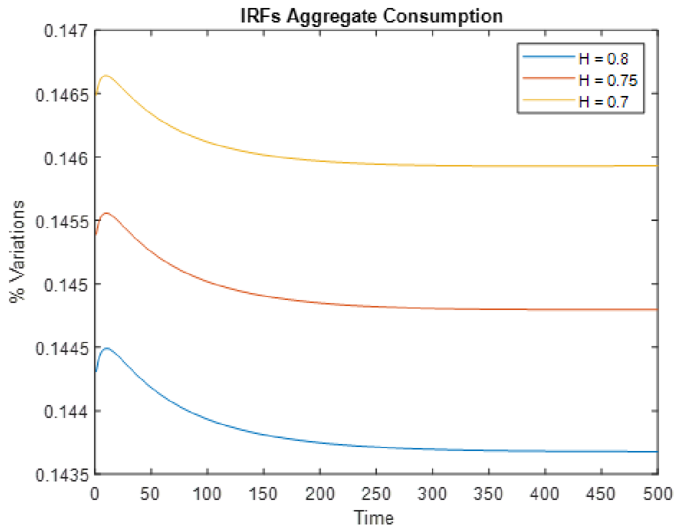

Figure 2.17: Robustness Check H, IRF Aggregate Consumption

As in the case of ω, different values of H affect the stationary values of

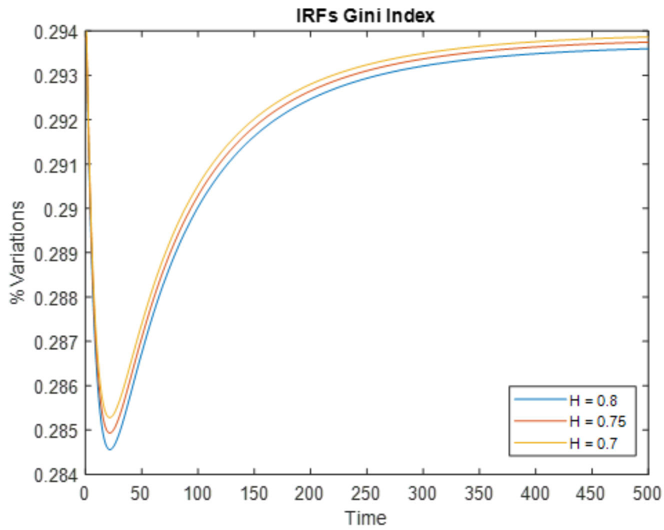

Figure 2.18: Robustness Check H, IRF Gini Index

all the variables of interest; in particular, a higher value for H implies a lower stationary aggregate consumption and a lower value for the Gini Index of the stationary net worth distribution. The implications of a reduction in the monetary balance rate r^b do not vary, as in the case of changes in the value of ω, then the results in this paper are also robust to different parametrization choices for H.

2.3.5 Contribution of the Collateral-based Borrowing Constraint

This paragraph assesses the relevance of a collateral-based borrowing constraint by a comparison with a simplified version of the model that features an exogenous borrowing limit (similar to Kaplan et al., 2018). We compute the steady state of the simplified version of the model and then the IRFs for all the variables of interest and compare the dynamics following an expansionary monetary policy shock. Unreported results show that the steady state of this simplified model is characterized by lower aggregate consumption, higher private debt, higher aggregate capital and higher inequality. We interpret these results as the consequence of "switching-off" the collateralization mechanism, which entails an increase in the borrowing opportunities to acquire new durable assets for households with zero (or close to) collateral assets. This generates a higher private debt and aggregate capital. This economy is more indebted than the one where the collateralization process is at work and constrains the borrowing opportunities. The MPCs are monotonic with respect to the net worth, meaning that the fraction of households who increases borrowing to purchase durable assets will be irrelevant for the evaluation of an expansionary monetary policy. For these reasons, we expect that an expansionary monetary policy shock in this simplified model has a smaller effect on private debt because it does not boost borrowing opportunities as in the model with the collateral-based constraint.

Table 2.3 compares the effects of a cut in the monetary balance policy rate for both models (with and without the collateralized borrowing constraint). As expected, the increase in private debt is significantly lower in the simplified

Table 2.3: Contribution of the collateral based borrowing constraint

	Consumption	Aggregate Capital	Private Debt	Gini Index
Collateral	0.5%	21.4%	53.0%	−2.9%
No Collateral	0.8%	20.0%	32.1%	−3.5%

version of the model; moreover, the increase in consumption and the decrease in the Gini Index are stronger. This means that a monetary policy application that ignores the collateralization mechanism may lead to an overestimation of the effects on consumption and inequality, and an underestimation of the effects on private debt that leads to an amplification of the instability of the financial system. The monetary policy authority must take into account these effects when evaluating their policies as their actions may be less effective than expected in terms of stimulating consumption and reducing inequality and potentially detrimental to the financial sector.

2.3.6 Redistribution and Monetary Policy

In the previous sections, I showed how the aggregates and the distribution are affected by an expansionary monetary policy shock. Namely, the shock boosts aggregates and reduces inequality, and this is directly linked to heterogeneity in households' behavior. In fact, a the reduction of inequality that implies a shift of net worth from low-MPCs households to high-MPCs households amplifies monetary policy transmission. In this respect, in this section, I further investigates this phenomenon considering two alternative model specifications; namely, while in the baseline model I assumed the net transfer to be equally distributed among the households, in this section, I consider what would happen if the net transfer was to be distributed among low-productivity ("poor-friendly" model) or high-productivity households ("rich-friendly" model).

Table 2.4 reports the (maximum) percentage variations of aggregate consumption, Gini Index, aggregate monetary balance and aggregate capital following the same expansionary monetary policy shock for the baseline and the two alternative versions of the model. The results are very interesting and can be interpreted along three dimensions. First, monetary policy transmission on the aggregate consumption is more effective in the "Poor-friendly" version and less effective in the "Rich-friendly" version of the model. Second,

Table 2.4: Redistributional effects of Monetary Policy

	Poor-Friendly	Baseline	Rich-Friendly
Consumption	1.14%	0.55%	0.31%
Gini Index	−4.26%	−2.99%	−2.44%
Monetary Balance	−21.63%	−52.88%	−92.32%
Capital	16.82%	20.01%	20.34%

the highest inequality reduction is obtained in the "Poor-friendly" version, and the lowest inequality reduction is in the "Rich-friendly" version. Third, the highest response in the aggregate consumption is coupled with the highest reduction in net worth inequality. Overall, this is an important message for monetary policy design: within this model, targeting net worth inequality improves monetary policy transmission.

2.3.7 Transitory Vs. Permanent Shock

The high decrease in the aggregate monetary balance (i.e., the increase in private debt) following an expansionary monetary policy shock should be a warning for monetary policy authorities given the role of private debt in financial crises. Transition dynamics after a transitory monetary policy shock is mitigated since households expect the change in the policy rate to be temporary (i.e., they have perfect knowledge of the shock) and anticipate that they will need to cut consumption in the future in order to repay their debts. However, the empirical literature reported a permanent increase in private debt and a permanent decrease in aggregate consumption following the increase in money supply before the financial crisis (see, e.g., Rajan 2010, Marzinotto 2016). For these reasons, I compare the aggregate and cross-sectional results of the baseline model with an alternative version of the model that features a one basis point permanent decrease in r^b. Table 2.5 summarizes the results.

Table 2.5: Permanent Monetary Policy Shock

	% Change
Consumption	-3.18%
Gini Index	1.39%
Monetary Balance	-45.4%
Capital	-1.9%

After a permanent change in r^b, the economy is in a different steady state that is characterized by a lower aggregate consumption level, while the aggregate capital is higher. The aggregate monetary balance is strikingly lower than in the baseline model and the net worth inequality is reduced. Therefore, despite the huge decrease in the monetary balance, the aggregate consumption will be permanently decreased. This happens mainly because of a shift of net worth distribution to the left as depicted in Figure 2.19.

In other words, even though the Gini Index is reduced, the households' sector is characterized by a lower net worth level and, given the shape of the consumption policy function, this results in a permanent fall of aggregate consumption. This is an important warning for macroeconomic policy: a permanent monetary policy action may lead to a permanent increase in private debt (both at the individual and the aggregate level) and, consequently, push the economy towards a low-consumption long run equilibrium. That is, monetary policy cannot foster the aggregate consumption in the long run.

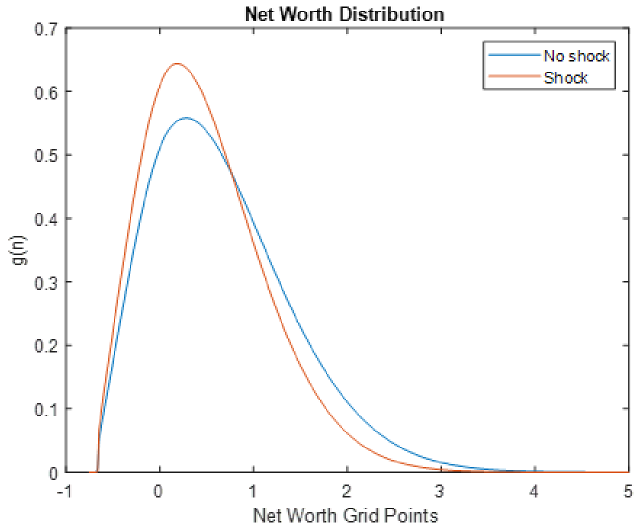

Figure 2.19: Permament shock, Net Worth Distribution

2.4 Conclusion

A proper characterization of the households' heterogeneity in their marginal propensities to consume, and in their borrowing attitude is crucial for macroeconomic policy design. On the one hand, it allows us to better understand the transmission of monetary policy shocks to the aggregate consumption; on the other hand, it is crucial to acknowledge the huge implications in terms of private debt and on net worth inequality. While the transmission channels have been largely explored by the most recent theoretical macroeconomic literature, the increase in private debt and the effects on inequality have been so far largely neglected. In this respect, I present a heterogeneous agent model with a collateral-based borrowing constraint, which provides an ideal framework to jointly investigate these issues. Notably, given the emphasis that I pose on understanding the consequences of a monetary shock on the net worth distribution the HA framework represents the obvious modelling choice. Therefore, I employ the Heterogeneous Agent Models in Continuous

Time framework developed by Achdou et al. 2021, who introduced a simple, efficient and sufficiently portable algorithm to solve numerically heterogeneous agent models that feature endogenous distributions. This algorithm is appropriate for handling borrowing constraints, which is crucial given the objective of introducing a collateral-based borrowing constraint in the model. This collateral-based constraint constitutes a key novelty with respect to the exogenous borrowing limit employed in most of the monetary policy applications featuring heterogeneous agents, and allows us to improve the description of individuals' MPCs as well as their debt profiles. In fact, this feature allows us to qualitatively replicate the empirical results in Crawley and Kuchler (2020); namely, if durable assets are included in the computation, the relationship between MPC and the net worth is non-monotonic. Moreover, the introduction of a collateral-based borrowing constraint allows catching up with the empirical literature, which emphasized the role of inequality in the vast increase in private debt before the GFC (see, e.g., Fitoussi and Saraceno, 2010; Rajan, 2010; Fazzari and Cynamon, 2013; Mian and Sufi, 2014). The attention that I pose to the distributional consequences of monetary policy, considered jointly with the ability to replicate the key empirical facts about the agents' MPC, represent an important contribution to the evaluation of the effects of a monetary policy shock. The transition dynamics following a cut in policy rates features a "hump-shaped" response in consumption, i.e., the highest consumption response is reached only a few periods following the intervention, and then reverts back to the steady-state level showing some persistence. The consumption response is backed by an increase in private debt and in aggregate durable assets, which sums up into a reduction in the net worth inequality (the Gini Index of net worth distribution). The relevance of a collateral-based borrowing constraint is assessed by a comparison with a simplified version of the model that relies on an exogenous borrowing limit. In particular, the same expansionary monetary policy shock leads to a smaller increase in aggregate consumption, a higher increase in private debt and a smaller decrease in net worth inequality when the model features a collateral-based borrowing constraint. This is crucial to understand for a monetary policy authority because, on the one hand, a model based on an exogenous constraint will overestimate the effects of the shock on aggregate consumption; on the other hand, the existence of a collateral-based borrowing attitude amplifies the instability of the financial sector through an increase in the aggregate private debt, and mitigate the positive effects on net worth inequality. The model offers additional, interesting results when exploited for two applications. The first links monetary policy redistributive effects with the efficiency in its transmission to aggregates, showing that the highest response in consumption is achieved when also the redistribution is maximized. The second focuses on the effects of a permanent reduction in the policy rates, showing that this may lead to a permanent increase in private debt (both at the individual and aggregate level) and, consequently, carry the economy towards a lower consumption long run equilibrium. My analysis remains mostly qualitative because of some simplifying assumptions in the model, such as the firms' sector and the interconnectedness between monetary and fiscal policy. However, I show that a framework that introduces a collateral-based borrowing constraint succeeds in achieving a proper characterization for private debt profiles and monetary policy's effects on inequality, in line with the empirical findings. Therefore, this work represents an interesting starting point for future quantitative research.

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Chapter 3

Endogenous Wealth Distribution and Labor Market Segmentation: a HACT Framework to set the Minimum Wage

Abstract

In this paper, I use a heterogenous agents in continuous time model to analyze the aggregate and distributional implications of the introduction of a minimum wage policy. In particular, I introduce an economy in which households differ for wealth and income and they can provide either skilled or unskilled work. Each household can become a skilled worker (and earn the higher skilled wage) through a fixed and indivisible investment in human capital; since the model features a borrowing limit, the households become skilled when the investment is both profitable and affordable. The government introduces a minimum wage and subsidizes the firms by taxing the households to restore the equilibrium in the labour market such that there is no unemployment. This policy is effective in boosting consumption and decreasing income inequality. However, it disincentives the investment in human capital and foster polarization among the households, thus increasing wealth inequality. Conversely, I show that an initial redistribution of wealth is more effective in reducing both wealth and income inequality, while also boosting aggregate consumption.

3.1 Introduction

The minimum wage policies have been subject to an intense debate both among academics and politicians. Nowadays, 22 out of the 28 states in the European Union (EU) have minimum wage laws and institutions, while the others (Austria, Cipro, Denmark, Finland, Sweden, and Italy) rely on collective bargaining and labour unions. The latter is problematic whenever some unregulated professions, such as riders, become a relevant part of the labour force: there is no collective bargaining and unions for this category of workers, which are generally at the bottom of the income and wealth distribution, so that setting a national minimum wage may be an alternative to reduce inequality and regulate the labour market.

However, there is an eternal debate about this issue that dates back to George Stigler's (1946) pioneering essay: in his framework, a wage floor implies a reduction in labour demand leading to unemployment. Intuitively, the minimum wage affects labour market equilibrium generating a wedge between supply and demand; in particular, it may cause the workers whose productivity is the smallest to be permanently unemployed. However, Stigler's analysis focuses on the effects of introducing a minimum wage in a single industry; in addition, the effects of the introduction of a minimum wage on the employment have been found to be at best small by part of the empirical literature in labour economics (see, e.g., Belman and Wolfson, 2016).

Given the ambiguity in the theory's predictions, the economists turned their attention to empirical studies but, as pointed out by Adams (1987), the impact of a minimum wage can only be understood in a macroeconomic theoretical framework. However, so far the literature investigating the effects of a minimum wage policy on macroeconomic aggregates has been rather scarce. Moreover, even less attention has been devoted by the theoretical literature to the cross-sectional effects of setting a minimum wage, notwithstanding the fact that the empirical literature has found evidence that higher minimum wages can help to reduce income inequality (see, e.g., Jaumotte and Buitron 2015). The effects of setting a minimum wage call for a proper theoretical investigation. I believe that the Heterogeneous Agent (HA) models represent a proper framework to investigate the implications of a minimum wage policy as they allow for a joint discussion of its aggregate and cross-sectional implications. In fact, a HA framework entails a full distribution of agents thus allowing for the discussion of inequality in terms of both concentration indexes and distributional ratios; in addition, it enables a proper characterization of the households' skill profile, which is crucial for the description of the minimum wage implications on the skilled and unskilled workers.

In this respect, the main contribution of this paper is to propose a HA framework to investigate the effects of the introduction of a minimum wage; this framework overcomes the obvious limitations of representative agent models and allows for the discussion of both the aggregate and the cross-sectional implications of the wage policy. More specifically, my goal is not only to understand whether a minimum wage policy is able to boost aggregate consumption, but also to shed light on its implication on the agents' skill profiles and its effects on income and wealth inequality. To this scope, I extend the Aiyagari (1994) model in two directions: on the one hand, households may offer skilled or unskilled work; each household may decide to become skilled through an indivisible human capital investment, similar to Galor and Zeira (1993); on the other hand, the Public Sector pursues a minimum wage policy that is strongly interconnected with fiscal policy. More precisely, when setting the minimum wage, the Public Sector subsidizes the firms' sector to enforce the equilibrium in the labour market and avoid unemployment in the low productivity sectors, similar to Dehez and Fitoussi (1996). In other words, the Public Sector taxes the households and uses the proceeds to pay to the firm sector the difference between the equilibrium and the minimum wage. The model is cast in continuous time to exploit the results in Achdou et al. (2021), who describe a simple and yet efficient algorithm to solve numerically the HA models.

The economy in the model is populated by a continuum of dynasties that are heterogeneous in wealth and income (i.e., they face either high or low income shocks). Each dynasty consists of a single household for each instant of time. As discussed above, households are divided into two groups, skilled and unskilled workers, and become skilled through an indivisible and fixed human capital investment. Households invest in human capital if this choice is both profitable and affordable. In fact, some households may need to borrow additional wealth to afford the human capital investment and, because there is a wedge between the borrowing and the lending rates, it could be the case that the investment is detrimental for their wealth. Moreover, some human capital investments may not be undertaken because of the presence of a borrowing limit. In this setting the households decide to invest in human capital if their individual wealth is above some specific threshold (which depends on the skilled and unskilled wages as well as the borrowing and lending rates), such that the size of the skilled and unskilled groups is endogenous.

Households have preferences over consumption and maximize their flow of utilities subject to the law of motion of wealth and the borrowing constraint. The firms' sector is kept purposely simple and it is populated by a representative firm that produces output using both skilled and unskilled labour. The Public Sector sets the minimum wage above the equilibrium unskilled wage rate. In this case, the representative firm should reduce the unskilled workforce in order to restore the equilibrium in the unskilled labour market. However, in this paper I assume that the Public Sector, in order to avoid the dismissals of part of the unskilled households, subsidizes the representative firm and taxes the households' sector. The model in section 3.2 assumes that the aggregate tax is equally distributed among the households; however, in section 3.3.6, I show the implications of setting a minimum wage when the tax is paid by some specific categories of households.

Setting the minimum wage generates an upward pressure on the skilled wage since it represents a disincentive to invest in human capital and therefore reduces the size of the skilled households' group. In fact, as a consequence of this policy the human capital investment became less profitable. Overall, the average wage rate in the economy increases, which boosts the aggregate consumption and reduces the Gini index of the income distribution. However, the disincentive to invest in human capital investment represents a crucial problem for a minimum wage policy, because it negatively affects the aggregate wealth and the Gini Index of the wealth distribution. Therefore, in this framework, setting the minimum wage do not succeed in reducing wealth inequality and, on the contrary, sharpen polarization among "rich" and "poor" households. The results remain qualitatively the same for different specifications of the fiscal policy, where the subsidy to the representative firm is financed by taxing a specific group of households; however, when the tax is paid only by the high income households, the positive effects on the aggregate consumption is stronger and the detrimental effects on the wealth inequality are mitigated.

Nevertheless, if the goal of the policymaker is to reduce wealth inequality, then a different policy should be implemented; in section 3.3.7 of this paper, I offer an alternative strategy. Namely, I investigate the effects of an initial redistribution of wealth in the model without the minimum wage policy, which I obtain by computing the stationary (steady state) wealth distribution two times starting from two different initial wealth distributions. This exercise exploits the fact that the model features multiple equilibria to disentangle the effects of redistribution on the aggregates and the income and wealth inequality. The reduction in the Gini Index of the initial distribution generates a downward pressure on the skilled wage, in contrast with the minimum wage policy, and an upward pressure on the unskilled wage because it shifts some of the unskilled workers to the skilled workers' group. In this setting, the Gini Index decreases both for the income and wealth distribution. Therefore, the initial redistribution represents a proper alternative for a policymaker interested in reducing wealth inequality and stimulating the aggregate demand. The rest of this paper proceeds as follows. Section 3.2 describes the model. Section 3.3 discusses the implications of setting the minimum wage and compares them to the implications of an alternative policy. Section 3.4 concludes.

3.2 The Model

3.2.1 Households

The economy is populated by a continuum of dynasties indexed by their wealth a and an income shock z. This shock follows a two-states Poisson process $z \in \{z_1, z_2\}, z_2 > z_1$, with intensities λ_1 and λ_2. Each dynasty consists of a single household for each instant t. Households are divided in two groups, skilled and unskilled: in particular, each household may become skilled through an indivisible human capital investment $\bar{a}$. The decision of becoming skilled if unskilled has two main drivers: profitability and afford-ability. On the one hand, a household will only decide to become skilled if this choice increases her wealth. On the other hand, households must have enough wealth to afford the payment of the indivisible amount $\bar{a}$; some of them may be able to borrow up to $\bar{a}$ while others will just be unable to become skilled. Therefore, three different types of households exist in this economy depending on their wealth level: the unskilled savers (USK, S), the skilled borrowers (SK, B) and the skilled savers (SK, S). The unskilled savers gain the unskilled wage w_{USK} and save their entire wealth earning a rate r; the skilled borrowers gain the skilled wage w_{SK} and borrow the difference between their wealth and the amount $\bar{a}$ at the rate $i > r$; the skilled savers gain the skilled wage w_{SK} and save the difference between their wealth and the amount $\bar{a}$ earning the rate r. In summary, the tree types of households earn:

$$USK, S : w_{USK} + ra$$

$$SK, B : w_{SK} + i(a - \bar{a}), a < -\bar{a}$$

$$SK, S : w_{SK} + r(a - \bar{a}), a \geq \bar{a}$$

The borrowing rate i is greater than the saving rate r to compensate the lender of the monitoring cost caused by the presence of a moral hazard issue (similar to Galor and Zeira, 1993). Households consider profitable to become skilled if this choice will increase her wealth. Since the third equation dominates the second for $a \geq \bar{a}$, the sufficient condition for the human capital investment to be profitable is given by $w_{SK} + i(a - \bar{a}) > w_{USK} + ra$, which yields:

$$a > h \equiv \frac{(w_{SK} - w_{USK}) - i\bar{a}}{r - i}$$

where h is the minimum wealth threshold above which the household will invest in human capital.

However, this choice needs also to be affordable. This is because the model features a borrowing limit $a_t \geq a_{min} = 0$, meaning that at each period in time wealth should be positive. In particular, the unskilled borrower households actually can become skilled if:

$$a + (a - \bar{a}) \geq a_{min}$$

where $(a - \bar{a})$ is the wealth amount they need to borrow. Therefore, becoming skilled is affordable if

$$a \geq f = \frac{\bar{a} + a_{min}}{2}$$

where f is the threshold above which the investment in human capital is affordable. The relationship between the two thresholds h and f depends upon the model calibration. More precisely, two distinct cases are possible. In the first case, $h > f$, which implies that, when the investment in human capital is profitable, it is also affordable, as showed in Figure 3.1. In this

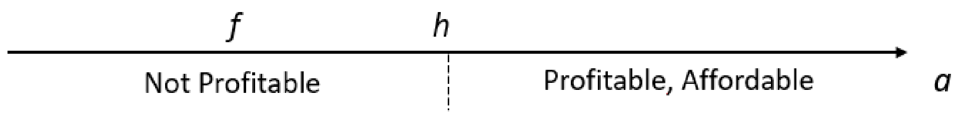

Figure 3.1: Human Capital Investment, $h > f$

case, households choose to become skilled if their individual wealth is greater or equal than h such that the three groups are as in Table 3.1. Then, skilled and unskilled labour offers are defined by:

$$L_{SK} = \int_{h}^{\infty} (g_1(a) + g_2(a))da$$

$$L_{SK} + L_{USK} = 1$$

Table 3.1: Households' group, first case

Individual Wealth	Category
$a \geq \bar{a}$	Skilled Savers
$h \leq a < \bar{a}$	Skilled Borrowers
$a < h$	Unskilled Savers

where $g_j(a)$ is the wealth distribution of the households with income type $j = 1, 2$.

In the second case, $h < f$, such that when the wealth is between h and f the investment is profitable but not affordable, as depicted in Figure 3.2. In this case, households choose to become skilled if their individual wealth is

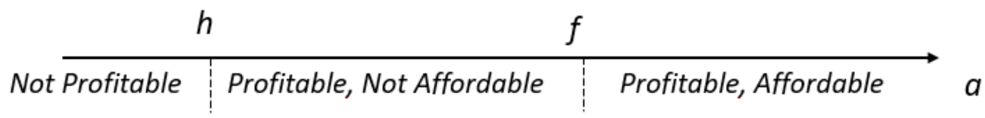

Figure 3.2: Human Capital Investment, $h < f$

greater or equal than f, as in Table 3.2. Then, skilled and unskilled labour

Table 3.2: Households' group, second case

Individual Wealth	Category
$a \geq \bar{a}$	Skilled Savers
$f \leq a < \bar{a}$	Skilled Borrowers
$a < f$	Unskilled Savers

offers are defined by

$$L_{SK} = \int_{f}^{\infty} (g_1(a) + g_2(a))da$$

$$L_{SK} + L_{USK} = 1$$

Section 3.3 will discuss the results of the model for the case $h > f$. The residual case of $h < f$ will be treated as a robustness check to show that the main implications of my paper do not depend on the specific calibration choices.

Households are characterized by rational expectations and have preferences over consumption c_t:

$$E_0 \int_0^\infty e^{-\rho t} u(c_t) dt$$

where $u(c_t)$ is a strictly increasing and strictly concave utility function and $\rho \geq 0$ is the intertemporal rate of substitution. Wealth evolves according to:

$$\dot{a}_t = w_{i,t} z_t - c_t + R(a_t) \tag{3.1}$$

where

$$w_i = w_{USK},\ R(a_t) = r a_t, \tag{3.2}$$

$$w_i = w_{SK},\ R(a_t) = i(a_t - \bar{a}), \tag{3.3}$$

$$w_i = w_{SK},\ R(a_t) = r(a_t - \bar{a}), \tag{3.4}$$

3.2.2 Firms

The firms' sector is kept purposefully simple. Namely, a representative firm uses both skilled and unskilled labour according to the following technology:

$$Y_t = A L_{SK,t}^{\alpha} L_{USK,t}^{1-\alpha}$$

where A is the total factor productivity and L_{SK} and L_{USK} are the skilled and unskilled labour offers as defined above. Inputs' markets are perfectly competitive; therefore, when the minimum wage is not set, the wage rates are equal to the marginal productivities:

$$w_{SK} = Y'_{SK}(.)$$

$$w_{USK} = Y'_{USK}(.)$$

3.2.3 Public Sector

The Public Sector sets the minimum wage $w_{min} > w_{USK}$; when a minimum wage is set, the equations of the model apply with w_{min} instead of w_{USK}. Setting w_{min} generates a disequilibrium in the unskilled labour market, because w_{min} is greater than the unskilled labour productivity, i.e.,

$$w_{min} > Y'_{USK}(.)$$

In this case, the representative firm should reduce L_{USK} to restore the equilibrium or, alternatively it bears the inefficiency costs C_t:

$$C_t = (w_{min} - w_{USK})L_{USK}$$

In this model, I assume that, when the Public Sector sets a value $w_{min} > w_{USK}$, it also restores the equilibrium in the unskilled labour market in order to avoid dismissals in the unskilled sector by subsidizing the representative firm with C_t and taxing the households' sector of the same amount, equally divided among households (the individual tax is τ_t). In this case, equation (3.1) becomes:

$$\dot{a}_t = w_{i,t}z_t - c_t + \tau_t + R(a_t)$$

In Section 3.3.6, I show how the main results are affected when the transfer is paid by some specific categories of households.

3.2.4 Stationary Equilibrium

A stationary equilibrium is fully characterized by the following system of equations:

$$v_j(a) = \max_c u(c) + v_j'(a)(w_i z_j - c + \tau + R(a)) + \lambda_j(v_{-j}(a) - v_j(a))$$

$$0 = -\frac{d}{da}(s_j(a)g_j(a)) - \lambda_j g_j(a) + \lambda_{-j} g_{-j}(a)$$

$$S_t = \int_{a_{min}}^{\infty} (a_1 g_1(a) + a_2 g_2(a))da$$

where $c_j(a) = (U')^{-1}(v_j'(a))$ and $s_j(a) = w_i z_j - c_j(a) + \tau + ra$ are the optimal consumption and savings, g_j is the wealth distribution of type j, and $-j$ is the opposite type of j. The wage rate w_i is defined by equations (3.2), (3.3) and (3.4). We define the functional form of $u(c_t)$ as:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

3.2.5 The Algorithm for the Stationary Equilibrium

The stationary equilibrium is computed iterating on the equilibrium system; the iteration starts defining an initial density $g(a, z)$ and a guess for the wage rates $w_{USK,i}$ and $w_{SK,i}$. Then, for $i = 0, 1, 2, ...$, the algorithm works as follows:

1. Set the minimum wage $w_{min} > w_{USK,i}$;

2. Given w_{min} and $w_{SK,i}$, compute the thresholds h and f;

3. Given $g(a, z)$ and the thresholds, compute L_{USK} and L_{SK};

4. Given w_{min},$w_{USK,i}$, and L_{USK}, compute the inefficiency costs C_i;

5. Given C_i, compute τ_i;

6. Given $w_{SK,i}$, w_{min} and τ_i solve the HJB equation using a finite difference method and derive the saving policy function $s_{j,i}(a)$;

7. Given the saving policy function, solve the KF equation for the density $g_i(a,z)$ using a finite difference method;

8. Given the density, compute L_{USK} and L_{SK};

9. Given L_{USK} and L_{SK}, compute the optimal wage rates w_{USK} and w_{SK};

10. Update $w_{USK,i}$ and $w_{SK,i}$ in the direction of the optimal wage rates;

11. Stop when $w_{SK,i+1}$ is "close enough" to $w_{SK,i}$.

The existence of a wealth threshold entails that the model is characterized by history dependence as in poverty trap models (see, e.g., Galor and Zeira 1993), and symmetry breaking models (see, e.g., Matsuyama 2004). This means that the stationary (steady state) wealth distribution depends on the initial wealth distribution (the initial state). In Section 3.3.3, I compute the steady state starting from two different specifications for the initial wealth distribution to show that the model features multiple equilibria.

3.2.6 Parametrization

Table 3.1 reports the parameters' value, which are mostly taken from the paper of Achdou et al. (2021). The Poisson process for the income shock is such that $z \in \{z_1 = 1, z_2 = 1.5\}$ with switching probabilities $\lambda_1 = 0.2$ and $\lambda_2 = 0.2$. The parameters affecting preferences and production take standard values. The coefficient of relative risk aversion is set to $\beta = 0.5$. The representative firm's total factor productivity is $A = 0.1$ and the skilled labour elasticity is set to $\alpha = 0.33$ (the unskilled labour elasticity is equal to

$1 - \alpha = 0.67$). The saving and borrowing rates are set equal to 0.04 and 0.1, respectively. As discussed above, the wealth limit is set to $a_{min} = 0$ and the indivisible human capital investment is $\bar{a} = 2$. This parametrization implies

Table 3.3: Parametrization

Parameter	Value
z_1	1
z_2	1.5
λ_1	0.2
λ_2	0.2
$\bar{a}$	2
a_{min}	0
r	0.04
i	0.1
α	0.33
A	0.1
ρ	0.05

that $h > f$, as discussed above. Section 3.3.5 will present some robustness checks on the parametrization that imply $h < f$.

3.3 Results

3.3.1 Consumption and Saving Policy Functions

In the steady-state, households take their optimal decisions depending on their rank in the joint distribution of wealth and income shock. Figures 3.3

and 3.4 plot the consumption and saving optimal policy functions.

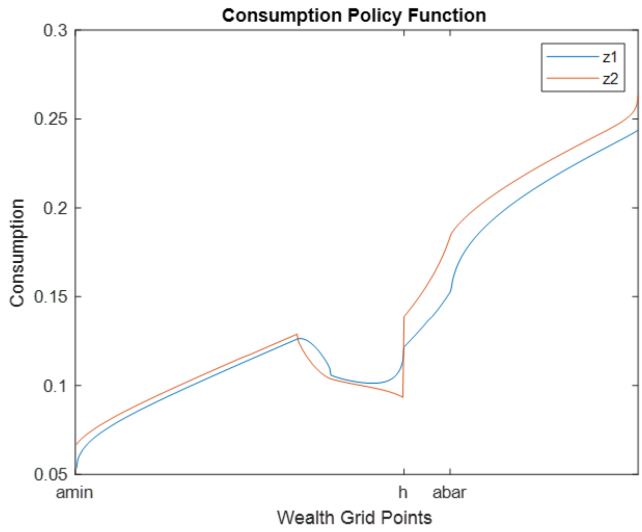

Figure 3.3: Consumption Policy Function

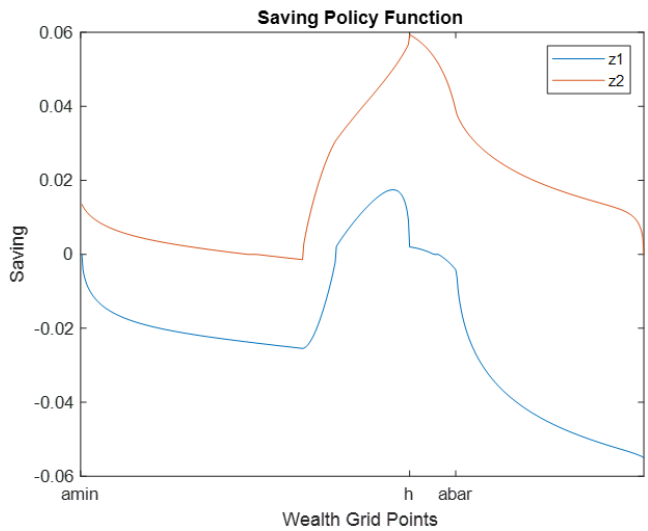

Figure 3.4: Saving Policy Function

Optimal consumption and saving policies have the typical features of problems with non-convexities: for each income type, there is a threshold level, which is commonly known in literature as the Skiba point, below which individuals decumulate wealth and decrease consumption; in Figure 3.4, this point is where the saving policy function intersects zero while being upward sloping. The Skiba point is strictly to the left of the non-convexity point h. In this model, the existence of the Skiba point has a crucial economic interpretation: the wealth threshold h separates the unskilled households from the skilled ones; therefore, households with individual wealth slightly below h choose to reduce consumption and save more to move to the right of the threshold in the long run and leave the unskilled group. Figure 3.5 represents the corresponding value functions.

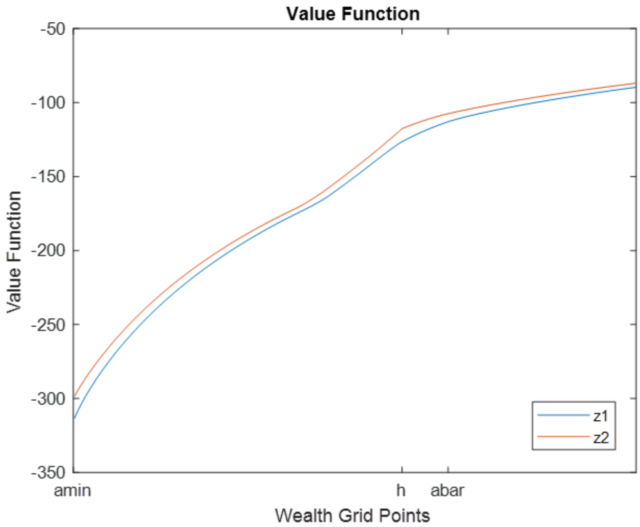

Figure 3.5: Value Function

Importantly, they feature convex kinks both at the Skiba point and at the non-convexity point h. There are no discontinuities.

3.3.2 Stationary Wealth Distribution

Figure 3.6 represents the stationary wealth distribution of the model. One

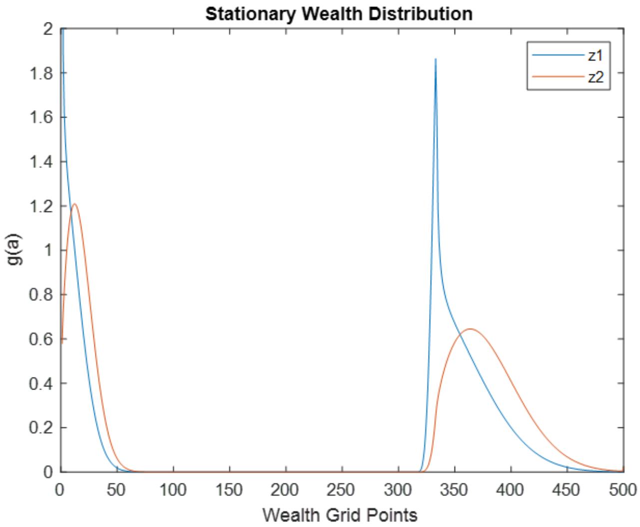

Figure 3.6: Stationary Wealth Distribution

crucial characteristic of this wealth distribution is that it is bimodal, so that the model features polarization as in the poverty trap and symmetry breaking models (see, e.g., Galor and Zeira 1993, and Matsuyama 2004). This feature is due to the non-convexity in the model; in particular, the threshold h separates the households into two groups: the unskilled group and the skilled one. Distributions have a Dirac point mass at the borrowing (wealth) limit: some individuals hit the limit in finite time after a long enough sequence of low-income shocks. This means that a huge mass of individuals will be characterized by wealth equal to a_{min} in steady-state. Distributions usually have an upper bound (see, e.g., Aiyagari 1994); because the model features a two-state Poisson process for the income shock, which is bounded, there exists a wealth level a_{max} such that savings are negative for both low and high income individuals for any $a \geq a_{max}$. Individuals hit the upper bound after a long enough sequence of high-income shocks, meaning that wealth accumulation requires both time and luck. The model in this paper reproduces these results both for the unskilled and skilled group, so that there exist a Dirac point mass and an upper bound for each of the groups; namely, the wealth distribution will feature two Dirac point mass at a_{min} and $\bar{a}$, and two upper bounds.

Table 3.4 summarizes the main cross-sectional results of the model. The

Table 3.4: Stationary Wealth Distribution

	USK,S	SK,B	SK,S
Total	42%	5.9%	52, 1%
Low Income	42.1%	10.7%	47.2%
High Income	39.9%	1.1%	59.0%

42% of the households are unskilled savers, meaning that they do not invest in human capital; the remaining part (58%) invests in human capital, becoming skilled, of which 5,9% of households have to borrow to afford the amount $\bar{a}$. Intuitively, the majority of the skilled borrowers are low income households, while the skilled savers are mostly represented by high income households. The Gini Index of the wealth distribution is equal to 0.424; in particular, the aggregate wealth is highly concentrated among low income households because the Gini Index of low income households' distribution is equal to 0.428, while the Gini Index of high income households' distribution is equal to 0.417).

3.3.3 Multiple Equilibria

Because the model may feature convex kinks in the thresholds h and f, similarly to what happens in the macroeconomic poverty trap literature, there can be multiple stationary wealth distributions. In this section, I numerically compute the steady-state wealth distribution starting from different specifications for the initial wealth distribution. In particular, I consider uniform initial wealth distributions with different supports such that, by construction, the change in the support reduces the Gini Index. Figure 3.7 plots two among the possible stationary wealth distributions. The dashed line represent the

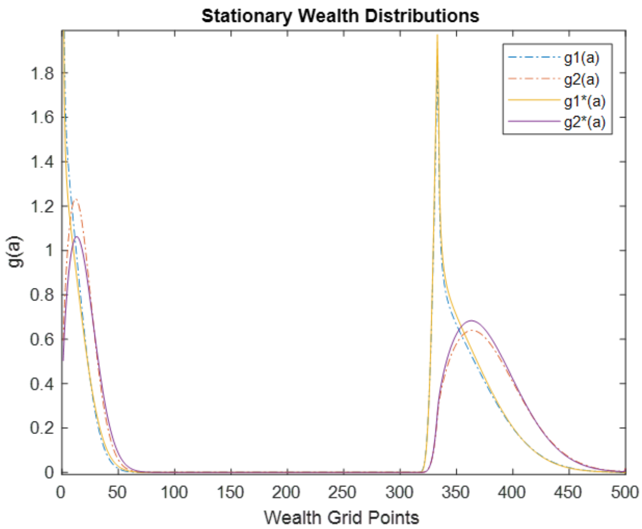

Figure 3.7: Stationary Wealth Distribution, Multiple Equilibria

stationary wealth distribution that is reached starting from the initial wealth distribution with the highest Gini Index. This means that if I change the support of the initial wealth distribution, reducing the respective Gini Index, some of the unskilled households shift to the skilled group such that the stationary wealth distribution will be characterized by a lower Gini Index. Table 3.5 summarizes these results considering five different specifications for the initial wealth distribution. It can be noted that the stationary Gini Index monotonically decreases with the initial Gini Index. This interesting result will be exploited in Section 3.3.7.

Table 3.5: Multiple Equilibria

Initial Gini Index	0.332	0.320	0.308	0.296	0.284
Stationary Gini Index	0.419	0.416	0.408	0.404	0.400

3.3.4 Setting the Minimum Wage

Table 3.6 summarizes the long-run effects of setting the minimum wage through a comparison between the steady-state of the model with (MW) and without (B) the minimum wage. The introduction of the minimum wage de-

Table 3.6: Minimum Wage Effects, Aggregates

Variable	MW vs. B
$w_{min,MW} > w_{USK,B}$	5%
$w_{SK,MW} > w_{SK,B}$	0.9%
$h_{MW} > h_B$	1.5%
$SK_{MW} < SK_B$	-1.7%
$C_{MW} > C_B$	2.9%
$S_{MW} < S_B$	-2.4%
$Gini_{Income,MW} < Gini_{Income,B}$	-1.9%
$Gini_{Wealth,MW} > Gini_{Wealth,B}$	1.8%

termines an increase in the skilled wage since it represents a disincentive to invest in human capital: in fact, this policy causes a shift to the right of the wealth threshold h, meaning that the investment becomes less profitable. As a result, the percentage of skilled workers will decrease. Overall, the increase in the average wage rate boosts aggregate consumption and a causes a reduction in the Gini index of the income distribution. The fact that the minimum wage represents a disincentive to human capital investment is a crucial issue for this policy, because it negatively affects the aggregate wealth and the Gini Index of the wealth distribution. Table 3.7 summarizes the main cross-sectional implications of the minimum wage policy.

Table 3.7: Minimum Wage Effects, Cross Sectional

	USK,S	SK,B	SK,S
Total	+5%	+1.7%	−3.9%
Low Income	+1.9%	+1.9%	−1.2%
High Income	+0.75%	+0.9%	−0.5%

The unskilled savers' group becomes larger in the case of both the low and the high income households; the same holds for the skilled borrowers. This happens at the expense of the skilled savers' group, which shrinks. These effects are more pronounced for the low income households, notwithstanding the group to which they belong. These implications are crucial for interpreting the effects of setting a minimum wage in the model: on the one hand, the minimum wage policy succeeds in effectively stimulating the aggregate demand and reducing income inequality; however, on the other hand, it strengthens wealth inequality and the polarization among social groups. If the objective of the policymaker is to reduce wealth inequality, the policy action discussed in this section is not effective; therefore section 3.3.7 discusses potential alternatives.

3.3.5 Robustness Check

The parameter choices assumed so far imply that $h > f$, as discussed above. This section discusses a set of robustness checks on the parametrization such that $h < f$ to show that the implications of setting a minimum wage do not differ qualitatively also in this scenario. Table 3.8 summarizes the long-run effects of setting the minimum wage through a comparison between the steady-state of the model with (MW) and without (B) the minimum wage, for this new parametrization.

Table 3.8: Robustness Check, Different Parametrization

Variable	MW vs. B
$w_{min,MW} > w_{USK,B}$	5%
$w_{SK,MW} > w_{SK,B}$	1.4%
$h_{MW} > h_B$	1.89%
$SK_{MW} < SK_B$	-2.6%
$C_{MW} > C_B$	3.67%
$S_{MW} < S_B$	-1.8%
$Gini_{Income,MW} < Gini_{Income,B}$	-1.74%
$Gini_{Wealth,MW} > Gini_{Wealth,B}$	4.3%

As anticipated, setting the minimum wage has the same qualitative implications in both of the cases, $h > f$ and $h < f$, as the variations in the variables of interest maintain their sign. Overall, the effects are amplified in the case of $h < f$.

3.3.6 Alternative Specifications for the Minimum Wage Policy

This section further investigates the implications of setting the minimum wage considering two alternative model specifications; namely, while the results in section 3.3.4 are derived under the assumption that the tax is equally distributed among the households, in this section I consider the effects of a net tax distributed among low-income ("Poor-friendly" model) or high-income households ("Rich-friendly" model). Table 3.9 reports the percentage variations of the variables of interest following the introduction of the minimum wage.

Table 3.9: Minimum Wage Effects, Different Specifications

Variable	Poor Friendly	Neutral	Rich Friendly
w_{min}	5%	5%	5%
w_{SK}	0.87%	0.9%	1.01%
h	1.45%	1.5%	1.71%
SK	-1.6%	-1.7%	-1.78%
C	3.3%	2.9%	2.65%
S	-2.25%	-2.4%	-2.61%
$Gini_{Income}$	-1.95%	-1.9%	-1.7%
$Gini_{Wealth}$	1.22%	1.8%	2.35%

The main implications of setting the minimum wage still hold in both the alternative specifications for the net tax distribution . In any case the minimum wage policy represents a disincentive to invest in human capital and the skilled workforce is reduced. The aggregate consumption is boosted at the cost of a reduction in the aggregate wealth; the income inequality is reduced but the wealth inequality increases.

However, this application leads to further considerations in terms of the design of the minimum wage policy. On the one hand, in the previous section, I showed that the distributional effects are stronger for the low income households. On the other hand, this paper assumes that the minimum wage policy is backed by a subsidy to the representative firm that is paid by the households through taxes. Therefore, it is expected that, when the tax is paid by the high income households, the detrimental effects of a minimum wage on the aggregate wealth and the wealth inequality may be reduced, as it is actually the case. In fact, Table 3.9 shows that introducing the minimum wage in a "Poor-friendly" specification of the model leads to a larger increase in consumption, a smaller reduction in the aggregate wealth and, most importantly, a more modest increase in wealth inequality. The opposite happens in the "Rich-friendly" specification of the model. Overall, this is an important message for a policymaker designing a minimum wage policy: the disincentive to invest in human capital may be reduced if distributional issues are taken into account.

3.3.7 Initial Wealth Redistribution

As already stated, the existence of a kink in the value function means that the model is characterized by history dependence as in poverty trap and symmetry breaking models; this implies that the initial state (i.e., the initial wealth distribution) determines the steady state, namely, the stationary wealth distribution. Therefore, a policymaker interested in reducing wealth inequality may act through an initial redistribution. This application considers a model without the minimum wage; the stationary wealth distribution is computed two times starting from two different initial wealth distributions. Namely, I consider two uniform initial wealth distributions with different supports such that, by construction, the change in the support shifts some of the unskilled households to the skilled group, thus reducing the Gini Index. Table 3.10 summarizes the long-run (steady state) effects of the reduction in the Gini Index. A reduction in the initial Gini Index determines a reduction in the

Table 3.10: Initial Redistribution Effects, Aggregates

Variable	MW vs. Benchmark
$w_{USK,IR} > w_{USK,B}$	5.3%
$w_{SK,IR} > w_{SK,B}$	−1.2%
$h_{IR} > h_B$	4.6%
$SK_{IR} < SK_B$	3.3%
$C_{IR} > C_B$	3.9%
$S_{IR} < S_B$	1.9%
$Gini_{Income,IR} < Gini_{Income,B}$	−3.6%
$Gini_{Wealth,IR} > Gini_{Wealth,B}$	−7.5%

skilled wage w_{SK} and an increase in the unskilled wage w_{USK} because the change in the support of the uniform distribution shifts some of the unskilled workers to the skilled workers' group. As a result, the profitability threshold h is also shifted to the right. However, the stationary aggregate consumption is greater because the average wage rate increases, and inequality decrease both in terms of income and wealth distributions.

Table 3.11 summarizes the main cross-sectional implications. The initial redistribution pushes a fraction of the household to acquire skills, therefore succeeding in escaping the poverty trap and decreasing the size of the un-

Table 3.11: Initial Redistribution effects, Cross Sectional

	USK,S	SK,B	SK,S
Total	-2.4%	$+2.7\%$	$+1.3\%$
Low Income	-1%	$+1.9\%$	$+0.2\%$
High Income	-3.5%	$+0.9\%$	$+2.2\%$

skilled savers' group. The skilled savers' group becomes larger both in terms of low and high income households; the same holds true for the skilled borrowers. These effects are more visible for the high income households, except for the skilled borrowers' group. In conclusion, the initial redistribution naturally leads to an increase in the unskilled wage and to an overall increase in the skilled workforce, successfully reducing both income and wealth inequality. Moreover, aggregate consumption and savings are boosted more than in the minimum wage policy. Therefore, the initial redistribution represents a proper alternative for a policymaker interested in reducing wealth inequality.

3.4 Conclusion

The implications of setting a minimum wage on the skill profile of the households and on the income and wealth distributions are crucial to understand for labour policy design. On the one hand, a minimum wage policy is a welfare measure for the households at the bottom of the wealth distribution that has the potential to reduce income inequality and boost the aggregate consumption; on the other hand, it represents a disincentive for the unskilled households to invest in their skill profile and therefore it may increase wealth inequality and reduce the aggregate wealth.

In this respect, I propose a heterogeneous agent model to jointly investigate the aggregate and the cross-sectional effects of a minimum wage policy. Given the emphasis that I pose on understanding the distributional implications of this policy, this represents the obvious modelling choice. More specifically, I cast the model in continuous time to exploit the results in Achdou et al. (2021), who introduced a simple, efficient and sufficiently portable algorithm to numerically solve HA models with endogenous distributions. Investigating the effects of minimum wage policies on macroeconomic aggregates and distributions is a key novelty. In fact, since the Stigler's pioneering paper (1946), macroeconomic theorists have devoted limited attention to minimum wage policies and most of the theoretical literature has focused on unemployment rather than cross-sectional effects. In contrast, most of the empirical literature on the minimum wage policies showed small and ambiguous effects on unemployment, but relevant effects on income inequality (see, e.g., Jaumotte and Buitron, 2015). Therefore, the lack of investigation of the distributional effects of minimum wage policies in the macroeconomic theoretical literature represents an important gap that my paper aims at filling. Besides, the attention that I pose to the distributional consequences of setting the minimum wage, as well as the description of its implications on the households' skill profile, its key for the discussion among policymakers. In this model, the introduction of the minimum wage triggers an increase in the skilled wage because it represents a disincentive to investing in skills. In fact, this policy causes human capital investment to become not profitable for the households at the bottom of the wealth distribution. Therefore, the percentage of skilled workers decreases.

Overall, the minimum wage policy boosts the aggregate consumption and reduces the Gini index of the income distribution. However, the minimum wage also reduces the aggregate wealth and the Gini Index of the wealth distribution as it sharpens the polarization among households' groups. The robustness of these implications is tested under alternative specifications where some specific households' categories are targeted; in particular, I discuss the implications of fiscal policies that specifically redistribute resources towards the low income or high income households. Also under these different specifications, the minimum wage policy still represents a disincentive to invest in human capital and the size of the skilled workforce is reduced. The aggregate consumption is boosted at the cost of a reduction in the aggregate wealth; the income inequality is reduced but the wealth inequality increases. However, when the minimum wage is supported by a fiscal policy that redistributes towards the low income households, it implies a higher boost in consumption, a smaller reduction in the aggregate wealth and, most importantly, a smaller increase in wealth inequality.

Therefore, while the disincentive to invest in skills may be reduced if distributional issues are taken into account, the wealth inequality still increases following the introduction of the minimum wage. If the goal of the policymaker is to reduce wealth inequality, a different approach is needed. To this purpose, I propose a potential alternative, which exploits the characteristics of the model, which features multiple equilibria. In this respect, I compute the stationary wealth distribution starting from two different wealth distributions, to mimic an initial redistribution that shifts some of the unskilled households to the skilled group, reducing the Gini Index of the wealth distribution. The results show that this initial redistribution naturally leads to an increase in the unskilled wage and an increase in the skilled workforce, successfully reducing both income and wealth inequality. Moreover, aggregate consumption and savings increase more than in the minimum wage policy.

Therefore, the initial redistribution represents a proper alternative measure for a policymaker interested in reducing wealth inequality.

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