A spatio-temporal model for events on road networks: an application to ambulance interventions in Milan

Un modello spazio-temporale per eventi su network stradali: analisi degli interventi delle ambulanze nel comune di Milano

Andrea Gilardi and Riccardo Borgoni and Jorge Mateu

Abstract The algorithms for optimal management and deployment of ambulances within a municipality require a spatio-temporal model to forecast hotspots and minimise the response times. Ambulance interventions represent an example of a point pattern occurring on a linear network, which was created starting from the main streets of Milan. The constrained spatial domain raises particular challenges and unique methodological problems that cannot be ignored for proper model development. Hence, this paper presents a non-separable spatio-temporal model for analysing the emergency interventions that occurred in the street network of Milan from 2015 to 2017. A dynamic latent factor model is adopted for capturing the temporal evolution, while the spatial dynamics are modelled using a network-readaptation of a kernel estimator.

Abstract Gli algoritmi per la gestione delle ambulanze all'interno di un comune necessitano di modelli statistici che possano prevedere l'insorgere di criticità, in maniera tale da poter minimizzare i tempi di intervento. Gli interventi in emergenza delle ambulanze rappresentano un esempio di processo di punto su network stradale, creato partendo dalla rete stradale di Milano. Il supporto spaziale del fenomeno sviluppa diverse problematiche sia da un punto di vista metodologico che applicato, che non possono essere ignorate per la creazione di un modello appropriato. In questo paper analizziamo la distribuzione degli interventi in emergenza delle ambulanze nel comune di Milano tra il 2015 ed il 2017, sviluppando un modello dinamico a fattori latenti per la componente temporale ed uno stimatore kernel non-parametrico per l'intensità spaziale, riadattato nel caso di dati su network.

Key words: ambulance interventions, point pattern on networks, spatial networks, spatio-temporal data

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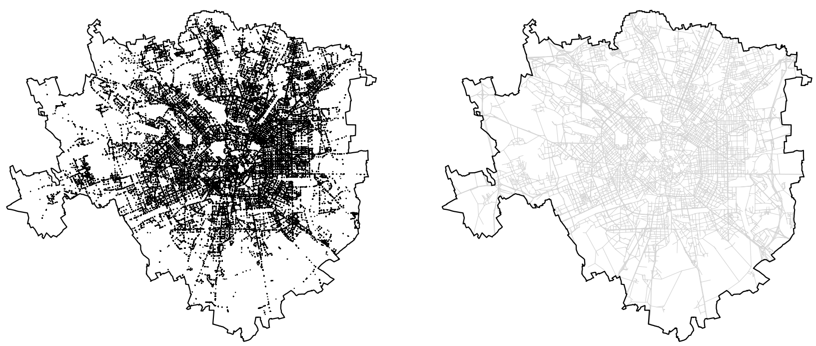
1 Introduction

The algorithms for optimal staff management and ambulances deployment within a municipality require a spatio-temporal model to forecast hotspots and minimise the expected response times. The predictions are required at a fine spatial and temporal resolution, due to intricate spatio-temporal patterns in emergency intervention data, which are particularly relevant for a hectic city like Milan.

Ambulance interventions represent a typical example of a point pattern occurring on a linear network, an increasingly popular type of events presenting several challenges related to the tangled and non-homogeneous nature of their spatial support [1]. Several authors explained the perils of re-adapting classical planar techniques, such as K-function or Kernel Density Estimator (KDE), to network data without considering the network's structure [5, 7]. The recent surge of interest can also be linked with the rapid development of several open-source spatial databases (such as Open Street Map), that provide the starting point for creating a computational representation of a road network.

2 Data: Ambulance Interventions

The data at hand included all emergency calls registered in the municipality of Milan (IT) from 2015-01-01 to 2017-12-31, which required an ambulance intervention and were handled by the regional Emergency Medical System (EMS). We removed all records with missing spatial or temporal coordinates, and we included only the first intervention when multiple ambulances were dispatched for the same (typically



(A) (B)

Fig. 1 *Left*: Locations of ambulance interventions in Milan from 2015 to 2017. *Right*: The most important streets of the road network. In both cases, we can recognise several white areas corresponding to parks (Parco Sempione), pedestrian areas (Citylife), and non-urban places.

life-threatening) event. The final sample included 495,950 interventions, 163,488 occurred in 2015, 165,368 in 2016 and 167,094 in 2017.

The spatial distribution of the EMS calls is reported in Figure 1(A). We note that the events resemble a street network structure, highlighting the city ring road and the most critical arterial thoroughfares. Hence, we argue that a spatio-temporal model of emergency interventions should not ignore their peculiar spatial support. The empty areas in Figure 1(A) correspond to non-urban places, mainly located in the south or the west. We can also clearly distinguish the shapes of several iconic locations of Milan, such as City Life, Parco Sempione or Scalo Farini.

We examined the temporal dimension of EMS interventions and determined the seasonal patterns that govern the total number of emergency calls. More precisely, we noticed that the average number of hourly events during the weekdays follows a particular trend: after a rapid increase in the early morning, the time series reaches its maximum around 10:00, slowly declines until 20:00 and then decreases until the night. The weekends present a similar distribution, with more interventions during the night hours (probably linked with the city's nightlife) and fewer events in the late morning. The time series of ambulance interventions also exhibits a weekly seasonal pattern, and the global minima are registered around August, in conjunction with national holidays. The dynamic latent factor model introduced in Section 3.1 was defined taking into account these seasonal patterns, which are discussed by [6, 4].

A linear network, typically denoted by L, is defined as the union of a finite set of segments, say l_i, lying in a planar region S:

$$l_i = [\mathbf{u}_i, \mathbf{v}_i] = \{\mathbf{s} : \mathbf{s} = t\mathbf{u}_i + (1-t)\mathbf{v}_i;\ 0 \leq t \leq 1\}; \quad \mathbf{u}_i, \mathbf{v}_i \in S \subseteq \mathbb{R}^2.$$

The endpoints of l_i are denoted by $\mathbf{u}_i$ and $\mathbf{v}_i$, and, in this paper, S denotes the polygonal boundary of Milan. The computational structure of the road network was created starting with data downloaded from Open Street Map (OSM) and selecting only the most important[1] street segments.

Spatial networks can also be seen as graph objects, where the edges correspond to the street segments, while the nodes are usually placed at road junctions [2]. We took advantage of the graph representation to simplify Milan's road network, excluding the small groups of isolated road segments. More precisely, we created a binary adjacency matrix between pairs of edges, defining two edges as *connected* if the corresponding road segments share one point at their geographical boundaries. Then, we clustered the segments and removed the isolated groups (typically denoted as *components* in the graph-analysis literature). This procedure creates a fully connected road network, which has relevant consequences on the kernel estimator presented in Section 3.2.

The linear network obtained after applying the pre-processing steps described above is depicted in Figure 1(B). It is composed of approximately 11,000 edges, and it covers more than 1850km, traversing almost every part of the city. We can

[1] We filtered only the street segments that, in the OSM jargon, are classified as *motorways*, *trunks*, *primary roads*, *secondary roads*, *tertiary roads*, and *unclassified roads*. Using the Italian classification, they range from *Autostrada* to *Strada Comunale*.

notice several similarities between Figure 1(A) and 1(B), and, once again, we can recognise several iconic places.

After creating the street network, we decided to exclude all ambulance interventions that occurred farther than 50 metres from the closest street segment, since we assumed that they occurred in other parts of the city network, and we projected the remaining ones into the linear network. We removed approximately 5% of the EMS data. Finally, we explored the spatio-temporal nature of EMS data, observing the presence of space-time interactions in the hourly distributions. More precisely, we noticed that from 08 AM to 08 PM the interventions are concentrated near the city centre, close to the office areas and the main buildings, while, during the night hours, they are scattered all around the municipality. These interactions are captured by the weighted network kernel estimator detailed in Section 3.2.

3 Statistical Methods

Following and extending the approach introduced in [9, 4], we consider a continuous one-dimensional linear network L and a discrete temporal dimension $\mathscr{T}$ divided into intervals of one hour. Let y_t denote the number of emergency calls that were recorded at time $t \in \mathscr{T}$, and let $\mathbf{s}_{i,t}, i = 1, \dots, y_t$ be the location of ith event. Then, we assume that, independently for each $t \in \mathscr{T}$, the point process $\{\mathbf{s}_{i,t} : i = 1, \dots, y_t\}$ can be modelled as a *Non-homogeneous Poisson Process* (NHPP) on a linear network with intensity function $\lambda_t(\mathbf{s})$ [3, 1]. Furthermore, we assume that

$$\lambda_t(\mathbf{s}) = \mu_t g_t(\mathbf{s}), \quad \mathbf{s} \in L;\ t \in \mathscr{T}, \tag{1}$$

where μ_t represents the temporal dimension of the EMS counts, while $g_t(\mathbf{s})$ is the spatial component of the process. Even though Equation 1 looks like the classical separability assumption for spatio-temporal point processes, the notation $g_t(\mathbf{s})$ implies that the spatial component depends on the temporal distribution of the data. These space-time interactions are taken into account adding a set of weights into the kernel function used to estimate $g_t(\mathbf{s})$, as detailed in Section 3.2.

In the next sections, we briefly introduce a time series model to capture the evolution of μ_t, and we describe with greater details the procedures for estimating $g_t(\mathbf{s})$ using a re-adaptation of the planar weighted kernel estimator for point pattern data on linear networks.

3.1 Temporal model

Following the approach detailed in [6, 4], we modelled the temporal component μ_t using a dynamic latent factor model. The hourly, daily, and weekly seasonalities were included by imposing a set of constraints on the factors and loadings matrices, while penalised and cyclic cubic regression splines were adopted to impose a smooth evolution on EMS counts.

3.2 Spatial model

As mentioned before, the spatial component of the EMS interventions is modelled using a network-readaptation of Jones-Diggle corrected weighted kernel estimator, which, given a location $\mathbf{s} \in L$ and a time period u, can be written as

$$\hat{g}_u(\mathbf{s}) = \frac{\sum_{t \in \mathscr{T}} \sum_{i=1}^{y_t} w_{\mathbf{s}_i}(t,u) K_N(\mathbf{s}, \mathbf{s}_{i,t})}{\sum_{t \in \mathscr{T}} \sum_{i=1}^{y_t} w_{\mathbf{s}_i}(t,u)}.$$

We assumed that the weight function, hereby denoted as $w_{\mathbf{s}_i}(t,u)$, depends only on the temporal lag between u and the historical data. The weights are used to incorporate a space-time interaction into the KDE, giving more importance to EMS calls that occurred in the temporal proximity of u, and creating a non-separable structure into $\lambda_t(\mathbf{s})$. We refer to [9, 4] for more details on the weights' estimation process.

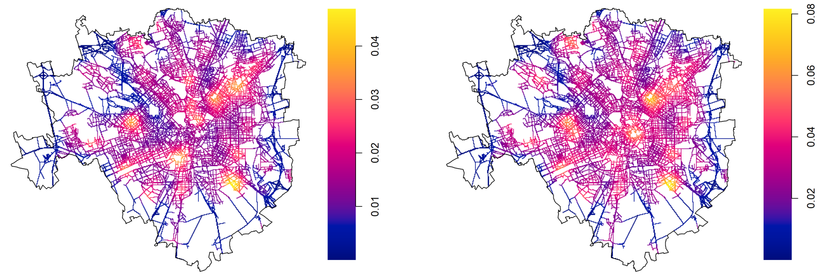
The function $K_N(\mathbf{s}, \mathbf{s}_{i,t})$ denotes the Jones-Diggle corrected network KDE, as introduced by [8]. More precisely, considering a location $\mathbf{s} \in L$ and a time period $t \in \mathscr{T}$, the estimator is defined as

$$K_N(\mathbf{s}, \mathbf{s}_{i,t}) = \frac{K(\mathbf{s} - \mathbf{s}_{i,t})}{c_L(\mathbf{s}_{i,t})}, \tag{2}$$

where K denotes a planar bivariate kernel function, $\mathbf{s}_{i,t}$ is an historical ambulance intervention, and $c_L(\mathbf{s}_{i,t})$ represents the convolution of the kernel K with arc-length measure on the network, defined as $c_L(\mathbf{s}) = \int_L k(\mathbf{v} - \mathbf{s})\, d_1\mathbf{v}$. Equation 2 is analogous to the planar KDE, where the Jones-Diggle correction is replaced using an integral over the network. Despite a slightly suboptimal statistical efficiency, the KDE estimator in Equation 2 can be computed rapidly using the fast Fourier transformation, which is essential considering the size of the network and the volume of EMS calls. The other statistical properties are extensively described in [8], whereas alternative approaches are discussed by [1].

4 Results and Conclusions

We exemplified the algorithm described in Section 3 considering two future temporal occasions: 2018-01-03 at 03:00 (left) and 2018-01-03 at 15:00 (right). The results are reported in Figure 2. The map on the left shows that EMS interventions are spread in several parts of Milan, highlighting nightlife areas such as Porta Genova or San Lorenzo, while the map on the right draws attention to other zones close to Duomo and significant working places. In both cases, the main train station, Pi-



(A) (B)

Fig. 2 Estimates of the spatial intensity function, $\hat{g}_u(\mathbf{s})$, considering two future time periods: 2018-01-03 at 03:00 (left) and 2018-01-03 at 15:00 (right).

azzale Loreto, and several retirement houses (such as Pio Albergo Trivulzio) are highlighted. The two maps are represented using different scales in order to better point out the temporal fluctuation of ambulance intervention intensity.

As further steps, we are developing a methodology for properly assessing the fit of the suggested model. Moreover, we plan to extend the planar spatio-temporal estimators for relative risk to network data to investigate and compare the spatial dynamic of EMS calls having different severity levels.

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