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Power corrections in a transverse-momentum cut for vector-boson production at NNLO: the qg-initiated real-virtual contribution

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Abstract We consider the production of a vector boson (Z, $W^{\pm}$ or γ^*) at next-to-next-to-leading order in the strong coupling constant α_S. We impose a transverse-momentum cutoff, q_T^{cut}, on the vector boson produced in the qg-initiated channel. We then compute the power corrections in the cutoff, up to the second power, of the real-virtual interference contribution to the cumulative cross section at order α_S^2. Other terms with the same kinematics, originating from the subtraction method applied to the double-real contribution, have been also considered. The knowledge of such power corrections is a required ingredient in order to reduce the dependence on the transverse-momentum cutoff of the QCD cross sections at next-to-next-to-leading order, when the q_T-subtraction method is applied. In addition, the study of the dependence of the cross section on q_T^{cut} allows as well for an understanding of its behaviour in the small transverse-momentum limit, giving hints on the structure at all orders in α_S and on the identification of universal patterns. Our result are presented in an analytic form, using the process-independent procedure described in a previous paper for the calculation of the all-order power corrections in q_T^{cut}.

Contents

1 Introduction

The recent years have witnessed an increasing growth in the accuracy of physics measurements at the Large Hadron Collider, on the one side, and the great efforts done by the theoretical community in order to provide theoretical results of increasing precision, on the other. The goal of these activities is not only important for the extraction of Standard Model (SM) parameters, but also for searches of signals of new physics that can appear as small deviations with respect to the SM predictions.

Reaching the highest possible level of precision is then the main goal and the calculation of perturbative QCD corrections plays a dominant role in this context. Until a few years ago, the standard for such calculations was next-to-leading-order (NLO) accuracy. In recent years, the goal has become next-to-next-to-leading-order (NNLO) accuracy, and even beyond for some processes.

The computation of higher-order terms becomes more involved due to the technical difficulties arising in the evaluation of virtual contributions and to the increasing complexity of the infrared (IR) structure of the real contributions. In order

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to expose the cancellation of the IR divergences between real and virtual contributions, the knowledge of the behaviour of the scattering amplitudes in the infrared limits is then crucial and it is indeed what is used by the subtraction methods in order to work.

At NNLO and beyond, several subtraction schemes have been proposed in the past years. These schemes mostly fit into two categories: local methods and slicing methods. The latter are based on partitions of the phase space into hard regions and infrared-sensitive regions, where the cancellation of divergences is performed with non-local subtraction terms. In order to apply these methods, one has to introduce a resolution parameter to identify the phase-space regions where the non-local subtraction acts. Slicing methods that have been successfully applied at NNLO and N^3LO are the transverse-momentum (q_T) subtraction method [1–5] and N-jettiness subtraction [6,7].

By applying non-local subtraction methods, the singular terms in the small-cutoff limit are cancelled. These terms have a universal nature and this allows to construct the subtraction terms on general grounds. After the cancellation has taken place, only finite and vanishing terms remain. These terms are, in general, process dependent. A residual dependence on the cutoff then remains as power corrections. While these terms formally vanish in the null-cutoff limit, they give a non-zero numerical contribution for any finite choice of the cutoff.

The knowledge of the power-correction terms greatly increases the numerical reliability of the final results. In fact, by subtracting the lowest powers in the cutoff makes the result less sensitive to the arbitrary cutoff, numerically approaching the theoretical limit of this parameter going to zero. This is not only valid when the subtraction method is applied to NLO computations, but it is numerically more relevant when applied to higher-order calculations, as pointed out, for example, in the evaluation of NNLO cross sections in Refs. [8,9].

Beyond reducing the dependence of the theoretical results on the cutoff, the study of power-suppressed terms in the infrared regions is a theoretically interesting subject, since it allows to deepen our knowledge of the universal and non-universal structure of the perturbative behaviour of QCD cross sections in the IR limits. Thus, several papers have tackled the study of power corrections in the general framework of fixed-order and threshold-resummed computations [10–19].

Power corrections at NLO have been extensively studied in Refs. [20–30] in the context of the N-jettiness subtraction method, and in Refs. [31–36] within SCET-based subtraction methods. Power corrections at NLO for the transverse momentum of a colour singlet have been derived for the first time at differential level in Ref. [37] within the SCET framework. In Ref. [38], we presented a method to compute the power corrections at all orders, for the inclusive production of a colourless final-state system, at NLO in QCD. Recently, the leading power corrections for the electroweak NLO corrections to the inclusive cross section for the production of a massive lepton pair through the Drell–Yan mechanism have been computed in Ref. [39].

N-jettiness power corrections at NNLO have been considered in Refs. [20,23]. In particular, analytic results are obtained for the dominant $\alpha_S\, \tau \log(\tau)$ and $\alpha_S^2\, \tau \log^3(\tau)$ subleading terms, where τ is the 0-jettiness, for $q\bar{q}$-initiated Drell–Yan production and for gg-, gq- and $q\bar{q}$-initiated Higgs boson production, along with a numerical fit for the subdominant terms.

In this paper we consider the production of a vector boson (Z, $W^\pm$ or γ^*) at NNLO in the strong coupling constant α_S. We impose a transverse-momentum cutoff, $q_T^{\rm cut}$, on the vector boson produced in the qg-initiated channel, and we compute, for the first time, the power corrections in the cutoff, up to the second power $(q_T^{\rm cut})^2$, of the real-virtual interference contribution to the cumulative cross section at order α_S^2, plus other terms with the same kinematics, originating from the application of the subtraction method to the double-real contribution. In order to perform this computation, we apply the general process-independent method that we have formulated in Ref. [38].

This is the first step in order to compute the power corrections, up to the second power, of the NNLO cumulative cross section for vector-boson production. In fact, together with the real-virtual qg-initiated channel that we consider in this paper, also the qq-initiated channel contributes to the real-virtual terms, together with all the double-real radiation contributions. We will consider these contributions in future works.

The outline of this paper is as follows. In Sect. 2 we introduce our notation and we briefly summarize the expressions of the partonic and hadronic cross sections, in a form that is suitable for what follows. In Sect. 3 we outline the calculation we have done and in Sect. 4 we present and discuss our analytic results. We draw our conclusions in Sect. 5. In Appendix A, we present some examples of the integrals we had to perform in order to compute the power corrections, and we give the result of the integration for a few of them. Finally, in Appendix B we collect the final results of our paper.

2 The hadronic and partonic cross sections

In this section we set the theoretical framework and introduce the notation used throughout the paper.

2.1 The hadronic cross sections

We consider the production of a colourless system F with quadri-momentum q and squared invariant mass Q^2, plus a coloured system X at a hadron collider

$$h_1 + h_2 \to F + X\,. \tag{2.1}$$

We call S the hadronic squared center-of-mass energy and we write the hadronic differential cross section for this process as

$$d\sigma = \sum_{a,b} \int_\tau^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2\, f_a(x_1)\, f_b(x_2)\, d\hat{\sigma}_{ab}\,, \tag{2.2}$$

where

$$\tau = \frac{Q^2}{S}\,, \tag{2.3}$$

$f_{a/b}$ are the parton densities of the partons a and b, in the hadron h_1 and h_2 respectively, and $d\hat{\sigma}_{ab}$ is the partonic cross section for the process $a + b \to F + X$. The dependence on the renormalisation and factorisation scales and on the other kinematic invariants of the process are implicitly assumed.

The hadronic cross section can be written as[1]

$$\sigma = \sum_{a,b} \int_\tau^1 dx_1 \int_{\frac{\tau}{x_1}}^1 dx_2\, f_a(x_1)\, f_b(x_2) \times \int dq_{\mathrm{T}}^2\, dz\, \frac{d\hat{\sigma}_{ab}(q_{\mathrm{T}}, z)}{dq_{\mathrm{T}}^2}\, \delta\!\left(z - \frac{Q^2}{s}\right), \tag{2.4}$$

where s is the partonic center-of-mass energy, equal to

$$s = S\, x_1\, x_2\,. \tag{2.5}$$

We have also made explicit the dependence on z, the ratio between the squared invariant mass of the system F and the partonic center-of-mass energy, and on q_{T}, the transverse momentum of the system F with respect to the hadronic beams. Using Eqs. (2.5) and (2.3) and integrating over x_2 we obtain

$$\sigma = \sum_{a,b} \tau \int_\tau^1 \frac{dz}{z} \int_{\frac{\tau}{z}}^1 \frac{dx_1}{x_1}\, f_a(x_1)\, f_b\left(\frac{\tau}{z\, x_1}\right) \frac{1}{z} \int dq_{\mathrm{T}}^2\, \frac{d\hat{\sigma}_{ab}(q_{\mathrm{T}}, z)}{dq_{\mathrm{T}}^2}\,. \tag{2.6}$$

We then introduce the parton luminosity $\mathcal{L}_{ab}(y)$ defined by

$$\mathcal{L}_{ab}(y) \equiv \int_y^1 \frac{dx}{x}\, f_a(x)\, f_b\!\left(\frac{y}{x}\right), \tag{2.7}$$

so that we can finally write

$$\sigma = \sum_{a,b} \tau \int_\tau^1 \frac{dz}{z}\, \mathcal{L}_{ab}\!\left(\frac{\tau}{z}\right) \times \frac{1}{z} \int dq_{\mathrm{T}}^2\, \frac{d\hat{\sigma}_{ab}(q_{\mathrm{T}}, z)}{dq_{\mathrm{T}}^2}\,. \tag{2.8}$$

2.2 The partonic differential cross sections

In this paper we consider the NNLO corrections to the production of a vector boson F, i.e. a $W^\pm$, a Z or a virtual photon γ^*. In particular, we deal with the qg-initiated partonic channel

$$q(p_1) + g(p_2) \;\to\; F(q) + X(k)\,, \tag{2.9}$$

where the quadri-momenta are given in parentheses. In Ref. [38], among other contributions, we considered the NLO cross section for F production, i.e.

$$q(p_1) + g(p_2) \;\to\; F(q) + q(k)\,, \tag{2.10}$$

where the final-state quark has the same flavour of the initial-state one, for Z/γ^* production, and different flavour, for W production. We generically indicate the initial- and final-state quark with the same letter q.

Introducing the kinematic invariants

$$s = (p_1 + p_2)^2\,, \qquad t = (p_1 - q)^2\,, \qquad u = (p_2 - q)^2, \tag{2.11}$$

we have the relation

$$s + t + u = q^2 + s_2 \tag{2.12}$$

where $s_2 = k^2$ is the squared invariant mass of the system recoiling against the F boson at parton level.

In the following, we use the same notation and the expressions computed in Ref. [40]. The couplings appearing in the differential cross sections follow this convention: if an electroweak boson F is emitted by a quark with flavour $f_1 = \{u, d, s, c, b\}$ which then changes into f_2, the vertex is described by the Feynman rule

$$-\,ie\gamma^\mu \left[\ell_{f_2 f_1} \frac{1-\gamma_5}{2} + r_{f_2 f_1} \frac{1+\gamma_5}{2}\right], \tag{2.13}$$

where the definitions of the left- and right-handed couplings ℓ and r depend on the F boson

$$W^- \;:\; \ell_{f_2 f_1} = \frac{1}{\sqrt{2}\,\sin\theta_W}\,(\sigma_+)_{f_2 f_1}\, V_{f_2 f_1}\,, \quad r_{f_2 f_1} = 0\,, \tag{2.14}$$

$$W^+ \;:\; \ell_{f_2 f_1} = \frac{1}{\sqrt{2}\,\sin\theta_W}\,(\sigma_-)_{f_2 f_1}\, V^\dagger_{f_2 f_1}\,, \quad r_{f_2 f_1} = 0\,, \tag{2.15}$$

$$Z \;:\; \ell_{f_2 f_1} = \frac{1}{\sin 2\theta_W}\,(\sigma_3)_{f_2 f_2} - \delta_{f_2 f_1}\, e_{f_1} \tan\theta_W\,, \quad r_{f_2 f_1} = -\delta_{f_2 f_1}\, e_{f_1} \tan\theta_W\,, \tag{2.16}$$

[1] For more details, see Appendix A of Ref. [38].

$$\gamma^* \; : \; \ell_{f_2 f_1} = r_{f_2 f_1} = \delta_{f_2 f_1}\, e_{f_1}\,, \tag{2.17}$$

where θ_W is the Weinberg angle, e_f is the fractional electric charge of the quark with flavour f, $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$ and σ_3 are the weak isospin Pauli matrices and V is the unitary Cabibbo–Kobayashi–Maskawa mixing matrix. In addition, in the following we abbreviate $\ell_{f_2 f_1}$ to ℓ_{21}, and the same for $r_{f_2 f_1}$.

The QCD NLO corrections to Eq. (2.9) were computed in Ref. [40]. We report here Eq. (2.12) of this reference, since we are going to use their results in $d = 4$ space-time dimensions, after correcting for some known typos[2]

$$\begin{aligned} E_q \frac{d\hat\sigma_{qg}}{d^3 q} = \frac{1}{s}\, \frac{C_{\rm F}}{N_c^2-1}\, \alpha\, \alpha_{\rm S}(\mu_{\rm R}) \Bigg\{ & \delta(s_2)\, A^{qg}(s,t,u) \\ & \times \sum_f \left(|\ell_{f1}|^2 + |r_{f1}|^2 \right) \\ & + \frac{\alpha_{\rm S}(\mu_{\rm R})}{2\pi} \Bigg\{ \Bigg[\delta(s_2) \Big(B_1^{qg}(s,t,u) \\ & + n_{\rm f}\, B_2^{qg}(s,t,u) + C_1^{qg}(s,t,u) + C_2^{qg}(s,t,u) \Big) \\ & + C_3^{qg}(s,t,u,s_2) \Bigg] \sum_f \left(|\ell_{f1}|^2 + |r_{f1}|^2 \right) \\ & + \delta(s_2)\, B_3^{qg}(s,t,u)\, (\ell_{11} - r_{11}) \sum_f \left(\ell_{ff} - r_{ff} \right) \Bigg\}\Bigg\}, \end{aligned} \tag{2.18}$$

where E_q is the energy of the F boson, $N_c = 3$ is the number of colours and $C_{\rm F} = (N_c^2 - 1)/(2N_c) = 4/3$. The functions A^{qg}, B_i^{qg}, C_i^{qg} $(i = 1, 2, 3)$ are defined in Eqs. (A4)–(A6), (A10)–(A12) of Ref. [40]. A^{qg} is the contribution at tree level of the process in Eq. (2.10). The functions B_i^{qg} receive contributions from the interference of the one-loop virtual corrections to Eq. (2.10), with the tree-level contribution. In particular, B_2^{qg} originates from the renormalisation counterterm, while B_3^{qg} is the contribution from the virtual diagrams with a triangular quark loop, which are present only for Z/γ^* production. These contributions are then multiplied by a $\delta(s_2)$ term, since the system recoiling against the F boson only comprises a single quark with momentum k, so that $s_2 = k^2 = 0$.

The functions C_i^{qg} originate from the diagrams contributing to the real corrections. In particular, C_1^{qg} and C_2^{qg} are the coefficient of a $\delta(s_2)$ term, leftovers of the subtraction method when dealing with initial- and final-state radiation. C_3^{qg} contributes instead for non-zero values of s_2, and corresponds to the double-real radiation contribution to qg-initiated F boson production. In the following we neglect all the infrared divergences appearing as poles in Eqs. (A4)–(A6), (A10)–(A12) of Ref. [40], since they cancel out when summing real and virtual contributions at this order in $\alpha_{\rm S}$.

The B_i^{qg} and C_i^{qg} are analytic functions of the kinematic invariants and contain logarithmic and dilogarithmic functions.

In this paper we present results for the calculation of the power corrections for all the terms proportional to $\delta(s_2)$ in Eq. (2.18), i.e. the virtual-correction terms and terms from the regularisation of the double-real radiation contributions.

Since the kinematics of these terms is equivalent to the one discussed in Ref. [38], we follow the same procedure described in its Appendix A (in particular Eqs. (A.16)–(A.20)), and we integrate all the terms proportional to $\delta(s_2)$ in Eq. (2.18), writing them in the form suitable to be inserted in Eq. (2.8), i.e.

$$\begin{aligned} \left.\frac{d\hat\sigma_{qg}(q_{\rm T}, z)}{dq_{\rm T}^2}\right|_{\delta(s_2)} = \frac{1}{16\pi}\, \frac{z^2}{Q^4}\, & \frac{1}{\sqrt{(1-z)^2 - 4z\, \dfrac{q_{\rm T}^2}{Q^2}}} \\ & \times \left[|\mathcal{M}(z, t_+, q_{\rm T})|^2 + |\mathcal{M}(z, t_-, q_{\rm T})|^2 \right], \end{aligned} \tag{2.19}$$

where $\mathcal{M}(s,t,u)$ is the sum of the functions A^{qg}, B_1^{qg}, B_2^{qg}, B_3^{qg}, C_1^{qg}, C_2^{qg}, as they appear in Eq. (2.18), together with the global factor in front, evaluated at

$$u = Q^2 - s - t\,, \quad s = \frac{Q^2}{z}\,, \quad t = t_\pm\,, \tag{2.20}$$

where

$$t_\pm = \frac{Q^2}{2z} \left[z - 1 \pm \sqrt{(1-z)^2 - 4z\, \frac{q_{\rm T}^2}{Q^2}} \right], \tag{2.21}$$

so that $\mathcal{M}$ becomes a function of z and $q_{\rm T}$, for a given vector-boson virtuality Q^2.

We can write Eq. (2.18), manipulated according to the previous steps, in a compact notation as

$$\begin{aligned} \frac{d\hat\sigma_{qg}(q_{\rm T}, z)}{dq_{\rm T}^2} = \frac{\alpha_{\rm S}}{2\pi}\, & \frac{d\hat\sigma_{qg}^{(1)}(q_{\rm T}, z)}{dq_{\rm T}^2} \\ & + \left(\frac{\alpha_{\rm S}}{2\pi}\right)^2 \frac{d\hat\sigma_{qg}^{(2)}(q_{\rm T}, z)}{dq_{\rm T}^2}, \end{aligned} \tag{2.22}$$

where the superscript $^{(1)}$ denotes the tree-level cross section, while the superscript $^{(2)}$ the virtual and real contributions. The choice is made in order to make contact with the labeling of the transverse-momentum resummation coefficients, that refer to F production as the zeroth term, to its NLO corrections as the first term, and to the NNLO corrections, i.e. the QCD NLO corrections to F + 1 parton, as the second one.

In the rest of the paper we focus on the contribution

$$\left.\frac{d\hat\sigma_{qg}^{(2)}(q_{\rm T}, z)}{dq_{\rm T}^2}\right|_{\delta(s_2)} \tag{2.23}$$

[2] See footnote § of Ref. [41].

and, with a little abuse of notation, when referring to Eq. (2.23), we sometimes drop the $\big|_{\delta(s_2)}$, to ease the notation.

3 Description of the calculation

In order to compute the power corrections of the cross section in Eq. (2.23), we follow the path along which we proceeded in Ref. [38] and which is described in Sect. 3 therein.

We recall here that, in the phase-space region where q_T is different from zero and much smaller than the invariant mass of the colour singlet, the cross section of Eq. (2.23) is characterised by a well-known perturbative structure. In fact, it contains logarithmically-enhanced terms that are singular in the $q_T \to 0$ limit [42–51], terms that are finite in the same limit, and power terms that vanish in the small-q_T limit.

It is customary in the literature [24,52] to compute the following cumulative partonic cross section, integrating the differential cross section in the range $0 \le q_T \le q_T^{\rm cut}$, in order to derive the perturbative behaviour of these terms at small q_T

$$\hat{\sigma}^{<}_{ab}(z) \equiv \int_0^{(q_T^{\rm cut})^2} dq_T^2\, \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}\,. \tag{3.1}$$

For $F + 1$ parton production at NLO, Eq. (3.1) receives contributions from the Born diagrams, that were analysed in Ref. [38], and from the virtual and real QCD corrections. The former are proportional to $\delta(s_2)$, while the latter describe the production of a further parton. Since the total NNLO partonic cross section for F production is finite, following what was done in Refs. [41,53], we compute the above integral as a difference

$$\hat{\sigma}^{<}_{ab}(z) = \hat{\sigma}^{\rm tot}_{ab}(z) - \hat{\sigma}^{>}_{ab}(z)\,, \tag{3.2}$$

with

$$\hat{\sigma}^{\rm tot}_{ab}(z) = \int_0^{(q_T^{\rm max})^2} dq_T^2\, \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}\,,$$
$$\hat{\sigma}^{>}_{ab}(z) = \int_{(q_T^{\rm cut})^2}^{(q_T^{\rm max})^2} dq_T^2\, \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}\,, \tag{3.3}$$

where $q_T^{\rm max}$ is the maximum transverse momentum allowed by the kinematics, $\hat{\sigma}^{\rm tot}_{ab}(z)$ is the total partonic cross section, and $\hat{\sigma}^{>}_{ab}(z)$ is the partonic cross section integrated above $q_T^{\rm cut}$, that can be then computed in four space-time dimensions.

In this paper we study the $q_T^{\rm cut} \ll Q$ behaviour of the NNLO real-virtual contribution to F production, by computing the $q_T^{\rm cut}$-expansion of

$$\hat{\sigma}^{>(2)}_{qg}(z) = \int_{(q_T^{\rm cut})^2}^{(q_T^{\rm max})^2} dq_T^2\, \left.\frac{d\hat{\sigma}^{(2)}_{qg}(q_T, z)}{dq_T^2}\right|_{\delta(s_2)}\,, \tag{3.4}$$

up to $\mathcal{O}\big((q_T^{\rm cut})^2\big)$ included. The integration goes from an arbitrary value $q_T^{\rm cut}$ up to the maximum transverse momentum $q_T^{\rm max}$ allowed by the kinematics of the event, given by

$$\left(q_T^{\rm max}\right)^2 = Q^2\, \frac{(1-z)^2}{4\,z}\,, \tag{3.5}$$

at a fixed value of z.

The integration in q_T is performed with dedicated changes of variables, in order to get rid of the square roots within the arguments of logarithms and dilogarithmic functions. The correct analytic continuation is then performed in order to obtain a real result. At difference with what was done in Ref. [38], we do not quote here the results of the integration, due to their length.

To lighten up the notation, we introduce the dimensionless quantity[3]

$$a \equiv \frac{\left(q_T^{\rm cut}\right)^2}{Q^2}\,, \tag{3.6}$$

that will be the expansion parameter in the rest of the paper. Then, in order to compute the hadronic cross section of Eq. (2.8), we need to integrate the partonic cross sections convoluted with the corresponding luminosities. In the calculation of the total cross sections, the upper limit in the z integration is unrestricted and equal to 1. When a cut on the transverse momentum q_T is applied the z-integration range is instead bound from above, i.e.

$$0 \le z \le z^{\rm max} \equiv 1 - f(a)\,,$$
$$f(a) \equiv 2\sqrt{a}\left(\sqrt{1+a} - \sqrt{a}\right). \tag{3.7}$$

However, in order to make contact with the transverse-momentum subtraction formulae, which allows us to recover the logarithmic-enhanced behaviour along with the power corrections, we need to extend the integration range of the z variable up to 1 and then expand our results in powers of a. To this aim, we used the same procedure presented in Ref. [38] in order to deal with the divergent terms in the $z \to 1$ limit. The procedure is very technical and all the details are presented in Appendix B of the same reference. Hence we refer the interested reader to that appendix for the description of the method.

4 Results

In this section we collect fully-analytic results for the NNLO power corrections in the transverse-momentum cutoff, up to order a. The results refer to the $\delta(s_2)$ contributions of the qg-initiated channel in Eq. (2.18).

[3] In the literature, the parameter a is also referred to as $r^2_{\rm cut}$ (see e.g. [8]).

We label the different contributions of Eq. (2.18) with the letter K, so that

$$K = \left\{A^{qg},\ B_1^{qg},\ B_2^{qg},\ B_3^{qg},\ C_1^{qg},\ C_2^{qg}\right\}. \tag{4.1}$$

Using Eq. (2.19) and following the discussion in Sect. 2.2, we integrate $K(s,t,u)$ in t to obtain a function of the transverse momentum of the vector boson, q_{T}, and z

$$\int dt\, K(s,t,u) = K(q_{\mathrm{T}}, z)\,. \tag{4.2}$$

As recalled in Sect. 3, the functions K are then integrated in q_{T} from an arbitrary value, $q_{\mathrm{T}}^{\mathrm{cut}}$, up to the maximum transverse momentum $q_{\mathrm{T}}^{\max}$ allowed by the kinematics of the event.

We further split the contributions of B_1^{qg} and C_1^{qg} according to their colour factor, C_{A} and C_{F}. We then introduce a further index, $c = \{C_{\mathrm{A}}, C_{\mathrm{F}}\}$, relevant for B_1^{qg} and C_1^{qg}, in order to distinguish the coefficients of the different colour factors.

The general procedure described in Appendix B of Ref. [38] is applied to the q_{T}-integrated K functions. In order to present the structure of the results, we refer to the definitions of I, $\tilde{I}_1$, $\tilde{I}_2$ and $\tilde{I}_3$ of Eqs. (B.8)–(B.11) in Ref. [38]. Moreover, we present the results for the sum $\tilde{I}_{23} \equiv \tilde{I}_2 + \tilde{I}_3$, and we do not give the two terms separately.

After dropping the qg superscript for ease of notation, we can then write

$$\begin{aligned} I^H &= \tilde{I}_1^H + \tilde{I}_{23}^H\,,\\ H &= \{A,\ B_1,\ B_2,\ B_3,\ C_1,\ C_2\}\,, \end{aligned} \tag{4.3}$$

where, if $H = \{B_1,\ C_1\}$,

$$\begin{aligned} \tilde{I}_1^H = \sum_{c=\{C_{\mathrm{A}}, C_{\mathrm{F}}\}} c \Bigg\{ &\int_0^1 dz\, l(z)\, {}^c g_0^H(z) \\ &+ \int_0^1 dz\, l(z) \left[\frac{{}^c g_1^H(z)}{1-z}\right]_+ \\ &+ \int_0^1 dz\, l(z) \left[\frac{{}^c g_2^H(z)}{1-z}\right]_{++} \Bigg\}, \end{aligned} \tag{4.4}$$

$$\tilde{I}_{23}^H = \sum_{c=\{C_{\mathrm{A}}, C_{\mathrm{F}}\}} c\, {}^c\mathcal{J}_{23}^H\,, \tag{4.5}$$

while, if $H = \{A,\ B_2,\ B_3,\ C_2\}$,

$$\begin{aligned} \tilde{I}_1^H = &\int_0^1 dz\, l(z)\, g_0^H(z) + \int_0^1 dz\, l(z) \left[\frac{g_1^H(z)}{1-z}\right]_+ \\ &+ \int_0^1 dz\, l(z) \left[\frac{g_2^H(z)}{1-z}\right]_{++}, \end{aligned} \tag{4.6}$$

$$\tilde{I}_{23}^H = \mathcal{J}_{23}^H\,, \tag{4.7}$$

where $l(z)$ is given in terms of the parton luminosity in Eq. (2.7), where we have dropped any subscript for ease of notation

$$l(z) \equiv \frac{1}{z}\, \mathcal{L}\left(\frac{\tau}{z}\right). \tag{4.8}$$

The functions ${}^c g_0^H(z)$, ${}^c g_1^H(z)$, ${}^c g_2^H(z)$, $g_0^H(z)$, $g_1^H(z)$, $g_2^H(z)$, ${}^c\mathcal{J}_{23}^H$ and $\mathcal{J}_{23}^H$ are the main results of this paper and are collected in Appendix B.

4.1 Technical details

We have written dedicated Mathematica parallel codes in order to apply the whole method to the different contributions. As already pointed out in Ref. [38], the hardest integrals are those to compute $\tilde{I}_2$, which requires the calculation of exact integrals in z, between 0 e $1 - f(a)$, where $f(a)$ is defined in Eq. (3.7).

The integrand functions have been classified into five groups, according to the number of logarithmic and polylogarithmic functions that appear at the integrand level. A sample of these integrals is collected in Appendix A. We have integrated $\mathcal{O}(800)$ integrals in order to compute the expressions in Eqs. (B.10) and (B.11) of Ref. [38], for all the contributions in Eq. (4.3). In general, the integrals require dedicated changes of variables and iterated integrations by parts, peculiarly for the ones involving polylogarithms and logarithms to the third power, that turned out to be the most difficult ones.

4.2 Comments

Due to the length of the intermediate results, in Appendix B we report only the final results, i.e. the functions ${}^c g_0^H(z)$, ${}^c g_1^H(z)$, ${}^c g_2^H(z)$, $g_0^H(z)$, $g_1^H(z)$, $g_2^H(z)$, ${}^c\mathcal{J}_{23}^H$ and $\mathcal{J}_{23}^H$ that appear in Eqs. (4.3)–(4.7).[4]

In agreement with what is found in Ref. [38], no odd-power corrections of $q_{\mathrm{T}}^{\mathrm{cut}}/Q = \sqrt{a}$ appear, i.e. the power expansion of the real-virtual interference terms for F production in the qg channel is in $(q_{\mathrm{T}}^{\mathrm{cut}})^2$.

In addition, we can define the $\hat{G}_{qg}^{(2)}(z)\Big|_{\delta(s_2)}$ function, starting from the integral of the cumulative cross section in Eq. (3.4), as

$$\begin{aligned} \sigma_{qg}^{>(2)}\Big|_{\delta(s_2)} &= \tau \int_\tau^{1-f(a)} \frac{dz}{z}\, \mathcal{L}_{qg}\left(\frac{\tau}{z}\right) \frac{1}{z} \sigma_{qg}^{>(2)}(z)\Big|_{\delta(s_2)} \\ &\equiv \tau \int_\tau^1 \frac{dz}{z}\, \mathcal{L}_{qg}\left(\frac{\tau}{z}\right) \hat{\sigma}^{(0)}\, \hat{G}_{qg}^{(2)}(z)\Big|_{\delta(s_2)}\,, \end{aligned} \tag{4.9}$$

[4] The intermediate results are available upon request to the authors.

and, from the structure of the power corrections we have computed in this paper, the general form of this function is given by[5]

$$\hat{G}^{(2)}_{qg}(z)\Big|_{\delta(s_2)} = \log^3(a)\,\hat{G}^{(2,3,0)}_{qg}(z) + \log^2(a)\,\hat{G}^{(2,2,0)}_{qg}(z) + \log(a)\,\hat{G}^{(2,1,0)}_{qg}(z) + \hat{G}^{(2,0,0)}_{qg}(z) + a\log^2(a)\,\hat{G}^{(2,2,2)}_{qg}(z) + a\log(a)\,\hat{G}^{(2,1,2)}_{qg}(z) + a\,\hat{G}^{(2,0,2)}_{qg}(z) + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right), \quad (4.10)$$

all the other coefficients being zero.

This also agrees with the calculation done in Ref. [20], although the observable is different. In fact, analytic results are therein obtained for the dominant $\alpha_S\,\tau\log(\tau)$ and $\alpha_S^2\,\tau\log^3(\tau)$ subleading terms for 0-jettiness (τ) for $q\bar{q}$-initiated Drell–Yan-like processes.

We do not expect this behaviour to be true in general when cuts are applied to the final-state boson. This was verified in Refs. [29,30], both for transverse momentum and N-jettiness. In fact, power corrections proportional to $\sqrt{a}$ and $\sqrt{\tau}$ are found therein.

5 Conclusions

In this paper we considered the production of a vector boson F ($Z, W^\pm, \gamma^*$) at next-to-next-to-leading order in the strong coupling constant α_S. We imposed a transverse-momentum cutoff, $q_T^{\rm cut}$, on the vector boson and we computed, up to the second power of $q_T^{\rm cut}$, the power corrections for the qg-initiated real-virtual contributions to the cumulative cross section, and for other contributions from double-real radiation, leftover of the subtraction scheme, having the same kinematics, i.e. $F + 1$ parton.

Although we studied Drell–Yan-type F boson production, the procedure we followed is general and can be applied to other similar cases, up to any order in the powers of $q_T^{\rm cut}$, as illustrated in our previous paper [38].

We presented analytic results for the power corrections in $q_T^{\rm cut}$ and we found that the logarithmic terms in $q_T^{\rm cut}$ show up at most to the third power in the power-correction contributions, as expected, and that no odd-power corrections in $q_T^{\rm cut}$ appear. This is in agreement with known results in the literature at a lower order in α_S, i.e. next-to-leading, and with what we found in Ref. [38] where we computed the power corrections at next-to-leading order up to $(q_T^{\rm cut})^4$. We do not expect this to be true in general when cuts are applied to the final state.

The knowledge of the power terms is crucial for understanding both the non-trivial behaviour of cross sections in the infrared limit, and the resummation structure at subleading orders. In addition, within the q_T-subtraction method, the knowledge of the power terms helps in reducing the cutoff dependence of the cross sections.

The result presented in this paper is the first step towards the full calculation of the power corrections of vector-boson production at NNLO. Work is ongoing to compute these corrections for the qq-initiated real-virtual contributions and for the double-real radiation contributions too.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Our paper is a theoretical study and there are no experimental data associated with it. All the results computed during this study are already contained in the article itself.]

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A Samples of integrals

According to the procedure first presented in Ref. [38], in order to compute the power corrections, one has to perform two integrations of the differential cross sections: an integration in q_T, in general easy to perform, and an integration in z from 0 to $1 - f(a)$. This second integration turned out to be challenging for some integrand functions.

We have classified the integrand functions into five groups, according to the number of logarithms and polylogarithms appearing in the expressions. We present here a sample of integrands for each group.

A.1 Integrand classification

Defining

$$r(z) \equiv \sqrt{(1-z)^2 - 4az}\,, \quad (A.1)$$

we have groups containing:

[5] The notation for the expansion of $\hat{G}^{(2)}_{ab}(z)\Big|_{\delta(s_2)}$ follows from the number of powers of α_S, $\log(a)$ and $a^{\frac{1}{2}}$, according to

$$\hat{G}^{(2)}_{ab}(z)\Big|_{\delta(s_2)} = \sum_{m,r} \log^m(a)\,\left(a^{\frac{1}{2}}\right)^r \hat{G}^{(2,m,r)}_{ab}(z)\,.$$

1. one logarithm:

$$\int_0^{1-f(a)} dz\, z^n \log\left[(1-z)\,\frac{1+z\pm r(z)}{1-z\pm r(z)}\right] \tag{A.2}$$

$$\int_0^{1-f(a)} dz\, z^n \frac{r(z)}{1+z\pm r(z)} \log\left[\frac{2z}{1-z\pm r(z)}\right] \tag{A.3}$$

2. two logarithms:

$$\int_0^{1-f(a)} dz\, z^n\, r(z) \log(z) \log\left[\frac{1-z-r(z)}{2(1-z)}\right] \tag{A.4}$$

$$\int_0^{1-f(a)} dz\, z^n\, r(z) \log^2\left[\frac{2z}{1-z\pm r(z)}\right] \tag{A.5}$$

3. three logarithms:

$$\int_0^{1-f(a)} dz\, z^n \log^2\left[\frac{1-z-r(z)}{1-z+r(z)}\right] \times \log\left[(1-z)\frac{1+z+r(z)}{1-z+r(z)}\right] \tag{A.6}$$

$$\int_0^{1-f(a)} dz\, z^n \log(z) \log^2\left[\frac{1-z\pm r(z)}{2(1-z)}\right] \tag{A.7}$$

$$\int_0^{1-f(a)} dz\, z^n \log^3\left[\frac{1\mp z\pm r(z)}{2z}\right] \tag{A.8}$$

$$\int_0^{1-f(a)} dz\, z^n \log(z) \times \log\frac{1-z\pm r(z)}{2\,(1-z)} \log\frac{1-z\mp r(z)}{2z} \tag{A.9}$$

4. one polylogarithm of order 2:

$$\int_0^{1-f(a)} dz\, z^n\, r(z)\, \mathrm{Li}_2\left[\frac{2z}{1+z\pm r(z)}\right] \tag{A.10}$$

$$\int_0^{1-f(a)} dz\, z^n \log\left[\frac{1-z\pm r(z)}{1-z\mp r(z)}\right] \times \mathrm{Li}_2\left[-z\,\frac{1-z\pm r(z)}{1-z\mp r(z)}\right] \tag{A.11}$$

5. one polylogarithm of order 3:

$$\int_0^{1-f(a)} dz\, z^n\, \mathrm{Li}_3\left[\frac{2z}{1+z\pm r(z)}\right] \tag{A.12}$$

$$\int_0^{1-f(a)} dz\, z^n\, \mathrm{Li}_3\left[-z\,\frac{1-z\pm r(z)}{1-z\mp r(z)}\right] \tag{A.13}$$

where $n = 1, \ldots, 4$.

A.2 Sample of integral expansion

After the z integration, the results are functions of a only, and have to be expanded around $a = 0$. A sample of these expansions is given in the following:

– **Example 1**

$$\begin{aligned}\int_0^{1-f(a)} & dz\, z \log^2\left[\frac{1-z-r(z)}{1-z+r(z)}\right] \times \log\left[(1-z)\frac{1+z+r(z)}{1-z+r(z)}\right] \\ &= \frac{1}{2}\, a \log^3(a) + 2\, a \log^2(a) + \left(\frac{17}{2}+\pi^2\right) a \log(a) \\ &+ \left[48 - 32\, C - 16 \log 2\right] \sqrt{a} \\ &+ \left[9\,\zeta(3) - \frac{15}{4} + \frac{4}{3}\pi^2 + 8 \log 2\right] a + \mathcal{O}\left(a^{\frac{3}{2}}\right),\end{aligned} \tag{A.14}$$

where C is the Catalan constant defined by

$$C = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots \approx 0.915965594\ldots \tag{A.15}$$

– **Example 2**

$$\begin{aligned}\int_0^{1-f(a)} & dz\, z^3 \log^2\left[\frac{1-z-r(z)}{1-z+r(z)}\right] \times \log\left[(1-z)\frac{1+z+r(z)}{1-z+r(z)}\right] \\ &= \frac{5}{6} a \log^3(a) + \frac{71}{12}\, a \log^2(a) + \left(\frac{1721}{72} + \frac{5}{3}\pi^2\right) a \log(a) \\ &+ \left[48 - 32\, C - 16 \log 2\right] \sqrt{a} \\ &+ \left[16\,\zeta(3) + \frac{8711}{864} + \frac{71}{18}\pi^2 + 16 \log 2\right] a \\ &+ \mathcal{O}\left(a^{\frac{3}{2}}\right),\end{aligned} \tag{A.16}$$

– **Example 3**

$$\int_0^{1-f(a)} dz\, z\, r(z) \log(z) \log\left[\frac{1-z-r(z)}{2(1-z)}\right]$$

$$= -\frac{5}{36}\log(a) + \frac{\pi^2}{18} - \frac{55}{108} + \left(\frac{\pi^2}{3} - \frac{5}{2}\right) a \log(a) + \left(\frac{5}{6}\pi^2 - \frac{25}{4}\right) a + \mathcal{O}\left(a^{\frac{3}{2}}\right), \quad \text{(A.17)}$$

– **Example 4**

$$\int_0^{1-f(a)} dz\, z \log(z) \log\left[\frac{1-z+r(z)}{2(1-z)}\right] = a\left[1 - \frac{\pi^2}{12} + 2\log^2 2 - 2\log 2\right] + \mathcal{O}\left(a^{\frac{3}{2}}\right), \quad \text{(A.18)}$$

– **Example 5**

$$\int_0^{1-f(a)} dz\, z \log(z) \log^2\left[\frac{1-z-r(z)}{2(1-z)}\right] = \frac{1}{2} a \log^2(a) - \frac{\log^2(a)}{4} + a\left(2 + \frac{3}{4}\pi^2 + 2\log^2 2 - 2\log 2\right) + \left(\frac{2}{3}\pi^2 - 4\right) a \log(a) + \left(\frac{\pi^2}{3} - \frac{7}{2}\right)\log(a) + \frac{2}{3}\pi^2 - \frac{59}{8} + \mathcal{O}\left(a^{\frac{3}{2}}\right). \quad \text{(A.19)}$$

We note that the intermediate integrals contain log(2) and $\sqrt{a}$ terms, and also terms proportional to the Catalan constant C. Despite this, once recombined to compose the whole behaviour of the physical cross section, all these terms disappear from the final answer, as illustrated in Appendix B. Something similar happened for the results at NLO we presented in Ref. [38].

B Final results

In this appendix we collect the results for the NNLO power corrections, up to order a in the transverse-momentum cutoff. The results refer to the $\delta(s_2)$ contribution of the qg-initiated channel to the inclusive cross section for the production of a vector boson F, i.e. the ${}^c g_0^H(z)$, ${}^c g_1^H(z)$, ${}^c g_2^H(z)$, $g_0^H(z)$, $g_1^H(z)$, $g_2^H(z)$, ${}^c\mathcal{J}_{23}^H$ and $\mathcal{J}_{23}^H$ functions in Eqs. (4.4)–(4.7).

In the following, we need $l(z)$, defined in Eq. (4.8), and its first derivative

$$l^{(1)}(z) \equiv \frac{d}{dz} l(z) = -\frac{1}{z^2}\mathcal{L}\left(\frac{\tau}{z}\right) - \frac{\tau}{z^3}\mathcal{L}^{(1)}\left(\frac{\tau}{z}\right), \quad \text{(B.1)}$$

both evaluated in $z = 1$. For sake of brevity, we introduce the following notation

$$\mathcal{L} \equiv l(1) = \mathcal{L}(\tau), \quad \text{(B.2)}$$

$$\mathcal{L}' \equiv l^{(1)}(1) = -\mathcal{L}(\tau) - \tau\mathcal{L}^{(1)}(\tau). \quad \text{(B.3)}$$

The renormalisation and factorisation scales are indicated with μ_R and μ_F, respectively, and $p_{qg}(z)$ is the zeroth-order Altarelli–Parisi splitting function, defined as

$$P_{qg}(z) = T_R\left[2z^2 - 2z + 1\right] \equiv T_R\, p_{qg}(z). \quad \text{(B.4)}$$

In addition, we recall the definition of a in Eq. (3.6): $a = \left(q_T^{\text{cut}}\right)^2 / Q^2$.

B.1 A^{qg}

$$g_0^A(z) = p_{qg}(z)\left[-\log(a) + \log\frac{(1-z)^2}{z}\right] + \frac{1}{2}(1+3z)(1-z) + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.5)}$$

$$g_1^A(z) = -z(1+3z)\, a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.6)}$$

$$g_2^A(z) = -2z\, p_{qg}(z)\, a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.7)}$$

$$\mathcal{J}_{23}^A = -\left(\mathcal{L} + \mathcal{L}'\right) a\log(a) - \left(\frac{3}{2}\mathcal{L} + \frac{10}{3}\mathcal{L}'\right) a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.8)}$$

B.2 B_1^{qg}: C_A coefficient

$$\begin{aligned} {}^{C_A}g_0^{B_1}(z) = {} & \frac{1}{6} p_{qg}(z)\log^3(a) - p_{qg}(z)\log(z)\log^2(a) \\ & + \left[z - p_{qg}(z)\left(\log^2\frac{z}{1-z} - 2\log(1-z)\log(z) + \frac{7}{6}\pi^2\right)\right]\log(a) \\ & + p_{qg}(z)\left(\frac{2}{3}\log^3(1-z) - \frac{1}{6}\log^3(z) + \log(1-z)\log^2(z)\right) \\ & - \left(6z^2 - 5z + \frac{5}{2}\right)\log^2(1-z)\log(z) \\ & - \left(\frac{15}{2}z^2 - 5z + 1\right)\log(1-z)\log(z) \\ & + \left(\frac{21}{4}z^2 - \frac{3}{2}z - \frac{1}{4}\right)\log^2(z) \\ & + \left(3z^2 - 4z + 1\right)\log^2(1-z) \\ & + \left(\frac{14}{3}\pi^2 z^2 + \frac{9}{2}z^2 - \frac{9}{2}\pi^2 z - 8z + \frac{9}{4}\pi^2 + \frac{3}{2}\right)\log(1-z) \end{aligned}$$

$$
\begin{aligned}
&-\left(\frac{4}{3}\pi^2 z^2+\frac{3}{2}z^2-\frac{3}{2}\pi^2 z-3z+\frac{3}{4}\pi^2-\frac{1}{2}\right)\log(z)\\
&-\left(\frac{3}{2}z^2+3z-1+\frac{1}{2}(2z-1)\,\log\frac{z}{1-z}\right)\\
&\times\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)-2(2z-1)\,\mathrm{Li}_3(1-z)\\
&-\Big(4z^2-2z+1\Big)\,\mathrm{Li}_3(z)\\
&+\zeta(3)\,\Big(4z^2-2z+1\Big)-\frac{15}{4}\pi^2 z^2\\
&-\frac{13}{4}z^2+\frac{23}{6}\pi^2 z+\frac{9}{2}z-\frac{2}{3}\pi^2-\frac{5}{4}\\
&+\left[\frac{11}{6}\,p_{qg}(z)\left(-\log(a)+\log\frac{(1-z)^2}{z}\right)\right.\\
&\left.+\frac{11}{12}(1-z)(3z+1)\right]\log\frac{\mu_{\mathrm{R}}^2}{Q^2}-2\,(1+z)\,a\log^2(a)\\
&+\left[\frac{39}{8}z-\frac{45}{16}+(11z+10)\,\log(1-z)\right.\\
&\left.-(16z+19)\,\log(z)\right]a\log(a)\\
&+\left[-\left(\frac{13}{8}z^3+9z^2+\frac{27}{16}z-\frac{1}{16}\right)\right.\\
&\times\log(1-z)+\left(\frac{5}{2}z+2\right)\log^2(1-z)\\
&+\left(\frac{13}{8}z^3+9z^2-\frac{63}{8}z-\frac{11}{2}\right)\\
&\times\log(z)-\left(\frac{19}{2}z+5\right)\log^2(z)\\
&-(z+9)\log(1-z)\log(z)+2\,(1+z)\\
&\times\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&\left.+\frac{23}{8}z^2-\frac{165}{16}z+\frac{\pi^2}{3}z-3+\frac{5}{6}\pi^2\right]a\\
&+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.9}
$$

$$
\begin{aligned}
{}^{C_{\mathrm{A}}}g_1^{B_1}(z)=&\left[-2z^3-\frac{1}{2}z^2+z+\frac{1}{2}\right]a\log^2(a)\\
&+\left[\Big(8z^3+12z^2-8z-4\Big)\log(1-z)\right.\\
&+\Big(-4z^3-11z^2+6z+1\Big)\log(z)\\
&\left.+4z^3-\frac{43}{8}z^2-\frac{75}{16}z+\frac{61}{16}\right]a\log(a)\\
&+\left[\left(-8z^3+\frac{1}{2}z^2+\frac{15}{2}z-8\right)\right.\\
&\times\log^2(1-z)+\left(6z^3-10z^2+\frac{11}{2}z\right)\log^2(z)\\
&+\left(12z^3-\frac{23}{2}z^2-2z+15\right)\log(1-z)\log(z)\\
&-(1+z)\left(\frac{13}{8}z^3+\frac{31}{4}z^2\right.\\
&\left.-\frac{177}{16}z+\frac{65}{16}\right)\log(1-z)\\
&+\left(\frac{9}{2}z^2-z-2\right)\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&+\left(\frac{13}{8}z^4+\frac{43}{8}z^3-\frac{29}{8}z^2\right.\\
&\left.-\frac{43}{8}z+\frac{17}{2}\right)\log(z)\\
&+2\pi^2 z^3-\frac{63}{16}z^3-\frac{79}{12}\pi^2 z^2\\
&-\frac{11}{4}z^2-\frac{\pi^2}{2}z+\frac{79}{16}z+\frac{4}{3}\pi^2+4\\
&\left.-\frac{11}{6}z\,(3z+1)\log\frac{\mu_{\mathrm{R}}^2}{Q^2}\right]a+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.10}
$$

$$
\begin{aligned}
{}^{C_{\mathrm{A}}}g_2^{B_1}(z)=&\left[\frac{1}{2}(1-z)(z+1)\Big(4z^2-6z+3\Big)\right]a\log^2(a)\\
&+\left[(z-1)\left(4z^3-\frac{21}{4}z^2-2z+3\right)\right.\\
&+2\Big(4z^4-4z^3-3z^2+7z-3\Big)\log(1-z)\\
&\left.+\Big(-4z^4+10z^3+2z^2-29z+18\Big)\log(z)\right]a\log(a)\\
&+\left[\left(12z^4-25z^3+\frac{43}{2}z^2+2z-6\right)\log(1-z)\log(z)\right.\\
&-2\Big(4z^4-4z^3-3z^2+7z-3\Big)\log^2(1-z)\\
&+\left(6z^4-5z^3-\frac{3}{2}z^2-5z+5\right)\log^2(z)\\
&+\frac{1}{4}\Big(16z^4-11z^3-32z^2+31z-12\Big)\log(z)\\
&+(1-z)\left(8z^3-4z^2-\frac{25}{4}z+6\right)\log(1-z)\\
&+\frac{z^2}{2}(2z-1)\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&+\frac{\pi^2}{12}\Big(24z^4-94z^3+45z^2+24z-26\Big)\\
&+\frac{1}{16}\Big(-109z^4+132z^3+83z^2-122z+48\Big)\\
&\left.-\frac{11}{3}z\,p_{qg}(z)\log\frac{\mu_{\mathrm{R}}^2}{Q^2}\right]a+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.11}
$$

$$
\begin{aligned}
{}^{C_{\mathrm{A}}}\mathcal{J}_{23}^{B_1}=&\frac{1}{6}\Big[\mathcal{L}+\mathcal{L}'\Big]a\log^3(a)\\
&+\frac{1}{4}\left[15\,\mathcal{L}-\frac{11}{3}\mathcal{L}'\right]a\log^2(a)\\
&+\left[\left(-\frac{11}{6}\log\frac{\mu_{\mathrm{R}}^2}{Q^2}-\frac{5}{3}\pi^2-\frac{43}{8}\right)\mathcal{L}\right.\\
&+\left(-\frac{11}{6}\log\frac{\mu_{\mathrm{R}}^2}{Q^2}-\frac{5}{3}\pi^2\right.\\
&\left.\left.+\frac{79}{6}\right)\mathcal{L}'\right]a\log(a)\\
&+\left[\left(-\frac{11}{4}\log\frac{\mu_{\mathrm{R}}^2}{Q^2}-5\zeta(3)-\frac{11}{3}\pi^2+\frac{897}{32}\right)\mathcal{L}\right.\\
&+\left(-\frac{55}{9}\log\frac{\mu_{\mathrm{R}}^2}{Q^2}-5\zeta(3)-\frac{185}{36}\pi^2\right.\\
&\left.\left.+\frac{565}{192}\right)\mathcal{L}'\right]a+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.12}
$$

B.3 $\boldsymbol{B_1^{qg}}$: $\boldsymbol{C_\mathrm{F}}$ coefficient

$$
\begin{aligned}
{}^{C_\mathrm{F}}g_0^{B_1}(z) &= p_{qg}(z)\log\frac{z}{1-z}\log^2(a)\\
&+\Big[16z^2-17z+8+p_{qg}(z)\\
&\times\Big(\frac{11}{6}\pi^2+\mathrm{Li}_2(z)-\mathrm{Li}_2(1-z)\\
&+2\log^2\frac{z}{1-z}+\log^2(1-z)\\
&-\log(1-z)\log(z)\Big)\Big]\log(a)\\
&-2p_{qg}(z)\Big(\log^3(1-z)-\frac{2}{3}\log^3(z)\\
&+2\log(1-z)\log^2(z)\Big)\\
&+\Big(8z^2-6z+3\Big)\log^2(1-z)\log(z)\\
&+\Big(-\frac{9}{2}z^2+7z-\frac{5}{2}\Big)\log^2(1-z)\\
&+\Big(-6z^2+3z-2\Big)\log^2(z)\\
&+\Big(-7\pi^2z^2-42z^2+\frac{20}{3}\pi^2z+48z\\
&-\frac{10}{3}\pi^2-20\Big)\log(1-z)\\
&+\Big(\frac{8}{3}\pi^2z^2+\frac{47}{2}z^2-3\pi^2z\\
&-24z+\frac{3}{2}\pi^2+9\Big)\log(z)\\
&+\Big(\frac{21}{2}z^2-10z+\frac{9}{2}\Big)\log(1-z)\log(z)\\
&+\Big[\frac{3}{2}z^2+4z-\frac{1}{2}+\Big(6z^2-8z+4\Big)\log(1-z)\\
&-\Big(4z^2-6z+3\Big)\log(z)\Big]\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&-8(1-z)^2\,\mathrm{Li}_3(1-z)+2\,(2z-1)\,\mathrm{Li}_3(z)\\
&+\frac{27}{4}\pi^2z^2+\frac{37}{2}z^2-4z\,\zeta(3)-\frac{23}{3}\pi^2z-\frac{33}{2}z\\
&+2\zeta(3)+\frac{7}{4}\pi^2-2+\frac{1}{2}z\,a\log^2(a)\\
&+\Big[-\frac{73}{8}z+\frac{25}{4}\\
&-(21z+20)\,\log(1-z)+(33z+38)\,\log(z)\Big]\\
&\times a\log(a)\\
&+\Big[\Big(\frac{19}{8}z^3+14z^2+3z-2\Big)\\
&\times\log(1-z)-12\,(1+z)\log^2(1-z)\\
&+\Big(-\frac{19}{8}z^3-14z^2+\frac{141}{8}z+\frac{31}{2}\Big)\\
&\times\log(z)+(16z+8)\log^2(z)\\
&+\Big(-\frac{9}{2}z+14\Big)\log(1-z)\log(z)\\
&-\Big(\frac{7}{2}z+4\Big)\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&-\frac{43}{8}z^2+\frac{175}{8}z+\frac{23}{12}\pi^2z\\
&+\frac{47}{8}+\pi^2\Big]a+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.13}
$$

$$
\begin{aligned}
{}^{C_\mathrm{F}}g_1^{B_1}(z) &= \Big[4z^3-\frac{1}{2}z^2-\frac{5}{2}z+3\Big]a\log^2(a)\\
&+\Big[\Big(-16z^3-27z^2+15z+8\Big)\log(1-z)\\
&+\Big(8z^3+19z^2-13z-2\Big)\log(z)\\
&-8z^3+\frac{13}{8}z^2+\frac{69}{8}z-\frac{25}{4}\Big]a\log(a)\\
&+\Big[\Big(16z^3-5z^2-15z+24\Big)\log^2(1-z)\\
&+\Big(-8z^3+19z^2-8z\Big)\log^2(z)\\
&+\Big(-22z^3+\frac{37}{2}z^2+\frac{11}{2}z-26\Big)\\
&\times\log(1-z)\log(z)\\
&+\Big(\frac{19}{8}z^4+\frac{125}{8}z^3\\
&+2z^2-12z+8\Big)\log(1-z)\\
&+\Big(-\frac{19}{8}z^4-\frac{61}{8}z^3+\frac{51}{8}z^2\\
&+\frac{89}{8}z-\frac{37}{2}\Big)\log(z)\\
&+\Big(2z^3-\frac{25}{2}z^2+\frac{5}{2}z+4\Big)\\
&\times\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&-\frac{13}{3}\pi^2z^3+\frac{125}{16}z^3+\frac{169}{12}\pi^2z^2+\frac{599}{16}z^2\\
&+\frac{5}{4}\pi^2z-\frac{51}{16}z-\frac{17}{3}\pi^2-\frac{55}{8}\Big]a\\
&+\mathcal{O}\Big(a^{\frac{3}{2}}\log(a)\Big)
\end{aligned}
\tag{B.14}
$$

$$
\begin{aligned}
{}^{C_\mathrm{F}}g_2^{B_1}(z) &= \Big(4z^4-4z^3-3z^2+7z-3\Big)a\log^2(a)\\
&+\Big[4\Big(2z^4-6z^3+14z-9\Big)\log(z)\\
&+\frac{1}{4}(1-z)\Big(32z^3-35z^2-13z+24\Big)\\
&-2\Big(8z^4-6z^3-8z^2+15z-6\Big)\\
&\times\log(1-z)\Big]a\log(a)+\Big[2z\Big(z^3-4z^2+3z-1\Big)\\
&\times\Big(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\Big)\\
&+2\Big(8z^4-6z^3-8z^2+15z-6\Big)\log^2(1-z)\\
&+\Big(-8z^4+10z^3-z^2+10z-8\Big)\log^2(z)\\
&-2\Big(11z^4-24z^3+21z^2+2z-6\Big)\log(z)\log(1-z)\\
&+4(z-1)(z+1)\Big(4z^2-6z+3\Big)\log(1-z)\\
&+\frac{1}{4}\Big(-32z^4-z^3+74z^2-65z+12\Big)\log(z)
\end{aligned}
$$

$$+\frac{\pi^2}{3}\left(-13z^4+42z^3-15z^2-17z+14\right) + \frac{1}{16}\left(211z^4+208z^3-566z^2+467z-96\right)\Bigg]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.15)}$$

$$^{C_F}\mathcal{J}_{23}^{B_1} = \left[-\frac{3}{4}\mathcal{L}+\frac{7}{2}\mathcal{L}'\right]a\log^2(a) + \left[\left(\frac{8}{3}\pi^2+\frac{429}{16}\right)\mathcal{L}+\left(\frac{8}{3}\pi^2-\frac{821}{72}\right)\mathcal{L}'\right]a\log(a) + \left[\left(8\zeta(3)+\frac{\pi^2}{3}+\frac{4627}{192}\right)\mathcal{L}+\left(8\zeta(3)+\frac{64}{9}\pi^2 + \frac{75667}{1728}\right)\mathcal{L}'\right]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.16)}$$

B.4 B_2^{qg}

$$g_i^{B_2}(z) = -\frac{1}{3}\log\frac{\mu_R^2}{Q^2}\; g_i^A(z) \quad i=0,1,2 \quad \text{(B.17)}$$

$$\mathcal{J}_{23}^{B_2} = -\frac{1}{3}\log\frac{\mu_R^2}{Q^2}\;\mathcal{J}_{23}^A \quad \text{(B.18)}$$

B.5 B_3^{qg}

$$g_0^{B_3}(z) = \frac{1}{2}z\log^2(z) + 2z(1-z)\log\frac{z}{1-z} - \frac{1}{2}z\log(1-z)\log(z) - \frac{1}{2}z \times\left(\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\right) - \frac{1}{12}\pi^2 z - z + 1 + \Big[(1+z) \times\log(1-z)\Big]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.19)}$$

$$g_1^{B_3}(z) = z\,a\log(a) + \Big[(z-1)\left(2z^2+2z+1\right) \times\log(1-z) - 2z^3\log(z) - z\Big]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.20)}$$

$$g_2^{B_3}(z) = (1-z)z^2\big[2+\log(z)\big]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.21)}$$

$$\mathcal{J}_{23}^{B_3} = -\frac{1}{2}\mathcal{L}\,a\log^2(a) - \mathcal{L}\,a\log(a) + \left[\left(\frac{\pi^2}{6}+\frac{1}{4}\right)\mathcal{L} - \frac{5}{9}\mathcal{L}'\right]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.22)}$$

B.6 C_1^{qg}: C_A coefficient

$$^{C_A}g_0^{C_1}(z) = -\frac{1}{6}p_{qg}(z)\log^3(a) + p_{qg}(z)\log(1-z)\log^2(a) - p_{qg}(z)\left[\frac{\pi^2}{6}+2\log^2(1-z)\right]\log(a) + \frac{1}{3}p_{qg}(z)\left(4\log^3(1-z)+\pi^2 \times\log\frac{1-z}{z} - \frac{1}{2}\log^3(z)\right) + \left(-\frac{3}{4}z^2+\frac{1}{2}z+\frac{1}{4}\right) \times\log^2(z) + \left(-\frac{1}{2}z^2+z-\frac{1}{2}\right)\log(z) + \frac{\pi^2}{6}\left(1+2z-3z^2\right) - \left[\frac{3}{2}z+\frac{3}{2}+z\log(z)\right]a\log(a) + \Big[2z\log(1-z)\log(z) - z\log^2(z) - \left(z+\frac{3}{2}\right)\log(z)\Big]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.23)}$$

$$^{C_A}g_1^{C_1}(z) = -\frac{z}{2}(3z+1)\,a\log^2(a) + \left[z(3z+1)\left(\log\frac{(1-z)^2}{z}+\frac{1}{2}\right)+\frac{3}{2}\right]a\log(a) + \Bigg[2z(3z+1)\left(\log(1-z)\log(z)-\log^2(1-z) - \frac{1}{2}\log^2(z)\right) - 3z\left(2z+\frac{1}{2}\right)\log(1-z) + \left(\frac{3}{2}z^2+z+\frac{3}{2}\right)\log(z) - \frac{\pi^2}{6}z(3z+1)\Bigg]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.24)}$$

$$^{C_A}g_2^{C_1}(z) = -z\,p_{qg}(z)\,a\log^2(a) + \left[2z\log\frac{(1-z)^2}{z}\,p_{qg}(z) + \frac{z}{2}(1-z)\right]a\log(a) + \Bigg[4z\left(\log(1-z)\log(z)-\log^2(1-z) - \frac{1}{2}\log^2(z)\right)p_{qg}(z) - \frac{z}{2}(1-z)\log(1-z) + \frac{1}{2}z\left(9z^2-10z+5\right)\log(z) - \frac{\pi^2}{3}z\,p_{qg}(z) + z\left(3z^2-2z-1\right)\Bigg]a + \mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \quad \text{(B.25)}$$

$$^{C_A}\mathcal{J}_{23}^{C_1} = -\frac{1}{6}\left[\mathcal{L}+\mathcal{L}'\right]a\log^3(a) - \left[\frac{3}{4}\mathcal{L}+\frac{5}{3}\mathcal{L}'\right]a\log^2(a)$$

$$
-\left[\left(\frac{\pi^2}{2}+\frac{7}{2}\right)\mathcal{L}
+\left(\frac{\pi^2}{2}+\frac{241}{36}\right)\mathcal{L}^{'}\right] a\log(a)
-\left[\left(\frac{\pi^2}{12}+12\right)\mathcal{L}
+\left(\frac{4}{3}\pi^2+\frac{1391}{108}\right)\mathcal{L}^{'}\right] a
+\mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \tag{B.26}
$$

B.7 C_1^{qg}: C_F coefficient

$$
{}^{C_F}g_0^{C_1}(z) = -\frac{1}{3}\,p_{qg}(z)\log^3(a)
+p_{qg}(z)\left[\log(1-z)+\frac{3}{4}\right]\log^2(a)
-p_{qg}(z)\left[\log^2(1-z)
+\frac{3}{2}\log(1-z)+\frac{7}{2}\right]\log(a)
+p_{qg}(z)\left(\frac{2}{3}\log^3(1-z)
-\log(1-z)\log(z)\log\frac{1-z}{z}-\frac{1}{3}\log^3(z)\right)
+\left(-\frac{3}{2}z^2+z+\frac{1}{2}\right)\log^2(1-z)
+\left(-3z^2+\frac{5}{2}z-\frac{1}{4}\right)\log^2(z)
+\left(6z^2-5z+\frac{1}{2}\right)\log(1-z)\log(z)
+\left(-\frac{51}{4}z^2+\frac{23}{2}z-\frac{9}{4}\right)\log(z)
+\left(\frac{79}{4}z^2-\frac{37}{2}z+\frac{23}{4}\right)\log(1-z)
-\frac{93}{8}z^2+\frac{37}{4}z+\frac{19}{8}
+\frac{1}{2}z\,a\log^2(a)
+\left[-z\log(1-z)-\frac{93}{16}z-\frac{19}{8}\right]a\log(a)
+\left[-\frac{1}{2}z\log^2(z)-z\left(\frac{23}{16}z^2+\frac{37}{8}z
+\frac{31}{16}\right)\log(z)+z\log(1-z)\log(z)
+\left(\frac{23}{16}z^3+\frac{37}{8}z^2+\frac{81}{8}z
+\frac{51}{8}\right)\log(1-z)+\frac{23}{16}z^2+\frac{19}{8}z\right]a
+\mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \tag{B.27}
$$

$$
{}^{C_F}g_1^{C_1}(z) = -\frac{z}{2}(3z+1)\,a\log^2(a)
+\left[z(3z+1)\log(1-z)+\frac{47}{16}z^2
+\frac{59}{16}z+\frac{19}{8}\right]a\log(a)
+\left[-z(3z+1)\left(\log(1-z)\log\frac{1-z}{z}
+\frac{1}{2}\log^2(z)\right)
+\left(\frac{23}{16}z^4+\frac{51}{16}z^3-\frac{9}{4}z-\frac{51}{8}\right)\log(1-z)
-\left(\frac{23}{16}z^4+\frac{51}{16}z^3+\frac{9}{16}z^2-\frac{3}{16}z\right)\log(z)
+\frac{139}{32}z^3-\frac{877}{32}z^2-\frac{105}{32}z\right]a
+\mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \tag{B.28}
$$

$$
{}^{C_F}g_2^{C_1}(z) = -z\,p_{qg}(z)\,a\log^2(a)
+\left[2z\,p_{qg}(z)\log(1-z)
+\frac{1}{4}z\left(19z^2-18z+5\right)\right]a\log(a)
+\left[z\left(2\log(z)\log(1-z)
-2\log^2(1-z)-\log^2(z)\right)p_{qg}(z)
-\frac{1}{4}z\left(19z^2-18z+5\right)\log(z)
+\frac{1}{32}z\left(93z^3-888z^2+878z-307\right)\right]a
+\mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \tag{B.29}
$$

$$
{}^{C_F}\mathcal{J}_{23}^{C_1} = -\frac{1}{3}\left[\mathcal{L}+\mathcal{L}^{'}\right]a\log^3(a)
-\frac{1}{2}\left[\mathcal{L}+\frac{11}{6}\mathcal{L}^{'}\right]a\log^2(a)
-\left[\frac{47}{32}\mathcal{L}+\frac{341}{72}\mathcal{L}^{'}\right]a\log(a)
+\left[\left(-\frac{\pi^2}{3}+\frac{1013}{128}\right)\mathcal{L}
-\frac{1}{36}\left(29\pi^2+\frac{58691}{96}\right)\mathcal{L}^{'}\right]a
+\mathcal{O}\left(a^{\frac{3}{2}}\log(a)\right) \tag{B.30}
$$

B.8 C_2^{qg}

$$g_i^{C_2}(z) = \frac{1}{3}\log\frac{\mu_F^2}{Q^2}\; g_i^A(z) \quad i=0,1,2 \tag{B.31}$$

$$\mathcal{J}_{23}^{C_2} = \frac{1}{3}\log\frac{\mu_F^2}{Q^2}\;\mathcal{J}_{23}^A \tag{B.32}$$

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