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A positional game for an overlapping generation economy

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ABSTRACT

We develop a model with intra-generational consumption externalities, based on the overlapping generation model by Diamond (1965). More specifically, we consider a two-period lived overlapping generation economy, assuming that the utility of each consumer depends also on the average consumption level by the consumers in the same generation. We suppose that such level is not taken as a parameter by agents, who behave strategically. We characterize the consumption and saving choices for the two periods in the Nash equilibrium path and we determine a dynamic equation for capital accumulation. For the associated dynamical system, we find a unique positive steady state for capital and we investigate how its position, as well as that of the steady states for consumption in both periods, change with respect to variations in the degree of interaction in the two periods. We finally compare the steady states for capital with and without social interaction.

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1. Introduction

The aim of the present paper consists in analyzing the role of the social dimension of consumption in an overlapping generation (OLG) model.

Traditional microeconomic accounts of consumption choices tend to characterize consumers in terms of a certain utility function that has to be maximized on the basis of the price structure and of the available budget; the possibility that preferences depend on the consumption choices of others is admitted but generally it is not considered an essential feature of the model. However, especially in the past two decades, there has been an increasing interest in introducing the social dimension of consumption into the core of the state-of-the-art microeconomic theory [11,12]. Introducing this element in the OLG framework amounts to something more than adding another element of dynamic complexity: it is a basic requisite for the consumption model to be realistic enough. In principle, introducing the social dimension of consumption could pave the way to both cooperative and competitive forms of interaction. The former includes, for instance, psychological benefits from the joint cultivation of common interests (e.g. sharing materials, information and emotions), and more generally, the so-called relational goods [21], i.e. goods whose enjoyment is enhanced by the simultaneous participation of others. The latter includes all kinds of positional competition, i.e. situations where the

level of satisfaction deriving from the enjoyment of a given good is determined to some extent by the level of consumption of the same good by individuals belonging to a given social reference group [10]. In this case, the social dimension, rather than being welfare-enhancing, easily becomes welfare destroying and is likely to be conducive to suboptimal over-consumption outcomes that closely replicate the social dynamics of arms races. The circumstances that cause the emergence of a cooperative or competitive social consumption attitude are generally complex and may be regarded as the outcome of a cultural evolution process acting on different motivational orientations [17].

In this paper, we do not want to tackle the general problem of attitude selection, but rather to explore the implications of a given social mode of consumption. Specifically, we will focus here on positional competition. For instance, current forms of post-industrial consumption are very sensitive to the positional dimension [13], which is often invoked as an explicit motivational leverage for prospective buyers of goods and services, especially in the luxury segments of the consumption spectrum. The literature concerning positional consumption is mainly addressed towards static aspects of interdependent preferences (see for instance [9,14,16]), while there are relatively few papers concerning dynamical aspects. In [19,20] it is shown how complex dynamics may arise in a dynamic setting, and in [3,18] the social interaction coefficient is endogenized.

Other approaches can be obtained by changing the kind of reference level of consumption. For instance, De la Croix [7] proposes an OLG model with production in which children inherit life standard aspirations from their parents, that is, the comparison reference level of consumption is given by parents' first period consumption level. Lahiri and Puhakka [15] introduce subtractive habit persistence preferences into the standard pure exchange OLG model. The reference level is here given by the agent's first period life consumption level. Bunzel [6] provides a complete characterization of the stationary and non-stationary monetary equilibria in a two-period pure exchange OLG model with multiplicative habit persistence preferences. Also in this case the reference level is represented by the agent's first period life consumption level.

Using the OLG version of the Diamond model with productive capital in [8], we develop a model with intra-generational consumption externalities. More specifically, we consider a two-period lived OLG economy, assuming that the utility of a consumer in each generation depends not only on his own consumption, but also on the benchmark consumption given by the average level of consumption of the consumers in the same generation. Such feature is common to a few other papers in the macro literature (see e.g. [1,2,4] and, more recently, [5]), that study consumption externalities in simple growth models with OLGs.

However we stress that, differently from the existing literature on the topic, we assume the average level of consumption not to be taken as a parameter by agents. On the contrary, they behave strategically, considering that the others' choices, together with their own choice, will influence their utility function. In this way we obtain a positional game embedded in an OLG economy, which displays strategic complementarity, i.e. each player increases his consumption strategy when the consumption strategies of other players increase.

For simplicity, we assume logarithmic utility functions and a Cobb–Douglas production function. Our formalization crucially relies on two parameters describing the influence of social interaction in the first and the second life periods, i.e. ρ_1 and ρ_2 , respectively. The main goal of the paper consists in analyzing the dynamical system generated by the capital

accumulation equation, in particular as concerns the stability of the positive steady state for capital and its position in dependence of variations in the parameters representing social interaction. We also compare the frameworks with and without social interaction.

The remainder of the paper is organized as follows. In Section 2 we introduce the model and characterize the Nash equilibrium path. Section 3 contains our main results. Section 4 concludes.

2. The model

We consider a two-period lived OLG economy, where in each period only two types of agents are alive, young and old. In their first period of life, when young, agents are endowed with one unit of labor, that they supply inelastically to firms. Their income is equal to the real wage, that they allocate between current consumption and savings, which are invested in the firms. In their second period of life, when old, agents are retired. Their income derives from the return of the savings made in the first life period. Agents are identical within each generation. Population is constant over time and each cohort is composed by N agents. The utility function of the representative agent i born at time t is given by

$$u_t^i = \log(c_{i,t} - \rho_1 \bar{c}_{i,t}^e) + \beta \log(c_{i,t+1} - \rho_2 \bar{c}_{i,t+1}^e) \quad (2.1)$$

where $\rho_1, \rho_2 \in [0, 1)$ are the social interaction coefficients in the first and second life periods, respectively, measuring the extent to which an agent is influenced by interaction; $\beta > 0$ is the given discount factor. We denote by $c_{i,t}$ consumption when agent i is young and by $\bar{c}_{i,t}^e$ the average consumption by all agents at the end of time t , expected by agent i at the beginning of the same period; similarly, $c_{i,t+1}$ denotes consumption when agent i is old and $\bar{c}_{i,t+1}^e$ denotes the average consumption of all agents at the end of time $t + 1$, expected by agent i at the beginning of period t . In symbols, $\bar{c}_{i,t}^e = \sum_{j=1}^N \frac{c_{ij,t}^e}{N} = \sum_{j \neq i} \frac{c_{ij,t}^e}{N} + \frac{c_{i,t}}{N}$, where $c_{ij,t}^e$ denotes the consumption by agent j at the end of time t , expected by agent i at the beginning of the same period; similarly, $\bar{c}_{i,t+1}^e = \sum_{j=1}^N \frac{c_{ij,t+1}^e}{N} = \sum_{j \neq i} \frac{c_{ij,t+1}^e}{N} + \frac{c_{i,t+1}}{N}$, where $c_{ij,t+1}^e$ denotes the consumption by agent j at the end of time $t + 1$, expected by agent i at the beginning of period t . Notice that when $\rho_1 = \rho_2 = 0$ we are in the case of an economy without social interaction as no positional elements enter the utility function, while when ρ_1 and ρ_2 are close to 1 we have a full form of positional consumption, in which utility partly depends on agent's own consumption, and partly from others' consumption. We also remark that the utility function in (2.1) is well-defined because ρ_1, ρ_2 are less than 1 and because, as we shall see in what follows, at the Nash equilibrium the consumption levels of all agents coincide. Indeed, with $\rho_1 = 1$ or $\rho_2 = 1$ the argument of one of the logarithms in the utility function in (2.1) would vanish at the Nash equilibrium.

The budget constraints of consumers in the young and old ages are respectively given by

$$w_t = c_{i,t} + s_{i,t} \quad (2.2)$$

and

$$c_{i,t+1} = (1 + r_{i,t+1}^e)(w_t - c_{i,t}), \quad (2.3)$$

where w_t is the real wage rate, $s_{i,t}$ are agent i savings and $r_{i,t+1}^e$ is the interest rate expected at time t for period $t + 1$. The agent born at the beginning of period t chooses $c_{i,t}$ and $c_{i,t+1}$

in order to maximize u_t^i subject to the lifetime budget constraints (2.2) and (2.3). We stress that, since all agents have identical strategy spaces $(0, w_t) \ni c_{i,t}$, for every t and i , and the payoff functions in (2.1) are symmetric, the decision problem is a symmetric game.

The first order conditions for our maximization problem are

$$5 \quad \frac{1 - \rho_1 \frac{\partial \bar{c}_{i,t}^e}{\partial c_{i,t}}}{c_{i,t} - \rho_1 \bar{c}_{i,t}^e} = \lambda (1 + r_{i,t+1}^e) \quad (2.4)$$

and

$$\beta \frac{1 - \rho_2 \frac{\partial \bar{c}_{i,t+1}^e}{\partial c_{i,t+1}}}{c_{i,t+1} - \rho_2 \bar{c}_{i,t+1}^e} = \lambda, \quad (2.5)$$

where $\lambda > 0$ is the Lagrange multiplier. Since $\frac{\partial \bar{c}_{i,t}^e}{\partial c_{i,t}} = \frac{\partial \bar{c}_{i,t+1}^e}{\partial c_{i,t+1}} = \frac{1}{N}$, inserting λ from (2.5) into (2.4), we easily obtain

$$10 \quad 1 + r_{i,t+1}^e = \frac{(c_{i,t+1} - \rho_2 \bar{c}_{i,t+1}^e)(1 - \frac{\rho_1}{N})}{\beta(c_{i,t} - \rho_1 \bar{c}_{i,t}^e)(1 - \frac{\rho_2}{N})} = \frac{\left(c_{i,t+1} - \rho_2 \left(\sum_{j \neq i} \frac{c_{j,t+1}^e}{N} + \frac{c_{i,t+1}}{N} \right) \right) (1 - \frac{\rho_1}{N})}{\beta \left(c_{i,t} - \rho_1 \left(\sum_{j \neq i} \frac{c_{j,t}^e}{N} + \frac{c_{i,t}}{N} \right) \right) (1 - \frac{\rho_2}{N})}.$$

Assuming perfect foresight for the agents, that is, $r_{i,t+1}^e = r_{t+1}$ and $c_{ij,t}^e = c_{j,t}$, for every t , every i and $j \neq i$, the expression above reads as

$$1 + r_{t+1} = \frac{\left(c_{i,t+1} - \rho_2 \sum_{j=1}^N \frac{c_{j,t+1}}{N} \right) (1 - \frac{\rho_1}{N})}{\beta \left(c_{i,t} - \rho_1 \sum_{j=1}^N \frac{c_{j,t}}{N} \right) (1 - \frac{\rho_2}{N})} = \frac{\left(c_{i,t+1} (1 - \frac{\rho_2}{N}) - \rho_2 \sum_{j \neq i} \frac{c_{j,t+1}}{N} \right) (1 - \frac{\rho_1}{N})}{\beta \left(c_{i,t} (1 - \frac{\rho_1}{N}) - \rho_1 \sum_{j \neq i} \frac{c_{j,t}}{N} \right) (1 - \frac{\rho_2}{N})} \quad (2.6)$$

and (2.3) becomes

$$15 \quad c_{i,t+1} = (1 + r_{t+1})(w_t - c_{i,t}). \quad (2.7)$$

Inserting then $c_{i,t+1}$ from (2.7) into (2.6), we find

$$1 + r_{t+1} = \frac{\left((1 + r_{t+1})(w_t - c_{i,t}) (1 - \frac{\rho_2}{N}) - \rho_2 \sum_{j \neq i} \frac{(1 + r_{t+1})(w_t - c_{j,t})}{N} \right) (1 - \frac{\rho_1}{N})}{\beta \left(c_{i,t} (1 - \frac{\rho_1}{N}) - \rho_1 \sum_{j \neq i} \frac{c_{j,t}}{N} \right) (1 - \frac{\rho_2}{N})}.$$

Making $c_{i,t}$ explicit in the last equation, we obtain the best response function for agent i

$$20 \quad c_{i,t} = \frac{w_t(1 - \rho_2)(1 - \frac{\rho_1}{N}) + \sum_{j \neq i} \frac{c_{j,t}}{N} [\beta \rho_1 (1 - \frac{\rho_2}{N}) + \rho_2 (1 - \frac{\rho_1}{N})]}{(1 + \beta)(1 - \frac{\rho_1}{N})(1 - \frac{\rho_2}{N})}. \quad (2.8)$$

We stress that, since for every t , for all i and $j \neq i$ it holds that

$$\frac{\partial c_{i,t}}{\partial c_{j,t}} = \frac{\beta \rho_1 (1 - \frac{\rho_2}{N}) + \rho_2 (1 - \frac{\rho_1}{N})}{N(1 + \beta)(1 - \frac{\rho_1}{N})(1 - \frac{\rho_2}{N})} > 0,$$

the game we are considering displays strategic complementarity due to positional consumption. More precisely, simple computations show that, for $j \neq i$, $\frac{\partial c_{i,t}}{\partial c_{j,t}} \in (0, 1)$ and thus the linear system (2.8) admits a unique positive solution. From the symmetry of the game it follows that such solution is symmetric and thus we can find it imposing $c_{1,t} = \dots = c_{N,t} = c_t$ in (2.8), which becomes

$$c_t = \frac{w_t(1 - \rho_2)(1 - \frac{\rho_1}{N}) + \sum_{j \neq i} \frac{c_t}{N} [\beta \rho_1(1 - \frac{\rho_2}{N}) + \rho_2(1 - \frac{\rho_1}{N})]}{(1 + \beta)(1 - \frac{\rho_1}{N})(1 - \frac{\rho_2}{N})},$$

from which we get

$$c_t = \frac{w_t(1 - \frac{\rho_1}{N})(1 - \rho_2)}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})}. \tag{2.9}$$

This is our game's symmetric Nash equilibrium, which is dynamic in nature as it depends on the real wage w_t , endogenously determined by the capital accumulation process. We stress that, although the real wage rate w_t varies from time to time, it is taken as a parameter by young agents, being fixed in the period in which they make their strategic choice.

From (2.2) and (2.9) we obtain the savings in the Nash equilibrium

$$s_t = w_t - c_t = \frac{\beta(1 - \rho_1)(1 - \frac{\rho_2}{N})w_t}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})}. \tag{2.10}$$

Moreover, since the equilibrium condition in the good market reads as

$$k_{t+1} = s_t, \tag{2.11}$$

by (2.10) and (2.11) we get

$$k_{t+1} = \frac{\beta(1 - \rho_1)(1 - \frac{\rho_2}{N})w_t}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})}. \tag{2.12}$$

Assuming a Cobb–Douglas production technology given, in intensive form, by

$$f(k_t) = Ak_t^\alpha,$$

with $A > 0$ and $0 < \alpha < 1$, we find that the real wage is

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) = A(1 - \alpha)k_t^\alpha. \tag{2.13}$$

Inserting this expression for w_t into (2.12), we obtain the following dynamic equation for the capital accumulation

$$k_{t+1} = \frac{A(1 - \alpha)\beta(1 - \rho_1)(1 - \frac{\rho_2}{N})k_t^\alpha}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})}. \tag{2.14}$$

We are going to study the main features of the dynamical system it generates in the next section.

Moreover, since

$$r_{t+1} = f'(k_t) = A\alpha k_t^{\alpha-1},$$

from (2.2), (2.3), (2.10) and (2.13) we obtain

$$\begin{aligned} c_{t+1} &= (1 + r_{t+1})s_t = \left(1 + A\alpha k_t^{\alpha-1}\right) \frac{\beta(1-\rho_1)(1-\frac{\rho_2}{N})w_t}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \\ &= \left(1 + A\alpha k_t^{\alpha-1}\right) \frac{\beta(1-\rho_1)(1-\frac{\rho_2}{N})A(1-\alpha)k_t^\alpha}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})}. \end{aligned} \tag{2.15}$$

Finally we observe that from (2.9) and (2.13) we find

$$c_t = \frac{A(1-\alpha)(1-\frac{\rho_1}{N})(1-\rho_2)k_t^\alpha}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})}. \tag{2.16}$$

3. Results

In this section we discuss the existence of steady states for capital in the presence and absence of social interaction and we compare their expressions. Then we analyse the system stability, showing that there exists a unique positive steady state, which is globally asymptotically stable. We finally perform some comparative statics exercises, in order to better understand the dependence of the globally asymptotically stable steady state with respect to some crucial parameters.

In view of the subsequent analysis, it is expedient to introduce the map $F : \mathbb{R}_+ \rightarrow \mathbb{R}$ defined as

$$F(k) = \frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})k^\alpha}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})}, \tag{3.1}$$

associated to the dynamic equation in (2.14). By our assumptions on ρ_1 , ρ_2 and α , it follows that F is positive. Moreover, since $0 < \alpha < 1$, the map is increasing and concave.

When we set $\rho_1 = \rho_2 = 0$ in (3.1), we find the classical increasing and concave function which describes the system in the absence of social interaction, i.e.

$$F_0(k) = \frac{A(1-\alpha)\beta k^\alpha}{1+\beta}.$$

Notice that we obtain the same expression also when setting $\rho_1 = \rho_2 = \bar{\rho} \in (0, 1)$ in F . This means that, if the degree of social interaction is the same in both life periods, the dynamic behaviour of the system coincides with that in the absence of social interaction.

Let us now show the existence of a unique positive steady state for (2.14).

Proposition 3.1: *In addition to the origin, the dynamical system generated by the map F in (3.1) has the unique positive steady state*

$$k^* = \left(\frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \right)^{\frac{1}{1-\alpha}}. \tag{3.2}$$

Proof: The expression for k^* immediately follows by solving the fixed point equation $F(k) = k$. \square

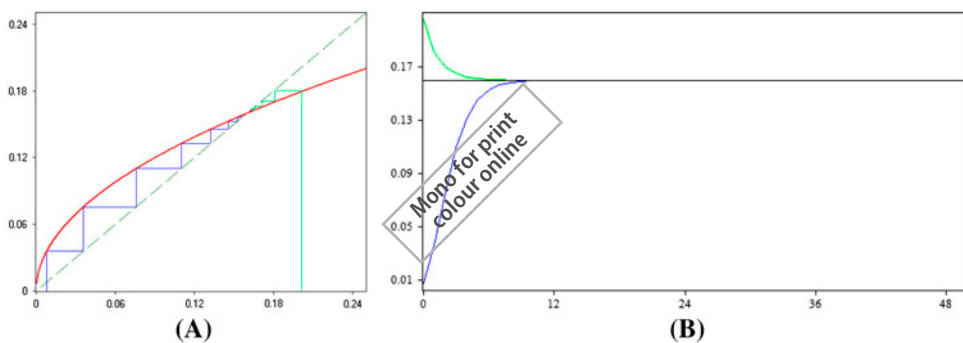


Figure 1. For F with $A = 1$, $\alpha = 0.5$, $\beta = 1$, $\rho_1 = 0.2$, $\rho_2 = 0.8$ and $N = 1000$, in (A) we represent the first iterates, starting respectively from $\bar{k}(0) = 0.008$ (in blue) and $\bar{k}(0) = 0.205$ (in green), and converging to the fixed point for F , while in (B) we represent the first 50 iterates of k_{t+1} in (2.14) starting from the same initial conditions, in blue and green, respectively.

Observe that, when $\rho_1 = \rho_2 = \bar{\rho} \in [0, 1)$, the expression for the steady state in (3.2) becomes

$$k_0^* = \left(\frac{A(1 - \alpha)\beta}{1 + \beta} \right)^{\frac{1}{1-\alpha}}. \tag{3.3}$$

As stated in the next result, the precise relationship between k^* and k_0^* depends on the relative values of the parameters ρ_1 and ρ_2 . Indeed we have the following:

Corollary 3.2: $k^* > k_0^*$ if and only if $\rho_2 > \rho_1$.

Proof: Recalling the definitions in (3.2) and (3.3), a direct computation shows that $k^* > k_0^*$ if and only if $(1 + \beta)(1 - \rho_1)(1 - \frac{\rho_2}{N}) > (1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})$, which is satisfied if and only if $(1 - \rho_1)(1 - \frac{\rho_2}{N}) > (1 - \frac{\rho_1}{N})(1 - \rho_2)$, and the latter holds true if and only if $\rho_2 > \rho_1$. This concludes the proof. \square

Since the relationship between the steady states for capital with and without social interaction has been clarified, in what follows we will focus on the more general case of k^* only and on the dynamics generated by the map F .

Proposition 3.3: *The dynamical system generated by the map F in (3.1) is globally asymptotically stable.*

Proof: We show that, for any starting point $\bar{k} > 0$, its forward F -trajectory tends to k^* . In fact, since $F(0) = 0$, F is strictly increasing, k^* is the unique positive fixed point of F and $F'(k^*) = \alpha \in (0, 1)$, then, by continuity, $F^n(\bar{k})$ will tend increasingly towards k^* as $n \rightarrow \infty$ for $0 < \bar{k} < k^*$, while $F^n(\bar{k})$ will tend decreasingly towards k^* as $n \rightarrow \infty$ for $\bar{k} > k^*$. The proof is complete. \square

Thanks to the global stability result shown in Proposition 3.3, and illustrated in Figure 1, the Nash equilibrium we found in (2.12) is dynamic in nature during the transient phase, while it becomes static in the long run.

Notice that Proposition 3.3 holds, in particular, when $\rho_1 = \rho_2 = 0$. In such case we are led back to the original framework in Diamond model without social interaction [8], in which the unique positive steady state in (3.3) is globally asymptotically stable. In this sense the dynamical features of the model are not affected by the introduction of consumption externalities. Nonetheless, the richer context we are dealing with allows us to perform

some new interesting comparative statics exercises. In fact, in the next results we analyse at first the dependence of the steady state for capital on the social interaction parameters $\rho_1, \rho_2 \in (0, 1)$ and on the population size of each cohort N ; then we derive the expression for the steady state values for consumption in both life periods and we investigate how such values change on varying ρ_1, ρ_2 and N .

Proposition 3.4: *It holds that $\frac{\partial k^*}{\partial \rho_1} < 0$ and $\frac{\partial k^*}{\partial \rho_2} > 0$.*

Proof: Direct computations show that

$$\begin{aligned} \frac{\partial k^*}{\partial \rho_1} &= \frac{1}{1-\alpha} \left(\frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \right)^{\frac{1}{1-\alpha}-1} \cdot \frac{-A(1-\alpha)\beta(1-\frac{\rho_2}{N})(1-\frac{1}{N})(1-\rho_2)}{\left((1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})\right)^2} \\ &= - \left(\frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{A\beta(1-\frac{\rho_2}{N})(1-\frac{1}{N})(1-\rho_2)}{\left((1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})\right)^2} < 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial k^*}{\partial \rho_2} &= \frac{1}{1-\alpha} \left(\frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \right)^{\frac{1}{1-\alpha}-1} \cdot \frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{1}{N})(1-\frac{\rho_1}{N})}{\left((1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})\right)^2} \\ &= \left(\frac{A(1-\alpha)\beta(1-\rho_1)(1-\frac{\rho_2}{N})}{(1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})} \right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{A\beta(1-\rho_1)(1-\frac{1}{N})(1-\frac{\rho_1}{N})}{\left((1-\frac{\rho_1}{N})(1-\rho_2)+\beta(1-\rho_1)(1-\frac{\rho_2}{N})\right)^2} > 0. \end{aligned}$$

□

The result above is quite intuitive and allows a clear interpretation. Indeed, an increasing interest towards the others' choices in the first life period makes savings in the first period decrease and thus the accumulated capital decreases. Vice versa, an increasing interest towards the others' choices in the second life period makes accumulated capital increase, due to an increase in the savings in the first period, in order to have the possibility to increase consumption in the second period.

Proposition 3.5: *It holds that $\frac{\partial k^*}{\partial N} > 0$ if and only if $\rho_2 > \rho_1$.*

Proof: A direct computation shows that

$$\begin{aligned} \frac{\partial k^*}{\partial N} &= \frac{1}{1-\alpha} \left(\frac{A(1-\alpha)\beta(1-\rho_1)(N-\rho_2)}{(N-\rho_1)(1-\rho_2)+\beta(1-\rho_1)(N-\rho_2)} \right)^{\frac{1}{1-\alpha}-1} \cdot \frac{A(1-\alpha)\beta(1-\rho_1)(1-\rho_2)(\rho_2-\rho_1)}{\left((N-\rho_1)(1-\rho_2)+\beta(1-\rho_1)(N-\rho_2)\right)^2} \\ &= \left(\frac{A(1-\alpha)\beta(1-\rho_1)(N-\rho_2)}{(N-\rho_1)(1-\rho_2)+\beta(1-\rho_1)(N-\rho_2)} \right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{A\beta(1-\rho_1)(1-\rho_2)(\rho_2-\rho_1)}{\left((N-\rho_1)(1-\rho_2)+\beta(1-\rho_1)(N-\rho_2)\right)^2} \end{aligned}$$

and the desired conclusion immediately follows. □

The latter result says that if the population size of each cohort increases, then savings in the first life period, and thus accumulated capital, increase if and only if the difference between the social interaction parameters $\rho_2 - \rho_1$ is positive. This means that an increase in the population size increases capital accumulation if and only if agents are more influenced by social interaction in the second life period. In particular, if agents are influenced in the same manner by social interaction in the two periods (i.e. if $\rho_1 = \rho_2$), then a variation in the population size has no effects on capital accumulation.

Let us now derive the steady state values for consumption in both life periods.

From (2.16) we find that the steady state value for consumption in the first period, we denote by $c^{1,*}$, is given by

$$c^{1,*} = \frac{A(1 - \alpha)(1 - \frac{\rho_1}{N})(1 - \rho_2)(k^*)^\alpha}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})},$$

with k^* as in (3.2). From (2.15) we find that the steady state value for consumption in the second life period, we denote by $c^{2,*}$, is given by

$$c^{2,*} = (1 + A\alpha(k^*)^{\alpha-1}) \frac{\beta(1 - \rho_1)(1 - \frac{\rho_2}{N})A(1 - \alpha)(k^*)^\alpha}{(1 - \frac{\rho_1}{N})(1 - \rho_2) + \beta(1 - \rho_1)(1 - \frac{\rho_2}{N})},$$

again with k^* as in (3.2).

By direct (heavy) computations, we obtain Propositions 3.6–3.10.

Proposition 3.6: It holds that $\frac{\partial c^{1,*}}{\partial \rho_1} > 0$ if and only if $\frac{\beta(1-\alpha)}{\alpha} \frac{N-\rho_2}{1-\rho_2} > \frac{N-\rho_1}{1-\rho_1}$.

Proposition 3.7: It holds that $\frac{\partial c^{1,*}}{\partial \rho_2} > 0$ if and only if $\frac{\beta(1-\alpha)}{\alpha} \frac{N-\rho_2}{1-\rho_2} < \frac{N-\rho_1}{1-\rho_1}$.

Hence, from Propositions 3.6 and 3.7 it follows that, as expected, $\frac{\partial c^{1,*}}{\partial \rho_1} > 0$ if and only if $\frac{\partial c^{1,*}}{\partial \rho_2} < 0$, that is, consumption in the first life period increases when the social interaction coefficient in the first period increases if and only if consumption in the first period decreases when the social interaction coefficient in the second life period increases.

Proposition 3.8: It holds that $\frac{\partial c^{1,*}}{\partial N} > 0$ if and only if

$$(\rho_2 - \rho_1) \frac{\beta(1 - \alpha)}{\alpha} \frac{N - \rho_2}{1 - \rho_2} < (\rho_2 - \rho_1) \frac{N - \rho_1}{1 - \rho_1}. \tag{3.4}$$

Thus, from Propositions 3.6–3.8 it follows that

if $\rho_2 > \rho_1$, then $\frac{\partial c^{1,*}}{\partial N} > 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_2} > 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_1} < 0$; if instead $\rho_1 > \rho_2$, then $\frac{\partial c^{1,*}}{\partial N} > 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_2} < 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_1} > 0$ (or, equivalently, $\frac{\partial c^{1,*}}{\partial N} < 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_2} > 0 \Leftrightarrow \frac{\partial c^{1,*}}{\partial \rho_1} < 0$).

Proposition 3.9: It holds that $\frac{\partial c^{2,*}}{\partial \rho_1} < 0$ and $\frac{\partial c^{2,*}}{\partial \rho_2} > 0$.

Comparing Proposition 3.9 with Propositions 3.6 and 3.7, we observe that an increase in the social interaction coefficient in the first period always decreases consumption in the second life period, and an increase in the social interaction coefficient in the second period always increases consumption in the second life period, independently of the value of the other parameters, while the behaviour of consumption in the first period with respect to variations in the social interaction coefficients is less sharp.

Proposition 3.10: It holds that $\frac{\partial c^{2,*}}{\partial N} > 0$ if and only if $\rho_2 > \rho_1$.

Hence, somewhat similarly to what happens to the steady state value of consumption in the first life period (see the comments after Proposition 3.8), it holds that if $\rho_2 > \rho_1$ then both $\frac{\partial c^{2,*}}{\partial N}$ and $\frac{\partial c^{2,*}}{\partial \rho_2}$ are positive and $\frac{\partial c^{2,*}}{\partial \rho_1}$ is negative, while if $\rho_1 > \rho_2$ then both $\frac{\partial c^{2,*}}{\partial N}$ and $\frac{\partial c^{2,*}}{\partial \rho_1}$ are negative and $\frac{\partial c^{2,*}}{\partial \rho_2}$ is positive.

Overall, we can conclude that the behaviour of consumption in the second life period with respect to variations in both the social interaction coefficients and the population size is more definite than the corresponding behaviour of consumption in the first period with respect to the same variations.

4. Conclusions

We have considered a two-period lived OLG economy, with the utility of each consumer depending also on the average level of consumption by the consumers in the same generation. In particular, differently from the existing literature on the topic, we have assumed that such average level of consumption is not taken as a parameter by agents. On the contrary, they behave strategically, considering that the others' choices, together with their own choice, will influence their utility function. In this way we obtained a positional game embedded in an OLG economy, for which we characterized the consumption and saving choices for the two life periods in the Nash equilibrium path and we determined a dynamic equation for capital accumulation coherent with the agents' choices in the Nash equilibrium. Hence, also the behaviour, both static and dynamic, described by the equation for the capital accumulation was coherent with the Nash equilibrium. For the associated dynamical system we found a unique positive steady state for capital, which is globally asymptotically stable, like in the original framework by Diamond [8]. Thus, we can infer that the qualitative dynamical behaviour of the standard OLG model is robust with respect to the introduction of social interaction through a positional game. The value of our steady state for capital turned out to be decreasing with respect to variations in the degree of interaction in the first life period, while the opposite relation holds with respect to variations in the degree of interaction in the second period. We then compared the steady states for capital with and without social interaction, showing that the steady state with social interaction is larger than the steady state in the absence of social interaction if and only if the degree of interaction in the second life period exceeds the degree of interaction in the first period. In particular, if the degrees of interaction in the two life periods coincide, the dynamical system is equivalent to the one without social interaction. We also performed several other comparative statics exercises, in order to understand how the position of the steady state for capital and for consumption in the two life periods varies with respect to the population size and with respect to the degree of interaction in both periods. Some of those exercises led to easily interpretable results, some others led instead to conclusions less straightforward to interpret, due to the presence of nontrivial inequalities.

Future research should focus for instance on the introduction in our model of a further degree of heterogeneity, by assuming that the agents within a generation may have different social interaction coefficients. In such context one could investigate how consumption varies according to the value of those coefficients and also compare consumption for all kinds of agents with the consumption level in the absence of social interaction. Another interesting possibility would consist in considering parental consumption as benchmark reference level for consumption.

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