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Non-Supersymmetric Space-Times and Renormalization Group Flows in String Theory

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Unless otherwise stated the results presented in this thesis are original. In particular, the results in Chapter 2, 3 and 4 are based on the published works [1, 2, 3] and on the ongoing works [4, 5].

Abstract

In this thesis we study solutions of string theories from different perspectives. We start in Chapter 1 with an introduction to the main ideas of string theory, focusing in particular on its low-energy description in terms of supergravity theories. We discuss the main ingredients of the supergravity theories derived from strings and we present their classical solutions corresponding to the physical objects we will use in the rest of the thesis. In Chapter 2 we begin the study of non-supersymmetric backgrounds by building explicit eight-dimensional Anti de Sitter (AdS) solutions of massive type IIA supergravity. As is common in non-supersymmetric settings, we are only able to solve the full set of equations of motion numerically. With these methods, we find AdS_8 solutions with a compact internal space having the topology of a two-sphere, with an orientifold plane (O8) sitting at its equator. In Chapter 3, we extend our study of non-supersymmetric backgrounds by looking for vacua with a positive cosmological constant. In particular, we find numerical four-dimensional de Sitter (dS) solutions of massive type IIA supergravity. Some of these vacua involve the same orientifold plane featuring in the AdS_8 backgrounds, which appears a particular singularity in the supergravity approximation. We analyze this singularity in detail before moving on and studying dS_4 solutions with a different orientifold plane (O6). The appearance of orientifold planes in classical de Sitter solutions of supergravity theories is required in order to evade a famous no-go theorem, which also applies to the AdS_8 backgrounds we describe in Chapter 2. For this reason, we review it in our particular setting at the beginning of the same chapter. Finally, in Chapter 4 we change our perspective, and we use supergravity as a tool to study the physics of the Renormalization Group (RG) flows. In particular, by using known building blocks, we assemble a seven-dimensional gravitational theory and we use it to construct the holographic duals of RG flows between six-dimensional superconformal field theories. Our construction is able to correctly characterize the physics of these RG flows by confirming, from the gravitational point of view, a conjecture on the literature regarding the allowed RG flows between these six-dimensional theories.

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Introduction

We know very well only a small corner of the physical world, the one we have been able to access experimentally. To be more precise, according to the standard model of cosmology, we know only the 5% of the constituents of our universe [6].

When we do not know how something works, we try to build theories that do not rely on these unknown parts of the world and are useful only up to the boundary of our ignorance, forgetting what lives outside. Technically, we construct *effective fled theories.* Far from being a sign of cowardice, this modesty in our approach to the natural world is what has fueled the huge progress science has been able to make in all its history. For example, we do not need to know how quarks and leptons work and interact with each other to understand the laws of thermodynamics [7] or how electricity works [8]. Nor even to build the electronic devices that connect our world, or to land on the moon, even though computers and the moon are made out of quarks and leptons. Effective theories are enough, and we have successfully enlarged the realm of the things we know by proceeding in this way. But as a great man once said (and another great man once taught to me) as the bubble of the things we know grows, also its boundary with the things we do not know gets bigger.

We have reached this boundary many times in the history of effective theories. For example, when we tried to compute the scattering of four fermions in the Fermi theory [9] at energy scales above 100 GeV, we got infinite probability for some events to happen. This clearly did not make sense. This breakdown of the effective theory was signaling that there was the physics of W and Z bosons outside the bubble, which has been then captured by the electroweak theory [10, 11, 12]. Then also quarks came into the play and eventually we built the Standard Model of Particle Physics, a beautiful theory able to explain the quantum properties of all the forces we observe in nature, but gravity. So we can ask ourselves where is now the boundary of our ignorance. The honest answer is that we do not know till we reach it, since surprises might always be around the corner. Even if we do not exactly know where this boundary is, we can put some bounds on it. If we want to use centimeters, we can say that for sure we do not know what happens at distances below 10^{-33} cm, the *Planck scale*. This is the scale where the quantum effects of gravity cannot be neglected anymore. By looking at this small number, one might think that it is too far away from what we could ever experience to be relevant, and that we can happily live in our bubble which thus seems pretty big. A conservative approach would suggest to just build and tune our effective field theories in order to match with the experimental data, for example with the astronomical observations. However, there are many indications that this approach is not satisfactory when dealing with gravity. First of all, the astronomical data are becoming more and more numerous and precise, making us approach the boundary of our bubble pretty fast. But even if we are able to build an effective model that explains well our current universe, and the fact that is expanding at an accelerating rate [13, 14], we have to explain how we got there. Our best candidate model [15, 16, 17] to explain the structures we observe today in our universe suggests that it has gone under another phase of (more violent) exponential expansion during its infancy, called *inflation*. We can build effective field theories for inflation but it turns out that they are often sensitive to the physics at the Planck scale [18], needing again to go beyond the boundary of our bubble.

Moreover, another current and very active line of research proposes that not all the effective field theories that look consistent below a certain energy scale can descend from a complete quantum gravity theory. Those that do not are said to belong to the *swampland* [19]. This is a highly debated topic but the main idea, regardless of the details of the various criteria that try to define the swampland, has striking effects for the effective field theory paradigm.

From different points of view it seems that quantum gravity has so many subtle and interesting physical implications that it would not be fair to its beauty and importance to relegate it to live outside of the bubble of the things we care about. And, as we have argued, doing so could also lead to incomplete or even wrong results. Thus it seems natural to use our current best candidate for a quantum theory of gravity, string theory, to investigate all these issues. To this end, we start in Chapter 1 by reviewing the basic ideas of string theory, focusing on its low-energy limits: the supergravity theories. The prefix *super* comes from the fact that these theories possess a peculiar symmetry that relates bosons and fermions, known as *supersymmetry*. Another feature that these supergravities share is that they are defined in ten (or eleven)-dimensional space-times. But our universe is not supersymmetric and only looks four-dimensional. If we want to use string theory to describe the world we live in, an important problem to solve is to understand solutions of string theory where a part of the space is compact, such that an observer which is not able to resolve the compact directions would only see the (four) non-compact ones. Similar constructions are called *string compactifications*[1], and part of this thesis will be devoted to the study of this problem. In particular we will construct *non-supersymmetric compactifications.*

In Chapter 2 we start attacking this problem by looking for non-supersymmetric solutions in a particularly restricted scenario, where direct progress can be made. In particular, we look for eight-dimensional Anti de Sitter space-times (AdS_8), where the high amount of symmetry constrains the problem enough to allow us to construct explicit numerical solutions of the theory's equations of motion.

We then continue our journey in the realm of non-supersymmetric string theory compactifications by trying to construct backgrounds more directly related to our observed universe. The simplest way to describe a universe expanding at an accelerated rate is to approximate it as a de Sitter space-time. Constructing de Sitter space-times introduces many challenges and in Chapter 3 we will confront ourselves with these problems by building different explicit dS_4 solutions of massive type IIA supergravity. Both the solutions we find here and the AdS_8 solutions we have introduced above involve some peculiar objects of string theory known as *orientifold planes*. However, near these objects the supergravity approximation is not reliable and thus more study is required in order to assess the validity of these solutions as true backgrounds of the full string theory.

Other than being useful as a framework to construct explicit phenomenological models, string theory has also been very fruitful as a playground to test various ideas on quantum gravity. In particular, it has provided many explicit realizations of the *holographic principle*, through the so-called AdS/CFT correspondence [21]. This correspondence relates a quantum gravitational theory on an Anti de Sitter space-time to a quantum conformal field theory defined on its boundary. In Chapter 4, we change our perspective and we use supergravity as a tool to study the physics of

[1] A pedagogical and extensive review of the various ideas and techniques used in the world of string compactifications can be found in [20].

Renormalization Group (RG) flows from the holographic point of view. In particular, by using known building blocks, we assemble a seven-dimensional gravitational theory and we use it to construct the holographic duals of RG flows between six-dimensional superconformal field theories. Our construction is able to correctly characterize the physics of these RG flows by confirming, from the gravitational point of view, a conjecture on the literature regarding the allowed RG flows between these theories.

Chapter 1

Foundations

In this chapter, we introduce the main ideas and tools we need in this thesis. In particular, we start in section 1.1 with a basic introduction to the string theory ideas, based on the classic book [22] and on the more modern review [23]. We hereby stress that this is not to be intended as a satisfactory introduction to the beautiful world of string theory, which would be well beyond the scope of this work, but merely as an introduction to the terminology we will need to develop this thesis. The interested reader can complement our discussion with aforementioned references. Other good references on this vast topic include [24, 25, 26].

In section 1.2 we will then focus on the low-energy limits of string theories, the ten-dimensional supergravity theories, and in particular in the type II theories. Again, we will just touch upon the main ingredients we will need later, and the interested reader is referred to [27, 28].

1.1 String theory introduction

Perturbative string theory is the study of the dynamics of one-dimensional extended objects called *strings*. The shift of paradigm from the study of point-like interactions (i.e. particles) proper of the standard Quantum Field Theory has some immediate benefits. First of all, the existence of a small, but finite, fundamental length scale, the string length l_s, automatically cures the ill-definiteness of interactions on arbitrarily small scales that plagues the study of point-like objects. In Quantum Field Theory various methods have been developed to overcome these difficulties, i.e. to *renormalize* the theory, but they fail when we try to apply them to the gravitational interaction. In other words, the gravitational interaction is *non-renormalizable*. This does not mean that without string theory we cannot study the quantum effects of the gravitational interaction at all, but that we can do it only in the context of *effective field theories*. Such theories can capture the physics at a low energy scale but will become meaningless above a certain scale. For the gravitational interaction this scale is the *Planck scale*. As a length scale, it is of order $l_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33}\,\text{cm}$. Surprisingly, in string theory the two issues are related. If on one side the appearance of a finite length scale spreads out the region of space-time where the interactions happen, on the other side it turns out that the quantum theory of one dimensional objects necessarily has to include gravity. Indeed, in string theory the different oscillation modes of the strings carry the quantum numbers of different particles, and all the consistent string theories always have an oscillation mode that corresponds to a massless *graviton*, the mediator of the gravitational interaction in the low-energy effective theory. A little bit more concretely, the study of free string theories is the study of the two-dimensional quantum field theory defined on the world-sheet σ of a string propagating in space-time. This theory is a Conformal Field Theory (CFT) arising (in the purely bosonic case) as a quantization of the Polyakov action

$$S_P = -\frac{1}{4\pi l_s^2}\int_\sigma \sqrt{-h}\partial X^\mu \cdot \partial X^\nu \eta_{\mu\nu} \ , \tag{1.1}$$

where the X^μ are the embedding coordinates of the string world-sheet in flat space-time (equipped with the Minkowski metric $\eta_{\mu\nu}$) and h is a metric on the world-sheet. The conformal symmetry of this action plays a central role in constraining the physics of string theory.

Notice that we have started the previous paragraph using the word *perturbative*. What we mean by it is that the theory we have just introduced is only defined as a perturbative expansion in a parameter which we call g_s, the *string coupling*, governing the strength of the interactions between strings. The interaction of strings can be defined as a weighted sum of over all the histories of the strings. This sum is naturally organized into a sum over Riemann surfaces with different topologies, weighted by powers of the string coupling with a factor

$$g_s^{-\chi} \ , \tag{1.2}$$

where χ is the Euler characteristic of the Riemann surface. The Euler characteristic χ appears naturally in this context as the interactions are defined by augmenting the Polyakov action (1.1) with the Einstein-Hilbert term on the world-sheet

$$S_{\text{EH}} = \frac{1}{2\pi}\int_\sigma \sqrt{-h}R \ . \tag{1.3}$$

In two dimensions this term is only sensitive to the topology of the space, giving as result of the integration the integer number χ.

Interestingly, in string theory g_s is not a free parameter, but consistency of the above construction requires it to be related to the expectation value of a dynamical massless scalar field ϕ already present in the spectrum. This scalar field is usually called *dilaton* and its relation to the string coupling is given by

$$g_s = \langle e^\phi \rangle \ . \tag{1.4}$$

We know a handful of perturbative string theories defined in this way, and the most studied among them share some peculiar properties: they are all supersymmetric and they require the existence of ten space-time dimensions. Supersymmetry on the world-sheet is required to have a consistent quantum theory, otherwise a tachyon is always present in the spectrum, signaling an instability. Consistency of the quantum theory in flat space then also requires the dimension of the space-time to be ten. The five perturbative string theories with these properties[1] are known as type IIA, type IIB, type I, heterotic SO(32) and heterotic $E_8 \times E_8$. They differ by the amount of supersymmetry they require ($\mathcal{N} = 2$ or $\mathcal{N} = 1$), the strings they describe (open and closed or only closed), and the

[1] Even though we will not comment more on these issues, let us notice that the five possibilities we have listed in the main text are not exhaustive. First of all, quantizing the theory on a background with non-constant dilaton (see also footnote 3) might alleviate the bound on the dimension of the space-time by introducing including a term proportional to $(D - 10)$ in equation (1.12). See for example [29, 30] for a general discussion and [31, 32] for applications to de Sitter backgrounds. Moreover, also a ten-dimensional non-supersymmetric heterotic string theory exists [33].

gauge group. Albeit from the perturbative description they appear to be completely independent theories, during the so-called *second superstring revolution* it has been understood that they are intimately related by a beautiful net of *dualities*. In this context, we use the word *duality* to denote an equivalence of two theories that from their definition are a priori different. Two important dualities in string theory that we will need in the rest of the discussion are known as *T-duality* and *S-duality*. T-duality is a perturbative duality, meaning that it works at each order in the string perturbation theory, that relates a string theory with some dimensions compactified on a certain space to a string theory compactified on a different space. S-duality, instead, relates a weakly-coupled string theory to a strongly-coupled one. Moreover, the web of dualities connecting the different theories hints to the existence of another mysterious theory, named *M-theory*. This theory is largely unknown, but we know that it is defined in an eleven-dimensional space-time and that at low energies it reduces to the unique eleven-dimensional supergravity.

Particularly important to us will be the low-energy limits of the string theories. We are going to describe these low energy limits better in section 1.2, but for now let us just quote here the most important term in their action, common to all the perturbative critical string theories we have introduced:

$$S[g,\phi] = \frac{1}{\kappa^2}\int_{M_{10}} \sqrt{-g}e^{-2\phi}(R + 4\nabla\phi\nabla\phi) \; , \tag{1.5}$$

where here and in the rest of the work we use the definition $\kappa^2 \equiv (2\pi)^7 l_s^8$. Notice the factor $e^{-2\phi}$ in the integrand. Being it a tree-level closed string action it is easy to understand the origin of this factor by combining equation (1.2) and equation (1.4). Indeed a tree-level action for a closed string comes from a spherical world-sheet, which has Euler characteristic equal to 2.

Other than connecting the different theories, the dualities have been also useful to understand some of the non-perturbative features of string theories. Of particular importance is the appearance of new dynamical objects known as Dirichlet branes or *D-branes*. In the perturbative string theories involving open strings, for the dynamics of the open strings to be well defined, some boundary conditions on their world-sheet have to be chosen. For a long time it has been thought that only Neumann boundary conditions (i.e. open strings whose endpoints can fluctuate but that cannot 'stretch') are compatible with Lorentz symmetry. However, it has been discovered that T-duality exchanges Neumann and Dirichlet boundary conditions, such that after a T-duality a string whose endpoints were free to move anywhere, now have endpoints fixed to move on some hyper-plane. This restriction does not break the Lorentz symmetry of the whole theory if the hyperplane itself is a dynamical object. Such an object has been called *D-brane*. These objects, indirectly described in open-string theories as the endpoints of open strings, are not directly visible in the perturbative spectrum of the theory and are thus *non-perturbative* degrees of freedom. By studying interactions of open and closed strings at low energies, one can obtain a low-energy action describing the dynamics of a Dp-brane. We are going to describe it better in section 1.2, but for now let us consider its gravitational part

$$S_{\text{DBI}} = -\frac{1}{(2\pi)^p l_s^{p+1}}\int_\Sigma e^{-\phi}\sqrt{-\det(g+\mathcal{F})_\Sigma} \; . \tag{1.6}$$

This action is known as *Dirac-Born-Infeld (DBI) action*. It is integrated on the $p+1$ dimensional world-volume Σ of the Dp-brane and involves the pullback of the space-time metric g. It also includes a two-form $\mathcal{F}$ which we will describe later. For the moment, let us just focus on the factor $e^{-\phi}$ that accompanies it. Since this action is the result of tree-level interactions involving open strings, its physics is captured by a Riemann surface with the topology of a disk, which has Euler characteristic equal to 1. From the action (1.6) we can see that the mass of a Dp-brane is

$$m_{D_p} \sim \frac{1}{g_s} \,. \tag{1.7}$$

Thus, at weak coupling ($g_s \ll 1$) these objects are very massive and do not appear in the perturbative spectrum. However, they become light at strong coupling where they can become the degrees of freedom of a dual-theory. As we are going to see in section 1.2.2, Dp-brane also have a charge and they couple to higher-dimensional generalizations of the electric potential.

Another important construction in string theory is that of an *orientifold projection*. An orientifold projection is a projection of the theory under a symmetry which is the result of a combined action on the world-sheet theory and on the space-time. The fixed loci in space-time of this action are called *orientifold planes*, or *Op-planes*, where p-denotes their space-time dimensions. These objects are not-dynamical, but the effect they produce on the-space time can be captured at tree-level by an action similar to the one we have introduced for Dp-branes. In particular, to these objects we can associate a tension T_{Op} (a mass per unit volume). The most common type of Op-plane has negative tension, which is related to the tension of the Dp-brane of the same dimensionality as

$$T_{Op} = -2^{p-5} T_{Dp} \,. \tag{1.8}$$

Moreover, if we conventionally take the charge of Dp-branes to be positive, these orientifold planes also have negative charge and are usually called Op_- planes. This is the most common kind of orientifold plane but, as we are going to see, also orientifold planes with positive tension and charge exist, and are usually denoted as Op_+ planes. An extensive review of orientifold planes in string theory can be found in [34].

1.2 Low-energy limit

As we have already discussed, the quantization of free strings produces a graviton and a massless scalar. These are not the only massless fields. The spectrum of all the string theories also includes the degrees of freedom associated to a space-time two-form B. Together, the metric g, the dilaton ϕ and the two-form B, form the part of the spectrum which is known as the massless Neveu-Schwarz (NS) sector.

In type II theories, on which we will focus from now on, there are other massless degrees of freedom associated to higher-degree p-forms C_p. In particular, odd values of p are allowed in type IIA string theory while even ones appear in type IIB string theory. This sector is called Ramond-Ramond (RR) sector. Other than the massless fields, there are massive states in both sectors with masses related to the string length as

$$M^2 \sim \frac{1}{l_s^2} \,. \tag{1.9}$$

If we take the string length to be very small, these masses are extremely high and the corresponding degrees of freedom are not excited at low-energies. Thus, we can try to derive an effective action describing only the massless degrees of freedom and, as such, valid only at energies well below (1.9). A procedure to obtain this effective action is to compute the scattering amplitudes for the particles in the spectrum at tree-level in string perturbation theory (the spherical world-sheet), restricting such computations to energy scales smaller than (1.9). One can then compare the results obtained in this way with the ones computed from an effective space-time action. With this procedure one can obtain all the supergravity actions we are going to describe. However, consistency of the theory gives us another way to compute the low-energy effective action of the NS sector. Indeed, we can consider strings propagating not on a flat-background, but on a background describing a coherent state of fields produced by other strings. The world-sheet action is now described by an interacting CFT, where the dimensionless coupling constant is given by

$$\frac{l_s}{r_c}\ , \tag{1.10}$$

with r_c the typical radius of curvature. Just like for the free-string world-sheet theory, consistency of the quantum theory requires that the Weyl symmetry, which is a gauge symmetry on the world-sheet, is non-anomalous. This in turn requires the β-function of this interacting CFT to vanish. We cannot compute it exactly, but if the radius of curvature is large (i.e. the curvature is small) the coupling constant (1.10) is small and we can compute the β-function perturbatively. For a spherical world-sheet (tree-level in string perturbation theory) the first order of the expansion in the coupling (1.10) is given by

$$\beta_{MN}(g) \;=\; \alpha'\left(R_{MN}+2\nabla_M\nabla_N\phi-\frac{1}{4}H^2_{MN}\right)+O(\alpha'^2)\ , \tag{1.11}$$

$$\beta(\phi) \;=\; \alpha'\left(-\frac{1}{2}\nabla^2\phi+\nabla\phi\nabla\phi-\frac{1}{24}H^2\right)+O(\alpha'^2)\ , \tag{1.12}$$

$$\beta_{MN}(B) \;=\; \alpha'\left(-\frac{1}{2}\nabla^P H_{PMN}+\nabla^P\phi H_{PMN}\right)+O(\alpha'^2)\ , \tag{1.13}$$

where, with a standard notation, we have defined $\alpha'\equiv l_s^2$. Notice that this is an expansion in derivatives. Our low-energy actions can now be defined such that they reproduce these β-functions as their equations of motion. We stress again that in regions of the space-time where the curvature is large, the world-sheet CFT is strongly coupled, and we cannot trust the β-functions we have computed perturbatively. Hence, the corresponding effective action breaks down in such regions. This problem is a technical problem but not much a conceptual one, in the sense that it is well-defined in terms of the world-sheet path integral. This is different from the problem of strongly-interacting strings ($g_s\gg 1$), for which we even lack a definition of the theory. Finally, notice that this procedure it not able to reproduce the dynamics of the RR fields, since we do not know how to couple the world-sheet theory to a background where the RR fluxes are non-trivial.

Another way to constrain the low-energy actions is by using supersymmetry. Since the world-sheet theory is supersymmetric, we can try to construct space-time theories with the same amount of supersymmetry and the correct field content. We this procedure we can constrain enough the low-energy actions and obtain, for example, the type IIA and type IIB supergravities which we are going to describe in the next section.

1.2.1 Type II supergravities

We now briefly describe the main properties of the ten-dimensional type II supergravities with a particular focus on their equations of motion. More details can be found in [26].

We start from the type IIA superstring. Its low-energy physics is described by massive type IIA supergravity [35], whose bosonic part of the action is given by:

$$
\begin{aligned}
S_{\mathrm{IIA,bos}} &= \frac{1}{\kappa^2}\int_{M_{10}}\sqrt{-g}\left[e^{-2\phi}\left(R+4\nabla\phi\nabla\phi-\frac{1}{12}H^2\right)-\frac{1}{2}\left(F_0^2+\frac{1}{2}F_2^2+\frac{1}{4!}F_4^2\right)\right] \\
&- \frac{1}{2}\left(dC_3\wedge dC_3\wedge B+\frac{F_0}{3}dC_3\wedge B\wedge B+\frac{F_0^2}{20}B\wedge B\wedge B\wedge B\wedge B\right) .
\end{aligned} \tag{1.14}
$$

We have defined the field strengths in terms of the potentials as

$$
H\equiv dB,\qquad F_2\equiv dC_1+F_0B,\qquad F_4\equiv dC_3-H\wedge C_1+\frac{F_0}{2}B\wedge B\ , \tag{1.15}
$$

and we have defined the square of an n-form A without any factor:

$$
A^2=A_{M_1\dots M_n}A^{M_1\dots M_n}. \tag{1.16}
$$

We are working in string-frame, but notice that the $e^{-2\phi}$ factor does not appear in front of all the terms. This is due to the fact that, as is customary, the RR gauge potentials C_p's have been rescaled with a factor of the dilaton in order to simplify the following Bianchi identities:

$$
dH = 0 \tag{1.17}
$$

$$
dF_2-H\wedge F_0 = 0 \tag{1.18}
$$

$$
dF_4-H\wedge F_2 = 0 \tag{1.19}
$$

$$
dF_0 = 0\ . \tag{1.20}
$$

Moreover, there is a new flux F_0, which is a 0-form and it is fixed from the Bianchi identity (1.20) to be a constant. This flux does not arise as the field strength of a gauge potential. Thus, there are no propagating degrees of freedom associated to it and it is not visible in the perturbative string spectrum. This constant is usually called *Romans mass*, hence the name *massive* type IIA.

From the action (1.15) we can derive the dilaton and Einstein equations of motion:

$$
R+4\nabla^2\phi-4\nabla\phi\nabla\phi-\frac{1}{12}H^2 = 0 \tag{1.21}
$$

$$
\begin{aligned}
R_{MN}+2\nabla_M\nabla_N\phi-\frac{1}{4}H^2_{MN} &= e^{2\phi}\left(\frac{1}{2}F^2_{2MN}+\frac{1}{12}F^2_{4MN}\right)+ \\
&- \frac{e^{2\phi}}{4}g_{MN}\left(F_0^2+\frac{1}{2}F_2^2+\frac{1}{24}F_4^2\right)\ .
\end{aligned} \tag{1.22}
$$

Notice that the Einstein equation (1.22) immediately reduces to the vanishing of the β-function for g (1.11) in the absence of RR fluxes. The same is true for the dilaton equation of motion (1.21) and the β-function for ϕ (1.12) once combined with the previous equation. The equations of motion for the fluxes, obtained by varying the action (1.15) with respect to the their potentials, are given by

$$
d(e^{-2\phi}\star H) = F_2\wedge\star F_4-\frac{1}{2}F_4\wedge F_4-F_0\star F_2 \tag{1.23}
$$

$$
d(\star F_2) = -H\wedge\star F_4 \tag{1.24}
$$

$$
d(\star F_4) = -H\wedge F_4\ . \tag{1.25}
$$

The action (1.15) can be made invariant under ten-dimensional $\mathcal{N}=2$ supersymmetry once fermions are added. However, since our focus in this work will be on non-supersymmetric solutions, we do not write here the complete action nor its supersymmetry variations. The interested reader can look in [26] and references therein.

When $F_0 = 0$ and fermions are added, the action (1.15) can be obtained as a dimensional reduction from the unique $\mathcal{N}=1$ supergravity in eleven-dimensions. In particular, the Majorana spinor ε generating the supersymmetry variations of the eleven-dimensional supergravity decomposes into the couple of ten-dimensional of Majorana-Weyl spinors of opposite chiralities generating the supersymmetries of type IIA supergravity.

However, there exists another maximal supersymmetric supergravity theory in ten dimensions, whose supersymmetry generators are instead two spinors with the same chirality. This theory is called type IIB supergravity and describes the low-energy dynamics of the string theory of the same name. Other than for the chirality of the spinors generating their supersymmetries, IIA and IIB differ for the degrees of the p-form potentials appearing in the RR sector, with type IIB having only odd values of p. In particular, there is also a self-dual five-form field strength

$$F_5 = \star F_5 \ . \tag{1.26}$$

Its presence prevents the formulation of a compact action like (1.15), since in general it is hard to write down a Lagrangian for self-dual forms.[2] However, the bosonic part of a pseudo-action can be written as

$$\begin{aligned} S_{\text{IIB,bos}} &= \frac{1}{\kappa^2}\int_{M_{10}}\sqrt{-g}\left[e^{-2\phi}\left(R+4\nabla\phi\nabla\phi-\frac{1}{12}H^2\right)-\frac{1}{2}\left(F_1^2+\frac{1}{3!}F_3^2+\frac{1}{2}\frac{1}{5!}F_5^2\right)\right]+ \\ &- \frac{1}{2}C_4\wedge H\wedge dC_2 \ , \end{aligned} \tag{1.27}$$

where the definition of the field strengths in terms of the potentials is

$$H \equiv dB, \qquad F_1 \equiv dC_0, \qquad F_3 \equiv dC_2 - C_0 H, \qquad F_5 \equiv dC_4 - H\wedge C_2 \ . \tag{1.28}$$

The naming *pseudo* of this action refers to the fact that, upon variation, it does not enforces the self-duality constraint (1.26), which has then to be imposed by hand once the equations of motion have been derived. Notice that also in this case the factor $e^{-2\phi}$ does not appear in front of the whole action, since the RR fields have again been redefined to simplify the following Bianchi identities:

$$dH = 0 \tag{1.29}$$
$$dF_1 = 0 \tag{1.30}$$
$$dF_3 - H\wedge F_1 = 0 \tag{1.31}$$
$$dF_5 - H\wedge F_3 = 0 \ . \tag{1.32}$$

The dilaton and Einstein equations of motion now read:

$$R + 4\nabla^2\phi - 4\nabla\phi\nabla\phi - \frac{1}{12}H^2 = 0 \tag{1.33}$$

$$\begin{aligned} R_{MN} + 2\nabla_M\nabla_N\phi - \frac{1}{4}H^2_{MN} &= e^{2\phi}\left(\frac{1}{2}F^2_{1MN}+\frac{1}{4}F^2_{3MN}+\frac{1}{96}F^2_{5MN}\right)+ \\ &- \frac{e^{2\phi}}{4}g_{MN}\left(F_1^2+\frac{1}{6}F_3^2\right) \ . \end{aligned} \tag{1.34}$$

[2] See for example [36] for an approach to this problem.

Since the above equations differ from the IIA equations (1.21), (1.22) only for the RR fluxes, they also reproduce the vanishing of the β-functions (1.11), (1.12). The equations of motion for the fluxes now read

$$\begin{aligned} d(e^{-2\phi} \star H) &= F_1 \wedge \star F_3 + F_3 \wedge F_5 && (1.35)\\ d(\star F_1) &= -H \wedge \star F_3 && (1.36)\\ d(\star F_3) &= -H \wedge F_5\ . && (1.37) \end{aligned}$$

Even if we will not need it in the following, for completeness let us notice that an important feature of the action (1.27) is its invariance under an $\mathrm{SL}(2,\mathbb{R})$ transformation. This transformation is more easily described by switching to the Einstein frame metric $g_E = e^{-\frac{\phi}{2}} g$ and defining the three-form

$$\widetilde{F_3} \equiv F_3 + C_0 H\ , \tag{1.38}$$

and the complex scalar (usually called *axio-dilaton*)

$$\tau \equiv C_0 + i e^{-\phi}\ . \tag{1.39}$$

Given an element $S\in$SL$(2,\mathbb{R})$ of the form

$$S \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}\ , \tag{1.40}$$

the classical action (1.27) is invariant under the transformation

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} \tilde{F}_3 \\ H \end{pmatrix} \to \begin{pmatrix} \tilde{F}_3' \\ H' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{F}_3 \\ H \end{pmatrix} \tag{1.41}$$

which leaves the Einstein-frame metric g_E and the five-form F_5 untouched. $\mathrm{SL}(2,\mathbb{R})$ is not a symmetry of the full quantum theory, but its $\mathrm{SL}(2,\mathbb{Z})$ subgroup is; the S-duality we have introduced in the previous section is an element of this discrete subgroup.

1.2.2 Coupling with the open string sector

We have described the closed string sector through its low-energy action. In the same spirit, a low-energy action can be obtained for the open-string spectrum, thus describing the dynamics of the Dp-branes. Its bosonic part is given by

$$S_{D_p} = -T_p \int_\Sigma e^{-\phi} \sqrt{-\det(g+\mathcal{F})_\Sigma} + Q_p \int_\Sigma C_{p+1} \wedge e^{\mathcal{F}}\ , \tag{1.42}$$

where Σ is the world-volume of the p-brane, and we have defined

$$\mathcal{F} = B + 2\pi\alpha' f\ , \tag{1.43}$$

where f is the curvature of the gauge field living on the p-brane, induced by the open strings ending on it. By the notation $(\ldots)_\Sigma$ we mean the pullback on the world-volume Σ of the space-time tensors, which does not apply to f being already defined on it. Explicitly, the metric on the brane is $g_{\alpha\beta} = g_{MN}\partial_\alpha X^M \partial_\beta X^N$, where the X^I's are the transverse scalars describing its embedding in the space-time. We have already encountered the first term appearing in (1.42), the DBI action, which describes the gravitational coupling of the brane with the background. The second term describes instead the electric coupling to the RR sector and it is called *Wess-Zumino* (WZ) action. It generalizes the usual coupling of a charged particle to an external electric field. The branes we will consider preserve half of the supersymmetry of the original type II theories, making them stable objects. For this to happen their charge has to be equal to their tension

$$Q_p = T_p. \tag{1.44}$$

Notice that, for dimensional reasons, from the WZ coupling in the action (1.42) we can see that only Dp-branes with even p appear in type IIA and with odd p in type IIB. Both of the terms in (1.42) arise from tree-level open-string interactions, so they both need to involve an $e^{-\phi}$ factor. In the WZ term this factor does not appear due to the redefinition of the RR potentials we have described in the previous section. As for the supergravity actions, being the action (1.42) the first term of an expansion in g_s, it is well-defined only in regions of the space-time where e^{ϕ} is small.

1.2.3 D-brane and O-plane solutions

We now describe an important class of solutions of the equations of motion of the supergravity theories we have introduced in section 1.2.1. These solutions are the low-energy avatars of the Dp-branes and Op-planes appearing in the full string theories. This description is based on [27].

The solution of the appropriate ten-dimensional type II supergravity theory describing a Dp-brane in flat space-time is given by

$$ds^2 = H_p^{-1/2} dx_{p+1}^2 + H_p^{1/2}(dr^2 + r^2 ds^2_{S^{8-p}}) \,, \tag{1.45}$$

where dx_{p+1}^2 is the space parallel to the Dp-brane and H_p is a harmonic function defined on its transverse flat $(9-p)$-dimensional space. Explicitly, in the various cases the H_p's are given by

$$H_p = 1 + \frac{h_p}{r^{7-p}} \qquad \text{for}\, p < 7, \qquad H_7 = 1 - h_7 \log(r), \qquad H_8 = 1 - h_8 |r| \tag{1.46}$$

where the constants h_i's are defined as

$$h_p = \frac{(2\pi l_s)^{7-p} g_s}{(7-p)\omega_{(8-p)}} \qquad \text{for}\, p < 7, \qquad h_7 = \frac{g_s}{2\pi}, \qquad h_8 = \frac{g_s}{4\pi l_s} \tag{1.47}$$

with $\omega_{(8-p)}$ being the volume of a unitary radius $(8-p)$-dimensional sphere. The other non-vanishing fields are given by

$$e^{\phi} = g_s H_p^{-\frac{p-3}{4}}, \qquad C_{p+1} = g_s^{-1}(H_p^{-1} - 1)\mathrm{vol}_{p+1} \,, \tag{1.48}$$

where vol_{p+1} is the volume form of the space parallel to the brane. In Figure 1.1 we plot the qualitative behavior of H_p and in Figure 1.2 the corresponding profile for the dilaton.

Let us briefly analyze the main features of the various cases.

- $p = 8$. In this case the transverse space is one-dimensional. Hence, the coordinate r is not a radial coordinate and the solution is defined also for $r < 0$. However, there is a maximum distance $r_0 = \pm 4\pi \frac{l_s}{g_s}$ where the solution stops making sense since H_8 becomes negative and the square roots appearing in (1.45) make the metric imaginary.

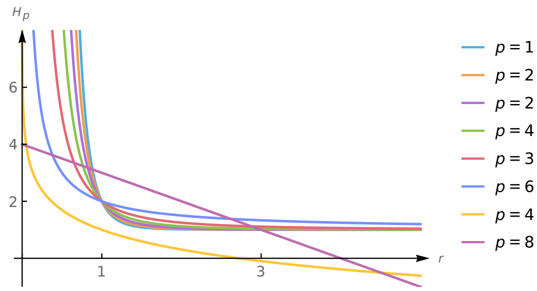


Figure 1.1: The function H_p for $1 \leqslant p \leqslant 8$.

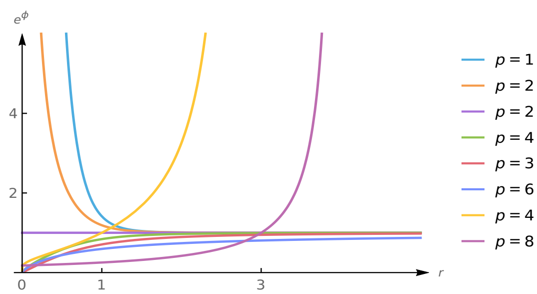


Figure 1.2: The behavior of the e^ϕ for the Dp-brane solutions. Here $g_s = 1$.

- $p = 7$. The situation is similar to the $p = 8$ case, with a maximum $r_0 = e^{\frac{2\pi}{g_s}}$.
- $3 < p < 7$. The string coupling e^ϕ is bounded by the constant g_s at $r = \infty$, which can be taken arbitrarily small such that the solution can be trusted at tree-level in the string coupling. However, the curvature blows up approaching $r = 0$.
- $p = 3$. The string coupling is a constant, and it can be taken to be arbitrarily small. The harmonic function has a pole, but $R = 0$ and also other curvature invariants remain finite. It has been argued that $r = 0$ is not a physical singularity and the solution could be continued beyond this point [37].
- $p < 3$. In this case the situation is reversed: the curvature goes to zero as $r = 0$ but the string coupling blows up.

Finally, we also observe that the appearance of the harmonic functions H_p's with a pole at $r = 0$ is due to the fact that for the particular ansatz (1.45), the equations of motion computed in the presence of the DBI action are schematically of the form

$$\Delta H_p = T_p \delta_p \, , \tag{1.49}$$

where δ_p are δ-functions localized at $r = 0$. The Laplacian in (1.49) is defined with respect to the flat metric on the transverse space and T_p is the tension of the brane. Thus the non-linear gravitational Einstein's equation reduce in this case to a simple linear equation. This phenomenon will not happen in more general cases.

Let us now discuss the solutions associated to Op-planes with negative tension. Such solutions are obtained by changing the signs in H_p (and introducing the relative factor 2^{p-5})

$$H_p^O = 1 - 2^{(p-5)}\frac{h_p}{r^{7-p}} \qquad \text{for } p < 7, \qquad H_7^O = 1 + 4h_7 \log(r), \qquad H_8^O = 1 + 8h_8|r|. \tag{1.50}$$

In Figure 1.3 we plot these functions and in Figure 1.4 we show the corresponding profile for the string coupling.

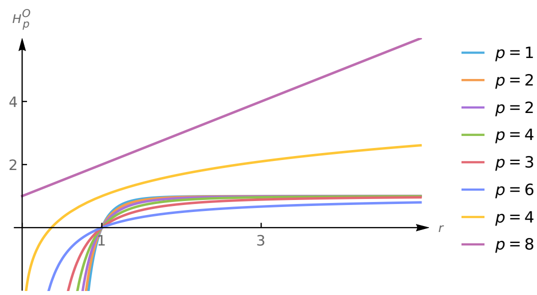


Figure 1.3: The behavior of the function H_p^O for the various Op-planes.

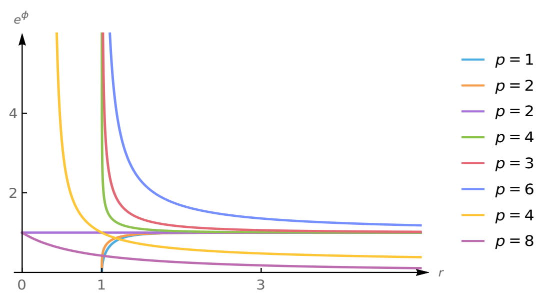


Figure 1.4: The behavior of e^ϕ for Op-planes in flat space. Here $g_s = 1$.

The qualitative behavior of the various cases is the following.

- $p < 8$. The solution ceases to make sense at a finite distance from the Op-plane. In contrast with the positive-tension case, the region where the object is located lies *outside the space-time*. We call this formal "hidden region" the *hole produced by an O-plane.* However, notice that this region is very small, being it of order $r_0 \sim l_s g_s^{\frac{1}{7-p}}$ for $p < 7$ and $r_0 \sim e^{-\frac{\pi}{2g_s}}$ for $p = 7$.
- $3 < p < 8$. The value of the string coupling blows up approaching the boundary of the hole produced by the Op-plane from outside. This makes the supergravity approximation alone not trustable when approaching this singularity.

- $p = 8$. The singularity is milder and it only amounts to a discontinuity on the derivatives of the various functions. Moreover, the string coupling is bounded everywhere. However, for $p = 8$ there is also a qualitative different solution to the equations of motion, which is given by
$$\widetilde{H_8} = 8h_8|r| \; . \tag{1.51}$$
Now at $r = 0$ the string coupling e^{ϕ} blows up:
$$e^{\phi} \sim |r|^{-\frac{5}{4}} \; , \tag{1.52}$$
making it more similar to the other Op-planes for $p > 3$, with the hole shrunk to a single point. The metric describing this solution reads:
$$ds^2_{10} = (h_8 r)^{-\frac{1}{2}} ds^2_{\|} + (h_8 r)^{\frac{1}{2}} dr^2 \; . \tag{1.53}$$

- $p = 3$. The solution still has the unphysical region around $r = 0$, but the string coupling is a constant which can be taken to be everywhere arbitrarily small.

- $p < 3$. Again the situation is reversed. These Op-planes still have the unphysical hole for $r < r_0$ but the string coupling goes to zero when approaching their boundary from outside.

Finally, let us introduce here the low-energy effective actions describing Op-planes. As we have already argued, these are non-dynamical objects, but their backreaction on the space-time can be taken into account with an action similar to the one describing Dp-branes. However there will be no gauge fields now living on their world-volume, and no scalars associated to transverse fluctuations:
$$S_{O_p} = -T_{Op} \int_{\Sigma} e^{-\phi} \sqrt{-\det(g+B)_{\Sigma}} + Q_{O_p} \int_{\Sigma} C_{p+1} \wedge e^{B} \quad , \tag{1.54}$$
where the WZ action takes into account the fact that they are also charged under the RR potentials. Their charge is related to the one of the corresponding Dp-branes by $Q_{Op} = -2^{p-5} Q_{Dp}$.

Notice that this action is now somewhat formal since, as we have just shown, in the supergravity approximation Σ often lives outside of the physical space-time, in a region where both the metric and the string coupling are imaginary. Moreover, for $p > 3$ the string coupling starts growing when reaching the boundary of this region from outside, making the validity of the tree-level action questionable. Nevertheless, the solutions we have just described are thought to be genuine solutions of the full string theory, even though they are dubious as solutions of supergravity, because of their world-sheet definition.

1.2.4 Type $\tilde{I}$

The orientifold constructions that will be more relevant to us are the one connecting the type II string theories to the so-called type $\tilde{I}$ model. In this section, we briefly describe them.

Starting from type IIB string theory, we can quotient the theory by the parity symmetry acting on the world-sheet, without doing anything explicit on the space-time. Thus, the orientifold plane associated to the quotient with respect to this symmetry is the whole ten-dimensional space-time of type IIB, or, equivalently, an O9-plane. Since orientifold planes carry RR charge, they couple to the corresponding RR potential through the WZ action we have introduced in (1.42). For an O9-plane this coupling introduces the term

$$S_{\rm WZ} \propto \int_{M_{10}} C_{10} \tag{1.55}$$

into the action. However, the C_{10} gauge form does not appear anywhere else in the action, thus its equations of motion can never be satisfied. This corresponds to a tadpole, and in particular to an RR tadpole since it comes from the RR sector. In order to avoid inconsistencies in the theory we need to cancel it. This can be done by adding a suitable number of D9 branes, such that the total charge vanishes.[3] In particular, since the charge of an O9-plane is 16 times the charge of a D9 brane we need 16 of them. In this way the coefficient in front of the term (1.55) in the action vanishes and its equations of motion are trivially satisfied. The addition of these space-time filling D9 branes brings the corresponding open strings attached to them but, since D9-branes fill the whole space-time, these open strings are free to move anywhere and the gauge fields associated to them now add a non-abelian gauge sector to the theory. This gauge group can be computed to be SO(32). Moreover, the quotient of the starting type IIB theory by the parity symmetry on the world-sheet has produced a theory of unoriented strings, since the only remaining Riemann surfaces are non-orientable. The theory obtained in this way has half of the supersymmetry of the original theory and it turns out to be one of the known perturbative type I string theories we have introduced in section 1.1. By performing a T-duality, this theory is dual to the so-called type I'.

The type I' can also be understood more directly from type IIA string theory. In this case, parity alone is not a symmetry of the world-sheet theory, and to obtain a symmetry we have to combine it with an involution acting on the space-time. This defines a theory on half of the space. If we combine this action with a compactification on a circle of the reflected coordinate, we obtain the type I' model, where the two fixed loci are two O8-planes. The RR tadpole now requires to cancel the total charge in the compact space, which in turn requires the addition of 16 D8-branes.

Let us describe this situation from the supergravity point of view. Since we only have nine-dimensional sources, we include only F_0 in the action:

$$\begin{aligned} \kappa^2 S &= \int_{M_{10}} \sqrt{-g} e^{-2\phi}(R + 4(\nabla\phi)^2) - \frac{1}{2}\int_{M_{10}} \sqrt{-g} F_0^2 + \\ &+ \sum_i -\kappa^2 \tau_i \int_{\Sigma_i} \sqrt{-g|_{\Sigma_i}} e^{-\phi} + \kappa^2 \tau_i \int_{\Sigma_i} C_9|_{\Sigma_i} \\ &\equiv \kappa^2 S_{\rm bulk} + \kappa^2 S_{\rm loc} \ , \end{aligned} \tag{1.56}$$

where the index i runs over all the sources and

$$F_{10} = dC_9 = \star F_0, \qquad F_0 = -\star F_{10}. \tag{1.57}$$

By writing the localized term in this way, we are assuming that there are only objects with opposite values for the tension and the charge. In the presence of sources, the Bianchi identity for F_0 will

[3] Notice that in this way also the total tension vanishes. If we instead consider configurations where the total charge vanishes but not the total tension, we could find a tadpole in the NS-NS sector, for example generating a potential for the dilaton. This does not signal an inconsistency of the corresponding string theory, but only that flat space-time with constant dilaton is not a solution of the field equations anymore. Some discussions on these models can be found in [38] [39] [40] [41].

now be modified, as can be easily checked by computing the equation of motion for the C_9 potential:

$$
\begin{aligned}
0 = \delta_{C_9}(\kappa^2 S) &= \delta\left(-\frac{1}{2}\int\sqrt{-g}F_0^2 + \kappa^2\tau\int\delta(z-z_0)C_9\wedge dz\right) &(1.58)\\
&= \delta\left(-\frac{1}{2}\int F_{10}\wedge\star F_{10} + \kappa^2\tau\int\delta(z-z_0)C_9\wedge dz\right) &(1.59)\\
&= -\int d\delta C_9\wedge\star dC_9 + \kappa^2\tau\int\delta(z-z_0)\delta C_9\wedge dz &(1.60)\\
&= -\int \delta C_9\wedge d\star F_{10} + \kappa^2\tau\int\delta(z-z_0)\delta C_9\wedge dz\ . &(1.61)
\end{aligned}
$$

Hence we get

$$
d\star F_{10} = \kappa^2\tau\delta(z-z_0)dz \qquad\Rightarrow\qquad dF_0 = -\kappa^2\tau\delta(z-z_0)dz\ . \tag{1.62}
$$

Integrating the above equation across the i-th source sitting at $z = z_i$ we thus obtain

$$
\Delta_i F_0 = -\kappa^2\tau_i \qquad \text{at}\ z = z_i. \tag{1.63}
$$

As we have already remarked, consistency of this compactification requires the presence of 16 D8-branes. We consider a simplified setting with only two stacks of D8-branes, with the circle described by the coordinate z divided in 4 regions as in Figure 1.5.

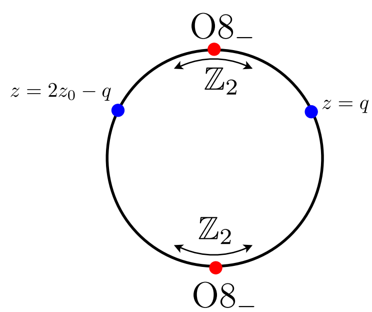


Figure 1.5: The circle described by the coordinate z in the type I' model. The two fixed loci of the $\mathbb{Z}_2$ involution are $z = 0$ and $z = z_0$. At $z = q$ and $z = 2z_0 - q$ two stacks of 8 D8-branes each are located.

Consistently with (1.63), in the four regions the Romans mass takes the values

$$
2\pi F_0 = \begin{cases} 4 & 0 < z < q \\ -4 & q < z < z_0 \\ 4 & z_0 < z < 2z_0 - q \\ -4 & 2z_0 - q < z < 2z_0 \end{cases} \quad , \qquad F_0^2 = \frac{4}{\pi^2}. \tag{1.64}
$$

The *local* solution of the equations of motion of the theory is given by

$$
ds^2 = H^{-1/2}ds^2_{\text{Mink}_9} + H^{1/2}dz^2 \tag{1.65}
$$

and

$$e^{\phi} = g_s H^{-5/4} \qquad H = a + g_s F_0 z. \tag{1.66}$$

Imposing the continuity of H at the location of all the sources fixes H in the four regions to be

$$H = \begin{cases} a + g_s|F_0|z & 0 < z < q \\ a + g_s|F_0|(2q - z) & q < z < z_0 \\ a + g_s|F_0|(2q - 2z_0 + z) & z_0 < z < 2z_0 - q \\ a + g_s|F_0|(2z_0 - z) & 2z_0 - q < z < 2z_0 \end{cases} , \qquad |F_0| = \frac{4}{2\pi}. \tag{1.67}$$

Notice that for the metric and the dilaton to be well-defined we also have to require $H > 0$ and thus

$$a \geqslant g_s(z_0 - 2q)|F_0|. \tag{1.68}$$

We can now consider the limit $q \to 0$ of this configuration. This would correspond to put all the D8-branes on top of the $O8_-$ plane at $z = 0$. The resulting object as the same tension and charge of an $O8_+$ plane, which for our purposes is simply an $O8_-$ with the sign of the tension and the charge reversed. From the supergravity point of view the description in terms of $O8_- + 16$ D8 or that of an $O8_+$ are indistinguishable, but not from the string theory point of view, where they differ because of the open-string degrees of freedom attached to the D8-branes. The configuration with an $O8_+$ and an $O8_-$ plane is known as type $\tilde{I}$ model, and it will be important for us in Chapter 3. Finally, let us conclude this section by noticing that, as already emphasized in [42], in the limit where the inequality (1.68) is saturated, the dilaton blows up on the $O8_-$ with fewer D8's on top of it. This particular case, corresponding to the case (1.51) of the previous section, would be suspicious from the supergravity point of view, but its definition through the world-sheet theory makes this singular supergravity solution trustable.

Chapter 2

AdS_8 solutions

We start our discussion of non-supersymmetric backgrounds of string theory by looking for solutions preserving all the symmetries of the eight-dimensional Anti de Sitter (AdS) space. The motivation for constructing backgrounds of this kind is twofold. On one hand, by enforcing the symmetries of AdS_8, we are guaranteed not to fall into supersymmetric solutions in disguise, since it is known that no superconformal group admits a bosonic subgroup which includes the isometry group of AdS_8. Let us be a bit more precise. The isomorphism between the $(d-1)$-dimensional conformal group and the isometries of the d-dimensional AdS space is at the hearth of the $\mathrm{AdS}_d/\mathrm{CFT}_{d-1}$ correspondence, as the first requirement for such a correspondence to hold is that the symmetries on both sides coincide. By virtue of this identification, the study of possible supersymmetric AdS solutions is mapped to the study of supersymmetric extensions of the conformal group. The fact that a certain superalgebra exists does not guarantee that a corresponding supersymmetric AdS background exists, but it is a necessary condition. Moreover, as anticipated, it has been shown long ago [43, 44] that for $d > 7$ no superconformal algebra whose bosonic subalgebra contains $\mathfrak{so}(d-1,2)_{\mathbb{C}}$, and whose fermionic subalgebra is in the spinor representation, exists. This result rules out supersymmetric AdS_8 solutions. Actually, as can be seen from [44], this is a fairly general feature: superconformal algebras are very constrained, and in the cases in which they exist it is due to 'accidental' isomorphisms between some low-dimensional Lie algebras.

We have thus learned that all the AdS_8 backgrounds, if any, are necessarily non-supersymmetric. From both the technical and the conceptual point of view this introduces some challenges. On the technical side, working without the help of supersymmetry requires us to directly solve the full set of second order non-linear partial differential equations that are the equations of motion. In supersymmetric cases this can be avoided, since it can be shown that it is often enough to impose the first order equations that enforce the symmetry, plus the equations of motion for the fluxes, to automatically obtain solutions to all the equations of motion (see e.g. [45] for massive type IIA and [46] for eleven-dimensional supergravity)

Due to to the powerful constraints imposed by supersymmetry, in some cases it possible to completely classify all the solutions of a given space-time dimension. For example, all the supersymmetric AdS_7 and AdS_6 solutions have been by now completely classified (see [47, 48, 49] for AdS_7 and [50, 51] for AdS_6). Actually this represents a small corner of the landscape problem, i.e. the study of all the possible consistent backgrounds of string theory. As we have argued, in the non-supersymmetric sector it is much harder, and various conjectures have been proposed to rule out various regions of the non-supersymmetric landscape. Regardless of these difficulties, we can hope that the AdS_8 case constitutes a constrained enough case where explicit progress towards a classification can be made. This is our second motivation to look at this particular example.

As we are going to see, the requirement that all the AdS_8 isometries are preserved highly constrains the possible fluxes, and leaves us with only a two-dimensional space (plus the dilaton) to be determined. In such a restricted scenario, a general no-go theorem, which we review in section 2.1, forbids the existence of smooth solutions. Albeit quite constrained, the full equations of motion still constitutes a system of coupled non-linear PDEs in two dimensions which we are not able to solve in full generality. In order to make progress and exhibit explicit solutions, we then propose an extra ansatz which imposes and extra U(1) symmetry. This ansatz will be motivated by the physical source that can introduce the possible singularities needed to evade the no-go theorem. With this simplification we get a system of ODEs, which we are then able to solve numerically.

As anticipated, all the solutions we find have to involve some kind of singularity. The physical significance of these solutions depends on the fact that these singularities can be understood as the breakdown of the supergravity approximation near a physical object described by the full string theory. For most of the singularities we will encounter we will not be able to make such an identification and we discard them. However, we also find a class of singularities for which this identification is possible. In particular, the leading order behavior of the non-trivial fields (the metric and the dilaton) matches at first order in the distance from the object the behavior that we recognize to be the one of an $\mathrm{O8}_-$ in flat space, the one of diverging-dilaton type described in (1.51).

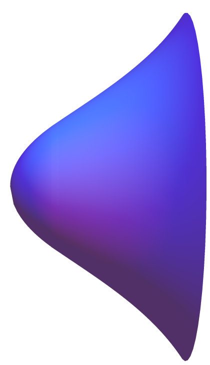


Figure 2.1: Half of the internal space for a typical AdS_8 solution. The topology of the full internal space is the one of an S^2, with an O8-plane where both the metric and the dilaton diverge sitting at its equator.

This behavior is not entirely unexpected, since diverging dilaton O8-planes also appear in many known AdS supersymmetric solutions, e.g in AdS_6 [52], AdS_7 [53] and AdS_3 [54]. The fact that the dilaton blows up makes the validity of all these solutions questionable, since they have been found in supergravity, a weakly coupled description of string theory only valid in regions of the space-time where the dilaton is small.

In some cases, it is possible to find independent arguments to believe that similar solutions can be lifted to full-fledged string theory solutions. For example, for the singularity associated to Op-planes in flat space we tend to believe them due to their world-sheet description. For supersymmetric AdS solutions, we have the possibility to address the validity of these backgrounds through holography. For example, for AdS_7 this has been done in [55], where a perfect match with the field theoretic results has been found. In our non-supersymmetric situation the question is still open and the existence of these backgrounds in a fully UV completed theory is not yet known.

We can take these open questions as an opportunity to investigate non-supersymmetric holography, for example with the approach of the conformal bootstrap, in a well-defined setup. For a pedagogical review see e.g. [56]. Roughly speaking, conformal field theories are so constrained that they are specified by a set of data called *CFT data*. For the most part, these data are just real numbers. However, not every choice for this set of real numbers gives a consistent theory, and the idea of the conformal bootstrap is to find which numbers work by starting from some 'guesses' and checking if they satisfy all the constraints of a unitary CFT. The first proposals of this idea go back to the 70's [57, 58]. In practice, it is not yet possible to impose *all* the constraints, which are not even known in general. What we are currently able to do is to put some bounds on the CFT data, obtaining some allowed/not-allowed regions in the space of putative CFT data. Remarkably, by exploiting just a small subset of the constraints, these allowed regions have shapes that point towards the existence of interesting CFTs. For example, in the three-dimensional case by considering just single [59] and multiple [60] scalar correlators it has been possible to sharply isolate a region in the parameter space that it is thought to describe the three-dimensional Ising model. The power of these methods is that they do not rely on a Lagrangian weakly coupled description of the theory, going beyond the perturbative definition. For example in [61] bootstrap techniques have been used to study six-dimensional the $\mathcal{N} = (2, 0)$ supersymmetric CFT describing the IR limit of a stack of M5 branes in M-theory, which has no known Lagrangian description.

However, the possibility that non-supersymmetric holography is realizable in string theory is not settled (see [62] for a recent proposal). Indeed, among the proposed criteria to rule out non-supersymmetric parts of the string landscape we find the conjecture that all the non-supersymmetric AdS backgrounds are unstable. In particular, it is claimed that non-supersymmetric AdS holography is not realizable in consistent quantum theories that at low energies reduce to Einstein gravity coupled to a finite number of fields [63]. As we are going to show in section 2.4, the proposed decay channels that would make these backgrounds unstable are not present in our solution. An interesting complement to this stability analysis would be to perform a full Kaluza Klein reduction. Albeit our background is quite simple, the presence of a non-trivial warping and its numerical nature background makes it complicated to analyze the full spectrum in detail. We leave all these interesting open questions to a future work [4].

In the meantime, a paper trying to address the validity of these singular solutions in supergravity itself has been put out [64], in which the authors argue that they are instead not allowed. As we are going to carefully review in section 3.2 of the next chapter, where we encounter these O8-planes in a different setup, the proposed argument to invalidate these solutions states that identifying the leading-order behavior near the singularities is not enough, and it shows that trying to match also the sub-leading one with the flat space case will result in no allowed solutions. We think that this is well beyond the regime of applicability of the supergravity approximation, as even the leading order behavior is going to be corrected by the full equations of motion of string-theory near the strongly-coupled region. We postpone a more detailed discussion to section 3.2. This chapter is based on the published work [1], where also more general solutions than the ones we present here can be found, including an analysis of the construction of AdS_8 solution in type IIB supergravity.

2.1 No smooth solutions

In this section we specialize to our setting a classical no-go theorem due to Maldacena and Núñez, [65] which is used in many contexts to exclude various regions of the allowed string-theory landscape.

For concreteness, we specialize it to our case, and we show that it forbids the existence of AdS_8 backgrounds with smooth compact internal spaces without boundaries. We work in massive type IIA supergravity, but it is be immediate to see that the same result also holds in type IIB. For the purposes of this section, it is more convenient to work in the Einstein frame, which is obtained with the following rescaling of the metric:

$$g_E \equiv e^{-\frac{\phi}{2}} g_S . \tag{2.1}$$

Applying this rescaling to the type IIA string-frame action in (1.15) we obtain

$$\begin{aligned}\kappa^2 S^E_{\mathrm{IIA}} &= \int_{M_{10}} \sqrt{-g}\left(R - \frac{1}{2}\nabla\phi\nabla\phi - \frac{1}{12}e^{-\phi}H^2 - \frac{1}{2}\left(e^{\frac{5}{2}\phi}F_0^2 + \frac{1}{2!}e^{\frac{3}{2}\phi}F_2^2 + \frac{1}{4!}e^{\frac{1}{2}\phi}F_4^2\right)\right) + \\ &- \frac{1}{2}\left(dC_3\wedge dC_3\wedge B + \frac{F_0}{3}dC_3\wedge B\wedge B\wedge B + \frac{F_0^2}{20}B\wedge B\wedge B\wedge B\wedge B\right) ,\end{aligned} \tag{2.2}$$

where now all the metric-related quantities are written with respect to the metric g_E. This change of frame has brought the gravitational part of the action to be the canonical Einstein-Hilbert term. As we are going to see in section 3.2.2 the action (2.2) is equivalent to (1.15) up to boundary terms, which we discard since here we are interested in internal spaces without boundaries.

The equation of motion for the dilaton derived from the action (2.2) reads

$$\nabla^2\phi = \frac{5}{4}e^{\frac{5}{2}\phi}F_0^2 + \frac{3}{4}\frac{1}{2!}e^{\frac{3}{2}\phi}F_2^2 + \frac{1}{4}\frac{1}{4!}e^{\frac{1}{2}\phi}F_4^2 - \frac{1}{12}e^{-\phi}H^2 , \tag{2.3}$$

where

$$\nabla^2\phi = -\Delta\phi = \frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}\partial_N\phi\right) . \tag{2.4}$$

We now specialize these equations to our case at hand. First of all, notice that the NS-flux H and the RR-flux F_4 have to vanish, otherwise they would kill the AdS_8 symmetry:

$$H = F_4 = 0 . \tag{2.5}$$

The most general ten-dimensional metric that preserves the AdS_8 isometries can then be written in the form

$$ds^2_E = e^{2A}ds^2_{\mathrm{AdS}_8} + ds^2_{M_2} , \tag{2.6}$$

with the function A only depending on the coordinates of M_2. Observe that with this definition

$$\sqrt{-g} = e^{10A}\sqrt{-g_8}\sqrt{g_2} , \tag{2.7}$$

where g_8 and g_2 are respectively the determinants of the AdS_8 metric and of the metric on M_2. Moreover, to preserve the AdS_8 symmetry also ϕ cannot depend on the space-time coordinates, so that its second derivative becomes simply

$$\nabla^2\phi = \frac{e^{-10A}}{\sqrt{g_2}}\partial_m\left(\sqrt{g_2}e^{10A}g_2^{mn}\partial_n\phi\right) , \tag{2.8}$$

where m, n are indices in the internal space.

We can then plug these results into the dilaton equation of motion obtaining

$$\frac{e^{-10A}}{\sqrt{g_2}}\partial_m\left(\sqrt{g_2}e^{10A}g_2^{mn}\partial_n\phi\right) = \frac{5}{4}e^{\frac{5}{2}\phi}F_0^2+\frac{3}{4}\frac{1}{2!}e^{\frac{3}{2}\phi}F_2^2\ . \tag{2.9}$$

Assuming everything is smooth, we can multiply the above equation by $\sqrt{g_2}e^{10A}$ obtaining a total derivative

$$\partial_m\left(\sqrt{g_2}e^{10A}g_2^{mn}\partial_n\phi\right) = \sqrt{g_2}e^{10A}\left(\frac{5}{4}e^{\frac{5}{2}\phi}F_0^2+\frac{3}{4}\frac{1}{2!}e^{\frac{3}{2}\phi}F_2^2\right)\ . \tag{2.10}$$

We can now integrate (2.10) on the internal space, and since it is compact and without boundaries we get

$$0=\int_{M_2}\sqrt{g_2}e^{10A}\left(\frac{5}{4}e^{\frac{5}{2}\phi}F_0^2+\frac{3}{4}\frac{1}{2!}e^{\frac{3}{2}\phi}F_2^2\right)\ . \tag{2.11}$$

Given that both terms in the brackets are non-negative we conclude that the only possibility for equation (2.11) to hold is to set

$$F_0=F_2=0\ . \tag{2.12}$$

With this result at hand, we can now look at the Einstein equation. In the cases where all the fluxes vanish it can be simplified to

$$R_{MN}=\frac{1}{2}\nabla_M\phi\nabla_N\phi\ . \tag{2.13}$$

We now specialize it to directions along AdS_8. Before doing so we recall that the Ricci tensor of the warped product (2.6) decomposes as

$$\begin{aligned} R_{\mu\nu} &= \Lambda g_{\mu\nu}^{\mathrm{AdS}_8}-e^{2A}g_{\mu\nu}^{\mathrm{AdS}_8}(8(\nabla A)^2+\nabla^2A) &(2.14)\\ &= g_{\mu\nu}^{\mathrm{AdS}_8}\left(\Lambda-\frac{1}{8}e^{-6A}\nabla^2(e^{8A})\right), &(2.15)\end{aligned}$$

where μ, ν are indices on AdS_8 and Λ is defined by the equation

$$R_{\mu\nu}^{\mathrm{AdS}_8}\equiv\Lambda g_{\mu\nu}^{\mathrm{AdS}_8}\ . \tag{2.16}$$

Finally, we notice that the right hand side of equation (2.13) evaluates to zero since for symmetry ϕ cannot depend on the coordinates of AdS_8. Equating the two sides of (2.13) we then get

$$\Lambda=\frac{1}{8}e^{-6A}\nabla^2(e^{8A})\ . \tag{2.17}$$

Similarly to what we did for the dilaton equation in (2.9), we can expand the second derivative and, assuming everything is smooth, multiplying by the appropriate power of the warping factor we obtain a total derivative. Integrating on the internal space then gives $\Lambda=0$.

From this result, we learn that AdS_8 solutions have to be necessarily singular if the internal space is compact and without boundaries. In the next section we will obtain and analyze solutions with this property.

2.2 Equations of motion

In this section we derive the equations of motion that AdS_8 backgrounds have to satisfy and we describe some of their features. We already know that they only admit singular solutions, which we need to interpret, if possible, as the presence of localized sources. However, for the time being we focus on the local equations of motion, away from the sources.

From now on, we will always work in the string frame. The most general metric that preserves the isometries of AdS_8 is

$$ds^2 = e^{2W} ds^2_{\mathrm{AdS}_8} + ds^2_{M_2} \ , \tag{2.18}$$

where W is a function only depending on M_2, which we will call *warp factor*. This warp factor is related to the function A we defined in the previous section by

$$W = A - \frac{\phi}{4} \ . \tag{2.19}$$

We fix the radius of AdS_8 such that it is an Einstein space with Einstein constant -1. Finally, the manifold M_2 is assumed to be compact. By looking at the equations of motion for the fluxes we immediately see some constraints. Plugging into the equation (1.23) the requirement that both F_4 and H vanish by symmetry, we are left with the condition

$$F_0 \star F_2 = 0 \ , \tag{2.20}$$

from which we see that either F_0 or F_2 has to vanish. We will focus on the case $F_2 = 0$ for the rest if this thesis, since the case where $F_0 = 0$ does not gives interesting solutions. More details can be found in [1].

The full set of the remaining equations of motion then reads

$$\begin{aligned}
0 &= R - 8e^{-2W} - 72(\nabla W)^2 - 16\nabla^2 W + 4\nabla^2\phi + 32\nabla W \nabla\phi - 4(\nabla\phi)^2 && (2.21)\\
0 &= e^{-2W} + 8(\nabla W)^2 + \nabla^2 W - 2\nabla W \nabla\phi - \frac{1}{4}F_0 e^{2\phi} && (2.22)\\
0 &= R_{\alpha\beta} - 8(\nabla_\alpha W \nabla_\beta W + \nabla_\alpha\nabla_\beta W) + 2\nabla_\alpha\nabla_\beta\phi + \frac{1}{4}F_0 e^{2\phi} g_{\alpha\beta} \ , && (2.23)
\end{aligned}$$

where all the metric-related objects are computed with respect to $g_{\alpha\beta}$, the metric on M_2. Unfortunately we are not able to find solutions of this system of partial differential equations as it is, hence we are going to simplify it in the next section.

2.2.1 Reduction to ODEs

In order to get explicit results, we will now *assume* an extra symmetry, namely an $U(1)$ isometry. This choice will reduce the system of PDEs we have obtained in the previous section to a more tractable system of ODEs. This ansatz is motivated by the fact that the only flux turned on is F_0. This flux couples to D8-branes and O8-planes, which are co-dimension 1 objects. By symmetry, these objects are extended along AdS_8 and one of the internal coordinates. We take this internal coordinate to be periodic, realizing the S^1 where this extra $U(1)$ symmetry acts. Thus, we make a cohomogeneity 1 ansatz, where we assume that all the functions depend on a single coordinate z. Our reduced ansatz for the metric now reads

$$ds^2_{10} = e^{2W} ds^2_{\mathrm{AdS}_8} + e^{-2Q}(dz^2 + e^{2\lambda} d\theta^2) \ , \tag{2.24}$$

where in these local coordinates θ parametrizes an S^1 and all the functions, including the dilaton, now depend on the coordinate z only. This ansatz gives the following system of equations of motion:

$$\begin{aligned}
0 &= -\lambda'' + 2\lambda'\phi' - (\lambda')^2 + Q'' + \lambda' Q' - 4e^{-2(Q+W)} - 8W'' - 8W'(\lambda' - 2\phi') + \\
&+ 2\phi'' - 2(\phi')^2 - 36(W')^2 \qquad (2.25)\\
0 &= e^{-2(Q+W)}(F_0^2 e^{2(W+\phi)} - 4) - 4W'' - 4W'(\lambda' - 2\phi') - 32(W')^2 \qquad (2.26)\\
0 &= \frac{1}{4}F_0^2 e^{2\phi - 2Q} - \lambda'' - (\lambda')^2 + Q'' + Q'(\lambda' - 8W' + 2\phi') - 8W'' - 8(W')^2 + 2\phi'' \qquad (2.27)\\
0 &= \frac{1}{4}F_0^2 e^{2\phi - 2Q} - \lambda'' + 2\lambda'\phi' - (\lambda')^2 + Q'' + Q'(\lambda' + 8W' - 2\phi') - 8\lambda' W' \,. \qquad (2.28)
\end{aligned}$$

Before discussing its proprieties, let us try to simplify it. As usual in general relativity, we can extract from the full set of the equations of motion a first order equation which acts as a constraint. In our case it is obtained as $\frac{1}{2}((2.25)-(2.27))$ and it reads:

$$(\phi')^2 = \phi'(\lambda' - Q' + 8W') - 2W'(2\lambda' - 2Q' + 7W') - 2e^{-2(Q+W)} - \frac{1}{8}F_0^2 e^{2\phi - 2Q}. \qquad (2.29)$$

We can then trade one of the equation entering in the linear combination, say the first one, for the first order equation (2.29). Moreover, it can be easily shown that one of the remaining second order equations can be eliminated, since it is a combination of the first order one, its derivative and the remaining ones. Before doing so, we fix the gauge redundancy in the definition of Q with the choice

$$Q = W. \qquad (2.30)$$

Then, eliminating one equation with the previous procedure, say (2.27)[1], and taking a linear combination of the remaining equations, we are left with the simpler system

$$\begin{aligned}
(\phi')^2 &= -\frac{1}{8}e^{-4W}(F_0^2 e^{2(W+\phi)} + 16) + \phi'(\lambda' + 7W') - 2W'(2\lambda' + 5W') \qquad (2.31)\\
W'' &= \frac{1}{4}e^{-4W}(F_0^2 e^{2(W+\phi)} - 4) - W'(\lambda' + 8W' - 2\phi') \qquad (2.32)\\
\lambda'' &= 2W'' + (2W' - \lambda')(\lambda' + 8W' - 2\phi') + e^{-4W} \,. \qquad (2.33)
\end{aligned}$$

Some comments on this system are in order. First of all, notice that λ never appears without derivatives. This means that it could be shifted by a constant amount without changing the system, thus producing a new solution from a given one. However, this shift could be reabsorbed into a redefinition of the coordinate θ, whose periodicity will be fixed in the next section, where we look for solutions with a regular point.

We want to find solutions of the system (2.31)-(2.33) where the behavior of the functions make up a compact space. A first possibility that springs to mind is to make also the coordinate z periodic. However, it is possible to extract a combination of the equations of motion and their derivatives that reads:

$$\partial_z(2\phi' - \lambda' - 8W') = (2\phi' - \lambda' - 8W')^2 + 9e^{-4W} \,. \qquad (2.34)$$

From the above equation we see that the derivative of the function $(2\phi' - \lambda' - 8W')$ has always the same sign, and as such it cannot be made periodic. This fact was expected as a periodic solution would have been smooth, a possibility we have already excluded in the previous section.

[1] This works only if $\phi' \neq 4W'$, but it can be easily shown that if $\phi' = 4W'$ there are no solutions.

Another important feature of the system (2.31)-(2.33) is that it is invariant under the constant rescaling

$$W \to W + c, \qquad \phi \to \phi - c, \quad \lambda \to \lambda + 2c, \qquad z \to z^{2c} \ . \tag{2.35}$$

From the ten-dimensional point of view this rescaling has the combined effect:

$$ds^2 \to e^{2c} ds^2, \qquad e^{\phi} \to e^{-c} e^{\phi} \ . \tag{2.36}$$

Thus, given any solution we can act on it with c to generate a new solution with smaller curvature and smaller string coupling, without changing the value of F_0. However, as we are going to see in section 3.2.3, this rescaling cannot be seen as a field of the eight-dimensional theory.

Since we are not able to find analytic solutions of the system (2.31)-(2.33) we will start looking for perturbative and numerical solutions in the next section.

2.3 Perturbative and numerical solutions

We now study the equations perturbatively, in order to find if they admit local solutions. If they do, we will then proceed to extend these solutions numerically and try to understand their global properties.

We start our analysis by looking for regular endpoints, i.e., points where the S^1 shrinks regularly. For a point where an S^1 shrink not to be singular, the local geometry should look like the one of $\mathbb{R}^2$ near the origin of the polar coordinates. Thus we require for the metric the local behavior

$$ds^2_{M_2} \sim dz^2 + z^2 d\theta^2 \ , \qquad \text{for} \quad z \sim 0 \ . \tag{2.37}$$

Notice that, in order to avoid conical singularities, the periodicity of the radial coordinate has to be 2π. This requirement fixes a possible rescaling of λ alone, which is a symmetry of the full system, since it would be equivalent to a rescaling of θ, spoiling its periodicity. Summing up, the requirements for the S^1 to shrink regularly imposes the following local behavior for λ:

$$e^{2\lambda} = z^2 + O(z^3) \ . \tag{2.38}$$

Notice that the rescaling c does not spoil the regularity conditions since it also rescales z with an e^{2c} factor, which is then factorized in front of the whole metric.

Since near $z = 0$ we are thinking of z as a radial coordinate, the requirement of smoothness for all the functions is that they do not have linear pieces in z. Indeed, if they do, such a linear term in z becomes a square root in Cartesian coordinates x, y

$$z = \sqrt{x^2 + y^2} \ , \tag{2.39}$$

making the functions not smooth at the origin. We will then impose

$$W'(0) = \phi'(0) = 0 \ . \tag{2.40}$$

With these requirements we get the local solution

$$W = \frac{\log(c_1)}{2} + \frac{z^2(c_1 c_2^{5/2} F_0^2 - 4)}{16 c_1^2} + \frac{z^4(c_1^2 c_2^5 F_0^4 + 6 c_1 c_2^{5/2} F_0^2 - 20)}{128 c_1^4} + O(z^6) \tag{2.41}$$

$$\phi = \frac{5\log(c_2)}{4} + \frac{5 c_2^{5/2} F_0^2 z^2}{16 c_1} + \frac{5 z^4 (c_1 c_2^5 F_0^4 + 4 c_2^{5/2} F_0^2)}{128 c_1^3} + O(z^6) \tag{2.42}$$

$$e^{2\lambda} = z^2 + \frac{z^4(c_1 c_2^{5/2} F_0^2 + 4)}{4 c_1^2} - \frac{z^6(-5 c_1^2 c_2^5 F_0^4 - 28 c_1 c_2^{5/2} F_0^2 - 96)}{80 c_1^4} + O(z^8)\ . \tag{2.43}$$

Albeit the system involves a quadratic equation, on the regular point the two solutions coincide, and we end up with a unique local solution. We have truncated the above local solution to the first orders, but it can be easily extended to arbitrarily high orders. This local solution only depends on two positive numbers, c_1 and c_2, on which the rescaling c acts as

$$c_1 \to e^{2c} c_1, \qquad c_2 \to e^{-\frac{4}{5}c} c_2\ . \tag{2.44}$$

In order to decouple its effect we define the new variables

$$u = \frac{1}{4}\log(c_1) + \frac{5}{8}\log(c_2), \qquad v = \frac{1}{4}\log(c_1) - \frac{5}{8}\log(c_2)\ , \tag{2.45}$$

which are such that u is not affected by the rescaling and $v \to v + c$. At this point we can choose different values for the parameter u and for the flux constant F_0 and start the numerical evolution. By making some experiments, we have found that for an u below a certain threshold the evolution stops at points where $e^W \to 0$. We are not able to identify singularities with this behavior with any physical object. However, above a certain threshold the system is generically attracted to singular points where the functions behave as

$$e^W \sim t^{-\frac{1}{4}}, \qquad e^\phi \sim t^{-\frac{5}{4}}, \qquad e^\lambda \sim t^{-\frac{1}{2}}, \qquad\qquad \text{with} \quad t \equiv |z - z_0|\ . \tag{2.46}$$

Near such a point, the local form of the full solution is then

$$ds^2_{10} \sim t^{-\frac{1}{2}}(ds^2_{\mathrm{AdS}_8} + d\theta^2) + t^{\frac{1}{2}} dt^2, \qquad e^\phi \sim t^{-\frac{5}{4}} \quad . \tag{2.47}$$

By comparing (2.47) with the solution (1.53) we interpret it as a diverging-dilaton O8-plane extended along $\mathrm{AdS}_8 \times S^1$, with possibly D8-branes on top of it. The number of D8's on top of the O8-plane is related to the value of F_0 on its left. Indeed, it is constrained both by the fact that the $\mathbb{Z}_2$-symmetry requires F_0 to be odd around the O8-plane, which gives

$$\Delta F_0 = -2F_0^{\mathrm{left}}\ , \tag{2.48}$$

and by the Bianchi identity

$$\Delta F_0 = -\frac{1}{2\pi}(n_{D8} - 8)\ , \tag{2.49}$$

where the 8 comes from the charge of the O8-plane. Combining (2.48) and (2.49) we obtain the relation

$$n_{D8} = 8 + 4\pi F_0^{\mathrm{left}}. \tag{2.50}$$

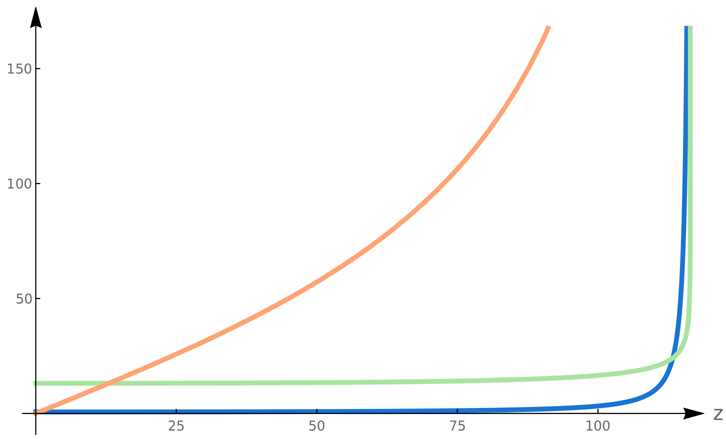


Figure 2.2: A numerical solution with $u = 1$, $v = 1.5$ and $F_0 = -\frac{2}{2\pi}$. The functions are e^{λ} (orange), e^{ϕ} (blue) and e^{W} (green). On the left, the internal S^1 shrinks regularly. On the right, the solution ends on a diverging-dilaton O8-plane, with 4 D8 branes on top of it, where the functions diverge as in equation (2.46).

In this language, a negative number of D8-branes would correspond to anti-D8 branes, since we are only looking at their charge. In Figure 2.2 we show a typical solution with this behavior.

In order to be sure that we have correctly identified the diverging behavior of the various functions in the numerical solution, we can try to analytically solve the equations near the singular point. Identifying the subleading behavior is not immediate, but after some hints from the numerical solutions we obtain the following expansions

$$e^{-4W} = \frac{a_2^5 t}{F_0^4} - \frac{(a_2^5 a_3) t^2}{3(a_1 F_0^4)} - \frac{5(a_2^3(a_2^2 a_3^2 - 12 a_1 a_2 a_4 a_3 - 45 a_1^2 a_4^2)) t^3}{108(a_1^2 F_0^4)} + O(t^4)\,, \tag{2.51}$$

$$e^{-\frac{4}{5}\phi} = a_2 t + a_4 t^2 + \frac{1}{54}\left(-\frac{7 a_2 a_3^2}{a_1^2} + \frac{3 a_4 a_3}{a_1} + \frac{72 a_4^2}{a_2}\right) t^3 + O(t^4)\,, \tag{2.52}$$

$$e^{-2\lambda} = a_1 t + a_3 t^2 + \frac{1}{108}\left(\frac{31 a_3^2}{a_1} - \frac{120 a_4 a_3}{a_2} + \frac{225 a_1 a_4^2}{a_2^2}\right) t^3 + O(t^4)\,. \tag{2.53}$$

Notice that this solution depends on four free parameters a_i. If we had started the numerical evolution from here we would have needed to tune them in order to hit a point where the S^1 shrinks regularly.

A way to check if the singularity we are identifying with an $O8_-$ has the correct tension would be to couple the bulk action to the localized action for such an object and compute its effect on the equations of motion. To avoid repetitions, we postpone this analysis to next chapter, where we will encounter the same singularity in another class of solution, and we extensively study it from this point of view. For the time being, let us anticipate that this prescription is somewhat ambiguous, due to the fact that the localized action for such an object is divergent, introducing a δ-function with a diverging coefficient in the equations of motion. A possibility to make sense of this expression is to rewrite the equations of motions such that the coefficient of the delta function is just a number, the tension appearing in the localized action. Applying this procedure to our solutions reproduces this tension, and it can be explicitly checked both on the numerical solutions and in their analytical expansions (2.51)-(2.53).

2.4 Stability

In this section, we study the stability properties of the numerical solutions we have constructed in the previous section. In general, there are two kinds of instabilities, perturbative and non-perturbative ones. Perturbative instabilities can be understood as the solution not being a local minimum of the action, such that small perturbations to it can make it leave the critical point forever. A physical way to understand perturbative instabilities is to perform a Kaluza-Klein reduction around the background. The main idea is to decompose the fluctuations of the full ten-dimensional fields into fields of different spins on the external space-time, and to analyze their stability by expanding the action at the quadratic order around the vacuum. Perturbative instabilities are then detected as tachyonic modes.[2] This is in general a hard computation since very often different modes are coupled to each other. In our case the main technically difficulties arise from the presence of a non-trivial warp factor and from the numerical nature of the background. We have started a perturbative study [4], and preliminary results seem to indicate that spin 2 and spin 1 modes do not introduce any instability, but the full spectrum has not yet been analyzed.

Non-perturbative instabilities can instead be pictured as the solution not being a global minimum of the potential, such that tunneling effects to lower vacua may occur. For some early work on these effect in the gravitational setting see [69], and for extensions to the case with p-form fields see [70]. Non-perturbative instabilities, being related to genuinely quantum effects, are under less control and are not completely understood. Nevertheless, we can try to investigate some of the proposed decay channels. In particular, we will look at *bubbles* in AdS, where a bubble is defined as a spherical configuration that it is nucleated at a given a time, and we try to understand their evolution. A first type of bubble that one can consider is a spherical of Dp-brane. As proposed in [71], this bubble can be thought of as the higher-dimensional analogous of the Schwinger pair production, where in presence of an external electric field a pair of particle and anti-particle is produced. In the higher-dimensional case, this effect is due to the p-form fields filling a region of the space-time and leads to the production of the p-brane that couples to it through the WZ action. For an $\mathrm{AdS}_d \times M_{10-d}$ compactification, such a bubble is described by a Dp-brane extended along a submanifold of AdS_d of the form

$$\mathbb{R} \times S^{d-2} \subset \mathrm{AdS}_d \,, \tag{2.54}$$

where $\mathbb{R}$ is the time-like direction. The remaining $p-(d-2)$-dimensions of the brane would then wrap a submanifold of the internal space:

$$\Sigma_{p-(d-2)} \subset M_{10-d}. \tag{2.55}$$

Since the RR flux coupled to the brane jumps across it, the vacuum inside the bubble is different from the vacuum outside. If after the creation the bubble expands, eventually reaching the boundary of AdS_d in finite time, this process completely destroys the original vacuum, leaving us with a new vacuum. We interpret this process as a tunnel effect from the original vacuum to the new one. In

[2]For a comprehensive review of this approach see [66]. As an example, an early explicit computation in the context of $\mathrm{AdS}_5 \times S^5$ backgrounds can be found in [67, 68].

general, the potential energy has two contributions, the gravitational one, coming from the DBI action, which would make the brane to collapse, and the 'electric' one, coming from the WZ action, which would make the bubble to expand. In supersymmetric configurations, these two effects cancel each other, and the resulting configuration, if admitted, represents a static BPS domain wall. In more general non-supersymmetric cases, one of the two forces will dominate and the bubble would either expand or collapse. An extension [63] of the Weak Gravity Conjecture [72] suggests that there is always a Dp-brane for which the gravitational force is weaker. Thus, it suggests that in non-supersymmetric configurations such bubbles would always expand reaching the boundary of AdS in finite time and completely destroying any non-supersymmetric vacuum. In some examples this conjecture has been explicitly tested in a test-brane approximation. For example, in [71] this effect has been computed for a non-supersymmetric $\mathrm{AdS}_3 \times S^3 \times K_2$ background of type IIB supergravity, and in [73] for a non-supersymmetric vacuum of massive type IIA. In both cases such brane configurations, if created, tend to expand.

Let us now analyze what happens in our non-supersymmetric AdS_8 backgrounds. A spherical Dp-brane bubble would wrap a submanifold of AdS_8 of the form

$$\mathbb{R} \times S^6 \times \Sigma_{p-6} \ , \tag{2.56}$$

which only leaves the possibilities $p = 6$ and $p = 8$, i.e. D6-branes and D8-branes. However, D6 branes can be readily excluded. Indeed such a brane would electrically couple to $F_8 \equiv \star F_2$, which is absent on our background. Therefore ,only the gravitational term is present, which will make the brane collapse, if created. This leaves us with the possibility of instabilities mediated by D8-branes. However, we now argue that such a brane cannot even appear in our solution, since it would intersect transversely the O8-plane already present in the background, a situation which is not allowed as we readily show. Working in global coordinates in AdS_8,

$$ds^2_{10} = e^{2W}(-\cosh\rho^2 dt^2 + d\rho^2 + \sinh^2\rho ds^2_{S^6}) + e^{-2W}(dz^2 + e^{2\lambda}d\theta^2) \ , \tag{2.57}$$

we can take the bubble to be at a fixed $\rho = \rho_0$. Since the background O8-plane is located at $z = z_0$, and completely fills the AdS space, the test D8-brane and the background O8-plane necessarily have to intersect. However, consistency of such a configuration would require both $n_0 = 2\pi F_0$ to change its sign when crossing the O8-plane and to jump of one unit when crossing the D8. From Figure 2.3 we see that satisfying these two conditions simultaneously is indeed not possible.

Thus, it seems that the Dp-branes bubbles are either absent or collapsing on our background, preventing its decay. Notice that in contrast to the non-supersymmetric AdS_4 vacuum in [73], in this case there is no supersymmetric AdS_8 solution on which our vacuum can decay to.

We conclude this section considering another possible bubble instability, known as *bubble of nothing*. Such a bubble is defined as the locus on the external space-time where a sphere on the internal space shrinks smoothly. These kinds of bubbles have been first proposed in [74] for the simple Kaluza-Klein vacuum $\mathrm{Mink}_4 \times S^1$. In that case the surface of the bubble is a locus where the internal S^1 shrinks smoothly. More recently, bubbles of nothing have been considered in other non-supersymmetric compactifications of string theory. For example, in [75] the authors show that these bubbles destabilize some non-supersymmetric quotients of the $\mathrm{AdS}_5 \times S^5$ vacua in type IIB [76], where the quotient introduces a one-cycle that can then shrink producing the bubble. In another recent example [77], it has been shown that it is the internal S^2 of the S^2 fibration over S^4 producing the internal $\mathbb{C}P^3$ space of a class of AdS_5 solution in M-theory [78] that can shrink smoothly producing this instability. In our case one could imagine the case where the internal S^2

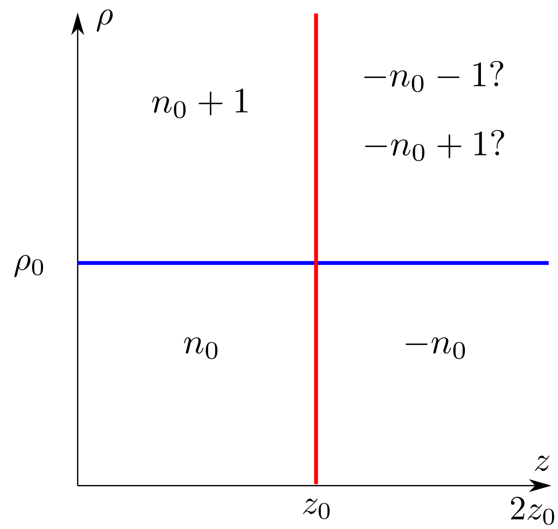


Figure 2.3: A hypothetical configuration describing a bubble of D8-branes (in blue) nucleated at $\rho = \rho_0$ in AdS_8. The $\mathrm{O8}_-$ of the background solution (in red) is located at $z = z_0$ for all the values of the coordinate ρ. For this intersecting configuration it is not possible to consistently satisfy all the conditions on $n_0 = 2\pi F_0$ in all the regions of the space-time.

shrinks smoothly, but the resulting geometry cannot be smooth being the sphere not round. A possible way out is to imagine a geometry that asymptotes to our backgrounds for $\rho \to \infty$, but where the S^2 becomes smooth going into the interior of AdS. This possibility seems unlikely given the presence of the O8-plane at the equator of the sphere.

Chapter 3

dS_4 solutions

We now turn our attention to solutions of string theory more directly related to our observed universe. Various astronomical observations indicate that our universe is expanding at an accelerating rate [13, 14]. This feature is well described by a de Sitter space-time, a solution of the Einstein's equation with a positive cosmological constant. However, observations also constrain this cosmological constant to be very small. We do not know how to explain this value within the actual framework of quantum field theory other than adding it by hand. As we have suggested in the introduction, understanding very important features of our world, such as the nature of the cosmological constant and the physics of the early universe, requires us to embed our effective models into an UV complete theory. String theory provides such a framework.[1]

In this chapter, we will explore a little bit this realm by trying to construct four-dimensional de Sitter space-times in string theory. This goal introduces many challenges. First of all, de Sitter solutions with a compact internal space cannot be supersymmetric.[2] Thus, we will be confronted again with the problem of constructing non-supersymmetric backgrounds. Moreover, the general no-go theorem [65], which in section 2.1 we have specialized to a particular case, can be formulated also in the presence of explicit physical sources. In this setting [65] shows that the existence of de Sitter vacua in the supergravity approximation requires the inclusion of orientifold planes to generate the required positive potential energy. However, as we have seen both in the flat space solutions in section 1.2.3 and in the AdS_8 models in the previous chapter, supergravity breaks down near orientifold planes and we might need to go beyond the supergravity approximation in order to understand the physics of these backgrounds. Given all the difficulties in building explicit de Sitter solutions of ten-dimensional string theory, an efficient way to explore the huge space of possibly allowed configurations is to use four-dimensional effective theories derived from string theory (see e.g. [18] for a review). This approach has produced many interesting scenarios for de Sitter compactifications in string theory, most notable [81]. However, the validity of the four-dimensional approximation to describe similar settings is still under active scrutiny. For example, understanding from the ten-dimensional perspective the various ingredients that lead to the four-dimensional construction in [81] turned out to be a very active area of research by its own. In particular, solutions obtained from the lower-dimensional point of view are usually constructed

[1] For a pedagogical introduction to the construction of cosmological models in string theory see for example [18, 79].

[2] They can be supersymmetric when the internal space is non-compact, as can be easily seen by writing the AdS_n factor of a supersymmetric solution as a non-compact foliation of a dS_{n-1} space [80].

by balancing different physical effects. These are captured as individual contributions which are on their own well understood, but for which the simultaneous effect is not yet completely under control.[3]

The difficulties in the construction of de Sitter space-times have led some authors to conjecture that vacua with a positive cosmological constant cannot be realized at all in string theory, or perhaps in any quantum theory of gravity, putting them in the swampland [89, 90]. In this chapter, we tackle this problem by constructing four-dimensional de Sitter space-times as solutions of the ten-dimensional equations of motion of massive type IIA supergravity. The models we will construct will be very simple and far from being complete phenomenological models. Nevertheless, we hope they are useful playgrounds to quantitatively test these various ideas and conjectures, starting from the questions on their existence.

We start in section 3.1 with a modification of the type $\tilde{I}$ model introduced in section 1.2.4, where we now take four out of the nine directions of the world-volume of the orientifold planes to be extended along a four-dimensional de Sitter space. In this construction, we will soon find the requirement for the internal space to involve a negatively curved internal space M_5 fibered over the circular direction. This is not unexpected since from the four-dimensional point of view it is known that negatively curved internal manifolds contribute with positive energy [79]. We will not be able to solve the equations of motion analytically, but we will find numerical solutions where the negative-tension orientifold plane is of the diverging-dilaton type. This is the same object we have already encountered both in the AdS_8 construction of the previous chapter and in a limit of the type $\tilde{I}$ model at the end of section 1.2.4.

In section 3.1.2 we enlarge our class of de Sitter solutions by adding an extra flux in the internal space and by splitting the internal manifold M_5 into a product of two Einstein spaces, M_2 and M_3. In doing so, we learn that a rescaling modulus c of the solutions where F_0 is the only flux turned on is generically quantized by the flux quantization conditions. We also find that the constraint on the cosmological constant now requires at least one of M_2 and M_3 to be negatively curved. We explicitly build the numerical solutions realizing all the allowed possibilities. All these solutions again feature a diverging-dilaton $\mathrm{O8}_-$ plane. As we have already remarked, the appearance of a strongly-coupled and strongly-curved region on a supergravity solution makes its validity questionable. For the flat space solutions where this happens, we have the alternative description in terms of the world-sheet theory that makes us confident that such singular solutions can be lifted to solutions of the full string theory. As we have already observed, for supersymmetric AdS solutions involving orientifold planes one can use holography to check the validity of a singular supergravity solution by comparing its prediction with results obtained in the dual quantum field theory. For the non-supersymmetric AdS_8 solutions we have found in the previous chapter, we have suggested that some hints might come from conformal field theory techniques such as the conformal bootstrap. For the de Sitter constructions of this chapter we can not immediately use these tools, since holography for de Sitter space is less developed.[4] Instead of viewing it as a limitation, we can see this as an opportunity. In the AdS case much progress in holography has been made possible thanks to the explicit construction of supergravity solutions. Hopefully, the efforts in the construction of simple models like the ones we try to build in this chapter, might also be useful to this end. In any case we need a better understanding of these singularities in a UV complete theory and we cannot rely on supergravity alone to confirm the validity of the solutions we are going to present.

[3] A necessarily incomplete list of references on this vast topic is [82, 83, 84, 85, 86, 87]. See also [88] for a critical review on various dS constructions.

[4] There are various approaches to this ambitious task [91, 92, 93, 94, 95, 96]. See also [97, 98] for recent progress.

In the meantime, a criticism to the solutions we will describe in this chapter arose in the literature [64]. As we are going to extensively review in section 3.2, this criticisms is based on the analysis of the uncorrected supergravity equations of motion near the singularity that we have identified as an $O8_-$. The claim is that it is not enough to look at the leading behavior of the fields near the singularity, but that also the sub-leading behavior has to be analyzed in the supergravity approximation. As we will review, the discrepancy between the two approaches can be recast into different boundary conditions imposed on the fields on top of the diverging-dilaton $O8_-$. By analyzing this issue from the point of view of the action, we relate it to a question of which fluctuations for the fields are allowed near these sources. Ultimately, we will discover that supergravity alone is not able to answer these questions, signaling again that it needs to be UV-completed in the strongly coupled region.

Finally, in the last part of this chapter we start exploring a richer class of de Sitter solutions. Building on the results of section 3.1.2, we allow for more fluxes in the internal space and we replace the ill-defined $O8_-$ plane with an $O6_-$ plane. We are able to do it explicitly by deforming a known class of analytic AdS_7 solutions where the same sources appear. Our strategy consists in separating the seven-dimensional space into a product $AdS_4 \times M_3$, with M_3 negatively curved, and then bringing the cosmological constant up numerically until it reaches positive values. At the end of this procedure we end up with dS_4 solutions involving an $O8_+$ plane (the well understood one) and an $O6_-$ plane. This $O6_-$ plane is identified by the behavior of the fields near the boundary of its 'hole' (see section 1.2.3). Even though we have not yet completely analyzed the complicated moduli space of these solutions, this procedure seems to leave some unfixed parameters. It would be interesting to understand if there exists a physical procedure to fix them. As a heuristic procedure, one might try to analytically continue the solution through the boundary of this hole, going inside the unphysical region, where both the metric and the dilaton become imaginary, and to check whether also the typical pole of the flat-space solution is reproduced. For the analytic AdS_7 solutions this turns out to be true, but we were not able to reproduce it in the numerical dS_4 solutions we have obtained so far. However, it is not clear that this procedure is meaningful and clearly a better understanding of these backgrounds should rely only on properties of the physical space-time. In the absence of Romans mass one can go further. For example it is known that in flat space the $O6_-$ singularity gets resolved by the lift in M-theory, where it becomes the non-singular Atiyah-Hitchin solution [99, 100, 101]. In other cases, thanks to supersymmetry, through the analysis of the quantum space-time seen from a probe Dp-brane [102] it has also been possible to resolve these singularities in the quantum theory. It would be interesting to understand how these construction change for non-supersymmetric de Sitter solutions. This chapter is based on the published work [2] and on the ongoing work [5].

3.1 Solutions with $O8_-$ planes

In this section we discuss two simple modifications of the type $\tilde{I}$ solutions we described in section 1.2.4. In the simplest class introduced in section 3.1.1, we keep F_0 as the only non-vanishing flux and we find that the equations of motion require a negatively curved internal five-manifold. In section 3.1.2 we extend this class by allowing for a non-vanishing four-form flux in the internal space. As we are going to see, the main effects of this extra flux will be to quantize the rescaling modulus of the solutions with only F_0 turned, and to naturally introduce a splitting of the internal space which allows for a positively curved factor.

3.1.1 Only $F_0 \neq 0$

We build upon the type $\tilde{I}$ model described in section 1.2.4 by looking for solutions where the geometry is described by the following local metric

$$ds^2 = e^{2W} ds^2_{M_4} + e^{-2W}(dz^2 + e^{2\lambda} ds^2_{M_5}) \ , \tag{3.1}$$

where W and and λ are functions on the internal space and M_4 is the external four-dimensional vacuum. The internal space can be seen as a fibration of a five-dimensional Einstein space M_5 over a one-dimensional space described by the coordinate z. As in the type $\tilde{I}$ model, and unlike the AdS$_8$ solutions of the previous chapter, we periodically identify the coordinate z as

$$z \sim z + 2z_0 \ . \tag{3.2}$$

All the functions we consider depend on the coordinate z only, on which a $\mathbb{Z}_2$ involution acts as in Figure 3.1, with the two fixed loci being $z = 0$ and $z = z_0$. At these points two orientifold planes with opposite charge and tension sit.

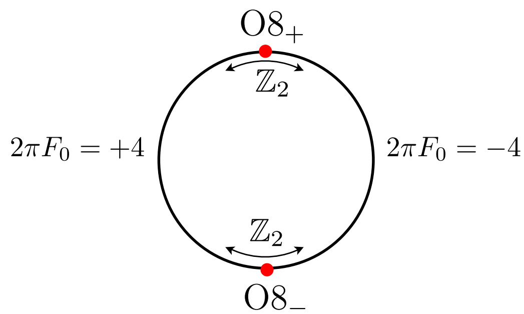


Figure 3.1: The coordinate z in the metric of the ansatz (3.1). The fixed loci of the $\mathbb{Z}_2$ involution are $z = 0$ and $z = z_0$, where two O8-planes with opposite charge and tension sit. The value of F_0 in the two regions is also indicated.

The full set of equations of motion in this setting then reads

$$\begin{aligned}
0 &= -2F_0^2 e^{2(\lambda+W+\phi)} + 16 e^{2\lambda+4W}(-5(\lambda')^2 - \phi'(W' - 5\lambda') + 2(W')^2 - (\phi')^2) + \\
&+ 20\rho e^{4W} + 16 e^{2\lambda}\Lambda && (3.3)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= e^{-3W-\phi}(-F_0^2 e^{2(W+\phi)} - 4\Lambda + 4e^{4W}(W'' + W'(5\lambda' - 2\phi'))) && (3.4)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= -\frac{2}{5} e^{-2\lambda-3W-\phi}(2F_0^2 e^{2(\lambda+W+\phi)} + 2e^{2\lambda+4W}(5\lambda'(\phi' - 2\lambda') - 2W'\phi' + 4(W')^2 - \phi'') + \\
&- \frac{2}{5} e^{-2\lambda-3W-\phi}\left(5\rho e^{4W} + 4e^{2\lambda}\Lambda\right) && (3.5)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= e^{-3W-\phi}(F_0^2(-e^{2(W+\phi)}) - 2\Lambda + 2e^{4W}(\lambda'' - 2\lambda'\phi' + 5(\lambda')^2) - 2\rho e^{4W-2\lambda}) \ , && (3.6)
\end{aligned}$$

where 4Λ and 5ρ are respectively the Ricci scalars of M_4 and M_5. Notice that their value is not physical, as it can be rescaled with a redefinition of the function W and λ. In the equations of motion the symbols $\delta_\pm$ indicate δ-functions localizing the O8$_\pm$ at the two fixed loci. Their tensions in our conventions where $l_s = 1$ are given by

$$\kappa^2 \tau_\pm = \pm \frac{8}{2\pi} \ . \tag{3.7}$$

Finally, the equations of motion have to be supplemented by the Bianchi identity for F_0,

$$dF_0 = -\sum_{i=\pm} \kappa^2 \tau_i \delta_i \ , \tag{3.8}$$

whose role is to make F_0 jump when crossing the sources, as shown in Figure 3.1. In writing the system (3.4)-(3.6) we have already taken an appropriate linear combination of the original equations coming from the ten-dimensional equations (1.21), (1.22) in order to extract the first-order equation (3.4), which thus does not involve δ-functions. The localized contributions of the sources to the equations of motion come from the corresponding contributions to the action. These are given by the localized terms

$$S_{\text{loc}} = \sum_{i=\pm} -\kappa^2 \tau_i \int_{\Sigma_i} \sqrt{-g|_{\Sigma_i}} e^{-\phi} + \kappa^2 \tau_i \int_{\Sigma_i} C_9|_{\Sigma_i} \ , \tag{3.9}$$

where C_9 is defined by

$$F_0 \equiv - \star F_{10} \equiv - \star dC_9 \tag{3.10}$$

and the Σ_i are the submanifolds where the O8-planes are located. With the symbol $|_{\Sigma_i}$ we indicate the pullback of the space-time fields on the sources. In the case where all the quantities remain finite, we can take into account the contributions of these actions by rewriting them as integrals on the whole space-time, which then get localized on the given loci by well-defined δ-functions, obtaining in this way the equations (3.4), (3.5), and (3.6). As we are going to discuss in section 3.2, the situation is more subtle for diverging-dilaton O8-planes.

We now solve the full set of equations of motion in the vicinity of the O8$_+$ at $z = 0$. Assuming that the functions are continuous, we can take into account the contributions of the δ-functions as the requirement for the first derivatives of the functions appearing in the equations to be discontinuos, with a discontinuity given by the coefficients of the δ-functions. Let us be more precise by working out the conditions imposed by equation (3.4) on W. Assuming that W is continuous, but with a first derivative W' which has a finite jump $\Delta W'$ at $z = 0$, its second derivative can be defined in the distributional sense to be

$$W'' = W_0'' + \Delta W' \delta(z) \ , \tag{3.11}$$

where $W_0^{''}$ is a (possibly discontinuous) function and $\delta(z)$ is the Dirac delta distribution centered in $z = 0$.

This relation is an immediate consequence of the well-known result that the distributional derivative of the Heaviside step function is the Dirac delta. Indeed, any function W' with a finite jump $\Delta W'$ at $z = 0$ can be decomposed as

$$W' = W_0' + \theta(z) \Delta W', \tag{3.12}$$

where W_0' is a continuous function and $\theta(z)$ is the Heaviside step function. By taking the derivative of this expression we then get (3.11). Plugging then the decomposition (3.11) into equation (3.4) we obtain a distributional equation with a δ-functions on both sides. To solve it we then match their coefficients, obtaining the relation

$$e^{W-\phi}\Delta W' = \frac{1}{4}\kappa^2\tau_+ \qquad \text{at } z=0\ . \tag{3.13}$$

An equivalent way to obtain the same result is by dividing both sides of equation (3.4) by the function $e^{W-\phi}$, obtaining an expression of the form

$$e^{\phi-W}\kappa^2\tau_+\delta(z) = W'' + \cdots\ , \tag{3.14}$$

where the terms in the dots include at most first derivatives. Integrating this equation on the interval $[-\varepsilon,\varepsilon]$, and integrating by parts the second derivative, we then get in the limit $\varepsilon \to 0$ the condition (3.13) multiplied by the function $e^{W-\phi}$. As we are going to carefully review in section 3.4, for a diverging-dilaton O8-plane these manipulations require some care. For example, the function $e^{W-\phi}$ for which we have divided both sides of the equation of motion (3.4) to obtain (3.14) vanishes where this object is located. Thus by following this procedure we are effectively dividing both sides of an equation by zero on the point where we are studying it. We will discuss these subtle issues in section 3.2, where we carefully analyze the jump conditions for diverging-dilaton O8$_-$ planes. For the time being, let us notice that both sides of equation (3.13) are always finite, and can thus be evaluated and used as boundary conditions also for the diverging-dilaton case.

Integrating the Bianchi identity (3.8) across the source we obtain the jump for F_0 :

$$\Delta F_0 = -\kappa^2\tau_i\ , \tag{3.15}$$

which, plugged into equations like (3.13), relates the jump of the derivatives to the jump of F_0:

$$e^{W-\phi}\Delta W' = -\frac{1}{4}\Delta F_0\ . \tag{3.16}$$

Finally, since our sources are orientifold planes, we have to take into account the projection they enforce on the string states. In our case, this is translated into the requirement for the metric functions (and the dilaton) to be even and for F_0 to be odd around them.[5] These requirements relate the jump of the various functions to their value on one side:

$$\Delta W' = 2\lim_{z\to 0^+} W', \qquad \Delta F_0 = 2\lim_{z\to 0^+} F_0\ . \tag{3.17}$$

Putting together conditions (3.16) and (3.17), and repeating the same analysis also for the equations (3.5) and (3.6), we obtain the full set of conditions that the derivatives of the functions have to satisfy at $z=0$:

$$e^{W-\phi}W' = -\frac{1}{4}F_0, \qquad e^{W-\phi}\phi' = -\frac{5}{4}F_0, \qquad e^{W-\phi}\lambda' = -\frac{1}{2}F_0, \qquad \text{for } z\to 0^+\ . \tag{3.18}$$

These conditions fix the value of the derivatives approaching $z=0$ from the right. Taking the same limit of the first order equation (3.4) and plugging the conditions (3.18) we finally obtain a constraint on the cosmological constant

$$\Lambda = -\frac{5}{4}\rho e^{4W-2\lambda}, \qquad \text{for } z\to 0^+\ . \tag{3.19}$$

[5] See for example [103] for a review on the parity conditions imposed by O8 planes on the fields.

From this equation, we now see that in order to allow for de Sitter solutions ($\Lambda > 0$), we need to take ρ, the curvature of the internal five-manifold M_5, to be negative. In order to simplify all the following expressions, from now on we make the choice

$$\rho \equiv -\frac{4}{5}\Lambda \ . \tag{3.20}$$

Finally, we notice that as in the AdS$_8$ construction, the full system of equations of motion is invariant under the rescaling

$$W \to W + c, \qquad \phi \to \phi - c, \quad \lambda \to \lambda + 2c, \qquad z \to z^{2c} \ . \tag{3.21}$$

As we are going to see in section 3.1.2, the constant c in (3.21) is continuous only in the degenerate case where F_0 is the only flux turned on. The effect of the rescaling (3.21) is to generate a new solution with arbitrarily small curvature and small string coupling from a given solution. However, in section 3.2 we are going to argue that even in the cases where the rescaling c is continuous it cannot be associated to a field of the lower-dimensional effective theory.

We now to solve the equations of motion. Since we were not able to find analytical solutions, we have started the analysis of the system (3.4)-(3.6) by studying it in a power-series approach near the vicinity of the O8$_+$ at $z = 0$. Imposing on the derivatives the conditions (3.18) results in the following local solution

$$e^{-4W} = c_1 + \frac{F_0 z}{\sqrt{c_2}} - 2z^2(c_1^2\Lambda) - \frac{17z^3(c_1F_0\Lambda)}{6\sqrt{c_2}} + z^4\left(3c_1^3\Lambda^2 - \frac{17F_0^2\Lambda}{24c_2}\right) + O(z^5) \ , \tag{3.22}$$

$$e^{-\frac{4}{5}\phi} = c_1c_2^{2/5} + \frac{F_0 z}{\sqrt[10]{c_2}} - \frac{z^3(c_1F_0\Lambda)}{6\sqrt[10]{c_2}} - \frac{z^4(F_0^2\Lambda)}{24c_2^{3/5}} + \frac{39c_1^2F_0\Lambda^2z^5}{200\sqrt[10]{c_2}} + O(z^6) \ , \tag{3.23}$$

$$e^{-2\lambda} = c_1 + \frac{F_0 z}{\sqrt{c_2}} - \frac{1}{5}z^2(c_1^2\Lambda) - \frac{13z^3(c_1F_0\Lambda)}{30\sqrt{c_2}} + \Lambda z^4\left(\frac{9}{25}c_1^3\Lambda - \frac{13F_0^2}{120c_2}\right) + O(z^5) \ . \tag{3.24}$$

Notice that, since F_0 is odd, all the functions are even around $z = 0$, as required by the O8-plane projection. Moreover, for $\Lambda = 0$ they truncate to the usual flat space solutions we described in section 1.2.3.

We can now choose some values for the constants c_1 and c_2 and start the numerical evolution. The situation is similar to the AdS$_8$ case: for an open set in the space of c_1 and c_2 we obtain solutions that end at $z = z_0$ on a diverging-dilaton O8$_-$, where the functions behave as

$$e^{\lambda} \sim |z - z_0|^{-\frac{1}{2}}, \qquad e^{W} \sim |z - z_0|^{-\frac{1}{4}}, \qquad e^{\phi} \sim (z - z_0)^{-\frac{5}{4}} \ .$$

In Figure 3.2 we show an example of a numerical solution. Near the diverging point the local form of the metric is

$$ds^2_{10} \sim t^{-\frac{1}{2}}(ds^2_{M_4} + ds^2_{M_5}) + t^{\frac{1}{2}}dt^2, \qquad e^{\phi} \sim t^{-\frac{5}{4}}, \qquad \text{with} \quad t \equiv |z - z_0| \ , \tag{3.25}$$

which can be interpreted as an O8-plane extended along the four-dimensional de Sitter space and the negatively curved Einstein space M_5. Moreover, we can also check explicitly that the tension appearing in the action is reproduced by these solutions by checking that the conditions (3.18) are satisfied also at $z = z_0$, with an opposite value for the tension. In section 3.2 we comment with more on the details of this singularity.

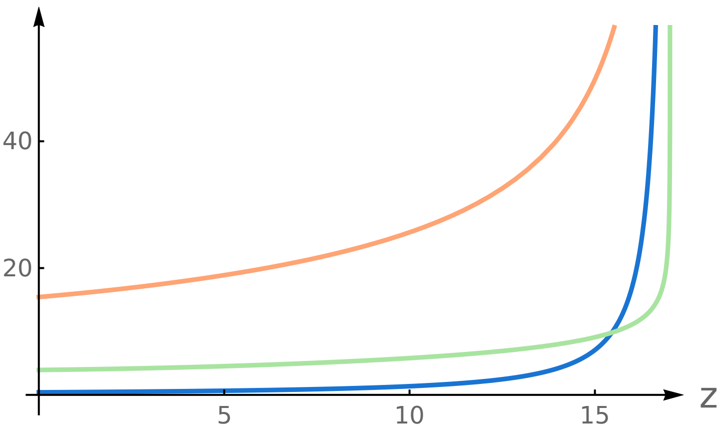


Figure 3.2: A numerical solution in the region where $F_0 = -\frac{4}{2\pi}$. The functions are e^{λ} (orange), e^{W} (green) and e^{ϕ} (blue). On the left, it starts from an $O8_+$ plane, where the functions are continuous but not differentiable. On the right, it ends at $z_0 \sim 17$ on a diverging-dilaton O8-plane,

3.1.2 Solutions with F_4

In this section we start adding more richness to our simple dS_4 model. In particular, we are going to see that the rescaling labeled as c in the previous section is due to a degeneracy in the equations of motion that appears when the only flux turned on is F_0. In more general situations this rescaling is quantized by the flux quantization conditions of the other fluxes, and we explicitly show how this possibility is realized by finding solutions with F_4 turned on. While doing so, we also learn that the five-dimensional negatively curved internal space can be split in two spaces, with the requirement that at least one of them is negatively curved. We then find explicit numerical solutions realizing all the four possibilities.

We generalize our ansatz (3.1) by splitting the five-dimensional space M_5, thus considering the more general metric

$$ds^2_{10} = e^{2W} ds^2_{M_4} + e^{-2W}(dz^2 + e^{2\lambda_2} ds^2_{M_2} + e^{2\lambda_3} ds^2_{M_3}) \ , \tag{3.26}$$

where now M_2 and M_3 are two Einstein spaces whose Ricci scalars are normalized as

$$R_{M_2} = 2\rho_2, \qquad R_{M_3} = 3\rho_3 \ . \tag{3.27}$$

As a consequence of this splitting, we can also easily allow for the presence of a flux F_4 in the internal space with legs along $dz \wedge \mathrm{vol}_{M_3}$ where vol_{M_3} is the volume form of M_3. Imposing then the Bianchi identity (1.19) and the equation of motion for F_4 written in (1.25), we obtain that its most general form is given by

$$F_4 = f_4 e^{-6W+3\lambda_3-2\lambda_2} dz \wedge \mathrm{vol}_{M_3} \ , \tag{3.28}$$

with f_4 a real constant. Again, notice that constant rescalings of ρ_2, ρ_3, and Λ are unphysical, since they can be reabsorbed with a redefinition of λ_2, λ_3, and W respectively. In this more general setup the equations of motion now read

$$
\begin{aligned}
0 &= -4((\lambda_2')^2 + 6\lambda_3'\lambda_2' + 3(\lambda_3')^2) + 8\phi'(2\lambda_2' + 3\lambda_3' - W') + 16(W')^2 - 8(\phi')^2 + \\
&+ f_4^2 e^{-4\lambda_2 - 6W + 2\phi} - F_0^2 e^{2\phi - 2W} + 4e^{-2\lambda_2}\rho_2 + 6e^{-2\lambda_3}\rho_3 + 8\Lambda e^{-4W} \qquad (3.29)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= -e^{-4\lambda_2 - 5W + \phi}(f_4^2 + F_0^2 e^{4(\lambda_2 + W)}) + 4e^{W-\phi}(W'' + W'(2\lambda_2' + 3\lambda_3' - 2\phi')) + \\
&- 4\Lambda e^{-3W-\phi} \qquad (3.30)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= -4e^{W-\phi}(2(\lambda_2')^2 + 3(\lambda_3')^2 + 2\lambda_2'' + 3\lambda_3'' - W'' + W'(-3\lambda_3' + 8W' - 2\phi') - 2\lambda_2' W' - 2\phi'')\\
&+ e^{\phi - 5W}(F_0^2 e^{4W} - f_4^2 e^{-4\lambda_2}) \qquad (3.31)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= 4e^{-2\lambda_2 + W - \phi}(e^{2\lambda_2}(\lambda_2'' - W'' + (2\lambda_2' + 3\lambda_3' - 2\phi')(\lambda_2' - W')) - \rho_2) + \\
&- e^{-4\lambda_2 - 5W + \phi}(f_4^2 + F_0^2 e^{4(\lambda_2 + W)}) \qquad (3.32)\\
\sum_{i=\pm} \kappa^2 \tau_i \delta_i &= e^{-4\lambda_2 - 2\lambda_3 - 5W - \phi}(4e^{4\lambda_2 + 6W}(e^{2\lambda_3}(\lambda_3'' - W'' + (2\lambda_2' + 3\lambda_3' - 2\phi')(\lambda_3' - W')) - \rho_3)) + \\
&+ e^{-4\lambda_2 - 5W + \phi}(f_4^2 - F_0^2 e^{4(\lambda_2 + W)})
\end{aligned}
$$

We have isolated a first order equation (3.29), which we will use to constrain the parameters of our solution. Notice that we now have an extra equation, which comes from the legs of the Einstein's equations along then new split internal direction. The system of equations is invariant under the rescaling

$$W \to W + c, \qquad \phi \to \phi - c, \qquad \lambda_i \to \lambda_i + 2c, \qquad z \to e^{2c} z, \qquad f_4 \to e^{6c} f_4 \;, \tag{3.33}$$

whose effect on the ten-dimensional fields is

$$g \to e^{2c} g, \qquad e^{\phi} \to e^{\phi} e^{-c}, \qquad F_4 \to e^{4c} F_4 \;. \tag{3.34}$$

This rescaling can be easily understood by noticing that the massive type IIA supergravity action (1.15) transforms homogeneously as

$$S_{\text{IIA}} \to e^{10c} S_{\text{IIA}} \tag{3.35}$$

under the combined rescaling

$$g \to e^{2c} g, \qquad e^{\phi} \to e^{\phi} e^{-c}, \qquad H \to e^{2c} H, \qquad F_k \to e^{kc} F_k \;. \tag{3.36}$$

Thus, when there are fluxes other than F_0 turned on, their quantization can quantize the constant c appearing in (3.36). In particular, in our case we have to require that

$$\int_{S^1 \times M_3} F_4 \in \frac{\mathbb{Z}}{(2\pi)^3} \;. \tag{3.37}$$

Before looking for new solutions, let us describes within our ansatz (3.26) the supersymmetric AdS_6 solution of [52], which describe the near-horizon limit of a system of D4-D8 branes. As far as the equations of motion are concerned, this AdS_6 solution can be seen as an $\text{AdS}_4 \times \mathbb{H}_2$ solution with

$$\rho_2 = \Lambda < 0 \;. \tag{3.38}$$

Indeed, the product of Einstein spaces with the same Einstein constant is again an Einstein space with the same constant. Since there are no fluxes extended along AdS_6 and no fields depend non-trivially on its coordinates, it enters in the equations of motions only through its cosmological constant, and as such it is not distinguishable from $\mathrm{AdS}_4 \times \mathbb{H}_2$. Notice that this is presumably false from the point of view of the supersymmetry variations, since they involve the Killing spinors of AdS_6. Thus when seen as an $\mathrm{AdS}_4 \times \mathbb{H}_2$ configuration it is likely a non-supersymmetric solution. Following the notation of [104], the AdS_6 solution is described by the metric

$$ds^2_{10} = e^{2W} L^2 \left(\frac{9}{4} ds^2_{\mathrm{AdS}_6} + ds^2_{S^4} \right) = e^{2W} L^2 \left(\frac{9}{4} ds^2_{\mathrm{AdS}_6} + dz^2 + \sin^2 z ds^2_{S^3} \right) , \tag{3.39}$$

and the other non-trivial fields are

$$e^{\phi} = \frac{2}{3L(F_0 \cos z)^{\frac{5}{6}}} , \qquad e^W = (F_0 \cos z)^{-\frac{1}{6}} , \qquad F_4 = 5L^2 (F_0 \cos z)^{\frac{1}{3}} \sin^3 z dz \wedge \mathrm{vol}_{S^3} . \tag{3.40}$$

Notice that at the equator of the S^4, which is located at $z = \frac{\pi}{2}$, the dilaton and the metric blow-up. At this locus a diverging-dilaton O8 sits, making this solution similar to our AdS_8 solutions described in chapter 2. To describe the solution (3.39), (3.40) within our ansatz (3.26), we first have to change gauge as

$$dz \to e^Q dz. \tag{3.41}$$

This redefinition transforms all the derivatives in the system (3.29)-(3.33) as $\partial_z \to e^{-Q} \partial_z$. The resulting system is then solved by taking M_3 to be a radius one S^3 ($\rho_3 = 2$) and setting

$$Q = 2W, \quad \lambda_2 = 2W, \quad \lambda_3 = 2W + \log(\sin z), \quad \phi = 5W + \log \left(\frac{2}{3L^2} \right) \tag{3.42}$$

and

$$W = -\frac{1}{6} \log(F_0 \cos(z)) + \log L, \qquad f_4 = 5L^6, \qquad \Lambda = -\frac{20}{9}. \tag{3.43}$$

In our more generic situation we are not able to find analytic solutions to the system (3.29)-(3.33). We will thus follow the steps described in the previous section to find numerical solutions. First of all we derive the conditions on the first derivatives of the fields at $z = 0$. In the present case, they read:

$$e^{W-\phi} W' = -\frac{1}{4} F_0, \qquad e^{W-\phi} \phi' = -\frac{5}{4} F_0, \qquad e^{W-\phi} \lambda_2' = e^{W-\phi} \lambda_3' = -\frac{1}{2} F_0, \qquad \text{for } z \to 0^+ . \tag{3.44}$$

Taking the limit for $z \to 0^+$ of the first order equation (3.29), and plugging the conditions (3.44) we get a generalization of the constraint (3.19):

$$\Lambda = -\frac{1}{8} f_4^2 e^{-4\lambda_2 - 2W + 2\phi} - \frac{1}{2} \rho_2 e^{4W - 2\lambda_2} - \frac{3}{4} \rho_3 e^{4W - 2\lambda_3} , \qquad \text{for } z \to 0^+ . \tag{3.45}$$

We see that it is now possible to solve the above constraint when at least one among ρ_2 and ρ_3 is negative. Imposing the conditions (3.44), we obtain the local solution

$$
\begin{aligned}
e^{-4W} &= c_1 + \frac{c_1^{5/4} F_0 z}{c_2^{5/4}} - z^2 \left(\frac{b^2 c_1^{5/2} f_4^2}{2 c_2^{5/2}} + 2 c_1^2 \Lambda \right) + \\
&- \frac{z^3 \left(c_1^{9/4} F_0 \left(4 b^2 \sqrt{c_1} f_4^2 + 17 c_2^{5/2} \Lambda \right) \right)}{6 c_2^{15/4}} + O(z^4)
\end{aligned} \tag{3.46}
$$

$$
\begin{aligned}
e^{-\frac{4}{5}\phi} &= c_2 + \frac{\sqrt[4]{c_1} F_0 z}{\sqrt[4]{c_2}} - \frac{z^2 (b^2 c_1^{3/2} f_4^2)}{10 c_2^{3/2}} \\
&- \frac{z^3 \left(c_1^{5/4} F_0 \left(4 b^2 \sqrt{c_1} f_4^2 + 5 c_2^{5/2} \Lambda \right) \right)}{30 c_2^{11/4}} + O(z^4)
\end{aligned} \tag{3.47}
$$

$$
\begin{aligned}
e^{-2\lambda_2} &= b + \frac{b \sqrt[4]{c_1} F_0 z}{c_2^{5/4}} + z^2 \left(- \frac{b^3 c_1^{3/2} f_4^2}{2 c_2^{5/2}} - b^2 \rho_2 - b c_1 \Lambda \right) + \\
&- z^3 \frac{\left(b \sqrt[4]{c_1} F_0 (4b(b c_1^{3/2} f_4^2 + 2 c_2^{5/2} \rho_2) + 9 c_1 c_2^{5/2} \Lambda) \right)}{6 c_2^{15/4}} + O(z^4)
\end{aligned} \tag{3.48}
$$

$$
\begin{aligned}
e^{-2\lambda_3} &= c_3 + \frac{\sqrt[4]{c_1} c_3 F_0 z}{c_2^{5/4}} + \frac{1}{6} c_3 z^2 \left(b \left(\frac{b c_1^{3/2} f_4^2}{c_2^{5/2}} + 4 \rho_2 \right) + 2 c_1 \Lambda \right) + \\
&+ z^3 \frac{\sqrt[4]{c_1} c_3 F_0 (4b(b c_1^{3/2} f_4^2 + 4 c_2^{5/2} \rho_2) + 5 c_1 c_2^{5/2} \Lambda)}{18 c_2^{15/4}} + O(z^4) \,.
\end{aligned} \tag{3.49}
$$

In these expressions b is not a free parameter but it satisfies the quadratic constraint

$$
b^2 \frac{c_1^{3/2} f_4^2}{c_2^{5/2}} + 4 b \rho_2 + 8 c_1 \Lambda + 6 c_3 \rho_3 = 0 \,. \tag{3.50}
$$

The constraint (3.50) comes from the fact that the first order equation (3.29) is generically quadratic in the first derivatives. However, notice that in the case $f_4 = 0$ the above constraint becomes linear, consistently with the fact that in the previous section the local solution did not have branches.

At this point we can look for numerical solutions. Notice that a generic choice for the free constants is not consistent since we also have to require that the discriminant of the equation (3.50) is non-negative in order for b to be real. We have not yet analyzed in detail the resulting moduli space of solutions, but we have found that for an open set in the space of c_1, c_2, c_3 and f_4 one of the branches gives numerical solutions that consistently end on diverging-dilaton O8$_-$ planes. Near such a point the fields behave as

$$
e^W \sim t^{-\frac{1}{4}}, \qquad e^\phi \sim t^{-\frac{5}{4}}, \qquad e^{\lambda_2} \sim t^{-\frac{1}{2}}, \qquad e^{\lambda_3} \sim t^{-\frac{1}{2}}, \qquad \text{with} \quad t \equiv |z - z_0|, \tag{3.51}
$$

analogously to their behavior at $z = \frac{\pi}{2}$ in the analytical solution (3.40). The local form of the metric near this singular locus is

$$
ds^2_{10} \sim t^{-\frac{1}{2}} (ds^2_{\text{dS}_4} + ds^2_{M_2} + ds^2_{M_3}) + t^{\frac{1}{2}} dt^2, \qquad e^\phi \sim t^{-\frac{5}{4}}, \qquad \text{with} \quad t \equiv |z - z_0| \,, \tag{3.52}
$$

and we interpret it as describing a diverging-dilaton O8-plane extended along $\mathrm{dS}_4 \times M_2 \times M_3$.

Moreover, we find that all the possibilities compatible with the constraint (3.45) can be realized, finding numerical solutions for all the cases

$$\{\rho_2, \rho_3\} = \{-1, 0\}, \qquad \{\rho_2, \rho_3\} = \{-1, 1\}, \qquad \{\rho_2, \rho_3\} = \{0, -1\}, \qquad \{\rho_2, \rho_3\} = \{1, -1\} \, . \tag{3.53}$$

The case $\{\rho_2, \rho_3\} = \{1, -1\}$ is particularly interesting since it allows for an internal S^2. We are going to explore this possibility in section 3.3 where this S^2 will become the sphere transverse to an O6-plane which will replace the diverging-dilaton O8-plane.

3.2 Discussion of singularities

We now pause for a moment our business of looking for non-supersymmetric solutions in order to analyze the singularities we have spotted in the previous constructions, in particular the one we identify with a diverging-dilaton $\mathrm{O8}_-$ plane. We will come back to our main mission in the next section, where we will build solutions in which this singularity is not present.

First of all, to avoid confusion, we remark that in our constructions we have already discarded the singularities for which *all* the fields do not behave at leading order as string theory solutions under better control, e.g. the flat space O-planes and D-branes solutions we have described in 1.2.3. We are not going to discuss those singularity in this section. Instead, the singularities we will focus on are the ones we have identified as the backreaction of a diverging-dilaton $\mathrm{O8}_-$. They have been identified by comparison with the singular behavior of $\mathrm{O8}_-$ solution in flat space and in the type $\tilde{I}$ model we have described in 1.2.4. For these singularities we have also checked that boundary conditions of the form (3.13) are satisfied for all the fields, with the correct tension τ_-.

Singularities of this kind also arise in supersymmetric AdS solutions (e.g. the in the AdS_6 [52] case we have recovered in the previous section, in the AdS_7 solutions [53] we are going to describe later and in AdS_3 [54]). Whether these singular supergravity solutions can be lifted to solutions of UV-complete theories is not a question that can be answered by working in supergravity alone. As we have already discussed, in a strongly coupled and strongly curved region, supergravity is in fact the least important contribution to the full string theory equations of motion. Notice that, albeit in this discussion we are putting the strongly coupled and strongly curved breakdowns of supergravity on the same footing, they are conceptually very different problems. As we have explained in section 1.2, the perturbative expansion in the curvature arises as a result of a perturbative treatment of the world-sheet theory. In principle, this can be avoided if we are able to perform computations without resorting to the perturbative expansion. In practice, it has been used in [105] to show that without Ramond-Ramond fluxes it is impossible to construct de Sitter vacua, even at the full non-perturbative order in the curvature. The problem with large string coupling is instead conceptually different since we are not even able to formulate string theory beyond the perturbative description. For AdS backgrounds these difficulties can be overcome by working in a dual description, for example, by employing the AdS/CFT correspondence to compare the supergravity results with field theory computations, even in presence of singularities. As an example, for the AdS_7 solutions with O8 and O6 planes this has been successfully done [55]. For the AdS_8 solution we have studied in chapter 2 we have proposed that maybe CFT techniques, like the conformal bootstrap, can give hints on the reliability of these solutions. In the dS case holography is less developed but given the recent progresses (see footnote 4) we hope that simple constructions like the ones we have presented in this chapter might be amenable of a non-perturbative analysis.

In any case, we need a better understanding of the singularity described above (and, as we will see in the next section, of *all* the orientifold singularities) and we cannot rely on supergravity alone to resolve these issues. Another way of saying it is that the fate of the singular solutions we have found by working in the supergravity approximation depends on string theory correction of either type. Since our solutions depend on some moduli, we expect these solutions to survive for at most some choices for these moduli. As an example, in the degenerate solutions with only F_0 turned on, we have seen that there is a continuous 'rescaling modulus' c, that can be used to make the ill-defined region arbitrarily small but never make it disappear. This rescaling is a symmetry of the supergravity equations of motion but not of the full string theory equations. For example the eight-derivative corrections at tree-level in string coupling (see e.g. [106, 107] for a recent summary) have the form

$$S^{\text{tree}}_{R^4} \sim \int_{M_{10}} \sqrt{-g} e^{-2\phi} \left(t_8 t_8 + \frac{1}{8} \varepsilon_{10} \varepsilon_{10} \right) R^4 \dots , \tag{3.54}$$

where $t_8 t_8$ and $\varepsilon_{10}\varepsilon_{10}$ are short-hands for combinations of metric and Levi-Civita tensors contractions. The eight-derivative action (3.54) breaks the overall rescaling (3.35) of the two-derivative action as it scales as e^{4c}. Thus c will not be a symmetry of the corrected equations of motion, and a solution of the full equation will exists at most for some values of c. It would be very interesting to systematically explore the effect of the higher derivative corrections on these, or more general, solutions to see if they help to smoothen or even completely wash away these singularities. If they do it at small coupling then one would have obtained in this way a reliable solution.

In the meantime, a criticism on the validity of our singular solutions arose in the literature [64]. This criticism is obtained by integrating a combination of the non-corrected supergravity equations of motion even in the strongly-coupled region. This procedure implicitly assumes a stronger boundary condition than (3.13) for the strongly-coupled O8$_-$. As we are going to review in section 3.2.1, this stronger version is obtained by looking at the subleading behavior of the fields in the strongly-coupled region. We consider instructive to analyze this issue in supergravity, as we are going to do it extensively in the rest of this section, in order to understand the limitations of this approximation, but we do not think that the validity of these solutions in the full theory can be decided in one way or the other by working at the supergravity level alone. In section 3.2.2 we relate the ambiguity on the boundary conditions to the space in which the variations of the fields are taken to belong, and we conclude in section 3.2.3 with a suggestion where to look for a possible physical determination of the allowed fluctuations.

3.2.1 Different forms of the jump conditions

For convenience of the reader we rewrite here the boundary conditions (3.18), which have been derived from the equations of motion for a finite-coupling O8$_+$ sitting at $z = 0$:

$$e^{W-\phi} W' = -\frac{1}{4} F_0, \qquad e^{W-\phi} \phi' = -\frac{5}{4} F_0, \qquad e^{W-\phi} \lambda' = -\frac{1}{2} F_0, \qquad \text{for } z \to 0^+ . \tag{3.55}$$

Notice that the sign of the derivatives is related to the sign of F_0, and as such we can *formally* use the above expressions to describe O8$_\pm$-planes at the same time, thus making them valid also for $z \to z_0^+$. The specification 'formally' comes from the fact that we have computed the relations (3.55) by assuming that the all functions were continuous at $z = 0$ with only a finite a jump on their first derivatives such that, as in equation (3.11), their second derivative would produce a δ-function proportional to this jump. This procedure is not justified for strongly-coupled O8$_-$ plane since at $z = z_0$ the functions are not even continuous, and the 'jump' of the first derivatives would be infinite. We have also commented that the alternative procedure of isolating a total derivative such that one could define the jump through an integration by parts requires to divide both sides of the equations of motion by zero. Either way, we see that the equations of motion are ambiguous when evaluated on top of the infinite-coupling O8$_-$ plane. For example, one could argue that the form (3.55) is not the 'correct' extrapolation in this case and that the correct one is perhaps

$$W' \stackrel{?}{=} -\frac{1}{4}e^{\phi-W}F_0, \qquad \phi' \stackrel{?}{=} -\frac{5}{4}e^{\phi-W}F_0, \qquad \lambda' \stackrel{?}{=} -\frac{1}{2}e^{\phi-W}F_0, \quad \text{for} \quad z \to z_0^+ \ , \tag{3.56}$$

where both sides have been divided by $e^{W-\phi}$. Obviously, in the finite case the two expressions are equivalent, but in the infinite coupling case both sides of the equations (3.56) have both a diverging and a finite piece and one might be tempted to match both of them. Let us see it more explicitly. For a strongly-coupled O8$_-$ plane, the metric functions and the dilaton are logarithmically divergent at $t \equiv |z - z_0|$

$$W = q_1 \log(t) + q_2 + q_3 t + \cdots \ , \qquad \phi = q_4 \log(t) + q_5 + q_6 t \ldots , \qquad \lambda = q_7 \log(t) + q_8 + q_9 t \ldots \ , \tag{3.57}$$

such that

$$e^{\phi-W} = t^{q_3-q_1}(e^{q_4-q_2} + e^{q_4-q_2}(q_6 - q_5) + \cdots) \ . \tag{3.58}$$

Matching the leading order behavior of both sides of the equations (3.56) requires to impose

$$q_1 = -\frac{1}{4}, \qquad q_4 = -\frac{5}{4}, \qquad q_7 = -\frac{1}{2}, \qquad e^{q_5-q_2}F_0 = 1 \ . \tag{3.59}$$

Imposing that the they agree also at the subleading order would also require to set

$$q_3 = q_6 = q_9 = 0 \ , \tag{3.60}$$

resulting in more conditions. Since the conditions (3.55) and (3.56) are not equivalent in the strongly-coupled case, we give them two different names, calling them respectively the *permissive* and the *restrictive* boundary conditions.

From this point of view the restrictive boundary conditions look unnatural: we are extrapolating the supergravity equations of motion in a region where they are going to be heavily corrected (notice that also the localized actions (3.9) are going to receive corrections) and we use them to impose some conditions on a subleading behavior of the fields. Moreover, one could continue this procedure and multiply the original conditions (3.55) by an even more diverging function like $e^{2\phi-2W}$, which would require even the next coefficient to vanish.

However, there is another point of view from which the restrictive boundary conditions might look more natural. As in [64, Fig. 1], one can consider the combination

$$f = 5W - \phi \ . \tag{3.61}$$

Subtracting the first two conditions in (3.55) with the appropriate relative factor then gives

$$e^{-4W}e^f f' = 0 \qquad \text{for } z \to z_0^+ \ . \tag{3.62}$$

Since e^f remains finite and different from zero at $z = z_0$, the above condition is equivalent to

$$e^{-4W}f' = 0, \qquad \text{for } z \to z_0^+ \ , \tag{3.63}$$

and one might be tempted to impose

$$f' \stackrel{?}{=} 0 \, . \tag{3.64}$$

However, since $e^{-4W} \sim |z - z_0|$, the condition (3.63) is not equivalent to (3.64), the first being the permissive one and the second being the restrictive one. An alternative way of rephrasing this discussion is that evaluating (3.56) *once the conditions* (3.59) *have been imposed*, i.e. on a solution that already satisfies (3.55), will result in a mismatch. In [64] this mismatch is interpreted as a wrong coupling of the $O8_-$.

We will now try to carefully re-derive these boundary conditions from the action, from two different points of view.

3.2.2 Derivation from the action

In this section we try to understand if the ambiguities we have described in the previous section can be avoided with a careful re-derivation of the equations of motion from the low-energy action. We will soon understand that the answer is no, and this is a consequence of the fact that the low-energy action itself is incomplete in the strongly-coupled and strongly-curved region. In doing so, we will relate the different forms of the boundary conditions to different possible choices for the space in which we take the variations of the fields to belong when computing the equations of motion.

As a boundary contribution

With the aim of avoiding dubiously-defined δ-functions, in this section we derive the equations of motion in the setting where the internal space is seen as the product

$$M_5 \times [0, z_0] \, , \tag{3.65}$$

with the two boundaries corresponding to the loci where the $O8_\pm$ planes sit. In the process we keep track of all the boundary terms, which also have to vanish in order for a solution of the equation of motion to be a stationary point of the action. By imposing this condition we obtain the boundary conditions that the fields have to satisfy.

In this description we have to be careful to include all the relevant boundary terms. First of all, the variation of a bulk term in the action can yield a boundary term when we integrate it by parts in order to extract an equation of motion. Moreover, as usual when dealing with space-times with boundaries in General Relativity, the action should already contain a boundary term known as the Gibbons-Hawking-York (GHY) term [108, 109]. The presence of this term is needed in order to have a well-defined variational problem in cases where the Lagrangian is already formulated with two-derivatives of the fields, as it is the case in General Relativity. Indeed, the Ricci scalar appearing in Einstein-Hilbert (EH) action involves two derivatives of the metric. When varying the EH action in order to compute the equations of motion we obtain boundary contributions proportional to the derivative of the variation of the metric. Schematically, they are of the form $n^M \partial_M \delta g_{NP}$, where n is the unit vector normal to the boundary and δg_{MN} is the variation of the metric. The GHY term is defined such that its variation cancels these terms. Without its inclusion, it is necessary to restrict too much the variational problem by imposing boundary condition both on the variations of the metric at the boundary and on the derivatives of such variations. In the Einstein frame, the standard GHY term has the form

$$S^{\mathrm{GHY},E} = \frac{2}{\kappa^2} \int_{\partial M_{10}} \sqrt{-h_E} \nabla^E_M n^M_E \, , \tag{3.66}$$

where use the letter E to distinguish the quantities defined in the Einstein frame

$$g_E \equiv e^{-\frac{\phi}{2}} g \, . \tag{3.67}$$

The other quantities appearing in (3.66) are h_E, the determinant of the induced metric on the boundary, and n^E is the unit vector field normal to the boundary. More geometrically, this term is the integral of the trace of the extrinsic curvature of the boundary. In the string frame, the equivalent term producing the same cancellation is

$$S^{\mathrm{GHY}} = \frac{2}{\kappa^2} \int_{\partial M_{10}} \sqrt{-h} e^{-2\phi} \nabla_M n^M \, . \tag{3.68}$$

Let us explicitly compute it. First of all, notice that it does not arise as a simple change of frame of the GHY term in Einstein frame, since such a change would produce extra pieces involving the derivatives of the dilaton. Instead, these extra pieces cancel with the ones coming from the change of frame of the bulk Einstein action

$$S_E^0 \equiv \frac{1}{\kappa^2} \int_{M_{10}} \sqrt{-g_E} \left(R_E - \frac{1}{2} \nabla\phi\nabla\phi - \frac{1}{2} e^{\frac{5}{2}\phi} F_0^2 \right) \tag{3.69}$$

(we have simplified the bulk action to include only the fields relevant to this discussion). Indeed, applying the transformation (3.67), the Einstein frame bulk action then becomes

$$\begin{aligned} S_E^0 &= \frac{1}{\kappa^2} \int_{M_{10}} \sqrt{-g} \left[e^{-2\phi} (R + 4(\nabla\phi)^2) - \frac{1}{2} F_0^2 \right] + \frac{9}{2\kappa^2} \int_{\partial M_{10}} \sqrt{-h} e^{-2\phi} n \cdot \nabla\phi & (3.70) \\ &\equiv S^0 + \frac{9}{2\kappa^2} \int_{\partial M_{10}} \sqrt{-h} e^{-2\phi} n \cdot \nabla\phi \, . & (3.71) \end{aligned}$$

A simple computation then shows that the last piece in (3.71) cancels with a contribution coming from the change of frame GHY action

$$\begin{aligned} S_E^{\mathrm{GHY}} &= \frac{2}{\kappa^2} \int_{\partial M_{10}} \sqrt{-h_E} \nabla^E \cdot n^E & (3.72) \\ &= \frac{2}{\kappa^2} \int_{\partial M_{10}} e^{-\frac{9}{4}\phi} \sqrt{-h} \frac{1}{\sqrt{-g_E}} \partial_M \left(g_E^{MN} \sqrt{-g_E} n_M^E \right) & (3.73) \\ &= \frac{2}{\kappa^2} \int_{\partial M_{10}} e^{\frac{\phi}{4}} \sqrt{-h} \frac{1}{\sqrt{-g}} \partial_M \left(e^{-\frac{9}{4}\phi} g^{MN} \sqrt{-g} n_M \right) & (3.74) \\ &= \frac{2}{\kappa^2} \int_{\partial M_{10}} e^{-2\phi} \sqrt{-h} \frac{1}{\sqrt{-g}} \left[\partial_M \left(g^{MN} \sqrt{-g} n_M \right) - \frac{9}{4} \sqrt{-g} \partial_M \phi g^{MN} n_M \right] & (3.75) \\ &= \frac{2}{\kappa^2} \int_{\partial M_{10}} e^{-2\phi} \sqrt{-h} \nabla \cdot n - \frac{9}{2\kappa^2} \int_{\partial M_{10}} e^{-2\phi} \sqrt{-h} n \cdot \nabla\phi \, , & (3.76) \end{aligned}$$

where we have used that $n = e^{-\frac{\phi}{4}} n^E$. Combining (3.71) and (3.76) we immediately obtain

$$S_E^0 + S_E^{\mathrm{GHY}} = S^0 + S^{\mathrm{GHY}} \, . \tag{3.77}$$

Finally, we add to our action the contributions coming from the localized DBI actions on the two end-points, getting the combined action

$$S_{1/2} = S^0 + \frac{1}{2} S^{\mathrm{DBI}} + S^{\mathrm{GHY}} \, , \tag{3.78}$$

where $S^{\rm DBI}$ is the usual

$$S^{\rm DBI} = -\sum_{i=\pm}\tau_i\int_{\Sigma_i} e^{-\phi}\sqrt{-h}\ . \tag{3.79}$$

Notice the subscript 1/2 and the same factor in front of the DBI reminding us that we are working with just half of the space-time in this section.

We can now vary the action (3.78) to obtain the equations of motion. For simplicity of exposition we focus on the dilaton equation of motion. We start from S_0. Its variation will produce two terms: a piece proportional to the bulk equations of motion, which we denote by E_0, and a boundary contribution

$$\begin{aligned}\delta_\phi S^0 &= \int_{M_{10}}\delta\phi E_0 + \frac{8}{\kappa^2}\int_{\partial M_{10}}\delta\phi\sqrt{-h}e^{-2\phi}\nabla\phi\cdot n \\ &= \int_{M_{10}}\delta\phi E_0 + \frac{8}{\kappa^2}\int_{\partial M_{10}}\sqrt{-g_4}\sqrt{g_5}\delta\phi e^{5\lambda-2\phi}\phi'\ ,\end{aligned} \tag{3.80}$$

where in the second line we have evaluated all the quantities on the ansatz (3.1), for which

$$\sqrt{-h} = e^{-W+5\lambda}\sqrt{-g_4}\sqrt{g_5}, \qquad \text{and} \qquad n = e^W\partial_z\ . \tag{3.81}$$

The variation of the DBI then gives

$$\frac{1}{2}\delta_\phi S^{\rm DBI} = \frac{1}{2}\sum_{i=\pm}\tau_i\int_{\Sigma_i}\delta\phi e^{-\phi}\sqrt{-h} \tag{3.82}$$

$$= \frac{1}{2}\sum_{i=\pm}\tau_i\int_{\Sigma_i}\sqrt{-g_4}\sqrt{g_5}\delta\phi e^{-\phi-W+5\lambda}\ . \tag{3.83}$$

Finally, the variation of the GHY term contributes with

$$\delta_\phi S^{\rm GHY} = -\frac{4}{\kappa^2}\int_{\partial M_{10}}\sqrt{-h}\delta\phi e^{-2\phi}\nabla_M n^M \tag{3.84}$$

$$= -\frac{4}{\kappa^2}\int_{\partial M_{10}}\delta\phi e^{-2\phi}\sqrt{-h}\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}n_N\right) \tag{3.85}$$

$$= -\frac{4}{\kappa^2}\int_{\partial M_{10}}\sqrt{-g_4}\sqrt{g_5}\delta\phi e^{-2\phi+W}\partial_z(e^{5\lambda-W}) \tag{3.86}$$

$$= -\frac{4}{\kappa^2}\int_{\partial M_{10}}\sqrt{-g_4}\sqrt{g_5}\delta\phi e^{-2\phi+5\lambda}(5\lambda'-W')\ . \tag{3.87}$$

Collecting now all the boundary terms, and evaluating them at the location of the O8$_-$ plane, we obtain

$$\kappa^2\delta_\phi S|_{{\rm O8}_-} = -4\int_{\partial M_{10}}\sqrt{-g_4}\sqrt{g_5}\delta\phi e^{-W+5\lambda-\phi}\left(e^{W-\phi}(5\lambda'-W'-2\phi')-\frac{1}{8}\kappa^2\tau_-\right)\ . \tag{3.88}$$

On a solution of the bulk equations of motion $E_0 = 0$ we should also make sure that (3.88) vanishes. However, observe that on top of the diverging-dilaton O8$_-$ plane the function in front of the of the parenthesis behaves as

$$\delta\phi e^{-W+5\lambda-\phi} \sim \frac{\delta\phi}{|z-z_0|}\ . \tag{3.89}$$

Thus, the two versions of the boundary conditions, the restrictive and the permissive one, arise as different possibilities for the behaviors of $\delta\phi$, i.e. the space in which we are taking the variations of the fields to belong. Assuming for $\delta\phi$ a power-like behavior

$$\delta\phi \sim |z - z_0|^\alpha \, , \tag{3.90}$$

and noticing that the expression in the parenthesis behaves as

$$\left(e^{W-\phi}(5\lambda' - W' - 2\phi') - \frac{1}{8}\kappa^2\tau_- \right) \sim k_1 + k_2|z - z_0| + O(|z - z_0|^2) \, , \tag{3.91}$$

we have to satisfy the condition

$$|z - z_0|^{\alpha-1}(k_1 + k_2|z - z_0| + O(|z - z_0|^2)) = 0 \, . \tag{3.92}$$

We then recognize the two natural possibilities:

1. $0 < \alpha < 1$. We only have to impose one condition for this boundary term to vanish, i.e. $k_1 = 0$. This gives a linear combination of the permissive boundary conditions (3.55).
2. $\alpha = 0$. We have to impose also the cancellation of the subleading term in the parenthesis, by imposing the two constraints $k_1 = k_2 = 0$. This gives a linear combination of the restrictive boundary conditions (3.56)

In addition we also have two more peculiar possibilities:

a) $\alpha > 1$. This choice would impose no condition at all on the fields since the boundary contribution automatically vanishes.

b) $\alpha < 0$. In this case one could also allow for the fluctuations to arbitrarily diverge. This would impose an arbitrary number of boundary conditions.

Intuitively, the bigger the space in which one takes the variations to belong, the smaller is the space of allowed solutions, since one is varying along more directions in the field space, imposing in fact more constraints.

Finally, we also notice that a natural requirement might have been for the fluctuation to be square integrable, i.e.

$$\delta\phi \in L^2(M_{10}, g) \quad \Rightarrow \quad \int_{M_{10}} \sqrt{-g}(\delta\phi)^2 < \infty.$$

Since $\sqrt{-g} = e^{-2W+5\lambda}\sqrt{-g_4}\sqrt{g_5}$, on top of the O8$_-$ it behaves as $\sqrt{-g} \sim \frac{1}{|z-z_0|^2}$, thus requiring $\alpha > \frac{1}{2}$, which is included in the possibility 1.

We now derive the same results in the language of distributions, before proposing in section 3.2.3 a physical way to understand in which space the variations $\delta\phi$ have to be taken.

As a δ-function source

In this section we consider the internal space to include the full S^1 in Figure 3.1. Hence our internal space is now

$$M_5 \times S^1 \tag{3.93}$$

and as such it does not have boundaries. The full action is then simply given by the two contributions

$$S = S^0 + S^{\rm DBI} \,. \tag{3.94}$$

The DBI action is evaluated on the loci where the two orientifold plane sit, but we can transform it on an integral on the full space-time that get localized by δ-functions. With this prescription the variation of the full action (3.94) is

$$\begin{aligned}
\kappa^2 \delta_\phi S_\phi &= \int_{M_{10}} \sqrt{-g} e^{-2\phi} [(-2\delta\phi)(R + 4(\nabla\phi)^2) + (8\nabla\phi\nabla\delta\phi)] + \\
&+ \sum_{i=\pm} \kappa^2 \tau_i \int_{\Sigma_i} \delta\phi \sqrt{-h} e^{-\phi} \qquad (3.95)\\
&= \int_{M_{10}} \delta\phi \sqrt{-g} e^{-2\phi} (-2R + 8(\nabla\phi)^2 - 8\nabla^2\phi) + \\
&+ \sum_{i=\pm} \kappa^2 \tau_i \int_{\Sigma_i} \delta\phi \sqrt{-h} e^{-\phi} \qquad (3.96)\\
&\propto \int_{M_{10}} \delta\phi e^{-W+5\lambda-\phi} \left(e^{W-\phi} (W + 2\phi - 5\lambda)'' + \cdots - \frac{1}{4} \sum_{i=\pm} \kappa^2 \tau_i \delta_i \right) \,, \qquad (3.97)
\end{aligned}$$

where in the dots we have included terms with at most one derivative. This is a combination of the equations of motion (3.4)-(3.6) we have used to derive the conditions on the derivatives in the first place. However, observe again the factor

$$\delta\phi e^{-W+5\lambda-\phi} \sim \frac{\delta\phi}{|z - z_0|} \,. \tag{3.98}$$

With different choices for $\delta\phi \sim |z - z_0|^\alpha$ we can interpret the δ-function in the bracket in different ways. In particular, the number of conditions we have to impose in the various cases is the same we have presented in the previous section. However, notice that in the case where only one constraint is imposed it can be done by using a well defined δ-function, by rewriting the equation (3.97) as

$$e^{W-\phi}(W + 2\phi - 5\lambda)'' + \cdots - \frac{1}{4} \sum_{i=\pm} \kappa^2 \tau_i \delta_i = 0 \,. \tag{3.99}$$

Instead the cases that require more than one condition to be imposed do so with the use of a not well-defined product of distributions of the form

$$\frac{1}{|z - z_0|} \delta(z - z_0) \,. \tag{3.100}$$

A possibility to avoid these ill-defined expressions is to work with a different set of variables that does not diverge on top of the O8$_-$. A natural set of variables with this property is the one directly inspired by the flat space solution:

$$H_1 \equiv e^{-4W}, \qquad H_2 \equiv e^{-\frac{4}{5}\phi}, \qquad H_3 \equiv e^{-2\lambda} \,. \tag{3.101}$$

In this language the ansatz (3.1) now reads

$$ds^2_{10} = H_1^{-1/2} ds^2_{M_4} + H_1^{1/2} (dz^2 + H_3^{-1} ds^2_{M_5}) \qquad e^\phi = H_2^{-5/4} \,, \tag{3.102}$$

and the full set of the equations motion turns out to be

$$\begin{aligned}
\Lambda &= \frac{F_0^2\sqrt{\frac{H_1}{H_2}}}{8H_1H_2^2} - \frac{5H_3\kappa}{4H_1} - \frac{(H_1')^2}{8H_1^3} + \frac{25(H_2')^2}{16H_1H_2^2} + \frac{5(H_3')^2}{4H_1H_3^2} + \\
&+ \frac{5H_1'H_2'}{16H_1^2H_2} - \frac{25H_2'H_3'}{8H_1H_2H_3}
\end{aligned} \tag{3.103}$$

$$\begin{aligned}
\sum_{i=\pm}\kappa^2\tau_i\delta_i &= -\frac{H_2^{5/4}H_1''}{H_1^{5/4}} - \frac{F_0^2\sqrt[4]{H_1}}{H_2^{5/4}} - 4H_1^{3/4}H_2^{5/4}\Lambda + \\
&+ \frac{H_2^{5/4}(H_1')^2}{H_1^{9/4}} - \frac{5\sqrt[4]{H_2}H_1'H_2'}{2H_1^{5/4}} + \frac{5H_2^{5/4}H_1'H_3'}{2H_1^{5/4}H_3}
\end{aligned} \tag{3.104}$$

$$\begin{aligned}
\sum_{i=\pm}\kappa^2\tau_i\delta_i &= -\frac{H_2^{5/4}H_3''}{\sqrt[4]{H_1}H_3} - \frac{F_0^2\sqrt[4]{H_1}}{H_2^{5/4}} - \frac{2H_2^{5/4}H_3\kappa}{\sqrt[4]{H_1}} + \\
&- 2H_1^{3/4}H_2^{5/4}\Lambda + \frac{7H_2^{5/4}(H_3')^2}{2\sqrt[4]{H_1}H_3^2} - \frac{5\sqrt[4]{H_2}H_2'H_3'}{2\sqrt[4]{H_1}H_3}
\end{aligned} \tag{3.105}$$

$$\begin{aligned}
\sum_{i=\pm}\kappa^2\tau_i\delta_i &= -\frac{\sqrt[4]{H_2}H_2''}{\sqrt[4]{H_1}} - \frac{4F_0^2\sqrt[4]{H_1}}{5H_2^{5/4}} - \frac{2H_2^{5/4}H_3\kappa}{\sqrt[4]{H_1}} - \frac{H_2^{5/4}(H_1')^2}{5H_1^{9/4}} + \\
&+ \frac{2H_2^{5/4}(H_3')^2}{\sqrt[4]{H_1}H_3^2} + \frac{\sqrt[4]{H_2}H_1'H_2'}{2H_1^{5/4}} - \frac{5\sqrt[4]{H_2}H_2'H_3'}{2\sqrt[4]{H_1}H_3} + \frac{(H_2')^2}{\sqrt[4]{H_1}H_2^{3/4}} - \frac{8}{5}H_1^{3/4}H_2^{5/4}\Lambda\ .
\end{aligned} \tag{3.106}$$

The first equation is a first order equation so that it does not involve δ-functions. Near the O8$_-$ the variables H_i remain finite, and they behave as

$$H_1 = a_1|z-z_0| + a_2(z-z_0)^2 + O(|z-z_0|^3) \tag{3.107}$$

$$H_2 = \sqrt[5]{a_1F_0^4}|z-z_0| + a_4(z-z_0)^2 + O(|z-z_0|^3) \tag{3.108}$$

$$H_3 = a_3|z-z_0| + \frac{a_2a_3}{5a_1}(z-z_0)^2 + O(|z-z_0|^3)\ . \tag{3.109}$$

Notice that now their behavior is similar to the behavior near the finite-coupling O8. In particular, these functions are continuous with a discontinuous derivative and we can compute their second derivatives as in equation (3.11):

$$H_i'' = (H_i)_0'' + \Delta H_i'\delta(z-z_0)\ . \tag{3.110}$$

The jump is now finite and we can match the coefficients of the δ-functions on both sides of the equations (3.103)-(3.106). This procedure is equivalent to the permissive boundary conditions, so that our numerical solutions correctly reproduce the δ-function in these equations.

3.2.3 Finite masses

We have seen from two different points of view that the difference between the permissive and the restrictive boundary conditions can be traced back to the space in which we take the variations of the fields to belong when computing the equations of motion. In this section we will try to understand if we can give some physical motivation to prefer a particular choice. A place where the fluctuations become relevant is the study of perturbations around a given background, for example in the context of a Kaluza-Klein reduction. Indeed, the Kaluza-Klein action will be an action for the fluctuations $\delta\phi$ we have been discussing in the previous sections. In the following we analyze it at the quadratic level in the picture where the internal space does not have boundaries. Notice that, in general, this is a convoluted computation where first the different spins need to be decoupled to extract the physical information. We do not perform this complete analysis here, but we only focus on a single block.

The second variation of the action with respect to ϕ is

$$\delta^2_\phi S = \frac{8}{\kappa^2}\int_{M_{10}} e^{-2\phi}\sqrt{-g}\nabla\delta\phi\nabla\delta\phi + \sum_i \tau_i \int_{\Sigma_i} e^{-\phi}\sqrt{-h}\delta\phi\delta\phi \ . \tag{3.111}$$

Now we expand the dilaton perturbation on a not-yet specified basis of functions on the internal space:

$$\delta\phi(x,y) \equiv \sum_k \varphi_k(x) f_k(y) \ , \tag{3.112}$$

where x and y denote coordinates on the external and internal space-time respectively. Plugging this decomposition in (3.111) we get

$$\begin{aligned}
\delta^2_\phi S_\phi &= \frac{8}{\kappa^2}\int_{M_4}\sqrt{-g_4}g_4^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^k\int_{S^1}dz\int_{M_5}\sqrt{g_5}e^{-4W+5\lambda-2\phi}f_i f_k + &(3.113)\\
&+ \frac{8}{\kappa^2}\int_{M_4}\sqrt{-g_4}\varphi^i\varphi^k\int_{S^1}dz\int_{M_5}\sqrt{g_5}e^{5\lambda-2\phi}\partial_z f_i\partial_z f_k + &(3.114)\\
&+ \frac{8}{\kappa^2}\int_{M_4}\sqrt{-g_4}\varphi^i\varphi^k\int_{S^1}dz\int_{M_5}\sqrt{g_5}e^{3\lambda-2\phi}g_5^{ab}\partial_a f_i\partial_b f_k + &(3.115)\\
&+ \sum_j \tau_j\int_{M_4}\sqrt{-g_4}\varphi^i\varphi^k\int_{M_5}\sqrt{g_5}f_i f_k e^{-W+5\lambda-\phi} \ . &(3.116)
\end{aligned}$$

When the integrals in the internal space are performed (and the last term is evaluated on top of the sources) we get an action for a tower of scalar fields φ^i's in the external space-time. In particular, the first line gives their kinetic terms and the other pieces give their mass matrix. Observe that all the pieces involving an integral in dz converge, if the f_k's do not diverge. The only potentially diverging term is (3.116) since on top of the $\mathrm{O8}_-$

$$e^{5\lambda-W-\phi} \sim \frac{1}{|z-z_0|} \ . \tag{3.117}$$

Thus in order for (3.116) to give a finite contribution to the mass-matrix we need the f_i's to behave as

$$f_i \sim |z-z_0|^\alpha, \qquad \alpha \geqslant \frac{1}{2} \ . \tag{3.118}$$

This requirement is in agreement with the permissive boundary conditions but not with the restrictive ones. However, as we have already emphasized, this is only a piece of the full KK reduction, and it might be possible that in the full computation different naively divergent term combine together to give a finite contribution even without imposing the condition (3.118) on the fluctuations.

This may require some regularization of the various divergent pieces and thus the result might depend on the 'scheme' chosen to regularize them. We have not analyzed these possibilities yet as it would require to look at the full KK reduction. Thus we do not consider the condition (3.118) to be a proof, but just as an hint that this procedure might give useful constraints on the space of allowed fluctuations for the supergravity problem to be well-defined. However, we stress again that a confirmation of the validity of these solutions has to be found beyond the supergravity limit.

To recap, we have seen that different forms of the boundary conditions are mapped to different choices for the space of the allowed fluctuation of the fields around a given background. These fluctuations become the four-dimensional fields of the effective theory. This has also consequences for the moduli of the solutions. For example, we have noticed that in the case where F_0 is the only flux turned on there is a continuous rescaling of the supergravity equations of motion, which we have labeled by c in (3.21), that can be used to generate new solutions. We have already argued that this modulus is going not to be free when stringy corrections are considered and it will be fixed in a fully corrected solution, if any. However, now we can see that it cannot be interpreted as a four-dimensional field already at the two-derivative level. Indeed, a variation corresponding to the action of c acts on the dilaton as

$$\phi \to \phi + c \,, \tag{3.119}$$

thus $\delta\phi = c$. This fluctuation is not in the class (3.118), as it does not vanish on the O8$_-$ plane.

3.3 Solutions with O6$_-$ planes

In this section, we come back to the study of dS$_4$ solutions of massive type IIA supergravity. Building on the previous results, we will start the construction of richer models with extra fluxes threading the internal cycles. Given the uncertainty surrounding its definition, we will try to avoid the use of diverging-dilaton O8$_-$ planes. However, thanks to the famous no-go theorem [65] we know that orientifold planes are important ingredients for constructing classical de Sitter solutions. In this section, we will find solutions where the O8$_-$ planes are replaced with O6$_-$ planes.

3.3.1 Setup

We explore the metric ansatz (3.26) with the addition of more internal fluxes. The idea is to impose again the presence of O8$_+$ planes, and to see if we can get new consistent endpoints, different than the diverging-dilaton O8$_-$. In particular, we also allow the presence of the RR flux F_2 on the internal space. This flux couples electrically to D6-branes and O6-planes. As we have seen in equation (3.45), when fluxes other than F_0 are present, it is possible to allow for a positively curved factor in the internal space. Anticipating a solution with an O6-plane we then take this factor to be a sphere S^2, which will be the sphere transverse to the O6-plane. All in all, we work with the ansatz

$$ds^2_{10} = e^{2W} ds^2_{M_4} + e^{-2W}(e^{2Q}dz^2 + e^{2\lambda_3} ds^2_{M_3} + e^{2\lambda_2} ds^2_{S^2}), \tag{3.120}$$

with all the functions only depending on the coordinate z. Again M_3 is an Einstein space with curvature $R_{M_3} = 3\rho_3$, and the cosmological constant of M_4 is normalized as $R_{M_4} = 4\Lambda$. Notice that in the ansatz (3.120) we have introduced a gauge redundancy Q, which will help us to describe within the metric ansatz (3.120) a known class of AdS$_7$ solutions which will guide our construction.

The most general choice for the fluxes compatible with the symmetries of the metric is

$$\begin{aligned} H &= h dz \wedge \mathrm{vol}_2 + h_2 \,\mathrm{vol}_3, && (3.121)\\ F_2 &= f_2 \,\mathrm{vol}_2, && (3.122)\\ F_4 &= f_{41} \,\mathrm{vol}_3 \wedge dz + f_{42} \,\mathrm{vol}_4, && (3.123)\\ F_0 &\neq 0, && (3.124) \end{aligned}$$

where a priori h, f_2, f_{41} and f_{42} are all functions of the coordinate z.

Equations of motion

Away from sources, the unique solution of the equations of motion for the fluxes (1.23)-(1.25) and their Bianchi identities (1.17)-(1.20), with non-vanishing F_0, is given by

$$h = f_2'/F_0, \quad h_2 = 0, \qquad f_{42} = \mathrm{cost}, \quad f_{41} = \frac{1}{F_0} e^{Q-6W-2\lambda_2+3\lambda_3}(F_0 c_1 - f_{42} f_2), \tag{3.125}$$

where c_1 is an integration constant. The equations for the fluxes are then completely satisfied up to the differential equation

$$f_2'' = e^{2(Q-5W+\phi)}(F_0 c_1 f_{42} + (e^{8W} F_0^2 - f_{42}^2) f_2) + f_2'(Q' - 4W' + 2\lambda_2' - 3\lambda_3' + 2\phi'). \tag{3.126}$$

This local form of the equation of motion has to be supplemented with appropriate boundary conditions for the fluxes, which we are going to discuss in the next section where we focus on a specific choice for the sources. To simplify the analysis we restrict to the case $F_4 = 0$ even though the more general case might reveal interesting new features. In particular, we now set $f_{41} = c_1 = 0$. Moreover, we reduce the gauge redundancy to a constant parameter by setting

$$Q(z) = q_0 \,. \tag{3.127}$$

Specializing the dilaton (1.21) and the Einstein (1.22) equations of motion to our ansatz (3.120) we obtain

$$\begin{aligned} -8\Lambda e^{2q_0-4W} &= \frac{(f_2')^2 e^{4W-4\lambda_2}}{F_0^2} - f_2^2 e^{-4\lambda_2+2q_0+2W+2\phi} - F_0^2 e^{2q_0-2W+2\phi} && (3.128)\\ &- 4(\lambda_2')^2 - 12(\lambda_3')^2 - 24\lambda_2'\lambda_3' + 6\rho_3 e^{2q_0-2\lambda_3} + 4e^{2q_0-2\lambda_2} \\ &- 8(\phi')^2 + 16\lambda_2'\phi' + 24\lambda_3'\phi' - 8W'\phi' + 16(W')^2 \\ -W'' e^{2\lambda_2-W-\phi} &= -\frac{1}{4} f_2^2 e^{-2\lambda_2+2q_0+W+\phi} - \frac{1}{4} F_0^2 e^{2\lambda_2+2q_0-3W+\phi} && (3.129)\\ &- \Lambda e^{2\lambda_2+2q_0-5W-\phi} - 2W'\phi' e^{2\lambda_2-W-\phi} + 2\lambda_2' W' e^{2\lambda_2-W-\phi} \\ &+ 3\lambda_3' W' e^{2\lambda_2-W-\phi} - \frac{e^{q_0}}{16\pi}\kappa^2\tau_6\delta_6 - \frac{1}{4}\delta_8\kappa^2\tau_8 e^{2\lambda_2+q_0-2W} \\ -\lambda_2'' e^{2\lambda_2-W-\phi} &= \frac{(f_2')^2 e^{-2\lambda_2+3W-\phi}}{2F_0^2} - \frac{1}{2} F_0^2 e^{2\lambda_2+2q_0-3W+\phi} - e^{2q_0-W-\phi} && (3.130)\\ &- \Lambda e^{2\lambda_2+2q_0-5W-\phi} - 2\lambda_2'\phi' e^{2\lambda_2-W-\phi} + 2(\lambda_2')^2 e^{2\lambda_2-W-\phi} \\ &+ 3\lambda_2'\lambda_3' e^{2\lambda_2-W-\phi} - \frac{1}{2}\delta_8\kappa^2\tau_8 e^{2\lambda_2+q_0-2W} \end{aligned}$$

$$\begin{aligned}
-\lambda_3'' e^{2\lambda_2-W-\phi} &= -\frac{1}{2}f_2^2 e^{-2\lambda_2+2q_0+W+\phi} - \frac{1}{2}F_0^2 e^{2\lambda_2+2q_0-3W+\phi} - \Lambda e^{2\lambda_2+2q_0-5W-\phi} && (3.131)\\
&- \rho_3 e^{2\lambda_2-2\lambda_3+2q_0-W-\phi} - 2\lambda_3'\phi' e^{2\lambda_2-W-\phi} + 3(\lambda_3')^2 e^{2\lambda_2-W-\phi} + \\
&+ 2\lambda_2'\lambda_3' e^{2\lambda_2-W-\phi} - \frac{e^{q_0}}{8\pi}\tau_6\delta_6\kappa^2 - \frac{1}{2}\delta_8\kappa^2\tau_8 e^{2\lambda_2+q_0-2W}\\
-\phi'' e^{2\lambda_2-W-\phi} &= \frac{(f_2')^2 e^{-2\lambda_2+3W-\phi}}{4F_0^2} - \frac{1}{2}f_2^2 e^{-2\lambda_2+2q_0+W+\phi} - F_0^2 e^{2\lambda_2+2q_0-3W+\phi} && (3.132)\\
&- \frac{3}{2}\rho_3 e^{2\lambda_2-2\lambda_3+2q_0-W-\phi} - 2\Lambda e^{2\lambda_2+2q_0-5W-\phi} - 3\lambda_3'\phi' e^{2\lambda_2-W-\phi}\\
&+ 2W'\phi' e^{2\lambda_2-W-\phi} - 4(W')^2 e^{2\lambda_2-W-\phi} - 2\lambda_2'\phi' e^{2\lambda_2-W-\phi}\\
&+ (\lambda_2')^2 e^{2\lambda_2-W-\phi} + 3(\lambda_3')^2 e^{2\lambda_2-W-\phi} + 6\lambda_2'\lambda_3' e^{2\lambda_2-W-\phi}\\
&- e^{2q_0-W-\phi} - \frac{3e^{q_0}}{16\pi}\tau_6\delta_6\kappa^2 - \frac{5}{4}\delta_8\kappa^2\tau_8 e^{2\lambda_2+q_0-2W}\ .
\end{aligned}$$

Equation (3.128) is a first order equation which will act as a constraint. Each of the other four equations involves a second derivative of a different function. To distinguish between the tension of the $O8_+$ and of the $O6_-$, we have changed the notation with respect to the previous section by defining $\tau_8 \equiv \tau_+$. In particular, the tension of the $O6_-$ plane (τ_6) enters in the equations of motion since we have also included in the full action the corresponding localized term. However, as in the flat space solutions in section 1.2.3, the formal δ-function localizing the $O6_-$ plane (δ_6) is located outside of the physical space-time and it is thus not clear how it really affects the equations of motion. For this reason, we will take into account the presence of an $O6_-$ plane by imposing its charge as in equation (3.136), and we will recognize it from the behavior of the fields at the boundary of its 'hole'. The same procedure has been employed to find the AdS_7 solutions we will review in section 3.3.2. Trying to use the uncorrected supergravity equations of motion beyond this boundary does not appear to be physically meaningful since approaching this locus the string coupling starts diverging. Moreover, trying to cross it would eventually lead to an imaginary metric and dilaton.

Flux quantization

From now on, we will focus on setups where the coordinate z starts from an $O8_+$ plane sitting at $z = 0$ and ends at $z = z_0$ at the boundary of the hole produced by an O6. The effect on the space-time due to the $O8_+$ projection is to impose on the solution the symmetry $z \to -z$. The O6 action instead acts as an antipodal map on the transverse S^2, which is thus $\mathbb{RP}^2$. More details on settings with combined orientifold planes can be found in [110, 111, 112].

The presence of an $O8_+$ makes the flux F_0 jump according to its Bianchi identity:

$$\Delta F_0 = -\kappa^2\tau_8. \qquad (3.133)$$

Since F_0 is odd across an O8-plane, we have $\Delta F_0 = 2F_0|_{z\to 0^+}$. Combining the two equations we get in our conventions

$$F_0|_{z\to 0^+} = \frac{n_0^+}{2\pi}, \qquad n_0^+ = -4. \qquad (3.134)$$

The behavior of F_2 on the O8-plane requires some care. Away from O6/D6 and NS5/ONS5 we have to satisfy the Bianchi identities

$$dF_2 = F_0 H, \qquad dH = 0. \qquad (3.135)$$

In particular, H does not have to jump. Since on top of an $O8_+$ plane F_0 jumps as in (3.133), then dF_2 has to jump accordingly. The O6 at $z = z_0$ is not defined through a δ-function, since the δ_6 is outside of the space-time, but through the boundary condition

$$f_2(z_0) = 1. \tag{3.136}$$

This choice fixes the flux quantization for F_2. Finally, we have to impose also the flux quantization for H:

$$\int_{M_3} H = (2\pi)^2 N. \tag{3.137}$$

To impose it, we integrate the Bianchi equation on half of the internal space,

$$\int_{\frac{M_3}{2}} dF_2 = \int_{\frac{M_3}{2}} F_0 H, \tag{3.138}$$

and we use (3.137) and the fact that H is even across the $O8_+$ to obtain

$$4\pi(f_2(z_0) - f_2(0)) = F_0^+ \frac{1}{2}(2\pi)^2 N. \tag{3.139}$$

By writing $F_0 \equiv \frac{n_0}{2\pi}$ and using (3.136), we get

$$f_2(0) = 1 - \frac{n_0^+ N}{4}\ , \tag{3.140}$$

where for a simple $O8_+$ (i.e. without D8s on top of it) $n_0^+ = -4$. Summing up, for a solution of the type $O8_+$-$O6_-$ we have to impose the conditions (3.134), (3.136) and (3.140), which account for the flux quantization of F_0, F_2 and H.

We conclude this section deriving a constraint on the cosmological constant. By integrating the equations of motion across the $O8_+$plane at $z = 0$, we obtain the usual boundary conditions

$$\lambda_i' = -\frac{1}{2}F_0 e^{q_0 - W + \phi}, \qquad W' = -\frac{1}{4}F_0 e^{q_0 - W + \phi}, \qquad \phi' = -\frac{5}{4}F_0 e^{q_0 - W + \phi}, \qquad \text{at } z = 0\ . \tag{3.141}$$

By plugging these conditions into the first order equation (3.128) we get

$$\begin{aligned} \Lambda \;&=\; \frac{1}{8}f_2^2 e^{-4\lambda_2 + 6W + 2\phi} - \frac{3}{4}\rho_3 e^{4W - 2\lambda_3} - \frac{1}{2}e^{4W - 2\lambda_2} - \frac{(f_2')^2 e^{-4\lambda_2 - 2q_0 + 8W}}{8F_0^2} + \\ &+\; \frac{c_1 f_2 f_{42} e^{-4\lambda_2 - 2W + 2\phi}}{4F_0} - \frac{1}{8}c_1^2 e^{-4\lambda_2 - 2W + 2\phi} - \frac{f_2^2 f_{42}^2 e^{-4\lambda_2 - 2W + 2\phi}}{8F_0^2} - \frac{1}{8}f_{42}^2 e^{2\phi - 6W} \\ &\quad \text{at } z = 0\ . \end{aligned} \tag{3.142}$$

For completeness, in the second line of (3.142) we have reinstated the contribution of F_4. Similarly to (3.45) we observe that with a negative ρ_3 we can obtain a positive cosmological constant.

3.3.2 Review of AdS_7 solutions

A notable class of solutions that can be descried within our ansatz (3.120)-(3.126) is given by AdS_7 solutions. They have been first found numerically in [47], then analytically in [48] and finally put in the following form in [49]. Similarly to our description of the AdS_6 solutions around equation (3.38), these AdS_7 backgrounds are seen in our ansatz as $AdS_4 \times H_3$ solutions, where H_3 is a compact hyperbolic space with the same Einstein constant of AdS_4

$$\rho_3 = \Lambda < 0\ . \tag{3.143}$$

In our language, the local form of the metric functions is given by

$$e^{2W} = \sqrt{2}\pi\sqrt{-\frac{\alpha}{\ddot{\alpha}}}, \qquad e^{2\lambda_2} = \frac{2\pi^2 X^{5/2}\alpha^2}{X^5\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}, \qquad \lambda_3 = 2W, \qquad q_0 = 2\pi^2 X^{-5/2}, \tag{3.144}$$

the cosmological constant has the value

$$\rho_3 = \Lambda = -\frac{2+X^5}{4X^{5/2}},$$

and F_2 and the dilaton are obtained from

$$e^{\phi} = X^{5/4}\frac{2^{5/4}3^4\pi^{5/2}\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{3/4}}{\sqrt{X^5\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}}, \qquad f_2 = \frac{\ddot{\alpha}}{2\times 3^4\pi^2} + \frac{F_0\pi X^5\alpha\dot{\alpha}}{X^5\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}. \tag{3.145}$$

The equations of motion then force α to be a piece-wise degree 3 polynomial satisfying the condition

$$\dddot{\alpha} = -162\pi^3 F_0. \tag{3.146}$$

If eight-dimensional sources are present, F_0 changes accordingly to its Bianchi identity (3.133), and $\dddot{\alpha}$ jumps. Nevertheless one can impose that the metric and the fields are continuous. The metric is then given by

$$\frac{1}{\sqrt{2}\pi}ds^2_{10} = \sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}X^{-5/2}\left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2X^{-5}\alpha\ddot{\alpha}}ds^2_{S^2}\right). \tag{3.147}$$

Supersymmetric solutions are obtained for X=1, while non-supersymmetric ones are obtained for $X = 2^{1/5}$ [113]. However, when seen as $AdS_4 \times H_3$ solutions, both the choices give non-supersymmetric backgrounds[6]. From now on we adopt the $AdS_4 \times H_3$ point of view, which we will then generalize.

Different sources are chosen by specifying the correct boundary conditions for the polynomial

$$\alpha = a_0 + a_1 z + \frac{a_2}{2!}z^2 - \frac{162\pi^3 F_0}{3!}z^3\ . \tag{3.148}$$

The various possibilities are the following.

- $a_0 = a_2 = 0$. The metric locally reads

$$ds^2_{10} \sim (ds^2_{\mathrm{AdS}_4} + ds^2_{H_3}) + X^{-5/2}(dz^2 + z^2 ds^2_{S^2})\ . \tag{3.149}$$

  This corresponds to having a regular point at $z = 0$ since the internal S^2 shrinks regularly.

[6] There is a procedure for obtaining instead supersymmetric $AdS_4 \times H_3$ but it produces more involved configurations [114].

- $a_0 = 0, a_2 \neq 0$. The metric locally reads

$$ds^2_{10} \sim z^{\frac{1}{2}}(ds^2_{\mathrm{AdS}_4} + ds^2_{H_3}) + z^{-\frac{1}{2}} X^{-5/2}(dz^2 + z^2 ds^2_{S^2}) \ . \tag{3.150}$$

  Comparing with the solutions in section (1.2.3) we interpret it as describing at $z = 0$ a D6 extended along $\mathrm{AdS}_4 \times H_3$.

- $a_2 = 0, a_0 \neq 0$. The metric locally reads

$$ds^2_{10} \sim z^{-\frac{1}{2}}(ds^2_{\mathrm{AdS}_4} + ds^2_{H_3}) + z^{\frac{1}{2}} X^{-5/2}(dz^2 + k ds^2_{S^2}) \ . \tag{3.151}$$

  This is the metric describing an O6$_-$ plane extended along $\mathrm{AdS}_4 \times H_3$, expanded around the boundary of its hole, as described in section (1.2.3).

- $a_1 = a_2 = 0$. We recognize the local metric

$$ds^2_{10} \sim z^{-\frac{1}{2}}(ds^2_{\mathrm{AdS}_4} + ds^2_{H_3} + k ds^2_{S^2}) + z^{\frac{1}{2}} dz^2 \tag{3.152}$$

  to be the one of a diverging-dilaton O8-plane extended along $\mathrm{AdS}_4 \times H_3 \times S^2$.

- $a_1 = 0, a_2 \neq 0$. In this case the metric does not stop at $z = 0$, but it has a non-differentiable point if also F_0 jumps crossing this point. Different regions with different values of F_0 can be connected on the locus at $z = 0$. From the supergravity point of view the resulting configuration can thus per interpreted as a finite-coupling O8$_+$ plane or a stack of D8-branes. In the AdS_7 setting, solutions constructed in this way are the holographic duals of the six-dimensional conformal field theories we are going to study in Chapter 4.

In these examples we have only looked at the metric, but using (3.145) one can check that also the dilaton and the fluxes behave properly near the various endpoints. Notice that these are only the local behavior near the sources and one can build global solutions by specifying these conditions on the two endpoints.

We now describe explicitly how to build a non-supersymmetric $\mathrm{AdS}_4 \times H_3$ solution with O8$_+$ and O6$_-$ planes, which will then deform numerically to obtain de Sitter solutions. Other than letting the appropriate coefficient to vanish on the different endpoints, we have to impose the correct flux quantization conditions that we have studied in the previous section. Hence, if we want an O8$_+$ at $z = 0$ and an O6$_-$-plane at $z = z_0$ we have to impose

$$\dot{\alpha}(0) = 0, \qquad \ddot{\alpha}(z_0) = 0, \qquad f_2(z_0) = 1, \qquad f_2(0) = 1 - \frac{n_0^+ N}{4} \quad . \tag{3.153}$$

Moreover, for a simple O8+ (i.e. without D8's on top) we have to impose $n_0^+ = -4$. In the gauge fixed as in (3.147), z_0 depends on N as

$$z_0 = \frac{4 - n_0^+ N}{2 n_0^+} = -\frac{N+1}{2}, \tag{3.154}$$

and the requirement that $z_0 > 0$ forces $N < -1$. Thus, a part from the choice of the integer N, the solution is completely specified by the function α. In Figure 3.3 we plot the behavior of the various fields for a solution with a given N.

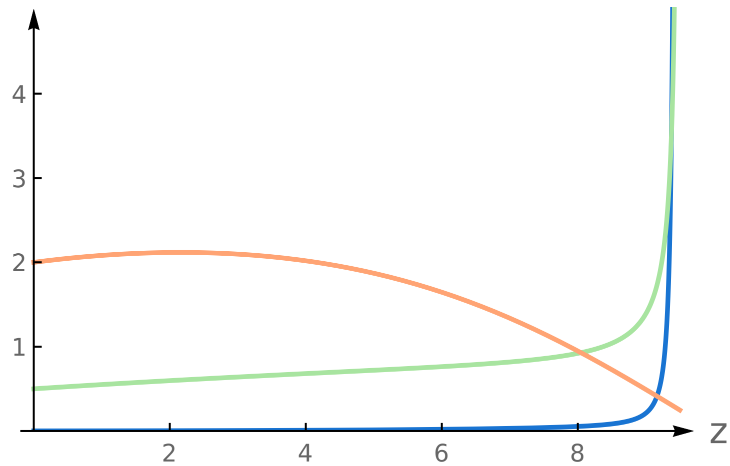


Figure 3.3: An $\mathrm{AdS}_4 \times H_3$ solution with $n_0^+ = -4$ and $N = -20$. The plotted functions are $e^{4\phi}$(blue), e^{4W}(green, rescaled), e^{λ_2}(orange, rescaled). For these solutions $\lambda_3 = 2W$. On the left it starts from an $\mathrm{O8}_+$ plane and on the right it ends on the boundary of the hole produced by an $\mathrm{O6}_-$ plane. Notice that the S^2 does not shrink there as e^{λ_2} does not reach zero.

3.3.3 Numerical solutions

In this section, we look for dS_4 solutions involving an $\mathrm{O8}_+$ plane and an $\mathrm{O6}_-$ plane. Our strategy is to take a large N (thus with small dilaton and small curvature) non-supersymmetric $\mathrm{AdS}_4 \times H_3$ solution described in the previous section and numerically rise the cosmological constant to positive values.

We start by building the local solution near an $\mathrm{O8}_+$ plane at $z = 0$. By imposing the boundary conditions (3.141) we obtain the expressions

$$e^{-4W} = 1 + \frac{F_0 e^{q_0} z}{a_1^{3/4}} + \frac{1}{2} e^{2q_0} z^2 \left(-\frac{f_{20}^2}{a_1^{3/2} a_2^2} - 4\Lambda \right) + O(z^3) \tag{3.155}$$

$$e^{-\frac{4}{3}\phi} = a_1 + \frac{5}{3} \sqrt[4]{a_1} F_0 e^{q_0} z + \frac{z^2 \left(\frac{6 a_1^{3/2} b^2}{F_0^2} + e^{2q_0} (10 a_2^2 F_0^2 - 9 f_{20}^2) \right)}{18 \sqrt{a_1} a_2^2} + O(z^3) \tag{3.156}$$

$$e^{-2\lambda_3} = 1 + \frac{F_0 e^{q_0} z}{a_1^{3/4}} + \frac{z^2 \left(2 e^{2q_0} \left(a_2 (a_2 \Lambda + 2) - \frac{2 f_{20}^2}{a_1^{3/2}} \right) + \frac{b^2}{F_0^2} \right)}{6 a_2^2} + O(z^3) \tag{3.157}$$

$$e^{2\lambda_2} = a_2 - \frac{a_2 F_0 e^{q_0} z}{a_1^{3/4}} + z^2 \left(e^{2q_0} \left(\frac{a_2 F_0^2}{a_1^{3/2}} + a_2 \Lambda + 1 \right) - \frac{b^2}{2 a_2 F_0^2} \right) + O(z^3) \tag{3.158}$$

$$f_2 = f_{20} + bz + \frac{F_0 e^{q_0} z^2 (f_{20} F_0 e^{q_0} - a_1^{3/4} b)}{2 a_1^{3/2}} + O(z^3). \tag{3.159}$$

Some comments are in order.

- Since in this case we preferred to keep Λ and ρ_3 as continuous parameters, we fixed the redundancy in the parametrization of the metric by setting e^{-4W} and $e^{-2\lambda_3}$ equal to 1 on top of the $O8_+$.

- q_0 is just a gauge redundancy and we can use it to rescale the length of the interval.

- f_{20} and F_0 are discrete parameters depending on N and n_0 as in (3.134) and (3.140):

$$f_{20} = 1 - \frac{n_0^+ N}{4}, \qquad F_0 = \frac{n_0^+}{2\pi}. \tag{3.160}$$

  For a simple $O8_+$ without D8-branes on top of it we have to take $n_0^+ = -4$.

- b is not a free parameter, but it satisfies a quadratic constraint coming from the first order equation (3.128)

$$b = \pm F_0 e^{q_0} \sqrt{\frac{f_{20}^2}{a_1^{3/2}} - 2a_2(3a_2\rho_3 + 4a_2\Lambda + 2)}. \tag{3.161}$$

  The two roots correspond to the two possible choices for the sign of $f_2'(0)$. We find that only the positive root gives solutions with a singularity we are be able to recognize as an O6-plane. Moreover, notice that in order to have real solutions the expression inside the square root has to be non-negative. This gives a constraint on the initial parameters of the solution, which therefore cannot be independently initialized.

Summing up, the local solution near the $O8_+$ plane depends on four continuous parameters a_1, a_2, ρ_3 and Λ and two discrete ones, N and n_0^+. These parameters have to be chosen such that b defined in (3.161) is real. To hit an $O6_-$ we then need to find a point where $f_2 = 1$, requiring one fine-tuning. Moreover, the $O6_-$ is identified by the behavior of the fields near the boundary of the 'hole', as in (3.151). Correspondingly, we have to tune the parameters (ρ_3, a_1, a_2) in order to reach a point z_0 where the functions behave as

$$f_2(z_0) = 1, \qquad e^{\lambda_2} \sim \text{const}, \qquad e^W \sim |z - z_0|^{-\frac{1}{4}}, \qquad e^{\lambda_3} \sim |z - z_0|^{-\frac{1}{2}}, \qquad e^{\phi} \sim |z - z_0|^{-\frac{3}{4}} \quad . \tag{3.162}$$

Near such a point, the metric, the dilaton and the fluxes have the same local expression as in (3.151), with AdS_4 replaced by dS_4. Notice that the supergravity approximation ceases being reliable already approaching the boundary of hole, since the dilaton starts growing and eventually diverges just like the $O6_-$ in flat space. Figure 3.4 shows a typical solution with this behavior.

Notice the similarities between Figure 3.4 and Figure 3.3. The main difference is that now $\lambda_3 \neq 2W$, allowing for the seven-dimensional space to be split into spaces with different curvatures. We have not-yet analyzed the more complicated space of solutions we obtain in this way, but it seems that there are some free parameters which are not determined by the conditions (3.162). One might wonder if also in this case more restrictive boundary conditions could fix them. Again, given the appearance of a strongly coupled region it is not clear that this question should be formulated within supergravity. When the Romans mass is turned off, it is known that the singularity is resolved in M-theory, where the $O6_-$ is lifted into a smooth geometry given by the Atiyah-Hitchin metric [99, 100, 101]. When F_0 is present a similar mechanism is not known.

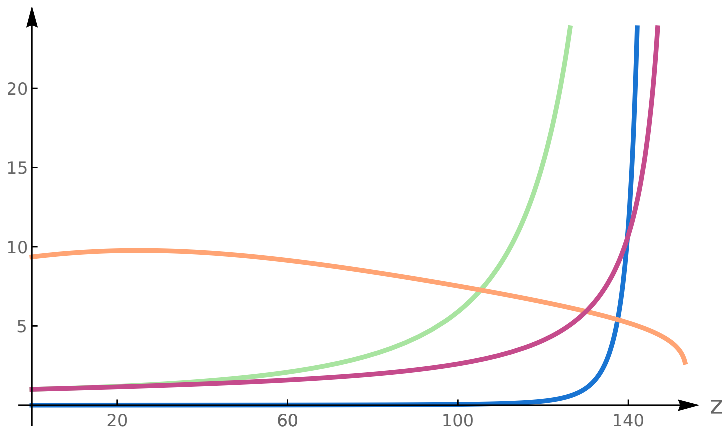


Figure 3.4: A numerical dS_4 solution with $\Lambda = 2.7 \times 10^{-3}$ and $\rho_3 = -2.1 \times 10^{-1}$. It starts from an $O8_+$ plane (on the left) and ends at the boundary of the hole produced by an $O6_-$ plane (on the right). The functions are $e^{4\phi}$(blue), e^{4W}(green), e^{λ_2}(orange, rescaled) and $e^{2\lambda_3}$(purple). On the right, the functions behave as in equation (3.162).

A heuristic test one might try perform, is to analytically continue solutions as the one in Figure 3.4 in the non-physical region, where the metric and dilaton are imaginary, and check whether they have the same pole the flat space solutions have. For the analytic AdS_7 solutions we are able to follow this procedure and the continuation of the solution in Figure 3.3 has a pole at $z_0 = -\frac{N}{2}$. Instead, for our numerical dS_4 solutions the situation is less clear. In Figure 3.5 we show the continuation inside the hole of the solution in Figure 3.4, zoomed into the region across the boundary.

Notice that the qualitative behavior of the functions in Figure 3.5 inside the hole is different from their behavior in the flat space solution. In particular $H \equiv e^{-4W}$, is not diverging and $e^{\lambda_2} \sim (z - z_{\text{fin}})^{0.1696}, e^{\lambda_3} \sim (z - z_{\text{fin}})^{0.562}$ and $e^{\phi} \sim (z - z_{\text{fin}})^{0.5126}$. It might turn out to be possible to choose the free parameters in the numerical solutions such that the pole in the unphysical region is reproduced, but it is not clear that this procedure is physically meaningful. Even if some extra conditions have to be imposed, we need to understand them by referring only to the physical space-time. An indication might come from a more careful analysis of the mass of the object behind the singularity.

Finally, we conclude with a comment on the physical value of the cosmological constant. Up to now Λ was only a numerical parameter. Its physical value in units of the four dimensional Planck's mass is obtained as

$$\Lambda_{\text{phys}} = \frac{\Lambda}{M_p^2}, \qquad \text{with} \quad M_P^2 = \kappa^2 \,\text{Vol}_2 \,\text{Vol}_3 \int dz e^{Q-4W+2\lambda_2+3\lambda_3-2\phi}. \tag{3.163}$$

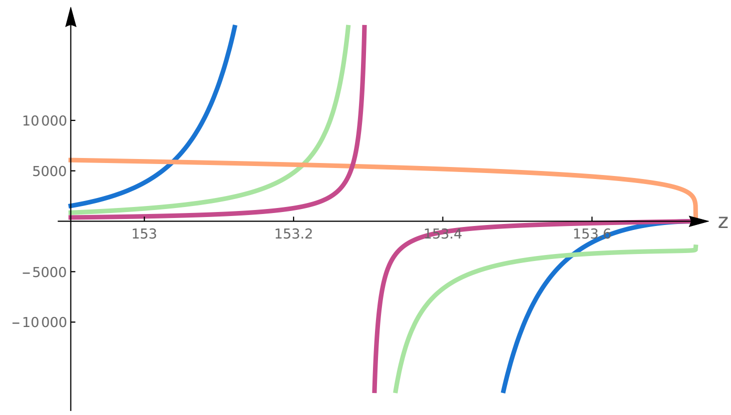


Figure 3.5: Zoom around the boundary of the hole of the analytic continuation of the solution in Figure 3.4. The functions are $e^{4\phi}$(rescaled, blue), e^{4W}(rescaled, green), $e^{2\lambda_3}$(purple) and e^{λ_2}(rescaled, orange). Notice that both the metric and the dilaton become imaginary on the right half of the plot.

Performing this integral[7] for the solution in Figure 3.4 we obtain

$$\Lambda_{\text{phys}} \sim \frac{1}{\kappa^2} \, \text{Vol}_3 \, 10^{-9}. \tag{3.164}$$

This small number is expected from the AdS$_7$ solutions, where at large N the Planck's mass scales as N^5 and Λ remains constant [49].

[7]There is almost no difference in stopping the integral on the boundary of the hole or on top of the O6 since the contribution of the hole is very small.

Chapter 4

Holographic RG flows

In this chapter we change our point of view and we use supergravity as a tool to study quantum field theories, by employing the so-called AdS/CFT correspondence.[1] In particular, we will analyze the physics of Renormalization Group (RG) flows between six-dimensional Superconformal Field Theories (SCFTs) through a holographic seven-dimensional supergravity description.

In section 4.1 we introduce the class of field theories we are interested in. As we are going to see, these six-dimensional theories are identified by the choice of an integer N, an ADE group G, and a pair of nilpotent elements in its (complexified) Lie algebra. We will refer to this set of information as the *theory data*. We focus on the $G = \mathrm{SU}(k)$ case, and in section 4.2 we propose a seven-dimensional gravitational holographic description of these SCFTs. Our strategy will be to add to a known universal consistent truncation the degrees of freedom that capture the physics of the D-branes. In section 4.3 we check the validity of our proposal by finding a correspondence between the theory data and the vacua of our gravitational theory. Moreover, we also perform extra quantitative tests of our proposal by comparing with known results both in ten-dimensional supergravity, where the holographic duals have been originally found, and in the quantum field theories. In doing so, we quantitatively understand the range of applicability of our theory.

We then try in section 4.4 to connect the vacua we have found through domain wall solutions, i.e. smooth geometries interpolating between them. When such solutions exist, they are interpreted as the holographic realization of RG flows connecting the corresponding dual field theories. We will find that in our theory these supersymmetric flows are described by a widely studied equation, known as *Nahm equation*. Applying known mathematical results on the space of solutions to this equation, we will be able to show that these interpolating geometries exist if and only if a certain condition on the nilpotent elements associated to the connected vacua is met. This condition matches the one previously conjectured on the quantum field theory side of the duality, which is then confirmed by our results.

An unexpected outcome of this work is that we have been able to perform very precise qualitative and quantitative checks on the physics of six-dimensional field theories by employing a supergravity theory which is not a consistent truncation of string theory. An interesting question is to understand if and to which extent this theory is reliable also 'far' from the original supersymmetric vacua around

[1] In this chapter we will use the main ideas and tools of AdS/CFT correspondence [21, 115, 116, 117] without reviewing them. The literature on this fascinating subject is breathtakingly vast and a good entry point is the famous review [118].

which it has been constructed. For example, it might turn out that it is also able to capture, in a more simple description, even some interesting non-supersymmetric sectors of the quantum field theories we are going to discuss. Perhaps, it might also contain non-trivial de Sitter backgrounds which are not just 'artifacts' of this effective approximation. This chapter is based on the published work [3].

4.1 The six-dimensional field theories

A lot of evidence has been accumulated in the literature on the existence of an interesting class of six-dimensional supersymmetric conformal field theories. These interacting fixed points with $\mathcal{N} = (1,0)$ supersymmetry[2] are labeled by an integer N, an ADE group G and two nilpotent elements $\mu_L, \mu_R \in \mathfrak{g}_{\mathbb{C}}$. We will refer to this set of data as the *theory data* and we denote the corresponding theories as

$$\mathcal{T}^N_{G,\mu_L,\mu_R}. \tag{4.1}$$

For $G = \mathrm{SU}(k)$, these theories have been proposed to describe the dynamics of D-branes probing orbifold singularities [120, 121] and more general systems of NS5-D6-D8 branes [122, 123]. For $G = \mathrm{SO}(2k)$ and E_k the theories were described in [124, 125, 126] and can be engineered by adding O6-planes to the IIA constructions of the $\mathrm{SU}(k)$ theories, or with F-theory constructions.

In this thesis, we are going to focus on the $G = \mathrm{SU}(k)$ case. The corresponding theories can be engineered in type IIA string theory by considering $N+1$ NS5-branes living inside the world-volume of k overlapping D6-branes, in a setting with or without D8-branes stacks where the D6-branes can end.

To discuss their properties in relation to the theory data, we first introduce some definitions and results in the theory of nilpotent orbits of semisimple Lie groups. For all the mathematical background we will refer to [127]. An element $\mu \in \mathfrak{g}_{\mathbb{C}}$ (the complexification of the Lie algebra $\mathfrak{g}$) is said to be *nilpotent* if it exists a $k > 0$ such that $\mu^k = 0$. Notice that with this definition we are implicitly working in the fundamental representation, but the same results hold in (and can be translated to) any finite-dimensional representation.

Given a nilpotent element $\mu \in \mathfrak{g}_{\mathbb{C}}$ we define its *nilpotent orbit* as the set

$$\mathcal{O}_\mu \equiv \{g\mu g^{-1} | g \in G\} \, , \tag{4.2}$$

and its *commutant*, or *centralizer*, as the set

$$C_G(\mu) \equiv \{g \in G | g\mu = \mu g\}. \tag{4.3}$$

The flavor symmetry of the generic theory (4.1) is then the group[3]

$$\mathrm{flavor}(\mathcal{T}^N_{G,\mu_L,\mu_R}) \quad = \quad C_G(\mu_L) \times C_G(\mu_R). \tag{4.4}$$

In particular, the flavor symmetry of the theory

$$\mathcal{T}^N_{G,0,0} \, , \tag{4.5}$$

[2]Since in six dimensions the superconformal algebra fixes all the supersymmetry generators to have the same chirality, the supersymmetry is usually denoted as $\mathcal{N} = (1,0)$ or $\mathcal{N} = (2,0)$, which have respectively 8 and 16 supercharges. More details can be found in [119].

[3]We will ignore possible extra abelian factors, since they will not play any role for us.

associated to the choice of the trivial element for both for μ_L and μ_R, is

$$\text{flavor}(\mathcal{T}^N_{G,0,0}) \quad = \quad G \times G \tag{4.6}$$

Notice that the flavor symmetry (4.4) is always a subgroup of (4.6).

It has been proposed in the literature [125, 128] that the theories with a given N can be connected by an RG flow if and only if the corresponding nilpotent orbits are related by *partial ordering*. This ordering is defined as follows. An orbit $\mathcal{O}_\mu$ is said to be *larger* than another orbit $\mathcal{O}_{\mu'}$ if $\mathcal{O}_{\mu'}$ belongs to the closure of $\mathcal{O}_\mu$ i.e.

$$\mathcal{O}_{\mu'} < \mathcal{O}_\mu \qquad \text{if} \qquad \mathcal{O}_{\mu'} \subset \overline{\mathcal{O}_\mu}. \tag{4.7}$$

The proposal on the RG flows states, in this case, that the theory associated to μ' can flow in the infrared to the SCFT labeled by μ, where the flavor symmetry has been partially Higgsed to (4.4). We can understand better the hierarchy of theories by starting from the theory (4.5) sitting at the top. The quaternionic dimension of its Higgs moduli space has been computed in [129] and it is given by

$$N + 1 + \dim(G). \tag{4.8}$$

The structure of this moduli space is not completely understood, but the $\dim(G)$ directions are supposed to be related to $\mathcal{N}$, the *nilpotent cone* of G, defined as the set of all its nilpotent elements. This space turns out to have many singularities and the type of singularity of a $\mu \in \mathcal{N}$ depends on its orbit. By giving a vacuum expectation value to fields corresponding to points μ_L and μ_R lying in singular orbits, one breaks the conformal symmetry and triggers an RG flow. This RG flow will eventually land on a new SCFT in the infrared (choosing a point on a smooth orbit is expected to lead to a free theory). This theory is conjectured to be (4.1) and its Higgs moduli space now has quaternionic dimension

$$N + 1 + \dim(G) - \dim(\mathcal{O}_{\mu_L}) - \dim(\mathcal{O}_{\mu_R}). \tag{4.9}$$

The starting nilpotent cone $\mathcal{N}$ is now reduced to a space where the $\mathcal{O}_{\mu_L}$ and the $\mathcal{O}_{\mu_R}$ directions have been quotiented away. A new RG flow can now be triggered by choosing a new nilpotent element in this space. By iterating this procedure we get the proposed hierarchy of RG flows and we can understand its connection with the partial ordering of orbits.

Let us describe more explicitly the partial ordering and the hierarchy of RG flows for the $G = \mathrm{SU}(k)$ case. By performing a Jordan block decomposition, each nilpotent element of $\mathfrak{su}(k)_\mathbb{C}$ can be written in the following form

$$\mu = \begin{pmatrix} J_{d_1} & & \\ & J_{d_2} & \\ & & \ddots \end{pmatrix} \qquad \text{with} \qquad J_d \equiv \left.\begin{pmatrix} 0 & 1 & & \\ 0 & 0 & 1 & \\ & & \ddots & \end{pmatrix}\right\} d\,. \tag{4.10}$$

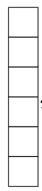
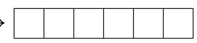
Two nilpotent elements whose J_{d_a} are related by a permutation are conjugated, and as such belong to the same orbit. In order to avoid repetitions, we can order the J_{d_a} such that the d_a are listed in increasing order

$$d_a \leq d_{a+1}\,.$$

In this way, we have identified a nilpotent orbit in $\mathrm{SU}(k)$ with a *partition* of k:

$$[d_1, d_2, \ldots], \qquad \text{such that} \quad \sum_a d_a = k. \tag{4.11}$$

For example, to the single big Jordan block J_d we associate the partition $[k]$, while to the trivial element $\mu = 0$ we associate the partition $[1, \ldots, 1] \equiv [1^k]$. These partitions can be described by Young diagrams, where the d_a are the number of blocks in the a-th *row* in the diagram. The opposite convention is also used in the literature in different contexts, where the emphasis is on the object which we call the *transpose partition* μ^t. It is easier to define μ^t from its Young diagram, which is simply obtained by reflecting the diagram of μ along a diagonal axis. For example, for k =6 we can easily see from the diagrams

$[1^6] \mapsto$ , $[6] \mapsto$  (4.12)

that $[1^6]^t = [6]$ and $[6]^t = [1^6]$.

The transpose partition can be used to more easily write the quaternionic dimension of $\mathcal{O}_\mu$

$$\dim(\mathcal{O}_\mu) = \frac{1}{2}\left(k^2 - \sum_a (\mu_a^t)^2\right) . \tag{4.13}$$

From this formula we can readily see that the trivial element $\mu = 0$ gives a trivial nilpotent orbit. Indeed $\mu = [1^k]$ and $\mu^t = [k]$ so that

$$\dim(\mathcal{O}_{\mu=[1^k]}) = \frac{1}{2}(k^2 - k^2) = 0. \tag{4.14}$$

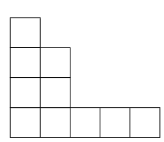
On the other side, a nilpotent element corresponding to a single k-dimensional block, $\mu = [k]$, has transpose partition $\mu^t = [1^k]$. The quaternionic dimension of its nilpotent is thus

$$\dim(\mathcal{O}_{\mu=[k]}) = \frac{1}{2}\left(k^2 - \sum_a (\mu_a^t)^2\right) = \frac{1}{2}(k^2 - k) = \frac{1}{2}k(k-1). \tag{4.15}$$

The combinatorial data encoded in the partition can also be used to readily write down the flavor symmetry associated to generic pair of μ_L and μ_R. To this end, we first define the set of numbers

$$f_a^{L,R} \equiv (\mu_{L,R})_a^t - (\mu_{L,R})_{a+1}^t. \tag{4.16}$$

As an explicit example, consider the diagram

 (4.17)

corresponding to the partition

$$\mu = [1, 2, 2, 5] \qquad \Rightarrow \qquad \mu^t = [4, 3, 1, 1, 1] \ . \tag{4.18}$$

From the definition (4.16) (with $f_6 = 0$) we compute

$$f_1 = 1, \quad f_2 = 2, \quad f_3 = 0, \quad f_4 = 0, \quad f_5 = 1. \tag{4.19}$$

Notice that the f_a can also be defined as the number of blocks J_a with dimension a. Alternatively, with our identification, this corresponds to the number of rows in the diagram having a blocks. The flavor symmetry of the theory can now be written as

$$\text{flavor}(\mathcal{T}^N_{G,\mu_L,\mu_R}) = \mathrm{S}\left(\prod_a (U(f_a^L))\right) \times \mathrm{S}\left(\prod_a (U(f_a^R))\right). \tag{4.20}$$

As an example, the theory associated to $\mu_L = \mu_R = 0$ has k blocks of dimension 1, so it has only $f_1^{L,R} = k$ and the corresponding flavor symmetry is

$$\text{flavor}(\mathcal{T}^N_{\mathrm{SU}(k),0,0}) \quad = \quad \mathrm{SU}(k) \times \mathrm{SU}(k).$$

Instead, if we keep one block trivial (say, the right one) and we take the other one to be associated to $\mu = [k]$, we have $\mu^t = [1, \ldots, 1]$ and so the only non-zero integer is $f_k^L = 1$. Thus, the flavor symmetry of the corresponding theory is just

$$\text{flavor}(\mathcal{T}^N_{\mathrm{SU}(k),[k],0}) \quad = \quad \mathrm{SU}(k). \tag{4.21}$$

From the Young diagrams we can also easily read the partial ordering between the corresponding orbits. We say that the diagram associated to μ *dominates* μ' ($\mu' < \mu$) if μ' can be obtained from μ by removing a box from an higher row and adding it to a lower row. In Figure 4.1 we can see an example of the full hierarchy for $k = 6$. From this figure it is also clear that the ordering is just partial, and as such not all the corresponding theories can be connected by one (or multiple) RG flows.

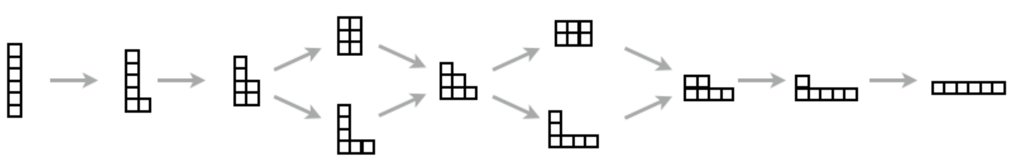


Figure 4.1: The partial ordering between Young diagrams corresponding to partitions with $k = 6$. The arrows go from the bigger element to the smaller one.

Finally, we introduce another point of view that allows to label the theories (4.1) and which will be important for the connection with the supergravity description. We start with some definitions. We call a *standard triple* $\{H, X, Y\}$ a triple of non-zero elements in $\mathfrak{g}_{\mathbb{C}}$ that satisfy the commutation relations

$$[H, X] = 2X, \qquad [H, Y] = -2Y \qquad \text{and} \qquad [X, Y] = H. \tag{4.22}$$

The elements H, X and Y are called *neutral*, *nilpositive* and *nilnegative* respectively. The importance of this definition comes from the Jacobson-Morozov theorem.

Theorem 1 (Jacobson-Morozov) *Let $\mathfrak{g}_\mathbb{C}$ be a complex semisimple Lie algebra. If X is a nonzero nilpotent element of $\mathfrak{g}_\mathbb{C}$, then there exists a standard triple for $\mathfrak{g}_\mathbb{C}$ whose nilpositive element is X.*

Thus, a given nilpotent element $\mu \in \mathfrak{g}_\mathbb{C}$ can always be thought of as part of a standard triple. From the definition in (4.22) we see that this triple spans an $\mathfrak{sl}_{2,\mathbb{C}}$ algebra. With a change of basis, we can bring the elements of the standard triple into three anti-hermitian matrices σ^i that satisfy the commutation relation

$$[\sigma^i, \sigma^j] = \varepsilon^{ijk}\sigma^k. \tag{4.23}$$

In other word, given a nilpotent element μ we can define an embedding

$$\sigma : \mathfrak{su} \hookrightarrow \mathfrak{g}_\mathbb{C}. \tag{4.24}$$

The matrices σ^i belong to $\mathfrak{g}_\mathbb{C}$ and satisfy the $\mathfrak{su}_2$ commutation relations, i.e. they provide an (in general) reducible representation of $\mathfrak{su}_2$. This representation can be decomposed as a direct sum of irreducible representations:

$$\sigma^i = \begin{pmatrix} \sigma^i_1 & & \\ & \sigma^i_2 & \\ & & \ddots \end{pmatrix}, \tag{4.25}$$

where the σ^i_a have dimensions d_a. We can label the irreducible representations with their spins l_a so that the blocks obey the usual properties of spin representations

$$d_a = 2l_a + 1, \qquad \mathrm{Tr}(\sigma^i_a\sigma^j_a) = -\kappa_a^2\delta^{ij}, \qquad \kappa_a^2 \equiv \frac{l_a(l_a+1)(2l_a+1)}{3} = \frac{d_a(d_a^2-1)}{12}. \tag{4.26}$$

The holographic dual of the $\mathcal{T}^N_{\mathrm{SU}(k),\mu_L,\mu_R}$ theories have been identified in [128] to be the AdS$_7$ solutions we have briefly introduced in section 3.3.2. As we have seen, the metric has a round S^2 factor, on which an SU(2) isometry acts realizing the R-symmetry of the dual theory. The local solution is fully determined by a function $\alpha(z)$ which satisfies the constraint (3.146):

$$\dddot{\alpha}(z) = -162\pi^3 F_0. \tag{4.27}$$

The Romans mass F_0 is a constant that jumps of an amount

$$\Delta F_0 = \frac{n_{D8}}{2\pi} \tag{4.28}$$

when crossing stacks of bound systems of D8/D6 branes. As such, the slope of $\ddot{\alpha}$ changes accordingly. In this language it is easy to describe the relation of a given AdS$_7$ configuration with a $\mathcal{T}^N_{\mathrm{SU}(k),\mu_L,\mu_R}$ theory: the μ^t_a give the slope of $\ddot{\alpha}$ in each region. This is identification is illustrated in Figure 4.2 with a picture taken from [49].

The U($f^{L,R}_a$) factors with $a \neq 1$ appearing in the flavor groups (4.20) are realized by the stacks of D8-branes, while the U($f^{L,R}_1$) are identified by stacks of D6 at the end of the interval. Thus, the theory $\mathcal{T}^N_{\mathrm{SU}(k),0,0}$ sitting at the top of the RG hierarchy is dual to a solution without D8 branes and with two stacks of k D6 branes on the two ends of the interval. The other theories are then obtained by moving some of the D6 branes from the poles of the S^3 into the interior, where they 'open up' into D8 branes. From this construction it is clear that to engineer the theories with $G = \mathrm{SU}(k)$ we have to consider for α only boundary conditions that describe D8-branes, D6-branes or regular points. If we instead consider boundary conditions that give O6-planes, as we did in section 3.3.2, we can engineer the theories with $G = \mathrm{SO}(2k)$. Considering also O8-planes boundary conditions gives theories outside the class (4.1) .

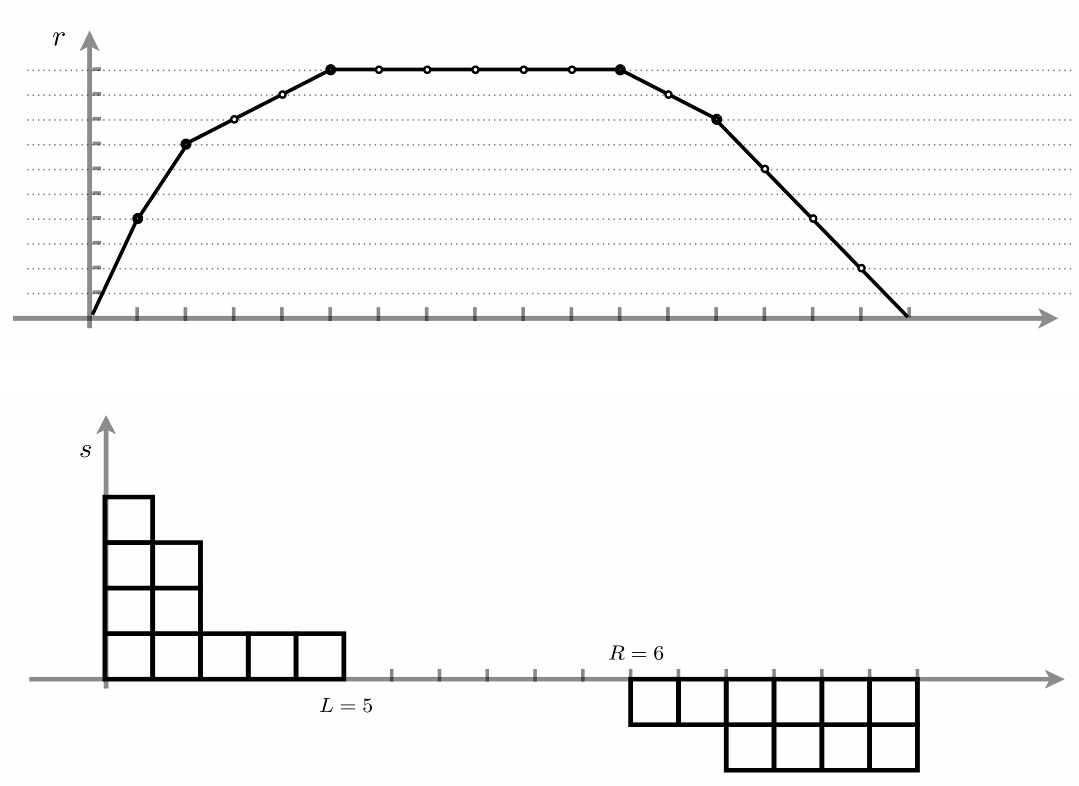


Figure 4.2: A picture taken from [49], where in each region $r \equiv -\frac{\ddot{\alpha}}{81\pi^2} = n_0 z$ and $s = n_0$.

4.2 Gauged supergravity description

As we have seen at the end of the previous section, the SCFTs we are considering have a ten-dimensional holographic description. However, we can ask if it is possible to repack the relevant degrees of freedom needed to capture the physics of RG flows into a simpler seven-dimensional gravitational theory. A possibility is to truncate the original theory by keeping only a subset of its degrees of freedom, and repack them into a set of seven-dimensional fields. In general, a truncation of a theory is said to be a *consistent truncation* when all the solutions of the reduced theory can be lifted to solutions of the original theory.

In [113] a consistent truncation of massive type IIA supergravity around its supersymmetric AdS_7 solutions has been constructed. This truncation leads to a theory called *minimal gauged supergravity*, which we are going to review in a moment. However, this theory does not contain enough information to study the RG flows we are interested in, since it has a single supersymmetric vacuum. In other words, it captures only a common sector of all the vacua, forgetting their individual details. This consistent truncation has also been derived from Exceptional Field theory in [130]. As a result, all the ten-dimensional AdS_7 solutions are identified in this description, which can thus be used only to study properties common to all the $\mathcal{T}^N_{\mathrm{SU}(k),\mu_L,\mu_R}$ theories, such as their compactification to four- and three- dimensional field theories, as it has been done in [113] .

To be able to study the connections between different AdS_7 vacua, we first need to tell them apart in the seven-dimensional theory. To this end, we would like to have a consistent truncation that keeps more modes of the internal manifold and includes the degrees of freedom living on the D-brane stacks. Up to now, this more general consistent truncation has not been found, and some attempts in the framework of the Exceptional Field Theory have shown that it is not possible to obtain it at the two-derivative level [131].

Instead of deriving a consistent truncation from the ten-dimensional theory, we can try to add to the minimal gauged supergravity the fields that can capture the extra degrees of freedom we need to study the RG flows. We are not trying to find a consistent truncation, but we want that our proposed theory satisfies some criteria that makes it suitable to our purposes:

- It should have some vacua in one-to-one correspondence with the choice of the two Young diagrams defining the $\mathcal{T}^N_{\mathrm{SU}(k),\mu_L,\mu_R}$ theories;
- The residual gauge symmetry on these vacua has to match (4.20), the flavor symmetry of the dual SCFT.

The second requirement comes from the general properties of the gauge/gravity duality, which states that a global symmetry of the field theory is mapped to a gauge symmetry of the gravitational theory.

In order to connect the different vacua with domain-wall solutions, we have to able to describe them in the same theory. Recalling that the generic flavor group (4.20) is always a subgroup of $\mathrm{SU}(k)\times\mathrm{SU}(k)$, our natural proposal is to couple the minimal gauged supergravity to two $\mathrm{SU}(k)$ vector multiplets, which we will use to gauge an $\mathrm{SU}(k)\times\mathrm{SU}(k)$ subgroup of the full symmetry group of the resulting theory. The different SCFTs in (4.1) would then correspond to vacua where the gauged group has been partially Higgsed to (4.4).

Let us be more precise by describing the details of the gauged-supergravity theory we are going to use. The field content of the complete theory is

- 1 gravity multiplet

$$(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma) \qquad i=1,2,3 \qquad A=1,2\,. \tag{4.29}$$

  This is the full field content of the minimal theory introduced in [132]. The R-symmetry is $\mathrm{SU}(2)_R$ and the multiplet includes a triplet of gauge fields A^i_μ and two doublets of fermions, ψ^A_μ and χ^A. The triplets and the doublets transform respectively in the adjoint and in the fundamental representation of $\mathrm{SU}(2)_R$ as their i and A indices denote. The other fields, neutral under the R-symmetry group, are the vielbein e^m_μ, a two-form $B_{\mu\nu}$ and a scalar σ which we will call *dilaton*.

- n vector multiplets

$$(A^R_\mu, \lambda^A_R, \phi^i_R), \qquad R=1,\dots n\,. \tag{4.30}$$

  The field content of each vector multiplet is composed by a vector, a doublet of fermions and a triplet of scalars. Since in the end we want to gauge an $\mathrm{SU}(k)\times\mathrm{SU}(k)$ subgroup of the global symmetry group, we take $n=2(k^2-1)$ vector multiplets.

The coupling of the minimal theory to the vector multiplets has been studied in [133] and reviewed e.g. in [134, 135] . We refer to the last two for the results on this gauged-supergravity theory. In total, we have $3n$ scalars coming from the vector multiplets. These scalars parametrize the scalar manifold

$$\mathcal{M}_\phi = \frac{\mathrm{SO}(3,n)}{\mathrm{SO}(3)\times\mathrm{SO}(n)}\ . \tag{4.31}$$

This manifold can be parametrized by matrices $L^I{}_J \in \mathrm{SO}(3,n)$. We follow the notation of [136]: the index I runs from 1 to $3+n$, and it is raised and lowered with $\eta_{IJ} = \mathrm{diag}(1,1,1,-1,\dots,-1)$. It splits as $I=(i,R)$, both of them raised and lowered with δ's.

Since in the theory we also have $3+n$ vectors, we could gauge a $(3+n)$-dimensional subgroup G of the $\mathrm{SO}(3,n)$ global symmetry group. This is possible only if the structure constants of G with all the indices lowered are antisymmetric:

$$f_{IJK} = -f_{JKL}, \qquad \text{with} \quad f_{IJK} \equiv f_{IJ}{}^L \eta_{JL}\ . \tag{4.32}$$

In our construction, we will gauge the particular subgroup

$$G = \mathrm{SU}(2)_R \times \mathrm{SU}(k)\times\mathrm{SU}(k), \tag{4.33}$$

so that the structure constants split as

$$f_{IJK} = (g_3\varepsilon_{ijk}, g_L f_{rst}, g_R f_{\hat{r}\hat{s}\hat{t}}), \tag{4.34}$$

where the f_{rst} and $f_{\hat{r}\hat{s}\hat{t}}$ are the structure constants of the two copies of $\mathrm{SU}(k)$, with indices going from 1 to k^2-1. This construction is consistent with the results of [135], where it is shown that, in order to allow for AdS vacua, the gauge group G has to be of the form $G = G_0\times H$ with H compact and semisimple and $G_0 \supset \mathrm{SU}(2)$. The Lagrangian of the theory is then completely fixed by supersymmetry, up to a small caveat. In order to allow for AdS vacua, the two-form B needs to be dualized into a three-form and the Lagrangian has to be supplemented by a topological mass term, which depends on a real constant h.

We will not need here the full Lagrangian, since we are only interested in supersymmetric solutions. Indeed, in order to find them we only need to set to zero the variations of all the fields with respect to fermionic variations, and then all the equations of motions automatically follow. As it is common in supersymmetric theories, the variations of the bosonic fields vanish if the background fermions are set to zero, which is the case for the solutions we are interested in. The only non-trivial equations are then given by the variations of the fermions

$$\delta\psi_\mu = 2D_\mu\varepsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\varepsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\varepsilon\ , \tag{4.35}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\varepsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\varepsilon - \frac{16}{5}e^{2\sigma}h\varepsilon\ , \tag{4.36}$$

$$\delta\lambda^R = i\gamma^\mu P^{iR}_\mu\sigma_i\varepsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{iR}\sigma^i\varepsilon\ , \tag{4.37}$$

where ε is the fermionic parameter of the supersymmetry transformation and we have suppressed the R-symmetry indices. To write down these expressions we have used the definitions

$$
\begin{aligned}
(P_\mu)^{iR} &\equiv L^{IR}\left(\partial_\mu \delta_I^K + f_{IJ}{}^K A_\mu^J\right) L_I^R \,, && (4.38)\\
C_{iR} &\equiv \frac{1}{\sqrt{2}} f_{IJ}{}^K L_j^I L_k^J L_{KR} \epsilon^{ijk} R \,, && (4.39)\\
C &\equiv -\frac{1}{\sqrt{2}} f_{IJK} L_i^I L_j^J L_k^K \epsilon^{ijk} \,, && (4.40)
\end{aligned}
$$

where we recall that the $L^I{}_J$ are the $\mathrm{SO}(3,n)$ matrices parametrizing the scalar manifold (4.31).

Finally, to compute the cosmological constant and the masses of the oscillations around our vacua, we also need the scalar potential of the theory:

$$
V = \frac{1}{4} e^{-\sigma}\left(C^{iR} C_{iR} - \frac{1}{9} C^2\right) + 16 h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3}{2}\sigma} C. \tag{4.41}
$$

4.3 Vacua

The first requirement that our gauged-supergravity description has to satisfy is to contain AdS_7 vacua in one-to-one correspondence with the choice of the nilpotent elements μ_L and μ_R defining the $\mathcal{T}^N_{\mathrm{SU}(k),\mu_L,\mu_R}$ theories. For simplicity, we will now focus on the case where μ_R is trivial, so that we can select different theories through the choice of a single nilpotent element $\mu \equiv \mu_L$. The more general case where both Young diagrams are non-trivial is a simple generalization, and can be found in [3, Section 6].

In order to preserve all the symmetries of the vacuum, all vector fields have to vanish and the scalars have to be set to constant values. Other than the dilaton, we have at our disposal the $3n$ scalars coming from the vector multiplets. In order to be able to identify the vacua with the choice of a nilpotent element μ, we want to find a map

$$
\mu \mapsto \phi_R^i \,. \tag{4.42}
$$

Thanks to the Jacobson-Morozov Theorem, we already know that we can associate to a generic nilpotent element μ a triplet of elements in $\mathfrak{su}(k)$, by building the σ^i matrices as in equation (4.25). Notice that the σ^i would generically belong to $\mathfrak{su}(k)_{\mathbb{C}}$. However, a standard theorem in the representation theory of Lie groups [137, Theorem 4.28] guarantees that every finite-dimensional representation of a compact Lie group G is equivalent to a unitary one. This implies that every finite-dimensional representation of its Lie algebra $\mathfrak{g}$ is equivalent to an anti-hermitian representation, meaning that we can take $\sigma^i \in \mathfrak{su}(k)$. We can then expand the matrices σ^i on a basis for the fundamental representation of $\mathfrak{su}(k)$:

$$
\sigma^i \equiv \sigma_r^i T_f^r, \qquad r = 1, \ldots, k^2-1. \tag{4.43}
$$

The numbers σ_r^i have the right structure to be associated to scalars ϕ_r^i.[4] More generally, given a choice of μ we can build the scalars

$$
\phi_r^i = \psi \sigma_r^i \,, \tag{4.44}
$$

where ψ is a constant. We could have in principle chosen different constants ψ_r in the above equation but the vacuum equations quickly set all of them to be equal.

[4] Recall that we are considering $\mu_R = 0$, so that all the $\phi_{\hat{r}}^i$ vanish.

With this identification, we can immediately build a generic element of the scalar manifold (4.31). Indeed, we can use the quotient with respect to $\mathrm{SO}(3)\times\mathrm{SO}(n)$, to bring an element of $\mathfrak{so}(3,n)$ in the form

$$\begin{pmatrix} 0 & \phi^i_R \\ \phi_i^R & 0 \end{pmatrix}, \tag{4.45}$$

which can then be exponentiated to build the $L^I{}_J$ as

$$L^I{}_J = \exp\left[\begin{pmatrix} 0 & \phi^i_R \\ \phi_i^R & 0 \end{pmatrix}\right]. \tag{4.46}$$

Finally, notice that this ansatz for the scalars automatically breaks the original gauge group to (4.4). Indeed, by definition, this group leaves μ invariant, and so also the σ^i in (4.44).

Before checking our ansatz against the BPS equations, let us point out that a very particular case of this ansatz has been already studied in [134]. That paper considered the particular case of the minimal theory coupled to only three vector multiplets, obtaining in this way the gauge group $G=\mathrm{SU}(2)\times\mathrm{SU}(2)$. This is a particular case of our construction where one of the two Young diagrams is not present and the other one has been specialized to $k=2$. The only non-trivial Young diagram for SU(2) then only leaves the possibility $\phi^i_r \propto \delta^i_r$ and [134] found that indeed this choice leads to a new vacuum.

We can now proceed to check if this ansatz can solve the BPS equations for supersymmetric vacua. We normalize the T^r_f such that $\mathrm{Tr}(T^r_f T^s_f) = -\delta^{rs}$ and, in order to compute the exponential in (4.46), we first compute

$$\begin{aligned}
\phi^i_r\phi^j_r &= -\,\mathrm{Tr}(\phi^i_r T^r_f \phi^j_s T^s_f) && (4.47)\\
&= -\,\mathrm{Tr}(\phi^i\phi^j) && (4.48)\\
&= \psi^2 \sum_a \kappa_a^2 \delta^{ij} && (4.49)\\
&\equiv \alpha^2\delta^{ij}\,, && (4.50)
\end{aligned}$$

where we have used the properties of the spin representations in (4.26) and we have defined

$$\alpha^2 \equiv \psi^2\kappa^2, \qquad \kappa^2 = \sum_a \kappa_a^2. \tag{4.51}$$

We also need to study the object

$$P^{st} \equiv \phi^s_j\phi^t_j, \tag{4.52}$$

which, by (4.50), we notice it is proportional to a projector:

$$\begin{aligned}
P^{rs}P^{st} &= \phi^r_j\phi^s_j\phi^s_k\phi^t_k && (4.53)\\
&= \alpha^2\phi^r_j\delta^{jk}\phi^t_k && (4.54)\\
&= \alpha^2 P^{rt}. && (4.55)
\end{aligned}$$

With these two properties the matrix exponential can be easily computed and we get

$$L^I_J = \begin{pmatrix} \cosh\alpha\,\delta^{ij} & \frac{\sinh\alpha}{\alpha}\phi^i_r \\ \frac{\sinh\alpha}{\alpha}\phi^s_j & \delta^{rs} + \frac{\cosh\alpha - 1}{\alpha^2}P^{rs} \end{pmatrix}. \tag{4.56}$$

To find BPS solutions we need set to zero the variations of the fermions in (4.35), (4.36) and (4.37). Since all the scalars are constant the P_μ^{ir} vanish, so that the equations $\delta\chi = \delta\lambda^r = 0$ give the conditions

$$C_{ir} = 0, \qquad C = 48\sqrt{2}he^{\frac{5}{2}\sigma}. \tag{4.57}$$

Evaluating these quantities on our ansatz we obtain

$$C = -3\sqrt{2}\left(-g_3(\cosh\alpha)^3 + \frac{g_L}{\kappa}(\sinh\alpha)^3\right) , \tag{4.58}$$

$$C_{ia} = \frac{\sqrt{2}}{\alpha}\cosh\alpha\sinh\alpha\left(-g_3\cosh\alpha + \frac{g_L}{\kappa}\sinh\alpha\right)\phi_a^i , \tag{4.59}$$

and imposing (4.57) we also get

$$\tanh(\psi\kappa) = \kappa\frac{g_3}{g_L}, \qquad e^{\frac{5}{2}\sigma} = \frac{g_3 g_L}{16h\sqrt{g_L^2 - g_3^2\kappa^2}}. \tag{4.60}$$

Finally the equation $\delta\psi_\mu = 0$ gives a differential equation for the Killing spinor ε, which we do not need here.

To recap, for each choice of a nilpotent element μ we have managed to find a corresponding BPS vacuum of the minimal theory coupled with n vector multiplets. The scalars in the vector multiplets are given by (4.44), where the matrices σ^i are the ones appearing (4.25). These matrices are associated to μ thanks to the Jacobson-Morozov Theorem. The constant ψ and the dilaton are then obtained from (4.60).

As already emphasized, since we do not have a consistent truncation there is no guarantee that these vacua correspond to the ten-dimensional ones. Nevertheless, this rich structure is encouraging. Before looking for the domain wall solutions connecting these vacua, we first perform additional cross checks on their validity, by matching with finer properties of the original ten-dimensional supergravity solutions and with known results on the dual quantum field theories. More precisely, we match the cosmological constant with the one obtained in type IIA supergravity and the masses of the scalar perturbations with the dimension of the dual operators. These two qualitatively different cross checks will help us to understand the regime of validity of our gravitational dual description.

4.3.1 Cosmological constant

The cosmological constant on our vacua can be evaluated by directly computing the value of the scalar potential (4.41), which gives

$$V = -240e^{4\sigma}h^2 . \tag{4.61}$$

Comparing this generic value with the value V_0 obtained for the trivial vacuum (which is obtained by taking the limit $\kappa \to 0$ in all the expressions) we get

$$\left(\frac{V_{\mu_L}}{V_0}\right)^{\frac{5}{4}} = \frac{e^{5\sigma}}{e^{5\sigma_0}} \tag{4.62}$$

$$= \frac{1}{1-\kappa^2\frac{g_3^2}{g_L^2}} . \tag{4.63}$$

We want to understand how the quantity (4.63) scales in the holographic limit

$$d_a \to \infty, \qquad N \to \infty, \qquad d_a/N \equiv \delta_a \text{ finite}, \tag{4.64}$$

where it has been computed in [49] for the type IIA vacua and matched with the field theory results for the anomaly coefficient a. From (4.26) we see that in this limit

$$\kappa_a^2 \sim \frac{d_a^3}{12}\,, \tag{4.65}$$

so that

$$\kappa^2 \sim \frac{1}{12}\sum_a d_a^3 = \frac{1}{12}\sum_a a^3 f_a\,, \tag{4.66}$$

where we have used the fact that the f_a are the numbers of blocks with dimension a, as noted below equation (4.19). The expression for the a anomaly given in [49] can be rewritten as

$$a_{\mu_L} = N^3\frac{k^2}{12} - N\frac{k}{6}\sum_a a^3 f_a + \dots. \tag{4.67}$$

Since the anomaly coefficient a is proportional to L^5_{AdS}, which in turn is proportional to $V^{-\frac{5}{2}}$, by taking the ratio of a for the different theories we can rewrite it as

$$\left(\frac{V_{\mu_L}}{V_0}\right)^{-\frac{5}{2}}_{\text{IIA}} = 1 - 2N^{-2}k^{-1}\sum_a a^3 f_a + \dots. \tag{4.68}$$

From (4.63) we see that instead in our gauged-supergravity description we have

$$\left(\frac{V_{\mu_L}}{V_0}\right)^{-\frac{5}{2}} \quad = \quad 1 - 2\kappa^2\frac{g_3^2}{g_L^2} + \dots\,. \tag{4.69}$$

To write this expression we have assumed that we are in the regime where $\kappa^2\frac{g_3^2}{g_L^2} \ll 1$. We are going to comment this assumption in a moment. This result coincides with (4.68) upon the use of (4.66) and the identification of the seven-dimensional parameters and the ten-dimensional ones as

$$\frac{1}{12}\frac{g_3^2}{g_L^2} = \frac{1}{Nk^2}. \tag{4.70}$$

However, this identification requires some comments. Since $k = \sum_a d_a = \sum a f_a$ we have that in the holographic limit k scales as

$$k \sim N\sum_a \delta_a\,, \tag{4.71}$$

so that all the terms in (4.67) actually scale like N^5, included some in the $\dots$ in equation (4.69). These terms are important and indeed in [49] they have been used to correctly match the gravitational computation with the field theory one. However, if we try to directly include these terms in our identification we fail. Since we are not working in a consistent truncation, a mismatch at a subleading order is inevitable. In particular, the vector multiplets we have added to the pure minimal supergravity in order to capture the physics of the degrees of freedom living on the D-branes are described here by the usual two-derivative quadratic action. However, in ten dimensions they arise from the DBI action, which is not quadratic in their field strength. As anticipated, our theory could never completely reproduce the results of massive type IIA. For a perfect matching one would ideally need to improve the vector multiplet action by adding higher-derivative terms to it. At the moment this is beyond the state of the art. However, it is natural to expect that these higher order terms are not relevant when the vacuum expectation values (vevs) of the fields in the vector multiplets are not too large. It is natural to interpret this requirement as the δ_a being not too large, even though the $d_a = \delta_a N$ are large in the holographic limit. With this requirement the terms in the ... are in fact subdominant. Related to it, with this assumption we can also see that upon our identification of the parameters we have that the quantity

$$\kappa^2 \frac{g_3^2}{g_L^2} \sim \frac{\sum_a \delta_a^3}{\sum_a \delta_a^2} \tag{4.72}$$

is indeed small, an assumption we have made in order to derive (4.69).

To recap, we have obtained a perfect match between our seven-dimensional supergravity and the IIA theory, in the regime where vevs of the fields in the vector multiplets are not too large and the use of the quadratic action can be quantitatively justified.

4.3.2 Masses and dimensions of the dual operators

We can now consider scalar perturbations around our backgrounds and compute their masses. We will then compare these results with the expectations from the field theory side. We can regard this comparison as a different consistency check on the ability of our seven-dimensional supergravity description to capture the relevant physics of the dual field theories.

The easiest perturbations we can consider are the ones associated to the dilaton σ and to the direction ψ in the scalar manifold, i.e. perturbations such that $\delta\phi^i \propto \phi^i$. For these two cases we can readily evaluate

$$m_\sigma^2 = \frac{4}{5}\partial_\sigma^2 V|_{\text{vac}} = -\frac{8}{L_{\text{AdS}}^2}, \qquad m_\psi^2 = \frac{1}{3\kappa^2}\partial_\psi^2 V|_{\text{vac}} = \frac{40}{L_{\text{AdS}}^2}\ , \tag{4.73}$$

where the factors $\frac{4}{5}$ and $\frac{1}{3\kappa^2}$ have been included in order to canonically normalize the fields in the Lagrangian. These masses satisfy the BF bound[5], and as such do not generate any instability, as is expected for supersymmetric solutions. We can then take the usual holographic relation

$$m^2 L_{\text{AdS}}^2 = \Delta(\Delta - 6)\ , \tag{4.74}$$

where Δ is the conformal dimension of the dual operator and it is defined to be the highest solution of the above equation. From it we obtain for these two modes with masses (4.73) the dimensions

$$\Delta = 4 \qquad \text{and} \qquad \Delta = 10. \tag{4.75}$$

There are many other scalars related to perturbations $\delta\phi^i$ that are not proportional to the ϕ^i. For the theory with only the $\mathrm{SU}(k)_L$ vector multiplet turned on there are $3(k^2-1)-1$ of them. In

[5]In AdS space the squared masses of scalar fields can be negative, if they are not too much negative. The scale is dictated by the cosmological constant and the precise bound is known as Breitenlohner-Freedman (BF) bound [138]. In our seven-dimensional case it reads $m^2 > -\frac{9}{L_{\text{AdS}}^2}$.

order to compute their masses we have to compute the variations $\delta L^I{}_J$ induced from the variations $\delta\phi^i$. We will not report here the full computation (more details can be found in [3, Section 4.4] , but we only quote the main results. The quadratic perturbation of the scalar potential is

$$\delta^2 V = -8\,\mathrm{Tr}(\delta\phi^i\delta\phi^i - 2[\sigma^i,\delta\phi^j][\sigma^i,\delta\phi^j] + 2[\sigma^i,\delta\phi^j][\sigma^j,\delta\phi^i] + [\sigma^i,\sigma^j][\delta\phi^i,\delta\phi^j])\ , \tag{4.76}$$

where the σ^i are the background triplets. Taking into account the factors coming from the kinetic term $\mathrm{Tr}(\partial_\mu\delta\phi^i\partial^\mu\delta\phi^i)$ we obtain the mass matrix for the canonically normalized scalars:

$$(M^{ij})_{\alpha\beta} = -8(\delta^{ij}(1+2j^k j^k) - 2j^{(i}j^{j)})_{\alpha\beta} \tag{4.77}$$

where we have defined

$$[\sigma^i, T^f_r] \equiv j^i_{rs} T^f_s \tag{4.78}$$

with T^f_r the basis of the fundamental representation of $\mathfrak{su}(k)$. From this expression we can compute all the masses and the corresponding conformal operator dimensions. An important feature of the full mass spectrum is the appearance of massless scalars, corresponding to marginal operators ($\Delta = 6$). They would suggest the presence of deformations of our vacua and their existence would be in tension with the general arguments of [135, 139] which forbid supersymmetric deformations of AdS_7 vacua. These putative marginal deformations appear in two classes (see [3, Table 1]). In the first class, the number of massless scalar is the same as the number of the broken generators in the Higgsing of the gauge group

$$\mathrm{SU}(k)_L \to \mathrm{S}(\Pi_a U(f^L_a))\ . \tag{4.79}$$

As such their are not independent modes but can be understood as being 'eaten' by the massless vectors that have acquired a mass in the process. Instead, the other class of marginal operators can be explicitly shown to be related to non-supersymmetric deformations. Combining these two effects we have no contradictions with the results of [135, 139].

4.4 Domain wall solutions

Let us recap the construction so far. We have taken a universal consistent truncation around the AdS_7 vacua of massive type IIA supergravity and we have modified it by adding non-abelian vector multiplets. The idea behind this construction is that these new fields could be able to tell the vacua apart by capturing the degrees of freedom living on the stack of D-branes. We have performed some quantitative checks on this proposal by showing that the resulting gravitational theory has many AdS_7 vacua in one-to-one correspondence with the data of the dual field theory. Moreover, we have also been able to use our theory to reproduce quantitative features of these vacua, by comparing both with ten-dimensional gravitational results and with quantum field theory results. In this process, we have also understood the limitations of our approach as being related to the restriction to the two-derivative action for the fields in the vector multiplet.

We now use our theory to study the physics of RG flows between the different SCFTs. We find that the equations governing these flows are a class of well studied differential equations known as Nahm equations, and we show that they have solutions with the correct boundary conditions only in the cases conjectured by the field theory arguments reviewed in section 4.1.

4.4.1 Equations governing the flows

A renormalization group flow connecting two conformal field theories with a holographic dual is described in the gravitational theory as a domain wall geometry interpolating between the two corresponding AdS vacua. In our case, these solutions, if they are allowed, are smooth geometries which are contained within the following class of seven-dimensional metrics

$$ds_7^2 = e^{2A(\rho)}ds^2_{\text{Mink}_6} + e^{2B(\rho)}d\rho^2. \tag{4.80}$$

In order for these metric to interpolate among two AdS$_7$ vacua, the function A is required to have the following asymptotic behavior:

$$A \to A_\pm \rho \qquad \text{for} \qquad \rho \to \pm\infty. \tag{4.81}$$

The constants $A_\pm$ are then related to the radii of the two AdS$_7$ spaces. Moreover in this gauge both B and all the scalars have to go constant values at the two endpoints of the coordinate ρ. Since in particular we are interested in flows connecting the vacua we have found in the previous sections, we will impose the boundary conditions

$$\phi^i(-\infty) = \phi^i_{\mu_L -}, \qquad \phi(\infty) = \phi^i_{\mu_L +}, \tag{4.82}$$

where the $\phi^i_{\mu_L\pm}$ are the values the scalars have in the vacua associated to the field theories at the endpoints of the RG flows. Notice that also in this section we are still keeping $\mu_R = 0$, so that the connected vacua are specified only by μ_L. The limits $\rho \to \pm\infty$ represent respectively the ultraviolet (UV) and infrared (IR) limit of the RG flow. For later convenience let us parametrize the gauge redundancy in (4.80) in terms of a function Q defined as

$$Q(\rho) \equiv B(\rho) - \frac{\sigma(\rho)}{2}. \tag{4.83}$$

On a generic point along the flow, the scalars ϕ^i will not be proportional to σ^i as in (4.44), so that the quantities C and C_{ia} are different from the expressions we computed in (4.58) and (4.59). Moreover the P^{ir}_μ do not vanish anymore since the various quantities are not constant along the flow. All in all, we obtain the expressions

$$\begin{aligned}
P^{ir}_\rho &= -\phi^i_r \frac{\sinh\alpha}{\alpha}\partial_\rho(\cosh\alpha) + \left(\delta_{rs} + \frac{\cosh\alpha - 1}{\alpha^2}P_{rs}\right)\partial_\rho\left(\frac{\sinh\alpha}{\alpha}\phi^i_s\right), &(4.84)\\
\frac{1}{\sqrt{2}}C^{ir} &= -\frac{g_3}{\alpha}\cosh^2\alpha\sinh\alpha\phi^i_r + g_L\frac{\sinh^2\alpha}{2\alpha^2}\left(\delta_{rs} + \frac{\cosh\alpha - 1}{\alpha^2}P_{rs}\right)[\phi^j, \phi^k]_s\varepsilon^{ijk}, &(4.85)\\
\sqrt{2}C &= 6g_3\cosh^3\alpha + g_L\frac{\sinh^3\alpha}{\alpha^3}\varepsilon^{ijk}\,\text{Tr}(\phi_i[\phi_j, \phi_k]), &(4.86)
\end{aligned}$$

where we recall that α was defined in (4.50) by $\text{Tr}(\phi^i\phi^j) \equiv -\alpha^2\delta^{ij}$ and P_{rs} was defined in (4.52). Since we are looking for supersymmetric flows we still need to solve the BPS equations (4.35), (4.36) and (4.37), where we now plug the quantities we have just computed. We start by imposing $\delta\lambda^r = 0$. In order to get rid of the projector appearing in P^{ir}_ρ, we notice that the object

$$\Pi_{rs} \equiv \left(\delta_{rs} + \frac{\cosh\alpha - 1}{\alpha^2}P_{rs}\right) \tag{4.87}$$

is an invertible operator, whose inverse is given by

$$\Pi^{-1}_{rs} \equiv \left(\delta_{rs} - \frac{\cosh\alpha - 1}{\alpha^2\cosh\alpha}P_{rs}\right). \tag{4.88}$$

Applying this inverse operator to both sides of the equation $\delta\lambda^r = 0$ we obtain the following set of first order differential equations

$$e^{-Q}\partial_\rho\Phi^i = \cosh\alpha\left(-g_3\Phi^i + \frac{1}{2}[\Phi^j.\Phi^k]\varepsilon^{ijk}\right), \quad \text{with} \qquad \Phi^i \equiv g_L\phi^i\frac{\tanh\alpha}{\alpha}. \tag{4.89}$$

We see that the gauge choice

$$e^{-Q} = \cosh\alpha \tag{4.90}$$

simplifies equation (4.89) that then reads

$$\partial_\rho\Phi^i = -g_3\Phi^i + \frac{1}{2}[\Phi^j.\Phi^k]\varepsilon^{ijk}. \tag{4.91}$$

This equation is a variant of a *Nahm equation*, to which it can be mapped through a change of variables. We will study it in more detail in the next section, where we will show that it admits solutions with the boundary conditions (4.82) if and only if the corresponding vacua are related by partial ordering, confirming in this way the field theory expectation.

For the remaining part of this section let us show that the other BPS equations can be solved with the correct asymptotics once a solution for $\Phi^i(\rho)$ (and so for $\alpha(\rho)$) is obtained. We impose the projection $e^B\gamma^\rho\varepsilon = \varepsilon$ on the Killing spinor and we replace the commutator by using (4.91), obtaining the differential equations

$$\partial_\rho\left(\frac{e^{-\frac{5}{2}\sigma}}{\cosh\alpha}\right) + g_3\frac{e^{-\frac{5}{2}\sigma}}{\cosh\alpha} - \frac{16h}{\cosh^2\alpha} \quad = \quad 0 \tag{4.92}$$

$$\cosh\alpha\partial_\rho A - \frac{1}{5}(g_3\cosh\alpha - \partial_\rho\cosh\alpha) - \frac{4}{5}he^{\frac{5}{2}\sigma} \quad = \quad 0\,. \tag{4.93}$$

Once $\alpha(\rho)$ is determined, these equations can be readily solved for $\sigma(\rho)$ and $A(\rho)$. The first one is analytically solved by performing an integral:

$$e^{-\frac{5}{2}\sigma(\rho)} = 16he^{-g_3}\cosh\alpha(\rho)\int_{\rho_0}^{\rho}\frac{e^{g_3 y}}{\cosh^2\alpha(y)}dy\,. \tag{4.94}$$

A linear combination of (4.92) and (4.93) then gives the equation

$$\partial_\rho\left(e^{4A+\frac{\sigma}{2}}\cosh\alpha\right) = g_3 e^{4A+\frac{\sigma}{2}}\cosh\alpha \tag{4.95}$$

which can be readily solved for A as

$$A(\rho) = \frac{1}{4}\left(g_3\rho - \log(\cosh\alpha(\rho)) - \frac{1}{2}\sigma(\rho)\right) + A_0\,, \tag{4.96}$$

where A_0 is an integration constant.

So, once the modified Nahm equation (4.91) is solved, a solution for A and σ is obtained explicitly. Moreover, it can be shown that they obey the appropriate boundary conditions for RG flows we have described at the beginning of this section. In particular, the precise values of $\sigma_\pm$ and $A_\pm$ agree with the ones we have computed in section 4.3. An interesting quantity to compute is the ratio of the cosmological constants between the two vacua:

$$\left(\frac{V_+}{V_-}\right)^{\frac{5}{4}} = \frac{g_L^2 - \kappa_-^2 g_3^2}{g_L^2 - \kappa_+^2 g_3^2} \, . \tag{4.97}$$

As we are going to see in the next section, our modified Nahm equation admits solutions only if $\kappa_+ < \kappa_-$, so that $V_+ < V_-$, in agreement with the expected behavior for a domain wall solution representing an RG flow.

4.4.2 Nahm equation

We have reduced the problem of finding BPS flows to the study of the modified Nahm equation (4.91) with the boundary conditions (4.82). With the simple change of variables

$$\Phi^i \equiv -\frac{g_3}{2} A^i, \qquad \rho \equiv \frac{2}{g_3} t, \tag{4.98}$$

our modified Nahm equation is mapped to the equation

$$\partial_t A^i = -2A^i - \frac{1}{2}[A^j . A^k]\varepsilon^{ijk}. \tag{4.99}$$

This equation has been already studied in the mathematical literature. In particular, its space of solutions has been analyzed in [140]. In the following, we will refer to [140] to describe this space, by translating the results which are relevant to us in our language.

The main result of [140] (Theorem 1) states that the moduli space of solutions of equation (4.91) with the boundary conditions (4.82) is the space

$$\mathcal{O}_{\mu_{L-}} \cap \mathcal{S}(\mu_{L+}) \, , \tag{4.100}$$

where $\mathcal{S}(\mu_{L+})$ is the so-called *Slodowy slice*: the affine space defined by

$$\mathcal{S}(\mu) \equiv \{Y + q \quad | \quad [X, q] = 0\} \, . \tag{4.101}$$

In the above definition Y and X are respectively the nilnegative and nilpositive elements of the triple σ^i associated to μ, as defined in and below equation (4.22). The space (4.101) has two important properties that are relevant for our construction:

1. it intersect $\mathcal{O}_\mu$ only at Y;
2. it meets only those nilpotent orbits whose closure contains $\mathcal{O}_\mu$.

By property 2, the space (4.100) is non-empty if and only if the closure of $\mathcal{O}_{\mu_{L-}}$ contains $\mathcal{O}_{\mu_{L+}}$, i.e., if and only if

$$\mathcal{O}_{\mu_{L+}} < \mathcal{O}_{\mu_{L-}} \, , \tag{4.102}$$

by the definition (4.7). In other words, the flow can go from the UV fixed point associated to μ_{L+} to the IR fixed point associated to μ_{L-} if and only if μ_{L+} dominates μ_{L-}. This result confirms the field theory conjecture we have described in Section 4.1.

For completeness, we now give an explicit example of such a flow. This example is taken from [141], where it used to study domain walls for $\mathcal{N}=1^*$ field theories in four dimensions. Adapted to our case, it describes the flows starting from the UV nilpotent element $\mu_{L+}=0$. In this case, a solution of the Nahm equation (4.91) with the correct boundary conditions is given by

$$\Phi^i = \frac{g_3}{1+e^{g_3\rho}}\sigma^i_{\mu_L -} \tag{4.103}$$

for any μ_{L-}. Let us check it explicitly. On a generic point of the solution we can compute

$$\begin{aligned}\mathrm{Tr}(\Phi^i\Phi^j) &= \left(\frac{g_3}{1+e^{g_3\rho}}\right)^2 \mathrm{Tr}(\sigma^i_{\mu_L -}\sigma^j_{\mu_L -}) && (4.104)\\ &= -\left(\frac{g_3}{1+e^{g_3\rho}}\right)^2 \kappa_-^2\delta^{ij}\ . && (4.105)\end{aligned}$$

By comparing this result with the definition of Φ^i,

$$\begin{aligned}\mathrm{Tr}(\Phi^i\Phi^j) &= g_L^2\frac{\tanh(\alpha)^2}{\alpha^2}\mathrm{Tr}(\phi^i\phi^j) && (4.106)\\ &= -g_L^2\frac{\tanh(\alpha)^2}{\alpha^2}\alpha^2\delta^{ij}\ , && (4.107)\end{aligned}$$

we can extract a relation for α

$$\tanh(\alpha) = \left(\frac{g_3}{g_L}\frac{\kappa_-}{1+e^{g_3\rho}}\right). \tag{4.108}$$

We can now invert the definition of Φ^i and substitute the solution (4.103) and the relation (4.108), obtaining the result

$$\begin{aligned}\phi^i &= g_L^{-1}\frac{\alpha}{\tanh(\alpha)}\Phi^i && (4.109)\\ &= \frac{g_3}{g_L}\frac{1}{1+e^{g_3\rho}}\frac{\alpha}{\tanh\alpha}\sigma^i_{\mu_L -} && (4.110)\\ &= \frac{\alpha}{\kappa_-}\sigma^i_{\mu_L -} && (4.111)\\ &= \frac{\alpha}{\psi_-\kappa_-}\phi^i_{\mu_L -}\ . && (4.112)\end{aligned}$$

By taking the limit $\rho\to-\infty$ of (4.108) and comparing the result with the expressions in the vacua (4.60) we get

$$\lim_{\rho\to-\infty}\alpha = \kappa_-\frac{g_3}{g_L} = \kappa_-\psi_- \qquad \text{and} \qquad \lim_{\rho\to\infty}\alpha = 0. \tag{4.113}$$

Taking the limits $\rho\to\pm\infty$ of (4.113) and using these results, we immediately see that the ϕ^i satisfy the correct boundary conditions. Finally, from the expressions (4.94) and (4.96) we can also extract the asymptotic behavior of A and σ and explicitly verify their asymptotics.

We conclude this section with another non-trivial check of our gauged-supergravity description, which consists in computing the conformal dimensions of the scalars triggering the flows determined by the Nahm equation. In order to do this we need to check the general behavior of the scalar fields around the UV fixed points. In particular, as shown in [139], we should find that six-dimensional SCFTs do not admit relevant deformations. With the change of variables $s \equiv e^{-g_3\rho}$ and $T^i \equiv \frac{\Phi^i}{g_3 s}$ the Nahm equation becomes

$$\partial_s T^i = -\frac{1}{2}[T^j, T^k]\varepsilon^{ik}. \tag{4.114}$$

We now study properties of the solutions near the UV, i.e., for $\rho \to \infty$ corresponding to $s \to 0$. It is easy to check that near such a point the Nahm equation (4.114) admits solutions consisting of a pole plus a regular part:

$$T^i = \frac{1}{s}T^i_{-1} + s^p T^i_p + O(s^{p+1}), \qquad p \geqslant 0 \tag{4.115}$$

where the T^i_p are constant matrices. Transforming back to the original coordinates we get

$$\begin{aligned} g_3^{-1}\Phi^i &= T^i_{-1} + s^{p+1}T^i_0 + O(s^{p+2}) && (4.116)\\ &= T^i_{-1} + e^{-(p+1)g_3\rho}T^i_0 + O(e^{-(p+2)g_3\rho}) && (4.117)\\ &= T^i_{-1} + e^{-(p+1)\frac{4}{L_+}\rho}T^i_0 + O\left(e^{-(p+2)\frac{4}{L_+}\rho}\right) && (4.118)\\ &= g_3^{-1}\phi^i_{\mu_+} + e^{-(p+1)\frac{4}{L_+}\rho}T^i_0 + O\left(e^{-(p+2)\frac{4}{L_+}\rho}\right) && (4.119) \end{aligned}$$

where in the last step we have rewritten g_3 in terms of the AdS$_7$ radius at the UV point:

$$g_3 = \frac{4}{L+}\,. \tag{4.120}$$

Since near the UV the space is asymptotically AdS, we compare expression (4.119) to the generic expansion of a scalar field

$$\delta\phi^i = \varphi_A e^{-(6-\Delta)\frac{\rho}{L}} + \varphi_B e^{-\Delta\frac{\rho}{L}} + \cdots\,, \tag{4.121}$$

where φ_A and φ_B are respectively the non-normalizable and the normalizable mode. Via the AdS/CFT correspondence they are associated respectively to a source for the dual operators or to a vacuum expectation values of the dual relevant operators triggering the flow. In order for our deformation to correspond to a source, we need to have

$$6 - \Delta = 4(p+1) \qquad \Rightarrow \qquad \Delta = 2 - 4p\,. \tag{4.122}$$

Assuming that the flow is triggered by an unitary operator, i.e with $\Delta \geqslant 2$, we see that the only possibility is $p = 0$ corresponding to $\Delta = 2$. However this would correspond to a free scalar field, which cannot trigger an RG flow. This result is in agreement with [139]. In particular, we see that in our case the flows are triggered by giving a vacuum expectation value for the scalar fields with conformal dimension

$$\Delta = 4(p+1). \tag{4.123}$$

In order for it to be associated to a relevant operator ($\Delta < 6$) the only possibility is then $p = 0$, for which $\Delta = 4$.

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