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# Heterogeneity in Space: An Agent-based Economic Geography model

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## Preface

This dissertation includes two chapters. The first one, "Heterogeneity and Economic Geography: The formation of Spatial Inequality" is where the main part of my research effort was dedicated during the post-exams period in the DEFAP program. This chapter proposes and develops an Agent-based model able to include the ingredients that govern the spatial distribution of economic agents. The second Chapter, "Local policy with multiple regions: an Agent-based Economic Geography analysis" is an extension of that model, and can be considered perhaps as an example of the type of analysis that could be performed in that framework. These works were born by a personal interest in developing new applications in Agent-based modelling after the surge of models dedicated to the financial crisis. I thought that the spatial dimension seemed relatively unexplored, but that the methodology overall was mature to be expanded in this direction. The emergence of works such as Petrovic et al. (2017) and Commendatore et al. (2015) seems to confirm this intuition.

The current version has undergone some improvement, including a much less bugged code, and an improved speed of execution. Some of the more puzzling results of the previous version have been solved also thanks to the suggestions of the reviewers. The work here is therefore less incomplete, but it still must be noticed that further measures should be taken to aim for scientific validity, such as performing MC analyses for all the proposed exercises, and to increase the number of the ones in the second Chapter. I regret moreover to not have made in time to include my secondary research work on macroeconomic modeling with heterogenous agents and automation: the DEFAP program permitted me nonetheless to acquire the skills to finish it some day.

Before presenting the chapters, I want to thank my supervisor Domenico Delli Gatti for the invaluable supervision work. Jakob Grazzini provided also very useful comments and discussions. I want to thank also Rosario Crinò and Giorgio Ricchiuti for suggestions on the Trade and Firms relocation literature. The feedback by the participants of the DEFAP Ph.D. seminars and the 2017 WEHIA conference was very valuable, as well as the ones by the participants at the 2018 ESCoS conference. The feedback by the two evaluators have been very useful to drive the research work forward in quality. I want then to thank Emilio Colombo and Vittoria Cerasi for organizing the DEFAP seminars, and Gianluca Femminis and Matteo Manera for coordinating the DEFAP Program. My fellow DEFAP Ph.D candidates have made for great colleagues and friends.

I thank my family and Alice for the support.

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## Heterogeneity and Economic Geography: The formation of Spatial Inequality

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### DEFAP

#### Abstract

Economic Geography studies the formation and the evolution of economic spatial inequality. This study wants to enrich this literature by investigating the case of heterogeneous agents operating under bounded rationality in complex spatial structures. To achieve this, I propose a flexible macroeconomic Agent-based model endowed with Households and Firms interacting on a given spatial network. The agents' decision heuristics embed and reproduce the features usually employed in mainstream Economic Geography models, as relocation, migration, trade and transportation costs. This permits to evaluate and compare the predictions of Economic geography into a disequilibrium bounded rationality setting, to check the emergence and stability of agglomerated configurations by the economic agents, and to test policies, such as the variation of the transportation costs. The flexibility of the spatial structure, which consists in a network of locations provided in the initial calibration of the model, allows to check these results on a wide variety of spatial environments, including realistic ones. I test different relocation protocols for the agents, as well as different spatial maps, and perform experiments on different Transportation costs level. I find that the spatial configuration assumed by the model is deeply influenced by the chosen relocation metric and by the chosen spatial network. Usual New Economic Geography patterns of agglomeration might emerge under some conditions, but seems to stem from different mechanisms.

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## 1 Introduction

If we were to look at a night satellite photo taken from space, we would observe a rather fascinating picture: in correspondence with cities, the agglomeration of electrical illumination systems will form a noticeable spot of light, signalling a wealth of people and economic activity.

What should hit us, is that these light spots will pattern the map in a very unequal way. If agglomeration means then an unequal distribution of activity in space, an economic treatment of space should be considered as a crucial factor when investigating a wide range of topics; indeed, in these days of shifting political and economic boundaries these includes (but are not limited to) globalization, trade, migration, and economic integration. To add to the complexity, these effects often interacts simultaneously and on multiple and diverse layers of governance.

The feedback of these interactions and their endogenous effects on the spatial distribution of economic activity has been researched by the discipline of Economic Geography and its ramifications (Thisse, 2011). New Economic Geography ("NEG") the current mainstream framework, originated from Krugman's Core-Periphery model of trade and geography (?), provided many remarkable results. These, both theoretical and empirical, underpinned the conditions that can lead an economy to assume an agglomerated form irrespective of natural advantages<sup>1</sup>. Having it been subject to requests for more realistic spatial structures and agents' depictions, many different works were able to provide many extensions and a very interesting debate.

This work wants to enter this debate by answering two research questions: first, can we observe the equilibrium effects of the typical features of Economic Geography in a non-equilibrium bounded rationality setting such as an Agent-based model? Second, are the policy recommendations of this body confirmed in such setting? The answer is not univocal (see Fowler, 2007 and for an early application, and the review by Commendatore et al. 2015). These questions could be considered part of the "refinement" stream of New Economic Geography, as an additional strand of extensions in Quantitative Spatial Economics framework Redding and Rossi-Hansberg (2016), the most recent embodiment of NEG ideas.

I expand this discussion in Section 2 and 3, where I briefly summarize the Economic Geography usual features, and argument why Agent-based modeling could become an useful tool to enrich this literature.

To answer these questions, I propose a macroeconomic agent-based model based

<sup>&</sup>lt;sup>1</sup>The "natural advantages" usually includes locations chosen for their defensive or agricultural prowess, or other factors such as available deep-water ports. The umbrella term for all these factors is "First Nature Geography". "Second Nature Geography" focuses instead on the relative relationship variables between locations (such as trade costs and distances), and so it can be depicted on a neutral space. For a work checking the influence of First Nature Geography on cities' locations choices, see Bosker and Buringh (2017). For an assessment of the effects on developing regions see Gallup et al. (1999).

on the works by Riccetti et al. (2015) and Caiani et al. (2016a). The key feature of this work is the decentralized matching mechanism at the core of the agents' interactions. The main difference is that I don't envisage a fixed regional or multi-country structure, but one where the agents are able to relocate, generating different configurations of economic activity. Compared with recent Multi-Country models (like for instance the EURACE OPEN model, Petrovic et al. (2017)) this is done far more parsimoniously, in terms of the economy structure and number and type of agents.

Space is added in this simple model through modeling the spatial environment as a network, where to each node corresponds a location, which is conceptualized like an agent. The model can accept different spatial network configurations to test for the most relevant structures investigated in the literature or to allow for more realistic ones. The economy of each location presents then different profiles depending on how many agents are currently residing over it. These profiles are used as an additional feedback in shaping the economic activity, as the local population of agents and the related locally-aggregated variables are relevant in shaping the decisions about trade and relocation.

In section 4 I describe the Agents and the timing. In section 5, 6, 7, 8 and 9 I detail the behavioral equations of the agents, and the functioning of the markets interaction protocols. In Section 10 I describe then the procedure to set the parameters values and to initialize the model.

I simulate first a benchmark version of the model, whose results are showed in Section 11. The model simulations shows that the spatial outcome is mainly driven by the relocation metrics employed by the mobile agents. To refine this result in Section 12 I propose a more general taxonomy of relocation metrics based on the interaction of their agglomerating or dispersive properties and perform various tests by modifying the benchmark model. I test moreover versions endowed with a less sensitive relocation protocol, and show that unrealistic and catastrophic agglomerations are dependent on the relocation protocol modelling choices.

Section 13 is devoted to different sensitivity experiments: I check the validity of the Economic Geography usual experiment of varying the transportation cost level, and perform this exercise on different spatial structures, and on a model with a greater number of spatial nodes and agents. These experiments confirms the relevancy of the metrics in determining the spatial behavior, but also shows that the shape of the network is not ineffectual on the final spatial configuration of the economy.

In section 15 I test the robustness of these results by performing some Monte Carlo simulations on the benchmark model with different metrics, and on the Trade costs experiment on two different spatial structures.

I find that patterns of agglomerations and agents movements are greatly influenced by the choice of the relocation protocol metrics, and reasonable configuration bring by more realistic maps. Increasing the number of Locations moreover can generate maps with multiple agglomerations. The spatial network chosen to run a model is effective in changing economic results, as it influences the model through different distances values and affecting the relocating behavior due to the different connectedness of nodes.

Different experiments run varying the transportation costs levels shows that it is possible to obtain an effect of increased costs in fostering dispersion, which is a main result of earlier New Economic Geography models. The explanation behind of these results seems to stem from different causes than theirs however, and are more related to the interaction of Households migrating incentives rather than Firms optimizing behavior.

Drifting away from the usual trade-cost driven agglomeration present in New Economic Geography models, this work still encompass the usual NEG features in a boundedly rational setting, and can therefore be used to study the situations where the usual assumptions of standard models might need some relaxation (e.g. costly and simultaneous relocation and migration, bounded rationality by the agents, realistic economic features such as involuntary unemployment and inventories, heterogeneity). In addition, as a contribution to Agent-based modeling, this effort provides a simple testing ground for heterogeneous trade, export production and relocation heuristics. These can be modeled to allow to test different insights from empirical evidence or behavioral and institutional theories before being considered in more complex Agent-based models.

## 2 A discussion on Economic geography, Rationality and Heterogeneity

Economic Geography is the discipline that investigates the determinants of the spatial distribution of economic activity (Thisse, 2011). In the past many thinkers were fascinated by this unequal distribution, animating different schools of thought<sup>2</sup>.

Despite these illoustrious predecessors, many reviews, such as (Fujita and Mori, 2005), evidence how major difficulties, mostly technical ones, prevented however these schools to become part of the mainstream focus of economic theory; a fitting example was a theoretical result such as the "spatial impossibility theorem", obtained by Starret (Starrett, 1978): within an homogeneous space, any agglomeration (from actual cities to regional specialized industries) cannot arise as a competitive equilibrium, given the presence of transportation costs. The solution to the conundrum was to relax the basic assumptions of the early models: the review by (Ottaviano and Thisse, 2004) categorize these attempts as by considering heterogeneous space (an example is the Neoclassical theory of trade), externalities (Urban Economics) and imperfect markets (Economic Geography).

Starting from this milieu, the theory of "New Economic Geography", henceforth NEG, conceived by Krugman, and expanded later by many other authors, was able to provide a synthesis between these insights. According to works such as Thisse (2011), it brought Economic Geography as a discipline towards a "renaissance".

The seminal NEG Core-Periphery model proposed by Krugman in "Increasing Returns and Economic Geography" ?, investigates the effect of prices and wages on the locating decisions of Firms and Workers between two different regions. Increasing returns of scale, employed through a monopolistic competition structure à la Dixit e Stiglitz, developed earlier by Dixit and Stiglitz (1977) and noticed by Krugman in previous trade-related works, cause contrasting effects of attraction and repulsion between the two regions, and space enters as a relevant variable through the assumption of transportation costs affecting trade. The model allows for multiple equilibria, the most interesting one being the Core-Periphery case, where one of the regions could attract most if not all of the economic activity.

In Figure 1 we can observe a typical Core-Periphery bifurcation diagram, where the Transportation Costs level, T on the Horizontal axis, mediates the different spatial configurations. Indeed for lower levels of T,  $\lambda$ , the share of manufacturing workers in one of the two model's regions is either zero or one, meaning that the economy is agglomerated in a Core-Periphery configuration. After a certain T threshold, the agglomerated equilibrium is no longer stable, and a dispersed economy, where  $\lambda = 0.5$  emerges. Notice that T(B) and T(S), the Break

 $<sup>^2 \</sup>rm Between the famous names that can be linked to the study of economics and space, we can highlight Von Thünen, Christaller, Lösch and Hotelling.$ 

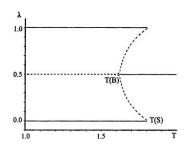


Figure 1: Example of a Core-Periphery bifurcation diagram, as shown in Fujita et al. (2001)

and sustain points, where the dispersed and agglomerated equilibrium becomes unstable are overlapping, such that the model might display multiple equilibria and hysteresis.

The attraction and repulsion effects can be rationalized as the backward and forward linkages that influence the location decision by firms and workers. It is interesting to summarize them here to explain the type of effects that I will try to include in the proposed model:

- *Backward linkages*, are incentives for the producers to locate into a bigger market. They imply that having more firms located into a region will result in also more workers, and consequently in an increased demand. Firms will realize that relocating there will grant better profits, as their goods could be sold in a wider market without the additional transportation costs.
- Forward linkages regards the incentives of workers or households also to relocate towards the bigger market. It could be modelled in a wide variety of ways, including search of employment, or imitation of neighbors. In NEG models, this is usually a mix stemming from workers preferences, that are in search of a wider variety of goods, and better wages.
- As more and more firms choose that location however, a Market crowding effect will take place; namely, the degree of competition in the region will rise, lowering the price index, and partially offsetting the two other effects.

It is an interesting point to take these effects as economic components in a bounded rationality framework, as in that case it is not clear if the dynamics shaped by these effects will be stable enough to guarantee realistic agglomeration patterns (this clearly would happen in an equilibrium model by definition). Fowler (2010) employ for instance an agent-based mapping of the Core Periphery equations to check this hypothesis, and finds that introducing "leakages" in rationality can completely offset these results.

It is moreover interesting to check how many of key features of New Economic Geography would be maintained in such setting. These feature are usually consequences of the modelling choices employed by this framework. Ottaviano (2003) summarize the more remarkable: home market magnification (trade liberalization affects market crowding, making Firms more footloose), circular causality (transition between equilibria follows a "snowball" cumulative effect), hump shaped agglomeration rents (the loss that a Firm will incur by relocating from an agglomeration, connected with the previous discussion about break and sustain points), endogenous asymmetry (spatial asymmetry not depending on ad-hoc First Nature factors like natural resources), catastrophic or "knife-edge" agglomeration (given the bifurcation diagram), hystheresis (transitory shocks might have permanent effects), and self-fullfilling expectations (a catastrophic agglomeration might be prompted by an expectation shocks about trade values). Another interesting point is how to model these linkages without making them "too dormitive". Krugman pointed out as a matter of fact that one of the achievements by NEG is indeed to avoid "dormitive" properties<sup>3</sup>, so to being able to explain agglomeration without assuming "agglomeration economies" like

information spillovers (Krugman, 2011).

Within a less than perfectly rational framework however, even simple heuristics could depict in theory the same relevant linkages that govern the economic agents' distribution in space in NEG models. For instance, Backward linkages could be modelled to consider firms to be able to compare their current profit with what is happening in the other locations. Forward linkages could emerge from workers tendencies of searching better wages, and market crowding could emerge by competition effects once a location get crowded. This possibility is not to be taken for granted however, as we gradually reduce the economic agents' capability to optimize a number of factors might act as a distortion.

Before moving on, it is fair to point out that the NEG literature greatly extended the basic Core-Periphery setting, as it was subjected to internal and external criticisms: works such (Ottaviano and Thisse, 2005; Krugman, 2011; Ascani et al., 2012) are good starting points to summarize the debate. Summarizing this debate even more, critiques from Economists tended to focus on how the different parts of the model were specified, like labelling the structure of consumer's taste or the transportation cost as unrealistic, and that the space itself was too abstract<sup>4</sup>. External criticism comes often directly from proper Geography departments, that question the use of abstract modelling, the reductionism, and the unrealistic assumptions (Neary, 2001; Fowler, 2007).

Many of the extensions of the basic Core-Periphery model and the subsequent directions of research are the direct consequences of these criticisms; an helpful review is in the book "The Spatial Economy: Cities, Regions, and International Trade" (Fujita et al., 2001), that collects the evolution of the theory in its first decade. More modern sources include Ascani et al. (2012) and Krugman (2011). Here it suffices to highlight that many of the issues have been tackled

 $<sup>^{3}</sup>$ The "dormitive property" is a reference to Moliere's doctor, who put people to sleep using Opium, and explains its power referring to the aforementioned properties.

 $<sup>^4\</sup>mathrm{In}$  this literature space consists in a set of discrete points; it is also often depicted as a line or a circle

effectively, such as the depiction of a more realistic 2-D space for multi-location NEG models (see Ikeda et al. 2016, 2014) or extending the tastes structures to allow taste heterogeneity between consumers (Tabuchi and Thisse, 2002).

Policy analysis in Economic Geography usually investigates the conditions that shapes the equilibrium of an agglomerated or a dispersed economy. This means acting on variables such as transportation costs and see the reaction of the economy, knowing that the intervention could perhaps even switch the model to another equilibria, triggering catastrophic<sup>5</sup> transitions to different spatial configurations (Puga, 1999; Ottaviano, 2003). Such works needs of course to be grounded on empirical applications, which has not been an easy task. Lack of relevant data, and disentangling the different effects at work proved indeed to be difficult for the researchers (Redding, 2010; Fowler, 2007). Many works had to rely for instance on real-life "natural experiments" such as the German reunification or the bombing of Japan's cities in WWII to test the theory predictions (Redding and Sturm, 2008; Davis and Weinstein, 2003, 2002). Such works were often limited at providing some confirmation evidence of the effects at play in New economic geography, downplaying then the necessarily considerations about active policy analysis (Cheshire and Magrini, 2009: Ascani et al., 2012).

More recent efforts worked hard therefore to obtain a better integration of data evidence to the model's parameters, resulting in the NEG offspring (but perhaps more general) field of Quantitative Spatial Economics (Allen and Arkolakis, 2014)<sup>6</sup>. Indeed, if we look at the taxonomy proposed by Redding and Rossi-Hansberg (2016), there are many combinations of core ingredients that can provide a quantitative spatial model, which could then be inverted to fully calibrate the model under scope consistently with known parameters, and provide realistic policy exercise by solving numerically the resulting system. The wide fine-grained grids used are moreover more realistic as a locations space, and can be tailored to reflects characteristics of the real space under investigation (very much as the model of this paper, see Subsection 3.1.1).

Finally, the current developments points toward extending the NEG models in multi-regional and realistically calibrated settings. The JRC and DG REGIO of the European Commission for instance conceived and are developing RHO-MOLO, a spatial computable general equilibrium model for the European Union (Lecca et al., 2018). Multi-regional settings are moreover much better suited for specific policy analysis: for instance Commendatore et al. (2018b) employ a three region setting to study the case of Brexit and of its consequences on the economy industry spatial distribution of the UK, the EU and within the EU.

This development in spatial theory and policy analysis indeed fully resonates with the research agenda of this work. In the following I will as a matter of

 $<sup>{}^{5}</sup>$ Catastrophic as the differential equation system of the model spans multiple equilibria which are non-smoothly connected, as typically studied in bifurcation theory.

<sup>&</sup>lt;sup>6</sup>This was perhaps helped in part by a far more diffused amount of geographical microdata. As formats as GIS become widely spread we can expect an even greater amount of applications.

fact present an Agent-based model, identified as a mean to look at spatial issues under the lenses of bounded rationality and heterogeneity (both between agents, and between spaces), in a very straightforward and uncomplicated model. At the light of the latest development in Quantitative Spatial Economics, providing an adaptive agent-based model with the ingredients of Economic Geography is not so different than considering a case where agents are not fully rational, and where the lack of equilibrium assumptions permits a more comprehensive look at the dynamics and the transitions from different states. This work could therefore be considered as a *trait-d'union* between the demand for a higher degree of complexities in Economic Geography models (Commendatore et al., 2015) and these new developments.

This could have relevant consequences also on the policy topic. Given the proven empirical resilience of agglomeration equilibria (Redding, 2010), it seems more interesting to focus on the additional effects that letting firms and workers flow would have onto variables such as productivity and unemployment when these "geographical" policies are applied; what would happen without the smooth shifts given by perfect rationality and foresight? And would national level policies have non-desired effects once we let agents free to roam?

## 3 Heterogeneous agents, bounded rationality and a spatial network environment: Agent-based models

As we have seen, Economic Geography models have been asked for more realistic assumptions regarding space and the depiction of its agents (Fujita and Mori, 2005). Moreover, the scope of this work is to employ the insights coming from those models into a bounded rationality setting. In this section I will detail the choice of an Agent-Based model to answer those questions by describing the key features of this methodology. In addition, I will elaborate the choice for the spatial environment on which the model will be built.

The diffusion of Agent-based modelling in economics was an answer to the demand to investigate the application of new computational approaches on a wide variety of topics that were limited by the available standard modelling tools (Fontana, 2006; LeBaron and Tesfatsion, 2008).

Agent-based modelling refers to the simulation and analysis of models composed by individual units, the agents, interacting in an artificial environment (Gilbert, 2008). The agents are defined as individual and often heterogeneous entities that acts according to different bundles of behavioural rules, which span from being simple arbitrary rule of thumbs to much more complicated forms. The interaction between agents, and between the agents and their environment can modify the variables of interest of the system (usually some aggregate measure), that will therefore evolve during time. While the basic idea was already diffused in the last decades, this methodology surged when computational power became more cheap and affordable. In fact, on a computer program it is relatively easy to implement a model. Every object-oriented computer language is then able in principle to build an agent-based model, and software and Graphical User Interface that ease the development of such models have becoming much more diffuse than before (Nikolai and Madey, 2009).

The appeal of this approach lies then in the "experimental chamber" property that this virtual system displays. After the system is started, even the interactions of the simplest rules can bring out complex and unexpected results at an aggregate level, results that may have been impossible to obtain or predict in an analytical way. Given the complex nature of most of the real world events this approach presents itself as a valuable tool for economics and other social and natural sciences. These features has been employed widely in macroeconomics, creating multiple approaches proposed by different research groups, as listed in the Computational Economics Handbook Chapter by Dawid and Delli Gatti (2018).

Limiting this discourse to spatial economics considerations, Agent-based modelling may provide useful into implementing a complex economic landscape with different actors (like workers, firms, institutions etc.) and can help when looking at a variety of economic decisions, like relocation and trade (Commendatore et al., 2015). An expansion of the research work detailed in Dawid and Delli Gatti (2018) in the field of regional and multi-country economies is therefore a natural and welcomed development.

A relevant feature linked with physical space, is that Agent-based models stresses particularly the importance of the environment where the agents operates. Whether in simple lattices or in more far complicated landscapes, this methodology can be a channel to include a far more detailed space conception. Let us not forget how one of the first seminal models in this field was the Schelling 1971 model of segregation (Schelling, 1971) which regarded the distribution of agents in space. The origin of the discipline, traced back to Von Neumann work on cellular automata still exhibit an inherent spatial structure, as the trigger for the automata behavioral rules was exactly the spatial proximity<sup>7</sup>. Again, when being introduced to NetLogo, one of the most famous platform for Agent-Based models development, one of the most used model is about ants foraging in space<sup>8</sup>.

Indeed, many Agent-based works has been developed on the top of spatial considerations, and hinted towards topics very close with the one proposed in this contribution. For instance, we can consider big multi-regional models like Wolf et al. (2013) or Dawid et al. (2014), as well as more focused applications, for instance to the housing market (Pangallo et al., 2016; Baptista et al., 2016).

The current frontier seems to points toward the development of big and complex multi-country and stock-flow consistent models, that can include multiple countries, spatial inequalities and related factors such as workers migration in economies with many sectors and agents types. Good examples are the stream of models developed from the Stock-flow consistent macroeconomic model Caiani et al. (2016a), that include Caiani et al. (2017) and Caiani et al. (017b), studying different fiscal policies effects in a model including an arbitrary number of countries, and Petrovic et al. (2017), that presents a multiple country version of the EURACE model, that hosts a monetary unions and two control countries.

These contributions are revealing and interesting, but highlight also one of the problems that affects this methodology, namely the lack of standardization and the wide variety of proposed solutions. This failure to provide a unified framework prevent researchers to exploit economies of scale as often other approaches do (Fagiolo and Roventini, 2016).

A look at a review like the remarkable chapter in Commendatore et al. (2015), detailing the recent development of Agent-based and network models on Economic Geography topics provides a confirmation of this view. The heterogeneity of approaches can go indeed as far as dividing school of modelling depending on the type of spatial structure adopted, providing a variety that in a sort of sense proposes again the divide between different spatial structures of NEG theory; with the additional obstacle, that on the different spatial structures there will be tested whole different models. To conclude, once models are developed, their

 $<sup>^7{\</sup>rm An}$  interesting excursus on the history of cellular automata: http://psoup.math.wisc.edu/491/CA<br/>origins.htm

<sup>&</sup>lt;sup>8</sup>http://ccl.northwestern.edu/netlogo/models/Ants

sheer size<sup>9</sup> can make it hard for researchers to be able to reproduce them for comparison purposes<sup>10</sup>.

An answer to these issues by this contribution is therefore to incorporate directly the features of Economic Geography (relocation, trade and transportation costs) in a simpler model and to allow different spatial structures on which to simulate the same model. For instance, the work by Fowler (2010) details an Agent-based model shaped to depict as close as possible the equations of the Core-Periphery model. The test is however limited to a three regions structure. One motivation for this work is therefore to add more realistic features such as multiple Locations, different topology, and additional sectors, and to make the Agent-based model more comparable with current macroeconomic Agent-based configurations.

Regarding the spatial structures, I will employ a network as the basic spatial environment on which the agents operates. The application of networks as a mathematical tool to model interactions and relationships has been widely applied in many different disciplines, including models that considers firms and industry relationships, like Acemoglu et al. (2010) and ecology models on fauna population (Fortuna et al., 2006).

As for Economic Geography, I consider that Network theory (Jackson et al., 2008) could provide a useful depiction of space, parsimonious with respect to some alternatives, like using a continuous landscape <sup>11</sup> but more interesting than using simpler fixed structures. I can better explain this thought with an analogy: as we know, the phenomenon of financial contagion was carefully modelled in the seminal paper of Allen and Gale (2000); subsequently, and in particular after the financial crisis, many works looked more in detail at how modifying the network of interactions provided different results about the diffusion of the contagion process (see for instance: Gai et al. (2011) and Battiston et al. (2012)). In this work modifying the spatial network configuration to reflect different shapes and statistics can be a channel on which to enrich the discussion. The importance of considering more complex forms of topology has been evidenced also in the framework of Multiregional NEG models. Barbero and Zofío (2016) for instance study two versions of a multiregional Core periphery model of 4 regions on a homogeneous circumference and heterogeneous star network configurations. They show that break and sustain points differ in the two spatial topology, and are affected by the network centrality.

In the next section I will present in detail the structure of the model.

 $<sup>^{9}</sup>$ Indeed Agent-based models are far more achievable thanks to modern computational power, but remain massive in terms of coding and computing power required with respect to other economic counterparts.

<sup>&</sup>lt;sup>10</sup> Luckily many efforts are being spent toward that direction, see for instance Dawid et al. (2016) for the Eurace@Unibi model and the JMABM model and its derivations (Caiani et al., 2016a), where the researchers are putting effort to share their code or to provide intuitive platforms on which to simulate their models.

 $<sup>^{11}\</sup>mathrm{Such}$  models are nonetheless viable with some even simple program, like NetLogo or JABM.

## 4 The model

I develop a micro-founded macroeconomic agent-based model, which I label "ABEG", short of Agent-based Economic Geography model. Employing as its core the decentralized matching mechanisms proposed by Riccetti et al. (2015), and employed in other articles such as Caiani et al. (2016a) and Catullo and Gallegati (2015). Indeed the model's main heuristics, matching mechanisms and calibration methodology are inspired by Caiani et al. (2016a), although there are substantial differences in the choice of sectors and agents. I trade the complexities stemming from the introduced spatial structure with the reduction in the economy sectors, therefore not including a capital good and a credit sector. In this sense, a comparable paper would be the "baseline model" proposed by Lengnick (2013).

In this section I will describe first the agents populating the economy and the timing. In the next section, I will present the behavioral equations, the markets and the mechanisms through which the agents interact.

### 4.1 Agents

In this model there are three types of agents: H Households, F Firms, and L Locations. The economy is also endowed with a global public sector, introduced for depicting a more realistic macroeconomic structure, and to pave the way for future policy analysis works.

#### 4.1.1 Locations

While not agents, strictly speaking, as they don't follow specific behavioural equations, Locations still interact passively with the other agents. To understand the effects that this "agent" have on the overall economy, I will explain here the basic intuition behind their implementation.

Each location  $l \in L$  is a node of a multi-layer network (Kivelä et al., 2014), that includes two types of edges. The first type of edge connects the locations, and the network of locations provides the spatial structure of the economy, which is fixed. The second type of edge connects each location with the other agents. If an agent is linked, then the agent resides in that location. By counting the number of existing edges between a specific location and the other agents, we can immediately retrieve the local population of both Firms and Households. The populations' size directly influences some very important features of the local economy, such as the size of the goods market and of the available workforce, the level of price competition, and so on. Since in this model relocation and migration are concrete possibilities, these edges are not fixed, and are updated every turn depending on the results of the agents' relocation and migration choice.

This spatial structure is very flexible, as it can accommodate different spatial configurations without the need to rewrite the entire model. For instance, by

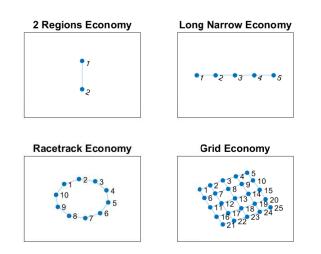


Figure 2: Different spatial network configurations that can be selected in the model

setting L = 1, the model would behave as it were a simple closed economy, without trade, migration and relocation. Instead, by declaring thousands of Locations on a regular grid, we would have something very similar to an approximated plane.

Most interestingly, by "injecting" the spatial structure of the Locations network, we can easily depict many different spatial structures, such as the most classical depiction used in New Economic Geography: two regions models, long narrow economies, and racetrack economies, alongside with several types of grids (see Fig.1). More complex structures could be used as well, including for instance star-shaped structures, multiple centers, and hub models. In principle one can generate a realistic distance network based on empirical data, or using theoretical ones such as the optimal spatial commodity distribution network seen in Gastner and Newman (2006). The spatial structure can be considered then as a sensitivity tool, answering the pledge for a comparable platform for different spatial analysis of the previous section. This is an interesting feature in Economic Geography, as spatial topology will have economic effects on NEG models, for instance by negating the original symmetry postulated in early works (Barbero and Zoffo, 2016).

This of course brings up the topic on how to spatially define locations, that are so named deliberately to keep some degree of abstraction in the model. Indeed, it would be possible (as it is often the case in Economic Geography), to consider them as Countries, Regions or Cities (and why not, even different neighborhoods in the same city). The present model can be most appropriately applied to regions within a country, as the spatial scale of countries to be more realistic would need the introduction of additional features, such as proper local Governments and exchange rates.

#### 4.1.2 Firms

The Firms hire Households as workers to produce and sell a homogeneous consumption good. Both the acts of hiring and of selling are influenced by the Firm's current location, as it can only hire workers that are currently in the same locations; this model therefore does not focus on commuting costs. Instead, a Firm can potentially "export" its produced good also to other locations, as it will be shown in detail in the next section.

Given the matching structure and the labor hiring, firms might as well obtain negative profits. If negative profits accumulate, the Firm might become bankrupt. In this case, the Firm is "bailed", or substituted by a new Firm, with a reboot of initial net worth and market price. To keep consistency in the model, the resources to bail the failed firm are obtained by a bill which is paid by the Public sector.

Firms are assumed to be owned by Households. Every firm will pay dividends to a common pool that is then divided among Households according to their relative wealth endowment.

A fraction of the population of Firms can relocate to another location, and therefore can choose in which domestic market to operate. The presence of a fraction of Firms unable to relocate rules out the case of Locations devoid of any economic activity. Given that some sectors, such as agriculture or tangible services cannot in a sense be physically somewhere else, this assumption is not too unrealistic.

#### 4.1.3 Households

The Households visit the local labour market to get employed and to obtain a wage; they then visit the consumption market to buy their desired share of consumption goods. To perform these actions, the Households are constrained to their current location, meaning that they can be hired only by local Firms and that they can buy only goods that are sold in that Location's goods market, which can include domestic production and/or imported goods from other locations.

If a Household does not find a job, or it is fired from its current Firm, it becomes unemployed and receives an unemployment subsidy, consisting of a fixed share of the economy's average wage; this subsidy acts also as a reservation level when setting expectations for the desired wage in the job market.

A fraction of Households are public workers. Public workers are employed by the Government, and receive a wage equal to the period's average wage. When unemployment is too high however, the public workers' wage is kept constant. Public workers cannot migrate, playing a role similar to non-relocating Firms: they help in avoiding Locations emptied from economic activity: compared to the work Riccetti et al. (2015), where public employees act as a an economic stabilizing force, here they will act also as a spatial stabilizing force. This permits to not assume a fixed agricultural sector as in NEG models.

Households which are not employed in the public sector can migrate to a different location, as it will be detailed in Section 9.

#### 4.1.4 Public sector

The economy is endowed with a public sector. In the benchmark model, a government collects taxes and pays salaries to Public Workers and unemployment benefits to the unemployed. Taxes are revised in order to keep a control on the level of deficit and public debt. Moreover the government bails bankrupt Firms. This government serves the whole economy, meaning that it acts on aggregate variables regardless of the specific Location. While its role is limited in this benchmark version, the structure of the model will make it possible to analyze

some interesting scenarios, such as adapting the response in taxation according to location-specific variables, or to have directly location-specific governments, which would fully "agentify" the Locations of the economy. Such situation is developed in the second Chapter.

## 4.2 Timing

The model consists of repeated iterations of the basic timing structure, starting from some given initial conditions (which are set during the initialization of the model). The iteration generates time-series that are stored at a micro and at an aggregate level. Aggregate data can be of two types: aggregates within a single location, and aggregates for the whole economy.

An iteration (turn, or period) of the economy can be divided in the following steps:

- Firms sets their desired production level, for both domestic and foreign markets. The production level determines the labor needed for production.
- Firms and Households visits the Labor market; Firms select and hire workers until they are able to fulfill their required labor demand through the Labor Matching Protocol.
- Production happens. If the Firms were not able to hire enough workers to complete their production, they apply a specific protocol to allocate their production to the various markets (see Section 5). Workers are paid.
- Households, having received their wages, set a desired consumption level, and visit the Goods market, where they buy goods following the Goods market's matching protocol. Unsold goods are added to the inventories of the firms.

- Both Firms and Households update their accounting variables, including profits, and wealth. The Government collects taxes.
- Finally, mobile agents operate their Relocation Matching Protocol, and decide if to relocate, and in which location to transfer. Location statistics are updated.

The cycle repeats until the final period.

## 5 Firms Behavioral equations

## 5.1 Production

The production process is applied to every market that the Firm wants to serve, although there is a difference between the domestic and the exporting ones. In the following I will show first the process for the domestic market to explain the main mechanisms at work. I will describe in detail the exporting process in Section 5.2.

I label desired production as  $y_{f,t}^d$  for firm f located in location l, at time t. Every Firm sets its desired production as its own expected production,  $y_{f,l,t}^e$ :

$$y_{f,l,t}^d = y_{f,l,t}^e \tag{1}$$

This expectation is defined as

$$y_{f,l,t}^{e} = y_{f,l,t-1}^{e} + \lambda(y_{f,l,t-1}^{s} - y_{f,l,t-1}^{e}) - Inventories_{f,l,t-1} + DT_{l,t}$$
(2)

. The first two terms on the right hand side are an adaptive response to the actual sales of the previous period, and follows Caiani et al. (2016a). Suppose that the current value of a variable  $x_t$  is unknown. The agent will therefore form an expectation  $x_t^e$ , that follows the adaptive mechanism:

$$x_t^e = x_{t-1}^e + \lambda(x_{t-1} - x_{t-1}^e)$$

where  $\lambda \epsilon [0, 1]$  is the degree of accuracy of the process, equal for all agents. By applying this heuristic on variables such as desired production we can see that there would be an effort by the agents to reduce the distance between their desired and actual outcomes. The third term  $Inventories_{t-1}$  refers to the unsold goods of the previous period, that do not depreciate. Finally, the term  $DT_{l,t}$  is a shift in demand, similar in spirit to the demand targeting component presented in Guerini et al. (2018). This increases or decreases  $y_{f,t}^e$  by random term  $\alpha$ , depending on the sign of the excess demand for the particular Locations' l Goods market.  $\alpha$  is based on a Uniform distribution with support  $[0, 250]^{12}$ .

$$DT_{l,t} = +\lambda\alpha$$

if the excess demand is positive, while

$$DT_{l,t} = -\lambda \alpha$$

if instead there is excess supply. This component acts as an additional notch of rationality, and it is assumed as in previous version the production setting was too imprecise, fact that joined with the less flexibility posed by the lack of a credit sector entailed an excessive number of failed Firms.

In this simple economy the firms produce only with a labor input; the desired level of labor for firm  $n_{f,l,t}^d$  depends then on the desired output and an exogenous labor productivity parameter,  $\eta$ , such that:

$$y_{f,l,t}^d = \eta n_{f,l,t}^d \tag{3}$$

. In a closed economy the production setting would stop here, but in this model Firms can operate in different markets by exporting goods. In the next section I will define the assumptions for Firms to do so.

## 5.2 Exporting

The Firms might decide to export their goods towards other locations.

In standard optimized models such as the ones in NEG or Trade theory, firms are able to perfectly forecast their profits in different locations and they will have enough labor to fulfill their desired production; these two conditions are not met in the present setting. In other agent-based models instead, exported goods are usually pooled in one global market (like in Catullo and Gallegati (2015) or Dawid et al. (2014)) accessible by all consumers. In the present model, each consumer can access only its local market. This means that Firms must explicitly target one market to export goods there, and that Firms can discriminate between markets.

This market structure makes the production process of a particular firm nontrivial. Indeed, the firm must decide if to export to a given market, how much to produce there, and how to allocate the production accross the markets if it were not able to carry out all the required production for that turn.

In the model, the firms follow then three different protocols:

 $<sup>^{12}</sup>$  The number 250 is not arbitrary: given the efficiency  $\lambda$  it means that a firm will never increase its production due to the demand effect to more than 12.5, which is the steady state production per firm in the benchmark model. Other calibrations would probably require different values, but I did not modify it in other versions of the model as this fixed value seems sufficient to reach a state of high Production and low unemployment.

- To manage production and sales into already served markets.
- To explore new markets and decide if to start exporting there.
- If production is not sufficient to serve all markets, to allocate that turn's production.

On the basis of empirical evidence on exporting and intra-regional trade decisions, I base these three protocols on a triplet of reasonable explanatory variables (see Lawless and Whelan, 2014 for an empirical application on Irish firms): Distance (that makes trade more expensive), the dimension of the target (bigger markets are more attractive), and path dependence (Firms prefers to focus on old exporting targets).

Path Dependence implies that Firms will then prioritize markets where profitable trade has already taken place. After deciding the production levels and the needed workforce for those markets, they will then start to explore new Locations, by drawing a sample of the non-served ones<sup>13</sup>. The sample dimension is set according to the available financial resources.

The new Locations are then sorted by Households population size, and production is initialized in each of them according to the Firm's resources. Indeed, the Firms will export in a new location only if they have enough resources (net worth) to pay the extra labour requirement needed to expand production.

If a production is started in a new location, it is initialized to simply one unit of production. Given this initial value, for recurring interactions on a particular market, production is instead updated by following the same adaptive processes described in the production subsection.

**Export protocol #1: Old Markets protocol** I expand the notation of the previous section to show the working of these protocols. I consider a suffix  $l^*$  for each location which is not the current residing one. The process described on the previous section is repeated for each location where the firm was already exporting profitably. This means that in addition to equation 2 the firm will set

$$y_{f,l^*,t}^e = y_{f,l^*,t-1}^e + \lambda(y_{f,l^*,t-1}^s - y_{f,l^*,t-1}^e) - Inventories_{f,l^*,t-1} + DT_{l^*,t} \quad (4)$$

if profits  $\pi_{f,l^*,t-1} \geq 0$ . This sets the need for an expanded workforce, and therefore the firms sets an additional level of labor  $n_{f,l^*,t}^d$ , such that

$$y_{f,l^*,t}^d = \eta n_{f,l^*,t}^d \tag{5}$$

to be able to produce additional goods. Now, since this economy is devoid of a credit sector, the firms might risk to employ too many workers without

 $<sup>^{13}</sup>$  In the first turn, no Firm is already exporting, therefore the process begins directly from this step.

enough resources. In order to check this, the firms register a prospect wage bill  $\hat{W_{f,t}}$  which is equal to

$$\hat{W_{f,t}} = n_{f,t}^{d,total} * \bar{W_{f,t-1}}$$
(6)

where  $n_{f,t}^{total}$  is the *total* labor needed by firm f in that turn for all the served markets, while  $W_{f,t-1}^{-}$  is the average wage paid by the firm in the previous turn. This is therefore an imperfect measure of the expenditures that the firm will face by recruiting additional workers, and it is expanded every time that a firm decides to serve a market. For instance, if a firm decides to serve a foreign location  $l_1^*$ 

$$n_{f,t}^{d,total} = \eta n_{f,l,t}^d + \eta n_{f,l_1^*,t}^d \tag{7}$$

and so on as additional locations are added. The prospect wage bill  $\hat{W}_{f,t}$  is therefore growing the more locations are served, and is compared with the current firm net worth to check if the recruitment of new workers is affordable. For instance, if for an additional location  $l_1^*$ 

$$\hat{W}_{f,t} > NW_{f,t-1}$$

then the exporting protocol is stopped, as the firm wouldn't want to fail even before sending their goods to the market  $^{14}$ .

**Export protocol #2: New Markets protocol** If a Firm was able then to fulfill the first protocol, and has still possibly positive resources as  $\hat{W}_{f,t} < NW_{f,t-1}$ , it can "scout" for new markets where to export its own goods. To do so the Firm samples locations, and orders them according to their local population, as we recall that firms prefer to target bigger markets. The firm than iterates trough this sample, and applies a protocol which is almost equal to the one used for serving the Old Markets. The only difference is that equation 4 is not valid for a newly served market, as there were no sales or production taking place there in the previous turn. Therefore the desired production is "initialized", by setting it as

$$y_{f,l^*,t}^d = \frac{y_{f,1}}{L}$$
(8)

which is simply the initial condition value for a firm's production (more on this in the parametrization section), divided by the number of locations. Having set an initial production target, the firm updates the prospect labor bill as well. The process stops when the prospect labor bill is no longer sustainable or when the sample of new markets is exhausted.

For the final exporting protocol to take place, the firm must have set actual production, and I will discuss it therefore in subsection 5.4.

 $<sup>^{14}\</sup>mathrm{This}$  occurrence is plausible given the non-existence of the credit sector.

### 5.3 Employment

When all the Firms have set their total desired production level, by summing the domestic and exporting ones, such that  $y_{f,t,total}^d = y_{f,l,t}^d + \Sigma y_{f,l^*,t}^d$ , they begin to search for the actual labor needed to process the production:

$$n_{f,t,total}^d = \frac{y_{f,t,total}^d}{\eta} \tag{9}$$

This happens trough the Labor Market Matching Protocol (details in Section 8.1). Notice that if not enough workers are recruited (due to the matching protocol) the actual  $n_{f,t,total}$  could be lower than the desired one, resulting moreover in a lower actual production  $y_{f,t}^{total}$ .

#### 5.4 Pricing and export allocation

After the Labor Market Matching Protocol each Firm observes its workforce size, and then sets its actual production,  $y_{f,t}$ , which needs to be priced before being sent to the Goods markets. The agent formulates its price  $p_{f,l,t}$ , which is modeled as a mark-up  $mk_{ft}$  over this unit labor  $\cos^{15}$ :

$$p_{f,l,t} = (1 + mk_{f,t}) \frac{W_{f,t} n_{f,t}}{y_{f,t}^{total}}$$
(10)

The mark-up is revised upwards to keep a reasonable ratio between sales and inventories, such that when  $Inventories_{t-1}/Sales_{t-1} \leq 0.1$ :

$$mk_{f,t} = mk_{f,t-1}(1+\zeta)$$
(11)

with  $\zeta \sim 0.05 * unif(0, 1)$ . If this condition is not met,

$$mk_{f,t} = mk_{f,t-1}(1-\zeta)$$
(12)

The price  $p_{f,l,t}$  needs however some adjustments whenever the l goods market is not the domestic one.

Export protocol #3: Pricing and production allocation in foreign markets As we have seen, given the Labor Market Matching Protocol, it is not guaranteed that a Firm would be able to hire enough workers to reach  $y_{f,t,total}^d$ . In the presence of labor shortages, the firm could not be able to serve all the exporting markets, and therefore it must decide where to allocate its production.

 $<sup>^{15}</sup>$ The presence of market power, even having an homogeneous good, is a consequence of the search and matching protocols employed to clear the Goods Market. Since the consumers can visit only a fraction of the firms to buy their goods, this amounts for the goods to act as imperfect substitutes (see Assenza et al. (2015) for a discussion).

I assume that there is a "pecking order": the firm will serve first the old markets (if profitable), in order of population, as I deemed relevant the size of the export's market, then the new locations based on prices level.

Why prices? To allocate production to the new locations selected in the exporting protocol, the Firms rank them by price to represent the discriminating effect of distance. This is because closer locations will display lower transportation costs, resulting in a lower and more competitive price. As it is standard in Trade and Economic Geography literature indeed, exporting entails in fact some additional costs related to distance. To represent this, the system adds a penalty for exporting into another location,  $T_{l^*}$ . This value depends on transportation costs and possibly other idiosyncratic factors that can hinder trade with other locations. In this simple model where the spatial structure is given by the number of edges between Locations, it is based on the relative distance between the one where the firm resides and the one where is exporting. This can be easily modelled after some statistics from the location network, the most easier example being the *shortest path* between the origin and target nodes.

These extra costs are then location-specific, and lead the Firm to apply a price discrimination between markets. This is internalized by the firm as a fixed component inside the pricing equation for each targeted market. Indeed, we label

$$T_{l^*} = T C^{D_{l,l^*}} \tag{13}$$

where  $T_i^{target}$  is the "exporting penalty",  $D_{l,l^*}$  is then the indicator of distances between l and  $l^*$ , TC is a scaling parameter representing the level of transportation costs of the economy, > 1. As usual in Economic Geography, the transportation costs acts as multiplicative term to the FOB one (Free on Board). Notice that the usual Gravity model for trade cost specifications is more detailed, including origin/destination cross and specific factors. In real life moreover values vary much across traded variety and transportation modes, see Lafourcade and Thisse (2011).

The price for the an exporting location  $l^*$  is then extended to

$$p_{f,l^*,t} = (1 + mk_{f,t})(\frac{w_{f,t}n_{f,t}}{y_{f,t}}) * T_{l^*}$$
(14)

and the firm will prioritize first the locations with the lower price. The priced goods are sent to the various local Goods Markets, where they are eventually bought by Households consumers.

#### 5.5 Accounting

After the Goods Market matching protocol, the firms evaluates the status of their goods on each market. Unsold goods are registered as current inventory, while sold goods provide revenues. Goods sold in any foreign location have a different price from the domestic one, such that total gross profits will take the form of:

$$\pi_{f,t}^{gross} = p_{f,l,t} s_{f,l,t} + \sum p_{f,l^*,t} s_{f,l^*,t} - W_{f,t} n_{f,t}$$
(15)

where  $s_{f,l,t}$  is the sale per locations goods market.

Given the foreign revenues for each served location,  $p_{f,l^*,t}s_{f,l^*,t}$ , the firms define also a "foreign" profit,

$$\pi_{f,l^*,t}^* = p_{f,l^*,t} s_{f,l^*,t} - W_{f,t} n_{f,l^*,t}$$

where  $n_{f,l^*,t}$  is the share of workers needed for the production of the exported goods in location  $l^*$ . As noted in the previous section, there might be cases where this "foreign" profit is negative, leading the firm to cease production in that particular market.

Positive total gross profits are taxed accordingly to the tax rate  $\tau_{f,t}$  to obtain net profits:

$$\pi_{f,t}^{net} = (1 - \tau_{f,t})(\pi_{f,t}^{gross}) \tag{16}$$

The firms then calculate dividends to be paid to their shareholders, defining then net profits after dividends (or retained earnings RE):

$$\pi_{f,t}^{RE} = (1 - div)(\pi_{f,t}^{net}) \tag{17}$$

Firms can account then the evolution of their Net Worth  $NW_{f,t}$ :

$$NW_{f,t} = NW_{f,t-1} + \pi_{f,t}^{RE} - rel * RC_{f,l^*,t}$$
(18)

, with the last term being the relocation costs due when the firm relocated and the flag variable rel = 1 (more on this in Section 9.1). Moreover, given the matching structure and the labor hiring, firms might as well obtain negative profits, therefore substituting  $\pi_{f,t}^{RE}$  with the negative  $\pi_{f,t}^{gross}$ .

**Bailing Firms** If negative profits accumulate over periods, the Firm might become bankrupt, with  $NW_{f,t} < 0$ . In this case, the Firm is "bailed", or substituted by a new firm, with a reboot of initial net worth updated for current price levels, and initial desired domestic production (both coming from the Calibration procedure). To keep consistency in the model, the resources to bail and reset the failed firms are obtained by a bill which is financed by the government trough the variable  $Bail_t^f$ .

## 6 Households Behavioral equations

### 6.1 Labor supply

Each Household could be a public employee, or a private one. In the latter case it can be employed or unemployed, with status  $u_{h,t} \in [0, 1]$ . The desired wage  $w_{h,t}^d$  evolves according to the employment status  $u_{h,t}$  to represent lowering expectations in prolonged spells of unemployment, when  $u_{h,t} = 1$ , and increasing bargaining power when employed  $(u_{h,t} = 0)$ . Then the desired wage is

$$w_{h,t}^d = w_{h,t-1}^d (1 - 0.05 * \zeta) \tag{19}$$

if  $u_{h,t} = 1$  and

$$w_{h,t}^d = w_{h,t-1}(1 + 0.05 * \zeta) \tag{20}$$

if  $u_{h,t} = 0$ , with  $\zeta \sim unif(0,1)$ .

If unemployed, the Household will receive an unemployment benefit, which is a fraction  $\bar{w}$  of the current average wage for the whole economy  $\bar{W_t}$ , and is provided by the Government, such that

$$w_{h,t} = \bar{w} * \bar{W}_t \tag{21}$$

if  $u_{h,t} = 1$ . This benefit works as a lower bound wage in the job market, as workers which have lowered their wage expectations under that level will prefer not to work and will not be available on the market. Once that level is hit, workers will reinitialize their expectations randomly above that level, in order to avoid being trapped forever in unemployment.

Only unemployed Households will visit their local Labor market, where they could be hired by the local firms by means of the Labor matching protocol (Section 8.1). In that case their desired wage is set as their actual wage, which still evolves according to the previous heuristic. Employed Households are indeed paid according to their desired wage, such that

$$w_{h,t} = w_{h,t}^d \tag{22}$$

if  $u_{h,t} = 0$ .

**Public Servants** Households that are employed in the public sector receives a wage

$$w_{h,t}^{public} = \bar{W}_t \tag{23}$$

These Households cannot be fired, and as shown in Riccetti et al. (2015) they help in reducing the economy output volatility. I extend this property to the

spatial distribution of households population by assuming them to be immobile, meaning that they are not able to relocate across the locations network. This assumptions is meant to avoid Locations empty from economic activity, and in this sense public servants fulfill a similar role to that of immobile firms<sup>16</sup>. In New Economic Geography models, that often lacks a public sector, this role is usually filled by agricultural workers (Neary, 2001).

## 6.2 Consumption and saving

The Household's level of desired consumption,  $c_{h,t}^d$ , is based on fixed propensities to consume out of wealth  $W_{h,t-1}$  and out of net income  $NI_{h,t}$ , respectively  $c_0$  and  $c_1$ :

$$c_{h,t}^d = c_0 W H_{h,t-1} + c_1 N I_{h,t} \tag{24}$$

When a good is bought through the Goods market matching mechanism (see Section 7.2), the price of the good is deducted from the desired consumption level. The Household iterates the matching until either its sample goods are exhausted or if the desired consumption level is too low to afford any additional transaction.

The Household's wealth is then updated as

$$WH_{h,t} = WH_{h,t-1} + S_{h,t} + (Tr_t(t)/H) - mig * RC_{h,l^*,t}$$
(25)

with  $S_{h,t}$  being the current savings, namely the difference between current net income  $NI_{h,t}$  and current expenditures  $XP_{h,t}$ :

$$S_{h,t} = NI_{h,t} - XP_{h,t} \tag{26}$$

while  $Tr_t/H$  is the transfer paid by the Government to each Households<sup>17</sup>, updated for current price levels, and  $RC_{h,l^*,t}$  is the relocation costs due when the Households migrate, and therefore the flag variable mig = 1.

Income net of taxes (with tax rate  $\tau_{h,t}$ ) is defined as

$$NI_{h,t} = (1 - \tau_{h,t})(w_{h,t} + d_{h,t})$$
(27)

where  $d_{h,t}$  is the received share of the firm's dividends and  $w_{h,t}$  is the actual period's wage taking the forms seen in section 5.1, according for the Household

<sup>&</sup>lt;sup>16</sup>It would be surely more realistic to let the public servants population per location to fluctuate accordingly to the local household population level, as for instance a big city will have more public servants than a small village. This assumption would probably just accelerate the agglomeration process for a location (by boosting local demand), so my guess is that it would only enhance the same results driven by the trade and relocation heuristics. It seems nonetheless that the effect of the public sector location on the spatial distribution of activity would be an interesting topic of study.

 $<sup>^{17}</sup>$ The transfer is introduced in this version to better match the simulation behavior with the original Calibration system. It can be rationalized as a supply of welfare services.

to be unemployed or a public servant. Expenditures  $XP_{h,t}$  are based on the actual consumption  $c_{h,t}$ .

$$XP_{h,t} = c_{h,t} \tag{28}$$

**Dividends** To simplify the structure of firm's ownership, the dividends paid by every single f,  $div * (\pi_{f,t}^{net})$ , are pooled in a common account:

$$Aggdiv_t = \sum_{f}^{F} div(\pi_{f,t}^{net})$$
<sup>(29)</sup>

that is then divided between Households according to their relative wealth endowment, such that Household h received dividends are equal to:

$$d_{h,t} = Aggdiv_t * \left(\frac{WH_{h,t-1}}{WH}\right) \tag{30}$$

## 7 Public sector

The global government collects taxes and pays the benefits for unemployed Households and the wages of the public workers in all locations l.

Aggregating all the taxes revenues, on income and profits, such that

$$T_{F,t} = \sum_{f}^{F} \tau_{f,t} * (\pi_{f,t}^{gross})$$
(31)

$$T_{H,t} = \sum_{h}^{H} \tau_{h,t} * (w_{h,t} + d_{h,t})$$
(32)

$$T_t = T_{F,t} + T_{H,t} \tag{33}$$

and subtracting them to the Government expenditures defines a deficit,

$$DF_{t} = \bar{W}_{t} + W_{Gt} + \bar{B}_{t}^{f} + Tr_{t} - T_{t}$$
(34)

where  $\bar{W}_t$  is the aggregate paid unemployed benefits,  $W_{Gt}$  the aggregate public workers wage and  $\bar{B}_t^f$  is the eventual remaining bill for rebooting new firms, while  $Tr_t$  is the Households transfer, adjusted for current inflation.

The deficit determines the path of public debt of the economy.

$$PD_t = PD_{t-1} + DF_t \tag{35}$$

This level of Public Debt corresponds to the level of liquidity held in the economy by agents. As such its self financed; it is however possible to set specific levels of it to be reached trough revision of the various fiscal instruments of the Government. I extend this concept in the next chapter.

## 8 Market protocols

In this Section I further describe the protocols at the core of the agent economic interactions.

## 8.1 Labor market matching protocol

The Firms try to meet their desired production level  $y_{f,t,total}^d$ . However, if the current workforce of a firm is less than its required  $n_{f,l,t}^d$ , then it will need to visit the local Labour market to post vacancies. The Local Labour Market is constituted by the pool of unemployed Households currently in the Location. Indeed, when a firm's workforce is greater than the needed production, excess workers will be fired from the Firm, becoming unemployed and available for hire.

The adjustment of the production level is not the only possible reason for firing workers. When firms relocate to other locations, their current workforce is dismissed completely, as the firm needs to find new employees in their new position. A sort of "voluntary" unemployment moreover exists, since when employed households choose to migrate they start in the new location as unemployed.

The matching mechanism for a local Labour market works as follows: After the Firms have updated the wages of their employed households, all local unemployed Households desired wages  $w_{h,t}^d$  are listed in a vector. To avoid a "first-mover" advantage, the local Firms are shuffled, such that their moving order is random. Every Firm selects then a random sample from the Labor Market, that is then sorted in increasing order, as lower wages are more convenient. If the needed workers are less than the sample dimension, the workers are selected up until the workforce is complete from the lowest wage available. If instead the needed workers are more, the firm must be limited to select all the workers of the sample.

After the labour pairings are formed, production takes place, and the expected wages and the unemployment subsidies are paid respectively to the employed and unemployed Households. Households will then proceed to their local Goods Market.

## 8.2 Goods market matching protocol

The Goods market is formed by pooling every produced and imported goods prices together for each Location. So, basically, the Goods market is similar to the Labor one, albeit being a vector of prices instead of wages. The matching protocol presents however some differences, as the Households will not select directly the cheapest sampled goods.

This happens because the Households present a degree of consumption habit preference. Therefore, each Household will search first the goods of the Firms acquired in the previous consumption turn. If these are found, their prices become a reference against which a "switching test" is performed, to define whether the change of supplier will take place. This assumption helps in stabilizing the Firms sales, making the model less volatile.

A way adopted in the literature to depict this switching test (see Caiani et al. (2016a)) is to define a probability value that governs the eventual change of goods. This is determined by the distance between the price of the current evaluated good and the one by the supplier chosen in the previous matching: suppose that  $p_t^{new}$  is a price currently selected from the market sample, and that p is the price of the good of the old seller. Then the probability to switch to the new price Pr decreases or increases exponentially depending on the distance between these two price levels as:

$$Pr = 1 - e^{(p_t^{new} - p_t)/p_t^{new}}$$
(36)

if  $p_t > p_t^{new}$ . If given this probability, the match is successful, the exchanged good is "sold" and removed from the market. If the probability test fails than the consumer "buys" the reference good. Of course when the reference good price is less then the new price, the consumer stays with the former.

If the old suppliers goods are not available<sup>18</sup>, the Households selects the goods directly from the random sample, going by price order. This market structure implies then that the matching process can finish either by satiation of demand, by exhaustion of supplies, or by ending of the protocol, leaving space for inventories to be formed.

## 9 Relocation and migration markets

After each turn's labor and goods markets interactions, the mobile agents are allowed to change their current location. I call the matching protocols between locations and other agents "market", because the mechanism is basically the same as the goods market. Agents will explore a sample of locations, rate them according to their preferred metrics, and then eventually switch to that location. The choice of the metric is fundamental to this step, as we will see in Section 12.

Since there exist relocation costs, assumed to avoid excessive relocation and for increased realism, to avoid "black holes" where resources vanishes, I collect these costs into a "profit for relocation" variable. This amount is then equally divided

 $<sup>^{18}\</sup>mathrm{The}$  firm might have stopped production or exporting to that location.

between all the Households: it can be conceived as having an economy-wide relocation agency jointly owned by all the Firms, that in turn pays complete dividends to them every turn.

## 9.1 Firms

Mobile firms might decide to relocate to other locations. This could happen for instance because a more populated area would present a bigger domestic market; or choosing a Location with fewer firms would mean to face less competition. Since the firm agent is not a perfectly rational profit optimizer, as in NEG models, the plurality of incentives requires the need to select some specific heuristics to drive the relocating behavior.

By looking at the literature on firms' relocation, one can indeed infer that a multitude of factors shape this decision. Usually concerns about the workforce, like size and costs are the most powerful drivers, (Pellenbarg et al., 2002). Rather interestingly, one can find in the literature other more exotic motivations that might still matter, like a declining reputation of the firms' neighbourhood (Weterings, 2014), or even the risk posed by climate change (Linnenluecke et al., 2011).

In this model it is possible to test all of these factors, as the basic relocation mechanism is implemented easily in the same form of the consumption market: a search, a ranking, and a probabilistic switching, and is a consequence of conceptualizing the Locations as an agent. The ranking itself is determined by the factor under study, and we will see the main examples in section 12. Here I will describe the mechanism; how it is used to form relocation's heuristics addresses then the problem of Firms not being so rational as to guess exactly the level of the local demand and costs of every location, as is explained in Fowler (2007).

To start the description, I assume that relocation is costly. In their empirical analysis, Brouwer et al. (2004) point out that in the optimizing neoclassical framework adopted by NEG the relocation costs are usually dismissed to base the relocation decision only on cost-push factors. In a bounded rationality framework this would be however unrealistic, especially at the light of the empirical evidence on how individual firm's factors differ in influencing the decision (without counting the institutional ones).

While firms are indeed footloose, their size and specific activity matters into the relocation decision, as they impact differently on the magnitude of the relocation cost. To be more specific, *older* and *bigger* firms are less prone to relocation. This result, evidenced also by Pellenbarg et al. (2002) and obtained also in the microsimulation exercise by Nguyen et al. (2013) can be rationalized by the simple fact that bigger firms needs to face higher costs due to sheer size of the workforce substitution and capital equipment to be transferred (to this we can add path dependence and institutional encroachment). However, in another work Sleuwaegen and Pennings (2006) evidence that for bigger firms it is easier to move to distant locations, which could depend instead on the availability of resources to be invested in this step.

To capture this trade-off, I model a cost of relocation for firm f to location  $l^*$ ,  $RC_{f,l^*,t}$ , as

$$RC_{f,l^*,t} = D_{l,l^*} * n_{f,t} * \bar{p_t}$$
(37)

where  $D_{l,l^*}$  is the distance between the current location and the targeted one,  $n_{f,t}$  is the current firm's workforce size, all multiplied by the economy average price. A relocating firm can reach the target location only if it can afford it, so if  $RC_{f,l^*,t} < NW_{f,t}$ . In this way, it will be costlier for larger firms to relocate, but a firm with large net worth would be able to reach farther distances.

The targeted locations are classified according to a Location Metric  $LM_{l^*,t}$ , that is ordered in increasing or decreasing order depending on the metric desirability by the Firm agents. For instance, the locations average wage will be in increasing order, while the average profit will be decreasing. These metrics are compared against the one of the current domestic location,  $LM_{l,t}$ . If the domestic metric is superior, the test is automatically failed. If instead the foreign metric is more attractive, a probabilistic switching test is applied. First, a probability of relocation  $Pr_r$  is generated based on the distance between the two metrics:

$$Pr_r = 1 - e^{(LM_{l*,t} - LM_{l,t})/LM_{l*,t}}$$
(38)

if  $LM_{l*t} < LM_{l,t}$ , or for cases where higher values of the metrics are more favorable for firms (like the average wage level),

$$Pr_r = 1 - e^{(LM_{l,t} - LM_{l^*,t})/LM_{l,t}}$$
(39)

with  $LM_{l*t} > LM_{l,t}$ . The probability  $Pr_r$  is tested against a value sampled from  $v \sim unif[0, 1]$ . If  $Pr_r > v$  the test is successful and the Firm relocates to the new location.

When a relocation is successful, the following happens: the Firm will now be located in the new location, modifying accordingly the Firm's population of both locations. The specific location relocation cost  $RC_{f,l^*,t}$  is deducted from the Firm's Net worth and accounted in the Global Relocation Firm revenues. Finally, the firm fires all of its current workforce, as it will need to hire a new one in the new location.

### 9.2 Households

Households that are not public workers can migrate. As in the Firms case migration is subject to a cost, that in the benchmark model is based on the distances of the Locations in the Spatial Network.

Distance is recognized as one of the main drivers of migration costs, even though in real life is far from being the only one. Indeed, the literature on the topic recognized many different factors at work; for instance, In addition to distance *(Kennan and Walker, 2011)* proposed a migration costs specification with additional factors: like age of the migrants, that increases the costs, while moving back to an original destination (return migration) and the target location sizes<sup>19</sup> decreases it. Estimating all the various factors could be quite difficult, given issues such as separating money and non-money costs (Sjaastad, 1962), and recover and harmonize data (Molloy et al., 2011). In this model the focus is therefore limited to distance; additional factors could be included easily in further work more tailored towards migration topics.

Every turn, the migration cost for moving to an adjacent location is defined as:

$$mc_{h,t} = \frac{\varepsilon_{mc} * (WH_{h,1})}{max(D_{l,l})} * \bar{p_t}$$

$$\tag{40}$$

where  $WH_{h,1}$  is the calibration-obtained real wealth per household,  $max(D_{l,l})$  is the maximum distance between nodes in the *L* spatial network, multiplied by the average price  $\bar{p}_t$ . This boils down to assuming that to travel the whole network an Household should spend the whole steady state wealth level at current prices. While the cost of migrating is indeed high as estimated in Kennan and Walker (2011), this assumption is taken mostly for simplicity, and the migrating cost mc can be scaled trough the parameter  $\varepsilon_{mc}$ , which is set = 1 in the benchmark model.  $mc_{h,t}$  is used to define a relocation span  $rs_{h,t}$ :

$$rs_{h,t} = \frac{\gamma_r * W H_{h,t}}{mc_{h,t}} \tag{41}$$

which defines the number of locations that can be reached by the Households, given the distances of the Location network. This acts as the relocation cost of the firm, meaning that richer Households can move farther away. Indeed, an Household that migrates to a location  $l^*$  will pay

$$RC_{h,l^*,t} = mc_{h,l^*,t} * D_{l,l^*}$$
(42)

Having defined  $rs_{h,t}$ , a location is extracted from the population of Locations under reach (the ones such that the distance from the target location  $l^* \leq rs_{h,t}$ ). As in the Firm's relocating protocol, Households will rank their potential targets according to a Location metric  $LM_{l^*,t}$ . What changes compared to Firms is the metric itself: indeed migration can be caused by many factors, including cultural and security reasons; as we are looking at Economic Geography ideas, the reasons behind Households relocation will be economic in nature.

In NEG models the relocation of workers stems by love of variety for goods provided by larger markets, and by replicator dynamics arguments based on the wage level of a location Krugman (1991). Other arguments might point towards

 $<sup>^{19}{\</sup>rm This}$  because more densely populated Locations makes more probable the existence of a supporting social network.

an expected income differential and unemployment issues as in development literature such as the famous Harris and Todaro model (Harris and Todaro, 1970). Molloy et al. (2011) lists other possible reasons as well: employment prospects, amenities, tax rates, as well as psychological factors.

Despite the different forms that the metric can take, the mechanism is the same of the previous section. Metrics are tested according to their attractiveness to the households and tested by defining a probability of migration  $Pr_m$  such that

$$Pr_m = 1 - e^{(LM_{l*,t} - LM_{l,t})/LM_{l*,t}}$$
(43)

when  $LM_{l*t} < LM_{l,t}$  or to

$$Pr_m = 1 - e^{(LM_{l,t} - LM_{l^*,t})/LM_{l,t}}$$

with  $LM_{l*t} > LM_{l,t}$ . The probability is again tested against a value  $v \sim unif[0,1]$ .

When a migration test is successful, the Households will change its location, and the Households population in the origin and destination locations are updated. The relocation cost (Eq. 42) is subtracted from the Household wealth, and as in the case of Firms accounted in the Global Relocation Firm's revenues. If the Household was employed, it gets fired and will start the following turn as an unemployed one in the new Location.

### 10 Parametrization

To simulate the model<sup>20</sup>, arbitrary values need be assigned to the exogenous parameters and the initial t = 1 period variables, a task that needs some discussion. Indeed, how to best calibrate a model is a compelling topic in macroeconomics, and in particular in agent-based modeling. As a matter of fact, in these types of models the parameters space could be so vast that often it is not clear, how and if, the overall results are depending on particular parameters' configurations. This explains why varying the values of the (deemed) relevant parameters is a necessary sensitivity analysis for most models.

Here the calibration is adapted from the procedure presented in Caiani et al. (2016a), which aims to provide a mixture of plausible-realistic values for some parameters and stock-flow consistent values for the remaining ones, also providing the values to initialize the model's variables. In this form, the aim of the calibration is twofold: first, to produce a benchmark version of the model against which different experiments could be run; and second, to present an initial "symmetrical" situation. Symmetry is an important feature to be reflected upon initial conditions, as even minimal differences in the values of the agent's initial endowments can be enhanced by the occurring random processes and

 $<sup>^{20}{\</sup>rm The}$  ABEG model has been developed on MATLAB R2018a. All codes are available upon request, and will be made publicly available soon.

lead to very unstable and volatile simulations. Initial symmetry means than that every agent starts with no advantage over its peers, like resources stocks such as Wealth, or for decision variables such as prices and mark-ups. Moreover, in this Economic Geography setup every economy is calibrated to have an homogeneous population of both firms and households per location. As a Caveat, this procedure is probably unnecessary for this work, as it was devised for a more complicated context endowing multiple assets and complex financial relationships. I maintained it however in this version because I consider it a first step toward developing more complex versions of this model. At the light of the first series of results it is quite possible that a simpler procedure would have stemmed more realistic macroeconomic behavior, for instance regarding the relative dimensions of Debt and GDP.

Before explaining the procedure in detail, it must be mentioned that recently other methods to estimate and calibrate models have been proposed and tested. Grazzini et al. (2017) compare Bayesian estimation techniques against previously tested methods like simulated minimum distance, and Lamperti et al. (2018) look for the application of machine-learning surrogates in the exploration of the parameters space. Employing such methods would be a nice continuation of the research work, also in order to better tail the model with empirical data as it is done in the Quantitative Spatial Economics literature.

Following for now the technique presented in Caiani et al. (2016a), the modeler needs to declare an "aggregate version" of the economy, thought to be at a "long-run" steady state. Lacking the heterogeneous stocks of the Caiani model the aggregated system here is considerably simpler, and is then solved to pin down endogenous variables by declaring some realistic values for some of the parameters. Other less immediate parameters can be treated as free ranging. The aggregate endogenous solutions are then equally divided according to the number of agents, being them the initial states for the recursive economy (so the value for a variable  $x_t$  at t = 1).

Notice that since the location structure of the economy is interchangeable, this calibration process could end up in designing initial conditions that do not foster trade between locations, for instance by postulating a "steady state" price level that is too low for Firms to face transportation costs. To allow for trade then every location structure should need a different calibration reflecting the different spatial structures. One relatively straightforward method to solve this issue reckons that the locations spatial structure acts upon trade through the main channel of prices, influenced by the distances between the location's nodes. To provide then a situation where trade would be available, the stationary steady state price is augmented to reflect the transportation costs mark-up multiplied by the average path length of the location's network, the average path length being an average measure of the distances of the network. This modification maintains the calibration procedure quite smooth, as it needs only to plug the value for the current network average path length<sup>21</sup> in the calibration price equation 30.

 $<sup>^{21}\</sup>mathrm{The}$  average path length is obtained thanks to the ave\_path\_length.m and sim-

In the benchmark model the spatial network is a rectangular grid<sup>22</sup> composed by nine different nodes. This choice stems from it being the simplest structure endowed with a central location.

Compared to the previous version draft I change the number of agents in the simulation: the employed Households are now 693, while Firms are 63. This lowers the share of employed Households in the calibrator to  $11^{23}$ , while public employee per location are set to 15, to have a value of  $\approx 20\%$  of the working force<sup>24</sup>. The unemployment rate in the steady state, is set again to 10, making the total Household population 909. I anticipate here that these values affect the simulation unemployment level, making it smaller, resolving one of the undesired features of the previous work calibration. Other advantages to adopt them is a greater comparability with the work developed in the second chapter, (I keep the same share of agents per location), and a much improved running speed of the simulations, due to the reduced number of total agents. Aside from these changes the results are very much comparable with the previous version, regarding for instance the behavior of the basic relocation metrics.

Table 1 provides the values for exogenous parameters for the benchmark model. Wages are arbitrarily initialized to 5 for private and public wages, and to a share  $\bar{w} = 0.4$  of them for the unemployment benefit. In the price equation, mark-ups are initialized at 0.20, while the exporting costs is set as trade cost share (1.1) times the average path length of the location network, 4. The dividend share is set at 0.2 of the profits.

The definition of Wealth includes the two propensity to consume, out of income and out of wealth, c0 and c1, usually to set to obtain 0 < c1 < c0 < 1. Injecting these parameters into the calibration system and solving it provides then the solutions for the initial values of the main variables.

In the following the economic system equations are listed. Capital letters stand for aggregate version of the variables. We will therefore, the definition of initial Production given the employed Households  $H_f$ 

$$H_f = Y/\eta \tag{44}$$

, the definition of initial price, mediated by the average transportation costs,

$$p = (1 + mk) * \left(\frac{(W * N)}{Y}\right) * TC^{AD}$$
(45)

ple\_dijkstra.m Matlab functions developed in the Matlab Tools for Network Analysis (2011) by MIT Strategic Engineering: Copyright (c) 2011, Massachusetts Institute of Technology. All rights reserved.

<sup>&</sup>lt;sup>22</sup>I generate the spatial structures of the ABEG model with the canonical\_nets function developed Matlab Tools for Network Analysis (2011) by MIT Strategic Engineering: Copyright (c) 2011, Massachusetts Institute of Technology. All rights reserved.

 $<sup>^{23}</sup>$ This makes the calibrated economy rather similar to a small-firm based one, (like Italy, with an average number of employees per firm (1989-94) of 10.5; comparable numbers are seen also for Canada 12.7, Denmark 13.3 and Finland 13 (Bartelsman et al., 2005), page 28. During the simulation firms might nonetheless hire more Households than this number.

 $<sup>^{24}</sup>$ Also here the number is arbitrary, however  $\approx 20\%$  is reported as the percentage of public sector employment for the OECD countries in 2009 (OECD, 2015).

Parameter	Description	Value			
1 arameter	Description	value			
L	Number of locations	9			
<i>L</i> type	Rectangular grid, one central location	/			
F	Number of firms (assumed 5 per location)	63			
$H_f$	Number of private sector working Households	693			
$H_g$	Number of public sector working Households	135			
Н	Total Households	909			
mk	Price markup	0.20			
TC	Transportation costs shares	1.1			
AD	Average path length				
$ au_y,  au_\Pi$	Income-Profit tax share	0.3			
<u>c0</u>	Propensity to consume out of income	0.5			
<i>c</i> 1	Propensity to consume out of wealth	0.3			
$\eta$	Labor productivity	0.5			
$\bar{w}$	Unemployment dole share of wages	0.4			
$\gamma$	Search intensity parameter	5			
$\lambda$	Accuracy of the adaptive processes	0.05			
$g_{ss}$	Nominal steady state growth rate	0.02			

Table 1: Calibration of the main parameters of the baseline model

the initial profits,

$$\Pi = p * Y - W * H_f \tag{46}$$

the Tax revenues collected from Firms,

$$T_f = \Pi * \tau_f \tag{47}$$

the dividends share,

$$Div = D * (\Pi * (1 - \tau_f))$$
 (48)

the number of employed Households, both from Firms and Government,  ${\cal H}_f$  and  ${\cal H}_g$ 

$$H_e = H_f + H_g \tag{49}$$

the unemployment rate

$$u = \frac{(H - H_e)}{H_e} \tag{50}$$

the net income, including the one by unemployed Households,

$$NI = (W * N_{tot} + (\frac{1}{1 + g_{ss}} * Div)) * (1 - \tau_h) + \bar{w} * W * (H - H_{tot})$$
(51)

the Tax revenues collected from the Households,

$$T_h = (W * N_{tot} + (\frac{1}{1 + g_{ss}} * Div)) * (\tau_h)$$
(52)

the real Consumption, defined as the level of initial production,

$$c_h = \left(\frac{c_0}{p}\right) * (NI) + \left(\frac{c_1}{p}\right) * \left(\frac{1}{1 + g_{ss}}\right) * (WH)$$
(53)

$$c_h = Y/p; \tag{54}$$

nominal consumption,

$$C_h = c_h * p \tag{55}$$

the delta of Households initial Net Wealth

$$\left(\frac{g_{ss}}{1+g_{ss}}\right) * WH = NI - C_h + \left(\frac{g_{ss}}{1+g_{ss}}\right) * Div + Tr$$
(56)

and the delta of initial Public Debt

$$\left(\frac{g_{ss}}{1+g_{ss}}\right) * Debt = W * H_g + \bar{w} * W * (H - H_{tot}) + Tr - (T_c + T_h)$$
(57)

. We can check the internal consistency in the Transaction flow matrix presented in Table 3. Table 2 presents then the initial asset situation for all the agents; being this a credit-less economy, the only existing asset is money, which existence is brought by the government. The economy is initialized by the government, that by issuing initial liquidity Tr permits the agent to operate in the economy<sup>25</sup>. This entails moreover that the liquidity stock owned by firms and households is inherently the public debt, which we could think of as a interest-less bond.

 $<sup>^{25}</sup>$ ..and mathematically, the system to be closed

Asset	Н	F	G	Σ
Money	8284.17	44730.10	-53014.27	0

Table 2: Initial Assets Matrix (Benchmark model Calibration)

Variables \ Agents	H	F	G	Σ
$C_h$	-5031.18	+5031.18	-	0
Tr	+1978.83	-	-1978.83	0
$W * H_g$	+675	-	-675	0
$W * H_f$	+3465	-3465	-	0
$\overline{w} * W * (H - H_e)$	+162	-	-162	0
Div	+219.26	-219.26	-	0
$T_h$	-1306.48	-	+1306.48	0
$T_f$	-	-469.85	+469.85	0
Y	-	[346.5]		-
$-\Delta NW, \Delta Debt$	-162.43	-877.0644	+1039.49	0
$\Sigma$	0	0	0	

Table 3: Transaction flow matrix (Benchmark model Calibration)

The simulated model is therefore initialized with the chosen parameters, and the generated endogenous variables, which are equally divided among the agents, acting as the first period of the simulation.

Notice that these values refers to the grid benchmark 9 Location structure, and that changing the spatial network slightly modifies the values of some initial variables.

# 11 Benchmark model, first computational results:

I define a Benchmark model as a model that follows the Calibration of section 9, employs a 9 Locations grid structure, and uses the location Average wage and location Consumption as metrics for the relocation protocols of Households and Firms respectively (equations 38 and 43). I simulate the model for 400 periods, and present the average values for the most relevant series.

Before discussing these statistics, it is interesting to check visually the behavior of agents. In Figure 3 I present then some selected plots of aggregate variables generated by the model. The first two panels show the results of some of the main economic series. Production raises towards the steady state level; sales fluctuate around the production level affecting inventories. Dips in production raise unemployment cyclically, events which as we will see, are amplified by the migration behavior of Households.

To check how the spatial distribution has evolved, we can observe the remaining panels of Figure 3. The population bar plots (third and fourth panels) display the population share per location for selected time periods, while the moving agents graphs (fifth panel) displays the share of relocating and migrating agents for each period. Finally, the sixth panel shows how the final agents distribution is effectively placed on the spatial network.

We can observe from these graphs that the economy agglomerated rather abruptly in one Location, due to Firm's behavior. The Households followed the Firms, but a small percentage of them kept on migrating till half of the simulation span. From that point onwards the main agglomeration local Average wage was so higher than the rest of the economy that only inconsiderable movement happened after t = 200. Rather interestingly, we can observe from the sixth panel that the main agglomeration (or megalopolis) did not form in the central node of the spatial network, where trade costs should be minimized.

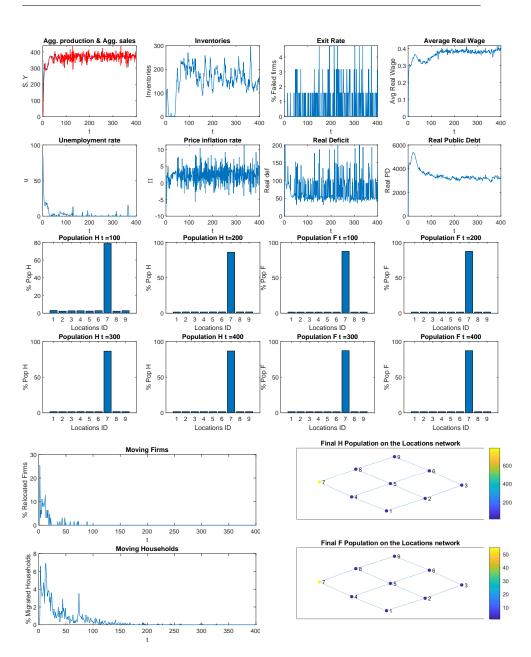


Figure 3: Benchmark model main plots. First panel: Aggregate Production and Aggregate sales, Aggregate Inventories, Unemployment rate and Consumption Goods Price Inflation rate. Second Panel: Firms' exit rate, Average Real Wage, Real Deficit and Real Public Debt. Third and Fourth Panel: Population share of Households, and Firms for Location at t = 100, 200, 300, 400. Fifth panel: Share of moving agents per period. Sixth panel: Final distribution of agents at t = 400 on the spatial network.

Is this configuration realistic? In figure 4, I plot the log of the locations ranks by Households population against the log of their population sizes.

This is a crude visual test to check if the location's population size distribution evolves towards a Zipf's Law (Zipf, 1949), a well known empirical regularity. Indeed, the city sizes distribution is well known to follow a power law distribution, as perhaps best exemplified by the work of Gabaix, that founds this pattern in the US system of cities, and proposes a stochastic growth model to account for it (Gabaix, 1999). This happens when the slope of an imaginary line interpolating the scatter plot takes a value of  $\approx -1$ .

It must be said that a debate emerged on the robustness of this empirical regularity, that seems to hold true only in the higher tails of the distribution. Competing explanations see the whole distribution of sizes as fitting a log-normal distribution. For the debate see Eeckhout (2004) and the following comment by Levy (2009). For some works that summarize the debate and tries to reconcile the different views see Malevergne et al. (2011) and Berry and Okulicz-Kozaryn (2012).

Giesen and Südekum (2010) find the Zipf's structure to hold in Germany, for different Economic Areas definitions, due to Gibrat's law holding regardless of how the spatial unit is defined<sup>26</sup>. While this law generates an overall log-normal distribution, in the upper tail the latter and the Pareto one are indistinguishable. Given the low number of locations of the benchmark model we could reasonably adopt the Zipf's law as a benchmark, as even if we were not sure about the underlying distribution, we would be still looking at the upper tail.

Nonetheless the benchmark model fails<sup>27</sup>, as we can see from the first panel of Figure 4: the economy never displays a distribution similar to the one predicted by Zipf's law. We can compare it with the second panel, that shows the same plot for the top 10 US cities by population in 2010<sup>28</sup>.

These unrealistic results fostered the need for further analysis with regards to the relocating behavior of the agents.

 $<sup>^{26}</sup>$  The authors point specifically to Gibrat's law (Gibrat, 1931) as the statistical explanation of the Zip's law rank-size stylized fact.

 $<sup>^{27}</sup>$ As expected: it was one the main shortcomings of this work's previous version.

 $<sup>^{28}\</sup>mathrm{The}$  data are from the 2010 United States Census

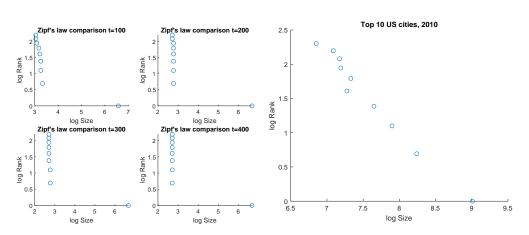


Figure 4: On the left panel, log rank plotted against the log Size of H population for selected time periods. On the right panel the same graph for the US top 10 cities in 2010.

## **12** Different Location Metrics

In this Section I will expand and generalize the results of the benchmark model by modifying the relocation protocol. This is a test to check how much the choice of the relocation metric affects the distribution of the spatial economy. Indeed, as anticipated in section 8, Firms and Households might have indeed different reasons to look for other locations.

I anticipate here that aside from the specific magnitude differences, what seems to really matter in this regard is the inherent attracting and repulsing nature of a given metric.

To better frame these natures, think of a Firm pursuing lower wages levels to lower its costs: it is basically escaping from a location that became too expensive (nature of the metric: Dispersion). If it were instead to target a location with a greater market it would be attracted (nature of the metric: Agglomeration).

The interaction between these two metrics types and the two mobile agents defines therefore four broad combinations that I characterize in Table 4. There we can observe that the benchmark model falls into the case where both metrics are agglomerating: it does not surprise us then that this model is converging towards a full agglomeration so starkly and irreversibly. I redefine therefore the Benchmark model as "AA", short of Agglomerating-Agglomerating and proceed in this section to test the other three main cases (DD, AD, DA). Subsequently, to further capture the heterogeneity in possible objectives, I will check a mixed case, to allow the agents to pursue simultaneous multiple Location metrics objectives.

As a summary in Table 5, I present averages for the main series. In the following subsections I will display and discuss the main plots for the different cases.

Metrics H / Metrics F	Dispersion	Agglomeration
Dispersion	Lower Unemployment/Lower wage	Lower Unemployment/Higher Consumption
Agglomeration	Higher wage/Lower wage	Higher wage/Higher Consumption

Table 4: Examples of different interactions by metric types.

MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark_AA	369.68	0.61	2.43	0.84	0	0.05
DD	260.95	24.25	1.20	0.06	5.35	13.91
DA	331.52	8.81	2.01	0.11	0	4.37
AD	295.07	18.69	1.74	0.19	0.81	1.56

Table 5: Average values for selected statistics, different relocation metrics. All averages after t = 100

### 12.1 Dispersion (H) -Dispersion(H) metrics: DD model

In this version the Firms prefer to relocate to Locations where the wage is lower (a sort of standard international relocation scenario), while Households leave locations with an higher unemployment rate. Notice that this last metric might not seem at first a Dispersion one, as one might think of more populated locations as having less unemployment. This can be true in the beginning of the simulation, but when a location attracts full agglomeration it becomes the one with the highest unemployment. A Location devoid of Households will still indeed still have an immobile Firm searching for workers, while the remaining Public Servants Households are counted as employed.

In figure 4 I report the same graphs generated for the benchmark model. The effect on the movement of the agents is significant: the economy maintains multiple agglomeration for the whole of the simulation, and most relevantly, the moving behavior of the agents is never stopped. This has consequences on the aggregate economics series; as agents keeps on switching locations, production and employment suffers, generating far less favorable statistics. Recall indeed that when a Firm relocates it fires its workers, while a migrating Household leave its current job, and starts in the new Location as an unemployed. This affects negatively the economic performance of the model, as both production and consumption are reduced considerably, given the high relocation and migration rate per period. This metrics combination is therefore inherently unstable, with agents "escaping" from each other. Rather interestingly, while the spatial structure is not realistic, as the main agglomerations are shifting constantly (see the third and Fourth panel of Figure 5), the log rank plot (last panel in Figure 5) presents a more realistic configuration than the AA model.

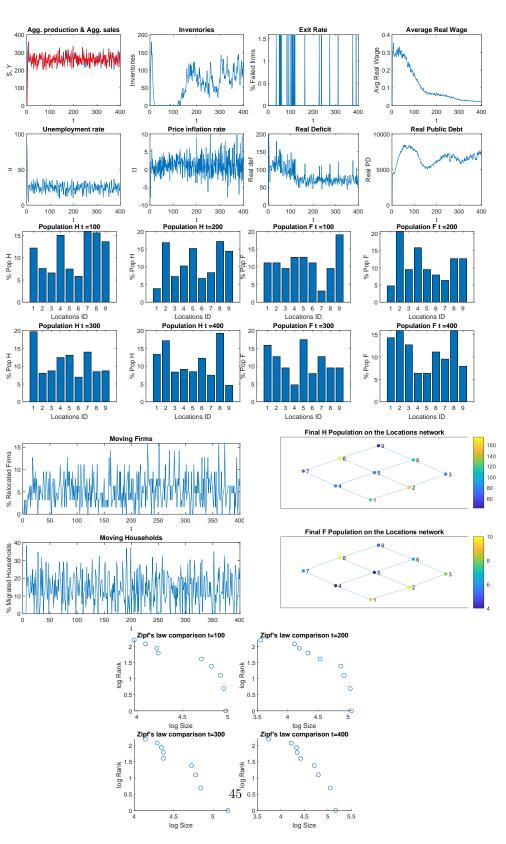


Figure 5: DD model main plots

### 12.2 Dispersion(H)-Agglomeration(F) metrics: DA model

Here Households are still looking for locations that present a lower unemployment rate, but the Firms prefer to target bigger markets as their location choice, employing the same metric of the benchmark AA model. I depict the same graphs as the previous models in Figure 6.

The interaction is unsurprisingly similar to a mixture of the previous two models: the Firms, endowed with an agglomerating metric quickly converge to a single agglomeration. The Households cyclically escape from this agglomeration, but they are not able to raise consumption enough in their new setting to break the agglomeration of the Firms. Moreover, when unemployed Households leave the agglomerated Location, they lower directly its unemployment rate by removing them from the local Labour market, triggering then a return migration until the next migration boom.

The boom in itself is caused by an almost nonexistent unemployment rate in depopulated Locations. It is interesting here how the the differences between locations accumulate until exploding in migration boosts of such magnitude (the highest almost regarding 80% of the H population). This is of course an unrealistic feature brought by the model structure, but provide some insights on how to include migration waves and return migration in other models with relatively simple heuristics.

Finally, this model performs less well than the benchmark model (AA), but better than the DD model: The migration oscillations corresponds to decreases in production and spikes in real deficit and real debt, as the higher unemployment rate and lower production and sales imply less revenues for the Global Government.

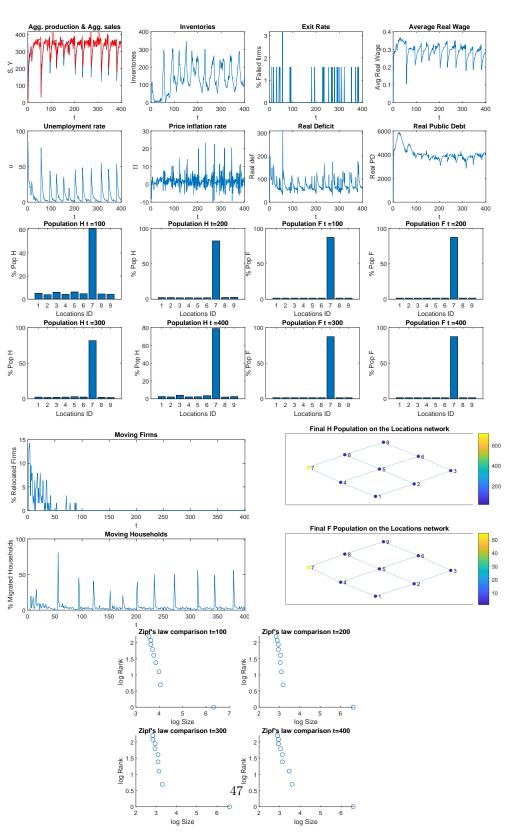


Figure 6: DA model main plots.

### 12.3 Agglomeration(H)-Dispersion(F) metrics: AD model

This version is the reverse to the previous one, in the sense that now Firms target the Dispersion metric, while the Households the agglomerating one. Both agents target the average wage, as Firms target lower average wages to minimize costs, while Households target higher wages to improve the living standard. While it seems redundant with the previous Sub-section, it is precisely interesting because of the same metric being tested.

As we can observe from Figure 7, the model undergoes a remarkable loss of production from period 200 onward, after simultaneous booms in agent movements. Since agglomeration here directly influences the dispersion metrics of one of the agents, the spatial configuration is quite unstable. The bar plots about the populations distribution shows that the Agents agglomerates in different Locations. When an agglomeration takes place Households migrate to that Location attracted by the rising wages of the working people. As this happens non-agglomerated locations become more attractive to Firms, that escapes from the Households.

Given these results and the ones coming from the similar Dispersion(H) Agglomeration(F) model we can see that two another scenarios emerge: one where the main agglomeration is never broken, and one where the main agglomeration changes over time. According to the summary statistics of Table 5, the former case performs better in the logic of this model, despite showing higher relocation rates by Households. This happens for two main reasons: recovering production in the main agglomeration is less disruptive, and second, higher but more temporally concentrated movement spikes by one agent are less distorting than by both agents.

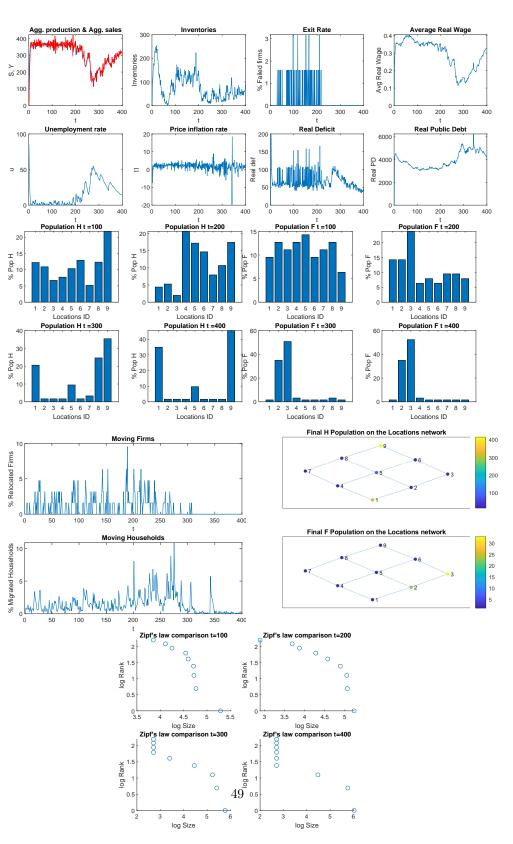


Figure 7: AD model main plots.

### 12.4 Mixed Metrics

The previous tests showed us that the choice of the relocation metric is not "innocent" in defining both the spatial distribution of agents and the macroeconomic performance of a model, which somewhat complicates the effort of testing the usual Economic Geography exercise of varying transportation costs. We have indeed already obtained a plethora of spatial possibilities, such as a full agglomeration, full agglomeration with cyclically escaping agents, unstable dispersion and agents chasing each other.

In this sub-section I will test the model under a more complex relocation rule for both agents, to see if it is possible to encompass all of these features in the same model. I extend therefore the relocation protocol for each type to have both a dispersion and an agglomeration metrics at play simultaneously.

The use of multiple metrics seems supported by the empirical literature, that, as we seen in section 8, considered different relevant determinants of the relocation behavior. An additional motive is the possibility of evaluating the interaction between agglomerating and dispersing forces, and moreover to allow for bidirectional flows, since a single metric would permit only movements towards a location more favorable in the metric dimension.

It would therefore be quite interesting to allow both type of forces to be tested simultaneously in the model, for both type of agents. Indeed, as evidenced mathematically by Akamatsu et al. (2017), is properly this interaction that characterize not only the presence of equilibrium agglomerations, but also their "shapes" in well-behaved optimized equilibrium models.

To perform this exercise, I modify the equations 38 and 43 , namely the "Probability of switching"  $Pr_r$  for firms, and  $Pr_m$  for Households. Both of them are extended from their general form

$$Pr = 1 - e^{(LM_{l*,t} - LM_{l,t})/LM_{l*,t}}$$

(if  $LM_{l*,t} > LM_{l,t}$ ) to a new form  $Pr_{mix}$ . Before showing the formula, I relabel the Location metric  $LM_{l,t}$  to  $LM_{l,t}^i$  with *i* as the index for the number of location metrics used. To keep the extension as simple as possible I employ only 2 metrics.

Since the scope is to analyze the interaction between agglomerating and dispersing forces, I define the two location metrics as  $LM_{l,t}^A$  and  $LM_{l,t}^D$ . This is perhaps a slight abuse of notation, as the main difference between the two is that agglomerating metrics will include variables where foreign higher levels will be seen as more favorable by the agent (example: consumption for Firms, wages for Households), while dispersing will include variables where foreign lower levels are preferred (example: wages for Firms, unemployment for Households).

Basically, this makes a location  $l^\ast$  more attractive with respect to the original location l when

$$LM_{l*,t}^A > LM_{l,t}^A$$

and when

$$LM_{l*,t}^D < LM_{l,t}^D$$

. As we have seen that both type of effects can be captured by the Pr formula by modifying the exponent of the exponential function (see Section 9), and combining the two we can therefore finally define  $Pr_{mix}$ :

$$Pr_{mix} = 1 - e^{((LM_{l,t}^A - LM_{l^*,t}^A)/LM_{l,t}^A + (LM_{l^*,t}^D - LM_{l,t}^D)/LM_{l^*,t}^D)}$$
(58)

Notice that this formulation permits different combinations of metrics to be evaluated, even if one of the metrics were not to be favorable, for instance like having  $LM_{l*,t}^A < LM_{l,t}^A$  or  $LM_{l*,t}^D > LM_{l,t}^D$ .

The condition for the protocol to bind is therefore

$$\left((LM_{l,t}^{A} - LM_{l^{*},t}^{A})/LM_{l,t}^{A} + (LM_{l^{*},t}^{D} - LM_{l,t}^{D})/LM_{l^{*},t}^{D}\right) < 0$$

, which allows some case of bidirectional agents flow between locations<sup>29</sup>.

The results (displayed in Figure 8) displays an oscillating behavior of migrations and relocation not too dissimilar to the DA model of section 12.2. In this version nonetheless the spatial distribution presents multiple agglomerations, and both agents keeps on moving for the whole simulation time. Moreover, contrary to the AD model of section 12.3 it does not present the mismatching issue between agents of different kind. Rather interestingly, if we observe the sixth panel of Figure 8 we can observe that the two main final Agglomerations are based at the opposite extremes of the Spatial network, namely at the maximal available distance between each other. The final panel presents moreover a well behaved slope, although one cannot dismiss the fact that the spikes of migration and relocation rates are still too much pronounced to be realistic.

Within this relocation framework I test additionally a different configuration of metrics, substituting local consumption with local Household population as an agglomerating metric for Firms. The results for this variation are very similar, with minimal variations probably caused by stochastic fluctuations (Robustness checks with multiple iterations are performed in Section 14).

MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Mixed	332.77	8.53	1.97	0.39	2.16	4.62
Mixed (alt. metric)	319.91	11.43	1.73	0.27	3.35	5.40

Table 6: Average values for selected statistics, Mixed models. All averages after t = 100

 $<sup>^{29}</sup>$  Notice that if a location metric is equal to zero, the value in the code is substituted with a 1e-07, to avoid the formation of NaNs in the code.

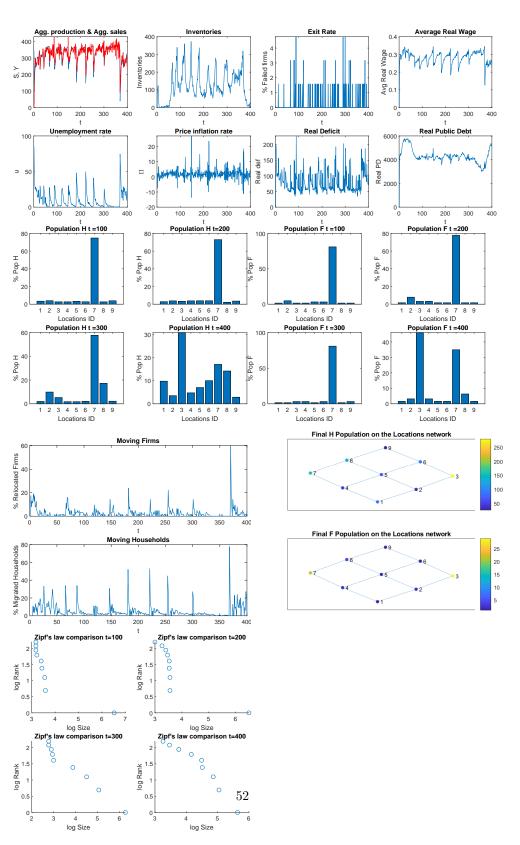


Figure 8: Mixed model main plots.

### 12.5 The issue of relocation intensity

The previous exercises displayed a propensity of this model to overestimate the relocation and migration behavior. Indeed even by accounting simultaneously agglomeration and dispersion forces in the behavioral metrics, it is not unusual to observe spikes of mobility by agents that are unrealistically high. We will observe later that this result is surely influenced by the structure of the spatial network, and the magnitude of the distances embedded in it, but it is still interesting to check simple modeling solutions to limit the "eagerness" of movements by the agents.

#### 12.5.1 Different relocation propensities

The definition of the various form probability of switching Pr is in itself an object that can be subjected to calibration. By defining an appropriate parameter in the Pr definitions is indeed possible to fix a given probability of switching, based on the difference of the two metrics. In the Caiani et al. (2016a) paper for instance the same probability mechanism (in the consumption goods market) is calibrated to obtain a probability of a 50% switch for a 20% difference in the compared values, e.g. the prices in the Consumption Goods market.

In the benchmark simulations, this parameter is not specified, or equal to 1. This means that in this case a difference of 20% between the two metrics gives a  $\approx 18\%$  probability of switching. By changing these reference values it would be possible to test directly probabilities taken from actual empirical works that estimates migration or relocation propensities. In this work I limit this feature as a test to see the effects of an higher and two lower propensities to move, by setting them equal to 3.46574, 0.526803 and 0.0502517, respectively a probability of 50%, 10,% and 1%. I apply then these different propensities to the Grid benchmark model, one with the AA and one with the Mixed Relocation protocol structures (with Population as a Firm metric).

In Table 7 I present the same average values for the economic series. We can see that changing these propensities might affect the economic performance, but that these effects are still affected by the shape of the relocation metric protocol.

As we can observe in Figure 9 for the AA model, rising the probability of relocation gives practically identical results, but there is a difference in the frequency of migration spikes: the enhanced probability made possible a second migration spike by Households, while the lowest value allowed some Firms to relocate for the whole simulation. The oscillations of these rates for the mixed models are dampened by the decreasing probability.

MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
AA	369.68	0.61	2.43	0.84	0.00	0.05
AA (50%)	369.51	0.79	2.42	0.83	0.00	0.10
AA (10%)	369.01	0.59	2.46	0.94	0.00	0.15
AA (1%)	366.92	1.32	2.46	0.54	0.22	0.16
Mixed	319.91	11.43	1.73	0.27	3.35	5.40
Mixed (50%)	325.93	10.11	1.72	0.37	4.20	4.61
Mixed (10%)	324.26	10.40	1.80	0.23	2.08	5.21
Mixed (1%)	324.92	10.34	1.80	0.08	0.45	4.77

Table 7: Average values for selected statistics, AA and Mixed models with different relocation propensities given a difference of 20% in the relocation compared metrics. All averages after t = 100.

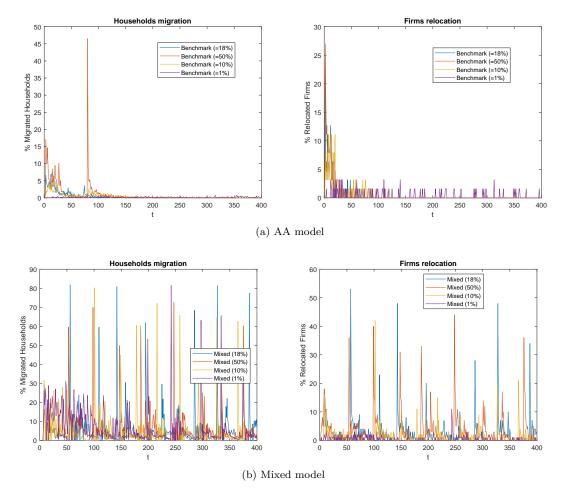


Figure 9: Relocation and migration rates for Firms and Households, different relocation propensities.

#### 12.5.2 Limiting the number of agents.

As shown in most Economic Geography models, the fixed agents plays a role in shaping the spatial equilibrium of the economy, as they act as a dispersing force. I consider therefore to generate more realistic spatial maps simply by assuming a greater share of fixed agents.

In this test I run therefore as an experiment the standard ABEG model, but limiting the number of agents that can move at every turn. This is done by making the relocation cycle to bind only for a tenth of both types of agents<sup>30</sup>. These agents will then apply the relocation protocol according to their own metrics and relocation possibilities given the costs. Such modeling modification might reflect the fact that the relocation protocol is missing some crucial factor that prompts the relocation behavior. To be neutral on this possible factor, I select randomly the agents whose protocol will be binding for that turn<sup>31</sup>. This modeling choice brings the model more closer to the Core-Periphery Agentbased model developed by Fowler, where location decision where taken each turn by a single agent of the whole population.

To not confound the possible effects with the ones stemming from changing the parameter governing the relocation probability, I set the latter equal to 1.

As before this is tested on the benchmark AA model, and on the mixed metrics Mixed model.

We can observe from Figure 10 that the migration and relocation rate collapsed to far more realistic figures, even for the AA metrics model. This permits the emergence and sustainability of additional agglomerations even with that unforgiving metric. It is also interesting to notice that different relocation metrics structure still affects the final outcome of the moving behavior, as the Mixed model, allowing for the additional dispersion metrics makes the migration behavior of Households more volatile.

In this section we evinced that simple modifications might help in fine-tuning the model towards more realistic behavior. In Policy analysis it would be highly recommended to exploit this feature to calibrate the model for specific relocation and migration rates, in order to match the considered spatial and timely economic context before performing policy experiments.

MODEL (10% relocating agent)	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark AA	366.74	1.18	2.45	0.60	0.16	0.18
Mixed	367.42	1.07	2.42	0.55	0.15	0.71

Table 8: RECAP table selected statistics all models- All averages after t =100

 $<sup>^{30}\</sup>mathrm{This}$  value is arbitrary, but it could be probably studied to depict desired migration and relocation rates

 $<sup>^{31}</sup>$ It must be noted that an interesting direction for modelling this feature would be to add some passive conditions on which the protocol becomes binding, like being unemployed for x periods for Households, or having low profits for Firms.

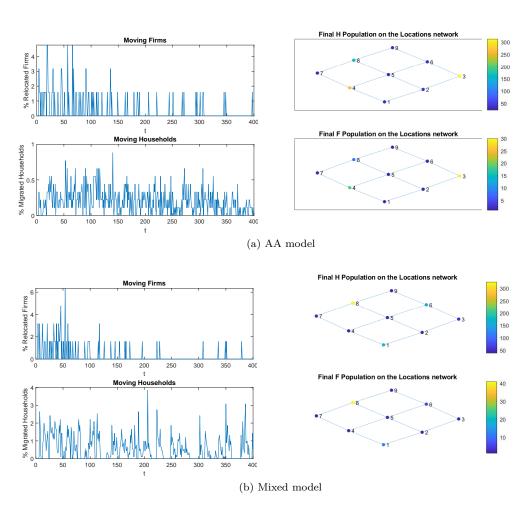


Figure 10: Relocation and migration rates and final spatial structures for models with fewer per-turn moving agents.

Having explored the consequences in modifying the relocation and migration protocols with different metrics, and different intensities, I will now proceed to test the effect of different Transportation costs and spatial structures.

## **13** Experiments

Following the tradition of Economic Geography models, I will test now how different transportation costs levels influence the simulation results. Moreover, the spatial structure of the benchmark model (the rectangular grid) is just one of the spatial configurations that this model can test. In this section I propose therefore a series of experiments focused on exploring the effects of different spatial networks on the economy.

Technically speaking, these experiments can be classified as a sensitivity analysis, obtained by varying relevant parameters. Sensitivity analysis is an important step in assessing the robustness of a model, and it is a usual step in Agent-based modeling (Caiani et al., 2016b). In the framework of the Agentbased Economic Geography model this imply to act on the Calibration procedure that generates the spatial network linking the Locations agent, and in modifying the level of the Transportation Costs TC parameter of Equation 13.

### 13.1 Different Transportation Costs

To check the influence of transportation costs on this economy, I perform a sensitivity analysis on the Transportation costs share parameter TC. This sensitivity analysis provides an interesting economic testing ground, as we can check if the agglomeration provided in the benchmark case follows the same determinants as the ones that emerge in Economic Geography models.

Indeed, Transportation costs are one of the key ingredients in Economic Geography (Lafourcade and Thisse, 2011). In a nutshell, transportation costs can span different configurations of the spatial structure of the economy, by triggering potential bifurcations in the underlying mathematical systems. This happens because these costs mediates the incentives of firms between exporting or relocating towards a particular location.

A direct consequence of this modeling feature is that transportation costs become one of the main policy dimensions of the Economic geography discipline. Indeed varying the level of transportation costs could be interpreted as a policy intervention on trade openness, like the introduction of barriers, tariffs and other instruments (Ottaviano, 2003; Commendatore et al., 2018a).

I test the benchmark model with three different levels of the TC parameter: low, medium and high, (1.01, 1.1, 1.5), where the medium one corresponds with the value used in the previous experiments. Following the previous section results on the Agents relocation metrics I employ the AA and the Mixed models metrics, with a restriction on the number of moving agents per turn as detailed in section 12.5.2.

To save space of exposition I present only the results on the spatial distribution, in Figure 11 (the other plots can be visioned in the appendix). Changing the transportation costs levels did not affect the agglomerating efficiency of the benchmark AA model, but we can observe in the second panel of Figure 8 that the economy was able to support a second agglomeration further compared with the higher costs cases.

In Table 9 I present the usual statistics: it seems that the Low TC level model was the one with worst economic performance, with an higher Firms exit rate, and higher unemployment rate. I will expand on this result later.

I perform then the same experiment with the benchmark grid model endowed with the Mixed relocation protocol (with population of Households as the Agglomerating relocation metric). Here it seems that changing the TC parameter has the same decreasing effect on Production and Unemployment as shown in table 10. By looking at Figure 12 we can see that the highest level of TC ended up in causing a dispersion of Households agents in the last periods of the simulation.

MODEL: AA	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	350.67	4.66	2.21	1.73	0.27	0.22
TC = 1.1	366.74	1.18	2.45	0.60	0.16	0.18
TC = 1.5	369.49	0.67	2.45	0.11	0.28	0.16

Table 9: Economic series average values - different TC levels. AA model, Grid

MODEL Mixed	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	342.86	6.33	2.23	1.56	0.27	1.01
TC = 1.1	367.42	1.07	2.42	0.55	0.15	0.71
TC = 1.5	369.74	0.54	2.53	0.11	0.21	0.35

Table 10: Economic series average values - different TC levels. Mixed model, Grid

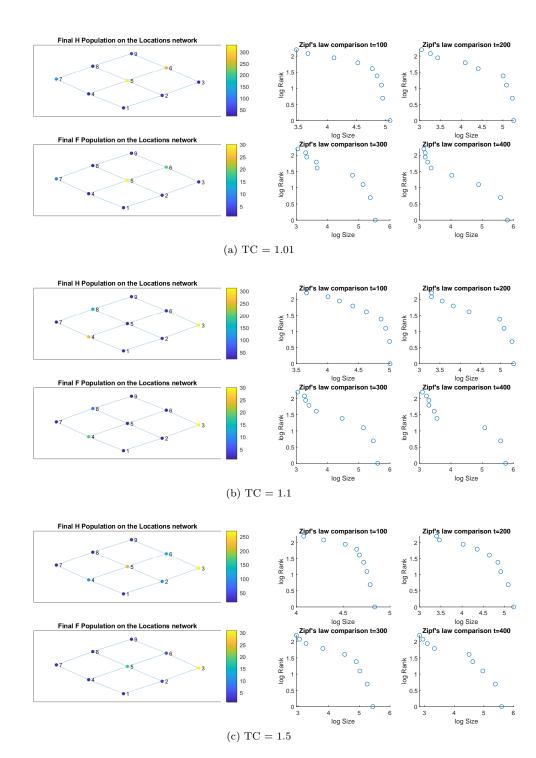


Figure 11: Spatial structures given different TC levels, AA model, Grid

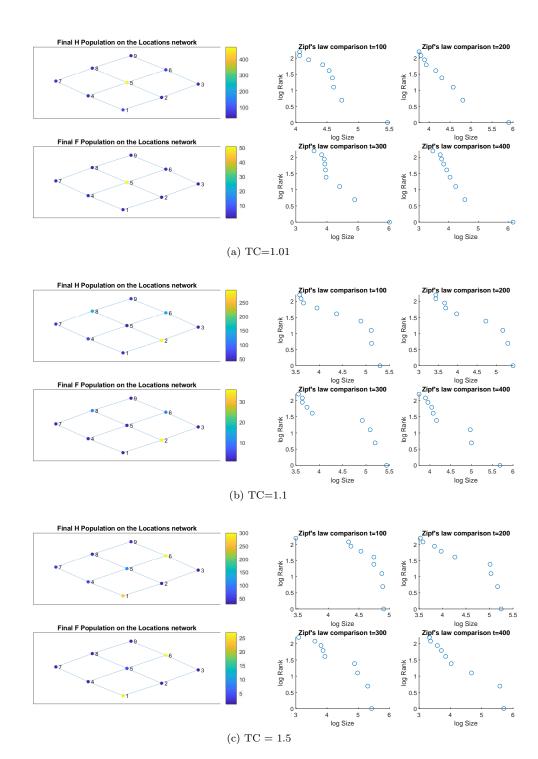


Figure 12: Spatial structures given different TC levels, AA model, Grid

[	MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
ſ	TC = 1.01	348.56	6.67	2.29	1.48	0.15	0.11
[	TC = 1.1	349.27	6.53	2.40	0.92	0.20	0.05
ſ	TC = 1.5	368.31	2.23	2.43	0.16	0.15	0.12

 Table 11: Economic series average values - different TC levels. AA model,

 Core-Periphery

MODEL Mixed	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	354.07	5.40	2.18	1.56	0.17	0.05
TC = 1.1	366.96	2.66	2.36	1.04	0.19	0.09
TC = 1.5	376.08	0.58	2.47	0.22	0.19	0.12

Table 12: Economic series average values - different TC levels. Mixed model,Core-Periphery

### **13.2** Different Spatial Structures

The benchmark version of the spatial structure is just one example of the spatial networks that could be employed in the model. Indeed, looking at other disciplines, many works in Quantitative Spatial Economics are able to embed different spatial structures, as these can be used to link the model with geographical data and to depict realistic networks (Allen and Arkolakis, 2014; Fajgelbaum and Schaal, 2017). In this section I reiterate therefore the Transportation costs exercises by modifying the underlying spatial structure.

I represent first some simple structures, which are nonetheless interesting since usually employed in the Economic Geography literature. I will test a Core-Periphery model with just two locations, a Long Narrow Economy, and a Racetrack economy. In all these experiments I will keep the number of Locations fixed to ease the comparison with the benchmark model (the only exception being the 2-region case). This is true also for the calibration procedure, and the number of agents. I simulate each different Spatial structure with two different relocation metrics (the AA and mixed) and for the three different levels of the TC parameter (low, medium and high).

From these experiments we can deduce that indeed the spatial structure has a relevant effect on the macroeconomic performance (compared to the benchmark model), as the interaction of relocation, migration and trade can affect economic variables ceteris paribus.

#### 13.2.1 Core-periphery model

In the classical 2 regions case, made popular by the seminal Krugman model Krugman (1991), we can observe that agglomeration is a recurring outcome, even by using the Mixed models. In the terminology of Krugman we do end up therefore with a Core-periphery situation.

Table 11 and 12 presents the average series for the two different relocation protocols models. We can see again lower TC prompting lower average Aggregate Production and average Unemployment rate, while the Mixed model performs better in all three tests. Agglomeration happens in all the scenarios, but its speed and dynamics varies across them.

As can be perhaps better seen in the Appendix plots, the Mixed model was able to keep Firms slightly more dispersed between the two Locations. The usual bifurcation graph relationship was not obtained for this range of parameters.

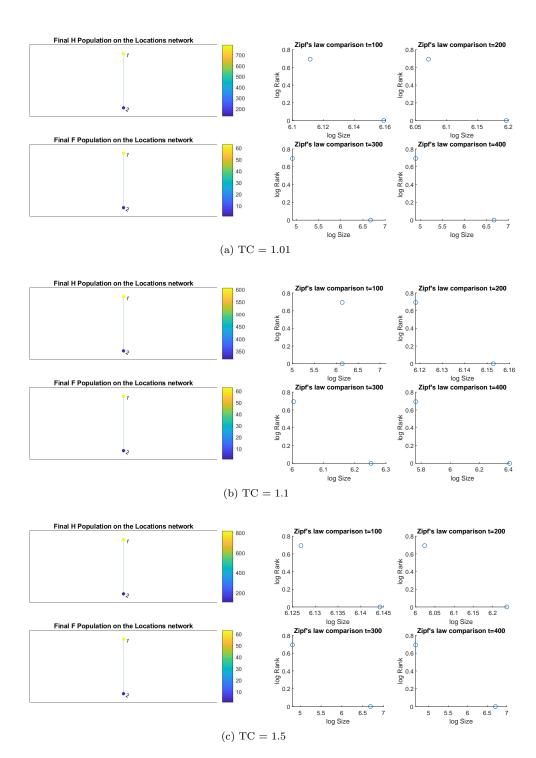


Figure 13: Spatial structures given different TC levels, AA model, Core\_Periphery

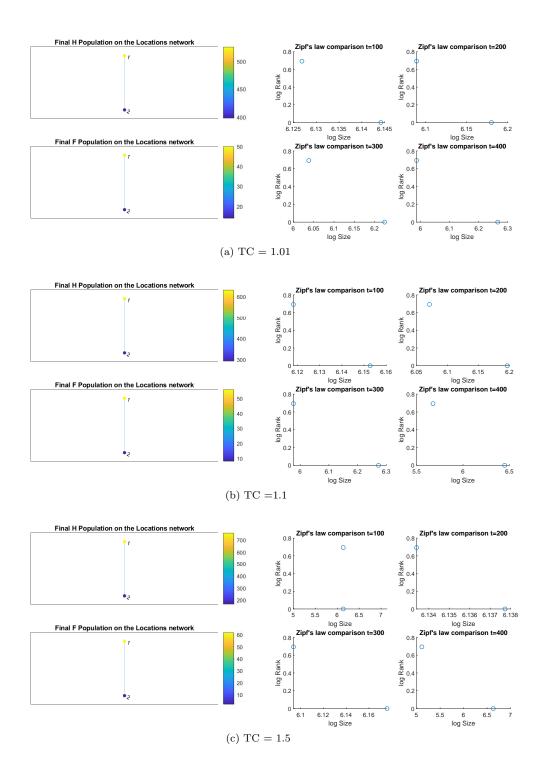


Figure 14: Spatial structures given different TC levels, AA model, Core\_Periphery

#### 13.2.2 Long Narrow Economy

The Long Narrow economy<sup>32</sup>, has been a well studied case, as the presence of "edge" locations with only one neighbor usually influences the model dynamics in interesting ways, since it breaks the embedded symmetry of the 2-Regions and Race track models; for instance these models present often situations of double-agglomeration near both ends of the line (Ikeda et al., 2016).

We can observe again the pattern of lower Average Aggregate Production and higher average unemployment rate for a lower level of TC in Tables 13 and 14.

For both models, multiple agglomerations emerge (Figures 15 and 16): two main with lower Transportation costs, while more homogeneous ones with increasing costs. Notice that these agglomerations does not seems to follow specific distances between them.

MODEL AA	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	351.61	4.45	2.24	1.53	0.19	0.24
TC = 1.1	368.75	0.89	2.45	0.29	0.27	0.20
TC = 1.5	369.16	0.49	2.55	0.07	0.21	0.16

Table 13: Economic series average values - different TC levels. Mixed model, Long Narrow Economy

MODEL Mixed	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	350.44	4.64	2.21	1.62	0.19	1.29
TC = 1.1	370.43	0.51	2.46	0.32	0.14	0.29
TC = 1.5	370.63	0.30	2.59	0.06	0.21	0.22

Table 14: Economic series average values - different TC levels. Mixed model,Long Narrow Economy

 $<sup>^{32}</sup>$ This structure is basically a line; I think that there could be interesting approximation of it in real life, if we think at economies such as Italy and Chile.

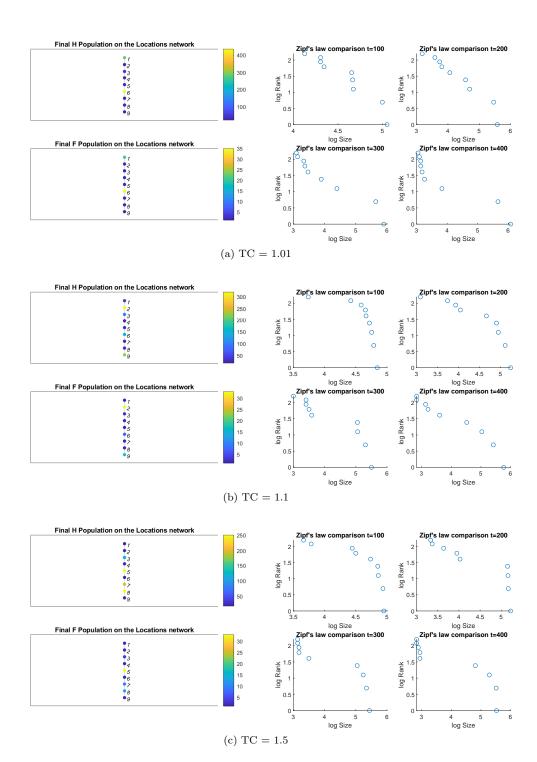


Figure 15: Spatial structures given different TC levels, AA model, Long Narrow Economy

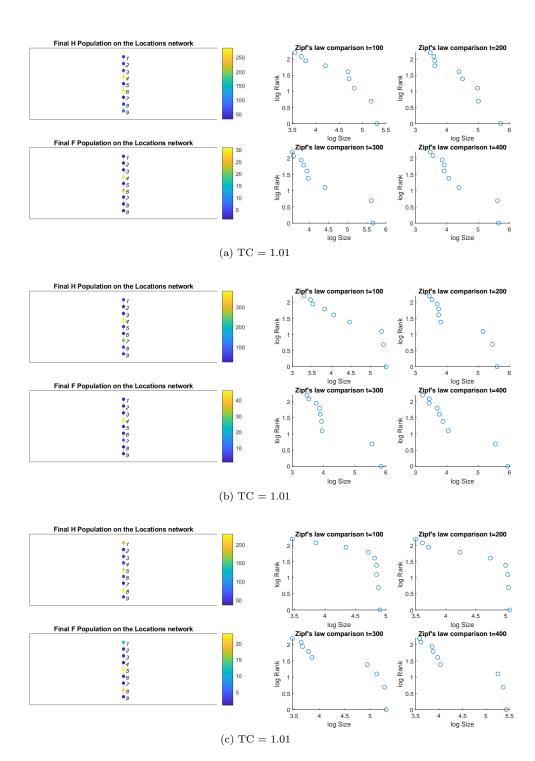


Figure 16: Spatial structures given different TC levels, Mixed model, Long Narrow Economy

#### 13.2.3 Race track economy

The Race track economy (also analyzed in Ikeda et al. (2016)) is an interesting testing ground, as no particular location has an advantage in terms of distance compared with the other, and it is a natural generalization of the Core-Periphery case. Depending on the agglomeration and dispersion forces interactions in the model, Akamatsu et al. (2017) shows that different patterns of agglomeration might arise in this setting, including equidistant multiple agglomerations or concentrations of agglomerations.

These models generates indeed multiple agglomerations, and the usual pattern for the average Aggregate Production and the Unemployment rate (Tables 15 and 16). Figures 17 and 18 reports the spatial distribution plots.

I will now summarize the results of this battery of simulations for a better interpretation of the results.

MODEL AA	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	350.14	4.75	2.24	1.55	0.22	0.22
TC = 1.1	368.48	0.84	2.45	0.45	0.20	0.17
TC = 1.5	366.89	1.08	2.56	0.09	0.27	0.23

Table 15: Economic series average values - different TC levels. AA model, Race Track Economy

MODEL Mixed	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC = 1.01	346.77	5.46	2.19	1.54	0.11	1.08
TC = 1.1	368.36	0.82	2.47	0.46	0.10	0.37
TC = 1.5	369.03	0.41	2.58	0.07	0.13	0.23

Table 16: Economic series average values - different TC levels. Mixed model,Race Track Economy

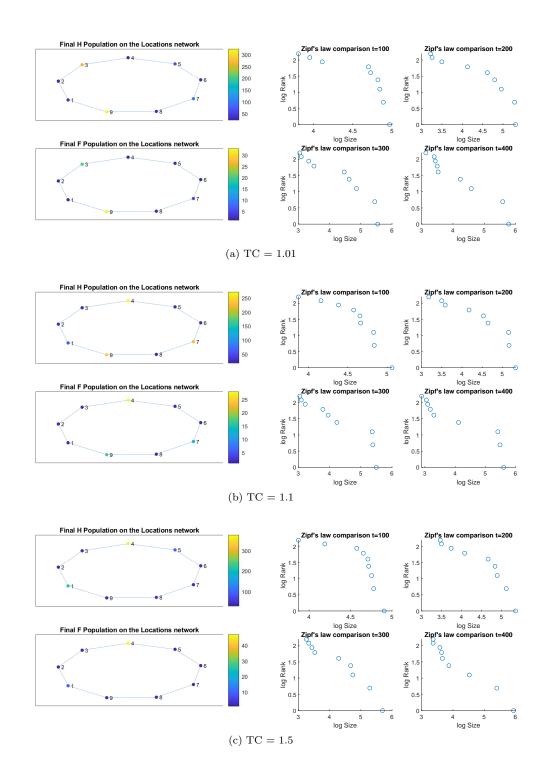


Figure 17: Spatial structures given different TC levels, AA model, Race Track

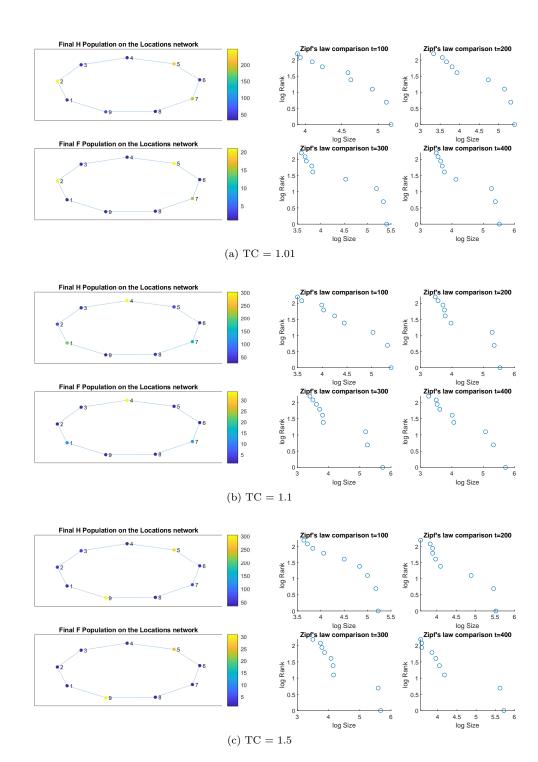


Figure 18: Spatial structures given different TC levels, Mixed model, Race Track

#### 13.2.4 Summing up:

Between the different spatial structures, the transportation costs levels, and the different Relocation protocols the number of models increased very swiftly. In this sub-section I will therefore synthesize the information obtained in the previous simulations, to compare more easily the effects of the various combinations, without getting lost in an endless list of plots.

To do so I condense the information about a specific model economy's spatial structure in one single box plot that is based on series composed by an Herfindahl-Hirschman a well known absolute concentration index<sup>33</sup>, calculated as

$$HH_{F,t} = (\sum (L_{F,t}/F) * 100)^2$$

for the Firms' Population and

$$HH_{H,t} = (\sum (L_{H,t}/H) * 100)^2$$

for the Hpuseholds one. The index is bounded between 0 and 10000, the higher the number, the higher the concentration. This index is obtained for a given model by calculating an HH index for every period, squaring the sum of shares of agents by each Location. Creating this series of indexes, and presenting a box plot, the time spent in different spatial configurations would be captured by the series, and we would be able to differentiate models that present the same final distribution, according to their average, median and variability values.

In every panel of Figure 19 I present these average HH index calculated for a given spatial network model and relocation protocol. Bars are grouped accordingly to the different Transportation costs values. For every group, the index is presented for both agents type, with Households being blue, and Firms being red. In a single box plot the horizontal line depicts the median, while the average value is a "x". The colored box includes values included between the 25th and 75th percentiles, while the whiskers are the lowest and highest values excluding the outliers. The latter are depicted instead with a red "+". Notice that the lowest values are always the same because the all the models begin the simulation with an equally distributed population of agents.

 $<sup>^{33}</sup>$ This index is perhaps more famous for its applications in market competitiveness analysis.

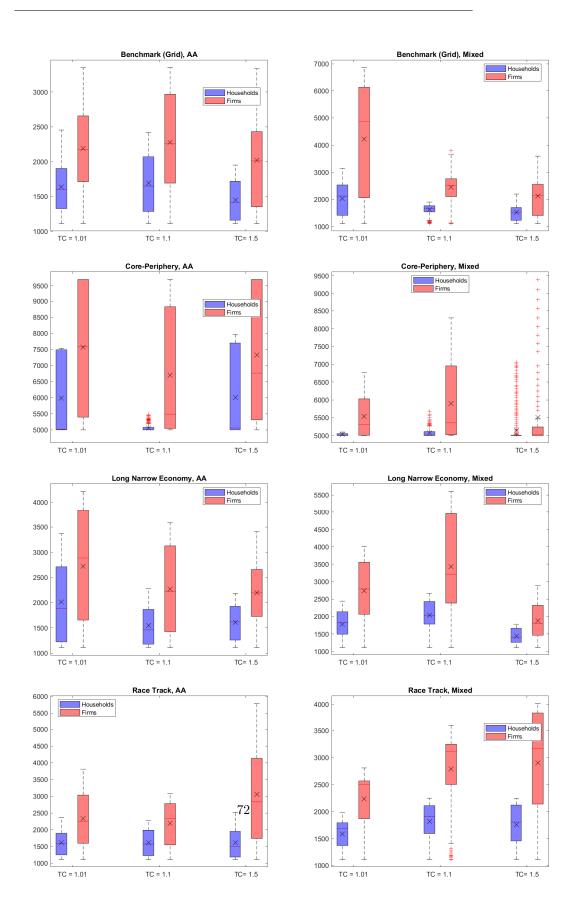


Figure 19: Box Plots for the HH indexes

We can see that the bifurcation effect of varying Transportation costs seems not systematically present. At least in this set of simulations and values for the TC parameter, it is not possible to derive a particular relationship between the degree of agglomeration of a given economy and the the level of the TC parameter. The usual Economic Geography result of linking the Transportation costs level (or in the other way, the degree of trade liberalization) with a dispersed or agglomerated state of the economy escape is lost in the boundedly rational heuristics.

In some configurations a comparable form seems present, like the case of the Grid economy with a mixed relocation metric. There, a lower level of Transportation costs supports a more concentrated economy (higher median, average and upper box line and whiskers). In the optimized logic of Economic Geography models, this happens because Firms can safely agglomerate without losing shares in foreign markets. Rising the transportation costs makes then Firms more likely to be dispersed, as Firms occupy safe havens where they suffer less competition. As we can see, this result is not robust to the changes in relocation metrics as well as of the spatial structures: for instance in the case of a Race Track economy with mixed relocation metrics this result is the opposite.

These plots helps moreover in showing that different spatial structures as well as different relocation metrics might affect usual predictions by Economic Geography.

#### 13.3 Robustness check: Logistic company

To test a version of the model where transportation costs could exert a greater influence I slightly modify the code to include an economy-wide "shipping company" that collects actual fees to ship goods to foreign markets for each Firm. To avoid these resources to be dispersed I assume that this company is jointly owned by all the Households population, and that it operates at a zero cost. The revenues collected by this company are equally divided to all Households population as dividends.

I test this model only with the mixed relocation metrics for the same three levels of the TC parameter and for the previously tested spatial structures. In Figure 20 I present the same box plots taken on the whole simulation concentration HH indexes, calculated for every period.

Also in this version I fail to identify a distinct pattern. the main difference with the previous case comes from the Race Track model. Agglomeration is higher with low transportation costs for the Benchmark grid and the race track economy, with the medium value in the Long Narrow economy, and with the higher value in the Core-Periphery model. The momentary conclusion is that probably the TC cost parameter is still not distorting enough to affect the agents heuristics at the point of generating the typical patterns of Economic Geography models. For sure, a more systematic sweep of parameters value and distance would be needed before taking definitive conclusions. The number of simulations is too low.

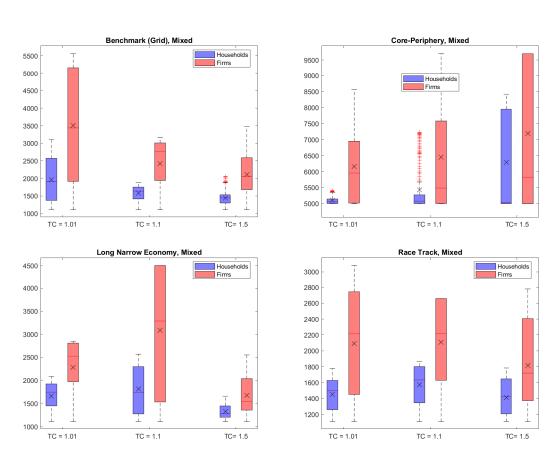


Figure 20: Box Plots for the HH indexes, model with a Logistic company.

### 13.4 Robustness check: TC after calibration

In this test I check the possibility that the including the Transportation Costs in the Calibration procedure might end up in dampening too much its effects. Indeed if the initial price of all Firms is already considering the role of distance, the discrimination between the price of exported goods and of domestic goods could not be as pronounced as to have a sensible effect on the economy.

I test here therefore a version of the model where the calibration step is modified to not account for the average distance of the spatial network multiplied by the TC parameter (which lowers the initial starting price for all Firms), and where the TC parameter is declared after the starting of the main cycle of the simulation. The model is run on the 9 Location grid structure, and the relocation metric is the Mixed metrics version employed in the previous sections.

The results in Figure 21 are in line with the previous versions, aside from an higher concentration index for the highest TC case. The average values for

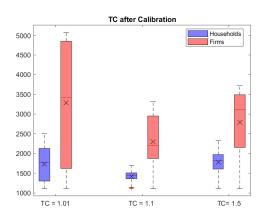


Figure 21: Box Plots for the HH indexes, TC set after the calibration

the economic series in Table 17 shows a behavior in line with the previous simulations, although now the medium TC values shows values for Average Aggregate Production and Average Unemployment rate more near the ones of the low TC value than the one with the high one. Overall these series are lower and higher than when TC is included in the calibration<sup>34</sup>, which points towards an effect at play by the price markups in shaping the future paths of the economy.

MODEL Mixed	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
TC after, $TC = 1.01$	306.53	14.32	1.67	0.76	2.72	6.88
TC after, $TC = 1.1$	309.34	13.70	1.57	0.76	2.81	6.83
TC after, $TC = 1.5$	336.25	7.75	1.89	0.18	2.09	4.24

Table 17: Economic series average values - different TC levels. Mixed model,Grid, TC after Calibration

It could be interesting to combine this robustness check with the one proposed in the previous Section. This implies assuming a Logistic company in a model where the TC parameter is not included in the Calibration step. I propose just the plot regarding the Concentration indexes for the mobile Agents (Figure 22). To stress additionally the model I test also values for TC that would generate unrealistic Price differentials between the different Locations, but it still would be interesting to see if there is some relationship emerging for higher values.

In this situation we can see that concentration decreases when Transport becomes more difficult, therefore being more in line with the predictions of Economic Geography bifurcation graphs. It shows moreover that are more systematic analysis for different TC values is needed, and that the difference in

 $<sup>^{34}</sup>$ I realized too late that this model was not endowed with the restriction on moving agents, and it is why it has higher relocation and migration rates. The series about production and unemployment are shifted downwards and upwards compared to right Mixed model.

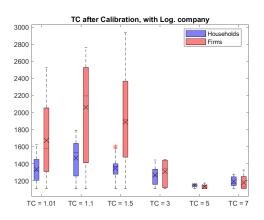


Figure 22: Box Plots for the HH indexes, model with a Logistic company and TC set after the calibration

concentration does not seem to follow the knife-edge property, but be more smooth. again, this number of iterations is still not robust.

### 13.5 The role of Distance: Increasing the number of Locations

The ABEG model could host indeed a far denser network of Locations, the limit being the available computational power. In this section I will propose an additional spatial sensitivity analysis by investigating a model with an increased number of Locations. Many of the results of the previous sections might indeed be driven by the relatively low size of the employed spatial networks. Distance, for instance, might become relevant enough to modify some of the exporting and relocating behavior by the agents, while the agents themselves might fail to converge so rapidly towards stable agglomerations.

I expand the basic benchmark structure from 9 to 100 Locations. To maintain comparability with the previous exercises I will keep the same initial ratio of agents for each location, resulting therefore in a wide increase in their populations.

The model is placed on a square Grid version<sup>35</sup>. I test here both the benchmark AA relocation metric, and the mixed one, for a total of two models. The number of agents is scaled up to reflect the same endowment of agents per location as the benchmark model. Given the 100 Locations the model is now endowed with 700 Firms and 10100 Households, feature that considerably slows the simulation.

The values reported in Table 18 shows a similar behavior of the model with respect to the version with less locations (aside from average production, which

 $<sup>^{35}\</sup>mathrm{I}$  modify slightly the canonical\_net function, as the original version did not create square grids

is naturally scaled up given the increased number of agents). It is quite remarkable to note that the agglomeration features brought by the AA metrics causes agglomerations locally, generating multiple centers. We can observe in Figure 24 that compared to the smaller Location Network analysis the number of agglomerations is higher and more heterogeneous as some of the smaller ones seems able to withstand the agglomeration shadow of the three main centers. Moreover, the share of moving Firms seems to be declining throughout the simulation, meaning that the spatial structure at the end should be stabler.

MODEL L =100	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
AA	4094.70	1.39	2.46	0.17	0.33	0.20
Mixed	4114.8	1.01	2.47	0.15	0.25	0.31
Low density Mixed	1133.08	1.67	2.38	0.12	0.24	0.58
Race Track Mixed	4123.073	0.275	3.18	0.05	0.16	0.18

Table 18: Main economic series, Models with 100 Locations

Another interesting feature, is that these agglomerations positioning (aside from the spatial competition effect between neighbors) seems completely random, and it is far from the neatly arranged patterns of models à la Christaller in Central place theory. Few elements of randomness and heterogeneity were then able to create an asymmetric spatial map, even without assuming any a priori first nature natural endowment or amenity for some of the Locations, aside from the centrality measures stemming from the network.

For additional robustness I test the same model with the mixed relocation metric, adding therefore a dispersion force. Some properties are maintained, such as the random positioning of agglomerations. The average statistics are very similar, although the protocol letting only a fraction of the agents to relocate surely dampens the differences between the different relocation metrics structures.

To check the effect of the spatial density of Agents I perform an additional test on the extended L = 100 grid, in order to test a version with less density of agents. In this test I reduce the initial number of Firms to 2 per Location, one mobile and one immobile for a total of 200. The immobile Firms are still assumed to not left any Location devoid of economic activity. Households still follows the number of 11 per Firms, while public servants are decreased at 1 per location. Counting the additional unemployed Households, there are a total of 2530 Households, Rounded up to 2500 to have an initial endowment of 25 Households for all Locations. This is therefore a far less crowded map. The relocation metrics follows the Mixed protocols. The result of this configuration are in line with the other cases, aside from a reduced average Production given the lower number of producing agents.

Finally, I test additionally a version of the model on Race Track map (with Mixed Relocation protocol), to see the effects of a purely symmetric map, which includes a higher average distance with respect to the grid maps.

The series for the Many L Mixed, Dispersed and Race Track Mixed structures are showed in the Appendix (Figures 48 to 50). To compare better the agglom-

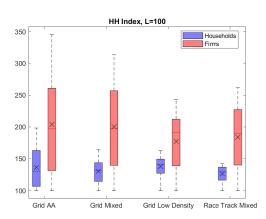


Figure 23: Average HH Indexes many L

eration differences of these models I grouped in Figure 24 the box plots for the HH index for both agents calculated for every model. In fact every model displayed multiple agglomerations, but there is the need to assess which one was the most agglomerated. The low density model should not be considered much here as it has a different number of agents. We can observe that the Mixed model displayed lower indexes for both agents compared to the AA model. The Race track model has a slightly lower values than the grid one, pointing towards the possibility of the increased distances to reduce the concentration of the economy.

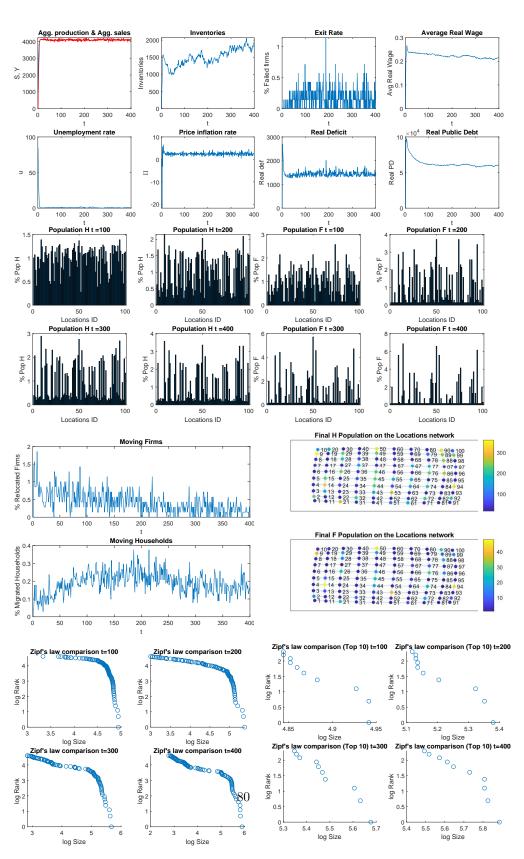


Figure 24: Main series with 100 L, AA model-Grid

# 14 Robustness check: Monte Carlo experiments

### 14.1 Benchmark models

Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Mean	369.72	0.60	2.45	0.79	0.04	0.13
SD	7.80	1.65	2.54	1.14	0.24	0.23
Median	372	0	2.45	0	0	0
MAD	4.50	0.90	1.89	0.97	0.09	0.14

Table 19: Selected statistics for the Monte Carlo simulation of the Benchmark AA model

Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Mean	323.09	10.69	1.78	0.28	2.71	5.21
SD	56.93	12.52	3.647	0.67	5.86	8.62
Median	348	5.28	1.611	0	1.58	2.75
MAD	43.15	9.46	2.379	0.47	3.00	4.65

Table 20: Selected statistics for the Monte Carlo simulation of the Benchmark Mixed model

To assess the effect of randomness, I simulate two versions of the benchmark model 100 times varying the initial Random Seed for each simulation<sup>36</sup>. The two versions are both endowed with the standard 9 Locations grid, and employs the AA and Mixed relocation protocol respectively.

In table 19 and 20 I present some selected statistics for the two models. The statistics includes the mean of the average, the mean of the standard deviation, the median of the median, and the mean of the Mean Absolute Deviation, calculated from period t=100 to the final 400 over the series generated by the Monte Carlo simulations.

To see the evolution of the spatial distribution, I compute the Herfindahl-Hirschman concentration index for the population shares of agents at every period. I then present here the statistics averaged for all the 100 simulations. The average values of the HH index (in Table 21) shows that the AA model is consistently more agglomerated with respect of the Mixed one.

Perhaps more interesting is to check if some of the Locations was able to attract most of the agents consistently across the simulations. To check this I calculate the average population for each location at the end of the simulation, with T = 400. As we can observe from table 22, in the case of the AA model there are some Locations endowed with a slightly higher average share across the simulation, but it is still not possible to say that this is a systematic feature and not a result of the random component.

With the mixed relocation metrics model we can observe that the Locations ending up with consistently less agents are the ones at L = 1, 3, 7, 9. If we check the sixth panel of Figure 3 we realize immediately that those are the Locations

 $<sup>^{36}{\</sup>rm the}$  random seeds are generated randomly by employing another random seed. This helps in making the Monte Carlo simulation reproducible.

Statistics	mean HH-index H	mean HH-index F
AA	5490.863	6304.501
Mixed	4296,481	5296.080

Table 21: Average HH index by agent for the Monte Carlo simulation of the benchmark AA and Mixed Model.

at the tips of the spatial network. Those are indeed the less connected ones, fact that makes them attract half or less of the final average share of agents. It seems therefore that the mixed relocation metric is more sensitive to the shape of the employed spatial network.

Location	Average share pop F	Average share pop H
1	8.49	8.67
2	11.06	11.11
3	10.15	10.03
4	11.77	11.69
5	11.88	11.98
6	15.38	15.27
7	10.12	10.15
8	9.23	9.27
9	11.87	11.79
Location	Average share pop F	Average share pop H
Location 1	Average share pop F 7.11	Average share pop H 7.98
		<u> </u>
1	7.11	7.98
1 2	7.11 18.41	7.98 17.56
$ \begin{array}{c} 1\\ 2\\ 3\\ \end{array} $	7.11 18.41 7.28	7.98 17.56 8.13
$ \begin{array}{c} 1\\ 2\\ 3\\ 4 \end{array} $	7.11 18.41 7.28 16.33	7.98 17.56 8.13 15.44
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ \end{array} $	7.11 18.41 7.28 16.33 15.34	7.98 17.56 8.13 15.44 14.75
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ \end{array} $	7.11 18.41 7.28 16.33 15.34 11.34	7.98 17.56 8.13 15.44 14.75 11.22

Table 22: Final (T=400) average share of population per Location after 100 simulations.

#### 14.2 Transportation costs and Spatial structures

I perform a Monte Carlo analysis also on two of the experiments performed by changing the Transportation costs parameter TC, for two different Spatial structures all tested with the Mixed relocation metrics. The spatial structures are the benchmark grid and a race track economy: given three levels of TC costs, this gives 6 models for a total of 600 simulated iterations. Tables 23 and 24 shows statistics for all the tested models. We can confirm the higher average Aggregate production levels and lower average unemployment rates (as well as less volatile) for a reduce TC value. The low TC case seems the one with an higher migration rate, which probably follows from the increased unemployment rate.

In Figure 25 I compare box plots for each map taken for the HH concentration indexes for three different TC costs levels. Higher Transportation costs has a lower concentration average and maximum values in both maps. In the case of the grid economy, the concentration is maximal for the medium value. Comparing the two spatial maps, the race track economy is less agglomerated for higher TC values, compared with the grid one.

It seems therefore that despite higher prices for exporting production, demand was binding enough to guarantee higher revenues and production for Firms. This lowered unemployment differentials between Locations, dampening the migration behavior of Households, that further lowered agglomeration. Rising the distance structure of the spatial map, such as considering a race track instead of a grid economy seems to enhance this effect.

$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$								
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	TC = 1.01							
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Mean	347.78	5.24	2.19	1.56	0.17	1.14	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	SD	13.03	2.86	2.36	1.56	0.51	0.61	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Median	350	4.84	2.23	1.58	0	1.1	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	MAD	10.34	2.27	1.81	1.16	0.30	0.48	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$				TC=1.1				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Mean	368.28	0.88	2.45	0.41	0.12	0.46	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	SD	5.70	1.16	1.52	0.81	0.43	0.59	
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Median	370	0.44	2.47	0	0	0.22	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	MAD	4.28	0.87	1.15	0.64	0.22	0.45	
Mean         369.94         0.42         2.53         0.08         0.19         0.26           SD         3.72         0.71         2.05         0.36         0.54         0.43				TC = 1.5				
SD         3.72         0.71         2.05         0.36         0.54         0.43	Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate	
	Mean	369.94	0.42	2.53	0.08	0.19	0.26	
Median 371 0 245 0 0 0	SD	3.72	0.71	2.05	0.36	0.54	0.43	
	Median	371	0	2.45	0	0	0	
MAD 2.74 0.53 1.57 0.15 0.34 0.32	MAD	2.74	0.53	1.57	0.15	0.34	0.32	

Table 23: Selected statistics for the Monte Carlo simulation of the Race track Mixed model,

	TC = 1.01							
Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate		
Mean	346.21	5.60	2.20	1.57	0.21	1.14		
SD	13.82	3.04	2.51	1.57	0.57	0.62		
Median	348	5.17	2.26	1.58	0	0.99		
MAD	10.92	2.40	1.92	1.17	0.36	0.48		
			TC=1.1					
Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate		
Mean	366.33	1.25	2.42	0.55	0.14	0.69		
SD	6.54	1.37	1.59	0.93	0.46	0.67		
Median	368	0.82	2.46	0	0	0.55		
MAD	4.99	1.04	1.21	0.78	0.25	0.54		
			TC = 1.5					
Statistics	Production	Unemployment rate	Price Inflation rate	Exit rate	Relocation rate	Migration rate		
Mean	370.02	0.45	2.49	0.10	0.19	0.26		
SD	3.80	0.73	2.02	0.40	0.55	0.43		
Median	371	0	2.42	0	0	0		
MAD	2.83	0.54	1.57	0.19	0.34	0.32		

Table 24: Selected statistics for the Monte Carlo simulation of the Grid Mixed model,

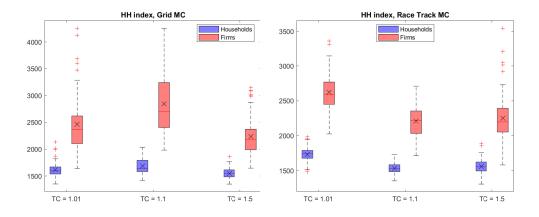


Figure 25: Box plots for the HH index for both agents. Left graph: results over 100 simulations run on a Grid map. Right graph: results over 100 simulations run on a Race Track map.

# 15 Concluding remarks

In this work I proposed a simple macroeconomic model endowed with Economic Geography features, including trade, transportation costs and a flexible spatial structure. The bounded rationality of the agents called for the development of specific exporting, relocation and migration protocols. This model is then employed to study different experiments to evaluate the emergence of particular patterns of agglomeration and the concurrent economic series under different relocation metrics, spatial structures, and levels of transportation costs.

The simulations exercises evidenced that this type of model structure (endowed with a simple consumption and labor market and a government) performs best under agglomeration, as unemployment is reduced and a positive wage and price inflation causes a nominal growth in the economic variables.

Similarly to the main optimized New Economic Geography class of models, agglomerated economies are quite easy to obtain, and robust to a series of changes in the relevant parameters. These agglomerations seems however to depend more on the metrics chosen for the relevant relocation and migration protocols than the parameters that are fundamental for NEG models, like the transportation costs level. This is a consequence of the behavioral rules of agents, that not being fully able to optimize their objectives, are not able to simultaneously locate in an equilibrium optimal setting. Following this, the economy might generate stable configurations, but they are based on random locations, notwithstanding the particular features of the employed spatial networks. Nonetheless the spatial network of choice is not ineffective in the determination of the final results, as it affects both the economic series and both the final distribution of agents. The model can indeed generate different spatial patterns, including migration cycles, return migration, and agglomeration shadows.

The tested models evidenced many interesting features. To begin with, relocation and migration metrics might cause oversensitive relocation behavior, in a way similar to the cumulative causation argument of NEG models: The benchmark AA model for instance agglomerates totally because once a threshold of consumption is reached, the Firms will totally stops the relocating behavior, and the other agent does not deviate. This call for the need to properly consider Dispersion forces to counterbalance this fact.

If the metrics tested have both a Dispersion nature, agents moves for the whole simulation, but in this model this distorts the economy performance. Movement is then a distortion, by lowering production due to firing of Households, which amplify this effect lowering consumption. High movement might not coincide therefore with a factor equalization, and both agents moving distorts the economy more than compared with the case of only a single mover. Comparing metrics that differs in their agglomeration or dispersion nature instead provides oscillatory behavior in the agents movement. If the agent with the agglomeration metric secures one Location, waves of escape and return movements will happen. These results depends on whether the main agglomeration is stable or shifting. Testing a mixed metrics model that considers metrics of both natures for each agent allows for multiple agglomerations, and both agents moving with less distorting effects. Tested on different spatial maps (inspired by the ones usually tested in NEG works), mixed metrics seems to reduces concentration, which is clearly a consequence of including a dispersion force in the model. Given the results for Monte Carlo simulation on a grid structure, the mixed metrics case seems more sensitive to local advantages of particular Locations benefiting from properties brought by the spatial network own statics, such as the node centrality.

It is moreover possible to scale up or down the relocation and migration rates of the different models by using controlling parameters, or limiting the number of relocation agents. The effects of particular metrics might be dampened, which could have some consequence in empirical works.

Lower TC parameter values, that scales the level of Transportation costs based on Location distances lower production and rise unemployment ceteris paribus. The effect seems to stem from the interaction of higher markups with a binding demand, that makes Firm agent able to expand trade also for the higher value of TC. The expanded production lowers unemployment differentials, slowing down Households migration, and bringing by less concentrated economies. Initial prices markups indeed increase production in a simulation, as proved in a robustness check. Stressing the TC value by rising fees and not calibrating it (such as the difference between domestic and exporting prices is higher) seems to bring results more in line with the NEG literature. Therefore it is possible to obtain the usual NEG result of having dispersion for increased trade barriers, but the mechanism follows another logic, with possibly different policy implications. Moreover, these shift in agglomerations seems far more gradual per TC value than compared with the knife-edge catastrophic nature of NEG model. This can be seen as a consequence of employing bounded rationality.

Relocation and migration costs helps in making agglomeration local when the number of Locations is increased. Indeed, even highly agglomerating relocation metrics end up in producing local agglomerations on a more expanded map. These agglomerations seems randomly distributed, and does not follow the neat geometrical shape of multi-regional NEG models. Bounded rationality therefore seems to cancel the "endogenous symmetry" of NEG models, also in completely symmetric spatial structures such as the Race Track economy.

In this sense policy recommendation based on trade interventions might not be as robust if we consider them to be based on Economic Geography insights, as, confirming the view of (Fowler, 2010), minimal deviations from the reference models ends up in making the economic model to behave differently than the clear-cut bifurcation maps.

These deviations are here far from being unrealistic and greatly enhances the analysis by Fowler, as the Agent-Based setting allows to include realistic instances such as involuntary unemployment and inventories, firm bankruptcies, relocation costs, and a fiscal sector. In an age of declining transportation costs (Hummels, 2007) and multiple firm-specific factors driving Firms relocating be-

havior, the flexibility of this model might perhaps prove as a valuable asset for studying these kind of situations. Recent developments in multi-regional NEG and quantitative spatial economic modeling will probably maintain the forefront of spatial economic research, but an approach such as the one proposed here might act as an alternative tool when competing explanations driven by agents heterogeneity and bounded rationality (for instance, the choice of a relocation metric) seems to work better in explaining the spatial behavior of the economy. Conversely, this approach could work as a suitable way to expand far more complicated Agent-based models to include a parsimonious spatial modeling and obtain therefore a more realistic economy.

Of course this model is still subject to many shortcomings. In particular much of the economic complexity is traded to allow the spatial element, but this prevents the model to match a validated macroeconomic behavior and the study of a series of issues that might natural candidates in this framework, like the behavior of multinational companies or regional specialization.

The validation part is also lacking, given the need for complete Monte Carlo analyses in all of the exercises, as well as a sensitivity test for the initial conditions. The results on transportation Costs and the different spatial structures could be made more precise by performing a more systematic parametric sweeps for different TC values and spatial networks. Spatial networks might moreover be selected more for some particular statistics features. For instance an interesting experiment could be made by comparing fully connected graphs, and gradually reducing the number of connected edges, as standard in network analysis theory.

After these issues are set, the future research effort will follow the direction to expand the model's characterization to make it more comparable to other macroeconomics agent-based models, by adding an additional sector for a goods capital and a credit sector. Relocation and migration protocols might moreover be endogeneized to bind for particular conditions, like for instance being triggered only in case of unemployment by the Household agent, and not by direct comparison of metrics. To conclude, it would be interesting to add some sort of congestion costs and negative externalities for Locations agglomeration to see if the model is able to reproduce the usual hump-shaped pattern of agglomeration given transportation costs values (Lafourcade and Thisse, 2011).

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A Appendix - Figures list

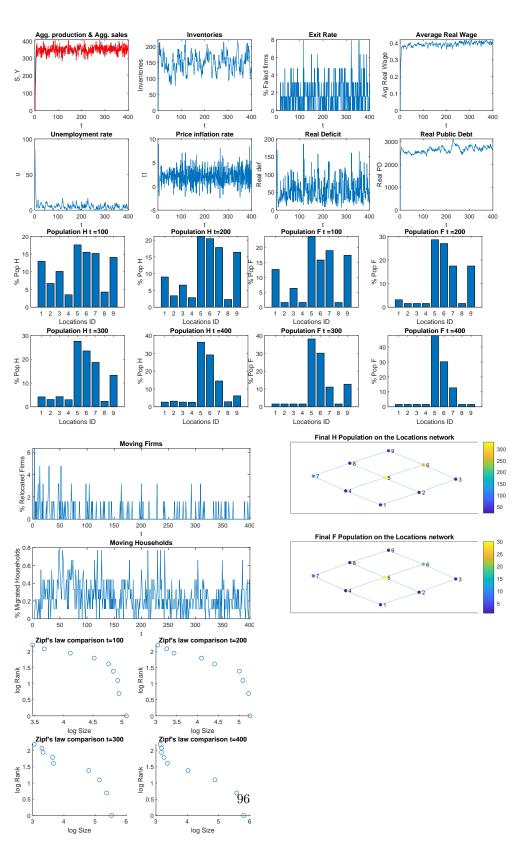


Figure 26: AA model, Grid - TC = 1.01

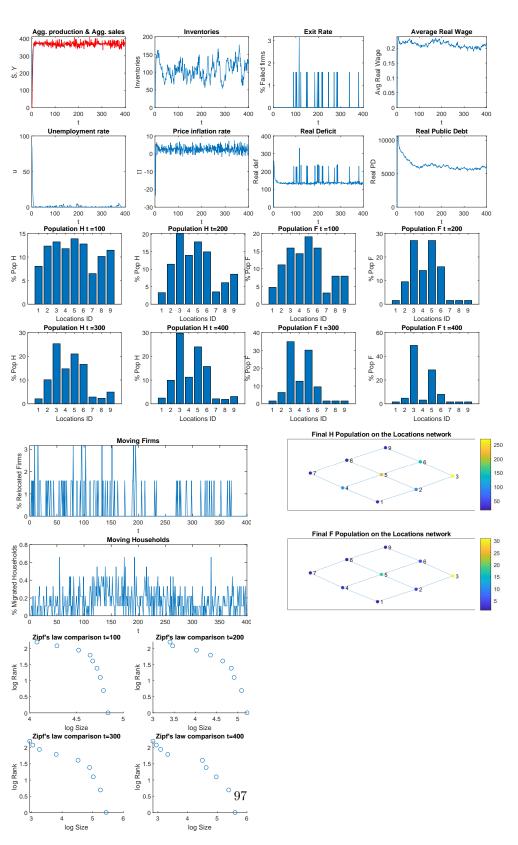


Figure 27: AA model, Grid - TC = 1.5

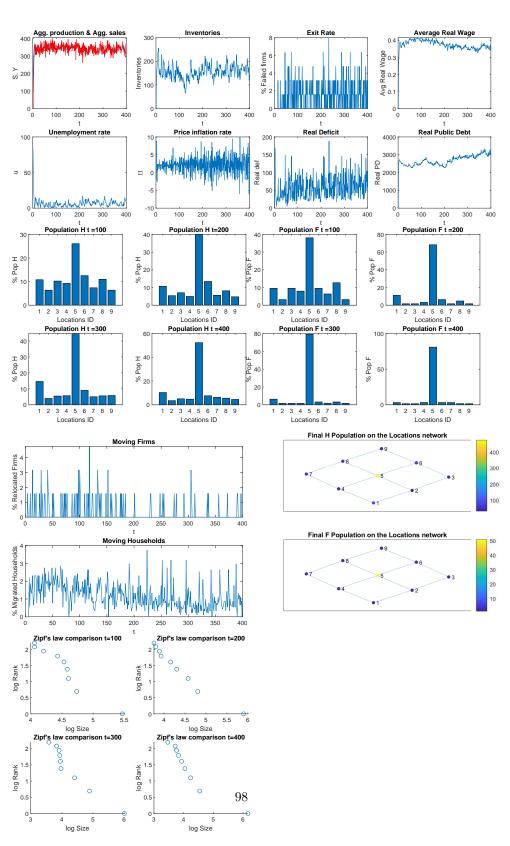


Figure 28: Mixed model, Grid - TC = 1.01

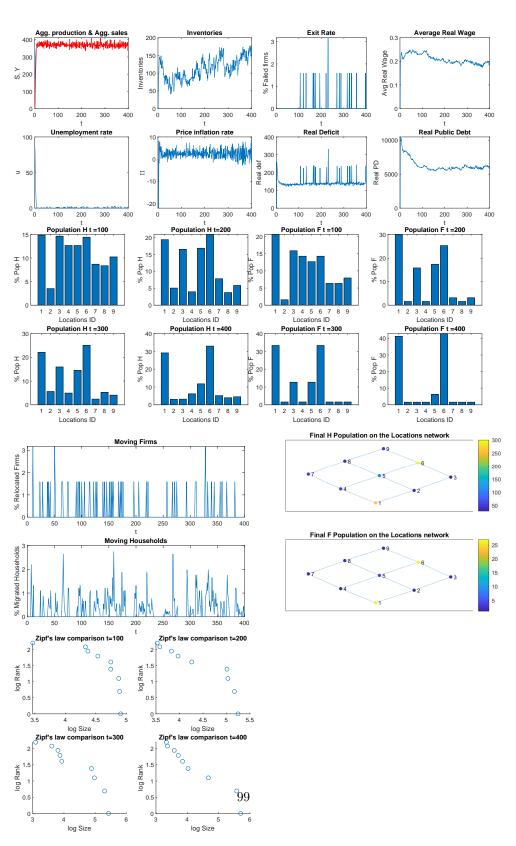


Figure 29: Mixed model, Grid - TC = 1.5

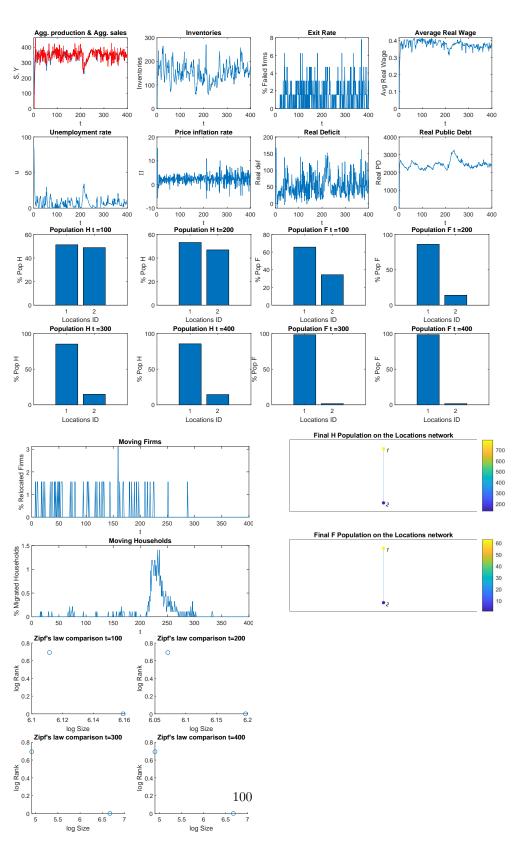


Figure 30: AA model, Core Periphery - TC = 1.01

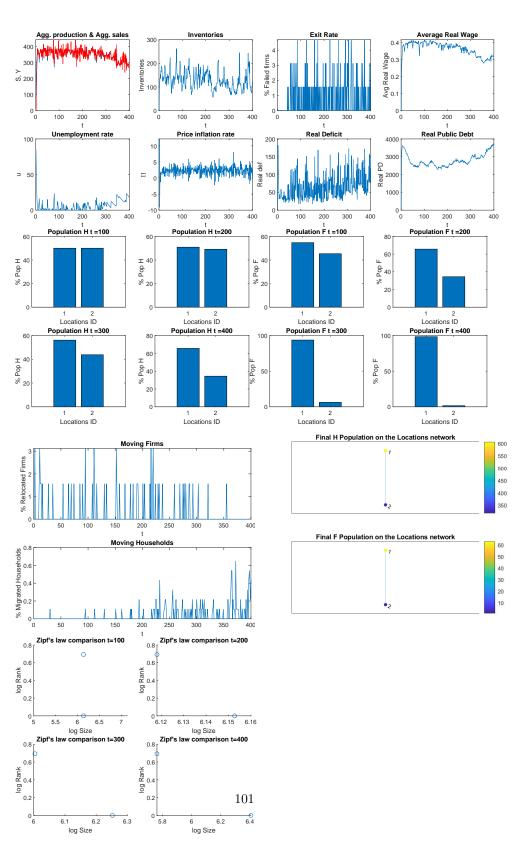


Figure 31: AA model, Core Periphery - TC = 1.1

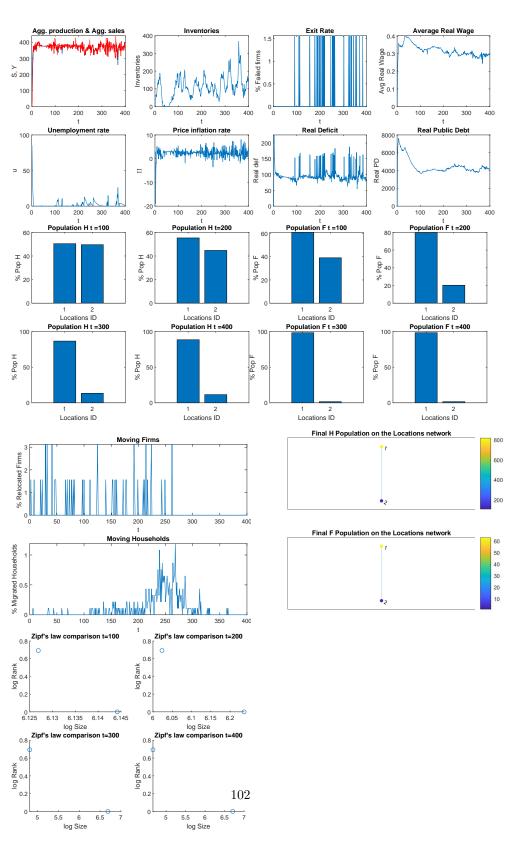


Figure 32: AA model, Core Periphery - TC = 1.5

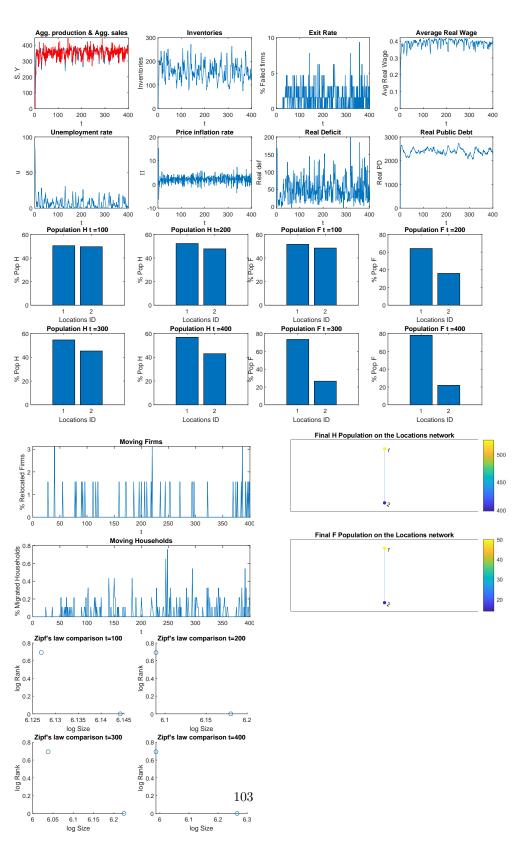


Figure 33: Mixed model, Core Periphery - TC = 1.01

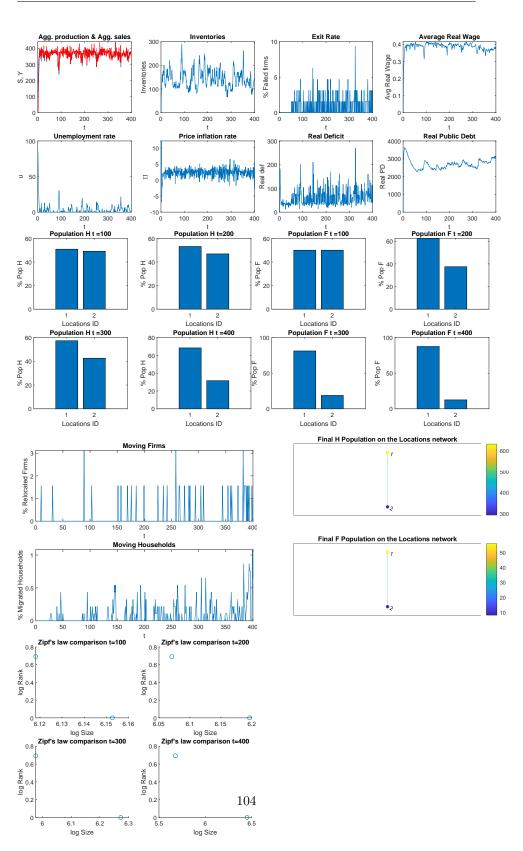


Figure 34: Mixed model, Core Periphery - TC = 1.1

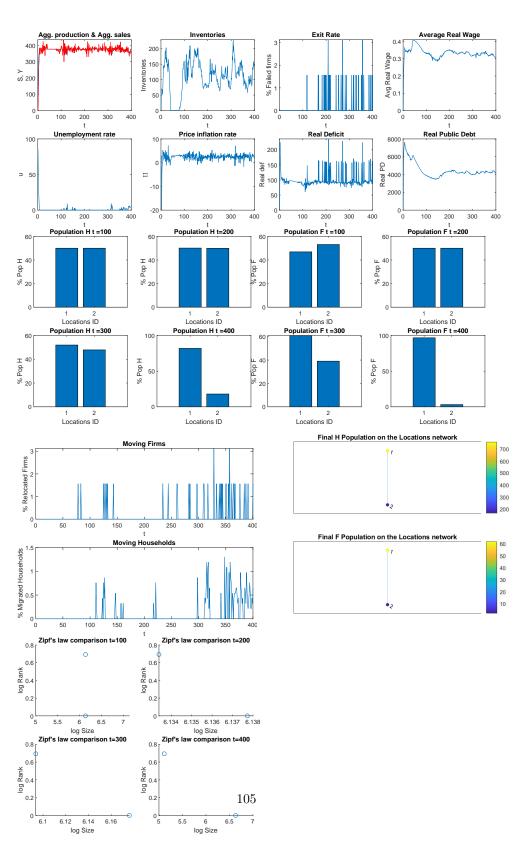


Figure 35: Mixed model, Core Periphery - TC = 1.5

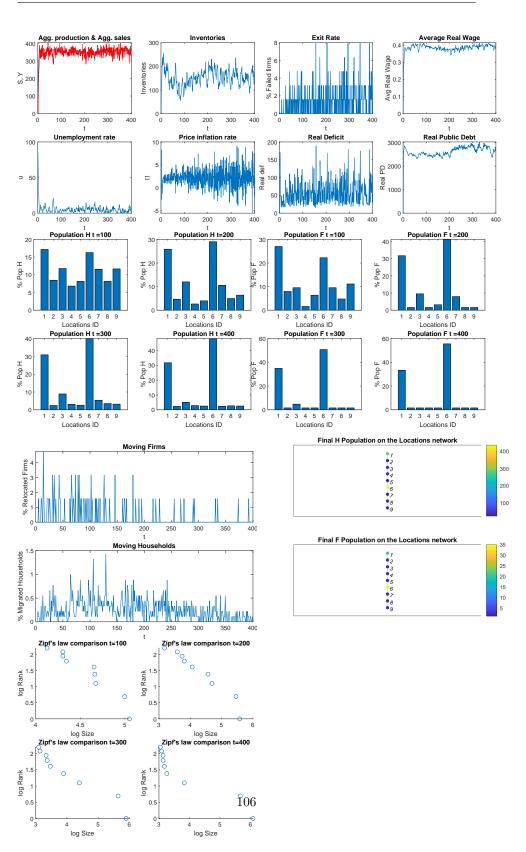


Figure 36: AA model, Long Narrow Economy - TC = 1.01

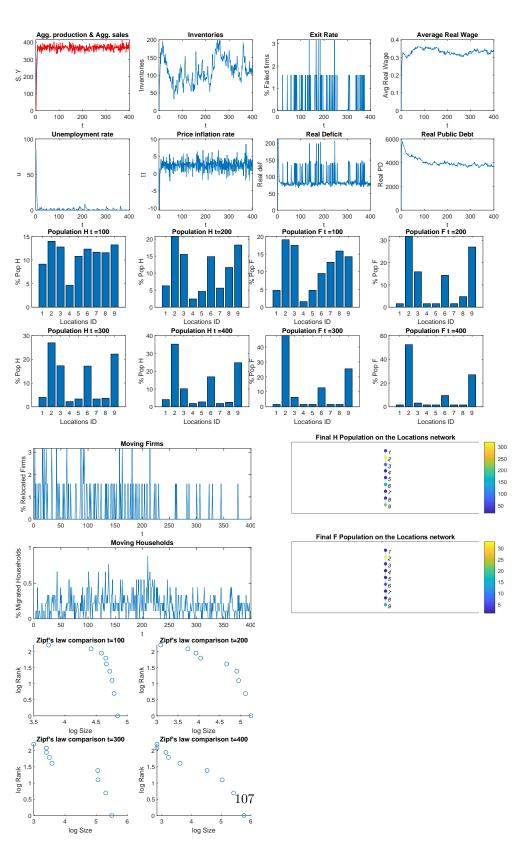


Figure 37: AA model, Long Narrow Economy - TC = 1.1

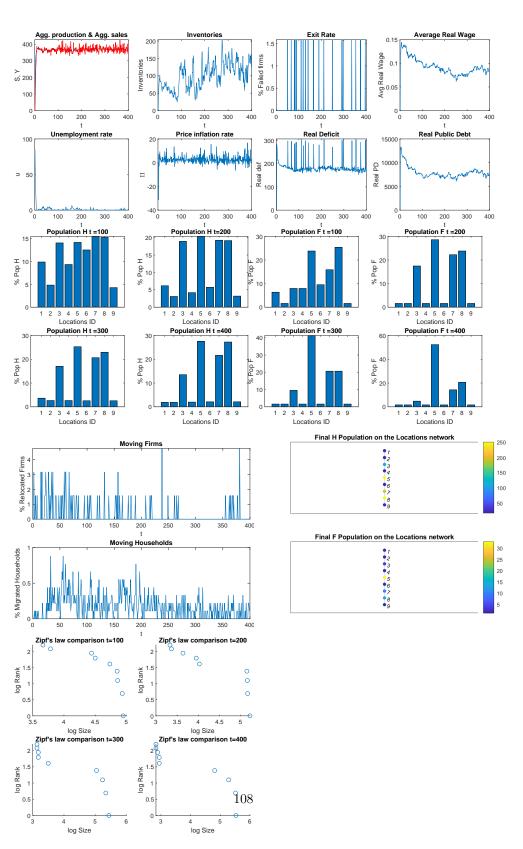


Figure 38: AA model, Long Narrow Economy - TC = 1.5

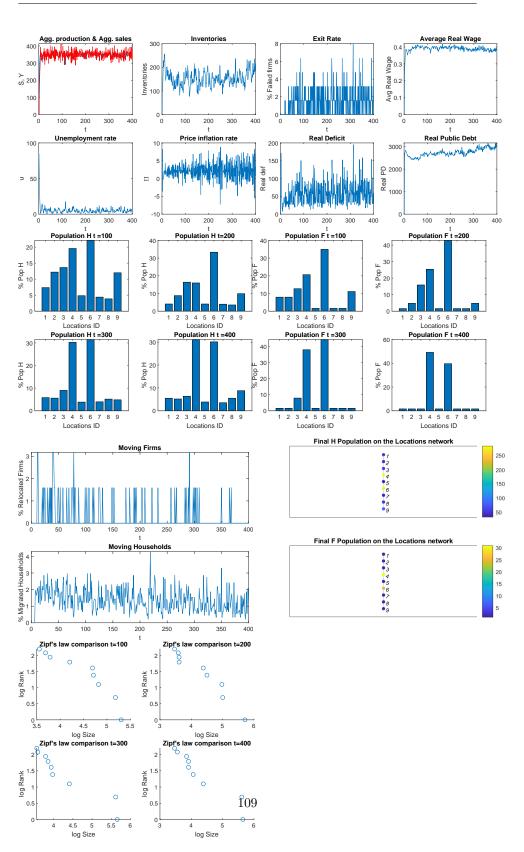


Figure 39: Mixed model, Long Narrow Economy - TC = 1.01

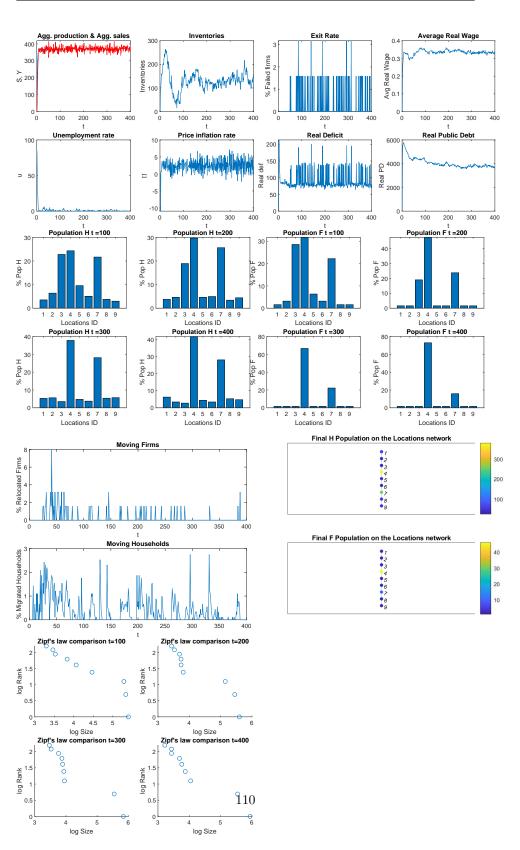


Figure 40: Mixed model, Long Narrow Economy - TC = 1.1

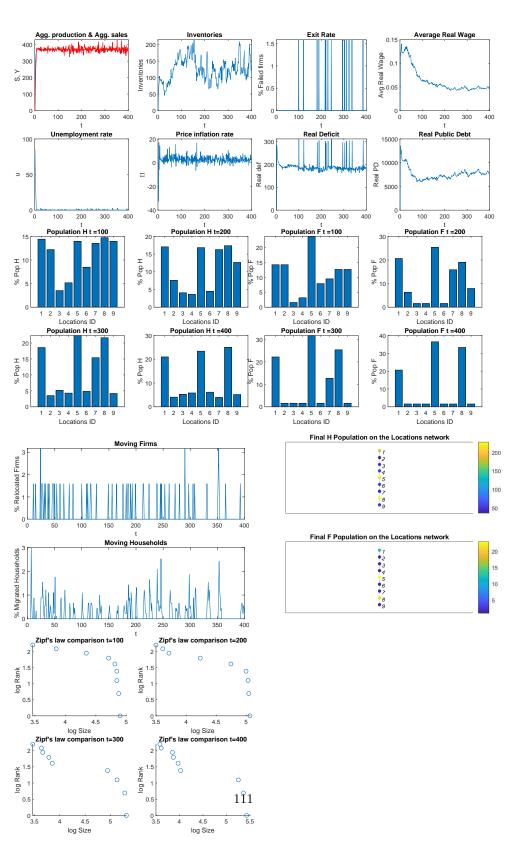


Figure 41: Mixed model, Long Narrow Economy -  $\mathrm{TC}=1.5$ 

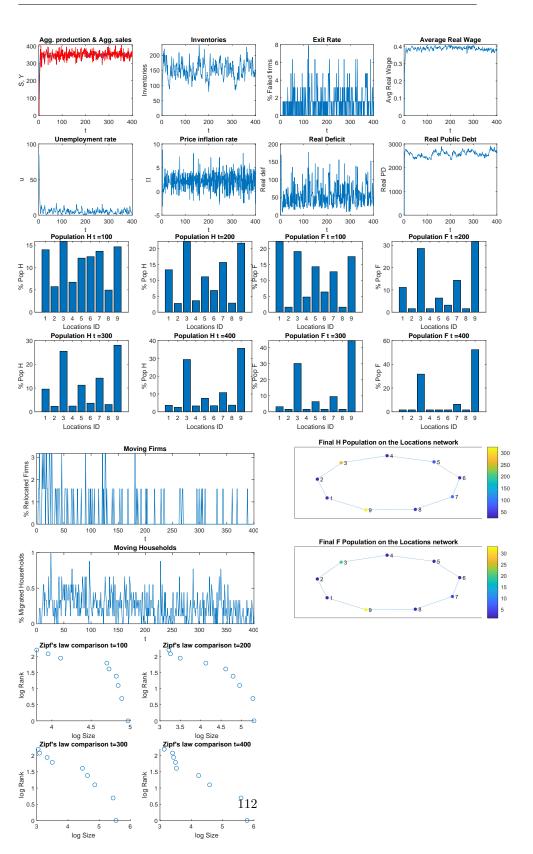


Figure 42: AA model, Race Track Economy - TC = 1.01

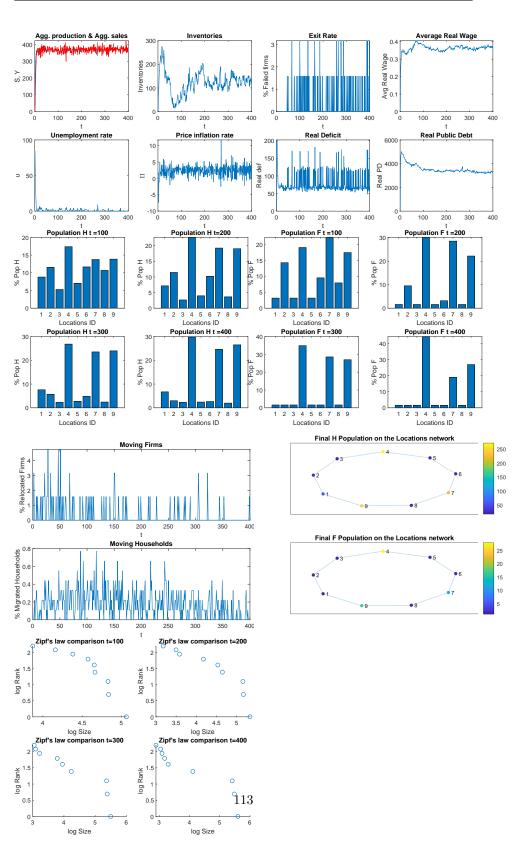


Figure 43: AA model, Race Track Economy - TC = 1.1

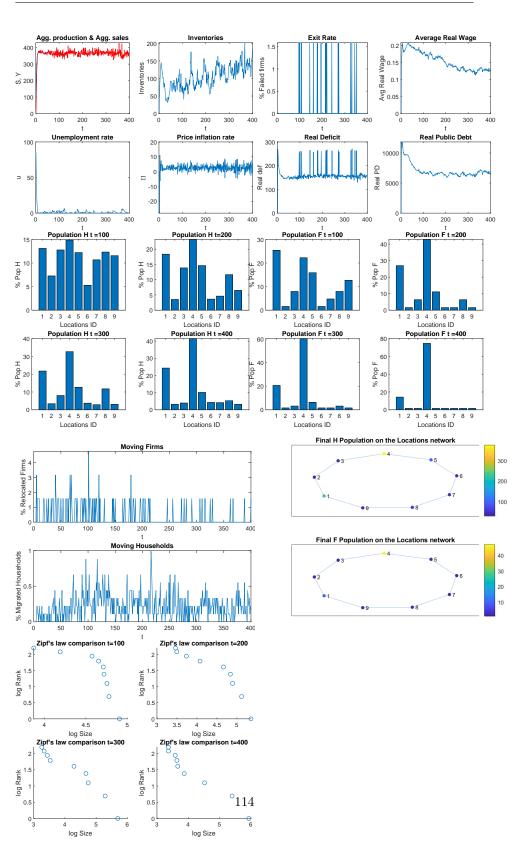


Figure 44: AA model, Race Track Economy - TC = 1.5

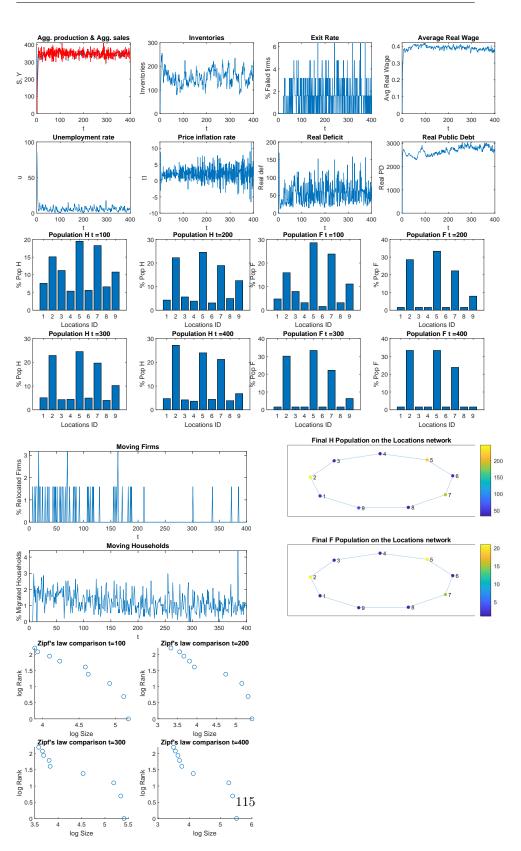


Figure 45: Mixed model, Race Track Economy - TC = 1.01

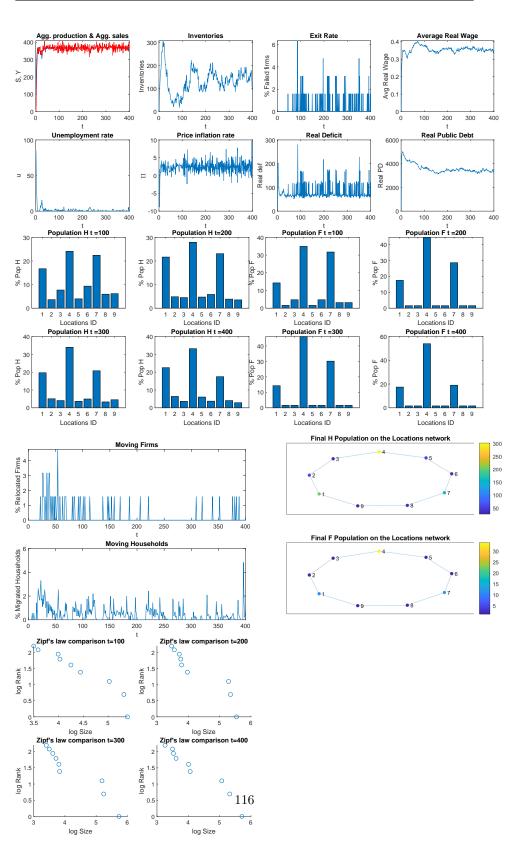


Figure 46: Mixed model, Race Track Economy - TC = 1.1

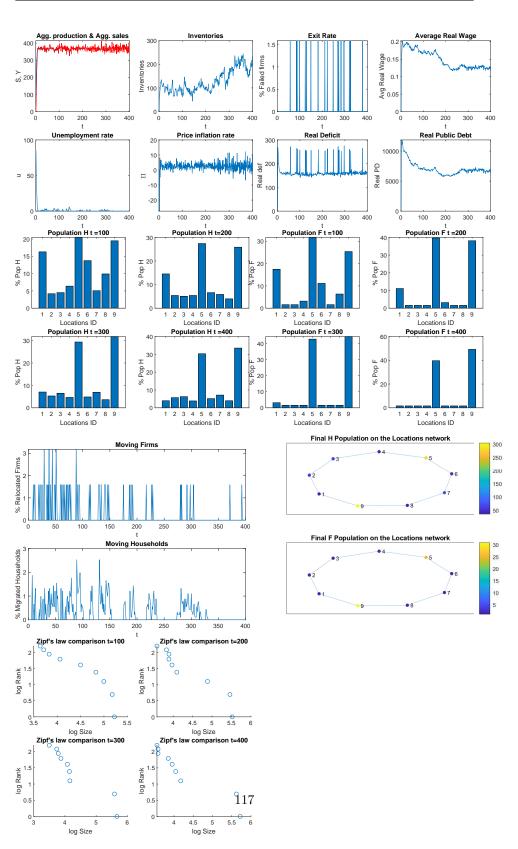


Figure 47: Mixed model, Race Track Economy - TC = 1.5

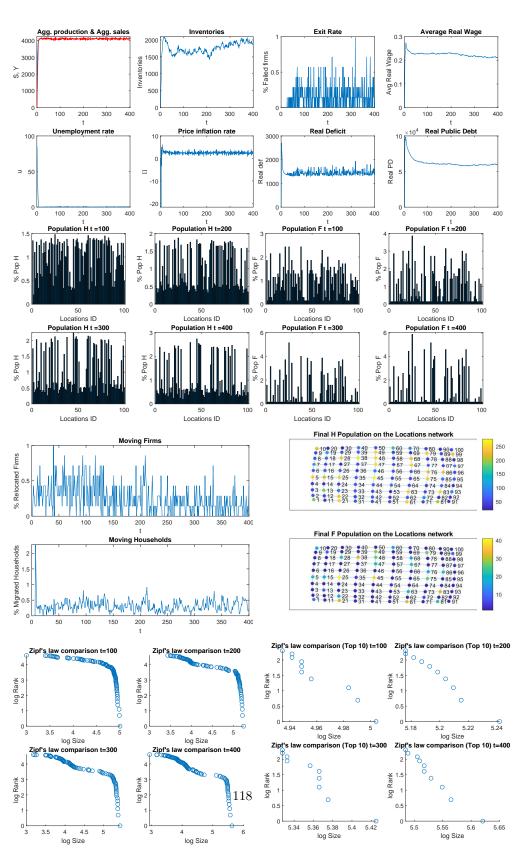


Figure 48: Main series with 100 L - Mixed model, Grid

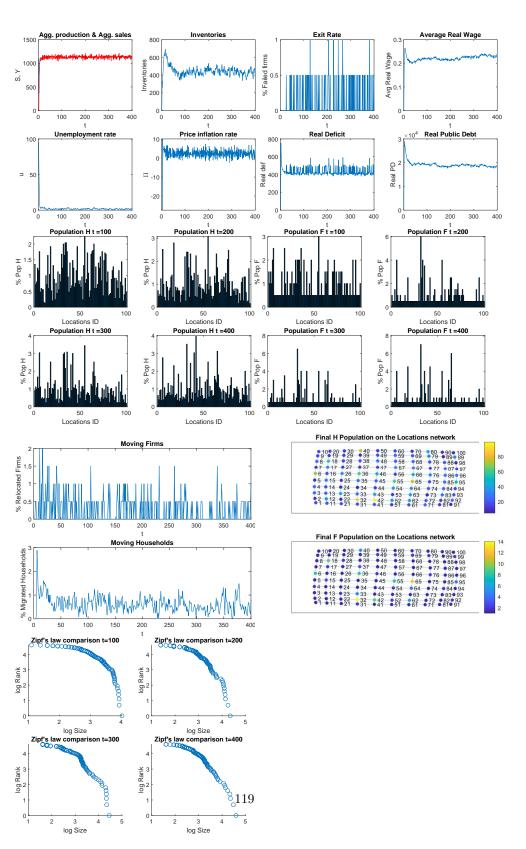


Figure 49: Main series with 100 L - Mixed model, Grid, Low Density

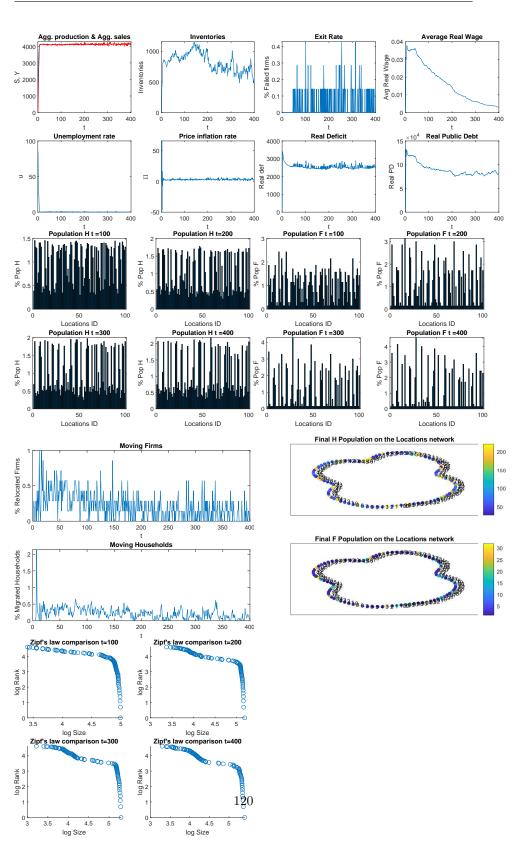


Figure 50: Main series with 100 L - Mixed model, Race Track

# Policy with multiple regions, an Agent-based Economic Geography analysis

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Supervisor: Domenico Delli Gatti

#### DEFAP

#### Abstract

This work wants to enrich the Economic Geography literature by investigating the case of local policy applied to heterogeneous agents operating under bounded rationality in complex spatial structures. I employ a flexible macroeconomic Agent-based model based on the decentralized matching protocol mechanism employed in models such as Riccetti et al. (2015) and Caiani et al. (2016a), developed in the previous Chapter. The agents' decision heuristics embed and reproduce the features usually employed in mainstream Economic Geography models, such as relocation, migration, trade and transportation costs Thisse (2011). This permits to check the emergence and stability of agglomerated configurations by the economic agents, and the spatial structure consists in a network of Locations which is set in the initial calibration of the model. This model is extended for the analysis of Local policy. In the proposed extensions, Locations are treated as "active" agents and can set their local tax collection and revenues usage according to adaptive heuristics. Moreover, I test the same measures on an asymmetric and realistically calibrated spatial structure. I test the most relevant local policy measures under different scenarios of tax competition, public spending and asymmetric shocks (Commendatore et al., 2018), which have been proven difficult to be compared in structures with multiple regions, bounded rationality and a dynamic setting. Results shows that the choice of the spatial map matters in determining the policy outcome, which often differs from the intended objectives of the Location Agents.

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# 1 Introduction

The aim of this paper is to investigate local policy results on a complex spatial setting. More specifically, I will propose an analysis on local public intervention and its effect on agglomeration and welfare in the context of a spatial agent-based model with bounded rationality and multiple locations. The aim is to extend the results of the existing theoretical literature in a framework endowed with multiple locations, multiple mobile mobile agents, and multiple layer of governance. The ongoing tensions between local, national and international level of governance, alongside an ongoing process of economic concentration in particular cities and areas could increase the demand for related research in the near future.

A particular feature of these results is that they are prone to be studied in a spatial framework. As a matter of fact, works in Economic Geography showed how adding spatial considerations to policy analysis could distort the predictions by traditional a-spatial models (Baldwin and Krugman, 2004). However, most of the models in the Economic geography tradition usually focus on one single

policy dimension at the time, failing to recognize a spatial economic environment as a complex and interconnected adaptive system. This means that in the majority of works, local policy is reduced to two or three regions models, with perfectly rational agents, engaging in static one-shot games and optimizations, in the presence of only one mobile factor. Nonetheless, these models provide indeed priceless theoretical results and insights: the objective of this work is see how these results fare in a more complex landscape.

Following Masahisa Fujita (one of the leading figures in contemporary Economic Geography) representation of economics knowledge creation (Berliant and Mori, 2017), we can consider this work then as an additional mapping cycle of Economic Geography models from a different set of assumptions. These assumptions are bounded rationality of agents, mobile agents, multiple locations, and multiple levels of government. This particular mapping is tractable trough the techniques of Agent-based modelling (Gilbert, 2008), which I adopt as a framework. Selecting two specific policy dimensions, namely local public spending and tax competition, I propose a series of experiments conducted in the Agent-based Economic Geography model (ABEG) developed in the first chapter of my Ph.D: dissertation Rizzati (2018).

Local policy dimensions can be examined in the ABEG framework by extending the Locations Agent. In addition to accommodate the economic interaction between Households and Firms, mediated trough their spatial network, Locations can now actively influence the economy by setting local taxation rates and deciding how to spend their revenues. The basic benchmark model of this extension boils down to the original source configuration, as the Locations revenues are fully aggregated and sent to the Central Government. However, changing the heuristic on the tax revision and on the use of these revenues can bring the model to different scenarios. These scenarios are stylized as the most relevant policy issues faced in New Economic Geography, as exemplified in the review by Commendatore et al. (2018); moreover, it can encompass also different situations emerging from different strands of literature. For instance, when considering tax competition between Locations, it is possible to replicate different economic behaviors, such as the tax "Race to the bottom" feared in the tax competition literature, compared to the "Race to the top" proposed in NEG models. Indeed, I also test a "Capture" scenario inspired by results grounded on political economy hypothesis about lobbying (Brulhart et al., 2012). Public spending policies are instead divided between a subsidy, affecting Firms costs, and public procurement, where the Location agents employ their revenues to buy local goods to support local demand.

All these different scenarios are tested on the same spatial structure, with the same calibration of the model's parameters. The different scenarios are set to affect the model after 100 periods in the simulation, in order to bind after that the transition phase of the dynamic system takes place. To test for these policies on an even more realistic situation, I propose moreover the same exercise on an asymmetric spatial structure, where the population of agents and the distances between Locations are modelled after real life data of a specific Geographical

system. Without presumption of generality  $^1$  I select Lombardy (Italy) as the reference system of the asymmetric model.

To expand the analysis to asymmetric policy interventions I propose moreover two additional exercises. The first assumes an asymmetric shock hitting one Location of the economy, while the second postulates the arrival of an exogenous migration boom. The exercises studies then the difference between a Local and Global answer to these events, the Local answer being based on the Previously cited Public spending structure, this time limited to the hit Locations.

I find that the spatial structure of the economy matters on the final outcomes of the different policies, as it is tightly linked with the relocation and migration protocols behavior. This might differentiate even policies with very similar economic consequences and produces effects that does not match the underlying theoretical results for the given policy rules, often defeating their original purposes. Finally, policy rues that does not affect the economy substantially might become much more effective just by considering a different agglomeration configuration in the economy.

## 2 Motivation

In this section I will present the main insights emerged from policy analysis in Economic Geography. I will present then the specific policies under analysis and some empirical evidence.

#### 2.1 Policy analysis in Economic Geography

Policy is one of the main debate points in Economic Geography modelling (Redding, 2010). As a matter of fact, considering economic systems to be shaped by the forces proposed in Economic Geography, makes them inherently posited to policy interventions (Commendatore et al., 2018). This happens because the mechanisms of second nature geography (for instance, the attraction of a location by a Home market effect) are built on inefficiencies such as monopolistic price setting and pecuniary externalities. Moreover, adding a spatial dimension to an economic model could lead to different, and perhaps unexpected outcomes of various policy interventions, as showed by (Baldwin and Krugman, 2004). For instance, while it is recognized that agglomerations are hubs of productivity, the question if these are the most welfare enhancing configuration of a spatial economy is not clear, leaving space for distributive policies between locations. Optimal policies on paper could also affect the mobile agents distribution, leading to sub-optimal outcomes.

The main channel through which this debate develops regards the incentives of mobile agents to keep their current position or to relocate. It is then not surprising that the main policies examined in these kind of models are the ones

 $<sup>^1\</sup>mathrm{but}$  nonetheless with a tidbit of parochialism, being it my home region

that mostly affects the trade-offs between agglomeration and dispersion: for instance Commendatore et al. (2018) identify trade freeness, tax competition and public spending as the most relevant policies typologies. Before discussing them, I will evidence the need for a policy tool able to analyze economies with multiple regions, heterogeneous agents and bounded rationality.

Fowler (2007) provides an early attempt at looking at the behavior of Economic Geography models in the lack of perfect rationality. The main motivation is that neoclassical models are endowed with perfectly rational agents, generating equilibrium configurations that depend on the values of relevant parameters. Given that most Economic geography models are bifurcation systems with multiple equilibria, there is concern for the presence of catastrophic "knife-edge" bifurcations. These are indeed quite unrealistic if we look at the persistence of agglomerations in real life (see Davis and Weinstein (2002); Redding (2010)). It must be said that some attempts have been successful in representing transitions that were not catastrophic, such as (Puga, 1999), where transitions are given by the interactions of sectors with different returns to scale or the use of different labor pools by firms. Nonetheless, these exercises are not focusing on the way with which the agents optimize. This could be leaving little space then to factors such as involuntary unemployment to play a role in shaping equilibrium. To investigate this, (Fowler, 2010) refine his previous analysis by proposing an agent-based model based on the basic Core-Periphery structure, but with bounded rationality. His baseline model is able to replicate the results of the Core-Periphery model (Krugman, 1991) about agglomeration depending on transportation costs, but even small modifications in the initial conditions are able to offset the obtained solution. In his case, this regards the initial distribution of agents, and the presence of heterogeneity in the workers' reservation wages, which are overall realistic assumptions.

Major factors in real life prevents then a direct applications of the insights of Economic Geography models. The asymmetry in real life spatial structures is a given fact, often due to already existing agglomerations, that can in turn be caused by first nature geography factors. Firms and workers are moreover greatly heterogeneous. The literature is of course aware of these limitations, that fostered works to comply in that direction: one example is the research agenda behind Heterogeneous firms in Economic Geography, the so called "New New economic geography" (Ottaviano, 2010), named so to draw a parallel with the research by Melitz in trade theory.

Even notwithstanding these limitations, a policy maker willing to incorporate insights from Economic Geography models would face additional difficulties: for instance understanding how the spatial structure, like the number of regions, could alter the final results, and how to disentangle all these effect from the empirical data at disposal.

But even more fundamental answers could be outside the scope of the usual models: Akamatsu et al. (2017) shows that given the mathematical properties of bifurcating systems, it is possible to classify dispersion forces as local and global, and that the final effects on a particular model agglomeration is the final sum

of these forces. This means that the final configuration of the economy resides in the convolution of parameters shaping these forces, but also that the policy maker might take a completely wrong move by considering her policy related to a type of agglomeration force that is not the one fostered by the current economy. Moreover, which type of agent is mobile (firms, workers or both), and frictions to the agents movement (relocation costs or migration quotas), and again, the number and type of connections between locations might all provides different outcomes. In this sense the flexibility of the model proposed in this chapter would allow the policy maker to draw a battery of simulations in order to encompass the various typologies that would not be so readily available in a custom Economic Geography model.

Indeed in the last decade many multi-country Agent-based models have been developed to answer policy challenges into a spatial context. The Eurace@Unibi model for instance, has been used to study different topics in the field of regional growth, and generated a stream of policy applications (Dawid et al., 2014a).

Dawid et al. (2008) studies a skill-enhancing policy on a 2 region economy with immobile labor. The configuration of the policy, distributed or agglomerated on only one region affects the growth of economy, respectively in the long and in the short run. (Dawid et al., 2009)studies the same policy experiment, but allowing for friction-less or costly movement of the working force. In the case of costly movements the concentrated policy case shows the higher economy output. Dawid et al. (2013) investigate a scenario inspired by the integration of EU labor markets, and propose a model endowed with two asymmetric regions, one with workers with higher skills and more firms. The analysis of different labor markets separation degrees shows that local preferences for open or closed labor market might change both trough which regions is being considered, and also on the time horizon. Dawid et al. (2014b) studies a scenario inspired by the Eu cohesion policy, where human capital and the technology frontier policy are targeted to the less developed region in an asymmetric two region model. The human capital policy is ineffective in fostering the less developed region when labor markets are integrated, as high skill workers migrate towards the high-tech one. The technology policy instead is always fostering convergence, but more efficiently with frictionless labor markets.

The Eurace open model Petrovic et al. (2017) study a structure with a monetary union and two control countries, to test whether joining a monetary union is convenient, and which are the consequences of migration flows. Another stream of models for Multi-Country analysis is the one based on the stock-fow consistent model of Caiani et al. (2016a), which include works such as Caiani et al. (2017), that can test policies on an arbitrary number of countries.

The advantage of using the ABEG model is to present a simplified context with respect to the previous works, as this model trades additional sectors and agents for a basic structure with just two types of mobile agents, and a simplified Public Sector. The model includes nonetheless simultaneous relocation behavior by both agents, localized and heterogeneous trade flows, and can span different spatial structures according to the selected relocation protocol structure. It is therefore a parsimonious solution to study a problem of multi-regional local policy.

# 2.2 Specific policies: Public spending and tax competition among regions

As highlighted before, the main policy dimensions of Economic Geography involve the parameters affecting the incentives to relocate of the mobile agents of the economy. The classic example is the trade costs parameter, which could also be seen as the inverse of trade freeness between locations. In NEG models this parameter usually determines the equilibrium spatial configuration, as it causes Firms to relocate. Moreover, most of their peculiar policy properties are due exactly because of the characteristics of the sustain and break points of the underlying bifurcation systems, as the presence of hysteresis (Ottaviano, 2003). As I already provided a sensitivity analysis based on transportation costs in the previous chapter, in this work I aim to analyze different policies dimensions, namely public spending and tax competition between locations, which are deemed as the other most relevant ones (Commendatore et al., 2018). These topics are wide enough to have generated a literature of their own, and are still relevant in the current policy debate. For instance, in the face of a slower growth of living standards, local governments have been subjected to pressures to stimulate business opportunities (Bartik, 2017), while the debate regarding tax harmonization as an instrument to mitigate tax competition fears continues on (Karmakar and Martinez-Vazquez, 2014).

An interesting way to frame these policies is to consider them as "attractive tools" by local governments. For instance, public spending by a local government could result in an additional demand for goods, which improves the local home market effect. Indeed, for a location unit, like a region, being an agglomeration is considered, at least in theory, a desired outcome. Agglomerations promote productivity, and entail a whole basket of positive factors which are at least enough to counterbalance congestion costs and other dispersion forces. Many of these factors are conceivably realistic policy objectives, like for instance, employment and higher wage levels. This of course should lead local governments to attract Firms to their own location, trough a variety of policy measures, which includes various form of tax benefits and the provision of local public spending. But, again, the location choice of Firms is crucial in the logic of Economic Geography models, as even few firms relocating can lead the system to reach a "tipping point", triggering a whole new spatial configuration with different agglomerations. An example of this mechanism is provided for instance by Andersson and Forslid (2003), which show that tax competition could be a very destabilizing force once considered on a symmetric dispersed equilibrium.

However, considering the effects stemming from agglomeration economies complicates the picture. Firms in agglomerated regions can rely on a series of additional advantages, labelled collectively as "agglomeration rent" (which can include for instance the access to a more skilled labor pool etc.) (Ludema and Wooton, 2000). One question that emerges then, once considering agglomerated economies, is that the local government of the most populated region could have the incentive to target this rent, reversing the logic of tax competition. Will local governments then engage in a "race to the bottom" in providing tax credits (or conversely a very high level of expenditures) or given the resilience of agglomerated configurations will they extract rents from the aforementioned "agglomeration rents", in a "race to the top"? This has consequences with regards to proposed policies such as tax harmonization. (Baldwin and Krugman, 2004) showed the existence of tax differentials between regions, while Borck and Pflüger (2006) extended this result in a setting with partial agglomeration, showing therefore that tax harmonization might not be welfare enhancing.

Indeed, Karmakar and Martinez-Vazquez (2014) detail the debate between the arguments favorable to tax harmonization issues against the risk of tax competition. The topic is very relevant, given also the call for major harmonization, especially in the European setting. The authors conclusion is that a series of results by the theoretical and empirical literature complicate the earlier visions on tax competition as completely adverse, and that much depends on the characterization of the tax game being played. It seems therefore that a definite conclusion on the effects of these policies has not been reached.

While tax competition models do not consider agglomeration economies, also Economic geography models are not immune from criticism. Gerritse (2014) critiques Economic Geography models as they usually avoid simultaneous policy making. By using sequential games, these models end up allowing a first mover advantage, usually to the already agglomerated region. Moreover, taxes revenues usually disappear from their model accounts, whether instead their uses, like financing public goods should be accounted for. By employing a Footloose Entrepreneur Vertical Linkages model the author tries to check if tax harmonization is the best outcome. They show that in contrast to Baldwin and Krugman (2004), it could be, as long that type of game which is played, is sequential or simultaneous.

Timing is then relevant for policy analysis, both in models with or without agglomeration effects. This issues is evidenced in Kato (2015). The author looks at a tax competition game in a 2-country setting with mobile firms. The game is dynamic, contrary to most Economic Geography models. In his analysis the focus is on the credibility of the chosen fiscal policy: a lack of commitment<sup>2</sup> by local governments is seen as a dispersion force, while commitment can foster agglomeration when trade costs are low. Interestingly, a "peripheral" position by the non-agglomerated country could be a better outcome than a dispersed one (holding half of the firm's population). This is because the fiscal expenditure to keep the Firms from relocating might more than offset the gains from the partial agglomeration. Overall, while commitment is then a strong instrument to foster stable outcomes, such a rigid policy could be vulnerable to sudden

 $<sup>^2\</sup>mathrm{An}$  example of fiscal commitment is Ireland keeping the 12.5% tax rate despite pressures from France and Germany to raise it.

events or to deviations by competitors. This would be indeed a motivation to check a scenario endowed with multiple locations and where agents could follow different strategies.

Empirical analysis on related topics do not offers much of a confirmation of either positions, but there is some evidence that local policy matters when explaining firms location, but also that rent extraction is not an implausible behavior. Of course, the difficulty stems from properly compartmentalize the relevant effects on the right spatial, administrative and policy units. Papke (1991) finds in the U.S. Establishment and Enterprise Micro-data File (USEEM) database for 1976-1982 that higher states taxes reduces the creation of new manufacturing firms for half of the industries considered. In more recent work focused on Swiss municipalities, Brulhart et al. (2012) finds that corporate taxes deters new firms' birth, but less so in more concentrated sectors. Becker et al. (2012) look at the consequences of profit taxation to MNEs investment decisions by looking at a database of 11.000 municipalities in Germany from 2001 to 2005. Higher tax rates affect negatively the location decision by MNEs, but it seems that the amount of tax credit to be given at the margin to attract one additional firms is too high to permit most of the municipalities to engage in tax competition. The interesting result is that municipalities with favorable environments for firms location (like having an abundant workforce), will have an higher return from a smaller cuts in tax rates. Freret and Maguain (2017) study a panel of local business taxes from 1995 to 2007 in France Départements to check the presence of rent taxing and slope sensitivity (agglomerated regions should react less to changes in policies by dispersed regions). They obtain a positive evidence of tax renting by attractive jurisdictions. Agglomerated regions are moreover less likely to copy other policies. Chen et al. (2017) looking at medium and large size manufacturing firms in China from 2002 to 2007 find also the evidence for higher taxation in agglomerated regions.

Finally, Bartik (2017) analyzes the New Panel Database on Business Incentives for Economic Development (covering 33 U.S. states). He finds that local business incentives are large, with an estimated value of 45 billion dollars in 2015. There is also a relevant variation by state, which seems to depend more on past practices than being tied to current local economic conditions. These incentives are not constant, having more than tripled since the 90's but being subject to cutbacks into some states more recently.

There are other issues when considering these policies: one is to consider that the agents might act trough multiple channels, some of which are unrelated to the effects of economic geography. In the paper by Brulhart and Simpson (2017), the authors looks at the evidence concerning the competing effects of the geographic "race to the top" and of a "government capture" by lobbying firms, more effective in agglomerated regions. In this sense the tax reduction would not be a tool to compete with other locations, but a consequence of more effective lobbying. This is evidenced for instance by Bohm et al. (2016), that finds an inverse relationship between firms size and concentration and local taxation. The other is that the central government intervention might offset the local ones, as its main objective could be directed towards less developed regions. This is more true given that regions might employ such a low level of equilibrium taxes as to offset the gains of being an economic core, which emerges as a result in the influential paper by (Ludema and Wooton, 2000). Moreover, current theoretical frameworks are somewhat limited in being able to analyze simultaneously the realistically proven fact that there are different types of policy intervention, even limiting the analysis to tax competition and public spending. Bartik (2017) shows that tax credits take many different forms: for instance, in his US analysis the most diffused ones are the JCTC (job creation tax credits) and property tax abatement. As detailed in Commendatore et al. (2018) there are differences in NEG models' results if considering different types of public spending: for instance buying local firms' goods is a boost to local demand, while the local effect of other interventions that raise firms' productivity depends on the transportation costs level. There are moreover differences in the way these interventions are financed: Dupont and Martin (2005) finds different effects on income inequality if subsidies to profits and production are financed locally or globally.

All of these factors concur to motivate the need for policy tools capable to provide additional insights to shed light on complex situations. The flexibility of the ABEG model permits to analyze all of these issues. The main contribution is that local governments are framed as agents, in the same fashion as Firms and Households; it is possible then to test scenarios where heterogeneity between the different location endowments and policy strategies could be fully represented in a dynamic setting, even including a central Government. Works such as Kato (2015) shows that the usual games representations of tax policies might be too limited in the case of dynamic models. His results rely on mixed strategies in the simultaneous game, but this might be affected by the number of regions and agents of the game. Allowing multiple dimensions of heterogeneity to arise simultaneously is also important, as they could better reflect realistic situations: as the prediction by standard models are however distorted (for instance by the "leakages" theorized by Fowler (2010)), flexible Agent-based models could provide policy insights (Dawid and Neugart, 2011) even for complex scenarios, and still be subjected to empirical validation (Fagiolo et al., 2017). Another fruitful feature is the stock flow consistency of the model, that addresses the critique by Gerritse (2014) over the disappearance of taxes revenues from the economy. To conclude, the structure of the Agent-based Economic Geography model inherit the macroeconomic fundamental structure of other MABMs, permitting to consider in the model the effects of realistic variables such as unemployment, inflation and involuntary inventories.

# 3 Model Extensions

I start my analysis from the Economic Geography Agent-Based model (ABEG), developed in the first chapter of my Ph.D. Thesis (Rizzati, 2018). This setting

permits to depict an economic system endowed with a spatial structure with an arbitrary number of regions and trade, relocation and migration processes. Agents, Firms and Households, are heterogeneous and subject to bounded rationality. For space reasons, I refer to the previous Chapter for the description of the relevant behavioral rules and matching protocols; will present here only the modifications introduced to perform the policy experiments.

#### 3.1 "Agentifying" Locations

The model includes a population of Households H, a population of Firms F and a spatial network of locations L and a Global Government. In the original ABEG model, a single location l constitutes the basic spatial unit for economic interaction. For instance, labor markets are local in the sense that only Households and Firms residing in the same Location could interact between each other. The feedback of locations L on the economic system is therefore indirect, the main one being the distance between the single l nodes.

Now, to study the aforementioned policy topics, the "local" level shall be able to actively intervene in the economic system. This is equivalent to fully "agentifying" the locations L. While this could also be achieved in similar fashion by imposing exogenous changes on the parameters values, the interesting feature of considering locations as agents is to link their responses to their own endogenous factors, e.g. the locations populations and other local economic variables.

This could be rationalized by considering a Local Government for each location, able to set fiscal policy variables with some degree of flexibility from the central Government. A relevant assumption of this analysis is therefore that the Locations will act as adaptive and heterogeneous agents.

To endow the Location agents with behavioral rules, I start from the fiscal income of the Central Government in the ABEG model, which was defined as:

$$T_t = T_{F,t} + T_{H,t} \tag{1}$$

where  $T_t$  was the collection of tax revenues collected from Firms profits and Households incomes according with the fixed tax rates  $\tau_{f,t}, \tau_{h,t}$ .

The first step is to consider the existence of a location specific tax rate,  $\tau_{l,t}$ , therefore decomposing the previous fiscal structure. Suppose for the moment that location specific taxes are aggregated to  $T_{L,t}$ . This means that, without assuming local spending activity by the Location (a situation that will be analyzed later), this tax income is fully sent to the Central Government, extending equation 1 to

$$T_t = T_{F,t} + T_{H,t} + T_{L,t}$$
(2)

For simplicity, suppose now that local taxation affects only Firms<sup>3</sup>. This means that net profits for a Firm f in location l at time t will be equal to

<sup>&</sup>lt;sup>3</sup>A different local taxation imposed to households would be an interesting setting, both as

$$\pi_{f,t}^{net} = (1 - \tau_{f,t} - \tau_{l,t})(p_{f,l,t}s_{f,l,t} + \sum p_{f,l^*,t}s_{f,l^*,t} - W_{f,t}n_{f,t})$$
(3)

with the revenues of the firms being the collection of domestic and exported goods sales  $p_{f,l,t}s_{f,l,t} + \sum p_{f,l^*,t}s_{f,l^*,t}$ . The tax income of location l then, will simply be the aggregation of all tax revenues from profitable firms located there.

$$T_{l,t} = \sum_{f} \tau_{l,t}(\pi_{f,t}^{gross}) \tag{4}$$

If the sum of the local and global tax rates is fixed, this specification does not modify the model compared to the benchmark ABEG case.

This local tax rate defines therefore the benchmark version of the Local Policy ABEG model used in this work. Since in this version the local tax rates are homogeneous, this starting structure could be conceived as a model of local tax harmonization.

Now, following the review of the previous section, I will let the locations L to be also able to revise their financing instruments and objectives. As we will see in the next sections, this permits to test the different scenarios. I will list therefore first the different Tax competition settings, and then extend it to include a local Public Spending. Finally, I propose a couple of exercises to test interesting applications of Local Public Spending in asymmetric scenarios.

Local resources and global financing An important modification of the models in this version, regards how the Global Government manages its finances. The lack of a credit sector, and the simplified public one, which considers the Global Government to act as a Central Bank, makes the Government deficit an injection of liquidity into the economy, basically an interest-less bond. This means that potentially this system might sustain different levels of debt without consequences, as the Debt would simply represent more resources held by the other Agents. To introduce here a more realistic public finance structure I test here a case where instead the Government has some limitations on the level of Deficit and Debt that is able to sustain.

To obtain this feature I act on the Transfer variable that the Government send to all Households every turn. Given a reference level for the Deficit, that I set as the Initial real value obtained in the Calibration step, I let the Global Government being able to revise this instrument upwards or downwards, according to a small revision parameter. Therefore

$$Tr_t = Tr_{t-1} + 0.05 * \frac{Tr_1}{P_1} * \bar{P}_t$$
(5)

an eventual robustness check, or to study additional policies. A scenario that comes to mind for instance is the Location of retired workers in Regions with lower taxation and price levels.

$$\frac{Def_{t-1}}{\bar{P_{t-1}}} \le \frac{Def_1}{P_1}$$

while

if

$$Tr_t = Tr_{t-1} - 0.05 * \frac{Tr_1}{P_1} * \bar{P}_t$$

if

$$\frac{Def_{t-1}}{P_{t-1}} > \frac{Def_1}{P_1}$$

Notice that the same type of exercise could have been done by making the Global Government to revise its global tax rate level. However, since in the tax competition exercises the Firms looks at Local tax rates as a relocation metric some confounding effect might have emerged, and therefore I choose the Transfer instrument as the main instrument of the Global Government.

#### 3.2 Tax competition: Harmonization, races or capture?

To study the dimension of tax competition I focus on Locations agents competing between each other to attract more Firms agents. To do so, I propose three different tax revision rules to be employed by locations: Race to the bottom, Race to the top and Capture.

Notice that in this scenario, for simplicity, taxation revenues are fully sent to the Government as the local use of this revenues will be studied in the next Public Spending section. Nonetheless these resources do not disappear from the economy, as the Government interacts back with the system trough wages of public workers, unemployment benefits, bailout for failed agents.

The models are parametrized in order to focus on the tax competition issues. I select as a relocation rule for Firms the "mixed" case described in the previous chapter. This rule is endowed with both an agglomerating and a dispersive metrics, whose trade-offs determines the final outcome of the Firms relocation behavior. To make Firms more responding to the tax issue I employ therefore directly the tax rate as the "dispersive" relocation metric, meaning that if a Location is endowed with a lower tax rate, it will result more desirable for the Firm relocation. This effect can however be balanced by the other "agglomerating" metric: if the other metric is location population, even with a very low taxation an unpopulated location will not be as effective in attracting firms.

Finally, another assumption needs to be discussed. Taking the policy as given and exogenous is taken here as measure to guarantee the comparability of the different scenarios. The consequence would be then for the policy maker to evaluate which scenario performs better, and then to devise appropriate interventions to reach it. Every scenario might indeed be modified by an appropriate intervention: for instance the "race to the bottom" might be limited by an imposed floor on taxation rate levels, while the "capture" one could be countered by stricter regulations on lobbying.

**Race to the top** In this scenario, Firms enjoys agglomeration rents, in the sense that they are willing "to pay" to remain in the main location (Baldwin and Krugman, 2004). In other words, such location government might raise local taxation, but Firms would still not relocate.

The Location agent will therefore take this assumption as given, and the tax revision rule will be for simplicity a linear increasing function of the relative Location Firms endowment.

$$\tau_{l,t} = 0.5 * \frac{F_l}{F} \tag{6}$$

with  $F_l$  being the Firms population in location l. This means that in case of full agglomeration in location l the tax rate would be maxed out at 0.5, which is an high value considering real tax rates.

**Race to the bottom** In the "Race to the bottom" scenario, local governments follows the prediction of standard Tax Competition theory, namely, they will tend to revise downwards their taxation rate to attract Firms, competing with each others. It is quite predictable that in such setting this taxation game would rapidly (perhaps simultaneously in a game theory setting) converge towards a situation where all local tax rates are cut to zero. Such scenario is not very interesting here, as this would simply leave the agglomerating metric to dominate the relocation protocol, providing no change with respect to a benchmark model endowed with a single relocation metric, as the benchmark one studied in the previous chapter. In this setting the focus is therefore more set towards the dynamic interaction between the locations behavior, and the tax revision is smoothed.

In this setting I assume therefore that the local tax rate of a single location is a negative function of the local Firms population dynamic. The main assumption here is that locations want to attract Firms, but needs an incentive to do so. The tax rate is therefore cut downwards only when the local population of Firms is decreasing, which means that nonetheless locations would prefer to keep their taxation rate constant in a favorable situation. The local tax rate for period t evolves then accordingly to the following rule:

$$\tau_{l,t} = \tau_{l,t-1} - \alpha \tag{7}$$
  
if  $F_{l,t} - F_{l,t-1} \le 0$ 

where  $\alpha$  is a small random revision parameter. To avoid situations of unrealistically high tax rates or negative ones (as public spending is studied later), the tax rate is set to belong to the interval  $\tau_{l,t} \epsilon[0, 0.5]$ . This last assumption is maintained for all exercises.

**Capture** To test an alternative revision rule not based on a Economic Geography argument, but rather a Political Economy one, the "Capture" scenario, represents a situation where the concentration of Firms enhances their capability to lobby the local government for a better fiscal treatment (Brulhart and Simpson, 2017). I model it therefore in a similar manner as the race to the top scenario,

$$\tau_{l,t} = 0.5 - 0.5 * \frac{F_l}{F} \tag{8}$$

but here full agglomeration totally erase the tax rate.

Each scenario is tested again the benchmark case, which given its structure could be considered as a "tax harmonization" situation, where local tax rates are invariant. To test for more realistic situations, the experiment is conducted both on a symmetric map, and on a asymmetric map with different transportation costs, and with already agglomerated regions.

#### 3.3 Public Spending

As evidenced in Commendatore et al. (2018), public spending could have different effects on the local economies whether is defined as a "productive" expenditure, or rather as an "unproductive" local demand-boosting one. For the public spending policy dimension I will test therefore two different spending policies by locations. These policies are financed in turn by the local tax revenues as I will briefly detail here.

**Local financing** We can label the local government eventual expenditures as  $XP_{l,t}$ . If these were financed by the central Government, they would be aggregated in a total bill, and become an additional entry in the Central Government deficit:

$$XP_{L,t} = \sum_{l} XP_{l,t} \tag{9}$$

$$DF_{t} = \bar{W}_{t} + W_{Gt} + \bar{B}_{t}^{f} + \bar{B}_{t}^{h} + XP_{L,t} - T_{t}$$
(10)

alongside with the unemployment benefits  $\overline{W}_t$ , the wage of public servants  $W_{Gt}$ , and the bills for bailing failed Firms  $\overline{B}_t^f$ . In the benchmark model however these are assumed to be zero, as the relevant source of local expenditures are indeed already accounted and financed by the central government. In this version, the presence of locally collected tax revenues makes it however realistic to conceive that at least a fraction of these resources could be used directly by the Locations agents to promote their objectives.

I define therefore a budget for the location agent,  $Bg_{l,t}$ . Collected revenues from local taxation,  $T_{l,t}$  are stored in this budget, and not sent to the Central Government. These resources are used instead as a public spending in order to enhance local economic activity. To avoid defaults and bailouts I assume that revenues from taxation are fully spent every turn, therefore keeping a balanced budget such that:

$$Bg_{l,t} = T_{l,t} = XP_{l,t} \tag{11}$$

The expenditures  $XP_{l,t}$  are then invested in the tested spending policy, that I will now detail.

**Public Procurement** "Unproductive spending" is defined here as a spending oriented to boost local demand, but without direct effects on the production process. This amount to the local Government acquiring goods on the local market, as a sort of public procurement. To represent this spending in the ABEG model I consider that after the consumption goods market interactions takes place, the Location agent acts as a consumer Household, going to the market, and fully spending its resources  $XP_{l,t}$  to buy the remaining goods. It could be that the government is not able to spend the whole amount of this sum; in this event, the remaining resources are saved and employed in the next turn:

$$Bg_{l,t} = T_{l,t} - XP_{l,t} \tag{12}$$

This spending should act in theory as a dispersive force in the economy.

**Subsidy** A "productive spending", is instead considered as a subsidy to local firms. Given the production structure of the Firms, it is modeled as a bonus to lower the unit labor cost of the Firms. The subsidy is then equally split between the firms of the location, and deduced from the labor cost.

$$Sub_{f,l,t} = XP_{l,t}/F_l \tag{13}$$

In the next section I will present the results of these experiments, conducted both on a symmetric unrealistic geometric spatial structure, and on an asymmetric but more realistic structure inspired by a real life geographical unit, in a scenario of a widespread use by all Locations.

#### 3.4 Asymmetric Public spending exercises

While the previous Sections study policy changes brought by all the Location Agents, there are many real life examples where the situation under study is asymmetric, so where the change in policy regards only one or a part of the agents under study. In this Section I present two preliminary studies to test potentially interesting applications of Local Policy, in particular regarding Public Spending as an answer to shocks. The first exercise tests the case of a Local answer to an Asymmetric Local shock, while the second hypothesize an exogenous migration boom in one of the Locations. Both exercises looks then at the consequences of letting the affected Location to answer its new situation with local fiscal resources.

#### 3.4.1 Asymmetric demand shock

In this experiment I assume that the economy is hit by an asymmetric local demand shock. The specific research question is what would happen by letting the Location agent (i.e. a local government) to react to this shock.

The shock is assumed to be exogenous, unexpected and permanent, and it is modeled by imposing a parameter change in the consumption goods market to take place at certain point in the simulation. Indeed, the formula that governs the probability of acquiring a new supplier

$$Pr = 1 - e^{\gamma_c (p_t^{new} - p_t)/p_t^{new}}$$
(14)

can be scaled trough the parameter  $\gamma_c$ , that amplifies or reduces its sensitivity to the different magnitudes of the two compared prices. Scaling down  $\gamma_c$  then reduces the probability of consuming a good from a new supplier given the same difference in prices between the two consumption goods. When the shock hit therefore, the hit location l will be endowed with a new parameter  $\gamma'_{l,c}$  in formula 1, such that  $\gamma'_{l,c} < \gamma_c$ . Notice that now the parameter shows a location suffix l, as it is now different from the economy wide  $\gamma_c$ . In a nutshell, this shock will end up in decreasing the local consumption demand.

In addition to the effects of the shock per se, the scope here is to see what will happen when a Location agent is let free to react. The "Public procurement" protocol is useful here then to depict a local government that boosts local demand to front the shock; this happens by acquiring goods directly from the local market, spending its own revenues. The difference with the scenario of the previous section, is that here only the location targeted by the shock will apply the protocol.

Another important caveat here is that Locations might display different sizes in different simulations. According to where the asymmetric shock might hit we could end up then with very different outcomes. While recognizing that a simulation sweep addressing each location in turn would be optimal, I limit this analysis to just two Locations. I identify one "big" and one "small" location, and test the policy experiments on both of them, in order to isolate some size effects. I simulate therefore a shock against a small and big location, and compare the overall series against a simulation where the shock is not addressed locally, i.e., the eventual effects of the shocks such as an increase of Firms failures and/or of the unemployment rate are all addressed by the Global government.

I select the "Big" and "Small" Locations by simulating the asymmetric benchmark model  $^4$  and choosing the main agglomeration and a second less populated Location.

#### 3.4.2 Exogenous Migration boom

Another interesting topic related to the local public spending is the analysis of migration flows. This topic is very debated recently, and at the center of heated policy controversies both in Europe and in the US. Petrovic et al. (2017) cite the migration crisis stemming from the Middle East crisis as an example of lack of coordination by EU countries.

Instead of focusing on the internal migration flows in this experiment I simulate therefore an exogenous migration shock hitting one of the locations.

The migration shock is obtained by injecting an amount of new households in the model starting from a periphery location. This permits to analyze two situations: first, the effect of this population increase on the overall economy, and second, if the spatial equilibrium distribution would be affected.

Another important issue on these topics is if the costs of this migration should be financed by the recipient alone, or to be spread across the members of the economy. The generation of new Households in a Location would indeed increase the public expenditures, as these new households would start as unemployed, and therefore would receive an unemployment subsidy. This is a voice of expenditure in the Government deficit accounting. In the benchmark ABEG model, this additional cost would be spread in the economy, as the wage benefits are paid by the Central Government. In an additional scenario I check what would happen if letting these costs to be faced by the local government. The experiment presents therefore two different versions of financing new Households.

Also for this exercise I employ again the Asymmetric map model. Is quite plausible that given a symmetric map an injection of Households in only one location would enhance an agglomeration process precisely in that location. It is therefore more interesting and more realistic to employ the asymmetric version of the ABEG model, and to let the Location being hit by the shock to not be the main agglomerated one<sup>5</sup>. As evidenced from the previous section, it is possible to select a 'small' Location by running the benchmark version of the exercise; in the asymmetric version this passage is further simplified, as the initial main agglomeration tends to maintain the status across different simulations.

 $<sup>^{4}\</sup>mathrm{I}$  employ only the Asymmetric map because the initial population will be the same for all the Monte Carlo iterations required by the exercise.

<sup>&</sup>lt;sup>5</sup>Although many different versions should be undoubtfully tested in a more targeted work.

Parameters	Values
Н	1219
H'	1284
$H^M$	65

Table 1: Changes in the H parameter.

To simulate the migration of new Households, the Household's population H is augmented to H', with H' > H and  $H' - H = H^M$ , the number of new Households. I set  $H^M$  with the arbitrary value of approximately 7% of the total working Households set in the Calibration step.

 $H^M$  initial location  $l^M$  is then the target of the migration shock. While H' acts as the new Households Population, the economy itself is calibrated with the old value H, the same used in the asymmetric map. Firms are unchanged.

The new Households  $H^M$  enters the economy with no resources (zero net wealth), and are not allowed to be hired from Firms until a given period of the simulation (100 in the simulation). While this assumption might be conceived as depicting these Households period of regularization, it is more of a technical feature to allow the underlying economy to employ first the local Households, and avoid them to be not hired given higher wage expectations. While under this burnout period,  $H^M$  Households can still consume Goods or relocate, depending on their income. Until they can participate in the Labor market, this income is modeled accordingly to the two different scenarios, labeled "Global" and "Local" respectively, short of "Global answer" and "Local answer".

In the first one, Global, the central (global) government finances the expenditures of the new Households. The latter receives then immediately an unemployment benefit as the old Households, which is fixed as a fraction  $\bar{w}$  of the economy average wage  $\bar{W}_t$ ; in a nutshell, they are treated exactly as the other unemployed Households, with their current wage being

$$w_{h,t} = \bar{w} * W_t \tag{15}$$

In the second, it is the Location Agent that finance the new Households expenditures, by spending local revenues generated from local taxation. The unemployment subsidy for the new Households  $H^M$ ,  $W_M$  is therefore in this case,

$$W_{h^M,t} = X P_{t,l^M} / H^M \tag{16}$$

as the resources are equally split between them. In this latter case the subsidy is not fixed, but cannot be higher than the one received by the other unemployed Households. In this situation, the reminder of the revenues is stored for the next turn.

In this exercise I will therefore compare the outcomes of three models simulation: One without the migration wave, one with the migration wave, with

costs sustained by the Global Government, and one with the costs bore by the recipient local government.

# 4 Parametrization

In this section I will describe the choices for the parametrization of the model. The exercises of this work are tested both on a symmetric and on a asymmetric spatial structures. The two different spatial structures host the same ABEG model, which include the modifications described in Section 3.

The ABEG model is endowed with a Mixed metric relocation protocol, limited to bind for 10% of the Agents each turn. The motivation for this configuration are better detailed in Rizzati (2018), but they can be summarized here with two main reasons: First, it allow the simultaneous evaluation of Agglomerating and Dispersing forces, and second, limiting the amount of moving agent per turn limits the distortion posed by the excess sensitivity of the original relocation protocol. Mobile Firms therefore employ Location Population and Local Average wage as an agglomerating and dispersive metric respectively, while Households employ Local Average Wage and Local Unemployment rate instead. In tax competition settings, the Dispersion metrics for the Firms is subsituted by the local tax rate.

The two different spatial maps present some minor differences in the model setup, that I will describe in the next two subsections.

#### 4.1 Symmetric spatial structure

The symmetric case is endowed with a population equally distributed between the Locations of the spatial structure, and follows directly the procedure employed in the original ABEG model in Chapter 1 (Rizzati, 2018). The procedure and the values chosen for the behavioral parameters are unchanged<sup>6</sup>. What I change, to make this exercise more comparable with the asymmetric one is to adopt the same number of Locations, 12, and a comparable number of Firms and Households agents. I consider a total population of employed Households of 931 (approximated to 924) Households and 89 Firms (approximated to 84). I add to the Household numbers the unemployed and public servants ones, keeping the same proportions of the previous Chapter simulations (10% and 20% respectively). The spatial structure employed is a Race track economy, as this structure does not embed any a priori asymmetry (Akamatsu et al., 2017).

#### 4.2 Asymmetric spatial structure

An interesting feature of the ABEG model is the possibility to analyze asymmetric spatial structures. The asymmetry regards two major dimensions: the dis-

 $<sup>^{6}</sup>$ I include the initial transaction flow matrix in the appendix to not burden the exposition

Parameter	Calibration
L	12
$H_f$	924
$H_G$	180
$H_u$	108
Н	1212
F	84
Spatial Network	Race Track

Table 2: Number of Agents for the Symmetric case.

tances between Locations, and the distribution of agents between them. Structures presenting these characteristics are intrinsically interesting, as they match more closely realistic situations. Recently, a number of works try indeed to generate their spatial structure starting directly from Geographical data, as for instance Fajgelbaum and Schaal (2017), and more generally, the field of Quantitative Spatial Economics (Redding and Rossi-Hansberg, 2016). Another example that comes to mind is the family of CGE models as best exemplified in Economic Geography by the RHOMOLO model (Lecca et al., 2018).

While at this stage this work is still not a fully empirical calibration effort, it is still nonetheless interesting to test the robustness of the findings on a more realistic spatial structure. To obtain this I generate a spatial structure reminding of the region of Lombardy (IT), where each Node of the Spatial network corresponds to a Province. Moreover to include the asymmetry dimension related to distances, the network structure is extended with weights for each edges. The weights are scaled to depict actual realistic distances. Population nodes of agents are calibrated to match the relative sizes of the already existing spatial units. This version of the model could be seen then as a toy version of a realistic spatial structure, and could in principle be replicated for different countries and different spatial scales: the scope here is not predict the policy effects on the economy of Lombardy, but rather to compare their outcomes between the asymmetric but more realistic spatial structure and the homogeneous unrealistic one.

It must be highlighted that assigning the relative population size to the Locations is a shortcut, as one could argue that it could be a good exercise to try to generate the realistic map directly from the model. This could be attempted in theory, but in practice it would require an additional extension with regards to First-Nature Geography effects (Bosker and Buringh, 2017). As a matter of fact, modern agglomerations locations, especially for Europe, are indeed affected by historical factors, which include defense and agricultural motivations. In short, it would be very unrealistic to try to generate such a structure without considering at least some sort of exogenous Endowment of the Location agent. As the focus of this Chapter is the comparison between policies, I leave therefore this interesting extension for future work, and I directly replicate the existing

Locations	Households	Firms
Bergamo	136	11
Brescia	147	14
Como	57	6
Cremona	34	3
Lecco	44	4
Lodi	15	1
Mantova	50	4
Milano	223	24
Monza e della Brianza	85	9
Pavia	34	4
Sondrio	13	1
Varese	93	8

Table 3: Number of Agents per Location, asymmetric case.

spatial composition.

It is relatively easy to form such network representation, as modern statistical offices provides contiguity and distances matrices for a variety of geographical units. For Lombardy, I employ the data provided by ISTAT<sup>7</sup>, which gives the network depicted in Figure 1, where each nodes corresponds to a Province. The edges of the adjacency matrix of the spatial network are moreover weighted to reflect the relative distances<sup>8</sup>.

I assign a population of Firms and Households to each Location to replicate the relative distribution of manufacturing firms for each of the Lombardy provinces. Approximating 1000 individuals as 1 household, and 1000 manufacturing Firms as 1 Firms I obtain the distribution<sup>9</sup> presented in table 1. As expected Lombardy is an agglomerated economy with most population and firms concentrated in Milan. There are however other significant centers, such as Bergamo and Brescia.

There is a number of improvement that should be added however before aiming for empirical validity: like weighting the distances through the geographical composition and the infrastructure endowment of the Locations (for instance, Sondrio is between the Alps Mountains, while southern cities are in flat land), and more trough subdivision of agents trough economic sectors. Moreover some location specificity, such as price and wage levels are not accounted for.

<sup>&</sup>lt;sup>7</sup>http://www.istat.it/it/archivio/157423

 $<sup>^8 \</sup>rm Data$  on distances comes from NUTS 3 Tercet app on Eurostat: http://ec.europa.eu/eurostat/tercet/distance.do

 $<sup>^{9}\</sup>rm http://www.asr-lombardia.it/ASR/imprese/imprese-attive-unita-locali-e-addetti-/lombardia-e-province/tavole/14400/2014/$ 

 $http://ec.europa.eu/eurostat/web/products-datasets/product?code=nama_10r_3empers/interval to the set of the$ 

# 5 Computational Results

The simulation methodology of agent-based models provides a fertile ground for policy experiments. There are nonetheless some best practices to be followed in order to provide comparable results. Given that the parametrization conducts usually to a stationary statistical equilibrium, one procedure to test for different policies is to declare a benchmark version of the model and to simulate it. Then different policies could be tested by modifying the relevant parameters or behavioral rules, simulating a new model and then comparing the resulting series. One important caveat is to introduce the modifications to take place after the transition phase, namely after the model converged to its quasi-steady state (Caiani et al., 2016b).

The most relevant series to be presented will regard production, unemployment, firm's exit rate and finally statistics about the spatial structure of the economy, including relocation and migration rates, the distribution of agent's population across the spatial network.

To try to rule out randomness disturbances that might confound the actual effect of the tested policy, I follow the methodology employed in Ponta et al. (2018). This work, run on the EURACE model investigates a setting where the policy change regards a feed-in tariff in the context of renewable energy production. The main difference is that here I am testing models where the change in policy regards different tax updating and spending protocols, instead of testing different values of a parameter. I run therefore 20 iterations of each scenario<sup>10</sup>, and by comparing the selected series of interest trough the use of box plots. I test moreover each series trough the Wilcoxon rank sum test, with the hypothesis of the two compared series coming from continuous distributions with the same median (Gibbons and Chakraborti, 2011)<sup>11</sup>.

## 5.1 Tax Competition Scenarios

Before focusing more in detail into the statistics, we can observe that the main difference between the two spatial maps is that the symmetric map presents a greater number of agglomerations than the asymmetric one. To provide just one first visual reference, in Figure 1 and 2 I present the plots for the main series of the benchmark model run on the symmetric and asymmetric spatial map respectively. The asymmetric one is therefore still too sensitive to the relocation behaviors of the agents to depict a stable and realistic configuration. Since the symmetric map starts with a fully dispersed agents population, more agglomerations are able to sustain the competition by the neighbors. These plots refers however to single simulations and are not as informative as the box plots presented later. I include therefore the additional ones of the various scenarios in the Appendix for the interested reader.

 $<sup>^{10}</sup>$ I will increase the number of repetitions at least to 100 in future versions.

 $<sup>^{11}</sup>$ A Nonparametric test is moreover less affected by the small size of the two compared series (20, as the number of the model iterations)

In Table 4 and 5 I summarize the main series averages for each Tax competition scenario. These series include the aggregate Production, the unemployment rate, the consumption good price inflation rate, the Firms exit rate (how many firms are failing for period), the relocation rate (the rate of Firms agents that change Location in a given turn) and the Migration rate (the rate of Households agents that changes Location in a given turn).

The averages in the tables show us that all the models behave quite similarly. In all the scenarios Aggregate production reaches and oscillates around a stable value, and no change in the tax policy of the countries seems to affect it.

The box plots from Figure 3 to 10 permits a better visual comparison of the compared series. In this format, the red line represent the median value of the series sample, the blue box top and bottom edges the 25th and 75th percentiles (respectively), and the black whiskers the extreme values of the data. Outliers are eventually marked with a red + symbol, while the x inside the box refers to the mean of the series.

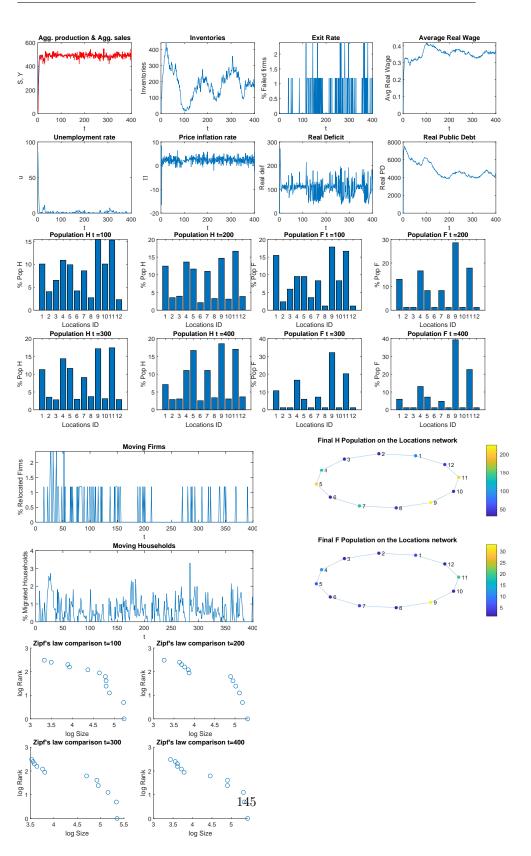


Figure 1: Main series for one iteration of the symmetric benchmark model

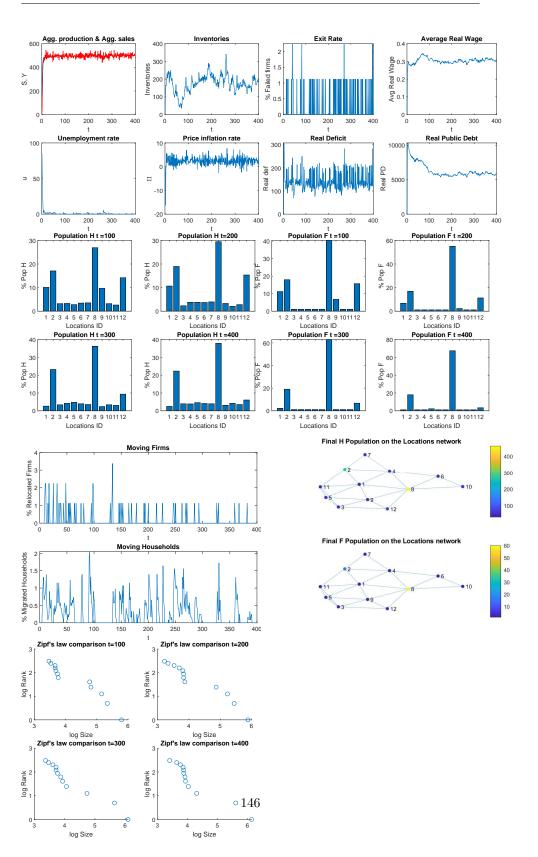


Figure 2: Main series for one iteration of the asymmetric benchmark model

MODEL\Series	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark Symmetric	487.730	1.628	2.461	0.353	0.223	0.531
RTT Symmetric	487.066	1.643	2.408	0.345	0.759	0.730
RTB Symmetric	487.320	1.685	2.453	0.312	0.237	0.511
CAP Symmetric	487.154	1.698	2.456	0.374	0.227	0.552

Table 4: Averages for the economic series, symmetric model

MODEL\Series	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark Asymmetric	492.592	1.314	2.465	0.241	0.166	0.328
RTT Asymmetric	490.654	1.433	2.406	0.412	0.776	0.593
RTB Asymmetric	492.173	1.369	2.469	0.239	0.170	0.352
CAP Asymmetric	490.849	1.597	2.448	0.287	0.186	0.400

Table 5: Averages for the economic series, Asymmetric model

In Figure 3 we can compare the box plots generated from the averaged Aggregate productions from each Monte Carlo simulation, for every tax competition scenario. Indeed the average values are very similar. The asymmetric map presents an higher value for every scenario, given the more agglomerated structure compared to the symmetric one. This seems to influence the unemployment rate, also slightly higher in the symmetric map (Figure 4).

In both the symmetric and asymmetric maps the highest value for Production is obtained by the benchmark scenario. The worst performing ones are the Race to the Bottom RTB and Capture CAP for the symmetric and asymmetric map respectively. Both displays a simulation with the lowest value of production and higher values of unemployment. In the case of the CAP for the Asymmetric map the low performing simulation value is not even considered as an outlier.

In Table 6 and 7 I list the p-values from the Wilcoxon rank-sum test for selected series, tested between each scenario. Production and unemployment rate p- values for the symmetric map are too high to reject the null hypothesis of the two series being sampled from a continuos distribution with an equal median, which confirms the similarity of performance by the different scenarios. This does not happen for the asymmetric map, where the null hypothesis is rejected at the 5% level for the RTT and CAP scenario with respect to the Benchmark one for both Production and Unemployment. Comparing the tax competition scenarios between each other shows that the null cannot be rejected for production between the CAP and RTT scenario, and for unemployment rate between the RTT and RTB scenario.

From these results we can summarize that the different local tax competition scenarios had a negligible effect on the main Economic variables of the model, but that this negligible effect is far more working in the asymmetric map: in my opinion, this is a consequence of the more agglomerated structure that this map displays when the policies become binding.

I will now comment results related to the spatial effects of these policies. I already presented averages for the relocation and migration rates of the mobile

Series	Bench/RTT	Benchmark	k/RTB	Ben	ch/Cap
Production	0.336	0.755	)	(	0.250
Unemployment	0.967	0.735	ó	(	).379
Migration H	0.000	0.323	3	(	0.364
Migration F	0.000	0.048	3	(	0.408
HH index H	0.000	0.004	1	(	0.019
HH index F	0.000	0.394	1	(	0.003
Real Deficit	0.009	0.000	)	(	0.714
Real Transfer	0.000	0.000	)	(	0.046
Series	RTB/RTT	RTB/CAP	RTT/	Cap	
Production	0.473	0.285	0.59	7	
Unemployment	0.839	0.379	0.45	6	
Migration H	0.000	0.155	0.00	0	
Migration F	0.000	0.260	0.00	0	
HH index H	0.000	0.579	0.00	0	
HH index F	0.000	0.126	0.00	0	
Real Deficit	0.147	0.000	0.04	3	
Real Transfer	0.655	0.043	0.08	1	

Table 6: P-values for Wilcoxon rank-sum test performed for the listed series between the scenarios of each Column, symmetric version. Values below 0.05 indicates that the null hypothesis (of the two series being sampled from a distribution with equal median) can be rejected at the 5% level

agents, and I will detail them more here by looking at the related Box plots of Figure 5 and 6. Both maps display an higher rate for both Households and Firms for the RTT scenario, which is an interesting result, especially given the very similar values for the series presented before. Indeed, the p-values for the Wilcoxon test rejects the null for each comparison of this scenario with the other ones.

This result directly helps in explaining the box plots of the following figures, Figure 7 and 8. There we can observe a box plot obtained from the series of average HH indexes taken at every turn for each of the mc simulation. it is then telling us a synthetic indicator of the degree of dispersion faced by the economy. Results are similar between all scenario besides the RTT one, that is lower. The interpretation is that therefore this scenario is more dispersed, with more homogenous Location's populations for both agents.

This does not happen in the Benchmark (tax harmonization) and RTB scenarios, because the taxation rates keeps on being too similar to act as a proper dispersion forces. In the capture scenario, where more agglomerated regions will have lower taxation rates this effect is absent. The contrary effect is not present, meaning that the Capture scenario is not prompting an higher agglomeration than the other scenarios.

Series	Bench/RTT	Benchmark	k/RTB	Ben	.ch/Cap
Production	0.000	0.223	3	(	).003
Unemployment	0.0179	0.273	3	(	0.000
Migration H	0.000	0.023	}	(	0.000
Migration F	0.000	0.115	ó	(	0.000
HH index H	0.000	0.126	j	(	0.000
HH index F	0.000	0.171	_	(	0.000
Real Deficit	0.126	0.755	0.755		0.524
Real Transfer	0.000	1	1		0.000
Series	RTB/RTT	RTB/CAP	RTT/	Cap	
Production	0.000	0.020	0.54	2	
Unemployment	0.250	0.005	0.06	7	
Migration H	0.000	0.001	0.00	0	
Migration F	0.000	0.000	0.00	0	
HH index H	0.000	0.007	0.00	0	
HH index F	0.000	0.000	0.00	0	
Real Deficit	0.147	0.597	0.13	3	
Real Transfer	0.000	0.000	0.00	0	

Table 7: P-values for Wilcoxon rank-sum test performed for the listed series between the scenarios of each Column, Asymmetric version. Values below 0.05 indicates that the null hypothesis (of the two series being sampled from a distribution with equal median) can be rejected at the 5% level

Finally, I investigate the fiscal consequences of the different local fiscal structures. In Figure 9 and in 10 we can check the box plots for the real Deficit and the real Transfer of the economy. Recall that given the simplified fiscal structure of the Government, that can sustain an arbitrary debt by printing money, I introduced a sustainability trade off for the Global government in the mean of an adaptive rule for the level of transfer given to all Households. Above or below of a certain threshold of Deficit, the transfer is increased or decreased in response to keep that level stable. This means that when the local revenues from local taxation are modified by the new policy rule, the answer by the Global government depends on how the Deficit level will be affected: for instance, if a location cuts its taxation to zero, and the global deficit rise, the Government will reduce its global transfer: the local action can therefore end up in having global consequences. This seems to have happened for the RTT scenario that shows a consistently lower real transfer value compared to the other scenarios: the higher dispersion entailed by this rule means that the agglomerations (with an higher agglomeration rent and therefore local tax revenues) where less populated: this affected the global transfer level trough the deficit adaptive rule of the Global government. In this simulation the reduction of the transfer was not enough to affect aggregate production and unemployment, but the consequences of this mechanisms should be probably investigated further as potentially harmful.

The second case where the transfer is lower is the CAP scenario for the asymmetric map: in this case the effect is more direct, as simply in the CAP scenario the most agglomerated locations, that hosts more Firms will have directly lower fiscal revenues given the lower taxation rates brought by lobbying.

This fiscal reasoning seems to be magnified in the situation of the Asymmetric map simulations, and the reasons stems from the higher degree of agglomeration displayed by this map at the time period when the new fiscal policy binds. The evaluation of Local fiscal policies therefore should account the relative density of the geographical unit being studied: scenarios that cannot be hypothesized to come from different distributions according to the rank sum test becomes different when tested on more asymmetric maps.

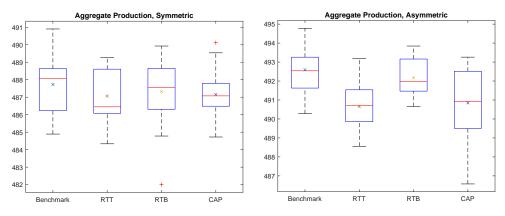


Figure 3: Box plots for the Aggregate Production for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

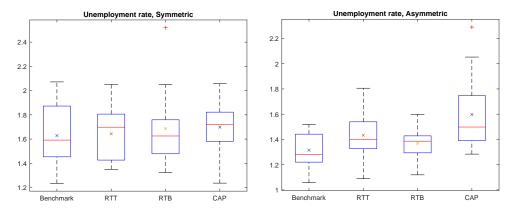


Figure 4: Box plots for the Unemployment rate for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

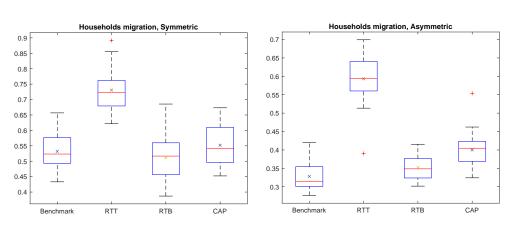


Figure 5: Box plots for the Households migration rate for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

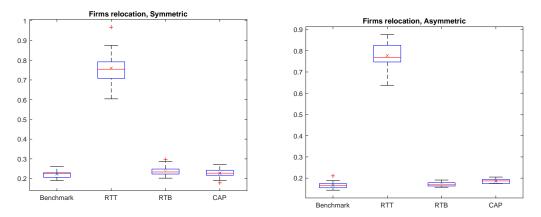


Figure 6: Box plots for the Firms relocation rate for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

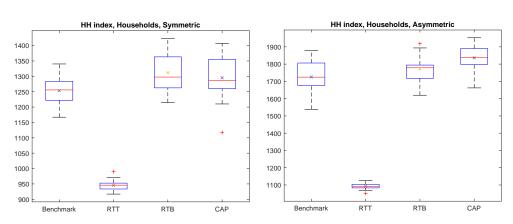


Figure 7: Box plots for the HH index for Households for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

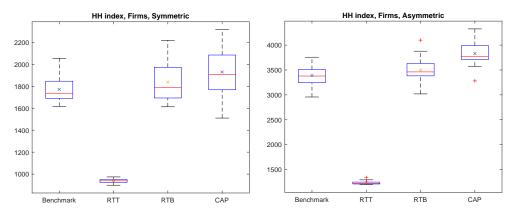


Figure 8: Box plots for the HH index for Firms for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

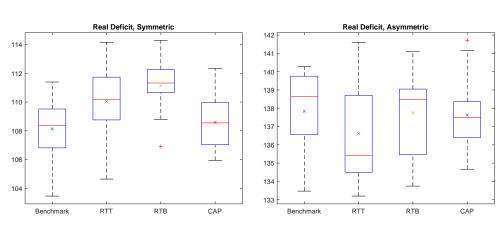


Figure 9: Box plots for the real Deficit for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

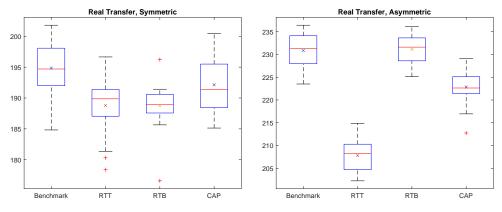


Figure 10: Box plots for the real Transfers for each Tax Competition scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

### 5.2 Public Spending Scenarios

In this section I will detail the results for the Public Spending scenarios. The series studied are the same of the previous section, and again the analysis is performed both on a symmetric and on an asymmetric map. I will summarize the results of the simulations trough average stastics, box plots and Wilcoxon rank-sum tests.

The main message of this section is that the policies tested had a more relevant effect only when tested on an symmetric map. This is immediately evident by looking at the average values in the Tables 8 and 9. This is also confirmed by the p-values of Tables 10: basically the models that run on the asymmetric map are behaving as one, as if the policy tested did not have any effects on the final outcome. So, some of the predicted effects of the policies, as the dispersion nature of local public procurement are lost in the asymmetric version.

Looking more carefully at the Asymmetric version, we can notice that there is one surviving difference: the fiscal variables. Given the loss in revenues of the Global Government, real Deficit rises, and the Transfer is reduced. Since the lack of effects on production and unemployment, we can hypothesize that the local expenditures, both as Subsidies and as local Public Procurement artificial demands compensated this loss.

In the Symmetric map the picture is less homogeneous, and the p-values in 10 seems to confirm that policies had an effect by modifying the benchmark model, but that the two policies tested generated two very similar models: Probably the subsidy scenario should be characterized differently.

The effects of the two policies on the symmetric map shows an increased Average Production and a reduced average Unemployment rate. There is also an effect present on the spatial side of the model, affecting the migration of Households: the average migration rate is lower, which explains the higher average HH index for the Households population. This means that the policies acted as an agglomerating force towards the Household agent, and surprisingly had no effect on the Firm agent. Indeed the lowered unemployment rate, which enters as a dispersive force in the mixed relocation metrics explain this behavior.

My main explanation for the lack of effects on the Firm Agent is that the policies where applied equally by each Location, probably counterbalancing their effects on the relocation metrics. Indeed, smaller Locations that might have gained by attracting Firms where not able to do so: the Households, that given the lower unemployment became less mobile, kept on staying in the greater ones.

From the fiscal side both Public Spending scenarios produced an higher Deficit, and the consequent reduction in Transfer did not affect Production and unemployment. The lower Real Transfer in the PP scenario compared to the SUB scenario is probably a consequence of the slightly higher average inflation rate. To conclude, this exercises displayed that the same policies works differently on differently agglomerated maps, even when the policy itself is not as disruptive,

MODEL\Series	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark Symmetric	487.730	1.628	2.461	0.353	0.223	0.531
SUB Symmetric	488.713	1.486	2.482	0.224	0.221	0.455
PP Symmetric	488.877	1.421	2.502	0.252	0.222	0.438

Table 8: Averages for the economic series, symmetric model

MODEL\Series	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark Asymmetric	492.592	1.314	2.465	0.241	0.166	0.328
SUB Asymmetric	492.523	1.308	2.461	0.153	0.170	0.321
PP Asymmetric	492.097	1.306	2.485	0.188	0.169	0.308

Table 9: Averages for the economic series, Asymmetric model

as was the case with the general Public Spending. The lack of effects of some Local Policies might not be replicated in different geographical contexts, and this stresses the importance for including proxies for agglomeration and spatial distributions controls when performing empirical work.

Series	Bench/Sub	Bench/pp	Sub/pp
Production	0.036	0.019	0.924
Unemployment	0.060	0.005	0.473
Migration H	0.000	0.0	0.409
Relocation F	0.577	0.892	0.838
HH index H	0.022	0.015	0.776
HH index F	0.198	0.180	0.714
Real Deficit	0.000	0.000	0.597
Real Transfer	0.755	0.010	0.007
Series	Bench/ Sub	Bench/pp	Sub/pp
Series Production	Bench/Sub 0.675	Bench/pp 0.180	Sub/pp 0.155
	,	, = =	
Production	0.675	0.180	0.155
Production Unemployment	$\begin{array}{r} 0.675 \\ 0.860 \end{array}$	0.180 0.967	$\begin{array}{c} 0.155 \\ 0.924 \end{array}$
Production Unemployment Migration H	$\begin{array}{r} 0.675 \\ 0.860 \\ 0.860 \end{array}$	0.180 0.967 0.155	$\begin{array}{r} 0.155 \\ 0.924 \\ 0.14 \end{array}$
Production Unemployment Migration H Relocation F	$\begin{array}{r} 0.675 \\ 0.860 \\ 0.860 \\ 0.348 \end{array}$	0.180 0.967 0.155 0.303	$\begin{array}{r} 0.155 \\ 0.924 \\ 0.14 \\ 0.817 \end{array}$
Production Unemployment Migration H Relocation F HH index H	$\begin{array}{r} 0.675\\ 0.860\\ 0.860\\ 0.348\\ 0.490\\ \end{array}$	$\begin{array}{c} 0.180 \\ 0.967 \\ 0.155 \\ 0.303 \\ 0.860 \end{array}$	$\begin{array}{c} 0.155\\ 0.924\\ 0.14\\ 0.817\\ 0.440\\ \end{array}$

Table 10: P-values for Wilcoxon rank-sum test performed for the listed series between the scenarios of each Column, symmetric version. Values below 0.05 indicates that the null hypothesis (of the two series being sampled from a distribution with equal median) can be rejected at the 5% level.

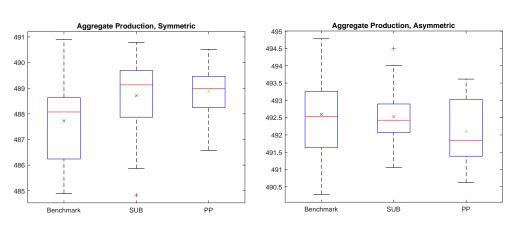


Figure 11: Box plots for the Aggregate Production for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

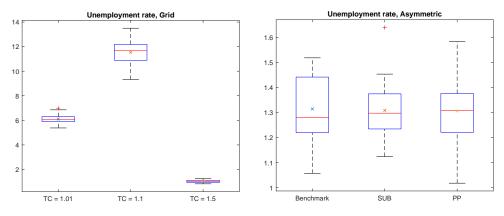


Figure 12: Box plots for the Unemployment rate for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

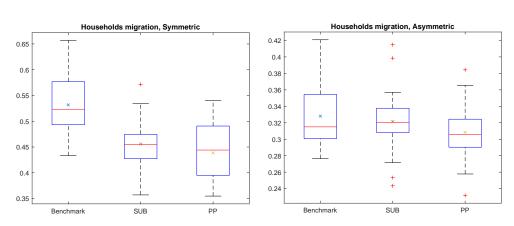


Figure 13: Box plots for the Households migration rate for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

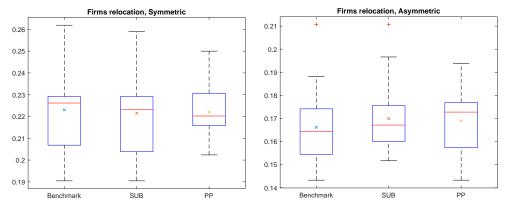


Figure 14: Box plots for the Firms relocation rate for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

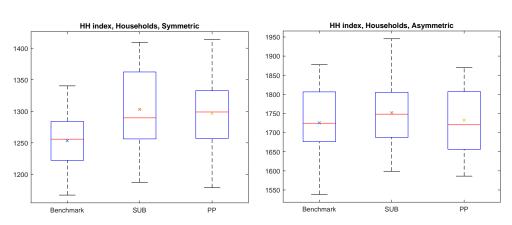


Figure 15: Box plots for the HH index for Households for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

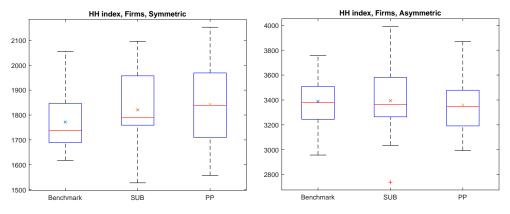


Figure 16: Box plots for the HH index for Firms for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

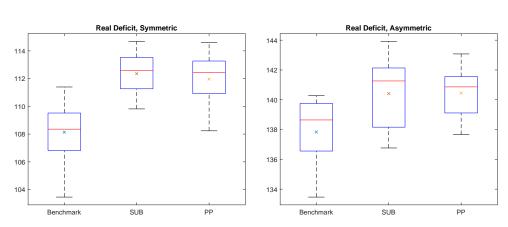


Figure 17: Box plots for the real Deficit for each Tax Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

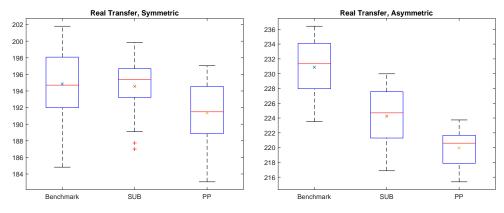


Figure 18: Box plots for the real Transfers for each Public Spending scenario. Left panel: symmetric spatial network model. Right panel: model run on an asymmetric map.

## 5.3 Asymmetric Public Spending Exercises:

#### 5.3.1 Asymmetric shock

In this section we can observe the computational results for the asymmetric demand shock policy experiment, as set up in Section 3.4.1.

Before presenting the results, I detail the selection of the "Small" and "Big" Locations. As already seen, the benchmark asymmetric model exasperates the initial agents endowments, ending up with three main agglomerations of Firms, but a more heterogenous distribution of Households. Location 8, the main initial agglomeration<sup>12</sup>, is therefore quite stable, and is selected as the "Big" one, owning up to 40% of Households population, and 60% of Firms. Location 2 is the second greatest agglomeration: it is not the initial smallest locations, but is the one that "survives" till the end, and it is considerably smaller than the previous one, maintaining a population of 20% of both agents.

The shock is set to hit at period 100, and to persist for the rest of the simulation. The shock lowers the parameter  $\gamma_c$  from the original value of 3 to 0.5268, reducing therefore the probability to buy from a new supplier from 50% to 10% given a difference of 20% in the compared consumption goods price, and inhibits the residual consumption protocol through which the Households satisfy their residual demand.

As we can observe from Figure 19, the shock drastically lowers local consumption. The effect is greatly magnified in the case of the small Location, as the shock is enough to drive out Firms and Households towards the adjacent and competing location  $1^{13}$ .

As a summary, Table 11 presents the averages taken over the averaged series for each of the Monte Carlo simulations. The asymmetric shock had a sizable and negative effect on the affected economies, decreasing the aggregate average production and rising the unemployment rate. Moreover, this economic slump affected also the exit rate of Firms, that is more than doubled. The economic dimension of the hit Location is influencing the effects of the shock, as, as expected, the shock is directly linked to the magnitude of the local consumption demand.

MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark	492.59	1.31	2.36	0.22	0.17	0.32
Asymmetric shock (Big L)	477.22	3.74	2.20	0.83	0.16	0.67
Asymmetric shock (Small L)	489.65	1.74	2.28	0.47	0.20	0.48
Local intervention (Big L)	473.22	4.37	2.17	0.87	0.17	0.69
Local Intervention (Small L)	483.70	2.66	2.24	0.57	0.20	0.59

#### Table 11: Main Series Averages

Given these results, I ask what could be the effect of letting the affected Location

 $<sup>^{12}</sup>$ Since this asymmetric map is based on Lombardy, the main initial (and final) agglomeration corrensponds to Milan, unsurprisingly.

 $<sup>^{13}</sup>$ It would be interesting here to test for locational hystheresis, so if a temporary shock would be able to permanently modify the spatial distribution (Ottaviano, 2003)

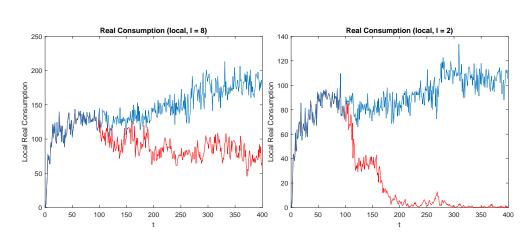


Figure 19: Effect of the shock on local consumption for location 8 ("Big"), and location 2 ("small")

to intervene, both on the general economy, on the spatial distribution of agents, and by the point of view of that specific location.

In Figure 21 and 22 we can observe some Box plots that compares specific series related to the previous questions. Figure 21 presents the results for the shock and the shock with policy scenario against the benchmark for the "big" Location (l=8), while Figure 22 for the "Small" one, l=2.

Regarding Aggregate Production and Unemployment rate, we observe that in addition to the decreasing effect of the shock on the mean value of the average series, the volatility is also enhanced. Moreover, for both Locations, Big and Small, the local spending by the Location agent to balance the asymmetric shock did not balance the negative consequences on production and unemployment, but exasperated it.

I hypothesize that this is brought by two distorting effects: the first is a change in the agents relocation behavior, and the second is the effect of deviating public revenues from the global government. If we observe the remaining panels of Figures 21 and 22 we can see that indeed there is a remarkable difference in both fields, especially regarding the migration of Households and the real Transfers.

The tables 12 and 13 presents the p-values for a Wilcoxon Rank-sum test, for Locations 8 and 2 respectively, and help in discerning in which case the presence of Local public procurement was affecting the behavior of the series compared with the case of just the asymmetric shock and no intervention.

At least in the case of the "Big" Location, the p-value rejects the null hypothesis for the Households migration rates, while not rejecting the difference in Transfer levels for both cases. Migration rates are moreover higher in the case of public spending for the Small region.

It seems therefore that the intervention of the Location agent not only was incapable to balance the negative effect of the asymmetric shock, but it worsened

Series	Benchmark/Shock	Benchmark/Shock w.S.	Shock/Shock w.S.
Production	0.000	0.000	0.003
Unemployment	0.000	0.000	0.006
Migration H	0.000	0.000	0.129
Migration F	0.000	0.030	0.775
HH index H	0.003	0.012	0.967
HH index F	0.000	0.009	0.101
Real Deficit	0.000	0.273(!)	0.000
Transfer	0.000	0.000	0.000
Avg. H (l shock)	0.000	0.000	0.001
Avg. F (l shock)	0.000	0.027	0.163

Table 12: 2-sided Wilcoxon rank-sum test for the relevant series. P-values reported refers to the test between the series of the benchmark model and each migration shock scenario. 0.05 (5%), 0.01 (1%)

Series	Benchmark/Shock	Benchmark/Shock w.S.	Shock/Shock w.S.
Production	0.000	0.000	0.000
Unemployment	0.000	0.000	0.000
Migration H	0.000	0.000	0.000
Migration F	0.000	0.000	0.775(!)
HH index H	0.000	0.000	0.839~(!)
HH index F	0.016	0.000	0.424 (!)
Real Deficit	0.000	0.001	0.001
Transfer	0.000	0.000	0.000
Avg. H (l shock)	0.000	0.000	0.507 (!)
Avg. F (l shock)	0.000	0.000	0.675(!)

Table 13: 2-sided Wilcoxon rank-sum test for the relevant series. P-values reported refers to the test between the series of the benchmark model and each migration shock scenario. 0.05 (5%), 0.01 (1%)

the overall effect of the shock through the fiscal channel. This happened because given the revenues subtracted from the fixed Deficit targeting Global government triggered a reduction of the economy-wide transfer, worsening the overall effect of the shock. It remains an open question whether with less stringent fiscal constraint the Location agent intervention would have been able to revert the shock effect.

Finally I analyze the effect of the shock and the local policy intervention on the size of the affected Location. Figure 20 presents Box plots for the two studied Locations regarding the average population for both type of Agent. The shock reduced the average population of both agents for each Location, with a more severe effect on the smaller Location. The effect of the local policy intervention seems to have been inconspicuous, with the related Wilcoxon test results rejecting the null hypothesis in the differences in population of agents between the two shock scenarios. The only case where the null is rejected is for the Household agent population in the case of the "Big" Location. In that case we can state that the Location agent was able to maintain an higher share of Households population, compared to the shock scenario.

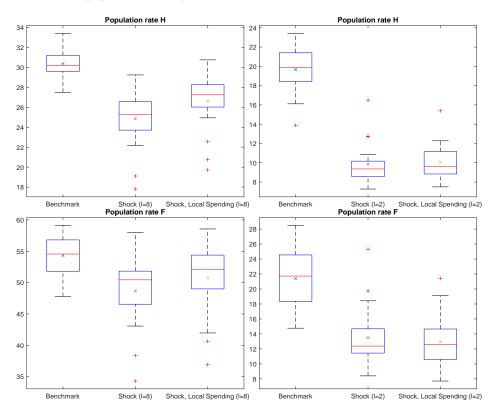


Figure 20: Average population rate in the different scenario for the Locations hit by the shock: Left side: Big Location (L = 8), Right side: Small Location(l=2)

To conclude, asymmetric shocks and related uncoordinated intervention by Local actors should be analyzed carefully by the policy maker, as multiple effects that are not immediate to disentangle might provide wrong answers. In this analysis we showed that an intervention by Local public authorities aimed at countering an asymmetric shock worsened the effect of the shock on the overall economy, mainly because it failed to limit an enhanced relocation behavior by the agents, and because of a reduction of global government transfers as an effort to keep a fixed deficit level. The size of the hit Location matters as well, both in the consequences of the shock, but also on the probability of success by the point of view of the Location agent; in fact, only in the case of the biggest agglomerated Location was the latter able to maintain a greater share of Households.

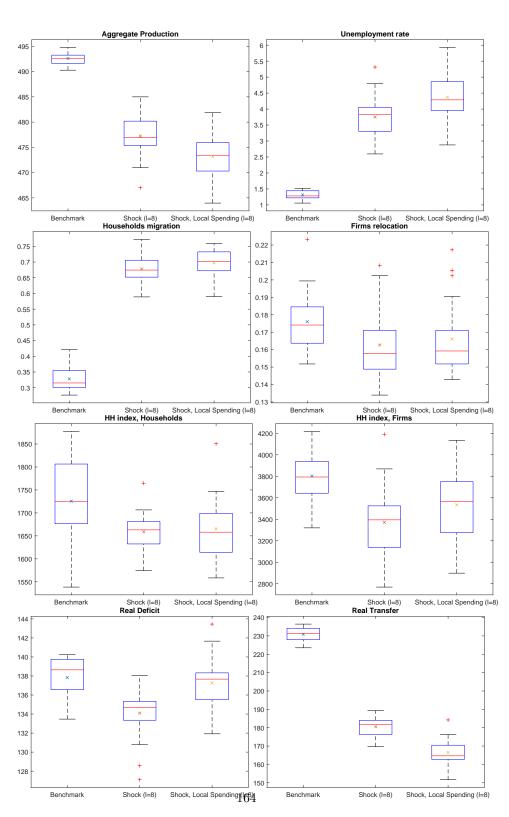


Figure 21: Series for shock to l=8

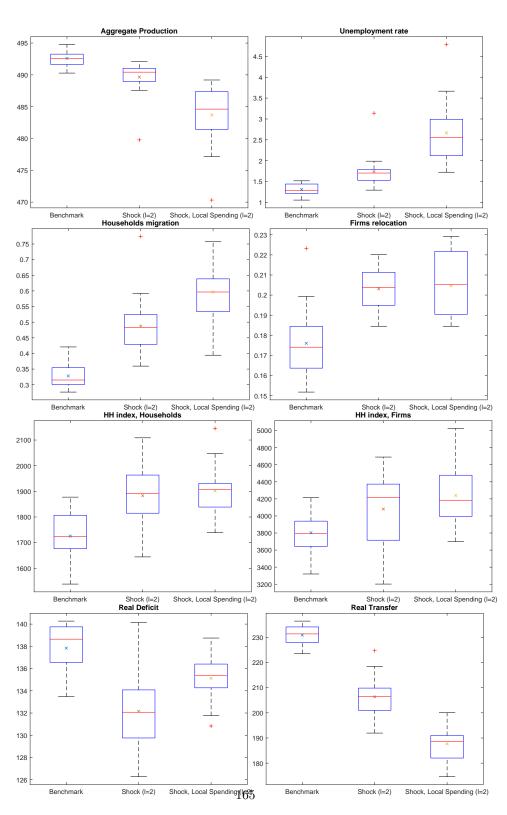


Figure 22: Series for shock to l=2

#### 5.3.2 Exogenous migration wave

In table 14 I present the averages for selected series averaged from 20 simulations for each scenario with varying random seed. Further information are presented in figure 23 where some of the series of table 14 are presented as box plots. In addition to production, unemployment rate and migration and relocation rates I present the box plots for the average HH index for each scenario studied.

From these data we can infer some first exploratory result: the increase of Households increased the average aggregate Production level of the model. This makes sense in the logic of the ABEG model, as more Households increase both production capacity and consumption demand. In terms of series the two scenarios with migration shocks are more "similar", between each other. There are however differences between them, and the main driver of these differences seems to be related to the relocation behavior of Households: in the Global answer scenario, as we can observe from the related box plots, the internal migration of Households is higher on average and more variable.

The higher average Unemployment rate in the migration shock scenarios is partially explained by the "burn-out" period to which  $H^M$  households are subjected. However the differences between the Global and the Local scenarios seems to derive from the enhanced mobile behavior of the Households agent.

In the 15 table I present the Wilcoxon rank-sum test performed on the relevant series, between the benchmark version and the other two policy scenarios, and between the two policy scenarios. Values under 0.05 (0.01) reject the hypothesis of series extracted from the same distribution at the 5% (1%). The main rejections concerns the HH index for Households, while Firm relocation HH index is accepted only at the 5% comparing the two different policy scenarios. Regarding fiscal variables, it is not possible to reject the null hypothesis for the real deficit and the transfer behavior between the Benchmark and the Local version of the policy analysis. In the Global version, the deficit is lower, mainly because of a lower Transfer, which was triggered by higher expenditures towards  $H^M$  Households.

MODEL	Production	Unemployment	Price Inflation rate	Exit rate	Relocation rate	Migration rate
Benchmark	492.59	1.31	2.36	0.22	0.17	0.32
Migration shock - Global answer	512.14	3.25	2.39	0.39	0.28	0.52
Migration Shock - Local answer	514.87	2.84	2.42	0.31	0.31	0.43

Table 14: Main Series Averages

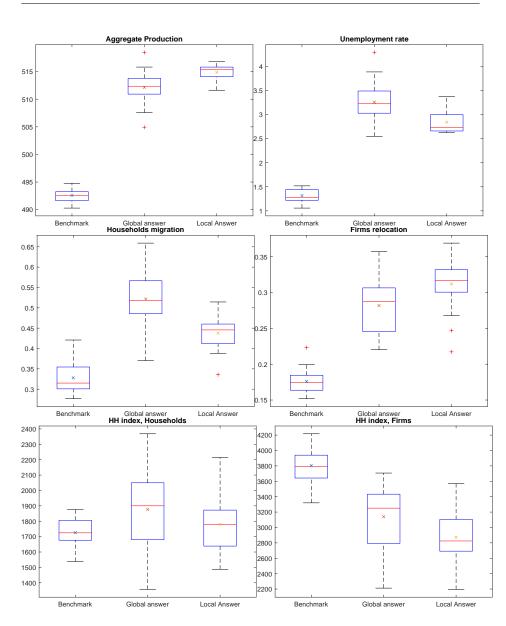


Figure 23: Migration shock, Box plots

Series	Benchmark/Global	Benchmark/Local	Global/Local
Production	0.000	0.000	0.000
Unemployment	0.000	0.000	0.000
Migration H	0.000	0.000	0.000
Migration F	0.000	0.000	0.012
HH index H	0.060	0.323	0.250
HH index F	0.000	0.00	0.027
Real Deficit	0.000	0.113	0.003
Transfer	0.000	0.579	0.000
Avg. H (l shock)	0.000	0.000	0.001
Avg. F (1 shock)	0.000	0.000	0.000

Table 15: 2-sided Wilcoxon rank-sum test for the relevant series. P-values reported refers to the test between the series of the benchmark model and each migration shock scenario. 0.05 (5%), 0.01 (1%)

What happened to the targeted location? How are the spatial long-run consequences of being hit by the migration shock? Surprisingly, the targeted Location did not benefit from the increase in Local population. Both in terms of Households and Firms population, the average results are lower compared to the benchmark model. Indeed it seems that the higher local unemployment rate became higher, prompting a migration of local Households. This effect dominated the lower wage level of the location, preventing Firms to be attracted: the effect of this shock was therefore a dispersion for the targeted location.

The level of spending that differences the global vs local scenario seemed to affect the differences in the spatial distribution. Here the cause seems to be mainly driven by the fact that the local financing can endow less resources to the  $H^M$  households. As we can see from Figure 25, until period 100, when migrants can start to be hired, and the local revenues are therefore enough to pay the same level as the wage benefit, the reduced resources influences negatively the (internal) migration rate of migrants from the arrival location, explaining therefore the lower volatility in Households migration and HH index box plots of figure 23. It seems therefore that the main effect here is more regarding the level of financing and not the original source (Global or local). For sure additional experiments should be run by varying the targeted location

To conclude, Migration booms might have both positive (increased production) as well as negative (increased unemployment) effect on the targeted economy. In a complex spatial setting, which location is targeted, and other features, such as the nature and level of financing for unemployed migrants should be considered as driving determinants of the final outcome of the shock. From a spatial distribution perspective, the change in population affects directly the relevant variables that might drive relocating and migrating behavior. An important role seems to stem also from duration of the regularization process, that affects exactly the level of financing needed to support the integrating population, and

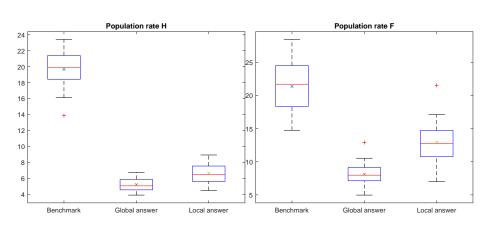


Figure 24: Agents average population rate in the targeted  $l^{\cal M}$ 

the other relevant variables such as the local unemployment rate. A more focused analysis on this dimension would surely provide an interesting endeavor.

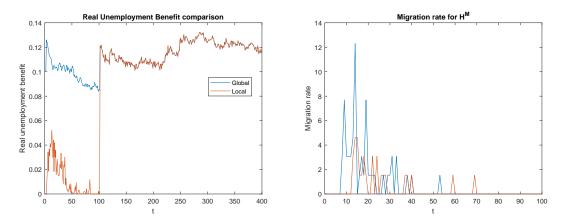


Figure 25: Left panel: Comparison between the real Unemployment benefit for new Households between the Global and Local scenarios. Right Panel: Differences in migration rates of new Households for the first 100 periods for the Global and Local scenarios.

# 6 Concluding remarks

In this work I tried to provide a tool to perform robustness checks about policies that can be influenced by the spatial structure of the economy. To do so I modify the Agent-based Economic Geography model presented in Chapter 1 to include Locations endowed with local policy behavioral rules. These rules can be declared as to follow specific heuristics that reflects different insights from Economic Geography and the tax competition literature.

I then test these scenarios onto two different spatial networks. One is a symmetric Racetrack economy, and the other an asymmetric discretization based on real region (Lombardy, Italy). The latter is calibrated as to present an already agglomerated economy that reflects the provinces relative endowments of Firms and Households. Moreover this spatial network is weighted to represent actual distances.

The simulation results about Tax competition shows that employing a realistic and more asymmetric map might exacerbate the model effects, compared to a symmetric map. This of course depends much on the relocation protocol employed. This is not unconsequential, as more agglomerated map will exacerbate policy differences that would be non-binding in more homogeneous mapping, with a process of cumulative causation not dissimilar to the one of NEG models, as we have seen in the Tax Competition Results section.

While different Tax revision rules had similar economic effects, The Race to the Top policy setting was the more dispersed one. It also presented the highest fiscal consequence. This happened given the major effect on the Relocation protocol metrics.

Public Spendings instead had a greater effect on a symmetric map. Spending policy was dispersive by rising production and lowering unemployment rate, that made Households to migrate less. Policies can therefore have an effect on the unintended agent, negating their purpose.

Some testing on Asymmetric Local policies was also performed. The asymmetric Location reaction to an asymmetric shock worsened its economic outcome. As regarding the local effectiveness in preventing the hit Location to lose Population shares of Firms, only the greatest tested Location had a moderate degree of success. The policy intervention should better match therefore the incentives for relocating agents, or the intervention might bring only the negative downsides.

Migration booms on a single Location rised average production. The hit Location was unable however to become an agglomeration in both cases tested, one with local and one with global financing of the initial Migrated Households income. The level of resources employed, as well as other factors such the duration of the regularization period might more influencing than the source of the financing.

Overall we observed that local policies might become uneffective when applied by a wide number of regions, and considering relocation as well as migration behavior might impact the final outcome of the policy. Also the nature of the spatial map on which the interaction takes place will have an effect: a more agglomerated or dispersed map will react differently to the same policy, often enhancing the nature of the policy being tested, but producing potentially opposite effects with respect to the policy objectives, as was the case of the public procurement scenario.

Employing a model with multiple regions was essential in this setting, since considering only few countries or regions might had lost these effects, missing changes in the distribution inside the chosen geographical unit.

Before taking this results as a given however, some further testing should be performed: in particular each scenario should be simulated more times. Moreover, as the previous analysis showed, the ABEG model is quite sensitive to the choice of the relocation metric, and therefore some sensitivity analysis should be performed, in particular with the metrics that might be more responsive to the local policy action, like the average local profit.

To conclude, an interesting future direction to which extend this framework would be to allow agents to be able to choose from multiple different policy instruments conditionally on relevant endogenous factors in the same model. In addition with the possibility to study different and tailored spatial systems, this would make making policy predictions into complex and hardly analytically tractable environments perhaps more immediate.

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# Appendix

Asset	H	F	G	Σ
Money	12472.78	84348.75	-96821.53	0

Table 16: Initial Assets Matrix (Symmetric model, benchmark Calibration)

Variables \Agents	Н	F	G	Σ
С	-7573.38	+7573.387	-	0
Tr	+3446.08	-	-3446.087	0
$W * H_g$	900	-	-900	0
$W * H_f$	+4620	-4620	-	0
$\overline{w} * W * (H - H_e)$	+216	-	-216	0
Div	+413.47	-413.474	-	0
$T_h$	-1777.61	-	+1777.61	0
$T_f$	-	-886.02	+886.02	0
Y	-	[462]	-	-
$-\Delta NW, \Delta Debt$	-244.56	-1653.89	1898.46	0
$\Sigma$	0	0	0	

Table 17: Transaction flow matrix (Symmetric model, benchmark Calibration)

Asset	Н	F	G	Σ
Money	15024.12.78	128518.82	-143542.94	0

Table 18: Initial Assets Matrix (Asymmetric model, benchmark Calibration)

Variables \ Agents	Н	F	G	Σ
С	-9119.95	+9119.95	-	0
Tr	+4875.84	-	-4875.84	0
$W * H_g$	+900	-	-900	0
$W * H_f$	+4620	-4620	-	0
$\bar{w} * W * (H - H_e)$	+230	-	-230	0
Div	+630	-630	-	0
$T_h$	-1841.29	-	+1841.29	0
$T_{f}$	-	-1350	+1350	0
Y		[462]		-
$-\Delta NW, \Delta Debt$	-294.59	-2519.97	2814.56	0
$\Sigma$	0	0	0	

Table 19: Transaction flow matrix (Asymmetric model, benchmark Calibration)

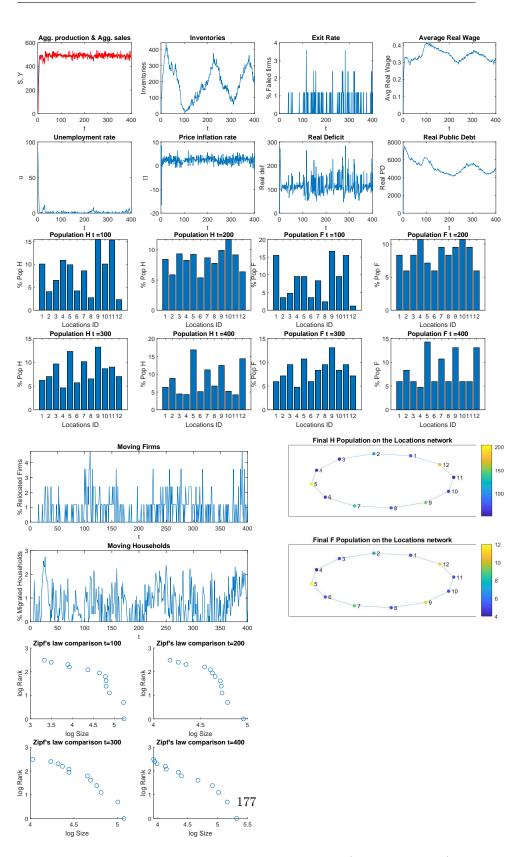


Figure 26: Symmetric map - Series for one iteration (Race to the Top)

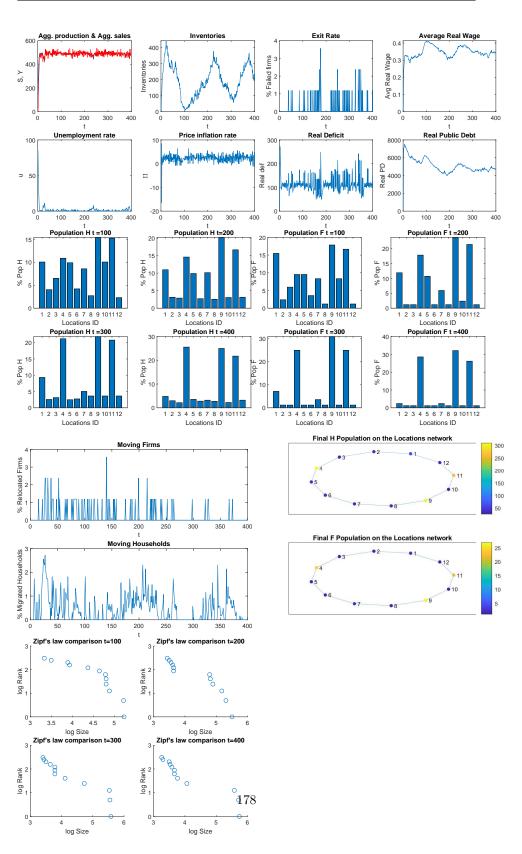


Figure 27: Symmetric map - Series for one iteration (Race to the Bottom)

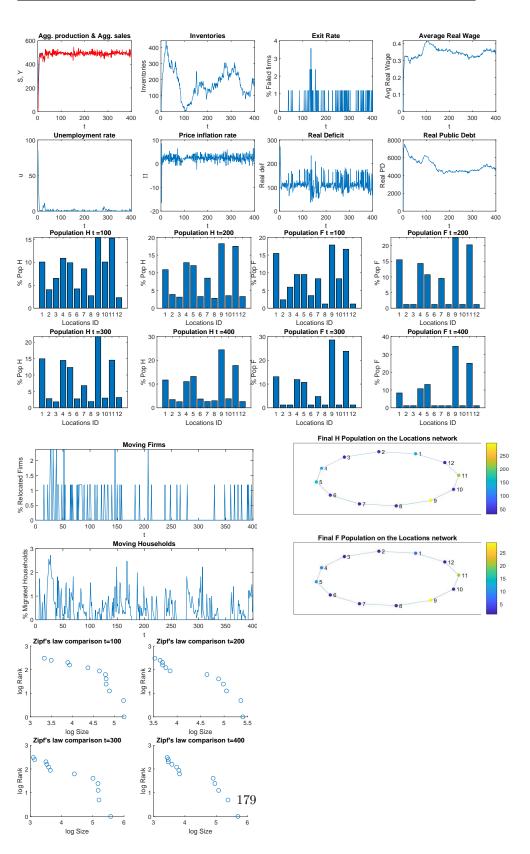


Figure 28: Symmetric map - Series for one iteration (Capture)

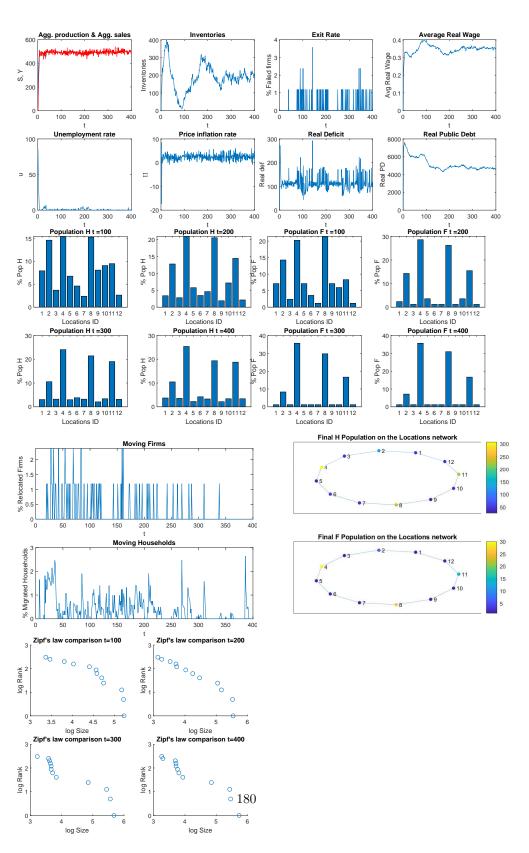


Figure 29: Symmetric map - Series for one iteration (Subsidy model)

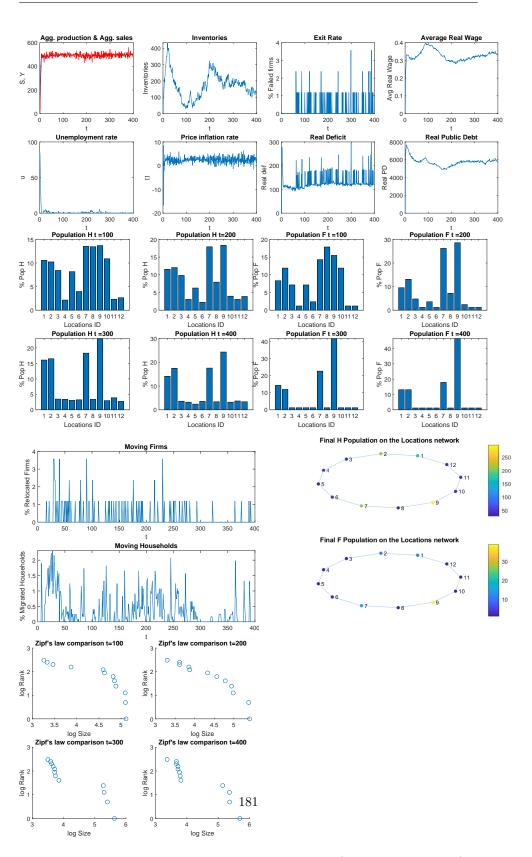


Figure 30: Symmetric map - Series for one iteration (Public Procurement)

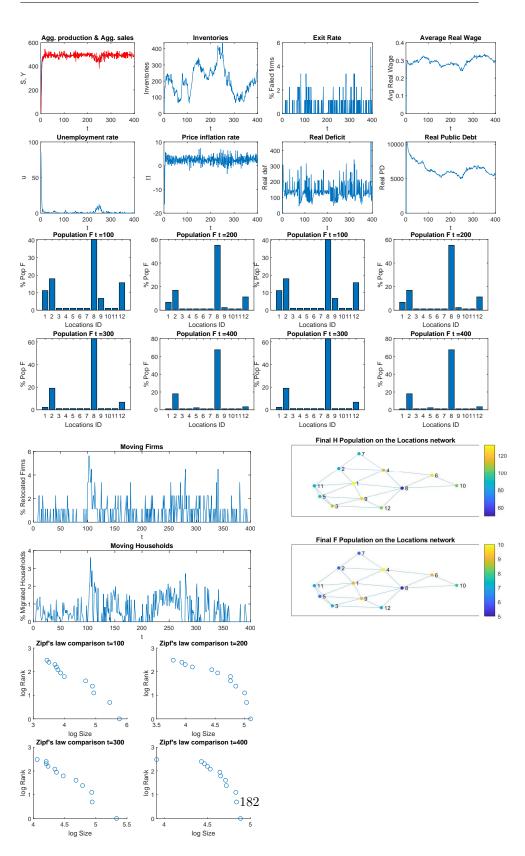


Figure 31: Asymmetric model - Series for one iteration (Race to the Top)

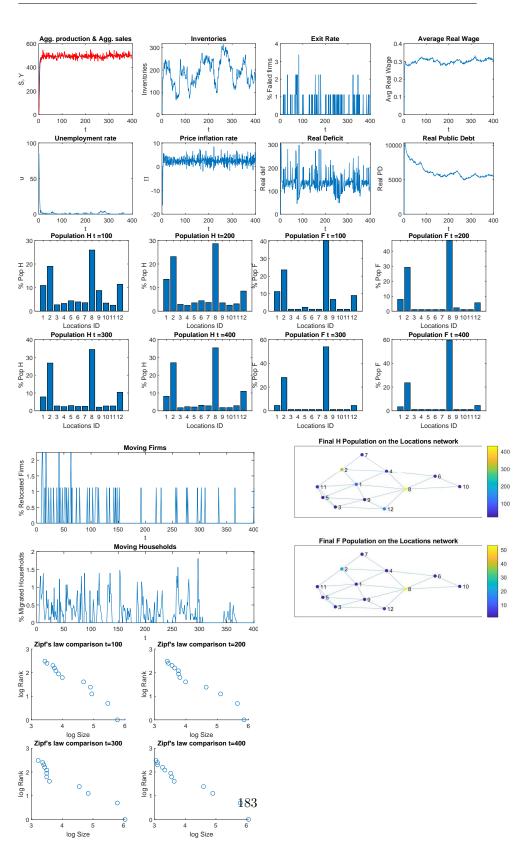


Figure 32: Asymmetric model - Series for one iteration of the Race to the Bottom

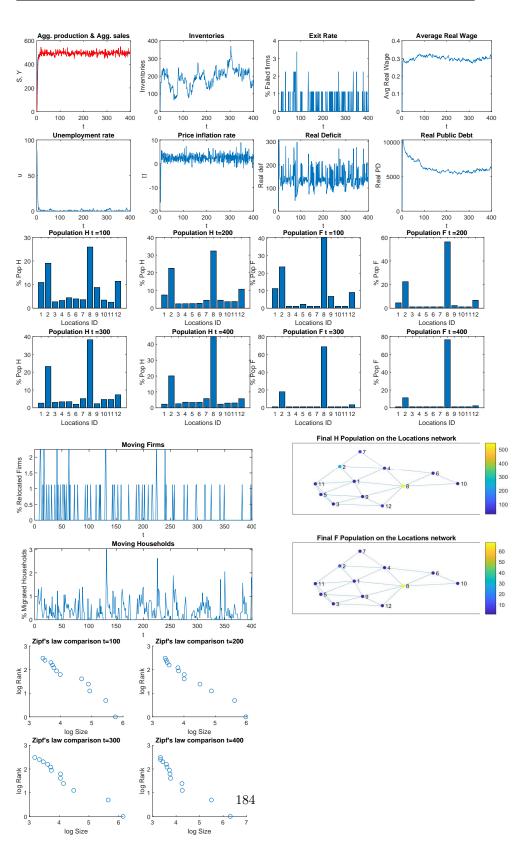


Figure 33: Asymmetric model - Capture

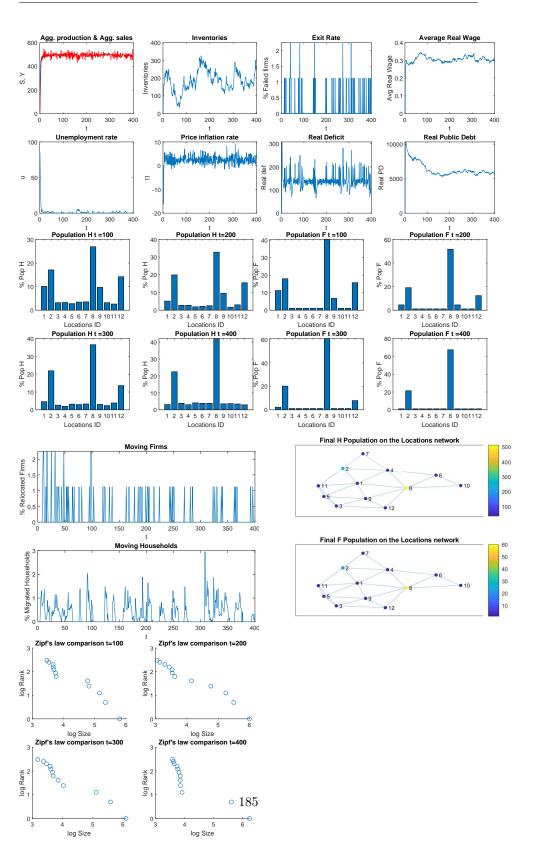


Figure 34: Asymmetric model - Productive Spending

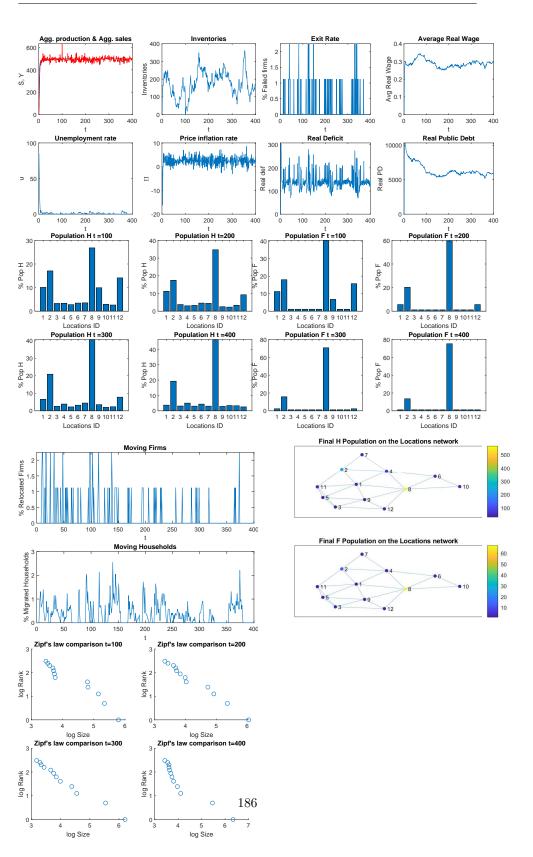


Figure 35: Asymmetric model - Unproductive Spending