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Thesis Title:

Essays on Investment-Specific Technology Progress and Endogenous Market Structure in DSGE Models

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Abstract

Productivity growth drives long run growth and plays a key role in determining business cycle fluctuations. Early stochastic growth models accounted for both aspects assuming investment-neutral technological change. Motivated by the spectacular increase of capital intensity in the production of final goods over the last 50 years, Greenwood, Hercowitz and Krusell (1997) argue in favor of the investment-specific technological (IST hereafter) change being the true driver of growth in the US.

While there is a broad consensus that permanent IST shocks are the key driver for long run growth, the relevance of such shocks at business cycle frequencies is controversial. Greenwood et al. (2000) suggest that IST shocks could play a prominent role for business cycle fluctuations. Fisher (2006) finds that the permanent IST shock is the major source of both growth and business cycle fluctuations. By contrast, Justiniano, Primiceri and Tambalotti (2011) (JPT hereafter) incorporate investment specific technology in an otherwise standard empirical DSGE model of the US. They draw a distinction between technological change that permanently affects the transformation of final into investment goods, and temporary shocks that affect the production of installed capital from investment goods (MEI shocks). They find that, different sectoral productivity trends drive IST relative prices and bear no relevance at busines cycle frequency, whereas MEI shocks are the most important driver of business cycle fluctuations.

The purpose of chapters 1 and 2 of the thesis is to provide new insights on the business cycle implications of permanent IST shocks.

In the first chapter we build a DSGE model incorporating endogenous firm entry and exit in the capital sector, idiosyncratic efficiency (productivity) levels and an endogenous technology diffusion process based on imitation. For sake of tractability, we model technological diffusion as participation to a lottery, where end-of-period incumbents draw their idiosyncratic efficiency from the latest technology frontier introduced by the most recent wave of new entrant firms. This is an innovative feature of our contribution which allows to model endogenous exit flows abstracting from the idiosyncratic evolution of each incumbent's efficiency.

The transmission mechanism we have in mind is as follows. In the sector producing producing investment goods (K-firms), firms are characterized by idiosyncratic efficiency, decreasing returns to scale, and by a fixed production cost. This allows to obtain a distribution of K-firms whose features are crucially determined by entry-exit conditions. In each period new entrants benefit from exogenous advances in the technology frontier, but entry and exit thresholds are affected by the endogenous relative price of investment goods. With a lag, the technology adopted by new entrants spreads to surviving incumbents. As a result, threshold dynamics, caused by endogenous variations in the relative price of investment goods, determine the average efficiency of new entrants and of surviving incumbents. K-firms production shrinks in response to a positive technology shock because the gain from new-entrants productivity is initially swamped by

the reduction in the probability mass of incumbents. This causes a creative destruction in the K-sector production which is then gradually reversed by technology spillovers.

The introduction of technology spillovers relies loosely on Schumpeterian growth theory. In this flavor, and according to the findings of Aghion et al. (2009), when sectors are initially close to the technology frontier (as it is the case for production of durables and equipment, i.e. the K-sector), the threat from innovative entrants triggers incumbents technological innovation which in turn steers (sectoral) productivity growth. We model this intriguing incentives scheme under the form of simple technology spillovers.

To support intuition, we sketch here the implications of a permanent IST shock in our model. The initial inflow of relatively more productive new entrants increases, shifting to the right the supply schedule for investment goods. A "creative destruction" event is triggered by fall in the relative price of investment, which raises the incumbent exit threshold and wipes out the least productive incumbents. However, the technology diffusion process eventually dominates, and production permanently increases in spite of the permanently lower price of investment goods. Numerical simulations show that transition to the new steady state is very persistent.

We can also evaluate the supply-side effects of MEI shocks, which raise demand for K-firms goods and therefore impact on entry-exit thresholds through their effect on K-sector prices. In fact we find that MEI shocks determine a strongly procyclical dynamics of the relative price of investment goods which is at odds with empirical evidence. This questions the plausibility of JPT celebrated result that MEI shocks are the main business cycle driver.

Our characterization of the K-sector endogenous evolution is loosely based on Asturias et al. (2017) who, inspired by the seminal work of Hopenhayn (1992), develop a growth model where firms entry and exit affect productivity through competitive pressures in the economy, but there is no endogenous technology diffusion. Clementi and Palazzo (2016) investigate the role that entry and exit dynamics play in the propagation of aggregate shocks, but neglect the role of sectoral productivity dynamics.

We contribute to a rapidly expanding literature on endogenous entry and exit in DSGE models based on Bilbiie, Ghironi and Melitz (2012) (see also Etro and Colciago (2010); Colciago and Rossi (2015); Devereux, Head and Lapham (1996); Chatterjee and Cooper (1993); Jaimovich and Floetotto (2008)). They focus on business cycle fluctuations whereas we emphasize the interaction between permanent technological change, technology diffusion and fluctuations at business cycle frequencies. In this regard our work is inspired by Sims (2011) and Canova (2014) who emphasize the importance of jointly considering the roles of the persistent but transitory productivity shocks of the RBC-DSGE literature and of the permanent shocks identified in the VAR literature (Galí (1999) and Fisher (2006)).

We also contribute to the literature on technology diffusion. Parente and Prescott (1994) build a model of barriers to technology adoption able to explain per capita income disparity across countries. Among others, Comin and Hobijn (2010) develop a neoclassical growth model of technology diffusion aiming to explain TFP differences at the country level. We are akin to Comin, Gertler and Santacreu (2009), who investigate the role of technology diffusion as business cycle driver and to Anzoategui et al. (2016) focus on the endogenous cyclicality of technology diffusion as to explain the slowdown in productivity following the great recession.

In chapter 2 I extend the previous model by considering also a non-trivial financial sector. In consequence of the great financial crisis, the financial accelerator framework, outlined in the seminal work of Bernanke, Gertler and Gilchrist (1999), has been adapted to feature the prolonged slump in the aggregate production of Western countries characterizing the great recession both in the US and the Euro-Area (see for instance Gertler and Kiyotaki (2010) and Gerali et al. (2010), respectively).

I consider the role of financial intermediaries in a multi-sector DSGE model where growth determinants in the Investment- and Final-goods (I- and F-sectors henceforth) production are allowed to differ. In this respect, we dig into the interactions between a financial friction and a stylized I-sector evolving endogenously in terms of firms entry/exit flows, technology spillovers, and thus, productivity dynamics.

The main focus is twofold. On the one hand, it concerns the role of financial intermediaries when the technologi̇̀cal change can take place in different production sectors. On the other, the study investigates how a financial crisis impacts on endogenous firm dynamics.

In this regard, the model has some specific features. First, I-sector firms are characterized as in chapter 2. Second, the financial sector is modeled as in Gertler and Karadi (2011) (GK henceforth), to emphasize how endogenous balance sheet constraints affecting financial intermediaries can limit the capacity of non financial firms to obtain investment funds. Most importantly, the stylized formulation of our I-sector allows to investigate how endogenous variations in the relative price of investment goods affect banks' balance sheets and impact on the financial transmission mechanism . We highlight how this channel is critical to shape the transmission of investment-specific technology (IST) shocks and banking crisis episodes.

Our first result concerns the transmission of an unexpected permanent investment-specific technology improvement, driven by an inflow of more efficient new firms. This depresses the relative price of investment goods and triggers a process of creative distruction as less efficient incumbents are driven out of the market. In cosequence of this banks assets lose value and the interest rate spread between loans and deposits increases, dampening the expansionary effect of the shock.

The second result concerns the slow recovery characterizing the aftermath of the Great Financial Crisis. Indeed, as documented by Siemer (2016), the financial constraints hitting the economy during the great recession led to an impressive contraction in the inflow of new firms, which contributed by a great extent to the slow recovery. The introduction of our stylized I-sector in an otherwise standard version of GK in fact leads to a milder subsequent recovery following the crisis episode. This is because, in the aftermath of a banking crisis, the inflow of new, more productive firms is dampened by endogenous variations in the relative price of investment goods which are otherwise neglected in the canonical GK model. The intuition is that, when new entrants do not bring any valuable innovation in the market (i.e. the entry flows are not led by any positive IST shock), the sectoral production increase is impaired because of a lower idiosyncratic efficiency of incoming market players.

With this work we mainly contribute, adopting a Schumpeterian growth theory perspective, to the financial friction literature as well as to the one which focuses on investment dynamics. In this regard, it must be said that the related economic literature in a general equilibrium framework is scant. Lorenzoni and Walentin (2007) study how the introduction of a financial constraint alters the comovement of gross investment and the Tobin's Q in an otherwise standard general equilibrium model as in Hayashi (1982). As a result, the authors are able to generate an empirically lower (and thus more plausible) correlation between these two variables even though they abstract from firms entry/exit dynamics. Another strand of the literature, instead, seems to point towards financial frictions impairing the relevance of the IST shock at business cycle frequencies. The first to argue in this direction were Justiniano, Primiceri and Tambalotti (2011) who, by drawing a distinction between a permanent shock affecting the transformation of final into investment goods (IST) and a transitory shock impacting the aggregation of gross investment into the stock of capital (MEI, i.e. marginal efficiency of investment), show that the latter is the major business cycle driver for the US. In particular, they identify the MEI shock as a disruption of the financial system, especially during the great recession, even if their model does not explicitly embed any financial sector formulation. However, Kamber, Smith and Thoenissen (2015) overthrow this view by introducing a collateral constraint a lÃ Kiyotaki and Moore (1997) in an otherwise standard DSGE model as in Smets and Wouters (2007). They find that, in the presence of binding collateral constraints, risk premium shocks dry up the contribution of more general investment shocks over the cycle. Thus, they suggest that investment and risk premium shocks are almost mutually exclusive since the relevance of the former is not robust to more explicit formulations of financial frictions. Similarly, Afrin (2017) introduces the MEI shock in a DSGE model akin to GK and shows that, in such a context, the shock transmission is impaired by the generated countercyclicality of the financial claims price. However, these last two contributions abstract from a true and permanent investment specific technology progress, which we adequately take into account instead, and do not allow for any distinction between the relative price of investment goods and the marginal Tobin's Q.

DISCLAIMER - LIBERATORIA

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I would like to thank my Supervisor Professor Patrizio Tirelli for the continuous support of my Ph.D study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D study.
All errors are and remain my own.

A mia madre e a mio padre che mi hanno insegnato la cultura del lavoro, a mia nonna che mi ha cresciuto, a tutte le persone che sono state e che sono importanti per me, e a Lou...

You know, some people got no choice
And they can never find a voice
To talk with that they can even call their own
So the first thing that they see
That allows them the right to be
Why they follow it
You know, it's called bad luck.

Grazie per avermi dato la possibilità di scegliere.

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Chapter 1

A DSGE Model with Creative Destruction and Knowledge Spillovers

Coauthored with Professor *Patrizio Tirelli*

JEL classification: E13, E22, E30, E32

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1.1 Introduction

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Here we plot the business cycle components of US real GDP and NIPA deflator for durable consumption and private investment from 1947:I to 2015:I (Figure 1.1).

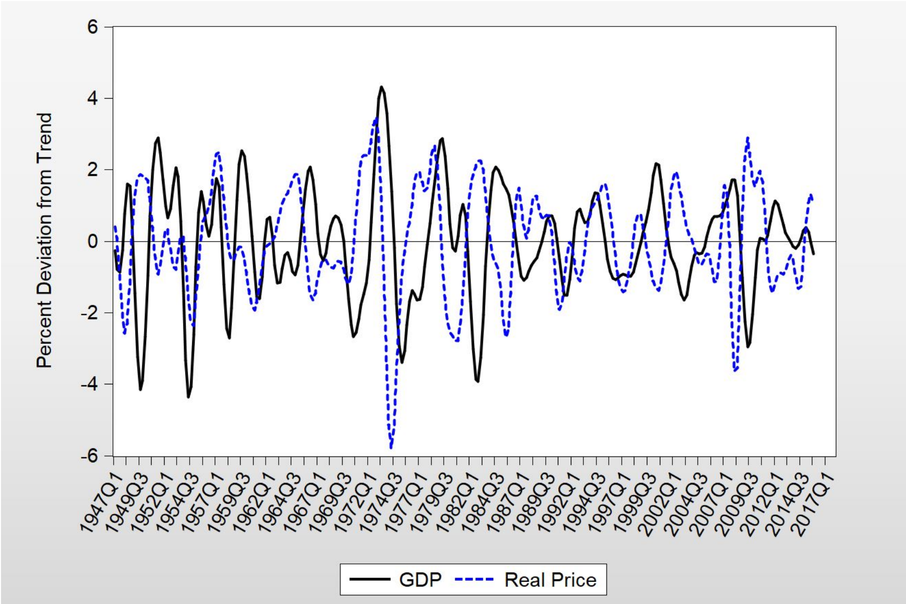

Figure 1.1: Real GDP and Real Price of Investment over the business cycle. Business cycle components are derived using Christiano and Fitzgerald (2003) implementation of the band pass filter as in Fisher (2006).

The unconditional correlation turns out to be −0.22, statistically significant at 5%. Such a negative comovement is typically rationalized by postulating exogeneity of new technology diffusion, i.e. highly persistent technology shocks. Fisher (2006) finds a correlation of −0.54 by using the GVC deflator for equipment as measure for the relative price of investment. Indeed our estimate should be considered a lower bound since, as documented in JPT, the NIPA deflator for gross investment is fairly less countercyclical than when only equipment are considered.

Our purpose here is to provide new insights on the business cycle implications of permanent IST shocks. We build a DSGE model incorporating endogenous firm entry and exit in the capital sector, idiosyncratic efficiency (productivity) levels and an endogenous technology diffusion process based on imitation. For sake of tractability, we model technological diffusion as participation to a lottery, where end-of-period incumbents draw their idiosyncratic efficiency from the latest technology frontier introduced by the most recent wave of new entrant firms. This is an innovative feature of our contribution which allows to model endogenous exit flows abstracting from the idiosyncratic evolution of each incumbent's efficiency.

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The paper is organized as follows. Sections 1.2 describes the model economy. Section 1.3 is devoted to the interpretation of our results. Section 1.4 concludes. Technical details are left to the Appendix.

1.2 The model economy

They key players in the economy are K- and F-firms, respectively producing investment and final goods. K-firms are endowed with a decreasing returns to scale technology, are characterized by idiosyncratic efficiency levels and face both variable and fixed production costs. The K-sector is characterized by entry and exit flows of firms. Our analysis emphasizes the distinct roles played by New Entrants (NEs), who draw their idiosyncratic efficiency levels from a new more productive technology distribution, and Incumbents (INCs), who remain in the market as long as their profits are non negative. Final goods producers are characterized by a permanent labor-augmenting stochastic technology shifter. This will allow to investigate the distinct roles played by capital augmenting and by neutral technological change.

Households supply labor in the competitive labor market and choose consumption and savings, determining the accumulation of capital. When chosing the desired stock of capital to be used in next-period production, households transfer savings to K-producers who transform them into investment goods. We also assume that when assembling investment goods into the capital stock households bear investment adjustment costs.

The sequence of events, summarized in the diagram below, is as follows. At time t, F-firms exploit factor services to produce goods which are sold to households. Households consume, save, supply labor and capital services. At the end of time t, our K-firms sector is made of a measure η_t of active firms distributed between new entrants, NE_t, and incumbents, INC_t, survived from period $t-1$.

$$\eta_t = NE_t + INC_t \tag{1.1}$$

K-firms are endowed with household savings, in the form of final goods, to produce investment goods which are then sold back to households who bear some adjustment costs in aggregating the new stock of capital.

At the beginning of period $t+1$ K-firms engage in technology updating, characterized as a random draw from the technology distribution introduced in the economy by NE_t firms.

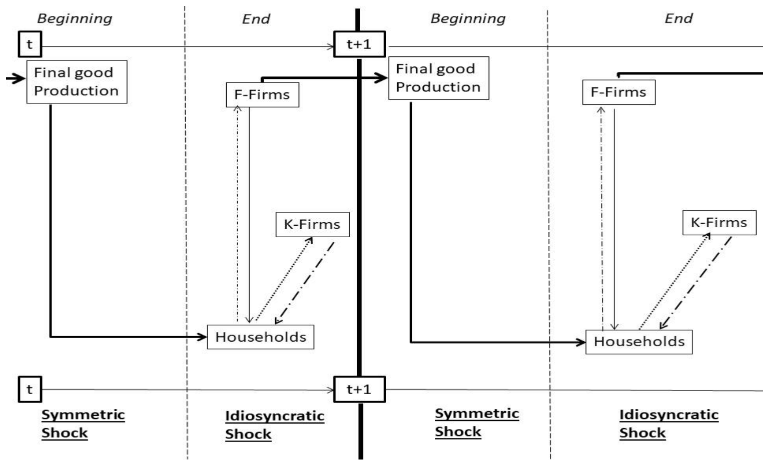

Figure 1.2: Flow of events in our economy.

1.2.1 Final Good Producers

Monopolistically competitive retail firms assemble the final good bundle Y_t using a continuum of intermediate inputs Y_t^h. The representative firm profit maximization problem is:

$$\max_{Y_t, Y_t^h} P_t Y_t - \int_0^1 P_t^h Y_t^h dh$$

$$\text{s.t. } Y_t = \left[\int_0^1 \left(Y_t^h \right)^{\frac{\nu-1}{\nu}} dh \right]^{\frac{\nu}{\nu-1}}$$

From the first order conditions, we obtain:

$$Y_t^h = \left(\frac{P_t^h}{P_t} \right)^{-\nu} Y_t \tag{1.2}$$

$$P_t = \left[\int_0^1 \left(P_t^h \right)^{1-\nu} dh \right]^{\frac{1}{1-\nu}} \tag{1.3}$$

Price stickiness is based on the Calvo mechanism. In each period retail firms face a probability $1 - \lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t^h = \pi_{t-1}^{\gamma_p} p_{t-1}^h \tag{1.4}$$

where $\gamma_p \in [0, 1]$ represents the degree of price indexation.

All the $1 - \lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$, the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t + s$ will read as $\widetilde{P}_t \left(\Pi_{t,t+s-1}^p \right)^{\gamma_p}$ where $\Pi_{t,t+s-1} = \pi_t ... \pi_{t+s-1} = \frac{P_{t+s-1}}{P_{t-1}}$.
$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s} \right) Y_{t+s}^h$$

subject to:

$$Y_{t+s}^h = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\chi}}{P_{t+s}} \right)^{-\nu} \tag{1.5}$$

where Y_t^d is aggregate demand and $\bar{\Lambda}_t$ is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\nu) \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{1-\nu} \widetilde{P}_t^{-\nu} \left(P_{t+s} \right)^{\nu} + \\ +\nu \widetilde{P}_t^{-\nu-1} P_{t+s}^{\nu+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{-\nu} \end{array} \right] = 0 \tag{1.6}$$

Intermediate good firms

Intermediate firms h are perfectly competitive, hire labor from households and exploit capital rented at the end of period $t-1$. Their production function is

$$Y_t^h = (z_t^n N_t)^{\chi} K_{t-1}^{1-\chi} \tag{1.7}$$

where N defines worked hours, K is the capital stock, z^n is a permanent labor augmenting technology shifter (LAT hereafter), such that $z_t^n = z_{t-1}^n g_{z,t}$ where

$$\ln(g_{z,t}) = (1-\rho_z)\ln(g_*) + \rho_z \ln(g_{z,t-1}) + \sigma^z \varepsilon_t^z \tag{1.8}$$

and $\varepsilon_t^z \sim N(0,1)$. The LAT shifter embeds a deterministic trend component, g_*, which is also the BGP gross rate of the economy.

Profits are as follows

$$\Pi_t^{fg} = mc_t Y_t^h - W_t N_t - r_{k,t} K_{t-1} \tag{1.9}$$

where W_t and $r_{k,t}$ respectively define the real wage and the rental rate of capital defined in consumption goods. Cost minimization implies

$$K_{t-1} = \frac{W_t}{r_{k,t}} \frac{(1-\chi)}{\chi} N_t$$

the real marginal costs are:

$$mc_t = \xi_t \left(\frac{r_{k,t}}{1-\chi}\right)^{1-\chi} \left(\frac{W_t}{z_t^n \chi}\right)^{\chi} \tag{1.10}$$

1.2.2 The Representative Household

We assume a standard characterization of households preferences,

$$U_t(C,N) = \sum_{i=0}^{\infty} \beta^{t+i} \left\{ \ln(C_{t+i} - aC_{t+i-1}) - \Phi \frac{N_{t+i}^{1+\theta}}{1+\theta} \right\} \tag{1.11}$$

Parameter a defines internal consumption habits. The flow budget constraint in real terms is

$$C_t + Q_t I_t + \frac{B_t}{P_t} = R_{n,t-1} \frac{B_{t-1}}{P_t} + r_{k,t} K_{t-1} + W_t N_t + \Pi_t^{K,F} \tag{1.12}$$

where Q is the relative price of investment goods, I defines investment goods, $\Pi^{K,F}$ are profits rebated by final and K-firms, and B is a nominally riskless bond of one-period maturity with gross nominal remuneration R_n.[1] The law of motion of capital is

$$K_t = (1-\delta)K_{t-1} + \mu_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{1.13}$$

[1] In this model there is no fiscal sector, therefore bonds are in zero net supply.

where δ is the capital depreciation rate, $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ defines investment adjustment costs, μ_t^i is a shock to the marginal efficiency of investment (i.e., MEI) as in JPT:

$$\ln\left(\mu_t^i\right) = \rho_{\mu^i} \ln\left(\mu_{t-1}^i\right) + \sigma^i \varepsilon_t^i. \tag{1.14}$$

and $\varepsilon_t^i \sim N(0,1)$ is an i.i.d. innovation term.

The FOCs are

$$\lambda_t = \left(C_t - aC_{t-1}\right)^{-1} - \beta a \left(C_{t+1} - aC_t\right)^{-1} \tag{1.15}$$

$$\frac{\Phi N_t^\theta}{\lambda_t} = W_t \tag{1.16}$$

$$Q_t = \left\{ \begin{array}{c} \varphi_t^k \mu_t^i \left[1 - \left(S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}}\right)\right)\right] + \\ +\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t}\varphi_{t+1}^k \mu_{t+1}^i S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array} \right\} \tag{1.17}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[\frac{r_{k,t+1}}{\varphi_t^k} + \frac{\varphi_{t+1}^k}{\varphi_t^k}(1-\delta) \right] \right\} \tag{1.18}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{R_{n,t}}{\pi_{t+1}} \right\} \tag{1.19}$$

Where $\varphi_t^k = \frac{\phi_t}{\lambda_t}$ is the shadow value of the capital stock in units of consumption goods (or in other words the marginal Tobin's Q), ϕ_t is the Lagrange multiplier of the law of motion of capital and $S'(\cdot) \equiv \gamma_I \left(\frac{I_{t+i}}{I_{t+i-1}} - 1\right)$.

1.2.3 The K-sector

At the end of period t, K-sector firm j is characterized by the following production function

$$I_t^{K,j} = A_t^{K,j}\left(S_t^{K,j}\right)^\alpha$$

where $K = NE, INC$ defines whether the firm is a new entrant or an incumbent one. $\alpha < 1$ implies that production occurs under decreasing returns to scale. $A_t^{K,j}$ is the idiosyncratic efficiency level. In terms of final goods, profits are

$$\Pi_t^{K,j} = Q_t I_t^{K,j} - S_t^{K,j} - f_t^K \tag{1.20}$$

where f_t^K is a fixed production cost such that $f_t^k = g_*^t f^k$, where we assume that K-firms fixed costs dynamics has the same deterministic trend of the LAT shifter, z_t^n [2]. K-producers operate only if they earn non-negative profits because idiosyncratic efficiency is fully observable

[2] This assumption is needed in order to avoid that the LAT shock impacts also the fixed costs structure.

by households and moral hazard problems do not arise by assumption. Therefore, the K-firm maximization problem boils down to a static one, and is solved maximizing profits with respect to $S_t^{K,j}$:

$$S_t^{K,j} = \left(Q_t \alpha A_t^{K,j}\right)^{\frac{1}{1-\alpha}} \tag{1.21}$$

By plugging (1.21) into the zero-profits condition (1.20) we obtain the efficiency threshold that defines operating K-firms:

$$\hat{A}_t^K = \left(\frac{f_t^K}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t \alpha^\alpha} \tag{1.22}$$

Note that $\hat{A}_t^K$ positively related to the fixed cost of production and is a negative function of the investment goods relative price. Thus entry and exit decisions are endogenous to any shock which affects Q_t.

New Entrants

The market entry decision at the end of time t is conditional to firm NE, j idiosyncratic productivity level $A_t^{NE,j}$. A unit probability mass of potential NEs draw their individual $A_t^{NE,j}$ every period from a new and more efficient Pareto distribution[3]

$$f_t(A_t^{NE}) = \int_{e_t}^{+\infty} \frac{\gamma e_t^\gamma}{\left(A_t^{NE}\right)^{\gamma+1}} d(A_t^{NE}) = 1 \;\; with \;\; A_t^{NE} \geq e_t \tag{1.23}$$

where γ is the tail index describing the distribution skewness. The NEs technology frontier embeds a stochastic trend: $e_t = e_{t-1} g_{e,t}$, where

$$\ln(g_{e,t}) = (1-\rho_e)\ln(g_e) + \rho_e \ln(g_{e,t-1}) + \sigma^e \varepsilon_t^e \tag{1.24}$$

and $\varepsilon_t^e \sim N(0,1)$.[4] The NEs efficiency draws technology shock consists of a sudden and unexpected shift to the right of the potential NEs' pfd virtually keeping the NEs cutoffs (1.22) fixed. This causes an inflow of a higher mass of more productive NEs in the market strengthening competition among all K-firms. Indeed, it is theoretically consistent with what is known in the DSGE literature as the Investment Specific Technology shock (IST). Moreover, notice that, in order to ensure the existence of a BGP, $g_*^{1-\alpha} = g_e$ must hold in the deterministic steady state[5].

The mean of (1.23)

$$\mu\left(A_t^{NE}\right) = \frac{\gamma}{\gamma-1} e_t. \tag{1.25}$$

is driven by the lower bound of the support defining the pdf which grows at the gross rate g_e in the deterministic steady state. The probability mass of effectively entering NE firms is

[3] The formulation of the potential NEs efficiency problem is a simplification of the one presented in Asturias et al. (2017)

[4] To ease the burden of notation, we abstract from the idiosyncratic index j.

[5] See section B.3.2 in Appendix.

obtained by cutting the pfd in (1.23) at the NEs threshold , $\hat{A}_t^{NE}$, obtained from eq. (1.22):

$$NE_t \equiv f_t(\hat{A}_t^{NE}) \equiv \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\gamma e_t^{\gamma}}{\left(A_t^{NE}\right)^{\gamma+1}}\, d(A_t^{NE}) = \left[Q_t \alpha^{\alpha} e_t \left(\frac{1-\alpha}{f_t^{NE}} \right)^{1-\alpha} \right]^{\gamma} \tag{1.26}$$

Incumbents

At the end of period $t-1$ the mass of active K-firms, is

$$\eta_{t-1} = NE_{t-1} + INC_{t-1}$$

At the beginning of t, INC_{t-1} firms observe NE_{t-1} K-firms technological level and update their plants accordingly. For sake of simplicity, we model the updating process as a lottery where all η_{t-1} firms draw their individual $A_t^{INC,j}$ from the Pareto distribution with support $\left[\hat{A}_{t-1}^{NE}, +\infty\right)$ that characterizes NE_{t-1} firms. Such a formulation brings the advantage of modeling endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent's efficiency level.

Thus, at the beginning of period t, survived NEs and INCs from $t-1$ are grouped into the Pareto pdf defining the new idiosyncratic productivity level for each incumbent K-firm

$$f_t(A_t^{INC}) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\gamma \left(\hat{A}_{t-1}^{NE}\right)^{\gamma}}{\left(A_t^{INC}\right)^{\gamma+1}}\, d(A_t^{INC}) \tag{1.27}$$

but only firms that satisfy the non negative profits condition (1.22) will survive in the market and thus be active in t. In other words, the mass of INC_t is obtained as the fraction of η_{t-1} computed over the support share $\left[\hat{A}_t^{INC}, +\infty\right)$ of (1.27), where $\hat{A}_t^{INC} = \left(\frac{f^{INC_t}}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t \alpha^{\alpha}}$ defines INC_t firms cutoff:

$$\begin{aligned} INC_t \equiv \eta_{t-1} f_t(\hat{A}_t^{INC}) \equiv \eta_{t-1} \int_{\hat{A}_t^{INC}}^{+\infty} \frac{\gamma \left(\hat{A}_{t-1}^{NE}\right)^{\gamma}}{\left(A_t^{INC}\right)^{\gamma+1}}\, d(A_t^{INC}) = \\ = \quad \eta_{t-1} \left[\hat{A}_{t-1}^{NE} \left(\frac{1-\alpha}{f_t^{INC}} \right)^{1-\alpha} Q_t \alpha^{\alpha} \right]^{\gamma} = \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}} \right)^{1-\alpha} \frac{Q_t}{Q_{t-1}} \right]^{\gamma} \end{aligned} \tag{1.28}$$

which allows us to rewrite the law of motion for the mass of active firms as

$$\eta_t = \left[Q_t \alpha^{\alpha} e_t \left(\frac{1-\alpha}{f_t^{NE}} \right)^{1-\alpha} \right]^{\gamma} + \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}} \right)^{1-\alpha} \frac{Q_t}{Q_{t-1}} \right]^{\gamma} \tag{1.29}$$

Finally, the mass of exiting K-firms in t is

$$EXIT_t = \eta_{t-1} \left\{ 1 - \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}} \right)^{1-\alpha} \frac{Q_t}{Q_{t-1}} \right]^{\gamma} \right\} \tag{1.30}$$

Needless say, the dynamics of the relative price of investment goods is key to determine the K-firms sector expansion.

K-firms production and the process of creative distruction

Supply functions for *NE* and *INC* firms are easily computed.[6]

$$
\begin{aligned}
I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE,j} \cdot \left(Q_t \alpha A_t^{NE,j}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{NE,j}) \\
&= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t \alpha)^{\frac{\alpha}{1-\alpha}} \\
&= \left[\alpha^\alpha e_t (1-\alpha)^{1-\alpha}\right]^\gamma \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{\left(f_t^{NE}\right)^{(1-\alpha)\gamma-1}}
\end{aligned} \tag{1.31}
$$

$$
\begin{aligned}
I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC,j} \cdot \left(Q_t \alpha A_t^{INC,j}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{INC,j}) \\
&= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t \alpha)^{\frac{\alpha}{1-\alpha}} \\
&= \eta_{t-1} \left[\frac{\left(f_t^{NE}\right)^{1-\alpha}}{Q_{t-1}}\right]^\gamma \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{\left(f_t^{INC}\right)^{(1-\alpha)\gamma-1}}
\end{aligned} \tag{1.32}
$$

From (1.31) it is easy to see that a shock to e_t shifts to the right the NE_t supply of investment goods. For any given demand for investment goods (1.17) this puts downward pressure on in the relative price Q_t. As a result, from (1.28) and (1.32) it is easy to see that both the mass of surviving incumbents and their supply of investment goods shrinks. This is the essence of the "creative destruction" process triggered by IST shocks. In Section 1.3.1 we fully characterize dynamics associated to such a shock.

1.2.4 Market clearing and policy rules

The K-goods and F-goods market clearing conditions respectively are

$$I_t = I_t^{NE} + I_t^{INC} \tag{1.33}$$

$$Y_t - NE_t f_t^{NE} - INC_t f_t^{INC} = C_t + S_t \tag{1.34}$$

where

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE,j}\right) dF(A_t^{NE,j}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC,j}\right) dF(A_t^{INC,j}) \tag{1.35}$$

is the amount of input demanded for K-goods production coinciding with households savings and thus

[6] See Appendix B.4 for details of the derivation.

$$I_t = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE,j} \cdot \left[S\left(A_t^{NE,j}\right)\right]^{\alpha} dF(A_t^{NE,j}) + \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC,j} \cdot \left[S\left(A_t^{INC,j}\right)\right]^{\alpha} dF(A_t^{INC,j}) \quad (1.36)$$

The K-sector average productivity level, $\bar{A}_t$, is

$$\bar{A}_t = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE,j} dF(A_t^{NE,j}) + \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC,j} dF(A_t^{INC,j}) \quad (1.37)$$

Finally, we assume that the Central Bank controls monetary policy by means of a simple Taylor rule with interest rate smoothing

$$\left(\frac{R_{n,t}}{R_n^{ss}}\right) = \left(\frac{R_{n,t-1}}{R_n^{ss}}\right)^{\rho_{R_n^{ss}}} \left[\left(\frac{\pi_t}{\pi^{ss}}\right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu-1)/\nu}\right)^{\kappa_y} \exp\left\{\sigma^r \varepsilon_t^r\right\}\right]^{1-\rho_{R^{n^{ss}}}} \quad (1.38)$$

1.2.5 Solution and Calibration

The trends in sectorial technologies render the model non-stationary even though in this formulation final output, capital and investment are assumed to grow at the same BGP rate, whilst hours worked are stationary as well as (for sake of simplicity) the relative price of investment goods[7]. The details on the existence of a BGP are left in Section A.3. For what concerns the stochastic simulation of the model, stationarity is obtained by appropriate variable transformation (see sections A.1 and A.2). The model is solved by means of a first order perturbation method and the list of stationary log-linearized equations is reported in Section A.7.

We assume the BGP rate of the economy to be $g_* = 1.004$ on quarterly basis. For sake of simplicity we set the *NE*s fixed cost of entry at 1% of final output. We calibrate the model at quarterly frequency, thus we impose $\beta = 0.99$ to obtain a yearly value of $r_k^{ss} = 0.0351$. We calibrate Φ at a conventional value such that $N^{ss} = 0.3333$ and $\theta = 0.276$. The capital depreciation rate is conventional: $\delta = 0.025$. We also set *NE*s as to be the 10% of total firms on annual basis in order to match the US business destruction rate as in Etro and Colciago (2010). Then, we normalize the steady state relative price of capital, $Q^{ss} = 1$, and the technology shifter in ss $z^n = 1$. The final goods elasticity of substitution, ν, is set equal to 11, the probability of not updating prices is $\lambda_p = 0.779$ and price indexation coefficient is $\gamma_p = 0.241$. Following JPT, the parameter governing investment adjustment costs is $\gamma_I = 3.142$.

The K-producers returns to scale, α, the *NE*s technology shifter, e^{ss}, and the tail index of the Pareto distribution, γ are calibrated to obtain that Their calibration must be consistent with the fact that, from the law of motion of capital, $I_t^{ss} = 0.2049$ and consumption output ratio is $\approx 80\%$. In other words K-producers must be distributed in a way such that in equilibrium $Q^{ss} = 1$. To do this, we impose $\alpha = 0.8$ and $\gamma = 6.1$, this pins down the initial condition for the NEs technology shifter, $e^{ss} = 0.3397$. The calibration of α is at the lower bound of Basu and Fernald (1997) estimates, whilst the value assigned to γ is set to resemble Asturias et al.

[7]The model could be easily extended to embrace a trend for Q.

(2017)[8]. Finally, concerning the Taylor rule, we set the interest rate smoothing coefficient, $\rho_{R_n^{ss}}$, equal to 0.8 and the relative coefficient on inflation and outputgap, κ_π and κ_y, are equal to 1.5 and 0.125, respectively. Finally, the MEI shock persistence, ρ_{μ^i}, is equal to 0.813 as in JPT and the IST shock peristence, ρ_e, is nil as to allow for the IST shock propagation fully due to our endogenous updating mechanism.

Parameters calibration is summarized in Table 1.1.

Table 1.1: **Parameters**

Households		
g_*	1.004	Gross BGP rate
β	0.994	Discount factor
a	0.815	Habit parameter
δ	0.025	Capital depreciation
γ_I	3.142	Investment adjustment costs
θ	0.276	Inverse Frisch elasticity of labor supply
N^{ss}	0.3333	SS labor
Retailers		
ν	11	Final goods elasticity of substitution
λ_p	0.779	Probability of not updating prices
γ_p	0.241	Price indexation parameter
Intermediate Producers		
z^n	1	L-shifter (LAT) initial condition
χ	0.67	Labor share of income
K-firms		
g_e	$g_*^{1-\alpha}$	Technology frontier (IST) BGP
α	0.8	K-producers returns to scale
e^{ss}	0.3397	Technology frontier (IST) initial condition
f^{NE}	0.01*Y	Entry Cost initial condition
$1 - H^{ss}$	0.025	Share of NEs over total K-firms
γ	6.1	Tail index of K-firms distributions
Central Bank		
κ_π	1.5	Taylor Rule inflation coefficient
κ_y	0.125	Taylor Rule output coefficient
$\rho_{R_n^{ss}}$	0.8	Interest rate smoothing
Exogenous Processes		
ρ_{μ^i}	0.813	MEI shock persistence
ρ_e	0	IST shock growth persistence
ρ_z	0	LAT shock growth persistence
σ^e	0.05	e_t shock sd
σ^z	0.01	z_t^n shock sd
σ^{μ^i}	0.05786	MEI shock sd

1.3 Impulse Response Analysis

1.3.1 The IST shock

To begin with, we exploit steady state derivations in section A.3 in Appendix to characterize the long run effects of the shock. Note that from conditions (A.78) and (A.79) it would be

[8] In order to show that our results are not specific to particular parametrizations of the K-sector, we perform a sensitivity analysis of our key results in Section A.6.

straightforward to show that the shock causes a permanent increase in the supply of investment goods which is associated to a permanent fall in their relative price. Numerical calculations show that a 5% white noise shock to the RW component of e_t causes a 6.91% increase in investment and a 4.63% fall in Q. This dynamics is qualitatively in line with the empirical evidence (see Greenwood, Hercowitz and Krusell (1997) and Fisher (2006)). Technology diffusion through incumbents' technology updating is crucial to determine this result. Given condition (A.69) this causes a fall in r_k and an increase in the capital labor ratio. As a result, in the medium run, higher wages determine an increase in the labor supply which then slowly reverts back to its old steady state value. Table 1.2 reports the steady state adjustments of key variables to a permanent 5% IST shock.

Table 1.2: **Variables deviations from old SS to 5% permanent IST shock**

Variable	**%Δ from old ss**
Y	2.28
C	2.28
I	6.91
K	6.91
Q	-4.63
W	2.28
η	2.28

Let us now turn to the analyis of IRFs, i.e. Figures 1.3 and 1.4. The stochastic trend is added back in order to visualize the steady state transition path from the old to the new steady state.

Given the stochastic process defining e_t, from condition (1.26) we know the IST shock implies an inflow of more productive NEs which shifts the supply schedule (1.31) to the right.

The fall in the relative price of investment goods raises the efficiency level of incumbent firms which is necessary to meet their zero-profit condition. As a result the mass of exiting incumbents increases. At the end of the initial period, surviving incumbents adopt the new technology shifting to the right the supply of investment goods and further lowering the the relative price of investment goods. This fall begins to raise the NEs cutoff, gradually bringing down their number. New technology adoption gradually allows more incumbent firms to survive in the market, but this process is very slow. In fact, it takes 20 periods before INCs mass returns to the initial steady state level.

The total mass of active firms, η, shrinks for a few periods and then begins to pick up again. This "creative destruction" effect characterizes the early phase of the adjustment to the shock and is reinforced by the sluggish demand for investment goods. In fact, in spite of the immediate fall in Q, which calls for greater demand, the expectation of further reduction in their relative price induces households to postpone investment, which remains below the initial steady state value for 10 quarters. Consumption remains almost constant for the first

20 periods and then begins to pick up. Weak investment demand implies that capital goods producers initialy decrease their demand for final goods. As a result output immediately falls, initiating a moderate four-years-lasting recession. This pattern, in turn, drives the evolution of employment.

The results we have just shown are not specific of the DSGE version of the model as an RBC formulation yields virtually the same impulse responses.

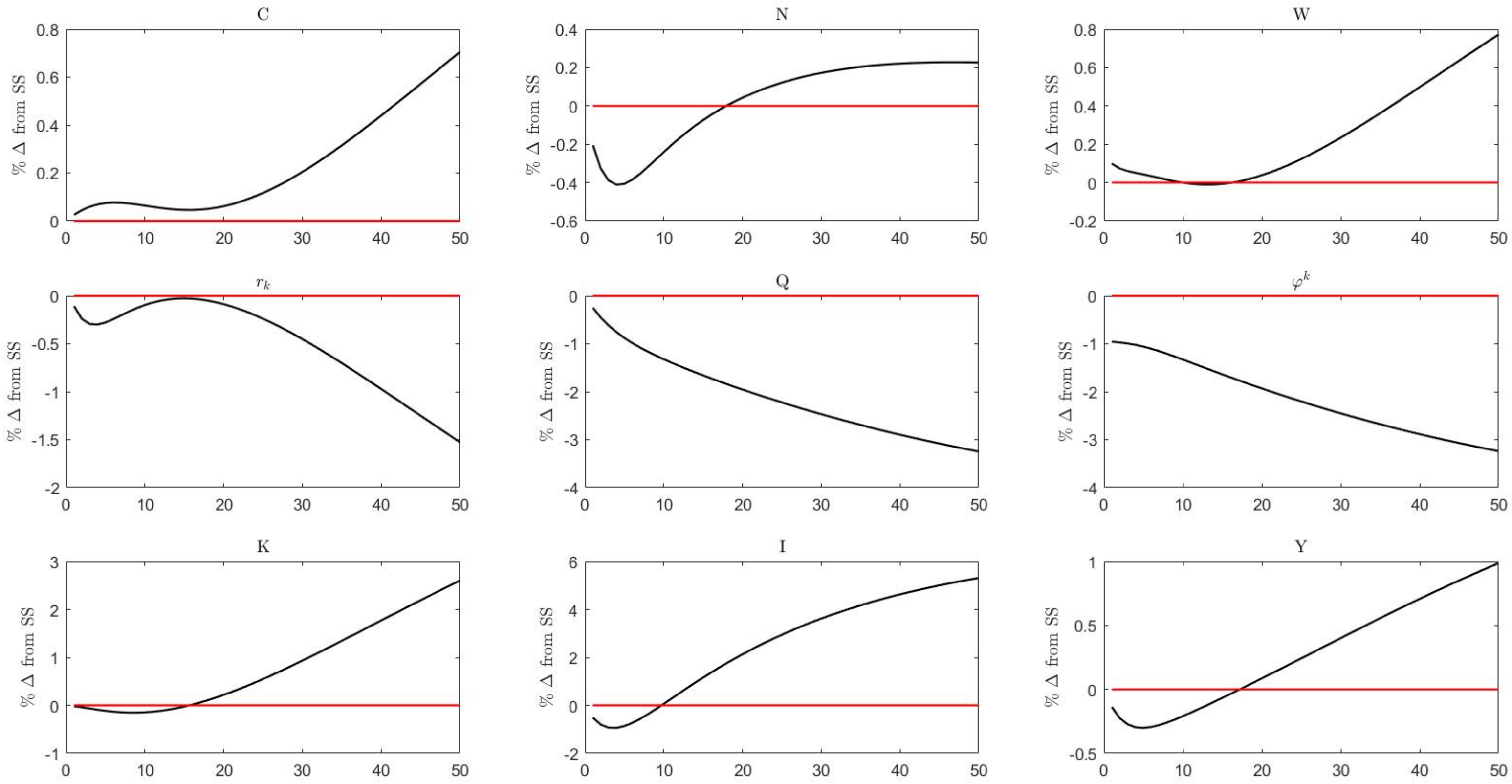

Figure 1.3: Impulse response functions to a permanent IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$, $\rho_e = 0$

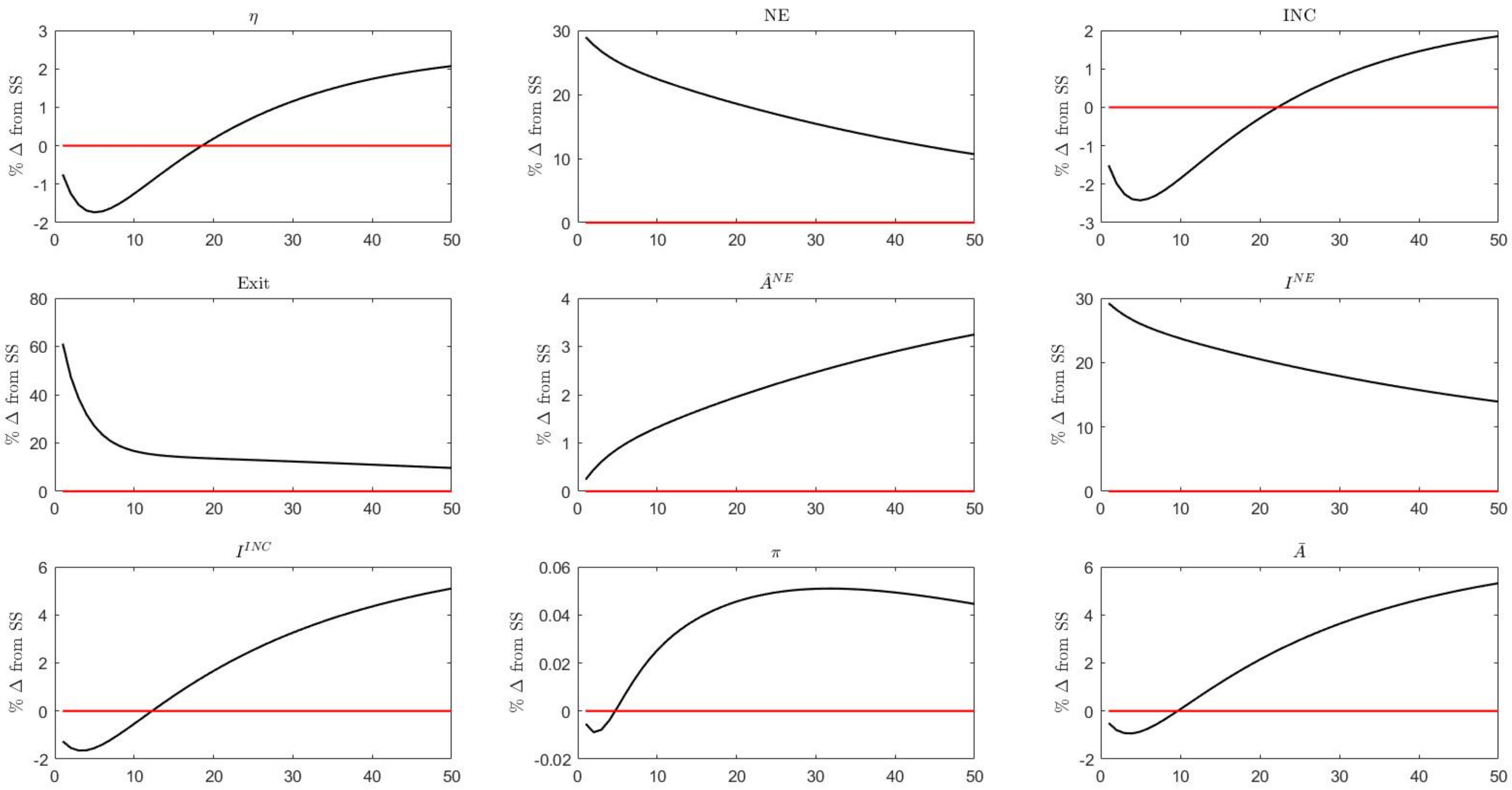

Figure 1.4: Impulse response functions to a permanent IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$, $\rho_e = 0$

The importance of technology spreading and endogenous entry and exit

As pointed out above, a pivotal role in this rich transmission mechanism is played by the incumbents' updating ability which generates an impressive degree of persistence in real variable dynamics. To benchmark the importance of knowledge spreading and endogenous entry/exit in general, we have also constructed a version of this model where ε_t^e shocks are unconsequential for incumbents' technology updating, which is now entirely driven by the deterministic trend g_e, and thus cyclical entry/exit flows in the K-sector are set off. For sake of clarity, we point out that what distinguishes our benchmark model from its no-entry/exit counterpart is the formulation of (1.31) and (1.32). Indeed, the relative K-firms mass and cutoff showing up there are now the ones prevailing in ss and thus do not vary over the business cycle.

This apparently slight modification has a non negligible effect. As a consequence, the model without endogenous entry/exit is now more in line with the canonical two-sector neoclassical growth model.

To match the variance of final output obtained in our canonical formulation where firm dynamics are endogenous and $\rho_e = 0$, we need to impose $\rho_e = 0.61$, i.e. the variance of $g_{e,t}$ is now roughly 1.6 times larger than in the baseline model.

For sake of clarity, stochastically detrended[9] impulse responses are displayed in Figures 1.5 and 1.6. In other words, the variables of interest are now expressed in terms of gaps from their "new" ss level. This is done also to show gap correlations which are the ones effectively discussed in the literature. Thus, for instance, a negative output gap does not necessarily mean that a recession is occurring (net of the initial "creative destruction" effect of course), rather that there is a transition from the "old" to the "new" ss and thus a likely output increase.

The first result is that the endogenous entry/exit mechanism is crucial to introduce a very high degree of persistence in the model in spite of a nil shock persistence. Indeed, the recovery from creative destruction is far slower. The second result hinges on the "creative destruction" itself being exacerbated by cyclical firm entry/exit flows. In fact, Q_{gap} and I_{gap} are negatively correlated in the benchmark model differently from the one where entry/exit is neglected. This is because in the former case the IST shock realization sweeps away the most inefficient incumbents thus destroying a sizable share of K-goods production (see the first impulse response on the third row in Figure 1.6) resulting in an excess of demand in the K-goods market. This makes Q_{gap} increase, since its stochastic trend is now permanently lowered, to then slowly roll back to its new ss level. When firm entry/exit flows are set off, no incumbent exits the market in response to the permanent sectorial technology advance and so K-goods production is now entirely driven by the shock persistence steering the demand of consumption goods necessary for sectorial production. This lack of "creative destruction" induces an initial fall in Q_{gap} via excess of supply in the K-sector which then overshoots it new ss level thanks to the higher demand of consumption goods for sectorial production induced again by the IST shock persistence.

[9]Stochastically detrended IRFs imply that variables are meant to be stationary with respect to their new stochastic trend (characterized by the new, permanent IST shock). By contrast when the stochastic trend is added back, as in Figures 1.3 and 1.4, IRFs are meant to be non-stationary with respect to the old stochastic trend, i.e. the one before the permanent IST shock realization.

In the end, entry/exit flows in the K-sector strongly characterize variables dynamics over the business cycle.

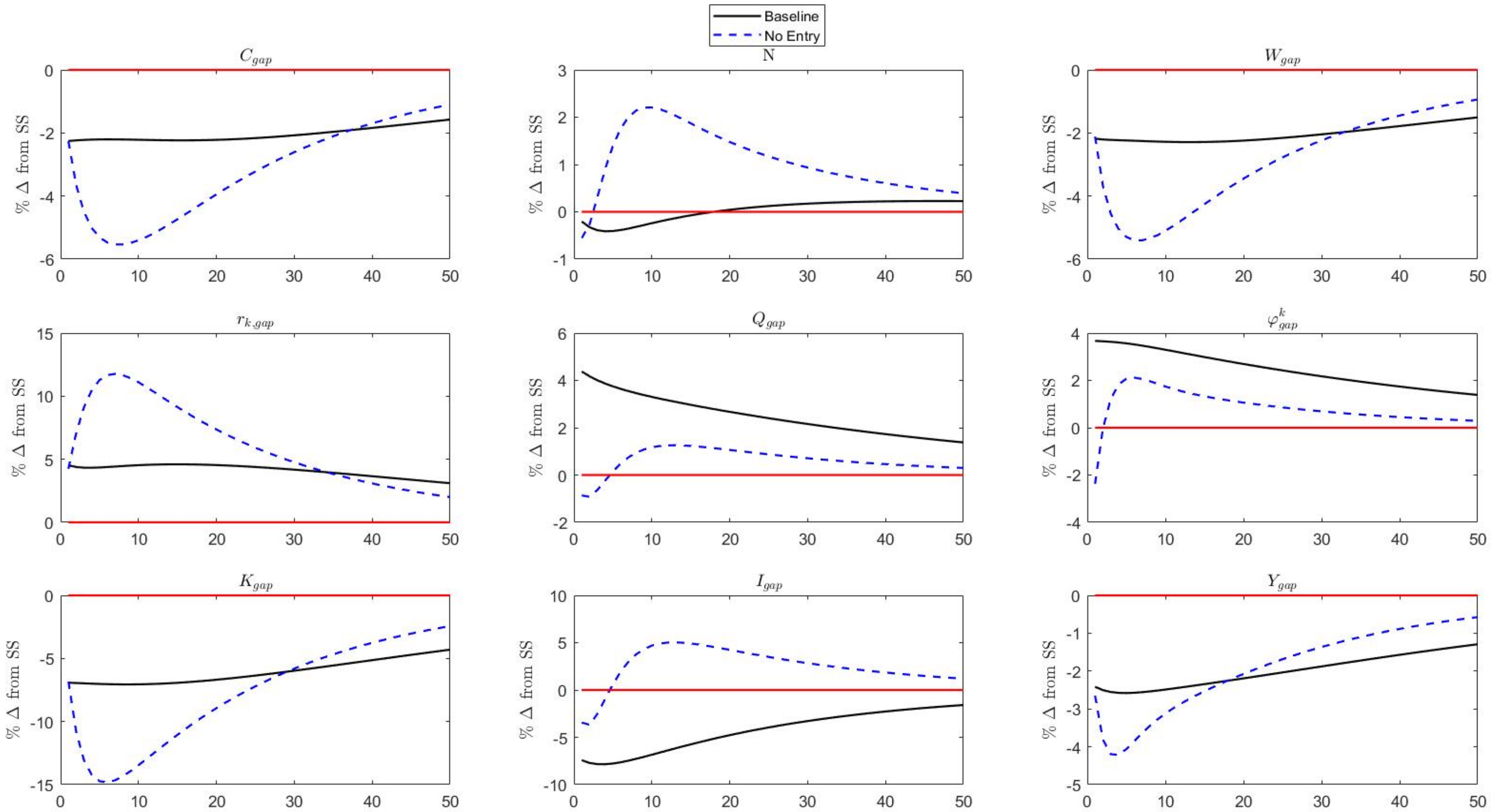

Figure 1.5: Impulse response functions to a stationary (stochastically detrended) IST shock. Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$.
$\rho_e = 0$ with endogenous entry/exit
$\rho_e = 0.61$ without endogenous entry/exit

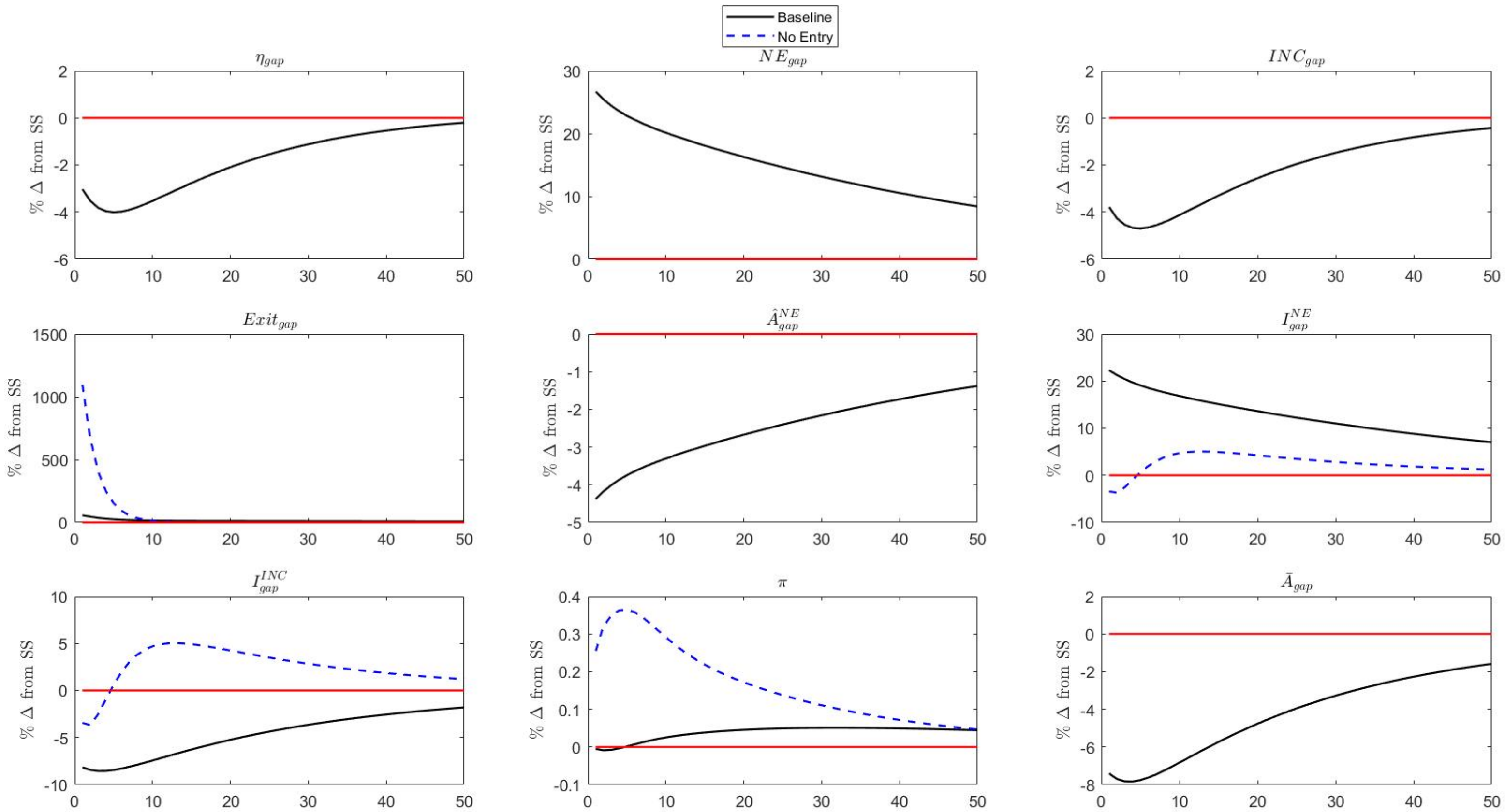

Figure 1.6: Impulse response functions to a stationary (stochastically detrended) IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$.
$\rho_e = 0$ with endogenous entry/exit
$\rho_e = 0.61$ without endogenous entry/exit

1.3.2 A permanent increase in the LAT shifter

Let us consider a white noise shock in the LAT shifter growth rate, $g_{z,t}$. Impulse responses are shown in Figures 1.7 and 1.8. Again, plotted variables incorporates the stochastic trend.

In line with the previous contributions (see Galí and Rabanal (2004)) we obtain that that sticky prices make labor demand shrink on impact in response to this shock. The LAT shock can be seen as a demand shock for the K sector. In this regard, we observe an increase in the relative price of investment goods, along with a reduction in the setor cutoffs, inducing a permanent increase in final production. This not only allows, as time goes by, for the gradual inflow of new less productive K-firms, but also makes less productive *INC*s survive. The netting out of these effects is in favor of the latter boosting investment production with the K-sector ending up being made of more less efficient firms. This mechanism downplays the quality of technology spillovers from *NE*s to *INC*s' as the impact of a true sectorial technology innovation is now absent.

Overall, the transition induced by the LAT shock is pretty fast and the correlation between Y (I) and Q is positive being at odds with empirical evidence and suggesting a minor role of such a shock as to explain business cycle.

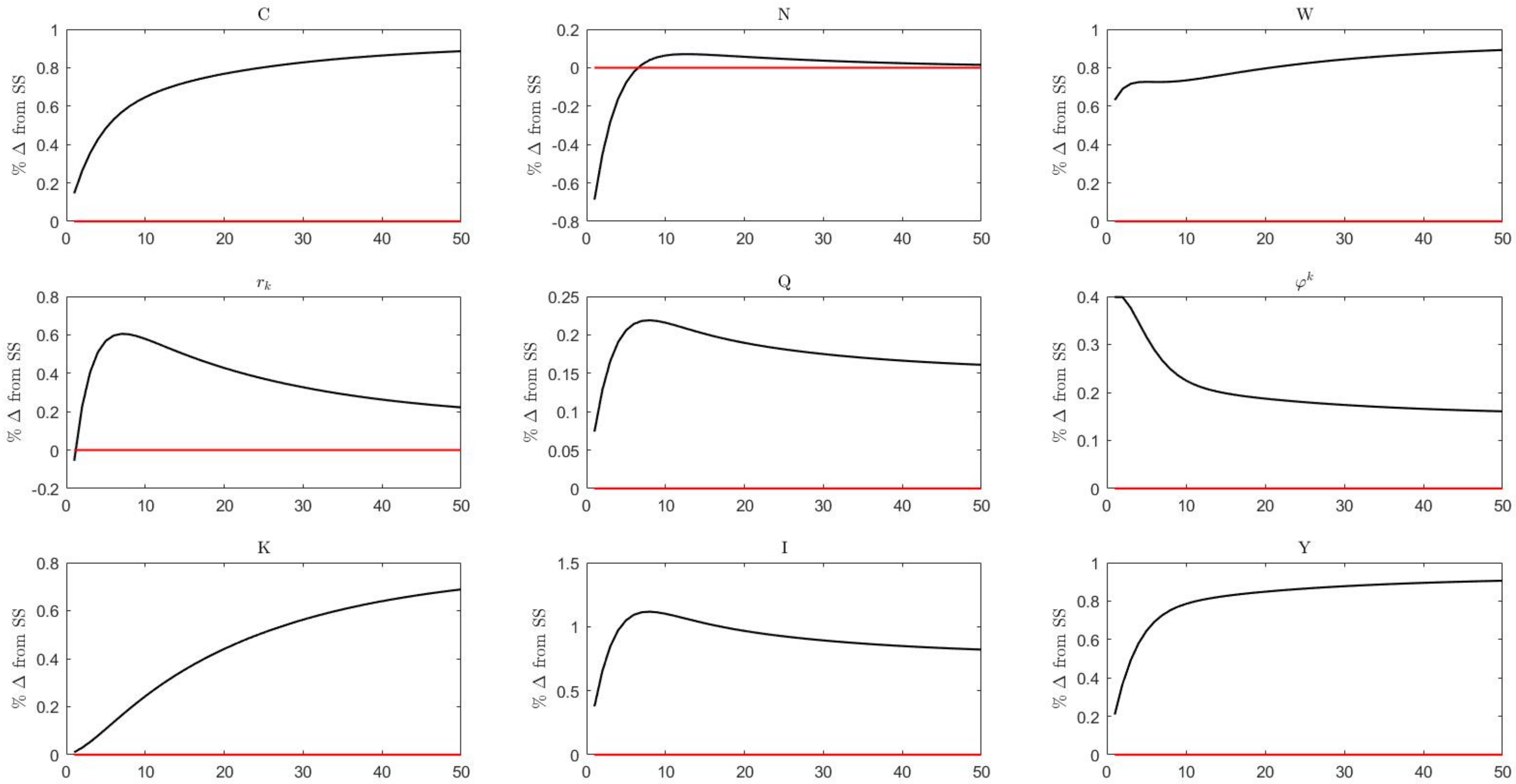

Figure 1.7: Impulse response functions to a Labor augmenting technology shifter permanent increase. Shock size of the white noise component of $g_{z,t}$ is $\sigma^z = 0.01$.
$\rho_z = 0$

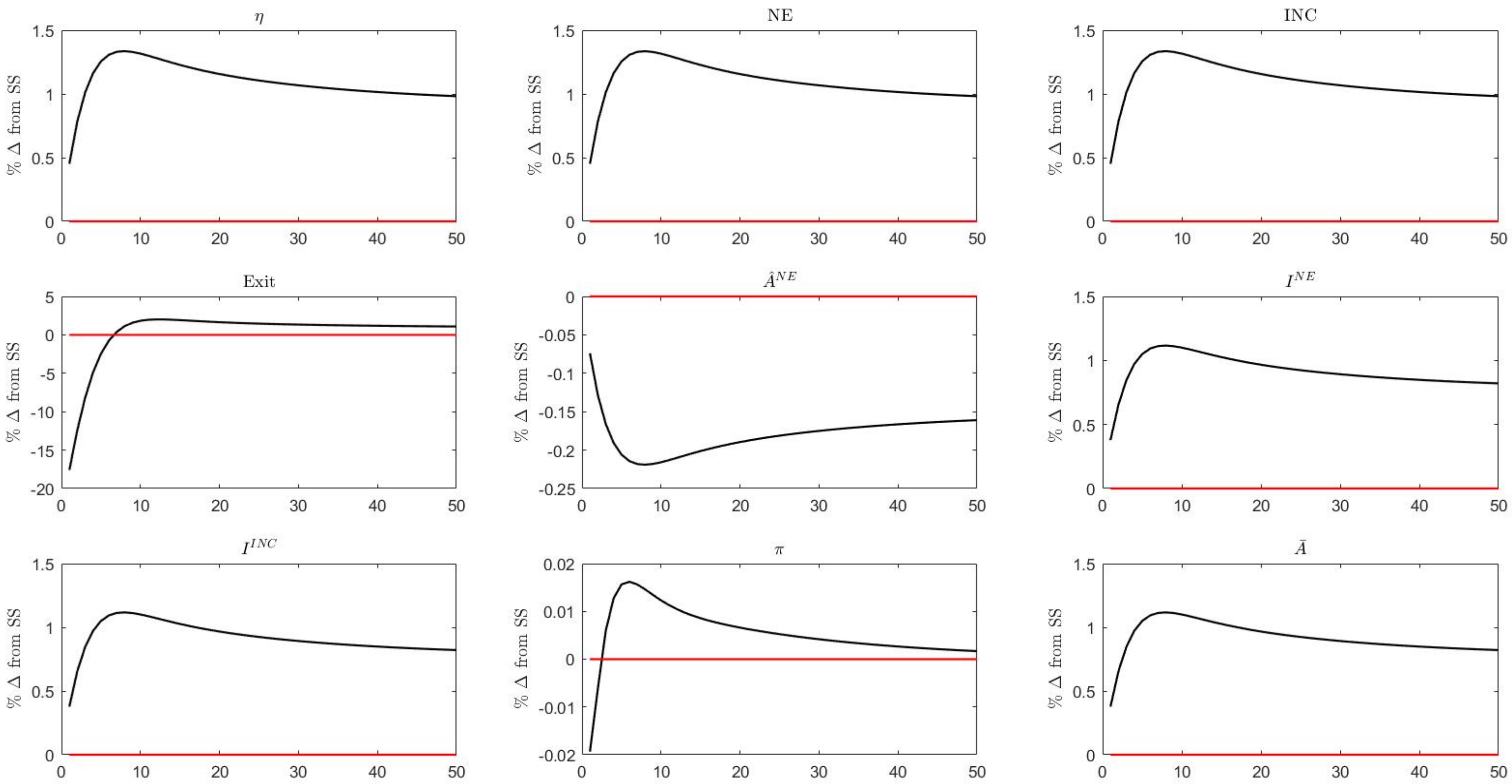

Figure 1.8: Impulse response functions to a Labor augmenting technology shifter permanent increase. Shock size of the white noise component of $g_{z,t}$ is $\sigma^z = 0.01$.
$\rho_z = 0$

1.3.3 MEI shock

We finally simulate a MEI shock of the kind documented in JPT by therefore assuming the same shock size.[10]

Impulse responses qualitatively resemble those in JPT, the transmission mechanism of the shock is not altered by our formulation of the K-industry. The MEI shock generates a persistent increase in the demand of capital and investment, this in turn lowers K-industry cutoffs. More inefficient K-firms survive and the production of investment goods increases as the composition effect prevails again in the K-industry. The persistent increase in capital accumulation in turn raises the demand of labor and thus final output. As usual consumption initially slightly decreases to leave room for the building up of investment to then start soaring when output peaks.

Further, we highlight how the relative price of investment, Q, and the shadow price of capital, φ^k (i.e., the marginal Tobin's Q), comove negatively (as can be seen from (1.17)). This is due to the fact that the MEI shock makes more convenient for households to buy investment goods as now their aggregation into capital is more efficient. This endogenous feedback from capital aggregation to the transformation of consumption into investment goods is completely neglected in JPT's analysis. Indeed, extending JPT to endogenous firms entry/exit and technology spillovers, triggers a strong procyclicality of Q in response to a positive MEI shock. This point is relevant as in the data the correlation between Q and Y is rather negative or eventually acyclical, but surely not positive. This implies that the MEI shock cannot be accounted for as the lion's share of business cycle fluctuations as the generated volatility of Q is at odds with the empirical evidence.

[10] It must be noted however that we abstract from both capital utilization costs and sticky wages as compared to JPT. In this regard however, the introduction of our stylized K-sector sensibly enhances the shock transmission.

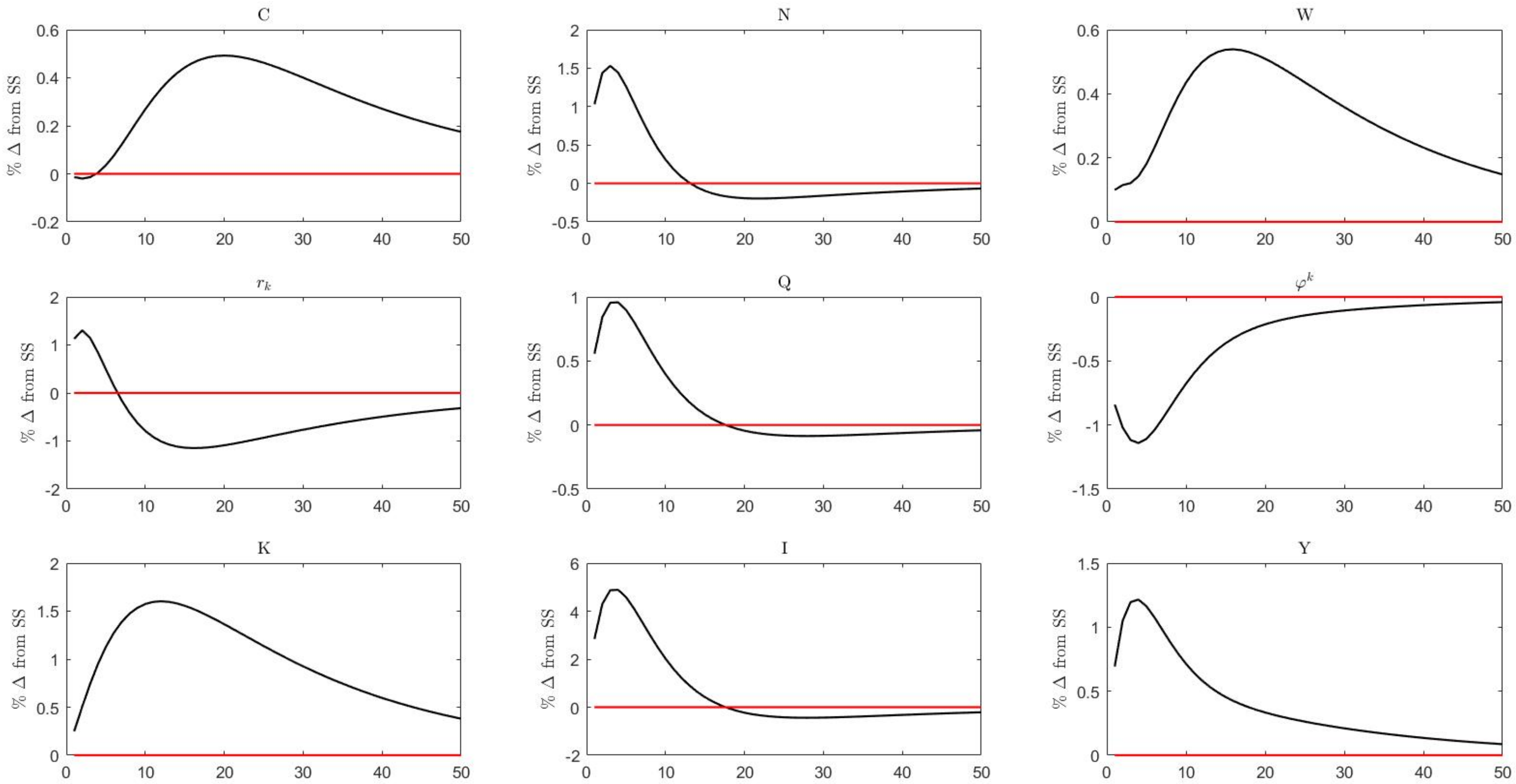

Figure 1.9: Impulse response functions to a MEI shock.
Shock size of the white noise component of μ_t^i is $\sigma^i = 0.05786$.
$\rho_{\mu^i} = 0.813$

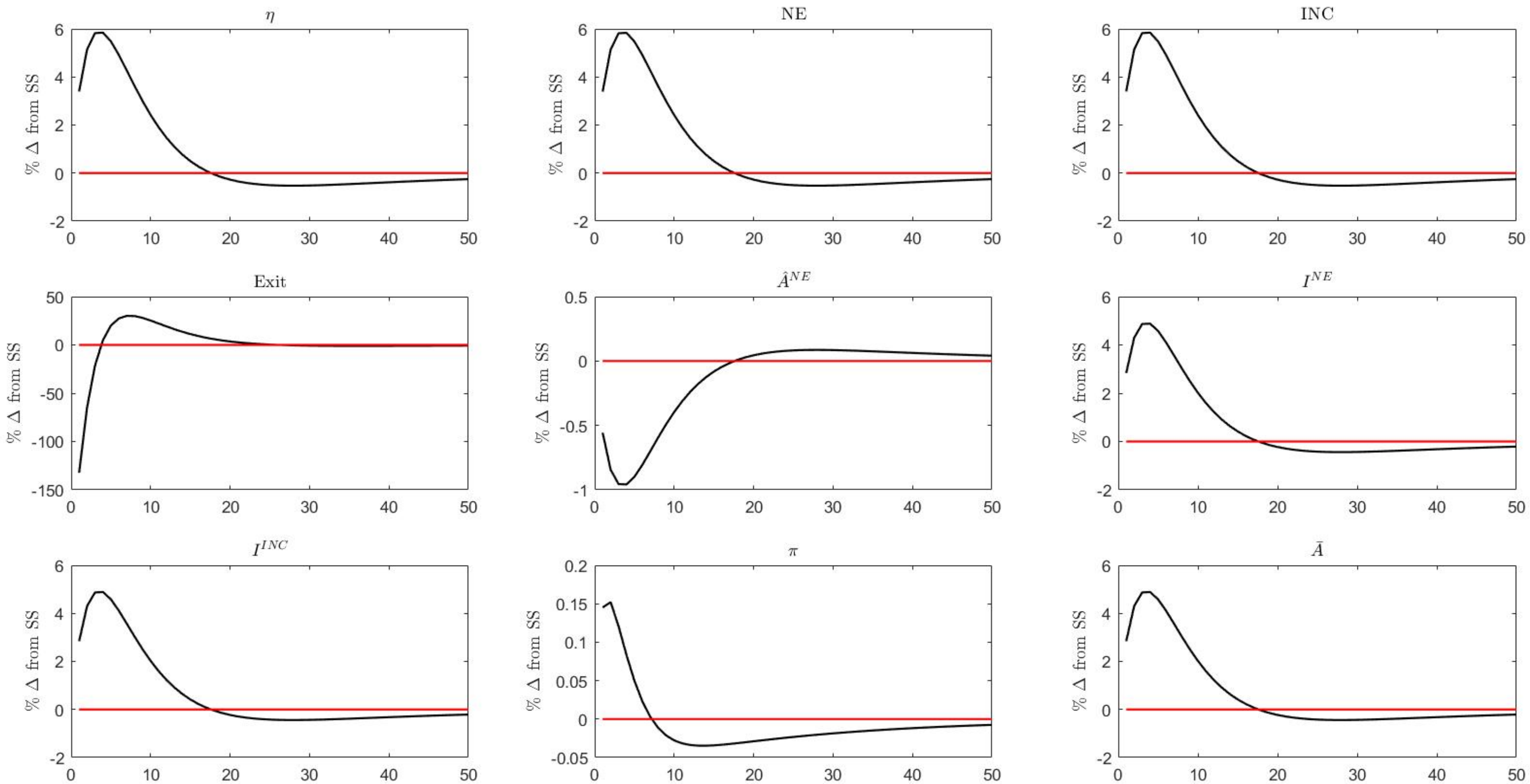

Figure 1.10: Impulse response functions to MEI shock.
Shock size of the white noise component of μ_t^i is $\sigma^i = 0.05786$.
$\rho_{\mu^i} = 0.813$

1.3.4 Variance Decomposition

We now try to give an assessment of the relevance of the IST shock as to explain US business cycle. In particular, we want to see whether the formulation of our stylized K-sector, which also allows for oscillations of Q coming from sources different from the IST shock, deals with the permanent IST shock playing a prominent role for business cycle. We exploit the theoretical variance decomposition of our model by calibrating the IST shock such that we are able to match a correlation between business cycle components of the relative price of investment goods and real GDP equal to -0.386. We choose to match such a correlation since it is something in the middle between the one obtained by Fisher (2006), -0.54, and our lower bound of -0.22 according to the different definitions of relative price of investment. We compute the variance decomposition by running the three shocks presented until now, that is permanent IST, permanent LAT and persistent MEI shock. We calibrate LAT and MEI as in JPT, and then tune the IST shock in order to match the above correlation between Q_{gap} and Y_{gap}. An issue arises however, and it concerns the parametrization of the LAT shock about which the empirical literature is scant. JPT estimate the permanent TFP shock for the production of consumption goods with a persistence of 0.287 and a standard deviation of the white noise component equal to 0.0094, yielding a standard deviation of the stochastic process growth rate equal to 0.0098. This is what is theoretically closer to our LAT shock, and thus it is calibrated it in the same way. Then, assigning a standard deviation equal to 0.012483 to the innovation of the IST shock growth rate, and imposing $\rho_e = 0$ because of the endogenous updating at work in the K-sector, we match the contemporaneous $corr(Y_{gap}, Q_{gap}) = -0.3865$. In Table 1.3 we summarize the relevant shocks calibration for the variance decomposition.

Table 1.3: **Shocks Calibration for Variance Decomposition**

ρ_{μ^i}	0.813	MEI shock persistence
ρ_e	0	IST shock growth persistence
ρ_z	0.287	LAT shock growth persistence
σ^e	0.012483	e_t shock sd
σ^z	0.0094	z_t^n shock sd
σ^{μ^i}	0.05786	MEI shock sd

According to the above parametrization, we obtain that the permanent IST shock is the major business cycle driver, in spite of a fairly lower magnitude than MEI shock, as we can see from Table 1.4 below.

Indeed, it explains barely half of the variation of final output, more than 60% of variation in consumption and almost 80% of variation in Q_{gap}. However, almost the totality of variation of investment, K-firms mass and labor is up to the MEI shock. Finally, as expected the LAT shock contribution is virtually not relevant.

At this point, it would be legitimate asking to what extent the endogenous K-firms entry/exit mechanism featuring our model contributed to obtain the observed variance decomposition. First, we observe that there is no way of obtaining a negative correlation between Q_{gap} and

Table 1.4: **Variance Decomposition in percentage points**

Variable	IST	LAT	MEI
Y_{gap}	49.95%	6.81%	43.24%
C_{gap}	61.53%	16.65%	21.82%
I_{gap}	31.99%	1.26%	66.74%
Q_{gap}	79.91%	0.37%	19.71%
N	1.88%	6.84%	91.28%
η_{gap}	6.41%	1.74%	91.85%

Y_{gap} even for a very high persistence of ρ_e. Thus, it follows that the natural comparison should be done by means of the same calibration in Table 1.3 but using our model without K-firms entry/exit flows. This experiment should give the extent to which entry/exit flows in K-sector soar the IST shock contribution to explain the business cycle.

Table 1.5: **Variance Decomposition in percentage points, No endogenous Entry/Exit**

Variable	IST	LAT	MEI
Y_{gap}	16.82%	13.08%	70.1%
C_{gap}	38.88%	27.9%	33.21%
I_{gap}	4.58%	1.78%	93.65%
Q_{gap}	4.58%	1.78%	93.65%
N	7.05%	7.47%	85.48%
η_{gap}	-	-	-

As it is possible to see from Table 1.5, the cyclical contribution of the IST shock to explain Y is far smaller in favor of the MEI shock which is now the main driver of all relevant variables but consumption. Notice also how the variance decomposition of Q_{gap} and I_{gap} is now identical. This is because Q_{gap} is the only determinant of variations in I_{gap}. Therefore, we can conclude that the endogenous K-firms entry/exit mechanism is key for the IST shock playing a relevant role to explain the business cycle.

1.4 Conclusions

Our results are theoretically consistent with the permanent IST shock playing a major role in explaining both business cycle and long run movements of aggregated variables in the US.

From the theoretical perspective we constructed a novel two-sector model where firms entry in the capital sector is endogenous to any aggregate shock hitting the economy. In particular, we have that comovements between the relative price of investment-goods and sectoral productivity depend on the specific features of the shock. Indeed, it is the endogenous response of the relative price of investment, along with aggregate variables, which suggests that the IST permanent shock must play a prioritary role. The novelty of our approach can be related to two main issues. first, differently from JPT, variations in Q are no longer attributable solely to the IST shock. Second,endogenous technology updating allows to obtain very persistent dynamics, especially Q, even though the shock persistence is nil.

We have also documented how the same, endogenous, market forces giving raise to the above result seem to rule out a relevant role of the labor augmenting technology improvement as long run source of growth given that it would imply a positive correlation between Q and Y growth.

Last but not least, we have also documented that endogenous entry downplays the relevance of the MEI shock as major business cycle driver, since it implies a strong procyclicality of the relative price of investment goods which is not observed in the data.

1

Chapter 2

IST shock transmission, Creative Destruction and Financial Constraints

JEL classification: E13, E22, E30, E32, E44, G10

keywords: Business cycles, Investment-specific technology, DSGE model, Creative destruction, Firm dynamics, Financial friction, Banking crisis

2.1 Introduction

In consequence of the great financial crisis, the financial accelerator framework, outlined in the seminal work of Bernanke, Gertler and Gilchrist (1999), has been adapted to feature the prolonged slump in the aggregate production of Western countries characterizing the great recession both in the US and the Euro-Area (see for instance Gertler and Kiyotaki (2010) and Gerali et al. (2010), respectively).

We consider the role of financial intermediaries in a multi-sector DSGE model where growth determinants in the Investment- and Final-goods (I- and F-sectors henceforth) production are allowed to differ. In this respect, we dig into the interactions between a financial friction and a stylized I-sector evolving endogenously in terms of firms entry/exit flows, technology spillovers, and thus, productivity dynamics.

Our main focus is twofold. On the one hand, it concerns the role of financial intermediaries when the technologìcal change can take place in different production sectors. On the other, we are interested in studying how a financial crisis impacts on endogenous firm dynamics.

In this regard, our model has some specific features. First, I-sector firms are characterized by idiosyncratic efficiency, decreasing returns to scale, and by a fixed production cost which is crucial to determine entry-exit conditions. In each period new entrants benefit from exogenous advances in the technology frontier, but entry and exit thresholds are affected by the endogenous relative price of investment goods. With a lag, the technology adopted by new entrants spreads to surviving incumbents. In this fashion, threshold dynamics, i.e endogenous variations in the relative price of investment, determine the average efficiency of new entrants and therefore the quality of the technology diffusion process. This allows for endogenous variations of the relative price of investment goods, which is no longer exogenously identified with the sectoral TFP (see Greenwood, Hercowitz and Krusell (1997)). Second, the financial sector is modeled as in Gertler and Karadi (2011) (GK henceforth), to emphasize how endogenous balance sheet constraints affecting financial intermediaries can limit the capacity of non financial firms to obtain investment funds. Most importantly, the stylized formulation of our I-sector allows to investigate how endogenous variations in the relative price of investment goods affect banks' balance sheets and impact on the financial transmission mechanism . We highlight how this channel is critical to shape the transmission of investment-specific technology (IST) shocks and banking crisis episodes.

Our first result concerns the transmission of an unexpected permanent investment-specific technology improvement, driven by an inflow of more efficient new firms. This depresses the relative price of investment goods and triggers a process of creative distruction as less efficient incumbents are driven out of the market. In cosequence of this banks assets lose value and the interest rate spread between loans and deposits increases, dampening the expansionary effect of the shock.

The second result concerns the slow recovery characterizing the aftermath of the Great Financial Crisis. Indeed, as documented by Siemer (2016), the financial constraints hitting the economy during the great recession led to an impressive contraction in the inflow of new firms, which contributed by a great extent to the slow recovery. The introduction of our stylized I-sector in an otherwise standard version of GK in fact leads to a milder subsequent recovery following the crisis episode. This is because, in the aftermath of a banking crisis, the inflow of new, more productive firms is dampened by endogenous variations in the relative price of investment goods which are otherwise neglected in the canonical GK model. The intuition is that, when new entrants do not bring any valuable innovation in the market (i.e. the entry flows are not led by any positive IST shock), the sectoral production increase is impaired because of a lower idiosyncratic efficiency of incoming market players.

With this work we mainly contribute, adopting a Schumpeterian growth theory perspective, to the financial friction literature as well as to the one which focuses on investment dynamics. In this regard, it must be said that the related economic literature in a general equilibrium framework is scant. Lorenzoni and Walentin (2007) study how the introduction of a financial constraint alters the comovement of gross investment and the Tobin's Q in an otherwise standard general equilibrium model as in Hayashi (1982). As a result, the authors are able to generate an empirically lower (and thus more plausible) correlation between these two variables even though they abstract from firms entry/exit dynamics. Another strand of the literature, instead, seems to point towards financial frictions impairing the relevance of the IST shock at business cycle frequencies. The first to argue in this direction were Justiniano, Primiceri and Tambalotti (2011) who, by drawing a distinction between a permanent shock affecting the transformation of final into investment goods (IST) and a transitory shock impacting the aggregation of gross investment into the stock of capital (MEI, i.e. marginal efficiency of investment), show that the latter is the major business cycle driver for the US. In particular, they identify the MEI shock as a disruption of the financial system, especially during the great recession, even if their model does not explicitly embed any financial sector formulation. However, Kamber, Smith and Thoenissen (2015) overthrow this view by introducing a collateral constraint a lÃ Kiyotaki and Moore (1997) in an otherwise standard DSGE model as in Smets and Wouters (2007). They find that, in the presence of binding collateral constraints, risk premium shocks dry up the contribution of more general investment shocks over the cycle. Thus, they suggest that investment and risk premium shocks are almost mutually exclusive since the relevance of the former is not robust to more explicit formulations of financial frictions. Similarly, Afrin (2017) introduces the MEI shock in a DSGE model akin to GK and shows that, in such a context, the shock transmission is impaired by the generated countercyclicality of the financial claims price. However, these last two contributions abstract from a true and permanent investment specific technology progress, which we adequately take into account instead, and do not allow for any distinction between the relative price of investment goods and the marginal Tobin's Q.

The remainder of the paper is organized as follows. Section 2.2 initially gives an overall picture of the flows of events in our economy and the describes in the details the model economy. Section 2.3 comments our main findings and a final section concludes. Technical details are left in Appendix.

2.2 The Model Economy

They key players in the economy are I- and F-firms, respectively producing investment and final goods, K-firms that assemble capital goods, and commercial banks. Right from the outset we characterize their specific roles in the model.

At the beginning of each period, systemic and idiosyncratic shocks are revealed. F-firms borrow from commercial banks to purchase from K-firms the capital stock they need to produce. At the end of the period, when production is sold, F-firms sell the undepreciated capital stock to K-firms and reimbourse the loans. At the end of each period, K-firms purchase from I-firms the investments goods necessary to produce the capital stock they will sell to F-firms. When aggregating the new stock of capital they bear some adjustment costs.

I-firms purchase from F-firms the final goods they need to produce the investment goods which are then sold to capital assemblers. The I-sector is characterized by entry and exit flows of firms. Our analysis emphasizes the distinct roles played by New Entrants (*NE*s), who draw their idiosyncratic efficiency levels from a new more productive technology distribution, and Incumbents (*INC*s), who remain in the market as long as their profits are non negative. In addition, our model accounts for endogenous technology updating. At the end of time t, our I-firms sector is made of a measure η_t of active firms, distributed between new entrants, NE_t, and incumbents, INC_t, survived from period $t-1$.

$$\eta_t = NE_t + INC_t \tag{2.1}$$

At the beginning of period $t+1$ I-firms engage in technology updating, characterized as a random draw from the technology distribution introduced in the economy by NE_t firms.

Commercial banks are modeled as in GK. There is a continuum of households of measure $i \in (0,1)$. Each household incorporates a continuum of individuals, $1-f$ workers and f bankers. Workers supply labor and return the wages earned to the household. Each banker instead manages a financial intermediary and similarly transfers earnings (bank profits) back to the household. There is an exogenous probability θ_b that bankers continue to perform their role in the following period. In turn, with probability $(1-\theta_b)$ bankers exit the financial sector and become workers; therefore for each individual engaged in banking, activity is expected to last $1/(1-\theta_b)$ periods.[1] At the beginning of each period the banker may choose to divert a fraction λ_b of available funds from the bank portfolio. To ensure that depositors are willing to supply deposits, bankers are required to cofinance bank activity with their own wealth, and their discounted continuation value must be no less than the value of divertible funds. In fact the supply of deposits sets a limit to the bankers' leverage ratio and allows them to earn an interest rate spread on the deposits rate which determines the banker's continuation value. Note that expected returns from banks loans are crucially determined by the evolution of the relative price of capital goods. The latter, in turn, is determined by the endogenous interaction between

[1] This assumption is typically made to prevent bankers from accumulating net worth up to the point where they would no longer need deposits to supply loans.

I-firms dynamics and financial market conditions. This is a novel feature of our model.

2.2.1 The Representative Household

The representative household's problem is

$$\max_{C_t,N_t,B_t} \sum_{i=0}^{\infty} \beta^{t+i} \left\{ \ln\left(C_{t+i} - aC_{t+i-1}\right) - \Phi \frac{N_{t+i}^{1+\theta}}{1+\theta} \right\} \qquad s.t. \qquad C_t = W_t N_t + \Pi^b + R_{t-1}^b B_{t-1} - B_t \tag{2.2}$$

Where Π^b is net payouts to the households from ownership financial intermediary, K-firms and I-producers net of the transfer the household gives to new bankers and B_t is the quantity of short term debt (deposits) the household acquires, remunerated at the real rate R^b.

First order conditions are standard

$$\lambda_t = \left(C_t - aC_{t-1}\right)^{-1} - \beta a \left(C_{t+1} - aC_t\right)^{-1} \tag{2.3}$$

$$\Phi \frac{N_t^\theta}{\lambda_t} = W_t \tag{2.4}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} R_t^b \right\} \tag{2.5}$$

Which are the usual marginal utility of consumption, Leisure-Consumption relationship and the riskless-bond Euler equation.

2.2.2 Financial Intermediaries

Define NW_{jt} as the amount of net worth that a banker j has at the end of period t; B_{jt+1} the deposit the intermediary gets from households; L_{jt} the quantity of financial claims on final producers that the intermediary holds and φ_t^k the relative price, in terms of final goods, of each claim. The banker's balance sheet is

$$\varphi_t^k L_{jt} = NW_{jt} + B_{jt+1} \tag{2.6}$$

The law of motion of the bankers' net worth is governed by the difference between the returns on assets and the interest payments on deposits

$$\begin{aligned} NW_{jt+1} &= R_{kt+1}\varphi_t^k L_{jt} - R_t^b B_{jt+1} \\ &= \left(R_{kt+1} - R_t^b\right)\varphi_t^k L_{jt} + R_t^b NW_{jt} \end{aligned} \tag{2.7}$$

As shown in GK, the banker participation constraint must be

$$E_t \beta^i \frac{\lambda_{t+1+i}}{\lambda_t}\left(R_{kt+1+i} - R_{t+i}^b\right) \geq 0, \quad i \geq 0 \tag{2.8}$$

Given the banker's moral hazard problem discussed above, the spread between loan and deposit interest rates wil induce the banker will build assets until she exits the market. Thus, the bankers' objective function is

$$\begin{aligned} V_{jt} &= E_t\left(1-\theta_b\right)\sum_{i=0}^{\infty}\theta_b^i\beta^{i+1}\frac{\lambda_{t+1+i}}{\lambda_t}NW_{jt+1+i} \\ &= E_t\left(1-\theta_b\right)\sum_{i=0}^{\infty}\theta_b^i\beta^{i+1}\frac{\lambda_{t+1+i}}{\lambda_t}\left[\left(R_{kt+1+i} - R_{t+i}^b\right)\varphi_{t+1}^k L_{jt+1} + R_{t+i}^b NW_{jt+i}\right] \end{aligned} \tag{2.9}$$

Condition

$$V_{jt} \geq \lambda_b \varphi_t^k L_{jt} \tag{2.10}$$

States that the banker's continuation value, V_{jt}, cannot be lower than the value of divertible funds. The left hand side of (2.10) can be expressed as

$$V_{jt} = \nu_t \varphi_t^k L_{jt} + \eta_t^{nw} NW_{jt} \tag{2.11}$$

where

$$\nu_t = E_t \left\{ (1-\theta_b)\beta \frac{\lambda_{t+1}}{\lambda_t} \left(R_{kt+1} - R_t^b \right) + \theta_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\phi_{t+1}^b}{\phi_t^b} z_{t+1}^{bk} \nu_{t+1} \right\}$$

$$\eta_t^{nw} = E_t \left\{ (1-\theta_b)\beta \frac{\lambda_{t+1}}{\lambda_t} R_t^b + \theta_b z_{t+1}^{bk} \eta_{t+1}^{nw} \right\}$$

Term ν_t defines the expected discounted marginal gain to the banker of expanding assets by a unit, and η_t^{nw} is the expected discounted value of having another additional unit of net worth.

Then, the constraint can be rewritten as $\nu_t \varphi_t^k L_{jt} + \eta_t^{nw} NW_{jt} \geq \lambda_b \varphi_t^k L_{jt}$ and assuming it binds, i.e. the banker will expand its assets as much as possible, we have

$$\varphi_t^k L_{jt} = \frac{\eta_t^{nw}}{\lambda_b - \nu_t} NW_{jt} = \phi_t^b NW_{jt} \tag{2.12}$$

where ϕ_t^b can be referred to as the intermediary's private leverage.

It follows that the evolution of the surviving banker's net worth can be expressed as

$$NW_{jt+1} = \left[\left(R_{kt+1} - R_t^b \right) \phi_t^b + R_t^b \right] NW_{jt} \tag{2.13}$$

from which

$$z_{t+1}^{bk} = \frac{N_{,jt+1}}{N_{,jt}} = \left(R_{kt+1} - R_t^b \right) \phi_t^b + R_t^b \tag{2.14}$$

Then, as all the elements constituting ϕ_t^b have no firms specific factor, the total intermediary's demand for assets can be easily aggregated and defined as

$$\varphi_t^k L_t = \phi_t^b NW_t \tag{2.15}$$

Where L_t is the aggregate quantity of intermediary assets and NW_t is the aggregate quantity of net worth in the banking sector held by surviving bankers.

At this point a law of motion for NW_t can be easily derived by aggregating across surviving banker and new ones.

$$NW_t = NW_{e,t} + NW_{n,t} \tag{2.16}$$

Where

$$NW_{e,t} = \theta_b \left[\left(R_{kt} - R_{t-1}^b \right) \phi_{t-1}^b + R_{t-1}^b \right] NW_{t-1} \tag{2.17}$$

$$NW_{n,t} = \omega \varphi_t^k L_{t-1} \tag{2.18}$$

And $\omega \varphi_t^k L_{t-1}$ defines the start up funds that new bankers receive from their households.

Thus, the evolution of aggregate net worth of bankers is

$$NW_t = \theta_b \left[\left(R_{kt} - R_{t-1}^b \right) \phi_{t-1}^b + R_{t-1}^b \right] NW_{t-1} + \omega \varphi_t^k L_{t-1} \tag{2.19}$$

2.2.3 Final Good Producers

Retailers

Monopolistically competitive retail firms assemble the final good bundle Y_t using a continuum of intermediate inputs Y_t^h. The representative firm profit maximization problem is:

$$\max_{Y_t, Y_t^h} P_t Y_t - \int_0^1 P_t^h Y_t^h dh$$

$$\text{s.t. } Y_t = \left[\int_0^1 \left(Y_t^h \right)^{\frac{\nu-1}{\nu}} dh \right]^{\frac{\nu}{\nu-1}}$$

From the first order conditions, we obtain:

$$Y_t^h = \left(\frac{P_t^h}{P_t} \right)^{-\nu} Y_t \tag{2.20}$$

$$P_t = \left[\int_0^1 \left(P_t^h \right)^{1-\nu} dh \right]^{\frac{1}{1-\nu}} \tag{2.21}$$

Price stickiness is based on the Calvo mechanism. In each period retail firms face a probability $1 - \lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t^h = \pi_{t-1}^{\gamma_p} p_{t-1}^h \tag{2.22}$$

where $\gamma_p \in [0, 1]$ represents the degree of price indexation.

All the $1 - \lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$, the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t + s$ will read as $\widetilde{P}_t \left(\Pi_{t,t+s-1}^p \right)^{\gamma_p}$ where $\Pi_{t,t+s-1} = \pi_t ... \pi_{t+s-1} = \frac{P_{t+s-1}}{P_{t-1}}$.
$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s} \right) Y_{t+s}^h$$

subject to:

$$Y_{t+s}^h = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\chi}}{P_{t+s}} \right)^{-\nu} \tag{2.23}$$

where Y_t^d is aggregate demand and $\bar{\Lambda}_t$ is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \bar{\Lambda}_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\nu) \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{1-\nu} \widetilde{P}_t^{-\nu} \left(P_{t+s} \right)^{\nu} + \\ +\nu \widetilde{P}_t^{-\nu-1} P_{t+s}^{\nu+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{-\nu} \end{array} \right] = 0 \tag{2.24}$$

Intermediate Producers

Intermediate producers finance capital acquisition each period by obtaining funds from banks. To do this, they issue L_t claims equal to the number of units of capital acquired, K_t and price each claim at the price φ_t^k. Therefore the following identity holds

$$\varphi_t^k L_t = \varphi_t^k K_t \tag{2.25}$$

However, intermediate good producers hire labor from households at the beginning of period t to produce their output after having bought the stock of capital at the end of $t-1$. Finally, they have the option of selling the stock of undepreciated capital to K-firms after production has occurred[2]. The production function is thus

$$Y_t^h = \mu_t \left(z_t^n N_t\right)^{\chi} \left(\zeta_t^k K_{t-1}\right)^{1-\chi} \tag{2.26}$$

Where $z_t^n = z_{t-1}^n g_{z,t}$ is a permanent labor augmenting technology shifter (LAT) driven by

$$\ln\left(g_{z,t}\right) = (1-\rho_z)\ln\left(g_*\right) + \rho_z \ln\left(g_{z,t-1}\right) + \sigma^z \varepsilon_t^z$$

and $\varepsilon_t^z \sim N(0,1)$.

We also allow for the presence of a transitory TFP shock

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \sigma^\mu \varepsilon_t^\mu$$

with $\varepsilon_t^\mu \sim N(0,1)$.

Intermediate producers maximization problem is

$$\max_{N_t} \Pi^h = mc_t Y_t^h - W_t N_t + (1-\delta)\varphi_t^k \zeta_t^k K_{t-1}$$

The first order conditions are

$$W_t = mc_t \chi \mu_t \left(z_t^n\right)^{\chi} \left[\frac{\zeta_t^k K_{t-1}}{N_t}\right]^{1-\chi} \tag{2.27}$$

$$R_{k,t} = \frac{mc_t}{\varphi_{t-1}^k}\mu_t(1-\chi)\left(\zeta_t^k\right)^{1-\chi}\left(\frac{z_t^n N_t}{K_{t-1}}\right)^{\chi} + \frac{\varphi_t^k}{\varphi_{t-1}^k}\zeta_t^k(1-\delta) \tag{2.28}$$

[2] Capital depreciation is not affected by the price of capital goods, differently from GK, because of a different structure of K-firms industry.

2.2.4 Capital Producing Firms

Capital assemblers behave differently from GK to the extent that they engage in transactions with I-firms. At the end of period t, K-firms buy back from F-firms the stock of capital from intermediate producers and simultaneously purchase investment-goods from I-firms in order to aggregate the stock of capital needed for intermediate F-firms next period production.

In this regard, K-firms choose the optimal amount of investment to be bought at the unit price, Q, considering also the cost of aggregating it into the stock of capital evaluated at its market value. The K-firms optimization problem reads

$$\max_{I_{t+i}} \sum_{i=0}^{\infty} \beta^{t+i} \lambda_{t+i} \left\{ -Q_{t+i} I_{t+i} - \varphi^k_{t+i} \zeta^k_{t+i} (1-\delta) K_{t+i-1} + \varphi^k_{t+i} \left[1 - S\left(\frac{I_{t+i}}{I_{t+i-1}} \right) \right] I_{t+i} \right\}$$

from which we obtain

$$\begin{aligned} Q_t = \varphi^k_t & \left\{ 1 - \left[S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}} \right) \right] \right\} + \\ & + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \varphi^k_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\} \end{aligned} \tag{2.29}$$

where $S'(\cdot) \equiv \gamma_I \left(\frac{I_{t+i}}{I_{t+i-1}} - 1 \right)$.
The law of motion of capital is

$$K_t = (1-\delta) \zeta^k_t K_{t-1} + I_t \tag{2.30}$$

Where, following GK, ζ^k_t is a quality of capital shock characterized by the following stochastic process

$$\ln\left(\zeta^k_t\right) = \rho_k \ln\left(\zeta^k_{t-1}\right) + \sigma^k \varepsilon^k_t.$$

With, as usual, $\varepsilon^k_t \sim N(0,1)$

2.2.5 I-firms

At the end of period t, the generic I-sector firm j is characterized by the following production function

$$I_t^{X,j} = A_t^{X,j} \left(S_t^{X,j}\right)^{\alpha}$$

where $X = NE, INC$ defines whether the firm is a new entrant or an incumbent. $\alpha < 1$ defines decreasing returns to scale. $A_t^{X,j}$ is the idiosyncratic efficiency level. In terms of final goods, profits are

$$\Pi_t^{X,j} = Q_t I_t^{X,j} - S_t^{X,j} - f_t^X \tag{2.31}$$

where f_t^X is a fixed production cost such that $f_t^X = g_*^t f^X$, where we assume that I-firms fixed costs dynamics has the same deterministic trend of the LAT shifter, z_t^n. [3] I-producers operate only if they earn non-negative profits because idiosyncratic efficiency is fully observable and moral hazard problems do not arise by assumption. Therefore, the I-firm maximization problem boils down to a static one, and is solved maximizing profits with respect to $S_t^{X,j}$:

$$S_t^{X,j} = \left(Q_t \alpha A_t^{X,j}\right)^{\frac{1}{1-\alpha}} \tag{2.32}$$

The efficiency threshold that satisfies non-negativity of (2.31) is

$$\hat{A}_t^X = \left(\frac{f_t^X}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t \alpha^{\alpha}} \tag{2.33}$$

The I-firms cutoff is thus positively related to the fixed cost of production and is a negative function of the investment-goods relative price. Thus, entry and exit decisions are endogenous to any shock affecting Q_t.

New Entrants

The market entry decision at the end of time t is conditional to firm NE, j idiosyncratic productivity level A_t^{NE}, j. However, to ease the burden of notation, we hereafter abstract from the idiosyncratic index j. A unit probability mass of potential NEs draw their individual A_t^{NE} every period from a new and more efficient Pareto distribution[4]

$$f_t(A_t^{NE}) = \int_{e_t}^{+\infty} \frac{\gamma e_t^{\gamma}}{(A_t^{NE})^{\gamma+1}} d(A_t^{NE}) = 1 \;\; with \;\; A_t^{NE} \geq e_t \tag{2.34}$$

Where γ is the tail index describing the distribution skewness, and $e_t = e_{t-1} g_{e,t}$ represents the technology frontier identifying the IST shock dynamics, where

[3] This assumption avoids that the LAT shock impacts also the fixed costs structure.

[4] The formulation of the potential NEs efficiency problem is a simplification of the one presented in Asturias et al. (2017)

$$\ln\left(g_{e,t}\right) = \left(1-\rho_e\right)\ln\left(g_e\right) + \rho_e \ln\left(g_{e,t-1}\right) + \sigma^e \varepsilon_t^e$$

And $\varepsilon_t^e \sim N(0,1)$. Also the NEs technology frontier embeds a stochastic trend. Moreover, notice that, in order to ensure the existence of a BGP, $g_*^{1-\alpha} = g_e$ must hold in the deterministic steady state[5].

The mean of (2.34)

$$\mu\left(A_t^{NE}\right) = \frac{\gamma}{\gamma - 1} e_t. \tag{2.35}$$

Is driven by the lower bound of the support defining the pdf which is increasing at the gross rate g_e in the deterministic steady state.

The probability mass of effectively entering NE firms is obtained by cutting the pfd in (2.34) at the NEs threshold , $\hat{A}_t^{NE}$, obtained from eq. (2.33):

$$NE_t \equiv f_t(\hat{A}_t^{NE}) \equiv \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\gamma e_t^{\gamma}}{(A_t^{NE})^{\gamma+1}} d(A_t^{NE}) = \left[Q_t \alpha^{\alpha} e_t \left(\frac{1-\alpha}{f_t^{NE}}\right)^{1-\alpha}\right]^{\gamma} \tag{2.36}$$

Incumbents

At the beginning of t, INC_{t-1} firms observe NE_{t-1} I-firms technological level and update their plants accordingly. For sake of simplicity, we model the updating process as a lottery where all η_{t-1} firms draw their individual $A_t^{INC,j}$ from the Pareto distribution with support $\left[\hat{A}_{t-1}^{NE}, +\infty\right)$ that characterizes NE_{t-1} firms. Such a formulation brings the advantage of modeling endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent's efficiency level.

Thus, at the beginning of period t, the η_{t-1} firms are grouped into the Pareto pdf defining the idiosyncratic productivity level for each incumbent I-firm at the beginning of period t

$$f_t(A_t^{INC}) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\gamma\left(\hat{A}_{t-1}^{NE}\right)^{\gamma}}{(A_t^{INC})^{\gamma+1}} d(A_t^{INC}) \tag{2.37}$$

But only firms that satisfy the non negative profits condition (2.33) will survive in the market. Thus the mass of INC_t is obtained as the fraction of η_{t-1} computed over the support

[5] See section B.3.2 in Appendix.

share $\left[\hat{A}_t^{INC}, +\infty\right)$ of (2.37), where $\hat{A}_t^{INC} = \left(\frac{f^{INC_t}}{1-\alpha}\right)^{1-\alpha} \frac{1}{Q_t\alpha^\alpha}$ defines INC_t firms cutoff:

$$INC_t \equiv \eta_{t-1} f_t(\hat{A}_t^{INC}) \equiv \eta_{t-1} \int_{\hat{A}_t^{INC}}^{+\infty} \frac{\gamma \left(\hat{A}_{t-1}^{NE}\right)^\gamma}{\left(A_t^{INC}\right)^{\gamma+1}} d(A_t^{INC}) =$$

$$= \eta_{t-1} \left[\hat{A}_{t-1}^{NE} \left(\frac{1-\alpha}{f_t^{INC}}\right)^{1-\alpha} Q_t\alpha^\alpha\right]^\gamma = \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^\gamma$$

which allows us to rewrite the law of motion for mass of active firms as

$$\eta_t = \left[Q_t\alpha^\alpha e_t \left(\frac{1-\alpha}{f_t^{NE}}\right)^{1-\alpha}\right]^\gamma + \eta_{t-1} \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^\gamma \tag{2.38}$$

Finally, the mass of exit in t is thus

$$exit2_t = \eta_{t-1} \left\{1 - \left[\left(\frac{f_{t-1}^{NE}}{f_t^{INC}}\right)^{1-\alpha} \frac{Q_t}{Q_{t-1}}\right]^\gamma\right\} \tag{2.39}$$

Needless say, the dynamics of the relative price of investment goods is key to determine I-firms dynamics.

I-firms production and the process of creative distruction

NEs and INCs supply functions are easily computed.

$$
\begin{aligned}
I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} \left(Q_t \alpha A_t^{NE}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{NE}) \\
&= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \\
&= \left[\alpha^{\alpha} e_t (1-\alpha)^{1-\alpha}\right]^{\gamma} \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{(f^{NE})^{(1-\alpha)\gamma-1}}
\end{aligned} \tag{2.40}
$$

$$
\begin{aligned}
I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC} \left(Q_t \alpha A_t^{INC}\right)^{\frac{\alpha}{1-\alpha}} dF(A_t^{INC}) \\
&= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \\
&= \eta_{t-1} \left[\frac{(f^{NE})^{1-\alpha}}{Q_{t-1}}\right]^{\gamma} \frac{\gamma}{\gamma(1-\alpha)-1} \frac{Q_t^{\gamma-1}}{(f^{INC})^{(1-\alpha)\gamma-1}}
\end{aligned} \tag{2.41}
$$

The details of the derivation are left in Appendix [6].

From (2.40) it is easy to see that a shock to e_t shifts to the right the NE_t supply of investment goods. For any given demand for investment goods (2.29) this puts downward pressure on in the relative price Q_t. As a result, from and (2.41) it is easy to see that both the mass of surviving incumbents and their supply of investment goods shrinks. This is the essence of the "creative destruction" process triggered by IST shocks. In Section 2.3.1 we fully characterize dynamics associated to such a shock.

[6] See section B.4.

2.2.6 Market clearing and policy rules

The investment-goods and final-goods market clearing conditions respectively are

$$I_t = I_t^{NE} + I_t^{INC} \tag{2.42}$$

$$Y_t = C_t + S_t + NE_t f_t^{NE} + INC_t f_t^{INC} + \varphi_t^k S\left(\frac{I_t}{I_{t-1}}\right) I_t \tag{2.43}$$

where

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE,j}\right) dF(A_t^{NE,j}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC,j}\right) dF(A_t^{INC,j}) \tag{2.44}$$

is the amount of input demanded for investment-goods production and thus

$$I_t = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE,j} \cdot \left[S\left(A_t^{NE,j}\right)\right]^{\alpha} dF(A_t^{NE,j}) + \int_{\hat{A}_t^{INC}}^{+\infty} A_t^{INC,j} \cdot \left[S\left(A_t^{INC,j}\right)\right]^{\alpha} dF(A_t^{INC,j}) \tag{2.45}$$

Finally, we assume that the Central Bank controls monetary policy by means of a simple Taylor rule with interest rate smoothing

$$\left(\frac{R_{n,t}}{R_n^{ss}}\right) = \left(\frac{R_{n,t-1}}{R_n^{ss}}\right)^{\rho_{R_n^{ss}}} \left[\left(\frac{\pi_t}{\pi^{ss}}\right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu-1)/\nu}\right)^{\kappa_y} \exp\left\{\sigma^r \varepsilon_t^r\right\}\right]^{1-\rho_{R_n^{ss}}} \tag{2.46}$$

Where of course the usual Fisher equations holds true

$$R_t^b = E_t\left\{\frac{R_{n,t}}{\pi_{t+1}}\right\} \tag{2.47}$$

2.2.7 Solution and Calibration

The trends in sectoral technologies render the model non-stationary even though in this formulation both final output, capital and investment are assumed to grow at the same BGP rate, whilst hours worked are stationary just like, for sake of simplicity, the relative price of investment goods[7]. The details on the existence of a BGP are left in Section B.3. For what concerns the stochastic simulation of the model, stationarity is obtained by appropriate variable transformation (see Sections B.1 and B.2). The model is solved by means of a first order perturbation

[7]The model could be easily extended to embrace a deterministic trend for Q.

method and the list of stationary log-linearized equations is reported in Section A.7.

We assume the BGP rate of the economy to be $g_* = 1.004$ on quarterly basis. For sake of simplicity we set the NEs fixed cost of entry at 1% of final output. Parameters calibration is meant to be on a quarterly basis. Thus, we impose $\beta = 0.99$ and set the risk premium equal to 0.1/4 as in GK. We calibrate Φ at a conventional value such that $N^{ss} = 0.3333$ and set the inverse of the Frisch elasticity θ at 0.276. Capital depreciation is also standard, $\delta = 0.025$. We also set NEs as to be the 10% of total firms on annual basis in order to match the US business destruction rate as in Etro and Colciago (2010). Then, we normalize the steady state relative price of capital, $Q^{ss} = 1$, and the technology shifter in ss $z^n = 1$. The final goods elasticity of substitution, ν, is set equal to 4.167, the probability of not updating prices is $\lambda_p = 0.779$ and price indexation coefficient is $\gamma_p = 0.241$. The parameter governing investment adjustment costs is the same as GK, $\gamma_I = 1.728$. We follow again GK and set bankers private leverage equal to 4, then the only differences with GK's calibration is up to the presence of growth in our model and are hence negligible. Policy rule coefficients are also standard.

The last parameters to be calibrated are non conventional for the DSGE literature. They are the I-producers returns to scale, α, the NEs technology shifter, e^{ss}, and the tail index of the Pareto distribution, γ. Their calibration must be consistent with the fact that, from the law of motion of capital, $I_t^{ss} = 0.141$ and consumption output ratio is $\approx 85\%$. In other words I-producers must be distributed in a way such that in equilibrium $Q^{ss} = 1$. To do this, we impose $\alpha = 0.8$ and $\gamma = 6.1$, this pins down the initial condition for the NEs technology shifter, $e^{ss} = 0.3177$. The calibration of α is at the lower bound of Basu and Fernald (1997) estimates, whilst the value assigned to γ is set to resemble Asturias et al. (2017). Finally, we set the interest rate smoothing coefficient, $\rho_{R_n^{ss}}$, equal to 0.8, the TFP shock persistence, ρ_μ, equal to 0.975, and the IST shock persistence, ρ_e, is equal to 0.

Parameters calibration is summarized in Table 2.1.

Table 2.1: **Parameters**

Households		
g_*	1.004	Gross BGP rate
β	0.99	Discount factor
a	0.815	Habit parameter
δ	0.025	Capital depreciation
θ	0.276	Inverse Frisch elasticity of labor supply
N^{ss}	0.3333	SS labor
Financial Intermediaries		
γ_I	1.728	Investment adjustment costs
λ_b	0.3166	Fraction of capital that can be diverted
ω	0.0031	Proportional transfer to entering bankers
θ_b	0.972	Survival rate of the bankers
$E[r_k] - r$	0.0025	Risk premium
Retailers		
ν	4.167	Final goods elasticity of substitution
λ_p	0.779	Probability of not updating prices
γ_p	0.241	Price indexation parameter
Intermediate Producers		
z^n	1	L-shifter (LAT) initial condition
χ	0.67	Labor share of income
I-firms		
g_e	$g_*^{1-\alpha}$	Technology frontier BGP
α	0.8	K-producers returns to scale
e^{ss}	0.3177	Technology frontier (IST) initial condition
f^{NE}	0.01*Y	Entry Cost initial condition
$1 - H^{ss}$	0.025	Share of NEs over total K-firms
γ	6.1	Tail index of K-firms distributions
Central Bank		
κ_π	1.5	Taylor Rule inflation coefficient
κ_y	0.125	Taylor Rule output coefficient
$\rho_{R_n^{ss}}$	0.8	Interest rate smoothing
Exogenous Processes		
ρ_μ	0.975	TFP persistence
ρ_e	0	IST growth persistence
ρ_z	0	LAT growth persistence
σ^μ	0.01	TFP shock sd
σ^e	0.05	e_t shock sd
σ^z	0.01	z_t^n shock sd

2.3 Impulse Response Analysis

The conventional wisdom dating back to Bernanke, Gertler and Gilchrist (1999) looks at the role of the financial sector in the RBC-DSGE literature mainly as an amplification mechanism for the TFP shock transmission. In Figure 2.1 we see the effects of a transitory TFP shock in our model as compared to its analog without financial intermediaries. As expected, the banking sector still enhances the transmission of the total factor productivity shock.

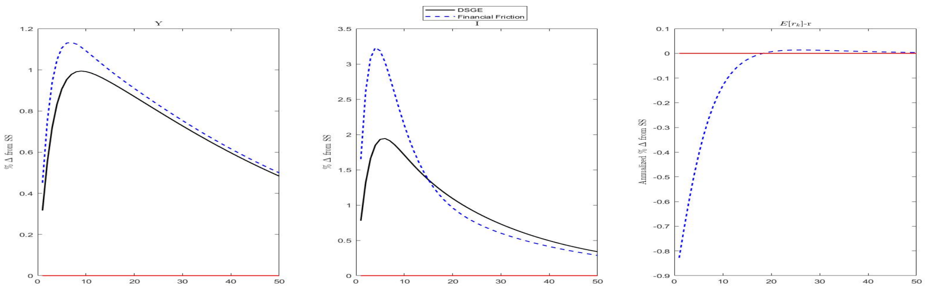

Figure 2.1: Impulse response functions to a transitory TFP shock.
$\sigma^{\mu} = 0.01$, $\rho_{\mu} = 0.975$

In this respect, it would be interesting to evaluate how the role played by the financial sector changes when the technology progress stems from different production sectors in the economy, especially when investment production evolves endogenously as in our model. This is relevant for at least a couple of reasons. First, we can exploit the interesting link between final goods production and financial intermediaries, typical of the GK's framework, as the former provides collateral to the latter in order to have the capital stock available in the production process. The second is directly related to the endogeneity of the relative price of investment, typical of our model, which strongly affects the first channel. Since in our model the optimal investment demand decision has also supply determinants, endogenous swings in Q directly affect financial intermediaries balance sheet through changes in the price of financial claims issued by intermediate producers in order to acquire the desired capital stock from K-firms. It follows that the endogenous feedbacks running from the I-sector to Final-goods production and financial intermediaries become relevant for the business cycle.

The most recent DSGE literature points at financial frictions as the right modeling choice to mimick developments in the aftermath of the latest financial crisis. However, those attempts failed to explain the weak recovery path undertaken by Western economies in spite of a quite high persistence in the several shocks simulating the occurrence of financial disruptions. To fill this gap, we argue that a relevant role can be played by the way endogenous responses of Q to financial shocks steer the production of investment goods.

In order to understand how financial frictions shape the impact of sectoral productivity shocks, we first benchmark our model against a DSGE version abstracting from the presence of the banking sector [8] and make then a comparison with its analog without entry and exit

[8] In this regard, in the simple DSGE version of our model the only difference is that households own the stock

flows in the I-sector. When we want to evaluate the occurrence of financial shocks instead, we compare our model with a simplified version of GK [9].

Finally, in this section we do not consider the LAT shock simulation since its effects are independent, in this model, from the presence of the banking sector[10].

2.3.1 Investment SpecificTechnology advances and financial frictions

The IST shock consists of a sudden and unexpected shift to the right of the potential NEs' pfd virtually keeping the NEs cutoffs (2.33) fixed. This causes an inflow of a higher mass of more productive NEs in the market strengthening competition among all I-firms.

In this respect, crucial to the attainment of the results we are going to present is the endogenous wedge betwee Q and φ^k as stated by K-aggregators optimal investment decision in (2.29).

The importance of financial frictions

In Figure 2.2 we display impulse responses to a permanent IST shock[11]. Blue dashed lines refers to our model with the inclusion of the financial sector, whilst black continuous ones refer to the simple DSGE version.

The realization of a positive IST shock triggers a supply increase in the I-sector as more productive NEs flow in the market. This puts downward pressure in the relative price of investment goods and pushes less productive incumbents out of the market. This is the well known "creative destrution" effect. In the (financially) frictionless version of the model, households understand Q will keep on declining and postpone their investment goods consumption, thus exacerbating the excess of supply in the investment market. When the financial sector is included in the model, K-firms who choose to postpone investment (see (2.29)). Their decision turns into a slack of demand for F-goods and ultimately lowers demand for bank loans. The ensuing fall in φ^k inevitably weakens banks profitability (the banker's continuation value). This turns into a higher interest rate spread which causes a deeper and more persistent creative destruction of I relative to what we observe in the DSGE model without financial frictions.

---

of capital and transfer funds to K-firms to give rise to the production gross investment.

[9] The version of GK we make use of abstracts from capital utilization rate and bears the same investment adjustment costs we have in our model along with an identical depreciation of capital. The only differences are thus up to the I-sector formulation.

[10] Results for the permanent LAT shock are left in Section B.6 in Appendix.

[11] The stochastic trend has been added back to impulse responses, thus percentage deviations in Figure 2.2 are to be read as from the "old" steady state.

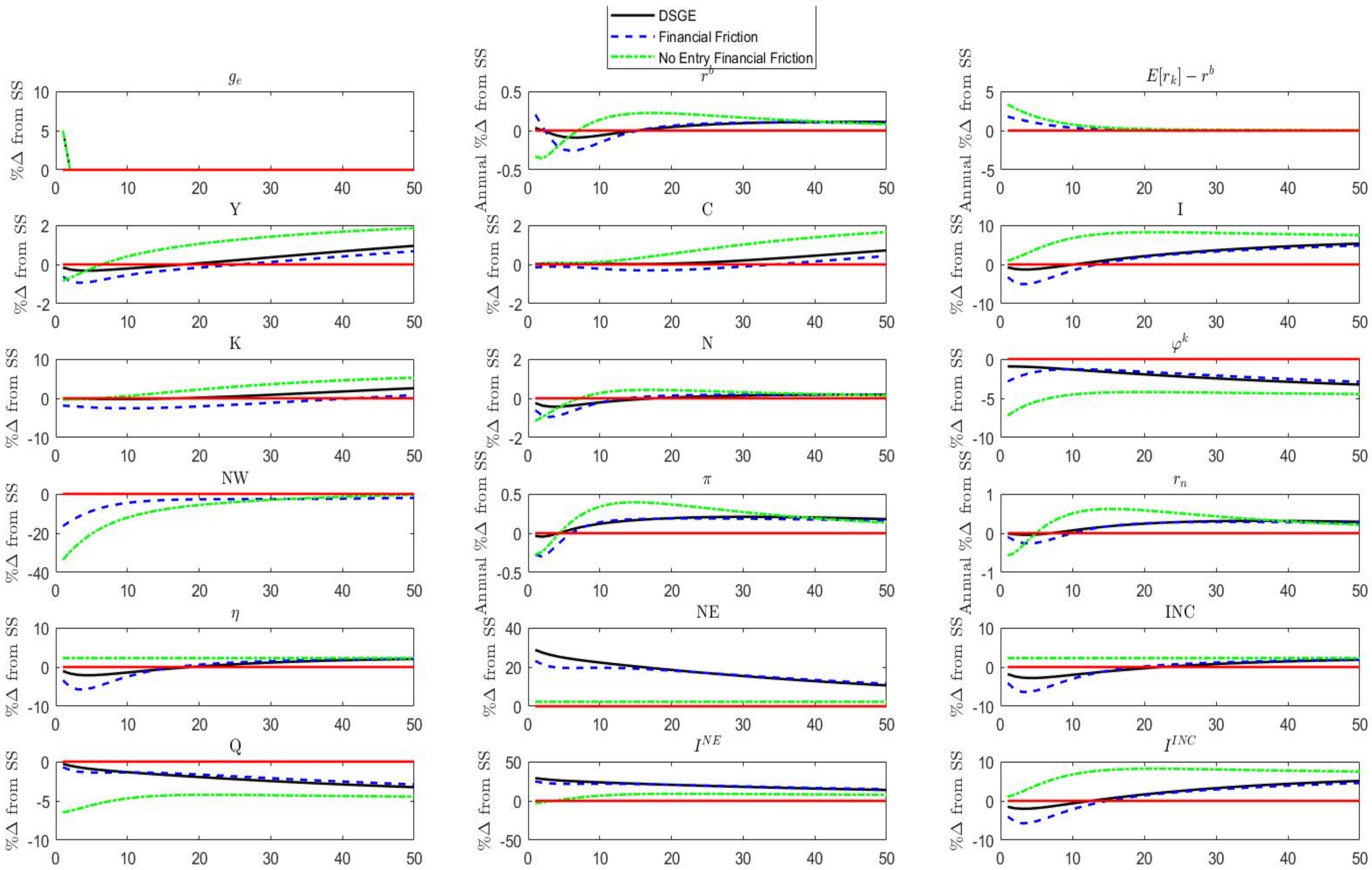

Figure 2.2: Impulse response functions to a permanent IST shock.
Shock size of the white noise component of $g_{e,t}$ is $\sigma^e = 0.05$, $\rho_e = 0$

The role of endogenous entry/exit flows

At this stage it is interesting to investigate the effects of a permanent IST shock when we remove endogenous firm dynamics. To model the shock, we assume that the mass of firms increses permanently. This, in turn, increaes the supply of I-goods at any given level of their relative price. Entry and exit flows are forced to remain constant at their ss level. As a result, the cyclical impact of the permanent IST does not affect I-firms distribution and productivity. This can easily be seen from dashed-dotted green lines in the fourth row of Figure 2.2: the mass of I-firms (and thus the I-sector thresholds) does not vary over the cycle, it just shifts exogenously as the shock materializes. [12]

Impulse responses are still depicted in Figure 2.2 (dashed-dotted green lines). Note that removing the "creative destruction" effect associated with endogenous firm dynamics strengthens the short run positive effect of the shock on the supply of I-firms. Such effect is also due to the fact that I-firms produce under decreasing returns to scale and incumbents react reducing their production without exiting the market. This in turn explains why the price of investment goods falls sharply on impact, whereas it barely moves in the benchmark model. Note also that the fall in Q steers the dynamics of φ^k via (2.29) and thus is associated to a much stronger fall in the net worth of bankers.

Overall, abstracting from endogenous entry and exit (and technology spillovers), removes a large part of the cyclical contribution of the permanent IST shock. Indeed, the variance of final output following the shock realization is now 3 times smaller.

For sake of clarity, stochastically detrended impulse responses are plotted[13] in Figure 2.3 below. In other words, the variables of interest are now expressed in terms of gaps from their "new" ss level[14]. This is done to visualize gap correlations which are the ones effectively discussed in the literature. Greenwood, Hercowitz and Krusell (2000) and Fisher (2006) show that the relative price of investment goods, Q, is countercyclical. As we can see, the model without endogenous entry/exit is not able to replicate the empirical evidence as abstracting from endogenous firm dynamics in the I-sector yields a procyclical dynamics of both φ^k_{gap} and Q_{gap}.

Again, this result stems from the absence of "creative destruction". In fact, Q_{gap} (φ^k_{gap}) and I_{gap} are negatively correlated in the benchmark model, differently from what happens when entry/exit is neglected.

[12] In the end, the only source of variation for investment supply functions in equations (B.28) and (B.29) is q. It turns out that I and Q are perfectly correlated as it is not the case in the benchmark model because of variations in NE, INC, $\hat{A}^{NE}$ and $\hat{A}^{INC}$.

[13] Stochastically detrended IRFs imply that variables are meant to be stationary with respect to their new stochastic trend (characterized by the new, permanent IST shock). By contrast when the stochastic trend is added back, as in Figure 2.2, IRFs are meant to be non-stationary with respect to the old stochastic trend, i.e. the one before the permanent IST shock realization.

[14] Thus, for instance, a negative output gap does not necessarily mean that a recession is occurring (net of the initial "creative destruction" effect, of course), rather that there is a transition from the "old" to the "new" ss and thus a final output increase in the medium run.

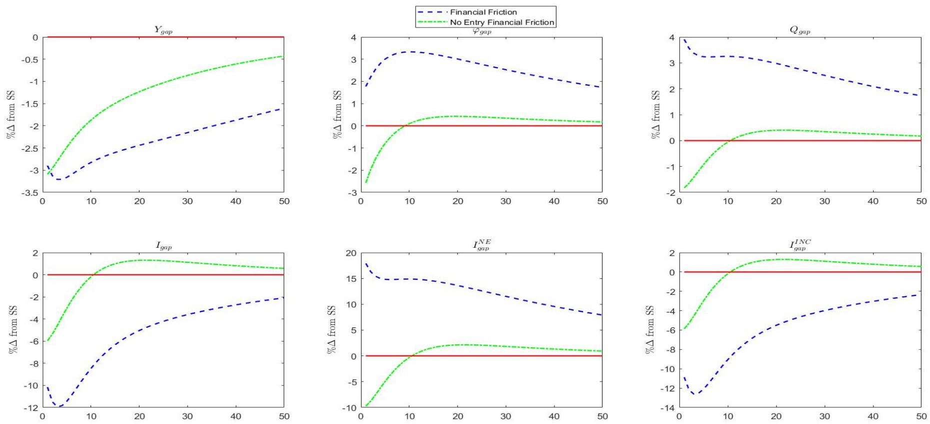

Figure 2.3: Cyclical Impulse response functions to a permanent IST shock. $\sigma^e = 0.05,\ \rho_e = 0$

2.3.2 The financial crisis and the slow recovery path

Similarly to GK, we run a shock to the quality of capital, and we assume that this shock occurrence is a rare event and, in order to mimic the post crisis response by the central bank, we abstract from interest rate smoothing in the Taylor rule. Impulse responses are depicted in Figure 2.4. Blue dashed lines refers to our model with financial sector, black continuous ones to the simplified version of GK. In our framework the transmission mechanism is reinforced due to the indirect feedbacks running between the I-sector and financial intermediaries (see condition (2.29)).

In GK a negative shock to the quality of capital triggers a slowdown in the financial claims price, φ^k, which is then associated to a fall in investment.[15] In our model, the slump in φ^k is exacerbated by (2.29) in two ways. First, the larger decline in φ^k ends up in a dramatically greater net worth reduction thus further tightening loans supply availability. Second, the initial fall in the price of investment goods triggers a sharp and persistent reduction in the inflow of new entrants and an increase in the outflow of incumbents weakening by a greater extent the demand of final goods for sectoral production. Even though there is a large outflow of I-firms, the following fall in I-sector production is cushioned by the higher average efficiency of I-firms during a recession. In fact, since Q initially falls, I-sector entry-exit thresholds are more binding. However, as Q turns positive (around quarter 10), this effect is reversed and the I-sector expansion is dampened by a lower average idiosyncratic productivity.

Overall, the initial fall in total investment is essentially unchanged relative to the standard GK model, but the subsequent investment recovery, when capital stock is driven back to steady state levels, is substantially slower. This leads to a much weaker recovery of GDP. In our exercise, during the first 50 quarters the accumulated output loss is 40% larger than in the standard GK model.

[15]It is important to point out that our framework differs from GK where $\varphi^k \equiv Q$.

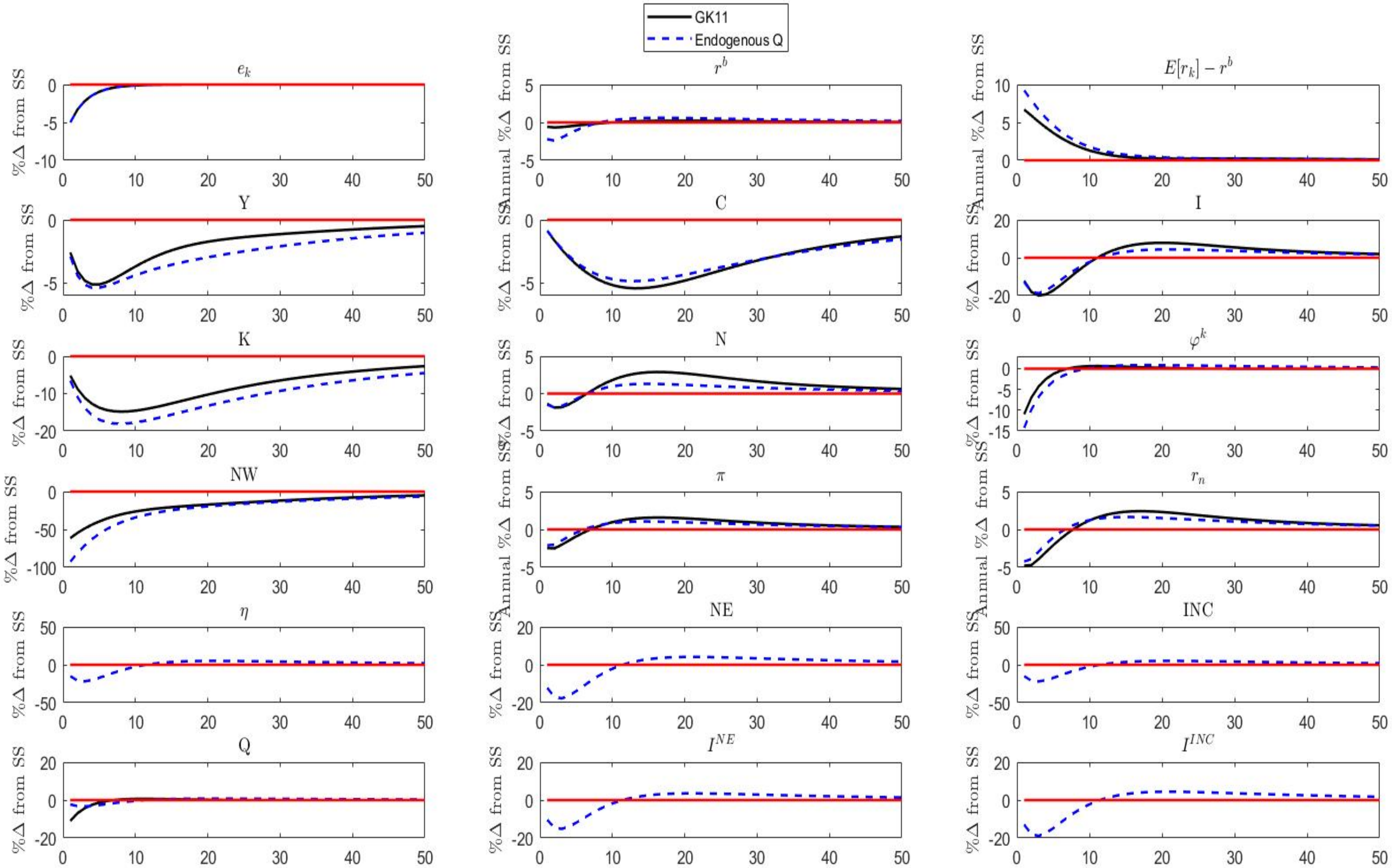

Figure 2.4: Impulse response functions to a Quality of capital shock.
Interest rate smoothing, $\rho_{r_n^{ss}} = 0$.

2.4 Conclusion

We developed a multi-sector DSGE model with a non-trivial financial banking sector and endogenous firm dynamics. The interaction between financial frictions and endogenous firm dynamics dampens the expansion following a positive IST shock and causes a larger output loss in consequence of a banking crisis.

To understand these results it is crucial to focus on the banks balance sheet implications of endogenous firm dynamics which turn out to be asymmetrical, working countercyclically in response to IST shocks and procyclically in case of a banking crisis.

2

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Appendix A

Appendix to Chapter 1

A.1 List of Detrended Equations

Here we list all the relevant equations in our model. The details of stochastic trend identification and removal are left in section A.2.

The stochastic trend governing aggregate variables as Y_t, S_t, C_t, W_t and governing inflation dynamics recursive components is $\Gamma_t = \frac{e_t^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{t(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}}$; the stochastic trend governing K_t and I_t is instead $\Lambda_t = \frac{e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{t[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}}$.

The relative price of investment and the shadow price of capital in consumption units, Q_t and φ_t^k respectively, share the same stochastic trend that is $\frac{\Gamma_t}{\Lambda_t}$, which also determines the dynamics of $r_{k,t}$.

The stochastic trend governining K-firms mass, η (but also NE and INC), is $\frac{e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} (z_t^n)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{t\gamma\left\{(1-\alpha)-\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}\right\}}}$, whilst the one governing K-firms cutoff, $\hat{A}_t^K$, is $\frac{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{(z_t^n)^{\frac{\chi}{1+\chi(\gamma-1)}} g_*^{t\left\{\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}-(1-\alpha)\right\}}}$. Finally, the K-firms fixed costs trend is by assumption deterministic and equals the BGP growth rate, g_*^t.

Lower case characters stand for stochastically detrended variables (i.e., variables expressed in terms of gaps from their ss in previous sections), the only exception concerns λ^*, and $\varphi_t^{*,k}$, $r_{k,t}^*$, and η_t^* which are stochastically detrended marginal utility of consumption and capital (in consumption units), rental rate of capital, and the probability mass of active K-firms.

A.1.1 Households

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \quad \text{Marginal utility of consumption} \tag{A.1}$$

$$w_t = \frac{\Phi N_t^{\theta}}{\lambda_t^*} \quad \text{Supply of Labor} \tag{A.2}$$

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\bar{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}} (1-\delta) \right] \right\} \quad \text{Capital Euler} \tag{A.3}$$

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \frac{R_{n,t}}{\pi_{t+1}} \right\} \quad \text{Bond Euler} \tag{A.4}$$

$$\begin{aligned} q_t &= \varphi_t^{*,k} \mu_t^i \left\{ 1 - \left[S' \left(\frac{i_t \bar{g}_t}{i_{t-1}} \right) \frac{i_t \bar{g}_t}{i_{t-1}} + S \left(\frac{i_t \bar{g}_t}{i_{t-1}} \right) \right] \right\} + \\ &+ \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^* \bar{g}_{t+1}} \varphi_{t+1}^{*,k} \mu_{t+1}^i S' \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} \right) \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} \right)^2 \right\} \end{aligned} \quad \text{Investment rule} \tag{A.5}$$

A.1.2 Intermediate Producers

$$k_t = (1-\delta) \frac{k_{t-1}}{\bar{g}_t} + \mu_t^i \left[1 - S \left(\frac{i_t \bar{g}_t}{i_{t-1}} \right) \right] i_t \quad \text{Law of motion of capital} \tag{A.6}$$

$$y_t = \frac{N_t^{\chi} \left(\frac{K_{t-1}}{\bar{g}_t} \right)^{1-\chi}}{\xi_t} \quad \text{Final Output} \tag{A.7}$$

$$r_{k,t}^* = \frac{mc_t}{\xi_t} (1-\chi) \left[\frac{\bar{g}_t N_t}{k_{t-1}} \right]^{\chi} \quad \text{Demand of Capital} \tag{A.8}$$

$$w_t = \frac{mc_t}{\xi_t} \chi \left[\frac{k_{t-1}}{\bar{g}_t N_t} \right]^{1-\chi} \quad \text{Demand of Labor} \tag{A.9}$$

A.1.3 Final Producers

$$d_t = \pi_t^* y_t + \beta \lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{\pi_t^*}{\pi_{t+1}^*} \left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}} \right)^{\nu-1} d_{t+1} \quad \text{First Recursive Inflation Term} \tag{A.10}$$

$$f_t = mc_t y_t + \beta \lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*} \left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}} \right)^{\nu} f_{t+1} \quad \text{Second Recursive Inflation Term} \tag{A.11}$$

$$d_t = \frac{\nu}{(\nu-1)} f_t \quad \text{Inflation dynamics} \tag{A.12}$$

$$1 = (1-\lambda_p) (\pi_t^*)^{1-\nu} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t} \right)^{1-\nu} \quad \text{Evolution of prices} \tag{A.13}$$

$$\xi_t = (1-\lambda_p) (\pi_t^*)^{-\nu} + \lambda_p \left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t} \right)^{-\nu} \xi_{t-1} \quad \text{Price Dispersion} \tag{A.14}$$

A.1.4 Capital Producers

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^{\alpha}} \qquad \text{NEs cutoff} \quad (A.15)$$

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^{\alpha}} \qquad \text{INCs cutoff} \quad (A.16)$$

$$\eta_t^* = ne_t + inc_t \qquad \text{Mass of active K-producers} \quad (A.17)$$

$$ne_t = \left(\frac{\hat{a}_t^{NE}}{e^{ss}}\right)^{-\gamma} \qquad \text{NEs mass} \quad (A.18)$$

$$inc_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t}\right)^{\chi} \frac{1}{g_*^{1-\alpha}}\right]^{\gamma} \qquad \text{INCs mass} \quad (A.19)$$

$$exit_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left\{1 - \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t}\right)^{\chi} \frac{1}{g_*^{1-\alpha}}\right]^{\gamma}\right\} \qquad \text{Exit Mass} \quad (A.20)$$

$$i_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (q_t \alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{NEs gross investment} \quad (A.21)$$

$$i_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} (q_t \alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{INCs gross investment} \quad (A.22)$$

$$i_t = i_t^{NE} + i_t^{INC} \qquad \text{K-producers gross investment} \quad (A.23)$$

$$s_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} +$$

$$+ inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} \qquad \text{K firms total input amount} \quad (A.24)$$

$$\bar{a}_t = \frac{\gamma}{\gamma-1} ne_t \hat{a}_t^{NE} + \frac{\gamma}{\gamma-1} inc_t \hat{a}_t^{INC} \qquad \text{K-sector productivity} \quad (A.25)$$

A.1.5 Market clearing conditions and policy rules

$$y_t = c_t + s_t + ne_t f^{NE} + inc_t f^{INC} \qquad \text{Market clearing}$$

(A.26)

$$\left(\frac{R_{n,t}}{R_n^{ss}}\right) = \left(\frac{R_{n,t-1}}{R_n^{ss}}\right)^{\rho_{R_n^{ss}}} \left[\left(\frac{\pi_t}{\pi^{ss}}\right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu-1)/\nu}\right)^{\kappa_y} \exp\left\{\sigma^r \varepsilon_t^r\right\}\right]^{1-\rho_{R_n^{ss}}} \qquad \text{Taylor rule} \quad (A.27)$$

A.1.6 Autoregressive processes

$$\ln\left(g_{e,t}\right) = (1-\rho_e)\ln(g_e) + \rho_e \ln\left(g_{e,t-1}\right) + \sigma^e \varepsilon_t^e \qquad \text{K-Tech growth rate} \tag{A.28}$$

$$\ln\left(g_{z,t}\right) = (1-\rho_z)\ln(g_*) + \rho_z \ln\left(g_{z,t-1}\right) + \sigma^z \varepsilon_t^z \qquad \text{L-Tech growth rate} \tag{A.29}$$

$$\ln(\bar{g}_t) = \frac{\gamma}{1+\chi(\gamma-1)}\ln\left(g_{e,t}\right) + \frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}\ln(g_{z,t}) + \\ - \frac{\gamma(1-\alpha)-1}{1+\chi(\gamma-1)}\ln(g_*) \qquad \text{K-Production stochastic growth} \tag{A.30}$$

$$\ln(\widetilde{g}_t) = \frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}\ln\left(g_{e,t}\right) + \frac{\chi\gamma}{1+\chi(\gamma-1)}\ln(g_{z,t}) + \\ - \frac{(1-\chi)\left[\gamma(1-\alpha)-1\right]}{1+\chi(\gamma-1)}\ln(g_*) \qquad \text{F-Production stochastic growth} \tag{A.31}$$

$$\ln\left(\mu_t^i\right) = \left(1-\rho_{\mu^i}\right)\ln\left(\mu^i\right) + \rho_{\mu^i}\ln\left(\mu_{t-1}^i\right) + \sigma^i \varepsilon_t^i \qquad \text{MEI shock} \tag{A.32}$$

A.2 Removing the Stochastic Trends Governing the Economy

Let us assume Γ_t is the stochastic trend governing Y_t, C_t, W_t and S_t, from which follows that, for instance, $Y_t = \Gamma_t y_t$, where smaller case characters are meant to be detrended variables if not differently specified. Moreover, assume that Λ_t is the stochastic trend governing K_t and I_t, thus $K_t = \Lambda_t k_t$. We claim that both Γ_t and Λ_t are convolutions of the labor augmenting and the *NE*s permanent technology shifters, z_t^n and e_t.

A.2.1 Final production

Without any loss of generality we can rewrite final production as

$$y_t \Gamma_t = \frac{(z_t^n N_t)^\chi \left(\frac{\Lambda_t k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{A.33}$$

where $\bar{g}_t = \frac{\Lambda_t}{\Lambda_{t-1}}$. Then we define

$$\Gamma_t = (z_t^n)^\chi \Lambda_t^{1-\chi} \tag{A.34}$$

Which can be interpreted as the non stationary stochastic evolution of TFP in our model. Thus, dividing (A.33) by (A.34) we obtain

$$y_t = \frac{(N_t)^\chi \left(\frac{k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{A.35}$$

In a similar fashion we can work out the detrended law of motion of capital by dividing both sides by Λ_t, that is

$$k_t = (1-\delta)\frac{k_{t-1}}{\bar{g}_t} + \mu_t^i \left[1 - \frac{\gamma_I}{2}\left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g}\right)^2\right] i_t \tag{A.36}$$

Again, without any loss of generality, the demand of capital can be rewritten as

$$r_{k,t} = \frac{mc_t}{\xi_t}(1-\chi)\left[\frac{z_t^n}{\Lambda_t}\frac{\bar{g}_t N_t}{k_{t-1}}\right]^\chi \tag{A.37}$$

Then, the stochastic trend leading $r_{k,t}$ is, manipulating (A.34), $\left(\frac{z_t^n}{\Lambda_t}\right)^\chi = \frac{\Gamma_t}{\Lambda_t}$, from which follows that the detrended rental rate of capital is

$$r_{k,t}^* = \frac{mc_t}{\xi_t}(1-\chi)\left[\frac{\bar{g}_t N_t}{k_{t-1}}\right]^\chi \tag{A.38}$$

where $r_{k,t}^* = r_{k,t}\frac{\Lambda_t}{\Gamma_t}$, is the stochastically detrended rental rate of capital.

Then we claimed that W_t shares the same stochastic trend as Y_t, therefore

$$w_t \Gamma_t = \frac{mc_t}{\xi_t}\chi (z_t^n)^\chi \Lambda_t^{1-\chi}\left[\frac{k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi}$$

implying that

$$w_t = \frac{mc_t}{\xi_t}\chi\left[\frac{k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi} \tag{A.39}$$

Where the marginal cost, mc_t, is stationary by itself since $\Gamma_t = (z_t^n)^\chi \Lambda_t^{1-\chi}$,

$$mc_t = \left(\frac{w_t \Gamma_t}{\chi z_t^n}\right)^\chi \left(\frac{r_{k,t}^* \Gamma_t}{(1-\chi)\Lambda_t}\right)^{1-\chi} \equiv \left(\frac{w_t}{\chi}\right)^\chi \left(\frac{r_{k,t}^*}{(1-\chi)}\right)^{1-\chi}$$

Retailers

For what concerns recursive inflation trend, they do have, by costruction, the same stochastic trend as Y. Therefore their detrended version is

$$d_t = \pi_t^* y_t + \beta E_t\left\{\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\pi_t^*}{\pi_{t+1}^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu-1} d_{t+1}\right\} \tag{A.40}$$

And

$$f_t = mc_t y_t + \beta E_t\left\{\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu} f_{t+1}\right\} \tag{A.41}$$

Thus,

$$d_t = \frac{\nu}{\nu-1} f_t \tag{A.42}$$

Finally, price dispersion and price evolution are unchanged.

A.2.2 Households

From before, we implicitly assumed $C_t = c_t \Gamma_t$, where we also define $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$. At this point we also have that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ where λ_t^* is the stochastically detrended MUC.

Then, MUC can be rewritten as

$$\frac{\lambda_t^*}{\Gamma_t} = \frac{1}{\Gamma_t c_t - a\Gamma_{t-1}c_{t-1}} - \beta a \frac{1}{\Gamma_{t+1}c_{t+1} - a\Gamma_t c_t}$$

Then multiplying on both sides by Γ_t and rearranging we have

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \tag{A.43}$$

The leisure-consumption relationship reads

$$w_t = \Phi \frac{N_t^\theta}{\lambda_t^*} \tag{A.44}$$

and from the Bond-Euler

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \frac{R_{n,t}}{\pi_{t+1}} \right\} \tag{A.45}$$

Before moving to the capital-Euler, we remark that the shadow price of capital in consumption units is $\varphi_t^k = \frac{\phi_t^k}{\lambda_t}$, where we know that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ and must also hold true that $\phi_t^k = \frac{\phi_t^{*,k}}{\Lambda_t}$ since Λ_t is the stochastic trend governing capital. This implies that $\varphi_t^k = \frac{\phi_t^{*,k}/\Lambda_t}{\lambda_t^*/\Gamma_t} \equiv \varphi_t^{*,k} \frac{\Gamma_t}{\Lambda_t}$. Plugging the latter into the capital-Euler yields

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}} \frac{\widetilde{g}_{t+1}}{\bar{g}_{t+1}} (1-\delta) \right] \right\}$$

which boils down to

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\bar{g}_{t+1}} \left[\frac{r_{k,t+1}^*}{\varphi_t^{*,k}} + \frac{\varphi_{t+1}^{*,k}}{\varphi_t^{*,k}} (1-\delta) \right] \right\} \tag{A.46}$$

Further, the optimal investment condition implies that the stochastic trend of φ^k is the same leading Q, implying $Q_t = q_t \frac{\Gamma_t}{\Lambda_t}$. Then, dividing both sides of (1.17) by $\frac{\Gamma_t}{\Lambda_t}$ and rearranging, we have

$$\begin{aligned} q_t = \varphi_t^{*,k} \mu_t^i \left\{ 1 - \left[\gamma_I \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right) \frac{i_t \bar{g}_t}{i_{t-1}} + \frac{\gamma_I}{2} \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right)^2 \right] \right\} + \\ + \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{\varphi_{t+1}^{*,k}}{\bar{g}_{t+1}} \mu_{t+1}^i \gamma_I \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} - \bar{g} \right) \left(\frac{i_{t+1} \bar{g}_{t+1}}{i_t} \right)^2 \right\} \end{aligned} \tag{A.47}$$

A.2.3 K-firms

We remark that by assumption $f_t^k = g_*^t f^k$ for $k = [NE, INC]$, i.e. the trend leading fixed costs is purely deterministic. This allows us rewriting the NEs cutoff as

$$\hat{A}_t^{NE} = \left(\frac{g_*^t f^{NE}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t \frac{\Gamma_t}{\Lambda_t}}$$

Then exploiting the fact that $\frac{\Lambda_t}{\Gamma_t} = \left(\frac{\Lambda_t}{z_t^n} \right)^\chi$ we have that

$$\hat{A}_t^{NE} = \hat{a}_t^{NE} \left(\frac{\Lambda_t}{z_t^n} \right)^\chi g_*^{t(1-\alpha)} \tag{A.48}$$

and thus

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t} \tag{A.49}$$

from which follows

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha}\right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t} \tag{A.50}$$

At this point we can easily rewrite the mass of active NEs as

$$NE_t = \left(\frac{e^{ss} e_t \left(z_t^n\right)^\chi}{\hat{a}_t^{NE} \Lambda_t^\chi g_*^{t(1-\alpha)}}\right)^\gamma = \left(\frac{e^{ss}}{\hat{a}_t^{NE}}\right)^\gamma \frac{\left(z_t^n\right)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$$

Implying that $NE_t = ne_t \frac{(z_t^n)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and so

$$ne_t = \left(\frac{e^{ss}}{\hat{a}_t^{NE}}\right)^\gamma \tag{A.51}$$

By BGP conditions we know that also η_t and INC_t share the same stochastic trend as NE_t, this implies

$$\eta_t^* = ne_t + inc_t \tag{A.52}$$

And

$$inc_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^\gamma} \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t}\right)^\chi \frac{1}{g_*^{1-\alpha}}\right]^\gamma \tag{A.53}$$

Where of course $INC_t = inc_t \frac{(z_t^n)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and $\eta_t = \eta_t^* \frac{(z_t^n)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and η_{t-1} in (A.53) must be expressed accordingly.

A.2.4 Stochastic Trends Identification

Notice that from the aggregate resource constraint in (1.34) it turns out that the stochastic trend leading $NE_t f_t^{NE}$ and $INC_t f_t^{INC}$ must be, by construction, the same leading Y_t, C_t and S_t, i.e. Γ_t. Thus, given that $NE_t f^{NE} g_*^t \equiv ne_t f^{NE} \frac{(z_t^n)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} g_*^t$, we can easily work out

$$\Gamma_t = \frac{\left(z_t^n\right)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}} \tag{A.54}$$

Then, plugging the relationship $\Gamma_t = \left(z_t^n\right)^\chi \Lambda_t^{1-\chi}$ into (A.54) allows for the identification of the stochastic trend leading both K_t and I_t, that is

$$\Lambda_t = \frac{e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} \left(z_t^n\right)^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{t[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{A.55}$$

Then, plugging (A.55) into (A.54) we have

$$\Gamma_t = \frac{e_t^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} \left(z_t^n\right)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{t(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{A.56}$$

Which is the stochastic trend leading aggregate variables but K_t and I_t. Thus the stochastic trend governing aggregate variables is a Cobb-Douglas of the the permanent shifters governing the NEs technology shifter and final goods production, respectively [1].

At this point we can also identify the stochastic trend leading K-firms cutoff. For instance, substituting for (A.55) into (A.48) we obtain that the corresponding stochastic trend is $\frac{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{(z_t^n)^{\frac{\chi}{1+\chi(\gamma-1)}} g_*^{t\left\{\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}-(1-\alpha)\right\}}}$.

Finally, plugging (A.55) into (A.51), (A.52) and (A.53) it turns out that the stochastic trend leading the K-firms industry composition is $\frac{e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}}(z_t^n)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{t\gamma\left\{(1-\alpha)-\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}\right\}}}$.

A.2.5 K-firms production

At this point, since we claimed that K_t and I_t are governed by the same stochastic trend, i.e. Λ_t, this implies that $I_t^{NE} = i_t^{NE}\Lambda_t$. Then

$$i_t^{NE}\Lambda_t = ne_t\frac{(z_t^n)^{\chi\gamma}e_t^{\gamma}}{\Lambda_t^{\chi\gamma}g_*^{t\gamma(1-\alpha)}}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\frac{\hat{a}_t^{NE}\Lambda_t^{\chi}g_*^{t(1-\alpha)}}{(z_t^n)^{\chi}}\right)^{\frac{1}{1-\alpha}}\left[\alpha q_t\left(\frac{z_t^n}{\Lambda_t}\right)^{\chi}\right]^{\frac{\alpha}{1-\alpha}}$$

From which rearranging

$$i_t^{NE}\Lambda_t = \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}ne_t\left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}}(\alpha q_t)^{\frac{\alpha}{1-\alpha}}\frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{\Lambda_t^{\chi(\gamma-1)}g_*^{t[\gamma(1-\alpha)-1]}}$$

But from (A.55) we know that $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$ which therefore implies

$$i_t^{NE} = ne_t\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}}(\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{A.57}$$

And thus it must also be that

$$i_t^{INC} = inc_t\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}}(\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{A.58}$$

And

$$i_t = i_t^{NE} + i_t^{INC} \tag{A.59}$$

A.2.6 Aggregate Resources Constraint

There is only one variable to be detrended yet. By construction it must be $S_t = s_t\Gamma_t$ which also implies $s_t\Gamma_t = s_t^{NE}\Gamma_t + s_t^{INC}\Gamma_t$.

Then it is sufficient to show that

[1] According to our parametrization $\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)} < 1$.

$$s_t^{NE}\Gamma_t = ne_t \frac{(z_t^n)^{\chi\gamma}\, e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \left(\frac{z_t^n}{\Lambda_t}\right)^{\chi} \hat{a}_t^{NE} \left(\frac{\Lambda_t}{z_t^n}\right)^{\chi} g_*^{t(1-\alpha)}\right]^{\frac{1}{1-\alpha}}$$

Can be rewritten as

$$s_t^{NE}\Gamma_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}} \frac{(z_t^n)^{\chi\gamma}\, e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}}$$

And again, since $\Gamma_t = (z_t^n)^{\chi}\Lambda^{(1-\chi)}$ and $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$, it must be that

$$s_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}} \tag{A.60}$$

And

$$s_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \hat{a}_t^{INC}\right]^{\frac{1}{1-\alpha}} \tag{A.61}$$

And

$$s_t = s_t^{NE} + s_t^{INC} \tag{A.62}$$

Thus, we have proven that

$$y_t - ne_t f^{NE} - inc_t f_t^{INC} = c_t + s_t \tag{A.63}$$

Holds true.

A.2.7 Stochastic Growth Rates Identification

We claimed that $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$, then exploiting (A.56) it turns out that

$$\widetilde{g}_t = \frac{g_{e,t}^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} g_{z,t}^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{A.64}$$

meaning that the stochastic BGP growth rate is a convolution of the stochastic growth rate of e_t and z_t^n. Similarly for $\frac{\Lambda_t}{\Lambda_{t-1}} = \bar{g}_t$ it follows that

$$\bar{g}_t = \frac{g_{e,t}^{\frac{\gamma}{1+\chi(\gamma-1)}} g_{z,t}^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{\gamma(1-\alpha)-1}{1+\chi(\gamma-1)}}} \tag{A.65}$$

Finally, in the deterministic steady state we have that $g_e = g_*^{1-\alpha}$ (see Section A.3 below). Moreover, also $\bar{g} = \widetilde{g} = g_*$ must hold true, i.e. the deterministic BGP is the same for all aggregated variables, which is verified plugging $g_e = g_*^{1-\alpha}$ into the deterministic formulation of (A.64) and (A.65).

A.3 Deterministic Steady State and the Existence of a Balanced Growth Path

In the deterministic steady state

$$\frac{z_t^n}{z_{t-1}^n} \equiv g_{z,t} = g_* \tag{A.66}$$

where z^n defines the ss value of the labor augmenting technology shifter and g_* is the BGP growth rate of the economy. Output, capital, investment, consumption and the real wage all grow at the BGP rate while the relative price of investment and the labor supply are constant. The latter is pinned down by the preference parameter Φ in (1.16). Variables without time index are detrended or, in a deterministic environment, implicitly stationary and investment adjustment costs are nil. Further, from (1.20) it is straightforward to show that the fixed costs $f_t^{K,ss}$ also grow at the BGP rate g^*. We assume that the monetary policy rule achieves $\pi = 1$. As a result, from (A.45), the real interest rate on the riskless bond is

$$\frac{g_*}{\beta} = \frac{R_n^{ss}}{\pi^{ss}} \equiv R_n^{ss} \tag{A.67}$$

From condition (A.47) the shadow price of capital is equal to the price of investment goods

$$\varphi^{k,ss} = Q^{ss} \tag{A.68}$$

where Q^{ss} is obtained when the investment goods market clears.

The steady state rental rate of capital stems from (A.46) and is

$$\frac{g_*}{\beta} - 1 + \delta = \frac{r_k^{ss}}{Q^{ss}} \tag{A.69}$$

Then, for what concerns the final producer's capital FOC we have that in steady state

$$mc^{ss}(1-\chi)\left[\frac{g_* N^{ss}}{K^{ss}}\right]^{\chi} = r_k^{ss} \tag{A.70}$$

Demand of capital for production is

$$K^{ss} = g^* N^{ss}\left[\frac{mc^{ss}(1-\chi)}{\left(\frac{g_*}{\beta} - 1 + \delta\right) Q^{ss}}\right]^{\frac{1}{\chi}} \tag{A.71}$$

From the capital accumulation condition

$$I_t^{ss} = \left(1 - \frac{1-\delta}{g_*}\right) K^{ss} \tag{A.72}$$

Given the monopolistic nature of the final goods market, in the zero net inflation steady state the marginal cost is

$$mc^{ss} = \frac{\nu - 1}{\nu}$$

And the real wage is obtained solving

$$mc^{ss} = \left(\frac{r_k^{ss}}{1-\chi}\right)^{1-\chi}\left(\frac{W^{ss}}{g_*\chi}\right)^{\chi} \tag{A.73}$$

To obtain closed form solutions for the above conditions (A.68) - (A.73) we need to solve for the K-sector market clearing condition.

A.3.1 K-sector

To begin with, bear in mind that $f_t^{NE,ss}$, $f_t^{INC,ss}$, e_t^{ss} respectively define fixed costs and the K-firms technology shifter where the latter gows at the BGP rate $g_e \neq g_*$. We therefore define $e_t^{ss} \equiv e^{ss} g_e^t$ and $f_t^{NE,ss} \equiv f^{NE} g_*^t$ as well as $f_t^{INC,ss} \equiv f^{INC} g_*^t$. The solution for NE^{ss} is thus

$$NE^{ss} = \left[Q^{ss}\alpha^{\alpha} e^{ss} g_e^t \left(\frac{1-\alpha}{f^{NE} g_*^t}\right)^{1-\alpha}\right]^{\gamma} \tag{A.74}$$

Thus, in order to have a constant non zero and non diverging mass of NEs, it turns out that $g_e = g_*^{1-\alpha}$ must necessarily hold true. In the end (A.74) boils down to the deterministic version of (A.51).

Then we can rewrite (A.52) to obtain η^{ss}

$$\begin{aligned}
\eta^{ss} &= NE^{ss} + INC^{ss} \\
\eta^{ss} &= NE^{ss} + \eta^{ss}\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma} \\
\eta^{ss} &= \frac{NE^{ss}}{1-\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}
\end{aligned} \tag{A.75}$$

And INC^{ss}

$$INC^{ss} = NE^{ss}\frac{\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}{1-\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}$$

Further it must also be that $\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma} < 1$ in order to have a positive exiting mass of incumbents ruling thus out the possibility of an exploding mass of active firms as it can be seen from (1.30).

We can now solve for ss investments. From condition (A.57) and (A.53) we get

$$I^{NE,ss} = NE^{ss}\frac{\gamma f^{NE,ss}}{[\gamma(1-\alpha)-1]\,Q^{ss}} \tag{A.76}$$

and

$$I^{INC,ss} = INC^{ss}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\hat{A}^{ss^{INC}}\right)^{\frac{1}{1-\alpha}}(Q^{ss}\alpha)^{\frac{\alpha}{1-\alpha}} \tag{A.77}$$
$$= \eta^{ss}\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}$$

As evident from above, when choosing K-firms returns to scale and tail index a condition must be respected, that is $\gamma(1-\alpha) > 1$. This is done to guarantee that gross investment production is positive as it appears clearly from (A.76) and (A.77).

A.3.2 Market clearing

Using (A.72), (A.66) and (A.71), we get

$$I^{ss} = \left(1-\frac{1-\delta}{g_*}\right)K^{ss}$$
$$= \left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} \tag{A.78}$$

Using (A.74), (A.75), (A.76), (A.77), (A.78) we get that Q^{ss} solves the following market clearing condition for the investment goods sector:

$$I^{ss} = I^{NE,ss} + I^{INC,ss} \Rightarrow$$
$$\left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} =$$
$$\left[\frac{e^{ss}Q^{ss}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left(f^{NE}\right)^{1-\alpha}}\right]^{\gamma}\left\{\frac{\gamma f^{NE}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}+\frac{\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}}{1-\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}\right\} \tag{A.79}$$

It is now possible to work out the closed form solutions for all endogenous variables.[2]

[2] As pointed out in section 2.2.7 we calibrate the model so that $Q^{ss} = 1$ and thus calibrate e^{ss} accordingly.

A.4 K-sector Production

Here we derive overall production in the K-sector.

A.4.1 Derivation of NEs total production

Let us start from new entrants. We know that the production function for the generic *NE* firm can be expressed as

$$I_t^{NE,j} = \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t \alpha)^{\frac{\alpha}{1-\alpha}} \tag{A.80}$$

Then, by exploiting the transformation theorem we can compute the expected value of *NE*s production

$$\begin{aligned}
I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \, dF(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} f(A_t^{NE,j}) \, d(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma e_t^{\gamma} \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}-\gamma-1} d(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \gamma e_t^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t^{NE}}^{+\infty} \\
\Rightarrow I_t^{NE} &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{A.81}$$

Where we exploited the fact that $NE_t = \left(\frac{\hat{A}_t^{NE}}{e_t}\right)^{-\gamma}$ and by assumption it must hold true that $\gamma(1-\alpha) - 1 > 0$.

Notice moreover that what we have computed is nothing different than the mean of a truncated distribution without normalizing it to a unit probability measure i.e., without dividing it by the probability share over which it is computed. This is done because in our model we want a measure of the total production in the *NE*s industry. Should one want to compute the idiosyncratic average production it would be sufficient dividing (A.81) by NE_t.

A.4.2 Derivation of INCs total production

Let us repeat the same computation for incumbents. The production function for the generic incumbent firm is

$$I_t^{INC,j} = \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t \alpha)^{\frac{\alpha}{1-\alpha}} \tag{A.82}$$

Then, as before we have

$$
\begin{aligned}
I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, dF(A_t^{INC,j}) \\
\Rightarrow I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, f(A_t^{INC,j})\, d(A_t^{INC,j}) \\
\Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}-\gamma-1} d(A_t^{INC,j}) \\
\Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t^{INC}}^{+\infty} \\
\Rightarrow I_t^{INC} &= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned}
\tag{A.83}
$$

Where we have exploited the fact that $\eta_{t-1} \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{INC}}\right)^{\gamma} = INC_t$.
Then, as the expected value of the sum is the sum of the expected values, we have that

$$
I_t = I_t^{NE} + I_t^{INC} \tag{A.84}
$$

A.5 K-firms profits derivation

Total revenues of the K-sector are $Q_t I_t$. Let us now define the the total amount of savings employed as input in the production of capital goods as

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \tag{A.85}$$

where $\int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) \equiv S_t^{NE}$ and $\int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \equiv S_t^{INC}$ are the total amount of inputs used in NEs and INCs sector production.

It follows that profits are respectively

$$\begin{aligned} \Pi_t^{NE} &= Q_t I_t^{NE} - S_t^{NE} - NE_t f^{NE} \\ &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(Q_t \hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - NE_t f^{NE} \end{aligned} \tag{A.86}$$

And

$$\begin{aligned} \Pi_t^{INC} &= Q_t I_t^{INC} - S_t^{INC} - INC_t f^{INC} \\ &= \; INC_t \frac{\phi(1-\alpha)}{\phi(1-\alpha)-1} \left(Q_t \hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - \; INC_t f^{INC} \end{aligned} \tag{A.87}$$

Which are always positive by construction as $\alpha < 1$. Then, define the total expenditures in fixed costs of the K-sector as

$$\overline{F}_t = NE_t f^{NE} + INC_t f^{INC} \tag{A.88}$$

Finally let us define the total amount of profits in the K-sector as

$$\overline{\Pi}_t = \Pi_t^{NE} + \Pi_t^{INC} \tag{A.89}$$

Then by substituting for (1.33), (A.86), (A.87) and (A.88) into equation (A.89) and rearranging we obtain the following identity

$$\overline{\Pi}_t + S_t + \overline{F}_t = Q_t I_t \tag{A.90}$$

Simply stating that the total amount of capital goods (in real terms) produced in the K-sector must be equal to the sum of profits, the input share of production and the total amount of fixed costs. Indeed households hold the stock of capital and use savings to buy the gross investment from the K-sector as a whole. The share of households revenues S_t is employed by K-firms in investment production at real good price cost, whilst the share $\overline{F}_t$ is devoted to fixed costs payment.

A.6 Sensitivity analysis for different parametrizations of the K-sector

Here we perform a sensitivity analysis for possibly different parametrizations of of our K-sector. The key parameters we play around with are the pareto tail index, γ, and the *NE*s initial condition for the technological shifter, e. The first thing to notice is that varying K-firms returns to scale, α is useless. This is can be easily seen, from the list of log-linearised equations in Section A.7, by plugging (A.104) and (A.105) into (A.110) and (A.111), respectively. It turns out that the dynamics of gross investments, and thus of other real variables, is never affected by changes in α.

Thus, the crucial parameter for our sensitivity analysis is γ. In particular it describes the rate at which the updated incumbents pdf decays. The lower it is, the slower the pdf approaches zero. This also implies that incumbents updating is more successful since, as compared with higher values of γ, there are more frequencies distributed on higher idiosyncratic productivity values. It follows that the recovery from creative destruction is faster for lower values of γ. We run our model for three different values of γ. In order to let the market clear, at any different value of γ is associated a different ss value for the *NE*s technology frontier, e. The table below shows how the lower bound of the potential *NE*s support, e, changes for different values of γ considered in the sensitivity analysis.

Table A.1: Different Tail Indexes Calibration

γ	e^{ss}
6	0.3401
9	0.4713
12	0.5228

Figures A.1 and A.2 show how impulses to a permanent IST shock with respect to different values of γ, the shocks magnitude is the same as before. For what concerns macro variables dynamics, they are slightly affected. Where the sensititvity is crucial is for K-sector specific variables. In particular, we can see how the lower is γ, the less *NE*s enter the market. This is because a more diffuse potential *NE*s distribution implies that there are more potential *NE*s on frequencies lower than $\hat{A}_t^{NE}$ who therefore remain out of the market. A specular reasoning applies to a lower mass of exit among updated incumbents.

Figures A.3 and A.4 show instead impulses to a permanent LAT shock. Also in this case the most affected variables are those specific to the K-sector, even if now gross investment, and thus capital, and Q_t dynamics are more responsive to changes in γ. This is why the lower is γ, the less *NE*s enter and the less *INC*s, who would have otherwise died, remain in the business. On aggregate the K-sector ends up being smaller for lower values of γ, and thus the relative price of investment reacts by increasing more strongly to clear the market. In the end, this calls for a smaller increase in the demand of investment goods.

Finally, Figures A.5 and A.6 show the same sensitivity analysis for the transitory MEI shock. In this case higher values of γ are associated with higher variations in final output. This is may seem counterintuitive as in principle all K-firms are on average less productive, hence are more concentrated towards the fat tail of the distribution. In addition the elasitcity of Q (and thus the one of $\hat{A}^k$) is lower for higher values of γ. However, the gain in the probability mass defining the K-sector dimension is higher when the pareto is more right-skewed and contingently the K-sector thresholds lower. This implies a more responsive production of investment goods even though the increase in Q is more muted. Then, capital accumulation, and thus the increase in the demand of labor, is faster.

In general, however, our results are virtually unaffected with respect to different reasonable parametrizations of the K-industry.

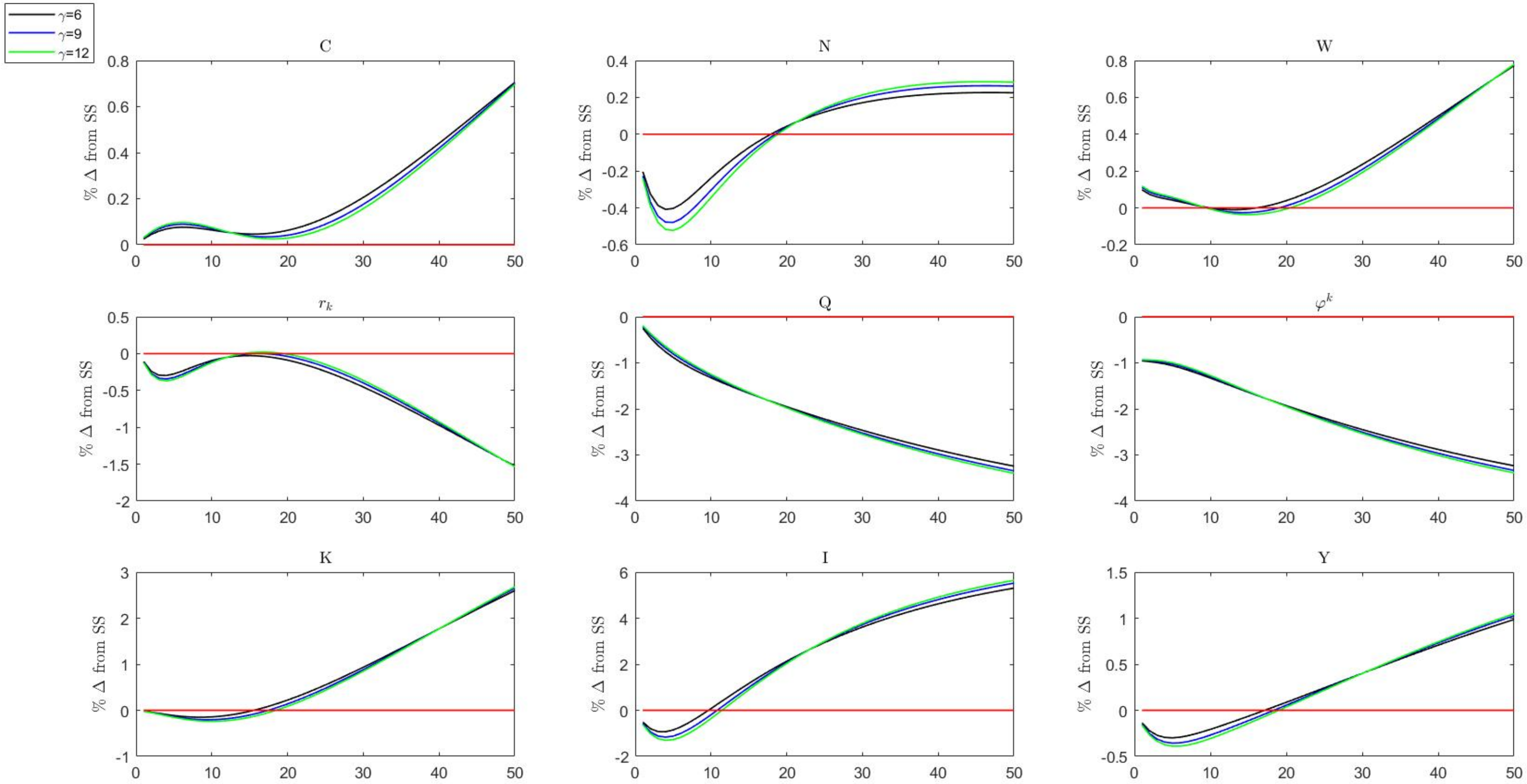

Figure A.1: Impulse response functions to a permanent IST shock for different values of γ.

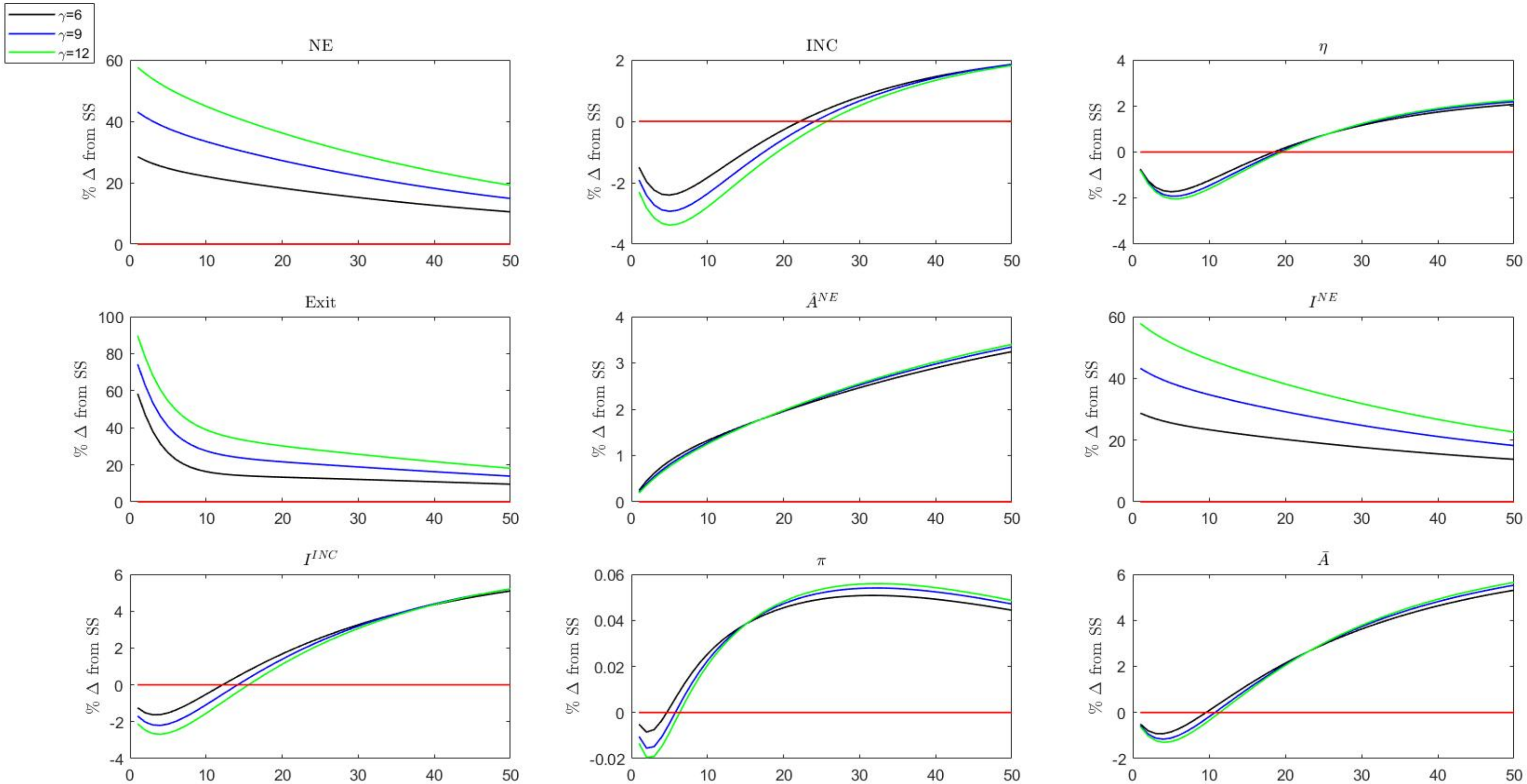

Figure A.2: Impulse response functions to a permanent IST shock for different values of γ.

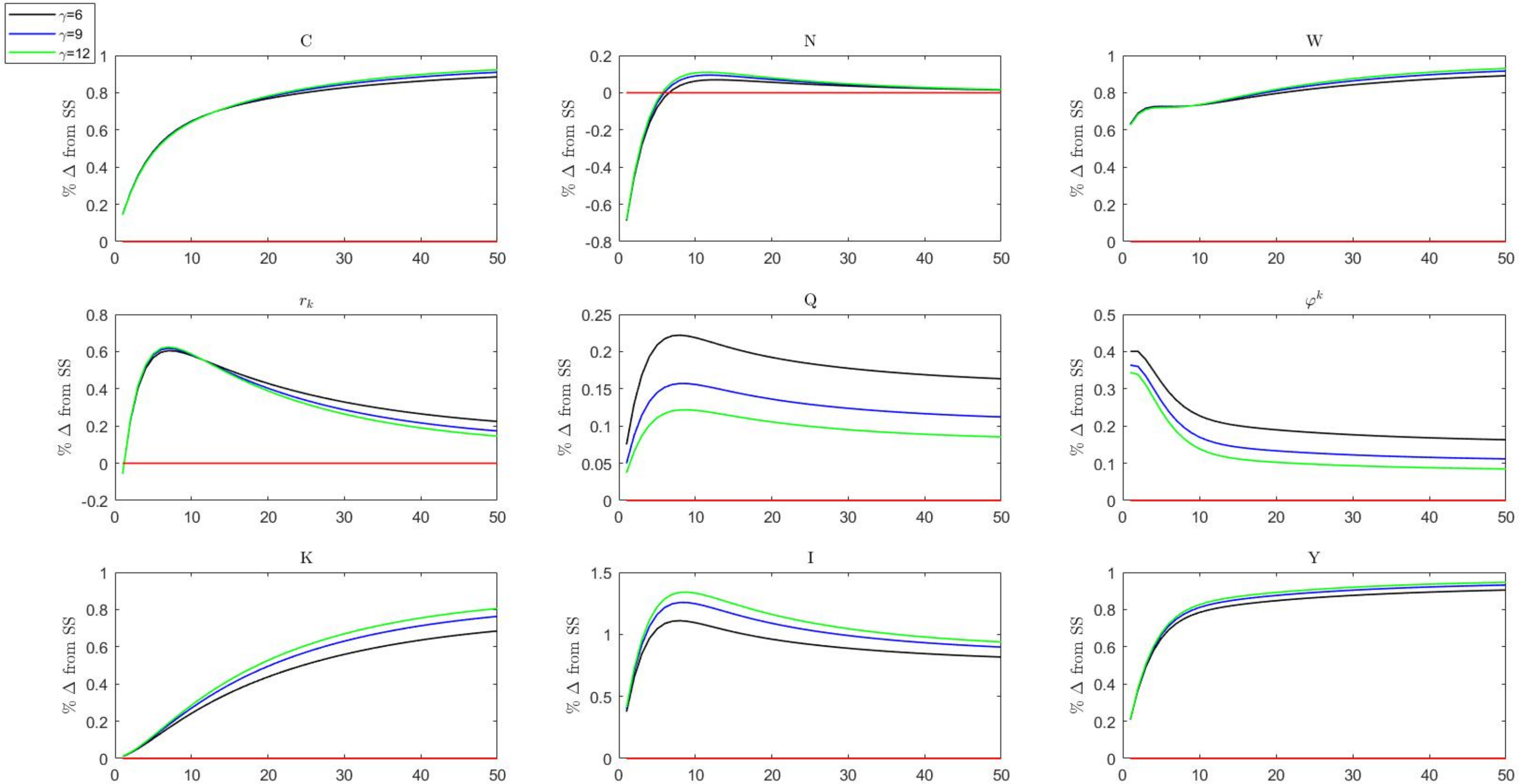

Figure A.3: Impulse response functions to a permanent LAT shock for different values of γ.

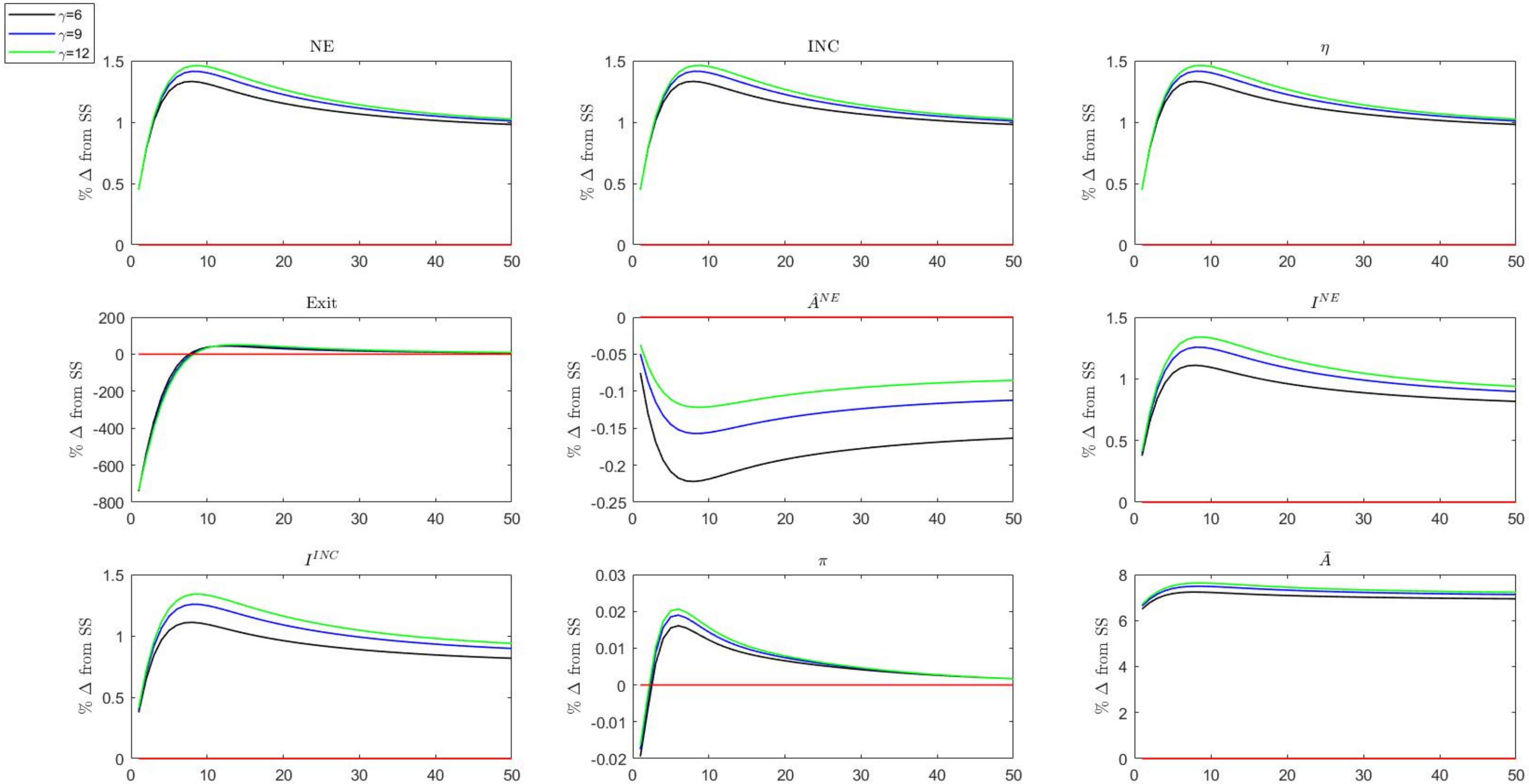

Figure A.4: Impulse response functions to a permanent LAT shock for different values of γ.

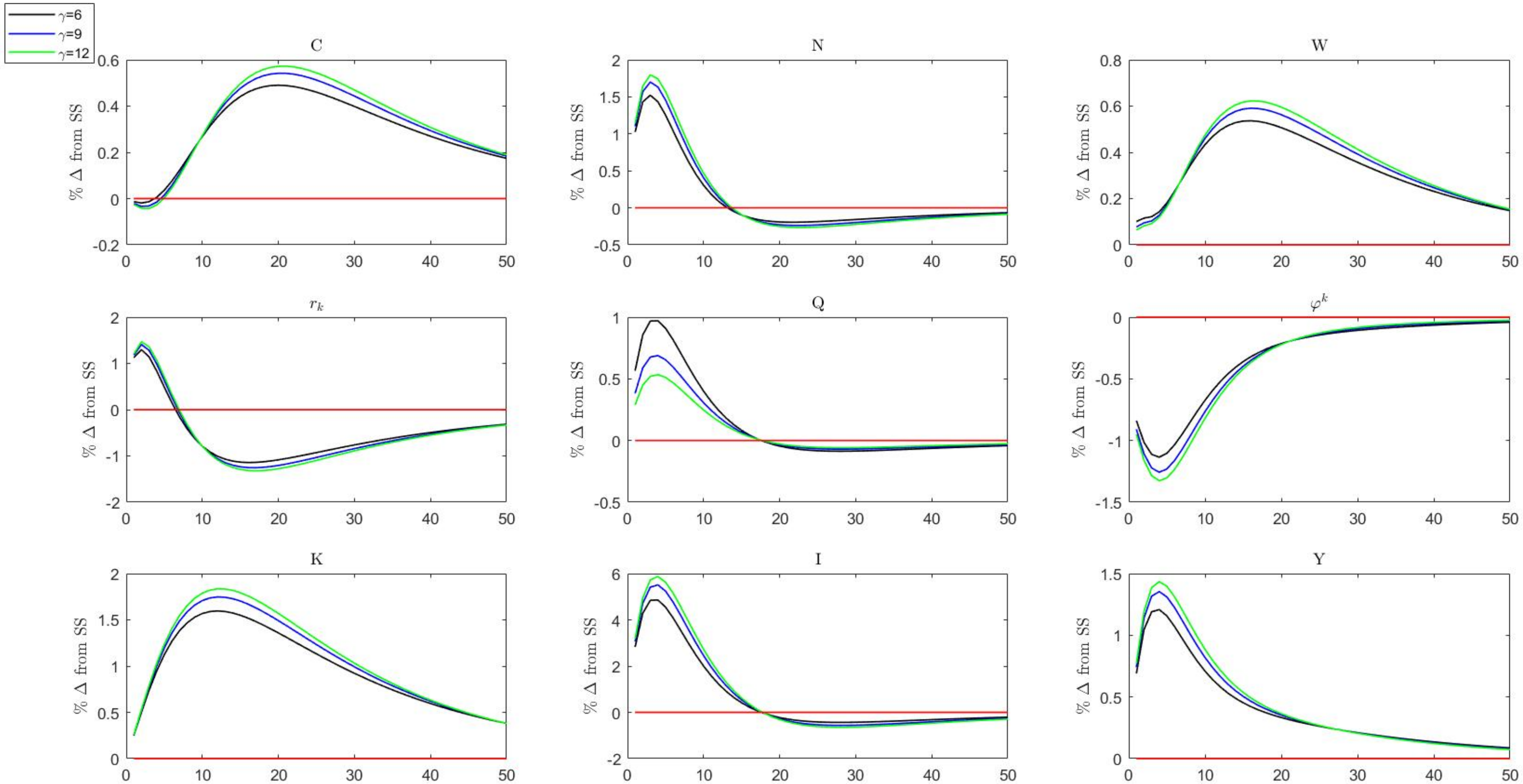

Figure A.5: Impulse response functions to a transitory MEI shock for different values of γ.

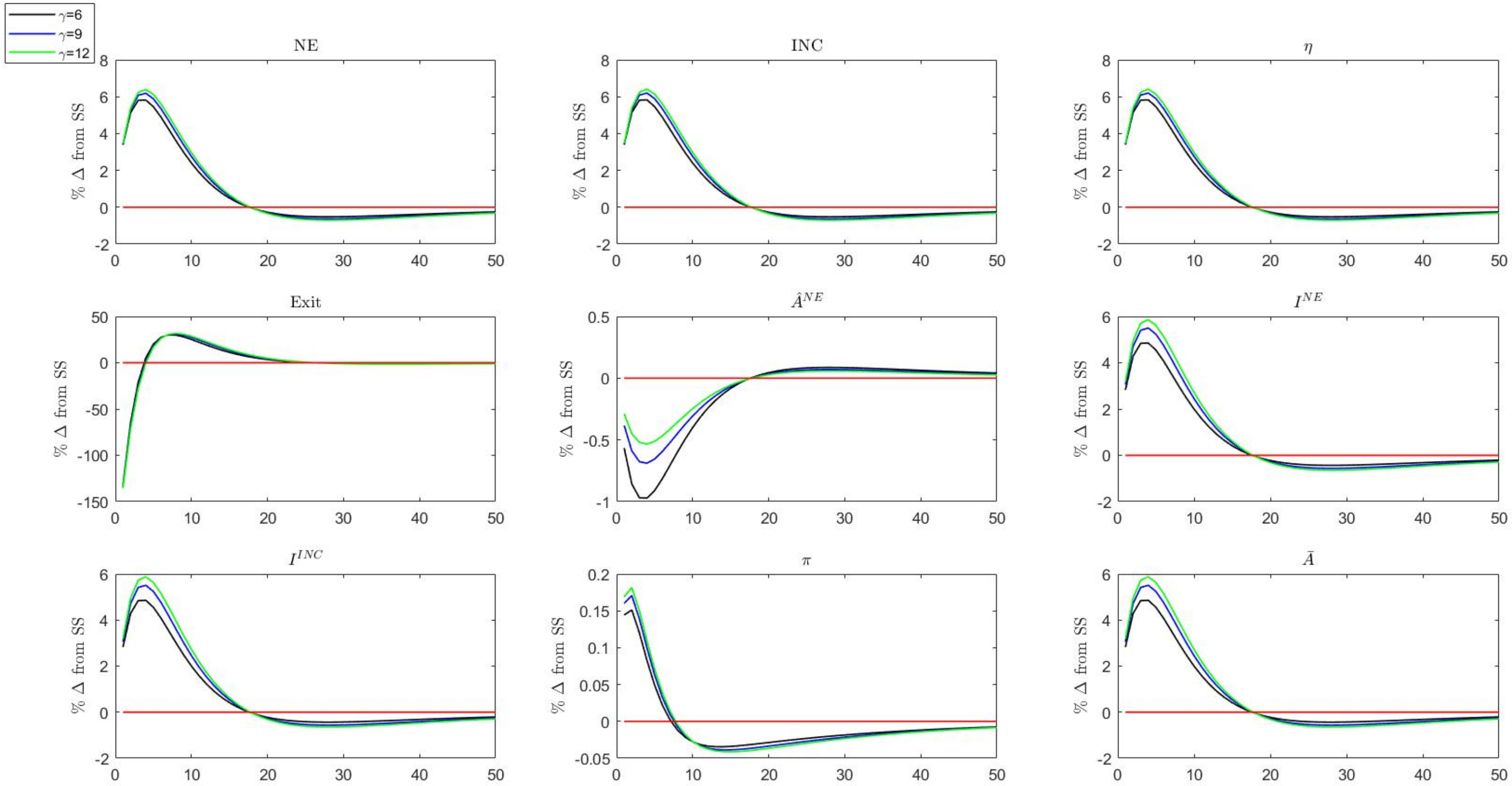

Figure A.6: Impulse response functions to a transitory MEI shock for different values of γ.

A.7 List of linearised equations

Here we list all the relevant linearised equations in our model. For sake of simplicity we abstract here from the expectations operator, stochastically detrended loglinearised variables are in small case and marked with a tilde.

A.7.1 Households

$$\widetilde{\lambda}_t^* = \frac{1}{(g_* - \beta a)(g_* - a)}\Big[a g_* (\beta \widetilde{g}_{t+1} - \widetilde{g}_t) - \left(g_*^2 + \beta a^2\right)\widetilde{c}_t + \tag{A.91}$$

$$+ g_* a (\widetilde{c}_{t-1} + \beta \widetilde{c}_{t+1}) \Big] \qquad \text{MUC}$$

$$\widetilde{w}_t = \theta \widetilde{N}_t - \widetilde{\lambda}_t^* \qquad \text{Labor Supply} \tag{A.92}$$

$$\widetilde{\lambda}_t^* = \widetilde{\lambda}_{t+1}^* - \bar{g}_{t+1} +$$

$$+ \frac{\beta}{g_*}\left[\frac{r_k^{ss}}{\widetilde{\varphi}^{k,ss}} \left(\widetilde{r}_{k,t+1}^* - \widetilde{\varphi}_t^{*,k}\right) + (1-\delta)\left(\widetilde{\varphi}_{t+1}^{*,k} - \widetilde{\varphi}_t^{*,k}\right)\right] \qquad \text{Capital Euler} \tag{A.93}$$

$$\widetilde{\lambda}_t^* = \widetilde{\lambda}_{t+1}^* - \widetilde{g}_{t+1} + \widetilde{r}_{n,t} - \widetilde{\pi}_{t+1} \qquad \text{Bond Euler} \tag{A.94}$$

$$\widetilde{q}_t = \widetilde{\varphi}_t^{*,k} + \widetilde{\mu}_t^i +$$

$$+ \gamma_I g_*^2 \left[-\frac{(1+\beta g_*)}{g_*}\widetilde{i}_t + \widetilde{i}_{t-1} + \beta\left(\widetilde{i}_{t+1} + \bar{g}_{t+1}\right) - \bar{g}_t \right] \qquad \text{Investment rule} \tag{A.95}$$

A.7.2 Intermediate Producers

$$\widetilde{k}_t = \frac{(1-\delta)}{g_*}\left(\widetilde{k}_{t-1} - \bar{g}_t\right) + \frac{I^{ss}}{K^{ss}}(\widetilde{i}_t + \widetilde{\mu}_t^i) \qquad \text{Law of motion of capital} \tag{A.96}$$

$$\widetilde{y}_t = \chi \widetilde{N}_t + (1-\chi)\left(\widetilde{k}_{t-1} - \bar{g}_t\right) - \widetilde{\xi}_t \qquad \text{Final Output} \tag{A.97}$$

$$\widetilde{r}_{k,t}^* = \widetilde{mc}_t - \widetilde{\xi}_t + \chi\left(\bar{g}_t + \widetilde{N}_t - \widetilde{k}_{t-1}\right) \qquad \text{Demand of Capital} \tag{A.98}$$

$$\widetilde{w}_t = \widetilde{mc}_t - \widetilde{\xi}_t + (1-\chi)\left(\widetilde{k}_{t-1} - \widetilde{n}_t - \bar{g}_t\right) \qquad \text{Demand of Labor} \tag{A.99}$$

A.7.3 Final Producers

$$\widetilde{d}_t = \widetilde{\pi}^*_t + (1-\lambda_p\beta)\,\widetilde{y}_t + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \widetilde{\pi}^*_{t+1} + (1-\nu)\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{d}_{t+1}\right] \quad \text{First Recursive Inflation Term} \tag{A.100}$$

$$\widetilde{f}_t = (1-\lambda_p\beta)\left(\widetilde{y}_t + \widetilde{mc}_t\right) + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \nu\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{f}_{t+1}\right] \quad \text{Second Recursive Inflation Term} \tag{A.101}$$

$$\widetilde{d}_t = \widetilde{f}_t \quad \text{Inflation dynamics} \tag{A.102}$$

$$\widetilde{\pi}^*_t = \frac{\lambda_p}{1-\lambda_p}\left(\widetilde{\pi}_t - \gamma_p\widetilde{\pi}_{t-1}\right) \quad \text{Evolution of prices} \tag{A.103}$$

A.7.4 Capital Producers

$$\widetilde{\hat{a}}^{NE}_t = -\widetilde{q} \quad \text{NEs cutoff} \tag{A.104}$$

$$\widetilde{\hat{a}}^{INC}_t = -\widetilde{q} \quad \text{INCs cutoff} \tag{A.105}$$

$$\widetilde{\eta}^*_t = (1-H^{ss})\widetilde{ne}_t + H^{ss}\widetilde{inc}_t \quad \text{Mass of active K-producers} \tag{A.106}$$

$$\widetilde{ne}_t = -\gamma\widetilde{a}^{NE}_t \quad \text{NEs mass} \tag{A.107}$$

$$\widetilde{inc}_t = \widetilde{\eta}^*_{t-1} + \chi\gamma(\bar{g}_t - g_{z,t}) - \gamma g_{e,t} + \\ + \gamma\left[\widetilde{\hat{a}}^{NE}_{t-1} - \widetilde{\hat{a}}^{INC}_t + (\chi+\alpha-1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{INCs mass} \tag{A.108}$$

$$\widetilde{exit}_t = \widetilde{\eta}^*_{t-1} + \chi\gamma(\bar{g}_t - g_{z,t}) - \gamma g_{e,t} + \\ - \frac{H^{ss}}{1-H^{ss}}\gamma\left[\widetilde{a}^{NE}_{t-1} - \widetilde{a}^{INC}_t + (\chi+\alpha-1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{Exit Mass} \tag{A.109}$$

$$\widetilde{i}_t^{NE} = \widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \qquad \text{NEs gross investment} \tag{A.110}$$

$$\widetilde{i}_t^{INC} = \widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \qquad \text{INCs gross investment} \tag{A.111}$$

$$\widetilde{i}_t = \frac{I^{NE,ss}}{I^{ss}}\widetilde{i}_t^{NE} + \frac{I^{INC,ss}}{I^{ss}}\widetilde{i}_t^{INC} \qquad \text{K-producers gross investment} \tag{A.112}$$

$$\begin{aligned}\widetilde{s}_t = & \frac{S^{NE,ss}}{S^{ss}}\left(\widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{1}{1-\alpha}\widetilde{q}_t\right) + \\ & + \frac{S^{INC,ss}}{S^{ss}}\left(\widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{1}{1-\alpha}\widetilde{q}_t\right) \qquad \text{K firms total input amount}\end{aligned} \tag{A.113}$$

$$\widetilde{a}_t = \frac{NE^{ss}\hat{A}^{NE,ss}}{\bar{A}^{ss}}\left(\widetilde{ne}_t + \widetilde{a}_t^{NE}\right) + \frac{INC^{ss}\hat{A}^{INC,ss}}{\bar{A}^{ss}}\left(\widetilde{inc}_t + \widetilde{a}_t^{inc}\right) \qquad \text{K-sector productivity} \tag{A.114}$$

A.7.5 Market clearing conditions and policy rules

$$\widetilde{y}_t = \frac{C^{ss}}{Y^{ss}}\widetilde{c}_t + \frac{S^{ss}}{Y^{ss}}\widetilde{s}_t + \frac{NE^{ss}f^{NE}}{Y^{ss}}\widetilde{ne}_t + \frac{INC^{ss}f^{INC}}{Y^{ss}}\widetilde{inc}_t \qquad \text{Market clearing} \tag{A.115}$$

$$\widetilde{r}_{n,t} = \rho_{R_n^{ss}}\widetilde{r}_{n,t-1} + (1-\rho_{R_n^{ss}})\left(k_\pi\widetilde{\pi}_t + k_y\widetilde{mc} + \sigma^r\varepsilon_{rt}\right) \qquad \text{Taylor rule} \tag{A.116}$$

Appendix B

Appendix to Chapter 2

B.1 List of Detrended Equations

Here we list all the relevant equations in our model. The details of stochastic trend identification and remotion are left in section B.2.

The stochastic trend governing aggregate variables as Y_t, S_t, C_t, W_t, NW_t and governing inflation dynamics recursive components is $\Gamma_t = e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}} (z_t^n)^{1-\frac{\gamma(1-\alpha)(1-\chi)}{1+\chi(\gamma-1)}}$; the stochastic trend governing K_t and I_t is instead $\Lambda_t = e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{1-\frac{\gamma(1-\alpha)}{1+\chi(\gamma-1)}}$.

The relative price of investment and the shadow price of capital in consumption units, Q_t and φ_t^k respectively, share the same stochastic trend that is $\frac{\Gamma_t}{\Lambda_t}$.

The stochastic trend governining K-firms mass is $\frac{e_t^{\gamma}(z_t^n)^{\gamma\chi+\gamma\alpha-\gamma}}{\Lambda_t^{\gamma\chi}}$, whilst K-firms fixed costs trend is z_t^n.

Finally, the stochastic trend governing K-firms cutoffs is $\frac{\Lambda_t^{\chi}}{(z_t^n)^{\chi+\alpha-1}}$.

Lower case characters stand for stochastically detrended variables, the only exception concerns λ^*, $\varphi_t^{*,k}$ and η_t^*which are stochastically detrended marginal utility of consumption and capital (in consumption units), and the total probability mass of active K-firms.

B.1.1 Households

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - ac_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - ac_t} \qquad \text{Marginal utility of consumption} \qquad (B.1)$$

$$w_t = \frac{\Phi N_t^{\theta}}{\lambda_t^*} \qquad \text{Supply of Labor} \qquad (B.2)$$

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} R_t^b \right\} \qquad \text{Bond Euler} \qquad (B.3)$$

B.1.2 Financial intermediaries

$$spr_t = \frac{R_{k,t+1}}{R_t^b} \qquad \text{Financial Spread} \quad \text{(B.4)}$$

$$z_{t+1}^{*,bk} = \frac{\left[\left(R_{kt+1} - R_t^b\right)\phi_t^b + R_t^b\right]}{E\left[\widetilde{g}_{t+1}\right]} \qquad \text{Private } NW_j \text{ evolution} \quad \text{(B.5)}$$

$$\varphi_t^{*,k} k_t = \phi_t^b nw_t \qquad \text{Aggregate Banks Balance sheet} \quad \text{(B.6)}$$

$$\phi_t^b = \frac{\eta_t^{nw}}{\lambda_b - \nu_t} \qquad \text{Aggregate Private Leverage} \quad \text{(B.7)}$$

$$\nu_t = E_t\left\{(1-\theta_b)\beta\frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}\lambda_t^*}\left(R_{kt+1} - R_t^b\right)\right\} + \\ + E_t\left\{\theta_b\beta\frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}\lambda_t^*}\frac{\phi_{t+1}^b}{\phi_t^b}z_{t+1}^{*,bk}\nu_{t+1}\right\} \qquad \text{Asset marginal incentive} \quad \text{(B.8)}$$

$$\eta_t^{nw} = E_t\left\{(1-\theta_b)\beta\frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}\lambda_t^*}R_t^b + \theta_b z_{t+1}^{*,bk}\eta_{t+1}^{nw}\right\} \qquad NW \text{ marginal incentive} \quad \text{(B.9)}$$

$$nw_t = \frac{\theta_b\left[\left(R_{k,t} - R_{t-1}^b\right)\phi_{t-1}^b + R_{t-1}^b\right]nw_{t-1}}{\widetilde{g}_t} + \\ + \frac{\omega\varphi_t^{*,k}\zeta_t^k k_{t-1}}{\widetilde{g}_t} \qquad NW \text{ Law of motion} \quad \text{(B.10)}$$

B.1.3 Final Producers

Retailers

$$d_t = \pi_t^* y_t + \beta\lambda_p\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\pi_t^*}{\pi_{t+1}^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu-1}d_{t+1} \qquad \text{First Recursive Inflation Term} \quad \text{(B.11)}$$

$$f_t = mc_t y_t + \beta\lambda_p\frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu}f_{t+1} \qquad \text{Second Recursive Inflation Term} \quad \text{(B.12)}$$

$$d_t = \frac{\nu}{(\nu-1)}f_t \qquad \text{Inflation dynamics} \quad \text{(B.13)}$$

$$1 = (1-\lambda_p)\left(\pi_t^*\right)^{1-\nu} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{1-\nu} \qquad \text{Evolution of prices} \quad \text{(B.14)}$$

$$\xi_t = (1-\lambda_p)\left(\pi_t^*\right)^{-\nu} + \lambda_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{-\nu}\xi_{t-1} \qquad \text{Price Dispersion} \quad \text{(B.15)}$$

Intermediate Producers

$$k_t = (1-\delta)\zeta_t^k \frac{k_{t-1}}{\bar{g}_t} + i_t \qquad \text{Law of motion of capital} \tag{B.16}$$

$$y_t = \frac{\mu_t N_t^{\chi} \left(\frac{\zeta_t^k k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi_t} \qquad \text{Final Output} \tag{B.17}$$

$$R_{k,t} = \frac{\widetilde{g}_t}{\bar{g}_t}\left[\frac{\varphi_t^{*,k}}{\varphi_{t-1}^{*,k}}\zeta_t^k(1-\delta) + \frac{mc_t}{\xi_t \varphi_{t-1}^{*,k}}(1-\chi)\mu_t\left(\zeta_t^k\right)^{1-\chi}\left(\frac{N_t \bar{g}_t}{k_{t-1}}\right)^{\chi}\right] \qquad \text{Demand of Capital} \tag{B.18}$$

$$w_t = \frac{mc_t}{\xi_t}\chi\mu_t\left[\frac{\zeta_t^k k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi} \qquad \text{Demand of Labor} \tag{B.19}$$

B.1.4 K-Firms

$$\begin{aligned} q_t = {}& \varphi_t^{*,k}\left\{1 - \left[S'\left(\frac{i_t \bar{g}_t}{i_{t-1}}\right)\frac{i_t \bar{g}_t}{i_{t-1}} + S\left(\frac{i_t \bar{g}_t}{i_{t-1}}\right)\right]\right\} + \\ & + \beta E_t\left\{\frac{\lambda_{t+1}^*}{\lambda_t^* \bar{g}_{t+1}}\varphi_{t+1}^{*,k} S'\left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t}\right)\left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t}\right)^2\right\} \end{aligned} \qquad \text{Investment rule} \tag{B.20}$$

B.1.5 I-Firms

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^\alpha} \qquad \text{NEs cutoff2} \tag{B.21}$$

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_t \alpha^\alpha} \qquad \text{INCs cutoff} \tag{B.22}$$

$$\eta_t^* = ne_t + inc_t \qquad \text{Mass of active I-producers} \tag{B.23}$$

$$ne_t = \left(\frac{\hat{a}_t^{NE}}{e^{ss}}\right)^{-\gamma} \qquad \text{NEs mass} \tag{B.24}$$

$$inc_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t}\right)^\chi \frac{1}{g_*^{1-\alpha}}\right]^\gamma \qquad \text{INCs mass} \tag{B.25}$$

$$exit_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left\{1 - \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t}\right)^\chi \frac{1}{g_*^{1-\alpha}}\right]^\gamma\right\} \qquad \text{Exit Mass} \tag{B.26}$$

$$i_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (q_t \alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{NEs gross investment} \tag{B.27}$$

$$i_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} (q_t \alpha)^{\frac{\alpha}{1-\alpha}} \qquad \text{INCs gross investment} \tag{B.28}$$

$$i_t = i_t^{NE} + i_t^{INC} \qquad \text{I-producers gross investment} \tag{B.29}$$

$$s_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} + $$
$$+ inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\alpha q_t \hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} \qquad \text{I firms total input amount} \tag{B.30}$$

$$\bar{a}_t = \frac{\gamma}{\gamma-1} ne_t \hat{a}_t^{NE} + \frac{\gamma}{\gamma-1} inc_t \hat{a}_t^{INC} \qquad \text{I-sector productivity} \tag{B.31}$$

B.1.6 Market clearing conditions and policy rules

$$y_t = c_t + s_t + ne_t f^{NE} + inc_t f^{INC} + \\ + \varphi_t^{*,k} \frac{\gamma_I}{2} \left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g} \right)^2 i_t \qquad \text{Market clearing} \qquad (B.32)$$

$$\left(\frac{R_{n,t}}{R_n^{ss}} \right) = \left(\frac{R_{n,t-1}}{R_n^{ss}} \right)^{\rho_{R_n^{ss}}} \times \\ \times \left[\left(\frac{\pi_t}{\pi^{ss}} \right)^{\kappa_\pi} \left(\frac{mc_t}{(\nu - 1)/\nu} \right)^{\kappa_y} \exp \{ \sigma^r \varepsilon_t^r \} \right]^{1-\rho_{R_n^{ss}}} \qquad \text{Taylor rule} \qquad (B.33)$$

$$R_t^b = E_t \left\{ \frac{R_{n,t}}{\pi_{t+1}} \right\} \qquad \text{Fisher equation} \qquad (B.34)$$

B.1.7 Autoregressive processes

$$\ln (g_{e,t}) = (1 - \rho_e) \ln(g_e) + \rho_e \ln (g_{e,t-1}) + \sigma^e \varepsilon_t^e \qquad \text{K-Tech growth rate} \qquad (B.35)$$

$$\ln (g_{z,t}) = (1 - \rho_z) \ln(g_*) + \rho_z \ln (g_{z,t-1}) + \sigma^z \varepsilon_t^z \qquad \text{L-Tech growth rate} \qquad (B.36)$$

$$\ln(\bar{g}_t) = \frac{\gamma}{1 + \chi(\gamma - 1)} \ln (g_{e,t}) + \frac{\chi(\gamma - 1)}{1 + \chi(\gamma - 1)} \ln(g_{z,t}) + \\ - \frac{\gamma(1 - \alpha) - 1}{1 + \chi(\gamma - 1)} \ln(g_*) \qquad \text{K-Production stochastic growth} \qquad (B.37)$$

$$\ln(\widetilde{g}_t) = \frac{\gamma(1 - \chi)}{1 + \chi(\gamma - 1)} \ln (g_{e,t}) + \frac{\chi\gamma}{1 + \chi(\gamma - 1)} \ln(g_{z,t}) + \\ - \frac{(1 - \chi) [\gamma(1 - \alpha) - 1]}{1 + \chi(\gamma - 1)} \ln(g_*) \qquad \text{F-Production stochastic growth} \qquad (B.38)$$

$$\ln \left(\mu_t^i \right) = \left(1 - \rho_{\mu^i} \right) \ln \left(\mu^i \right) + \rho_{\mu^i} \ln \left(\mu_{t-1}^i \right) + \sigma^i \varepsilon_t^i \qquad \text{MEI shock} \qquad (B.39)$$

$$\ln \left(\zeta_t^k \right) = \rho_k \ln \left(\zeta_{t-1}^k \right) + \sigma^k \varepsilon_t^k \qquad \text{Quality of Capital shock} \qquad (B.40)$$

B.2 Removing the Stochastic Trend Governing the Economy

Let us assume Γ_t is the stochastic trend governing Y_t, C_t, W_t,S_t and NW_t from which follows that, for instance, $Y_t = \Gamma_t y_t$, where smaller case characters are meant to be detrended variables if not differently specificated. Moreover, assume that Λ_t is the stochastic trend governing K_t and I_t, thus $K_t = \Lambda_t k_t$. We claim that both Γ_t and Λ_t are convolutions of the labor augmenting and the NEs permanent technology shifters, z_t^n and e_t.

B.2.1 Final production, K-firms and the Banking sector

Without any loss of generality we can rewrite final production as

$$y_t \Gamma_t = \frac{\mu_t \left(z_t^n N_t\right)^{\chi} \left(\frac{\Lambda_t \zeta_t^k k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{B.41}$$

where $\bar{g}_t = \frac{\Lambda_t}{\Lambda_{t-1}}$. Then we define

$$\Gamma_t = \left(z_t^n\right)^{\chi} \Lambda_t^{1-\chi} \tag{B.42}$$

Which can be interpreted as the non stationary stochastic evolution of TFP in our model. Then dividing (B.41) by (B.42) we obtain

$$y_t = \frac{\mu_t \left(N_t\right)^{\chi} \left(\frac{\zeta_t^k k_{t-1}}{\bar{g}_t}\right)^{1-\chi}}{\xi} \tag{B.43}$$

In a similar fashion we can work out the detrended law of motion of capital by dividing both sides by Λ_t that is

$$k_t = (1-\delta)\frac{\zeta_t^k k_{t-1}}{\bar{g}_t} + i_t \tag{B.44}$$

For the moment is useful moving to the K-sector. From (2.29) we know that φ_t^k is the shadow price of capital in consumption units (i.e. tha marginal Tobin's Q) and therefore $\varphi_t^k = \frac{\phi_t^k}{\lambda_t}$, where we know that, since marginal utility of Consumption is governed by the same stochastic trend as consumption, $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$, and must also hold true that $\phi_t^k = \frac{\phi_t^{*,k}}{\Lambda_t}$ since Λ_t is the stochastic trend governing capital. This implies that $\varphi_t^k = \frac{\phi_t^{*,k}/\Lambda_t}{\lambda_t^*/\Gamma_t}$ so that we can define $\varphi_t^k = \varphi_t^{*,k}\frac{\Gamma_t}{\Lambda_t}$ which is governed by the same stochastic trend as Q_t. Thus, dividing on both sides by $\frac{\Gamma_t}{\Lambda_t}$ and rearranging, we can rewrite (2.29) as

$$q_t = \varphi_t^{*,k}\left\{1-\left[\gamma_I\left(\frac{i_t\bar{g}_t}{i_{t-1}}-\bar{g}\right)\frac{i_t\bar{g}_t}{i_{t-1}}+\frac{\gamma_I}{2}\left(\frac{i_t\bar{g}_t}{i_{t-1}}-\bar{g}\right)^2\right]\right\}+$$
$$+\beta E_t\left\{\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\varphi_{t+1}^{*,k}}{\bar{g}_{t+1}}\gamma_I\left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t}-\bar{g}\right)\left(\frac{i_{t+1}\bar{g}_{t+1}}{i_t}\right)^2\right\} \tag{B.45}$$

At this point let us look for a while to the banking sector and notice that, without any loss of generality, we can rewrite (2.15) as

$$\varphi_t^k K_{jt} = NW_{jt} + B_{jt+1} \tag{B.46}$$

Where the LHS can be rewritten in terms of stochastically stationary variables as $\varphi_t^{*,k}\frac{\Gamma_t}{\Lambda_t}k_{j,t}\Lambda_t \equiv \varphi_t^{*,k}k_{j,t}\Gamma_t$ implying that both NW_{jt} and B_{jt+1} must be governed by Γ. Thus, we can rewrite the above equation dividing on both sides by Λ_t as

$$\varphi_t^{*,k}k_{j,t} = nw_{j,t} + \frac{b_{jt+1}}{\widetilde{g}_{t+1}} \tag{B.47}$$

from which we can easily work out

$$nw_{jt+1} = E_t\left\{\frac{\left(R_{kt+1}-R_t^b\right)\varphi_t^{*,k}k_{jt}+R_t^b nw_{jt}}{\widetilde{g}_{t+1}}\right\} \tag{B.48}$$

Then, without any loss of generality, we can rewrite (2.15) as

$$\varphi_t^k K_t = \phi_t^b NW_t$$

which in the stochastically detrended version reads

$$\varphi_t^{*,k}k_t = \phi_t^b nw_t \tag{B.49}$$

Implying that private leverage, ϕ_t^b, has no stochastic trend. This allows us rewrite (B.48) as

$$nw_{jt+1} = E_t\left\{\frac{\left[\left(R_{kt+1}-R_t^b\right)\phi_t^b+R_t^b\right]nw_{jt}}{\widetilde{g}_{t+1}}\right\} \tag{B.50}$$

Then dividing by nw_{jt} we have

$$z^{*,bk}_{t+1} = E_t \left\{ \frac{\left[\left(R_{kt+1} - R^b_t \right) \phi^b_t + R^b_t \right]}{\widetilde{g}_{t+1}} \right\} \tag{B.51}$$

where we exploited (B.49) where $nw_t = \frac{NW_t}{\Gamma_t}$.

At this point we can claim that no component of the private leverage ratio is governed by any stochastic trend as well, and thus must be just rearranged in terms of stochastically stationary variables, that is

$$\nu_t = E_t \left\{ (1-\theta_b)\beta \frac{\lambda^*_{t+1}}{\widetilde{g}_{t+1}\lambda^*_t} \left(R_{kt+1} - R^b_t \right) + \theta_b \beta \frac{\lambda^*_{t+1}}{\widetilde{g}_{t+1}\lambda^*_t} \frac{\phi^b_{t+1}}{\phi^b_t} z^{*,bk}_{t+1} \nu_{t+1} \right\} \tag{B.52}$$

$$\eta^{nw}_t = E_t \left\{ (1-\theta_b)\beta \frac{\lambda^*_{t+1}}{\widetilde{g}_{t+1}\lambda^*_t} R^b_t + \theta_b \frac{\lambda^*_{t+1}}{\widetilde{g}_{t+1}\lambda^*_t} z^{*,bk}_{t+1} \eta^{nw}_{t+1} \right\} \tag{B.53}$$

Thus, the basic relationship $\phi^b_t = \frac{\eta^{nw}_t}{\lambda_b - \nu_t}$ is unchanged.

Then, it is easy to show that the law of motion for aggregate net worth reads

$$nw_t = \frac{\theta_b \left[\left(R_{k,t} - R^b_{t-1} \right) \phi^b_{t-1} + R^b_{t-1} \right] nw_{t-1} + \omega \varphi^{*,k}_t \zeta^k_t k_{t-1}}{\widetilde{g}_t} \tag{B.54}$$

We can now come back to intermediate producers optimality conditions. We argue that the marginal product of capital is led by the same stochastic trend as φ^k_t, thus we rewrite (2.28) as

$$R_{k,t}\varphi^{*,k}_{t-1}\frac{\Gamma_{t-1}}{\Lambda_{t-1}} = \varphi^{*,k}_t \frac{\Gamma_t}{\Lambda_t}(1-\delta) + mc_t(1-\chi)\mu_t \left(\zeta^k_t\right)^{1-\chi} \left(\frac{N_t \bar{g}_t}{k_{t-1}} \frac{z^n}{\Lambda_t} \right)^{\chi}$$

where exploiting the fact that $\left(\frac{z^n}{\Lambda_t}\right)^{\chi} = \frac{\Gamma_t}{\Lambda_t}$, and dividing on both sides by such a ratio and rearranging we have

$$R_{k,t} = \frac{\widetilde{g}_t}{\bar{g}_t} \left[\frac{\varphi^{*,k}_t}{\varphi^{*,k}_{t-1}}(1-\delta) + \frac{mc_t}{\xi_t \varphi^{*,k}_{t-1}}(1-\chi)\mu_t \left(\zeta^k_t\right)^{1-\chi} \left(\frac{N_t \bar{g}_t}{k_{t-1}} \right)^{\chi} \right] \tag{B.55}$$

Where it turns out that $R_{k,t}$ has no stochastic trend as well as R^b_t.

Finally, we claimed that W_t shares the same stochastic trend as Y_t, therefore

$$w_t \Gamma_t = \frac{mc_t}{\xi_t} \chi \mu_t \left(z^n_t\right)^{\chi} \Lambda^{1-\chi}_t \left[\frac{\zeta^k_t k_{t-1}}{\bar{g}_t N_t} \right]^{1-\chi}$$

implying that

$$w_t = \frac{mc_t}{\xi_t}\chi\mu_t\left[\frac{\zeta_t^k k_{t-1}}{\bar{g}_t N_t}\right]^{1-\chi} \tag{B.56}$$

Where mc has no stochastic trend by construction.

Retailers

For what concerns recursive inflation trend, they do have, by costruction, the same stochastic trend as Y. Therefore their detrended version is

$$d_t = \pi_t^* y_t + \beta E_t\left\{\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\pi_t^*}{\pi_{t+1}^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu-1} d_{t+1}\right\} \tag{B.57}$$

In a similar fashion

$$f_t = mc_t y_t + \beta E_t\left\{\lambda_p \frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^{\nu} f_{t+1}\right\} \tag{B.58}$$

and thus

$$d_t = \frac{\nu}{\nu-1} f_t \tag{B.59}$$

Finally, price dispersion and price evolution are unchanged.

B.2.2 Households

From before we implicitly assumed $C_t = c_t\Gamma_t$, where we also define $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$. At this point we also have that $\lambda_t = \frac{\lambda_t^*}{\Gamma_t}$ where λ_t^* is the detrended MUC.

MUC can be rewritten as

$$\frac{\lambda_t^*}{\Gamma_t} = \frac{1}{\Gamma_t c_t - a\Gamma_{t-1}c_{t-1}} - \beta a \frac{1}{\Gamma_{t+1}c_{t+1} - a\Gamma_t c_t}$$

Multiplying on both sides by Γ_t and rearranging we have

$$\lambda_t^* = \frac{\widetilde{g}_t}{\widetilde{g}_t c_t - a c_{t-1}} - \beta a \frac{1}{\widetilde{g}_{t+1} c_{t+1} - a c_t} \tag{B.60}$$

Then the leisure-consumption relationship reads

$$w_t = \Phi\frac{N_t^\theta}{\lambda_t^*} \tag{B.61}$$

And from the Bond-Euler

$$\lambda_t^* = \beta E_t \left\{ \frac{\lambda_{t+1}^*}{\widetilde{g}_{t+1}} R_t^b \right\} \tag{B.62}$$

B.2.3 I-firms

We remark that by assumption $f_t^k = g_*^t f^k$ for $k = [NE, INC]$, i.e. the trend leading fixed costs is purely deterministic. This allows us rewriting the NEs cutoff as

$$\hat{A}_t^{NE} = \left(\frac{g_*^t f^{NE}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t \frac{\Gamma_t}{\Lambda_t}}$$

Then exploiting the fact that $\frac{\Lambda_t}{\Gamma_t} = \left(\frac{\Lambda_t}{z_t^n} \right)^\chi$ we have that

$$\hat{A}_t^{NE} = \hat{a}_t^{NE} \left(\frac{\Lambda_t}{z_t^n} \right)^\chi g_*^{t(1-\alpha)} \tag{B.63}$$

And thus

$$\hat{a}_t^{NE} = \left(\frac{f^{NE}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t} \tag{B.64}$$

From which follows

$$\hat{a}_t^{INC} = \left(\frac{f^{INC}}{1-\alpha} \right)^{1-\alpha} \frac{1}{\alpha^\alpha q_t} \tag{B.65}$$

At this point we can easily rewrite the mass of active NEs as

$$NE_t = \left(\frac{e^{ss} e_t \left(z_t^n \right)^\chi}{\hat{a}_t^{NE} \Lambda_t^\chi g_*^{t(1-\alpha)}} \right)^\gamma = \left(\frac{e^{ss}}{\hat{a}_t^{NE}} \right)^\gamma \frac{\left(z_t^n \right)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$$

implying that $NE_t = ne_t \frac{(z_t^n)^{\chi\gamma} e_t^\gamma}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and so

$$ne_t = \left(\frac{e^{ss}}{\hat{a}_t^{NE}} \right)^\gamma \tag{B.66}$$

By BGP conditions we know that also η_t and INC_t share the same stochastic trend as NE_t, this implies

$$\eta_t^* = ne_t + inc_t \tag{B.67}$$

and

$$inc_t = \eta_{t-1}^* \frac{\bar{g}_t^{\chi\gamma} g_*^{\gamma(1-\alpha)}}{g_{z,t}^{\chi\gamma} g_{e,t}^{\gamma}} \left[\frac{\hat{a}_{t-1}^{NE}}{\hat{a}_t^{INC}} \left(\frac{g_{z,t}}{\bar{g}_t} \right)^{\chi} \frac{1}{g_*^{1-\alpha}} \right]^{\gamma} \tag{B.68}$$

where of course $\eta_t = \eta_t^* \frac{(z_t^n)^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}}$ and η_{t-1} in (B.68) must be expressed accordingly.

B.2.4 Stochastic Trends Identification

Notice that from the aggregate resource constraint in (1.34) it turns out that the stochastic trend leading $NE_t f_t^{NE}$ and $INC_t f_t^{INC}$ must be, by construction, the same leading Y_t, C_t and S_t, i.e. Γ_t. Thus, given that $NE_t g_*^t f^{NE} \equiv ne_t f^{NE} \frac{(z_t^n)^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} g_*^t$, we can easily work out

$$\Gamma_t = \frac{(z_t^n)^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}} \tag{B.69}$$

Then, plugging the relationship $\Gamma_t = (z_t^n)^{\chi} \Lambda_t^{1-\chi}$ into (B.69) allows identifying the stochastic trend leading both K_t and I_t, that is

$$\Lambda_t = \frac{e_t^{\frac{\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{t[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{B.70}$$

Then, plugging (B.70) into (B.69) we have

$$\Gamma_t = \frac{e_t^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}} (z_t^n)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{t(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{B.71}$$

Which is the stochastic trend leading aggregate variables but K_t and I_t. Thus the stochastic trend governing aggregate variables is a Cobb-Douglas of the the permanent shifters governing the NEs technology frontier and final goods production, respectively [1].

At this point we can also identify the stochastic trend leading K-firms cutoff. For instance, substituting for (B.70) into (B.63) we obtain that the corresponding stochastic trend is $\frac{e_t^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{(z_t^n)^{\frac{\chi}{1+\chi(\gamma-1)}} g_*^{t\left\{\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}-(1-\alpha)\right\}}}$.

[1] According to our parametrization $\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)} < 1$.

Finally, plugging (B.70) into (B.66), (B.67) and (B.68) it turns out that the stochastic trend leading the K-firms industry composition is $\frac{e_t^{\frac{\gamma(1-\chi)}{1+\chi(\gamma-1)}}(z_t^n)^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{t\gamma\left\{(1-\alpha)-\chi\frac{[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}\right\}}}$.

B.2.5 I-firms production

At this point, since we claimed that K_t and I_t are governed by the same stochastic trend, i.e. Λ_t, this implies that $I_t^{NE} = i_t^{NE}\Lambda_t$. Then

$$i_t^{NE}\Lambda_t = ne_t \frac{(z_t^n)^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\frac{\hat{a}_t^{NE}\Lambda_t^{\chi} g_*^{t(1-\alpha)}}{(z_t^n)^{\chi}}\right)^{\frac{1}{1-\alpha}} \left[\alpha q_t \left(\frac{z_t^n}{\Lambda_t}\right)^{\chi}\right]^{\frac{\alpha}{1-\alpha}}$$

From which rearranging

$$i_t^{NE}\Lambda_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{\Lambda_t^{\chi(\gamma-1)} g_*^{t[\gamma(1-\alpha)-1]}}$$

but from (B.70) we know that $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$ which therefore implies

$$i_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{NE}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{B.72}$$

and thus it must also be that

$$i_t^{INC} = inc_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{a}_t^{INC}\right)^{\frac{1}{1-\alpha}} (\alpha q_t)^{\frac{\alpha}{1-\alpha}} \tag{B.73}$$

and

$$i_t = i_t^{NE} + i_t^{INC} \tag{B.74}$$

B.2.6 Aggregate Resources Constraint

There is only one variable to be detrended yet. By construction it must be $S_t = s_t\Gamma_t$ which also implies $s_t\Gamma_t = s_t^{NE}\Gamma_t + s_t^{INC}\Gamma_t$.

Then it is sufficient to show that

$$s_t^{NE}\Gamma_t = ne_t \frac{(z_t^n)^{\chi\gamma} e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t\gamma(1-\alpha)}} \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left[\alpha q_t \left(\frac{z_t^n}{\Lambda_t}\right)^{\chi} \hat{a}_t^{NE} \left(\frac{\Lambda_t}{z_t^n}\right)^{\chi} g_*^{t(1-\alpha)}\right]^{\frac{1}{1-\alpha}}$$

Can be rewritten as

$$s_t^{NE}\Gamma_t = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}} \frac{(z_t^n)^{\chi\gamma}\, e_t^{\gamma}}{\Lambda_t^{\chi\gamma} g_*^{t[\gamma(1-\alpha)-1]}}$$

And again, since $\Gamma_t = (z_t^n)^{\chi}\Lambda^{(1-\chi)}$ and $\Lambda_t^{1+\chi(\gamma-1)} = \frac{e_t^{\gamma}(z_t^n)^{\chi(\gamma-1)}}{g_*^{t[\gamma(1-\alpha)-1]}}$, it must be that

$$s_t^{NE} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left[\alpha q_t \hat{a}_t^{NE}\right]^{\frac{1}{1-\alpha}} \tag{B.75}$$

And

$$s_t^{INC} = ne_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left[\alpha q_t \hat{a}_t^{INC}\right]^{\frac{1}{1-\alpha}} \tag{B.76}$$

And

$$s_t = s_t^{NE} + s_t^{INC} \tag{B.77}$$

Thus, since we know that $\varphi_t^k I_t \equiv \varphi_t^{*,k} i_t \Gamma_t$, we have proven that

$$y_t - ne_t f^{NE} - inc_t f_t^{INC} = c_t + s_t + \varphi_t^{*,k}\frac{\gamma_I}{2}\left(\frac{i_t \bar{g}_t}{i_{t-1}} - \bar{g}\right)^2 i_t \tag{B.78}$$

Holds true.

B.2.7 Stochastic Growth Rates Identification

We claimed that $\frac{\Gamma_t}{\Gamma_{t-1}} = \widetilde{g}_t$, then exploiting (B.71) it turns out that

$$\widetilde{g}_t = \frac{g_{e,t}^{\frac{(1-\chi)\gamma}{1+\chi(\gamma-1)}}\, g_{z,t}^{\frac{\chi\gamma}{1+\chi(\gamma-1)}}}{g_*^{\frac{(1-\chi)[\gamma(1-\alpha)-1]}{1+\chi(\gamma-1)}}} \tag{B.79}$$

meaning that the stochastic BGP growth rate is a convolution of the stochastic growth rate of e_t and z_t^n. Similarly for $\frac{\Lambda_t}{\Lambda_{t-1}} = \bar{g}_t$ it follows that

$$\bar{g}_t = \frac{g_{e,t}^{\frac{\gamma}{1+\chi(\gamma-1)}}\, g_{z,t}^{\frac{\chi(\gamma-1)}{1+\chi(\gamma-1)}}}{g_*^{\frac{\gamma(1-\alpha)-1}{1+\chi(\gamma-1)}}} \tag{B.80}$$

Finally, in the deterministic steady state we have that $g_e = g_*^{1-\alpha}$. Moreover, also $\bar{g} = \widetilde{g} = g_*$ must hold true, i.e. the deterministic BGP is the same for all aggregated variables, which is verified plugging $g_e = g_*^{1-\alpha}$ into the deterministic formulation of (B.79) and (B.80).

B.3 Deterministic Steady State and the Existence of a Balanced Growth Path

In the deterministic steady state

$$\frac{z_t^n}{z_{t-1}^n} = g_* \tag{B.81}$$

Where z^n defines the ss value of the technology shifter and g_* is the BGP growth rate of the economy. Output, capital, investment, consumption and the real wage all grow at the BGP rate while the relative price of investment and the labor supply are constant. The latter is pinned down by the preference parameter Φ in (2.4). Variables without time index are detrended or, in a deterministic environment, implicitly stationary and investment adjustment costs are nil. Further, from (2.31) it is straightforward to show that the fixed costs $f_t^{X,ss}$ also grows at the BGP rate g^*. We assume that the monetary policy rule achieves $\pi = 1$. As a result, from (B.62), the real interest rate on the riskless bond is

$$\frac{g_*}{\beta} = R_n^{ss} \equiv R_b^{ss} \tag{B.82}$$

From which we define $R_k^{ss} = R_b^{ss} + 0.01/4$ where we impose the financial spread to be 25 basis point in steady state.

From condition (B.45) the shadow price of capital is equal to the price of investment goods

$$\varphi^{k,ss} = Q^{ss} \tag{B.83}$$

Where Q^{ss} is obtained when the investment goods market clears.

Then, for what concerns the final producer's capital FOC we have that in steady state

$$\frac{mc^{ss}}{Q^{ss}}(1-\chi)\left[\frac{g_* N^{ss}}{K^{ss}}\right]^{\chi} + 1 - \delta = R_k^{ss} \tag{B.84}$$

Demand of capital for production is

$$K^{ss} = g^* N^{ss} \left[\frac{mc^{ss}(1-\chi)}{\left(\frac{g_*}{\beta} + 0.01/4 - 1 + \delta\right) Q^{ss}}\right]^{\frac{1}{\chi}} \tag{B.85}$$

From the capital accumulation condition

$$I_t^{ss} = \left(1 - \frac{1-\delta}{g_*}\right) K^{ss} \tag{B.86}$$

Given the monopolistic nature of the final goods market, in the zero net inflation steady state the marginal cost is

$$mc^{ss} = \frac{\nu - 1}{\nu}$$

and the real wage is obtained solving

$$mc^{ss} = \left(\frac{Q^{ss} R_k^{ss} - 1 + \delta}{1 - \chi} \right)^{1-\chi} \left(\frac{W^{ss}}{g_* \chi} \right)^{\chi} \tag{B.87}$$

To obtain closed form solutions for the above conditions (B.83) - (B.87) we need to solve for the K-sector market clearing condition.

B.3.1 Financial Intermediaries

At first, following GK, we calibrate the steady state private leverage as $\phi^{b,ss} = 4$, then exploiting the fact that $R_k^{ss} - R^{b,ss} = 0.01/4$ allows us rewriting (B.51) as

$$z^{bk,ss} = \frac{1}{g_*} + \beta \tag{B.88}$$

Then, since in the detrministic steady state the marginal incentive to accumulate private assets is stationary by itself plugging (B.88) into (B.52), and still exploiting $R_k^{ss} - R^{b,ss} = 0.01/4$, after some manipulations yields

$$\nu^{ss} = \frac{(1 - \theta_b)\beta}{(g_* - \theta_b \beta)} \frac{0.01/4}{(1/g_* + \beta)} \tag{B.89}$$

similarly from (96) we obtain

$$\eta^{nw,ss} = \frac{(1 - \theta_b) g_*}{g_* - \theta_b (1/g_* + \beta)} \tag{B.90}$$

Then, from the definition of private leverage given in (2.12), and given the fact that we already imposed $\phi^{b,ss} = 4$, we can pin down the share of divertable funds by the bankers as

$$\lambda_b = \frac{\frac{(1-\theta_b) g_*}{g_* - \theta_b (1/g_* + \beta)}}{4} + \frac{(1 - \theta_b)\beta}{(g_* - \theta_b \beta)} \frac{0.01/4}{(1/g_* + \beta)} \tag{B.91}$$

Then, since $\varphi^{k,ss} = Q^{ss} = 1$ and $\phi^{b,ss} = 4$, from (B.49) we easily obtain that

$$NW^{ss} = \frac{K^{ss}}{4} \tag{B.92}$$

Finally, from the law of motion of aggregate net worth in equation (B.54) we can pin down the value of the proportional transfer to the entering bankers, that is

$$\omega = \frac{g_* - \theta_b 0.01 + g_*/\beta}{4} \tag{B.93}$$

B.3.2 K-sector

To begin with, bear in mind that $f_t^{NE,ss}$, $f_t^{INC,ss}$, e_t^{ss} respectively define fixed costs and the K-firms technology shifter where the latter gows at the BGP rate $g_e \neq g_*$. We therefore define $e_t^{ss} \equiv e^{ss} g_e^t$ and $f_t^{NE,ss} \equiv f^{NE} g_*^t$ The solution for NE^{ss} is thus

$$NE^{ss} = \left[Q^{ss} \alpha^\alpha e^{ss} g_e^t \left(\frac{1-\alpha}{f^{NE} g_*^t} \right)^{1-\alpha} \right]^\gamma \tag{B.94}$$

Thus, in order to have a constant non zero and non diverging mass of NEs, it turns out that $g_e = g_*^{1-\alpha}$ must necessarily hold true. In the end (B.94), it boils down to the deterministic version of (B.66).

Then we can rewrite (B.67) to obtain η^{ss}

$$\begin{aligned} \eta^{ss} &= NE^{ss} + INC^{SS} \\ \eta^{ss} &= NE^{ss} + \eta^{ss} \left(\frac{f^{NE}}{g_* f^{INC}} \right)^{(1-\alpha)\gamma} \\ \eta^{ss} &= \frac{NE^{ss}}{1 - \left(\frac{f^{NE}}{g_* f^{INC}} \right)^{(1-\alpha)\gamma}} \end{aligned} \tag{B.95}$$

And INC^{ss}

$$INC^{ss} = NE^{ss} \frac{\left(\frac{f^{NE}}{g_* f^{INC}} \right)^{(1-\alpha)\gamma}}{1 - \left(\frac{f^{NE}}{g_* f^{INC}} \right)^{(1-\alpha)\gamma}}$$

Further it must also be that $\left(\frac{f^{NE}}{g_* f^{INC}} \right)^{(1-\alpha)\gamma} < 1$ in order to have a positive exiting mass of incumbents ruling thus out the possibility of an exploding mass of active firms as it can be seen from (2.39).

We can now solve for ss investments. From condition (B.72) and (B.73) we get:

$$I^{NE,ss} = NE^{ss} \frac{\gamma f^{NE,ss}}{\left[\gamma(1-\alpha) - 1 \right] Q^{ss}} \tag{B.96}$$

And

$$I^{INC,ss} = INC^{ss}\frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1}\left(\hat{A}^{ss^{INC}}\right)^{\frac{1}{1-\alpha}}(Q^{ss}\alpha)^{\frac{\alpha}{1-\alpha}} \tag{B.97}$$
$$= \eta^{ss}\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}$$

As evident from above, when choosing K-firms returns to scale and tail index a condition must be respected, that is $\gamma(1-\alpha) > 1$. This is done to guarantee that gross investment production is positive as it appears clearly from (B.96) and (B.97).

B.3.3 Market clearing

Using (B.86), (B.81) and (B.85), we get

$$I^{ss} = \left(1-\frac{1-\delta}{g_*}\right)K^{ss}$$
$$= \left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}+0.01/4-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} \tag{B.98}$$

Using (B.94), (B.95), (B.96), (B.97), (B.98) we get that Q^{ss} solves the following market clearing condition for the investment goods sector:

$$I^{ss} = I^{NE,ss} + I^{INC,ss} \Rightarrow$$
$$\left(1-\frac{1-\delta}{g_*}\right)g_* N^{ss}\left[\frac{\frac{\nu-1}{\nu}(1-\chi)}{\left(\frac{g_*}{\beta}+0.01/4-1+\delta\right)Q^{ss}}\right]^{\frac{1}{\chi}} =$$
$$\left[\frac{e^{ss}Q^{ss}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(f^{NE})^{1-\alpha}}\right]^{\gamma}\left\{\frac{\gamma f^{NE}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}} + \frac{\left[\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{1-\alpha}\right]^{\gamma}}{1-\left(\frac{f^{NE}}{g_* f^{INC}}\right)^{(1-\alpha)\gamma}}\frac{\gamma f^{INC}}{\left[\gamma(1-\alpha)-1\right]Q^{ss}}\right\} \tag{B.99}$$

It is now possible to work out the closed form solutions for all endogenous variables.[2]

[2] As pointed out in section 2.2.7 we calibrate the model so that $Q^{ss} = 1$ and thus calibrate e^{ss} accordingly.

B.4 I-sector Production

Here we derive overall production in the I-sector.

B.4.1 Derivation of NEs total production

Let us start from new entrants. We know that the production function for the generic *NE* firm can be expressed as

$$I_t^{NE,j} = \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \tag{B.100}$$

Then, by exploiting the transformation theorem we can compute the expected value of *NE*s production

$$\begin{aligned}
I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, dF(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, f(A_t^{NE,j})\, d(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma e_t^{\gamma} \int_{\hat{A}_t^{NE}}^{+\infty} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}-\gamma-1} d(A_t^{NE,j}) \\
\Rightarrow I_t^{NE} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma e_t^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{NE,j}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t^{NE}}^{+\infty} \\
\Rightarrow I_t^{NE} &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{B.101}$$

Where we exploited the fact that $NE_t = \left(\frac{\hat{A}_t^{NE}}{e_t}\right)^{-\gamma}$ and by assumption it must hold true that $\gamma(1-\alpha) - 1 > 0$.

Notice moreover that what we have computed is nothing different than the mean of a truncated distribution without normalizing it to a unit probability measure i.e., without dividing it by the probability share over which it is computed. This is done because in our model we want a measure of the total production in the *NE*s industry. Should one want to compute the idiosyncratic average production, it would be sufficient dividing (B.101) by NE_t.

B.4.2 Derivation of INCs total production

Let us repeat the same computation for incumbents. The production function for the generic incumbent firm is

$$I_t^{INC,j} = \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \tag{B.102}$$

Then, as before we have

$$\begin{aligned} I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, dF(A_t^{INC,j}) \\ \Rightarrow I_t^{INC} &= \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, f(A_t^{INC,j})\, d(A_t^{INC,j}) \\ \Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \int_{\hat{A}_t^{INC}}^{+\infty} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}-\gamma-1} d(A_t^{INC,j}) \\ \Rightarrow I_t^{INC} &= (Q_t\alpha)^{\frac{\alpha}{1-\alpha}}\, \gamma\eta_{t-1} \left(\hat{A}_{t-1}^{NE}\right)^{\gamma} \left[\frac{1-\alpha}{1-\gamma(1-\alpha)} \left(A_t^{INC,j}\right)^{\frac{1}{1-\alpha}-\gamma}\right]_{\hat{A}_t^{INC}}^{+\infty} \\ \Rightarrow I_t^{INC} &= INC_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(\hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} (Q_t\alpha)^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{B.103}$$

Where we have exploited the fact that $\eta_{t-1}\left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{INC}}\right)^{\gamma} = INC_t$.

Then, as the expected value of the sum is the sum of the expected values, we have that

$$I_t = I_t^{NE} + I_t^{INC} \tag{B.104}$$

B.5 I-firms profits derivation

Total revenues of the I-sector are thus $Q_t I_t$. Let us now define the the total amount of savings employed as input in the production of capital goods as

$$S_t = \int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) + \int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \tag{B.105}$$

where $\int_{\hat{A}_t^{NE}}^{+\infty} S\left(A_t^{NE}\right) dF(A_t^{NE}) \equiv S_t^{NE}$ and $\int_{\hat{A}_t^{INC}}^{+\infty} S\left(A_t^{INC}\right) dF(A_t^{INC}) \equiv S_t^{INC}$ are the total amount of inputs used in NEs and INCs sector production.

It follows that profits are respectively

$$\begin{aligned} \Pi_t^{NE} &= Q_t I_t^{NE} - S_t^{NE} - NE_t f^{NE} \\ &= NE_t \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)-1} \left(Q_t \hat{A}_t^{NE}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) - NE_t f^{NE} \end{aligned} \tag{B.106}$$

and

$$\begin{aligned} \Pi_t^{INC} &= Q_t I_t^{INC} - S_t^{INC} - INC_t f^{INC} \\ &= INC_t \frac{\phi(1-\alpha)}{\phi(1-\alpha)-1} \left(Q_t \hat{A}_t^{INC}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) - INC_t f^{INC} \end{aligned} \tag{B.107}$$

Which are always positive by construction as $\alpha < 1$. Then, define the total expenditures in fixed costs of the K-sector as

$$\overline{F}_t = NE_t f^{NE} + INC_t f^{INC} \tag{B.108}$$

Finally let us define the total amount of profits in the K-sector as

$$\overline{\Pi}_t = \Pi_t^{NE} + \Pi_t^{INC} \tag{B.109}$$

Then by substituting for (2.42), (B.106), (B.107) and (B.108) into equation (B.109) and rearranging we obtain the following identity

$$\overline{\Pi}_t + S_t + \overline{F}_t = Q_t I_t \tag{B.110}$$

Simply stating that the total amount of capital goods (in real terms) produced in the I-sector must be equal to the sum of profits, the input share of production and the total amount of fixed costs. Indeed K-firms hold the stock of capital and use households saving to buy the gross investment from the K-sector as a whole. The share of households saving S_t is employed by I-firms in investment production at real good price cost, whilst the share $\overline{F}_t$ is devoted to fixed costs payment.

B.6 The LAT shock

In principle, the major effect of the LAT shock is that of inducing an excess of supply in the final goods market, this reduces the marginal product of both labor inducing a fall in hours worked and raises, ceteris paribus, that of capital. This happens in both models, however only in the financial accelerator version we observe an initial slump in the accumulation of capital. This is because capital producers anticipate intermediate producers behavior and demand more investment-goods, pushing up both Q and φ^k as they expect a future increase in the relative price of investment goods. This puts downward pressure on the intermediate producers demand of capital causing a sudden capital decumulation on impact. Because of this, however, there is a smaller fall in hours worked. The netting out of these effects is almost zero and thus virtually there is almost no difference between in the effects of the LAT shock on the two model versions.

Therefore, from the banking sector perspective, the increase in banking assets lowers the marginal incentive to accumulate risky assets via spread and private leverage reduction and boosting net worth.

Most importantly we notice how the financial friction is mostly effective to the propagation of supply rather than demand shock (observed from the K-sector perspective).

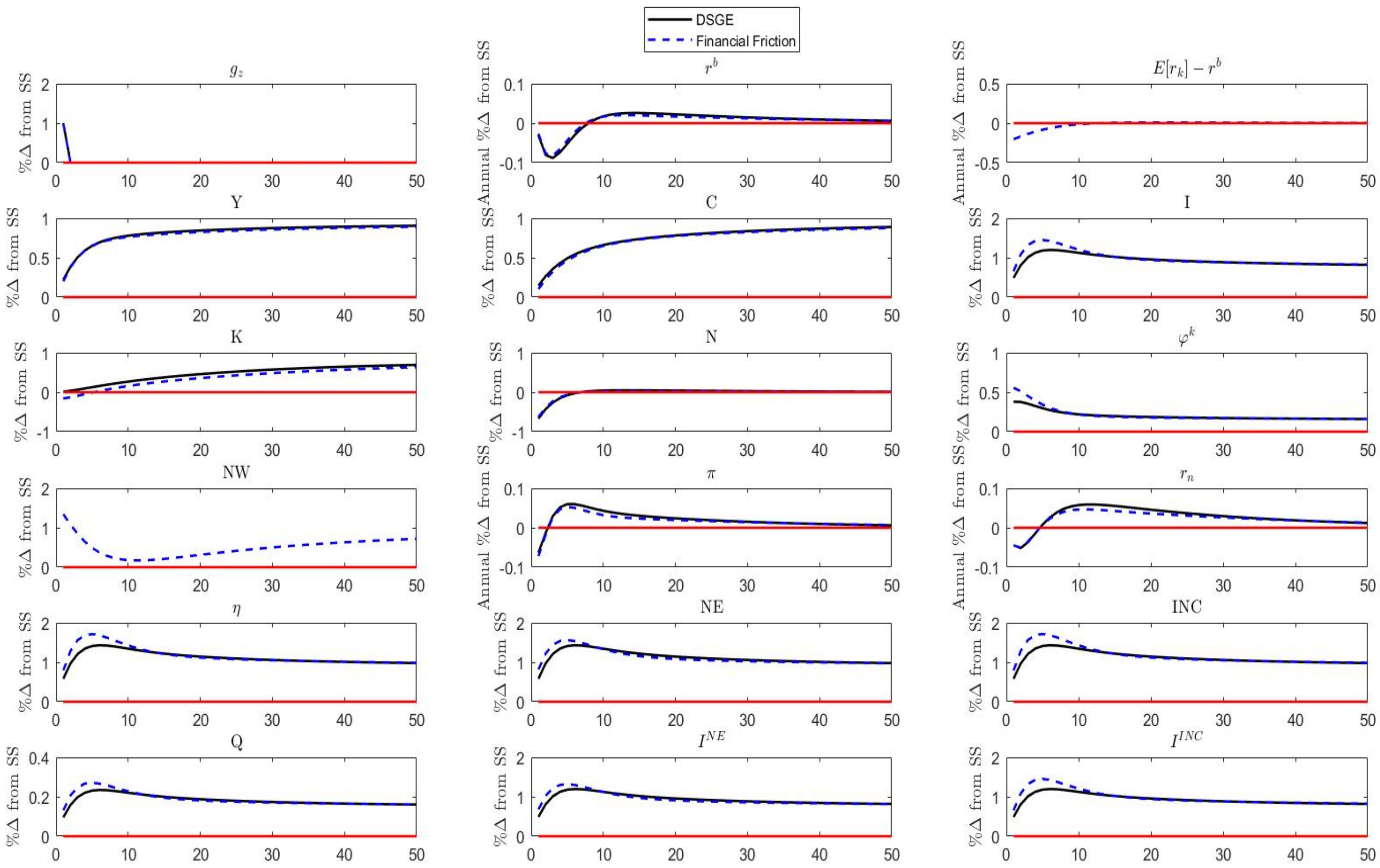

Figure B.1: Impulse response functions to a permanent LAT shock. Final goods production and banking sector. Shock size of the white noise component of $g_{z,t}$ is $\sigma^z = 0.01$, $\rho_z = 0$

B.7 The Investment Specific shock transmission abstracting from endogenous firms entry/exit

B.7.1 Permanent IST shock

Abstracting from endogenous firms entry/exit in the I-sector makes our model pretty similar to the canonical formulation of the two-sector neoclassical growth model even though a formal distinction between Q and φ is still maintained. In this respect the transmission of a permanent IST shock is sensibly weakened by the introduction of a financial friction via the generated higher procyclicality of the relative price of investment goods (because of the initial supply shock in the I-sector given the absence of "creative destruction") as it can be seen from Figure B.2. Magenta continuous lines stand for the financial frictionless model, green dash-dotted ones for its analog allowing for the financial sector presence.

However, the permanent IST shock cyclical impact is stronger when financial intermediaries are considered.

B.7.2 Persistent MEI shock

Introducing a MEI shock in this model would be equivalent to rewrite (2.29) as[3]

$$Q_t = \mu_t^{MEI}\varphi_t^k \left\{1 - \left[S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + S\left(\frac{I_t}{I_{t-1}}\right)\right]\right\} + \\ +\beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\varphi_{t+1}^k \mu_{t+1}^{MEI} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}$$

where

$$\ln(\mu_t^{MEI}) = \rho_{\mu^{MEI}} \ln(\mu_{t-1}^{MEI}) + \varepsilon_t^{\mu^{MEI}}$$

with $\varepsilon_t^{\mu^{MEI}} \sim N\left(0, \sigma^{\mu^{MEI}}\right)$.

Concerning the MEI shock impact, our model yields results which are perfectly consistent with what suggested by Afrin (2017)[4]. Indeed the generated countercyclicality of φ^k when a financial friction a lÃ GK is introduced into the model reduces the MEI shock propagation, as expected. Impulse responses are reported in Figure B.3. However, when endogenous entry/exit is considered, the cyclical impact of MEI shock is slightly increased both with and without financial frictions (see dashed blue and continuous black lines, respectively). This is intuitive. The MEI shock can be seen as a demand shock,

[3]This is true up to a first order linear approximation of the model.

[4]The shock calibration is the same as in Justiniano, Primiceri and Tambalotti (2011).

especially from the I-sector perspective. This implies the increase in the demand for I-goods calls for more I-firms to flow in the market (and less incumbents to die). Thus, the expansion following the MEI shock is boosted.

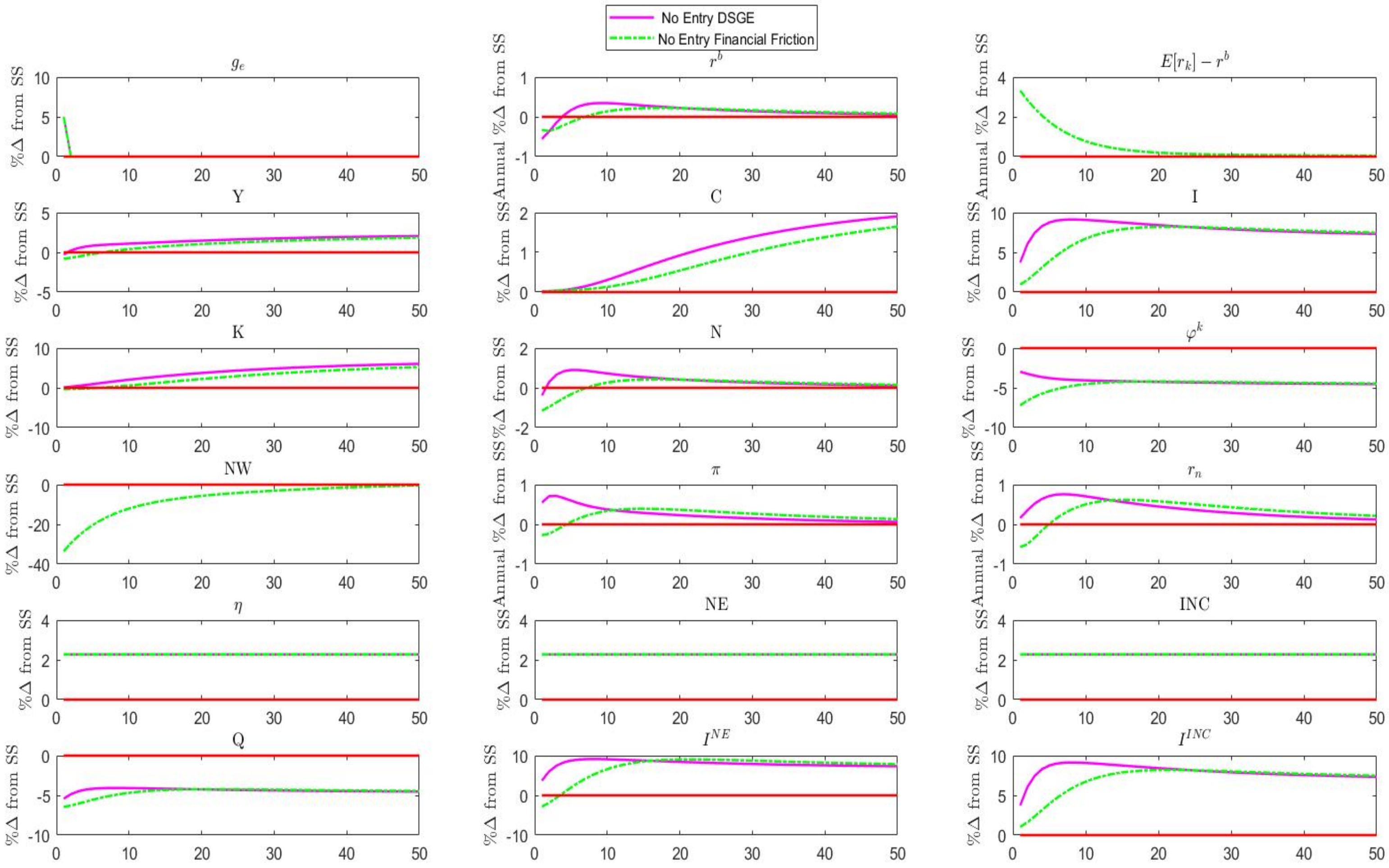

Figure B.2: Impulse response functions to a permanent IST shock abstracting from endogenous firms entry/exit in the I-sector.

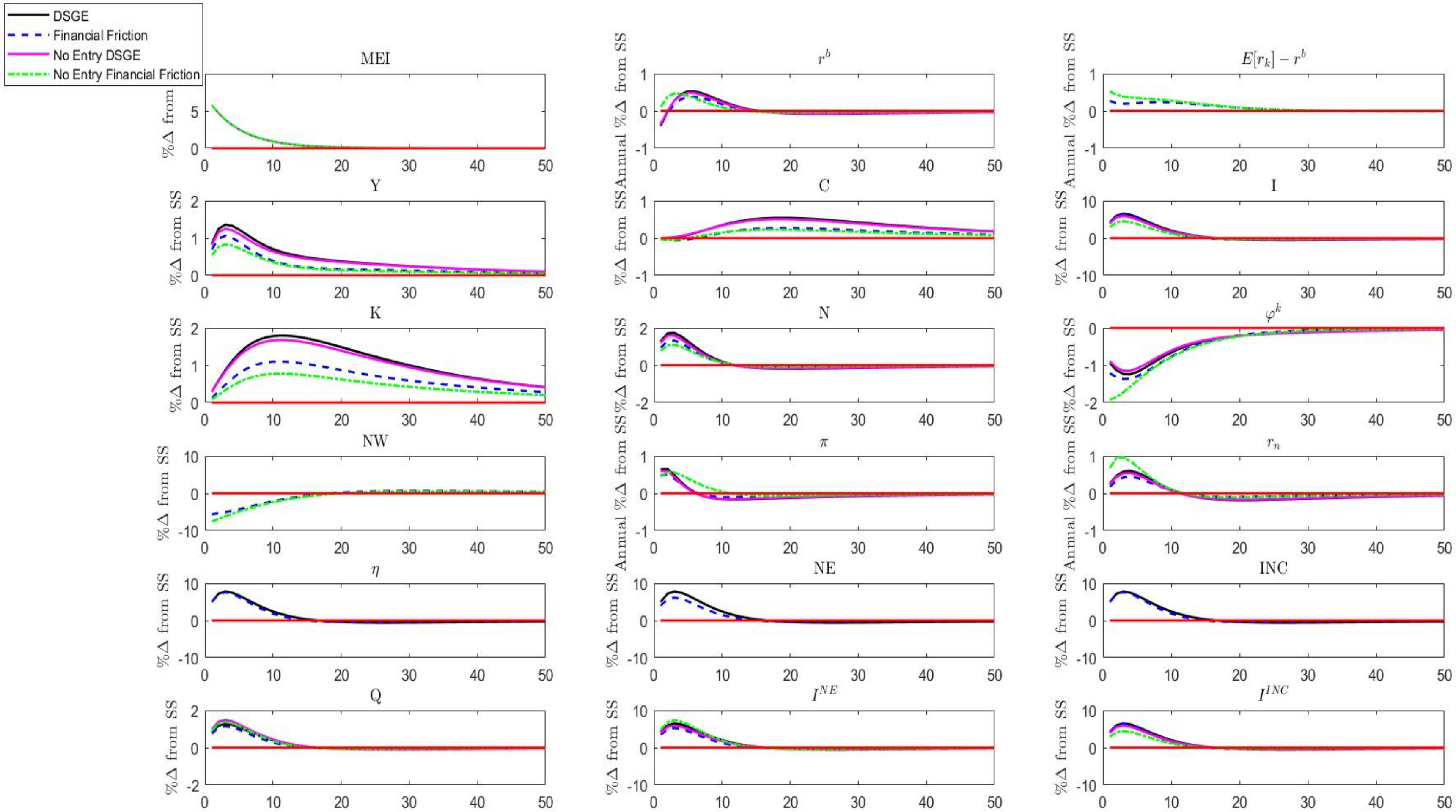

Figure B.3: Impulse response functions to a persistent MEI shock abstracting from endogenous firms entry/exit in the I-sector.

B.8 List of linearised equations

Here we list all the relevant linearised equations in our model. Detrended loglinearised variables are in small case and marked with a tilde.

B.8.1 Households

$$\widetilde{\lambda}_t^* = \frac{1}{(g_* - \beta a)(g_* - a)} \left[a g_* \left(\beta \widetilde{g}_{t+1} - \widetilde{g}_t \right) - \left(g_*^2 + \beta a^2 \right) \widetilde{c}_t + \right. \tag{B.111}$$

$$\left. + g_* a \left(\widetilde{c}_{t-1} + \beta \widetilde{c}_{t+1} \right) \right] \qquad \text{MUC}$$

$$\widetilde{w}_t = \theta \widetilde{N}_t - \widetilde{\lambda}_t^* \qquad \text{Labor Supply} \tag{B.112}$$

$$\widetilde{\lambda}_t^* = \widetilde{\lambda}_{t+1}^* - \widetilde{g}_{t+1} + \widetilde{r}_t^b \qquad \text{Bond Euler} \tag{B.113}$$

B.8.2 Financial Intermediaries

$$\widetilde{spr}_t = \widetilde{r}^{*,k}_{t+1} - \widetilde{r}^b_t \qquad \text{Financial Spread} \tag{B.114}$$

$$\begin{aligned}\widetilde{z}^{*,bk}_{t+1} &= \frac{\phi^{b,ss} R^{ss}_k}{g_* z^{bk,ss}} \widetilde{r}^{*,k}_{t+1} - \frac{(\phi^{b,ss}-1) R^{b,ss}}{g_* z^{bk,ss}} \widetilde{r}^b_t + \\ &+ \frac{\phi^{b,ss}\left(R^{ss}_k - R^{b,ss}\right)}{g_* z^{bk,ss}} \widetilde{\phi}^b_t - \widetilde{g}_t \qquad \text{Private } NW_j \text{ evolution}\end{aligned} \tag{B.115}$$

$$\widetilde{k}_t = -\widetilde{\varphi}^{*,k}_t + \widetilde{\phi}^b_t + \widetilde{nw}_t \qquad \text{Aggregate Banks Balance sheet} \tag{B.116}$$

$$\widetilde{\phi}^b_t = \widetilde{\eta}^{nw}_t + \frac{\nu^{ss}}{\lambda_b - \nu^{ss}} \widetilde{\nu}_t \qquad \text{Aggregate Private Leverage} \tag{B.117}$$

$$\begin{aligned}\widetilde{\nu}_t &= \frac{(1-\theta_b)\beta}{\nu^{ss} g_*}\left(R^{ss}_k - R^{b,ss}\right)\left(\widetilde{\lambda}^*_{t+1} - \widetilde{\lambda}^*_t - \widetilde{g}_{t+1}\right) + \\ &+ \frac{(1-\theta_b)\beta}{\nu^{ss} g_*}\left(R^{ss}_k \widetilde{r}^*_{k,t} - R^{b,ss}\widetilde{r}^b_t\right) + \\ &+ \frac{\theta_b \beta}{\nu^{ss} g_*}\left(\widetilde{\lambda}^*_{t+1} - \widetilde{\lambda}^*_t - \widetilde{g}_{t+1} + \widetilde{\phi}^b_{t+1} - \widetilde{\phi}^b_t + \widetilde{z}^{*,bk}_{t+1} + \widetilde{\nu}_{t+1}\right) \quad \text{Assets marginal incentive}\end{aligned} \tag{B.118}$$

$$\begin{aligned}\widetilde{\eta}^{nw}_t &= \frac{(1-\theta_b)\beta R^{b,ss}}{\eta^{ss,nw} g_*}\left(\widetilde{\lambda}^*_{t+1} - \widetilde{\lambda}^*_t - \widetilde{g}_{t+1} + \widetilde{r}^b_t\right) + \\ &+ \frac{\theta_b}{\eta^{ss,nw}}\left(\widetilde{z}^{*,bk}_{t+1} + \widetilde{\eta}^{nw}_{t+1}\right) \qquad NW \text{ marginal incentive}\end{aligned} \tag{B.119}$$

$$\begin{aligned}\widetilde{nw}_t &= \frac{\theta_b}{g_*}\left\{\phi^{b,ss} R^{ss}_k \widetilde{r}^*_{k,t} + (1-\phi^{b,ss}) R^{b,ss}\widetilde{r}^b_{t-1} + \right. \\ &+ \phi^{b,ss}(R^{ss}_k - R^{b,ss})\widetilde{\phi}^b_{t-1} + \\ &+ \left.\left[\left(R^{ss}_k - R^{b,ss}\right)\phi^{b,ss} + R^{b,ss}\right]\widetilde{nw}_{t-1}\right\} + \\ &+ \frac{\omega}{g_* nw^{ss}}\left(\widetilde{\varphi}^{*,k}_t + \zeta^k_t + \widetilde{k}_{t-1}\right) \qquad NW \text{ Law of motion}\end{aligned} \tag{B.120}$$

B.8.3 Final Producers

Retailers

$$\widetilde{d}_t = \widetilde{\pi}^*_t + (1-\lambda_p\beta)\,\widetilde{y}_t + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \widetilde{\pi}^*_{t+1} + (1-\nu)\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{d}_{t+1}\right] \quad \text{First Recursive Inflation Term} \tag{B.121}$$

$$\widetilde{f}_t = (1-\lambda_p\beta)\left(\widetilde{y}_t + \widetilde{mc}_t\right) + \\ + \lambda_p\beta\left[\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t - \nu\left(\gamma_p\widetilde{\pi}_t - \widetilde{\pi}_{t+1}\right) + \widetilde{f}_{t+1}\right] \quad \text{Second Recursive Inflation Term} \tag{B.122}$$

$$\widetilde{d}_t = \widetilde{f}_t \quad \text{Inflation dynamics} \tag{B.123}$$

$$\widetilde{\pi}^*_t = \frac{\lambda_p}{1-\lambda_p}\left(\widetilde{\pi}_t - \gamma_p\widetilde{\pi}_{t-1}\right) \quad \text{Evolution of prices} \tag{B.124}$$

Intermediate Producers

$$\widetilde{y}_t = \chi\widetilde{N}_t + (1-\chi)\left(\widetilde{k}_{t-1} - \bar{g}_t\right) - \widetilde{\xi}_t \quad \text{Final Output} \tag{B.125}$$

$$\widetilde{r}^*_{k,t} = \widetilde{g}_t - \bar{g}_t + \frac{(1-\delta)}{R^{ss}_k}\left(\widetilde{\varphi}^{*,k}_t - \widetilde{\varphi}^{*,k}_{t-1} - \zeta^k_t\right) + \\ + \frac{(1-\chi)}{R^{ss}_k}\left(\frac{Ng_*}{K}\right)^{\chi}\left[\widetilde{mc}_t - \widetilde{\xi}_t - \widetilde{\varphi}^{*,k}_{t-1} + \widetilde{\mu}_t + \right. \\ \left. +(1-\chi)\zeta^k_t + \chi\left(\widetilde{N}_t + \bar{g}_t + \widetilde{k}_{t-1}\right)\right] \quad \text{Demand of Capital} \tag{B.126}$$

$$\widetilde{w}_t = \widetilde{mc}_t - \widetilde{\xi}_t + (1-\chi)\left(\widetilde{k}_{t-1} + \zeta^k_t - \widetilde{n}_t - \bar{g}_t\right) \quad \text{Demand of Labor} \tag{B.127}$$

B.8.4 I-Firms

$$\widetilde{k}_t = \frac{(1-\delta)}{g_*}\left(\widetilde{k}_{t-1} - \bar{g}_t + \widetilde{\zeta}^k_t\right) + \frac{I^{ss}}{K^{ss}}\widetilde{i}_t \quad \text{Law of motion of capital} \tag{B.128}$$

$$\widetilde{q}_t = \widetilde{\varphi}^{*,k}_t + \widetilde{\mu}^i_t + \\ + \gamma_I g^2_*\left[-\frac{(1+\beta g_*)}{g_*}\widetilde{i}_t + \widetilde{i}_{t-1} + \beta\left(\widetilde{i}_{t+1} + \bar{g}_{t+1}\right) - \bar{g}_t\right] \quad \text{Investment rule} \tag{B.129}$$

B.8.5 I-Firms

$$\widetilde{\hat{a}}_t^{NE} = -\widetilde{q} \quad \text{NEs cutoff} \tag{B.130}$$

$$\widetilde{\hat{a}}_t^{INC} = -\widetilde{q} \quad \text{INCs cutoff} \tag{B.131}$$

$$\widetilde{\eta}_t^* = (1 - H^{ss})\widetilde{ne}_t + H^{ss}\widetilde{inc}_t \quad \text{Mass of active K-producers} \tag{B.132}$$

$$\widetilde{ne}_t = -\gamma \widetilde{a}_t^{NE} \quad \text{NEs mass} \tag{B.133}$$

$$\begin{aligned}\widetilde{inc}_t &= \widetilde{\eta}_{t-1}^* + \chi\gamma(\bar{g}_t - g_{z,t}) - \gamma g_{e,t} + \\ &+ \gamma \left[\widetilde{\hat{a}}_{t-1}^{NE} - \widetilde{\hat{a}}_t^{INC} + (\chi + \alpha - 1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{INCs mass}\end{aligned} \tag{B.134}$$

$$\begin{aligned}\widetilde{exit}_t &= \widetilde{\eta}_{t-1}^* + \chi\gamma(\bar{g}_t - g_{z,t}) - \gamma g_{e,t} + \\ &- \frac{H^{ss}}{1 - H^{ss}}\gamma \left[\widetilde{a}_{t-1}^{NE} - \widetilde{a}_t^{INC} + (\chi + \alpha - 1)g_{z,t} - \chi\bar{g}_t\right] \quad \text{Exit Mass}\end{aligned} \tag{B.135}$$

$$\widetilde{i}_t^{NE} = \widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \quad \text{NEs gross investment} \tag{B.136}$$

$$\widetilde{i}_t^{INC} = \widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{\alpha}{1-\alpha}\widetilde{q}_t \quad \text{INCs gross investment} \tag{B.137}$$

$$\widetilde{i}_t = \frac{I^{NE,ss}}{I^{ss}}\widetilde{i}_t^{NE} + \frac{I^{INC,ss}}{I^{ss}}\widetilde{i}_t^{INC} \quad \text{I-producers gross investment} \tag{B.138}$$

$$\begin{aligned}\widetilde{s}_t &= \frac{S^{NE,ss}}{S^{ss}}\left(\widetilde{ne}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{NE} + \frac{1}{1-\alpha}\widetilde{q}_t\right) + \\ &+ \frac{S^{INC,ss}}{S^{ss}}\left(\widetilde{inc}_t + \frac{1}{1-\alpha}\widetilde{a}_t^{INC} + \frac{1}{1-\alpha}\widetilde{q}_t\right) \quad \text{K firms total input amount}\end{aligned} \tag{B.139}$$

$$\widetilde{a}_t = \frac{NE^{ss}\hat{A}^{NE,ss}}{\bar{A}^{ss}}\left(\widetilde{ne}_t + \widetilde{a}_t^{NE}\right) + \frac{INC^{ss}\hat{A}^{INC,ss}}{\bar{A}^{ss}}\left(\widetilde{inc}_t + \widetilde{a}_t^{inc}\right) \quad \text{K-sector productivity} \tag{B.140}$$

B.8.6 Market clearing conditions and policy rules

$$\widetilde{y}_t = \frac{C^{ss}}{Y^{ss}}\widetilde{c}_t + \frac{S^{ss}}{Y^{ss}}\widetilde{s}_t + \frac{NE^{ss}f^{NE}}{Y^{ss}}\widetilde{ne}_t + \frac{INC^{ss}f^{INC}}{Y^{ss}}\widetilde{inc}_t \quad \text{Market clearing} \quad (B.141)$$

$$\widetilde{r}^n_t = \rho_{R^{n,ss}}\widetilde{r}^n_{t-1} + (1-\rho_{R^{n,ss}})\left(k_\pi\widetilde{\pi}_t + k_y\widetilde{mc} + \sigma^r\varepsilon_{rt}\right) \quad \text{Taylor rule} \quad (B.142)$$

$$\widetilde{r}_{b,t} = \widetilde{r}^n_t - \widetilde{\pi}_{t+1} \quad \text{Fisher equation} \quad (B.143)$$

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