Soap-film Möbius strip changes topology with a twist singularity

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Contributed by H. Keith Moffatt, October 26, 2010 (sent for review September 17, 2010)

It is well-known that a soap film spanning a looped wire can have the topology of a Möbius strip and that deformations of the wire can induce a transformation to a two-sided film, but the process by which this transformation is achieved has remained unknown. Experimental studies presented here show that this process consists of a collapse of the film toward the boundary that produces a previously unrecognized finite-time twist singularity that changes the linking number of the film's Plateau border and the centerline of the wire. We conjecture that it is a general feature of this type of transition that the singularity always occurs at the surface boundary. The change in linking number is shown to be a consequence of a viscous reconnection of the Plateau border at the moment of the singularity. High-speed imaging of the collapse dynamics of the film's throat, similar to that of the central opening of a catenoid, reveals a crossover between two power laws. Far from the singularity, it is suggested that the collapse is controlled by dissipation within the fluid film surrounding the wire, whereas closer to the transition the power law has the classical form arising from a balance between air inertia and surface tension. Analytical and numerical studies of minimal surfaces and ruled surfaces are used to gain insight into the energetics underlying the transition and the twisted geometry in the neighborhood of the singularity. A number of challenging mathematical questions arising from these observations are posed.

topological transition | contact line

In an elegant article in 1940 (1), the mathematician R. Courant laid out a number of fundamental questions about surfaces of minimal area that could be visualized with soap films spanning wire frames of various shapes. He noted that when the frame is a double loop it can support a film with a Möbius strip topology. Pulling apart and untwisting the loop leads to an instability whereby the film jumps with change of topology to a two-sided solution (Fig. 1). From splitting fluid drops (2) to reconnecting solar magnetic field lines (3), such topological transitions (4) abound in nature and are often associated with singular structures that evolve rapidly to a new state. The study of minimal surfaces dates back to the work of Euler (5) and Lagrange (6). "Plateau's problem," that of proving the existence of a minimal surface spanning a given contour, was solved in the 1930s (7–9), and subsequent mathematical work has focused chiefly on statics, involving, for example, proofs of the existence of such surfaces of prescribed topology in higher dimensions, classification of periodic minimal surfaces (10), and classification of embedded surfaces (11). With few exceptions, such as a study of the transition between the helicoid and the catenoid (12), little attention has been paid to transitions that take one surface to another. On the other hand, topological transitions have been studied extensively in fluid dynamics, with an emphasis on interface collapse in viscous flows (2, 4), and on the more inviscid problems of fluid and soap-film motion (13–18) and networks of film junctions (19). Yet one elementary question remains unanswered: *What is the process that takes a one-sided film to a two-sided one?*

We report experimental and theoretical results on four central issues raised by this question. We find that the singularity (*i*) occurs at the film boundary, (*ii*) lacks the cylindrical symmetry found in the collapse of a catenoid (13), or even the bilateral symmetry recently observed in bubble pinch off (20), and changes the linking number between the Plateau border and the centerline of the wire frame, (*iii*) displays an intriguing crossover from a nonclassical behavior in the collapse of the film to the more familiar one arising from a balance of capillary forces and air inertia (15, 21), and (*iv*) has energetics and geometry that may be understood through analysis of minimal surfaces and ruled surfaces spanning a family of parametrized frames (22).

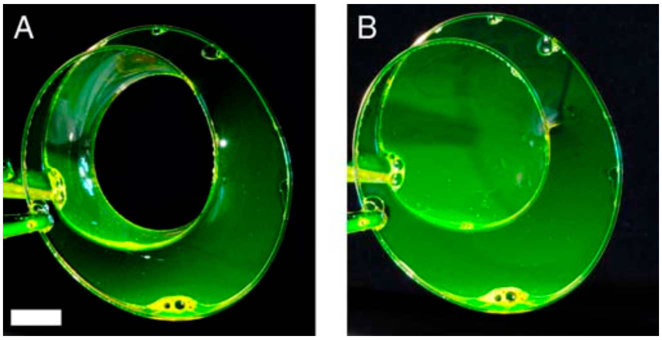

Fig. 1. Topological transition of a soap-film Möbius strip. As its frame is gradually distorted, a critical point is reached at which the one-sided film (*A*) transforms rapidly to a two-sided film (*B*). Scale bar in *A* is 2 cm.

Results and Discussion

Film Collapse Changes Linkage of Plateau Border and Frame. Removing the wire frame from the soap solution (see *Materials and Methods*), one typically finds (1) it necessary to destroy a small central disc-shaped film to obtain the Möbius strip. Gradually pulling apart the two loops then produces a narrow neck (or throat) (Fig. 1*A*) similar to a catenoid spanning two rings (13). Looking down this throat one sees an approximately circular boundary meeting the frame, a situation considered recently (23), but the geometry is now decidedly twisted and asymmetric. Once unstable the neck collapses, leaving a two-sided surface (Fig. 1*B*).

To illustrate the wire deformation during the process described above, it is helpful to introduce a particular representation (22) of the unfolding loop through the one-parameter family of curves $C\colon \mathbf{x}(\mu,\theta) = (x,y,z)$ with $\mu = -1$ and $0 \le \theta < 2\pi$, where

$$x(\mu,\theta) = [\mu\tau\cos\theta + (1-\tau)\cos 2\theta]/\ell(\tau), \quad \textbf{[1a]}$$

$$y(\mu,\theta) = [\mu\tau\sin\theta + (1-\tau)\sin 2\theta]/\ell(\tau), \quad \textbf{[1b]}$$

Author contributions: R.E.G., H.K.M., A.I.P., and R.L.R. designed research, performed research, analyzed data, and wrote the paper.

The authors declare no conflict of interest.

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This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1015997107/-/DCSupplemental.

$$z(\mu,\theta) = [2\mu\tau(1-\tau)\sin\theta]/\ell(\tau), \quad \textbf{[1c]}$$

and τ is a time-like parameter ($0 \le \tau \le 1$; $\tau = 0$ is the double-covered circle and $\tau = 1$ is a single circle in the *xy* plane, as shown in Fig. 2). The factor $\ell(\tau)$ normalizes the wire length to 2π,

$$\int_0^{2\pi} d\theta [x'(-1,\theta)^2 + y'(-1,\theta)^2 + z'(-1,\theta)^2]^{1/2} = 2\pi, \quad \textbf{[2]}$$

where $' \equiv d/d\theta$. It should be noted that because the wire has a finite diameter, this representation captures only the position of the centerline and does not address the twisting of the material frame of the wire. Of all the possible ways to join the two ends of a wire together to form a double loop, the one shown in Fig. 1 was constructed from fishing line, looped to form a double covering of a circle, the ends being glued together face-on to lock in zero twist Tw and one unit of writhe Wr. As is well-known (24–26), as the wire is untwisted and unfolded, $Tw + Wr$ is conserved; Wr decreases from 1 to 0, and Tw increases from 0 to 1, with $Tw + Wr = 1$ at all times.

Parametric Eqs. **1** actually provide a representation of a surface spanning C, the appropriate parameter range being ($-1 \le \mu < 1$, $0 \le \theta < \pi$). As the surface is swept out by a straight line as θ varies, this is a *ruled* surface (27). We may exploit the topological equivalence of this family of ruled surfaces and the actual soap film to study global properties of the surface before and after the singularity.

Throughout the family of Möbius strip minimal surfaces, the two closed curves tracking the Plateau border and the centerline of the frame are topologically linked, with linking number ± 2, a fact that does not appear to have been noted previously. This double linking of the wire and the Plateau border can be seen by graphing two nearby curves, one corresponding to $\mu = -1$ representing the wire, and the second, with say, $\mu = -0.9$, representing the Plateau border where the surface meets the wire. These are shown in Fig. 2*B*, where one can easily verify that there are four crossings of the red and gray curves representing, respectively, the border and the frame. Another visualization of the double linking can be achieved with a paper Möbius strip by cutting the strip through its midline, yielding a two-sided surface twice as long, with two half-twists, and then cutting it again (as if to separate the border from the wire) to yield two loops that are doubly linked. This result can be traced back to the classic work of Listing and Tait (28–30).

On the other hand, if the wire frame is a circle ($\tau = 1$), both the minimal surface that spans it and its Plateau border obviously lie in a plane; the latter traces the inner radius of the wire. This border clearly is *not* topologically linked with the wire and remains unlinked under all deformations of the wire that preserve the film's two-sidedness, as can be seen in Fig. 2*C*. Hence, not only does the bulk of the film undergo a transformation from one-sided to two-sidedness, but the boundary itself must experience a simultaneous topological transition.

Close inspection of the region near the singularity before and after the collapse (Fig. 3) shows the Plateau border twisting around the wire. The pitch of that twist is large before the collapse (Fig. 3 *A* and *B*), on the scale of the throat, and small after it (Fig. 3 *C* and *D*), on the scale of the wire, and the sense of the twist reverses at the critical instant t_p at which the hole disappears, when the throat envelops the wire and the border and the wire ceases to be linked. The strong deformation of the minimal surface near the Plateau border twist is visible through the deflection of light into a caustic. If the wire loop is further opened (as in Fig. 2*A*) toward a single planar circle, then the twist seen in Fig. 3*C* eventually disappears smoothly, but reappears if the frame is again deformed back toward a nonintersecting double loop. This process can be viewed as the interconversion of twist and writhe (here we are talking of the twist and writhe of the ribbon-shaped surface bounded by the Plateau border and the centerline of the wire). When the frame bounding the soap film is a single circle, its writhe is zero, and this ribbon has no twist, consistent with zero linking number. On the other hand, the nonintersecting double loop (e.g., $\tau = 0.3$ in Fig. 2*A*) has a writhe of nearly unity, and the ribbon now has a twist that balances that writhe (31). It is the change in the linking number of the ribbon, which occurs at the singularity, that has escaped previous notice.

Moreover, we have found that, contrary to Courant's original statements (1), the transition has a fundamental lack of reversibility. That is, although the change from one- to two-sidedness happens spontaneously beyond a critical surface configuration, a transition from two- to one-sidedness can be achieved only by forcing the wire to come in contact with itself, thus creating a region of the film where the necessary reconfiguration of the Plateau border can occur.

The twisting of the stationary film remaining immediately after the transition (Fig. 3*C*) is centered at the point of maximum curvature of the wire and extends over a region whose extent is comparable to the wire radius. The mean curvature $H = (1/R_1 + 1/R_2)/2$ of the surface is everywhere zero by virtue of the energy-minimization condition (where $R_{1,2}$ are the local principal radii of curvature), but, unlike everywhere else on the film, the Gaussian curvature $K = 1/R_1R_2$ in the region of the caustic is very large. This is a physical realization of a solution to two general classes of problems. The first was discussed by Courant (32), who proved the existence of a minimal surface, part of whose boundary lies on specified contours, the rest being constrained to lie on specified surfaces. Here, the particular shape of the Plateau border as it twists around the boundary is part of the unknown of the minimal-surface problem. Given the high Gaussian curvature, it is likely that singular perturbation methods, where the wire radius defines the small parameter, should be applicable.

The second class of problems, discussed by Douglas (9), is an extension of his results on Plateau's problem to one-sided surfaces bounded by a given contour. In this work, he uses the example of a contour in the form of a double-covered circle [e.g., Fig. 2*A* ($\tau \lesssim 2/3$), equivalent to his figure 1] to note that such a contour not only supports a film with the topology of a Möbius

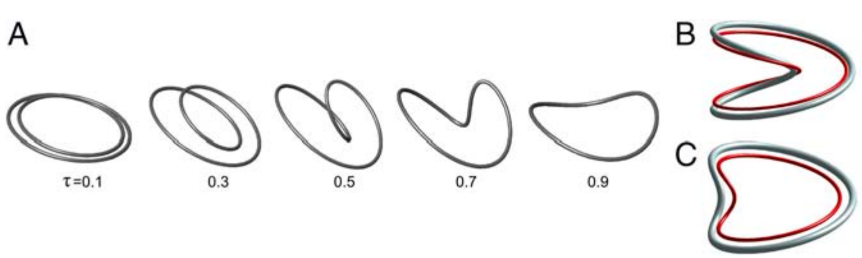

Fig. 2. Schematic of the complete unfolding of a wire frame. (*A*) Sequence of contours produced from the one-parameter family of curves, Eq. **1**, with $\mu = -1$ and increasing values of the parameter τ. (*B*) Double linking of the Plateau border (shown schematically in red) and the frame at low τ, before the transition. (*C*) After the transition the Plateau border and frame are unlinked.

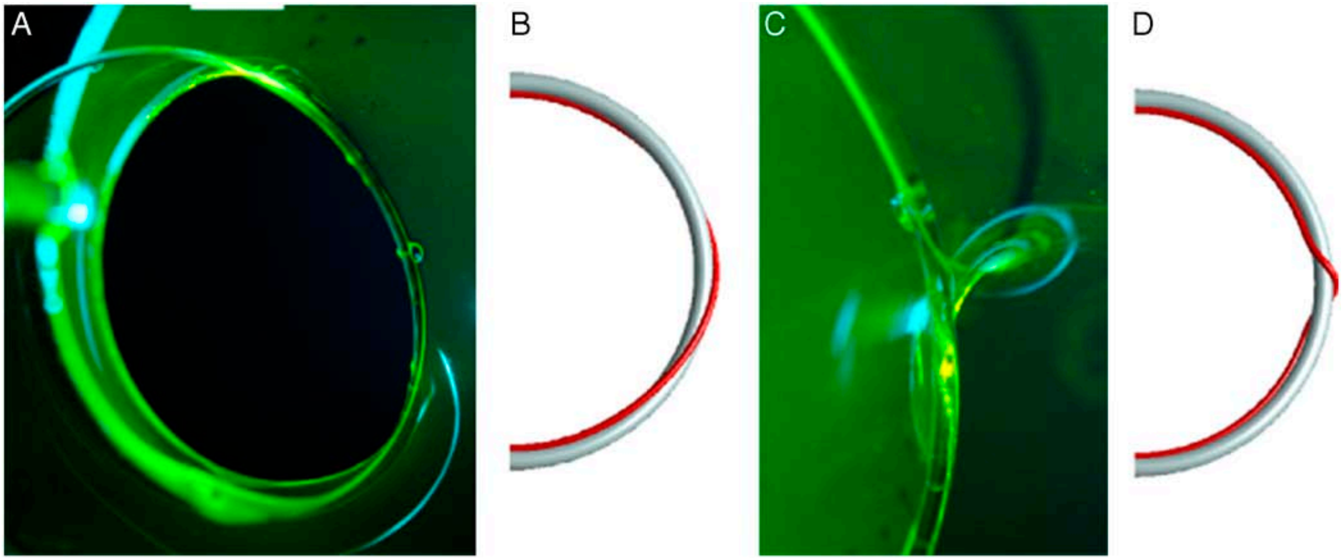

Fig. 3. Details of the twist singularity. The twisted Plateau border far from the collapse time (*A*) is shown schematically in *B* as a red line wrapping around the wire (gray). Just after the collapse (*C*) the border is twisted around the frame as shown in *D* and produces the caustic in *C*. Note the reversal of the sense of twist between *A* and *C*.

strip, but "…there also exists a minimal surface bounded by the same contour having the ordinary topology of a circular disc; but this other surface is self-intersecting, and it is rather the one-sided surface which is obtained in the actual experiment." It is certainly true in our experiments that upon removing a given wire frame of this shape from the soap solution one never finds a two-sided surface. It is the Möbius strip with the additional central film that is the most commonly obtained configuration; the Möbius strip by itself seldom happens. However, the two-sided solution for this configuration of the wire can be obtained, as described above, by distorting the frame and returning it to the initial position once the Möbius strip has collapsed. That this new configuration is stable is an indication of the hysteresis in this system.

The fact that we have obtained a nonintersecting two-sided minimal surface spanning the double-covered circular contour apparently contradicts Douglas's remark. The resolution of this apparent discrepancy resides in the finite thickness of the wire, which regularizes what would otherwise be a singularity. We should also note that Douglas studied the location of the singularities on the two-sided minimal surfaces bounded by a contour of zero size, whereas we have performed experiments where the singularity occurs when the film is supported by a finite size contour (the wire). To make a comparison between Douglas's results and the experimental ones, it would be necessary to study the limit of the wire radius approaching zero after the singularity has developed; but this limit inverts the order of the limiting processes implicitly considered by Douglas. Because we already know that there is hysteresis in the system, one would expect that the outcomes of these two different routes toward the singularity may not yield the same result. As is described in the following section, we observe that the singularity develops at the wire frame (surface boundary), a fact that should not be considered as a contradiction to Douglas's theorem stating that the two-sided surface spanning the contour has a singularity in its *interior*, but simply as a consequence of the different approaches to the singularity.

The Singularity Occurs at the Film Boundary. Careful manipulation of the wire frame has allowed us to trigger the instability of the Möbius strip film on demand, so that high-speed movies of that process can be reliably obtained (see *Materials and Methods*). The collapse process itself typically takes about 0.1 s. Fig. 4 *A*–*D* are frames from a high-speed movie at 5.4-ms intervals showing the approach to the singularity and the collision of the surface throat with the frame. The contrast enhancement achieved by the use of fluorescein in the film allows us to trace the whole projected shape of the collapsing throat with an edge-detection algorithm. Extensive experimentation has shown that the *only* route to the two-sided solution from the Möbius film is via a singularity at the boundary wire. In fact, the singularity never occurs away from the boundary, even in the presence of the second (disc-shaped) film. This is as one would expect because, as explained above, not only does the bulk of the film undergo a topological transition (one-sided to two-sided), but the boundary itself participates in this transition. We conjecture that in general any singularity associated with such a transition must always occur at the film boundary because, as we have noted, the linking number of the wire centerline and the Plateau border changes from ± 2 to zero at the transition, and this linking number, here defined for two closely adjacent curves (coincident in the limit as the wire radius tends to zero), clearly cannot be changed by a singularity on the film at any finite distance from the boundary. Translating this abstract argument to the experimental context is however nontrivial, for although the singular rearrangement of the Plateau border must occur at some point of the wire, it is by no means evident that the collapsing surface is always *directly* driven to this point; yet that is what is observed (Fig. 4).

Dynamics of the Finite-Time Singularity. When a minimal area soap film becomes unstable due to boundary deformation, there is a net resultant of surface-tension forces driving its motion. From movies of this collapse we extract a time-dependent throat diameter $D(t)$, the distance between the wire and the film (Fig. 4*A*). Fig. 5 shows the width $D(t)$, for three runs, measured with respect to the apparent pinch time t_p. In this log–log plot we see a crossover at $t_p - t \sim 5 \times 10^{-3}$ s between two distinct regimes of power-law behavior, $D \sim (t_p - t)^{\nu}$. Fitting the data asymptotically close to the singularity ($t_p - t \lesssim 0.003$) we find $\nu_1 = 0.67 \pm 0.02$, whereas far from the singularity ($t_p - t \gtrsim 0.015$), we obtain the exponent $\nu_2 = 0.33 \pm 0.02$. When the force on the film, proportional to the product of the surface tension σ and the local mean curvature H, is resisted only by the inertia of air, with mass density ρ then dimensional arguments (15) and boundary-integral formulations (21) show that the width should vanish as $D \sim (\sigma/\rho)^{1/3}(t_p - t)^{2/3}$, in good agreement with the observed exponent ν_1. To our knowl-

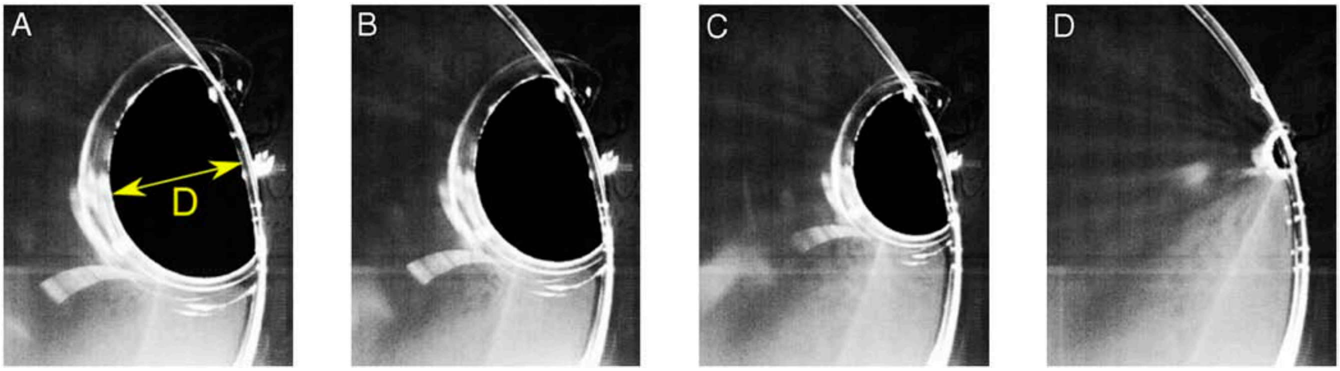

Fig. 4. Collapse of the Möbius strip throat. (*A*–*D*) Frames from a high-speed movie at 5.4-ms intervals show the approach to the singularity and collision with the frame. The throat diameter *D* used to quantify the collapse dynamics is indicated in *A*.

edge, the exponent ν_2 observed far from t_p has not been reported previously in contexts such as this. As this occurs when the interface is beginning its motion, it is reasonable to suspect that the Young–Laplace force would be balanced by dissipation. Examining movies of the collapse we observed that at the onset of collapse there is torsional motion of the Plateau border as the twist is focused toward an ever smaller segment of the wire (Fig. 4). An application of lubrication theory similar to ref. 33 to calculate the viscous forces within the wetting film on the wire yields the equation of motion $D^2\dot{D} \sim \sigma h^2/3\mu$, where h is a typical thickness of the film and μ is the fluid viscosity. This has the solution $D \sim (\sigma h^2/\mu)^{1/3}(t_p - t)^{1/3}$, an exponent consistent with the experimental value of ν_2 (Fig. 5).

Geometry of Twist Singularity and Surface Energetics. As equilibrium soap films minimize their surface area, the transition from one to two sides must lower the energy. The energetic competition among minimal surfaces can be studied numerically by using the computational routine Surface Evolver (34) to find the stable surface(s) that span curve C for various τ. Here we implement this calculation with a boundary of zero thickness [as in Douglas's analysis (9)] and therefore do not address any effects of the Plateau border. As described in *Materials and Methods*, the calculations are done on surfaces of a fixed topology, determined by the connectivity of the coarse initial grid that defines the surface. We find that stable two-sided solutions exist for $0.41 \lesssim \tau \leq 1$, whereas stable Möbius strip solutions are found in the range $0 < \tau \lesssim 0.42$ (within the small overlap of these ranges the relaxation scheme is extremely slow to converge, and it is difficult to be definitive regarding stability). The areas (surface energies) of these two branches of solutions are shown in Fig. 6*A* along with a number of minimal surfaces found numerically. The energy differences at the crossover point are extremely small. Continuing the numerical scheme beyond the point of instability of the one-sided solution allows one to approach the singularity through the very simplified dynamics of motion by mean curvature, the relaxation method of Surface Evolver. Interrupting this evolution before it becomes singular, we obtain an approximate representation of the twisted throat (Fig. 6*B*), similar to the experimental one (Fig. 4*D*).

For comparison, let us return to the ruled surfaces defined by Eq. **1**. As shown in Fig. 6*A*, and confirmed by analytical calculations, the energy $E(\tau)$ of the ruled surfaces spanning C approximates that of the minimal surfaces especially well for $\tau \lesssim 0.3$ and $\tau \gtrsim 0.8$. However, the value of τ at which the topological transition occurs differs between the two: $\tau = 2/3$ for the ruled surface versus $\tau \simeq 0.42$ for the minimal surface.

In detail, there are three critical values of τ in the evolution of the ruled surface as τ increases from 0 to 1. First, for $\tau > \tau_1 = 2 - \sqrt{2} \approx 0.586$, the surface is self-intersecting on a straight line segment (35). This may be shown by seeking parameter pairs (μ,θ) and (μ',θ') that yield the same values of x, y, and z in Eq. **1**; such pairs indeed exist for $\tau > \tau_1$. The minimal area soap film obviously cannot share this feature of self-intersection. Second, when τ increases through the critical value $\tau_2 = 2/3$, the projection of C on the plane $z = 0$ loses its loop, being cusped at $\theta = 0$ for $\tau = \tau_2$. Therefore, at τ_2, the hole through the one-sided surface $\mathcal{S}$ disappears: Its genus falls from 1 to zero (Fig. 7). Third, when τ is further increased through the critical value $\tau_3 = 4/5$, the bounding curve C passes through an "inflexional configuration"; i.e., there is instantaneously an inflexion point of zero curvature on C, Q say (36). At τ_3, the torsion at Q is infinite, but the singularity is integrable; in fact, it is at τ_3 that the internal twist of the wire jumps from zero to ± 1 (31).

Even though the ruled surface Eq. **1** self-intersects for $\tau > \tau_1$, i.e., even before the throat disappears, it nevertheless provides qualitative insight into features of the surface as it approaches the singularity. At $\tau = \tau_2$, as mentioned above, the projection of C on the plane $z = 0$ is cusped at $\theta = 0$; for $\tau > 2/3$, this projection has no reentrant loop. When $\tau = \tau_2$, C has the form near $\theta = 0$ of the twisted cubic with local parametric equations (with $\tilde{x} = \ell x$, etc.)

$$\tilde{x} \simeq -\frac{1}{3} - \frac{1}{3}\theta^2, \qquad \tilde{y} \simeq -\frac{1}{3}\theta^3, \qquad \tilde{z} \simeq -\frac{4}{9}\theta. \qquad [3]$$

Fig. 7 shows the projection of a portion of the surface $-1 < \theta < 1$, $-1 < \mu < -0.9$ on the *xy* plane for values of τ near the critical value $\tau_2 = 2/3$, showing how the hole disappears when the cusp in the boundary curve $\mu = -1$ is apparent. Close examination shows that the singularity that appears to propagate

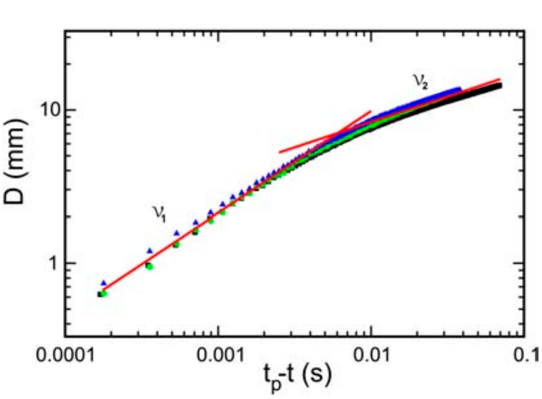

Fig. 5. Dynamics of the topological transition. The time dependence of the throat diameter *D*, for three typical separate events (colored symbols). Red lines indicate best-fit power laws, with exponent $\nu_1 \simeq 0.67 \pm 0.02$ close to the singularity and $\nu_2 \simeq 0.33 \pm 0.02$ far from the pinch time t_p.

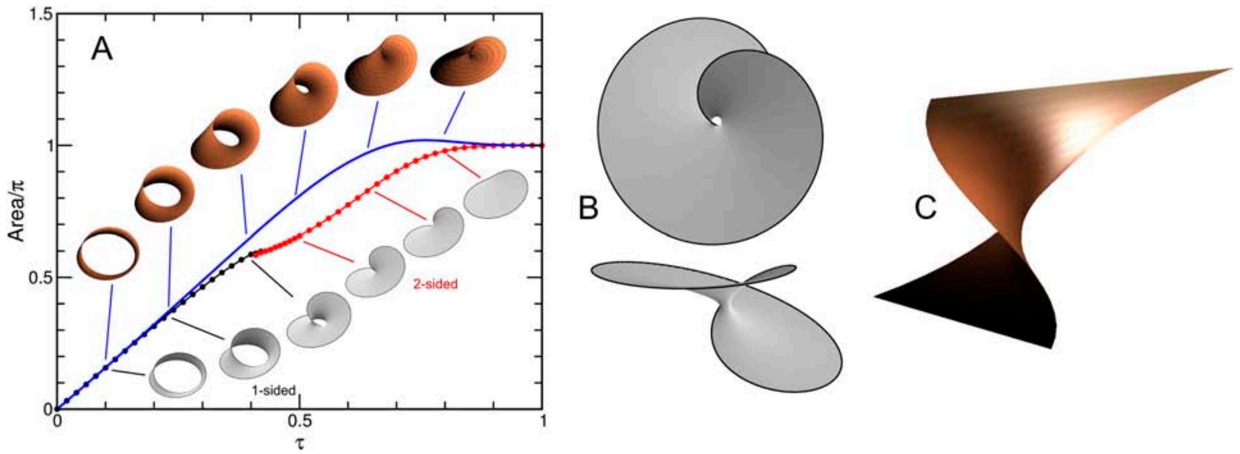

Fig. 6. Energy and geometry of surfaces. (*A*) Numerically obtained minimal surfaces (gray) and ruled surfaces (copper) as a function of the model parameter τ. Black and red filled circles indicate the minimal energy for the two classes of surfaces. The blue line is the energy of the ruled surface. (*B*) Two views of the surface geometry for τ beyond the limit of stability of the one-sided surface, where surface is evolving toward a singularity by the numerical relaxation scheme and the evolution has been terminated just prior to the singularity. (*C*) The universal "twisted saddle" geometry from the ruled surface model. Note similarity with the throat in *B*.

toward the center of the disc for $\tau > \tau_2$ is in fact a twisted fold (or cusp catastrophe); defining $\tau^* = 2(1-\tau)$ and scaled variables $X = \tau^*(\tilde{x} + \tau^*/2)$, $Y = \tau^{*2}\tilde{y}$, $Z = \tilde{z}/\tau^*$ its local equation can be expressed in the universal form

$$\frac{1}{2}Z^3 + XZ + Y = 0. \qquad [4]$$

This twisted saddle structure (Fig. 6*C*) constitutes an idealized approximation to the observed film structure, as well as that obtained numerically (Fig. 6*B*), and reveals how the twist of the surface imposed by the distant boundary may persist at the transition.

Conclusions

The results presented here indicate that the topological rearrangement of a soap film is far more subtle than previously thought. Although the transition from a one-sided to a two-sided surface was known, the role of the Plateau border and its linkage to the frame was not recognized. By establishing experimentally that there is a twist singularity involving reconnection of the Plateau border, a number of challenging mathematical and physical issues are raised. For example, can one prove rigorously our conjecture that the singularity must always occur on the boundary of the minimal surface for a wire of finite radius? For sure there must be a singular rearrangement of the Plateau border at the wire, but is the collapsing surface always driven directly to that point? What is the precise geometry in the region of concentrated twist left after the transition (Fig. 3*C*)? What is the nature of the singularity in the limit of vanishing wire radius?

On the more physical side, a major challenge is to develop an equation of motion for the collapsing surface that includes the singular change in the linking number. Such an equation of motion must account for the viscous dynamics of the Plateau border reconnection. Finally, and more generally, one would like a complete classification of the possible types of topological transition from one minimal surface to another and their associated singularities.

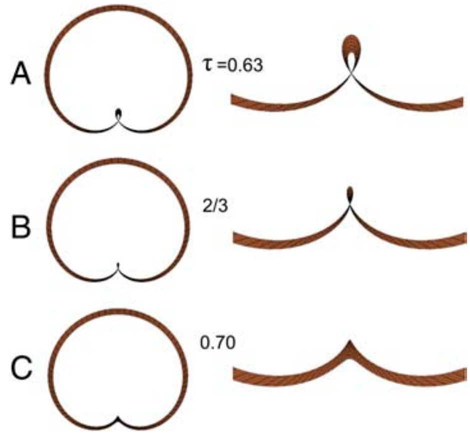

Fig. 7. Evolution of the ruled surface with τ. Section of ruled surface $-1 < \mu < -0.9$ (*Left*) and enlargement near $\theta = 0$ (*Right*) as τ passes from below (*A*) to above (*C*) the critical value $\tau_1 = 2/3$ (*B*) at which the genus changes.

Materials and Methods

Wire Frames and Soap Solutions. The films were formed on fishing line (diameter 0.73 mm), ∼32 cm long, looped to form a double covering of a circle and whose ends were glued together face-on, locking in one unit of twist. The frame was held by two chrome-plated alligator clips, each at the end of a long delrin handle. The soap film was composed of water, glycerine, and washing up liquid (1), to which fluorescein was added to allow visualization.

Imaging. Films were illuminated with high-power cyan light-emitting diodes (Luxeon Star, 505 nm) to maximize contrast when viewed against a background of black flocked paper (Thorlabs). High-resolution still images were obtained with a Nikon D3000S digital single-lens reflex and a 60-mm f/2.8 micro-Nikkor macro lens; high-speed imaging utilized a Phantom V310 and the same macro lens, at speeds up to 5,600 fps. Image sequences were analyzed with Matlab and ImageJ. A high-speed movie of the collapse dynamics can be found in Movie S1.

Surface Evolver Calculations. Here we note the initial vertices and edges used to create one- and two-sided surfaces within the Surface Evolver environment (34). For the two-sided surface the initial vertices v_n lie on the boundary curve Eq. **1** ($\mu = -1$) and with $\theta_n = (n-1)\pi/5$ for $n = 1,\ldots,10$, and $v_{11} = (0,0,0)$. The 20 edges are (1/2,2/3,…,9/10,10/1,1/11,2/11,…10/11), and the faces are (1/12/ − 11; 2/13/ − 12,…; 10/11/ − 20), where the minus sign signifies traversal in the opposite sense. The one-sided surface has the 10 vertices v_1–v_{10} above, the 15 edges (1/2,2/3,…,9/10,10/1,1/6,2/7,…5/10), and 5 faces (10/11/ − 5/15; 11/6/ − 12/ − 1; 12/7/ − 13/ − 2; 13/8/ − 14/ − 3; 14/9/ − 15/ − 4).

ACKNOWLEDGMENTS. We thank D. Page-Croft and J. Milton for device fabrication, Vision Research UK for technical assistance with high-speed imaging, and C. Wolgemuth for computational advice. This work was supported by the Schlumberger Chair Fund. R.L.R. acknowledges the support of Trinity College, Cambridge, for visits to the Department of Applied Mathematics and Theoretical Physics.

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