Successive Approximations, Propagation Algorithms and the Inverse Obstacle Problem

Giovanni F Crosta

DISAT (Environmental Sciences)
Università degli Studi
Milano, IT
giovanni@alpha.disat.unimi.it

Progress in Electromagnetics Research Symposium 1998
Session B06
Nantes, FR

DEDICATED TO

RALPH E KLEINMAN

A TEACHER,

A GUIDE,

AND AN EXAMPLE

OF FULL COMMITTMENT TO RESEARCH.

PLAN

Statement of the inverse electromagnetic problem
Spatial dimension: $n = 2$.
Data: IPSWICH {RCS, χ}.

Prior knowledge: {V polarization; $\{\mathbf{k}_q^{(\mathrm{inc})}\}$; DIRICHLET BC on $\partial\Omega$; Ψ_{adm}}.
Parameters: {obstacle; scattered field approximation} $= \{ I ; L \}$.
Unknowns: {shape parameters; phase offsets} $= \{ \vec{\psi} ; \gamma \}$.

Top – down heuristics of shape reconstruction:
minimization of the far zone defect
(*approximate forward propagation*, AFP).

Numerical results:
full *vs*. limited aperture data,
superposition of incident waves.

Bottom – up justification of the algorithm: an attempt

The *least squares* boundary coefficients, $\mathbf{c}^{(L)}$:
- * error bounds;
- * RAYLEIGH' s hypothesis.

Approximate forward propagation
- * successive approximations and fixed points;
- * convergence, trust and effectiveness;
- * the spectral radius of a propagator matrix.

Further problems.

INVERSION SCHEMES FOR VERTICAL POLARIZATION AND PERFECTLY ELECTRICAL CONDUCTING, STAR SHAPED OBSTACLE, $n = 2$

Obstacle: prior knowledge and parameterization

$$\Gamma = \{ \mathbf{x} \mid r = r[\phi],\ 0 \le \phi \le 2\pi,\ r[.] \in C^2([0,2\pi]),$$
$$0 < r_{\min} \le r[\phi] \le r_{\max},\ \forall\phi \in [0,2\pi] \}$$

$$\frac{1}{r^2}[\phi] = \psi_1 + \psi_2 \cos\phi + \psi_3 \sin\phi + \psi_4 \cos 2\phi + \dots \ (I \text{ terms})$$

$\Rightarrow$ { constraints on $r[.]$ } $\Leftrightarrow$ $\{\vec{\psi} \in \Psi_{\text{adm}}\}$.

Data

incident waves

$$\{\mathbf{E}_q^{(\text{inc})} = \hat{\mathbf{z}}\, e^{\,i\,\mathbf{k}_q \cdot \mathbf{r}},\ q \in Q\}$$

RCSs (σ_q) and phases (χ_q)

$$\underbrace{\left\{\frac{\sigma_q[\phi_j]}{\text{wavelength}};\ \chi_q[\phi_j;\gamma_q]\right\}}_{A[\mathbf{k}_q;\phi_j\ ;\gamma_q]}$$

scattering amplitude

Scattered waves $\{\mathbf{E}_q^{(\text{sc})}\}$

$$\hat{\mathbf{z}}(\Delta + k^2)E^{(\text{sc})}_{q,z} = \mathbf{0} \text{ in } I\!R^n \setminus \overline{\Omega} \ ; \ (E^{(\text{inc})}_{q,z} + E^{(\text{sc})}_{q,z})|_\Gamma = 0 \ ; \ \text{SRC}$$

Approximation of the scattered waves

... in the far zone ...

from $E^{(\text{sc})}_{q,z}[\mathbf{x}] = \sum_\lambda f_{q,\lambda} v_\lambda[\mathbf{x}],\ \mathbf{x} = \{r, \phi\};\ \lambda = \{p, l\};\ p = 0, 1;\ p \le l$,

where $v_\lambda := \sqrt{\epsilon_l}\, H_l^{(1)}[kr]\Big[(1-p)\cos l\phi + p \sin l\phi\Big]$,

to $\Lambda[L] := \{p, l \mid p = 0, 1;\ p \le l \le L\}$

and $E^{(\text{sc}, L)}_{q,z}[\mathbf{x}] := \sum_{\lambda \in \Lambda(L)} p^{(L)}_{q,\lambda} v_\lambda[\mathbf{x}]$, with $\{p^{(L)}_{q,\lambda}\}$ suitable ....

.. on the obstacle boundary ...

$$E^{(\text{sc}, L)}_{q,z}|_\Gamma := \sum_{\lambda \in \Lambda(L)} c^{(L)}_{q,\lambda} v_\lambda|_\Gamma \text{, with } \{c^{(L)}_{q,\lambda}\} \text{ suitable.}$$

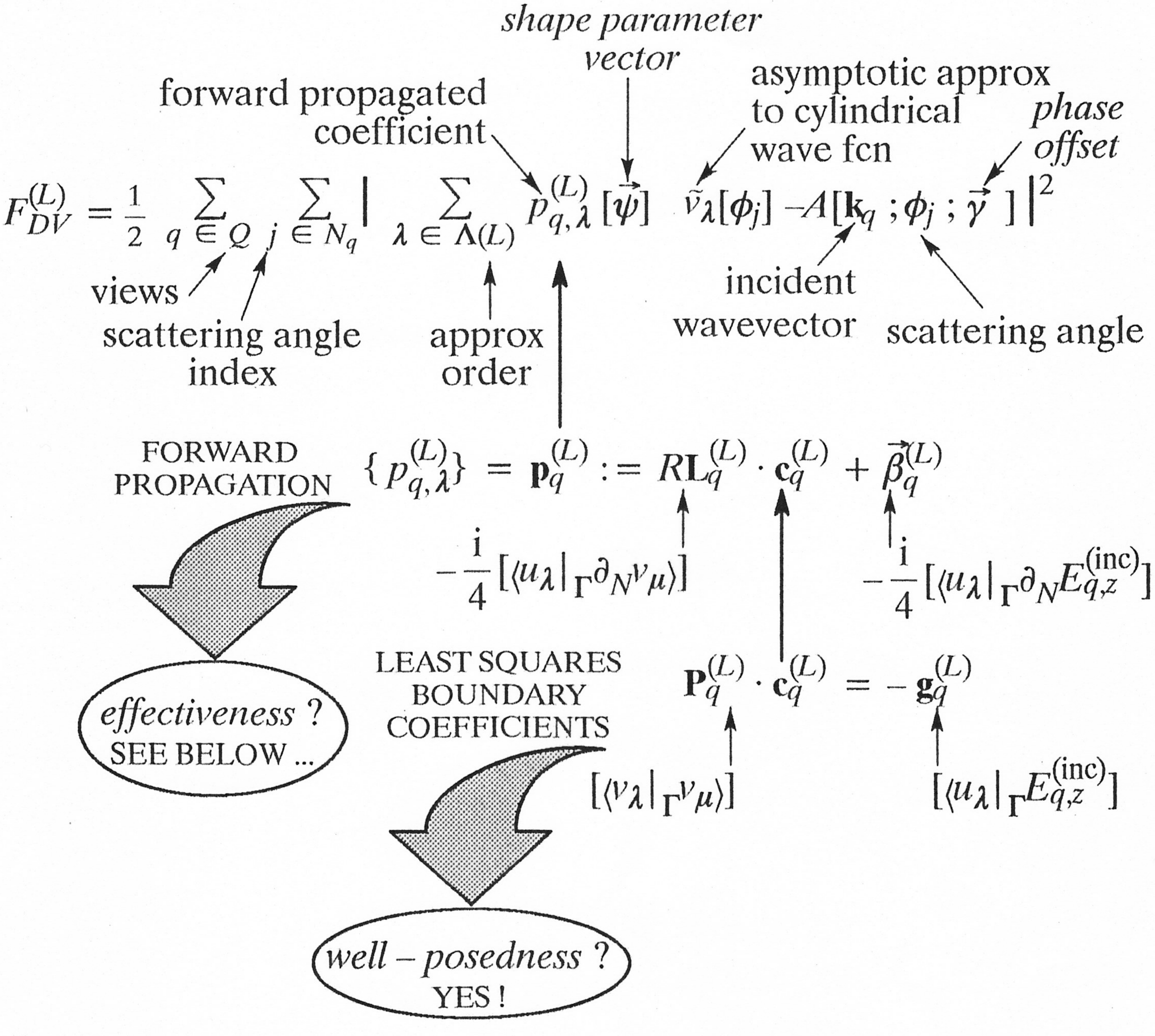
TOP – DOWN HEURISTICS: SHAPE RECONSTRUCTION BY $F_{DV}^{(L)}$ MINIMIZATION

PBM (shape reconstruction from card$[Q]$ sets of IPSWICH data)

Given I, $L, \{A[\mathbf{k}_q\ ;\ \phi_j\ ;\ \gamma_q],\ j \in N_q\ ,\ q \in Q\}$, *find* $\vec{\psi}^{\star} \in \Psi_{\text{adm}}$ *and* $\vec{\gamma}^{\star}$ *such that,*

$$F_{DV}^{(L)}[\vec{\psi}^{\star};\ \vec{\gamma}^{\star}] = \min$$

where



shape parameter vector

forward propagated coefficient

asymptotic approx to cylindrical wave fcn

phase offset

$$F_{DV}^{(L)} = \frac{1}{2} \sum_{q \in Q} \sum_{j \in N_q} \Big| \sum_{\lambda \in \Lambda(L)} p_{q,\lambda}^{(L)}[\vec{\psi}]\ \ \tilde{v}_\lambda[\phi_j] - A[\mathbf{k}_q\ ;\phi_j\ ;\vec{\gamma}\ ] \Big|^2$$

views

scattering angle index

approx order

incident wavevector

scattering angle

FORWARD PROPAGATION

$$\{p_{q,\lambda}^{(L)}\} = \mathbf{p}_q^{(L)} := R\mathbf{L}_q^{(L)} \cdot \mathbf{c}_q^{(L)} + \vec{\beta}_q^{(L)}$$

$$-\frac{\mathrm{i}}{4}[\langle u_\lambda|_\Gamma \partial_N v_\mu\rangle] \qquad -\frac{\mathrm{i}}{4}[\langle u_\lambda|_\Gamma \partial_N E_{q,z}^{(\mathrm{inc})}\rangle]$$

effectiveness ?
SEE BELOW ...

LEAST SQUARES BOUNDARY COEFFICIENTS

$$\mathbf{P}_q^{(L)} \cdot \mathbf{c}_q^{(L)} = -\,\mathbf{g}_q^{(L)}$$

$$[\langle v_\lambda|_\Gamma v_\mu\rangle] \qquad [\langle u_\lambda|_\Gamma E_{q,z}^{(\mathrm{inc})}\rangle]$$

well – posedness ?
YES !