Shape Reconstruction from {Radar Cross Section, Phase} Data

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THIS TALK WANTS TO BE

A TRIBUTE TO THE MEMORY OF

RALPH E KLEINMAN

A TEACHER,

A GUIDE,

AND AN EXAMPLE

PLAN

Statement of the inverse electromagnetic problem
Spatial dimension: $n = 2$.
Data: IPSWICH {RCS, χ}.

Prior knowledge: {V polarization; $\{\mathbf{k}_q^{(\mathrm{inc})}\}$; DIRICHLET BC on $\partial\Omega$; Ψ_{adm}}.
Parameters: $\{ I, L \}$.
Unknowns: $\{ \vec{\psi}, \gamma \}$.

PART ONE
The heuristics of shape reconstruction. Two classes of algorithms:

A1: minimization of the boundary defect
($\mathbf{M}^{(L)}$ *approximate back propagation*, ABP)

A2: minimization of the far zone defect
(*approximate forward propagation*, AFP).

Numerical results:
full *vs.* limited aperture data,
superposition of incident waves.

PART TWO
Towards the justification of the A1 and A2 algorithms.

J1 The *least squares* boundary coefficients, $\mathbf{c}^{(L)}$:
* error bounds;
* RAYLEIGH' s hypothesis.

J2 The *affine – least squares* (ALS) scheme &
the forward propagated coefficients, $\mathbf{p}^{(L)}$:
* consistency;
* ALS, successive approximations and fixed points;
* the spectral radius of a propagator matrix;
* the trusted method inequality.

Further problems.

INVERSION SCHEMES FOR VERTICAL POLARIZATION AND PERFECTLY ELECTRICAL CONDUCTING, STAR SHAPED OBSTACLE, $n = 2$

Obstacle: prior knowledge and parameterization

$\Gamma := \{ \mathbf{x} \mid r = r[\phi],\ 0 \le \phi \le 2\pi,\ r[.] \in C^2([0, 2\pi]),$
$0 < r_{\min} \le r[\phi] \le r_{\max},\ \forall\phi \in [0, 2\pi] \}$

$\frac{1}{r^2}[\phi] = \psi_1 + \psi_2 \cos\phi + \psi_3 \sin\phi + \psi_4 \cos 2\phi + \ldots$ (I terms)

$\Rightarrow$ { constraints on $r[.]$ } $\Leftrightarrow$ $\{ \vec{\psi} \in \Psi_{\text{adm}} \}$.

Data

incident waves

$\{ \mathbf{E}_q^{(\text{inc})} = \hat{\mathbf{z}}\, e^{\mathrm{i}\, \mathbf{k}_q \cdot \mathbf{r}},\ q \in Q \}$

RCSs (σ_q) and phases (χ_q)

$\{ \dfrac{\sigma_q[\phi_j]}{\text{wavelength}} ; \chi_q[\phi_j ; \gamma_q] \}$

$A[\mathbf{k}_q ; \phi_j ; \gamma_q]$

scattering amplitude

Scattered waves $\{ \mathbf{E}_q^{(\text{sc})} \}$

$$\hat{\mathbf{z}}(\Delta + k^2) E_{q,z}^{(\text{sc})} = \mathbf{0} \text{ in } I\!R^n \setminus \overline{\Omega} \ ; \ (E_{q,z}^{(\text{inc})} + E_{q,z}^{(\text{sc})})|_\Gamma = 0 \ ; \ \text{SRC}$$

Approximation of the scattered waves

... in the far zone ...

<u>from</u> $E_{q,z}^{(\text{sc})}[\mathbf{x}] = \sum_\lambda f_{q,\lambda} v_\lambda[\mathbf{x}],\ \mathbf{x} = \{ r, \phi \} ;\ \lambda = \{p, l\} ; p = 0, 1; p \le l$,

where $v_\lambda := \sqrt{\epsilon_l}\, H_l^{(1)}[kr] \big[(1-p)\cos l\phi + p \sin l\phi \big]$,

<u>to</u> $\Lambda[L] := \{ p, l \mid p = 0, 1 ; p \le l \le L \}$

and $E_{q,z}^{(\text{sc}, L)}[\mathbf{x}] := \sum_{\lambda \in \Lambda(L)} p_{q,\lambda}^{(L)} v_\lambda[\mathbf{x}]$, with $\{ p_{q,\lambda}^{(L)} \}$ suitable ....

... on the obstacle boundary ...

$$E_{q,z}^{(\text{sc}, L)}|_\Gamma := \sum_{\lambda \in \Lambda(L)} c_{q,\lambda}^{(L)} v_\lambda|_\Gamma \text{, with } \{ c_{q,\lambda}^{(L)} \} \text{ suitable.}$$

PART ONE
APPROXIMATION

Defect norms

A1 — A2

Boundary defect (one value of q only) — *Far zone defect*

$B_{DV}^{(L)}[\vec{\psi}\,;\gamma]:=$

$$= \frac{1}{2}\Big\| \sum_{\lambda \in \Lambda(L)} c_\lambda^{(L)} v_\lambda + E_{q,z}^{(\mathrm{inc})} \Big\|_{L^2(\Gamma)}^2$$

$F_{DV}^{(L)}[\vec{\psi}\,;\vec{\gamma}\,]:=$

$$= \frac{1}{2} \sum_{q \in Q} \sum_{j \in N_q} \Big| \sum_{\lambda \in \Lambda(L)} p_{q,\lambda}^{(L)} \tilde{v}_\lambda[\phi_j] - A[\mathbf{k}_q\,;\phi_j\,;\gamma_q] \Big|^2 .$$

PBM A1.1: *relate* $\{c_\lambda^{(L)}\}$ *to* $A[\mathbf{k}_q\,;.]$. $\Rightarrow$ **ABP**

PBM A2.1: *relate* $\{p_{q,\lambda}^{(L)}\}$ *to* $\{E_{q,z}^{(\mathrm{inc})}|_\Gamma\}$ *and* $\vec{\psi}$. $\Rightarrow$ **AFP**

SOLUTION A1.1 (the least squares boundary coefficients)
[BARANTSEV + KOZACHEK, 1969; RAMM, 1986]

Given $\vec{\psi}_{\mathrm{ref}} \in \Psi_{\mathrm{adm}}$, $E_{q,z}^{(\mathrm{inc})}$, L, *the coefficients*

$$\{c_\lambda^{(L)} \mid \lambda \in \Lambda[L]\} := \mathbf{c}^{(L)}$$

such that, $B_{DV}^{(L)}[\vec{\psi}_{\mathrm{ref}}\,;\mathbf{c}^{(L)}] = \min$ *are given by* $\mathbf{P}^{(L)} \cdot \mathbf{c}^{(L)} = -\mathbf{g}^{(L)}$
where $\mathbf{g}^{(L)} := [\langle v_\lambda \,|_\Gamma\, E_{q,z}^{(\mathrm{inc})} \rangle]$ *and* $\mathbf{P}^{(L)} := [\langle v_\lambda \,|_\Gamma\, v_\mu \rangle]$.

SOLUTION A2.1: **AFP** and **ALS** scheme in $X_1^{(L)} := \mathrm{span}\{v_\lambda \mid \lambda \in \Lambda[L]\}$.

Since $f_{q,\lambda} = -\frac{i}{4}\langle u_\lambda|_\Gamma\, \partial_N(E_{q,z}^{(\mathrm{inc})} + E_{q,z}^{(\mathrm{sc})})\rangle$, where $u_\lambda := \mathrm{Re}[v_\lambda]$, then

I) <u>project</u> $E_{q,z}^{(\mathrm{sc})}|_\Gamma$ <u>onto</u> $X_1^{(L)}$: $\quad E_{q,z}^{(\mathrm{sc},L)} := \sum_{\mu \in \Lambda(L)} c_{q,\mu}^{(L)} v_\mu$

II) <u>differentiate</u>: $\quad \partial_N E_{q,z}^{(\mathrm{sc},L)}\,|_\Gamma = \sum_{\mu \in \Lambda(L)} c_{q,\mu}^{(L)} \partial_N v_\mu$

III) <u>propagate</u> $\mathbf{c}_q^{(L)}$ <u>forward</u>: $\quad RL^{(L)}.\mathbf{c}_q^{(L)} + \vec{\beta}_q^{(L)} = \mathbf{p}_q^{(L)}$,

where $RL^{(L)} := -\frac{i}{4}[\langle u_\lambda|_\Gamma\, \partial_N v_\mu \rangle]$, $\vec{\beta}_q^{(L)} := -\frac{i}{4}[\langle u_\lambda|_\Gamma\, \partial_N E_{q,z}^{(\mathrm{inc})} \rangle]$.

THE HEURISTICS OF SHAPE RECONSTRUCTION BY $B_{DV}^{(L)}$ MINIMIZATION

Hp $\exists\ [R\mathbf{L}^{(L)}]^{-1}$

Def The $X_1^{(L)}$ **ABP** is the affine map

$$\mathbf{M}_q^{(L)}[.] := [R\mathbf{L}^{(L)}]^{-1}\left([.] - \vec{\beta}_q^{(L)}\right)$$

Def At fixed q, the far zone coefficients $\vec{\varphi}_q^{(L)}[\gamma_q]$ estimated from the <u>full aperture</u> scattering amplitude $\{A[\mathbf{k}_q\,;\,\phi_j\,;\,\gamma_q], j \in N_q\}$ are

$$\varphi_{q,\lambda}^{(L)} := \langle v_\lambda^{(\text{asympt})} \,|_{S_R^1} A[\mathbf{k}_q\,;\,.\,;\,\gamma_q]\rangle.$$

Pbm A1.2 (reconstruction from one set of Ipswich data)

Given $\{A[\mathbf{k}_q;\phi_j;\gamma_q], j \in N_q\}$ *at fixed* q*, find* $\vec{\psi}^\star \in \Psi_{\text{adm}}$ *and* $\gamma_q^\star$ *such that*

$$B_{DV}^{(L)}[\vec{\psi}^\star, \gamma_q^\star] =$$

$$= \frac{1}{2}\Big\| \sum_{\lambda \in \Lambda(L)} (\mathbf{M}_q^{(L)}[\vec{\varphi}_q^{(L)}[\gamma_q^\star]])_\lambda v_\lambda + E_{q,z}^{(\text{inc})} \Big\|^2_{L^2(\Gamma[\vec{\psi}^\star])} = \min$$

Features

1) ♠ The actual scattering <u>data</u> are $\vec{\varphi}_q^{(L)}[\gamma_q]$.
2) ♠ If $A[.\,;\,.\,;\,.]$ available, full aperture values required.
3) ♦ The <u>cost function</u> to be minimized is $B_{DV}^{(L)}[\vec{\psi}, \gamma_q]$ i.e., NO PENALTY TERM !
4) ♦ The <u>unknowns</u> are $\vec{\psi} \in \Psi_{\text{adm}}$ and $\gamma_q \in [0, 2\pi]$.
5) ♦ The far zone to boundary relation is represented by $\mathbf{M}^{(L)}[.]$.
6) ♦ $\forall$thing in $B_{DV}^{(L)}$, except $\vec{\varphi}_q^{(L)}[\gamma_q]$, depends on $\vec{\psi}$.
7) ♠ Nothing is known in general about the $\exists$ of $[R\mathbf{L}^{(L)}]^{-1}$.