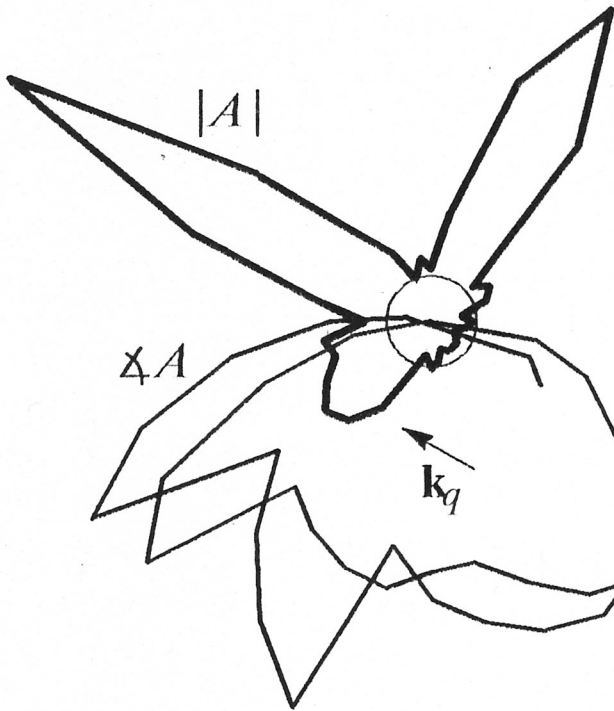


NUMERICAL RESULT FROM $B_{DV}^{(L)}$ MINIMIZATION



IPSWICH file *ips009/ips009FV.240*
 Full aperture data obtained via
 the reciprocity of $A[\mathbf{k}_q; \phi_j; \gamma_q]$.
 Ideally (no offset γ_q):

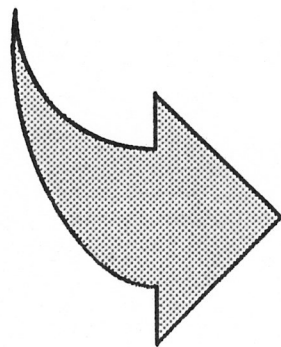
$$A[\{k_q; \phi_q\}; \phi_j] = A[\{k_q; -\phi_j\}; -\phi_q]$$

$$k_q = \frac{2\pi}{3} \text{ cm}^{-1}$$

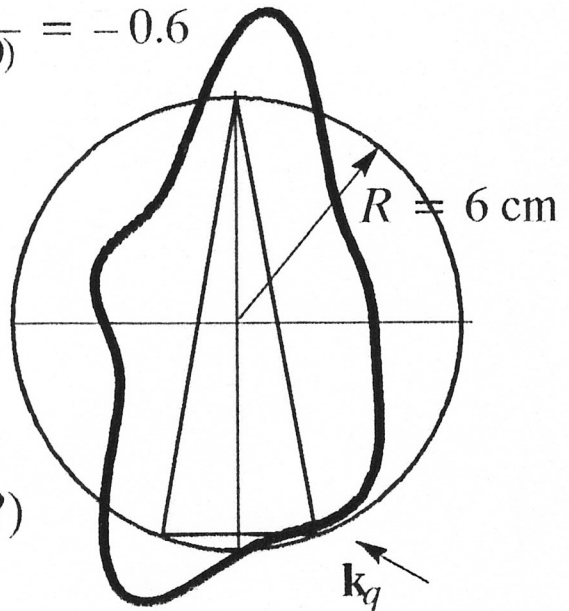
$$\phi_q = \frac{4}{3}\pi$$

$$L = 8, I = 10$$

$$\frac{\psi_4^{(0)}}{\psi_1^{(0)}} = -0.6$$



reference object :
 PEC prism.
 Height: indefinite
 Cross section: isosceles (?)
 triangle



m49248A-6 $r_0 = .453686D+01$ It. = 15;
 $B^{[8]} = .153205D+02$; $\|\nabla B\| = .968031D+03$
 $\gamma = .159121D+00$

THE HEURISTICS OF SHAPE RECONSTRUCTION

BY $F_{DV}^{(L)}$ MINIMIZATION

PROP A2.1 Given q , $E_{q,z}^{(inc)}$, $\vec{\psi}_{ref}$ and L , an approximate (far zone) solution to the exterior DIRICHLET BVP is represented by

$$E_{q,z}^{(sc,L)} = \sum_{\lambda \in \Lambda(L)} P_{q,\lambda}^{(L)}[\vec{\psi}_{ref}] v_{\lambda},$$

where $\{p_{q,\lambda}^{(L)}\} = \mathbf{p}_q^{(L)} = -RL^{(L)} \cdot [\mathbf{P}^{(L)}]^{-1} \cdot \mathbf{g}_q^{(L)} + \vec{\beta}_q^{(L)}$.

PBM A2.2 (reconstruction from card[Q] sets of IPSWICH data)

Given $\{A[\mathbf{k}_q; \phi_j; \gamma_q], j \in N_q, q \in Q\}$, find $\vec{\psi}^{\star} \in \Psi_{adm}$ and $\vec{\gamma}^{\star}$ such that

$$F_{DV}^{(L)}[\vec{\psi}^{\star}; \vec{\gamma}^{\star}] := \frac{1}{2} \sum_{q \in Q} \sum_{j \in N_q} \left| \sum_{\lambda \in \Lambda(L)} P_{q,\lambda}^{(L)}[\vec{\psi}^{\star}] v_{\lambda}[\phi_j] - A[\mathbf{k}_q; \phi_j; \vec{\gamma}^{\star}] \right|^2 = \min$$

REM. @ fixed q

$$\nabla_{\vec{\psi}} \mathbf{p}^{(L)} = -\nabla_{\vec{\psi}} RL^{(L)} \cdot (-\mathbf{c}^{(L)}) - RL^{(L)} \cdot (-\nabla_{\vec{\psi}} \mathbf{c}^{(L)}) + \nabla_{\vec{\psi}} \vec{\beta}^{(L)}$$

where

$$\mathbf{P}^{(L)} \cdot (-\nabla_{\vec{\psi}} \mathbf{c}^{(L)}) = \nabla_{\vec{\psi}} \mathbf{P}^{(L)} \cdot (-\mathbf{c}^{(L)}) - \nabla_{\vec{\psi}} \mathbf{g}^{(L)}.$$

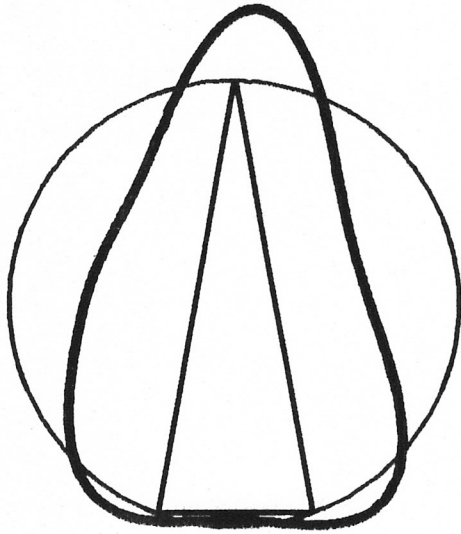
FEATURES

- 1) \blacklozenge The scattering data are $\{A[\mathbf{k}_q; \phi_j; \gamma_q], j \in N_q, q \in Q\}$.
- 2) \blacklozenge Superposition of limited aperture – multiple views allowed.
- 3) \blacklozenge The cost function to be minimized is $F_{DV}^{(L)}[\vec{\psi}, \vec{\gamma}]$.
- 4) \blacklozenge The unknowns are $\vec{\psi} \in \Psi_{adm}$ and $\vec{\gamma} \in ([0, 2\pi])^{\text{card}[Q]}$.
- 5) The DIRICHLET BC and the boundary to far zone relation are represented by AFP.
- 6) $\mathbf{P}^{(L)}$, $RL^{(L)}$ depend on $\vec{\psi}$ and on k_q (not on \mathbf{k}_q).
- 7) $\{\vec{\beta}_q^{(L)}\}$ and $\{\mathbf{g}_q^{(L)}\}$ depend on $\vec{\psi}$ and on \mathbf{k}_q .
- 8) \blacklozenge Only $\mathbf{P}^{(L)}$ has to be inverted; $[\mathbf{P}^{(L)}]^{-1} \exists$ unconditionally.

NUMERICAL RESULTS FROM $F_{DV}^{(L)}$ MINIMIZATION

IPSWICH files *ips009/ips009FV.**

Full aperture data obtained via reciprocity of $A[\mathbf{k}_q; \phi_j; \gamma_q]$

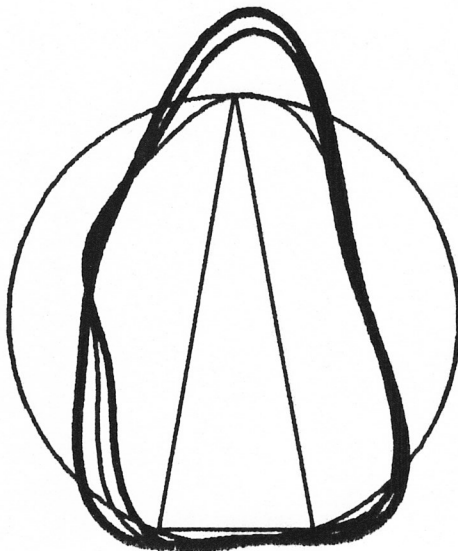


$$\{\mathbf{k}_q, 1 \leq q \leq 4\}$$

$$L = 12, I = 10$$

$$\frac{\psi_4^{(0)}}{\psi_1^{(0)}} = -0.4$$

MF9x4CA $r_0 = .495868D + 01$ It. = 15
 $B^{[12]} = .425680D + 03; \|\nabla B\| = .258242D + 05$

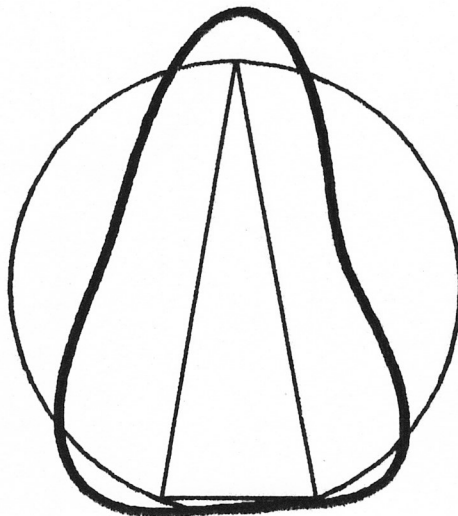


$$\{\mathbf{k}_q, 1 \leq q \leq 5\}$$

$$L = 12, I = 10$$

$$\frac{\psi_4^{(0)}}{\psi_1^{(0)}} = -0.4$$

MF9x5CA $r_0 = .495868D + 01$ It. = 15
 $B^{[12]} = .485822D + 03; \|\nabla B\| = .195682D + 05$
MF9x5CA $r_0 = .409808D + 01$ It. = 15
 $B^{[12]} = .494881D + 03; \|\nabla B\| = .177667D + 05$
MF9x5CA $r_0 = .450789D + 01$ It. = 15
 $B^{[12]} = .514020D + 03; \|\nabla B\| = .195457D + 05$



$$\{\mathbf{k}_q, 1 \leq q \leq 5\}$$

$$L = 14, I = 10$$

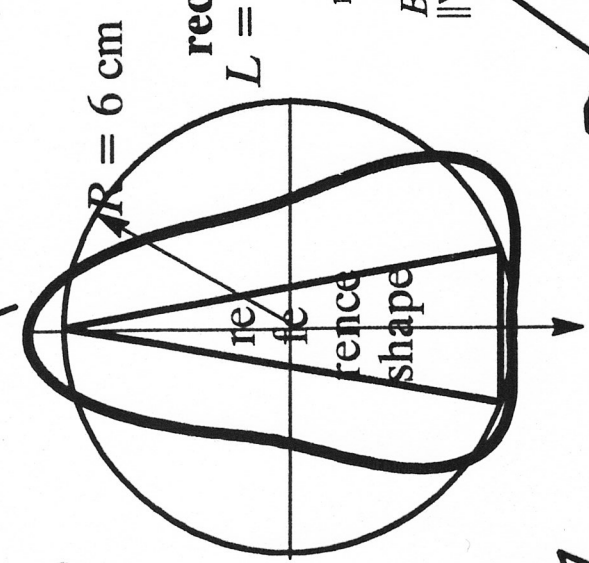
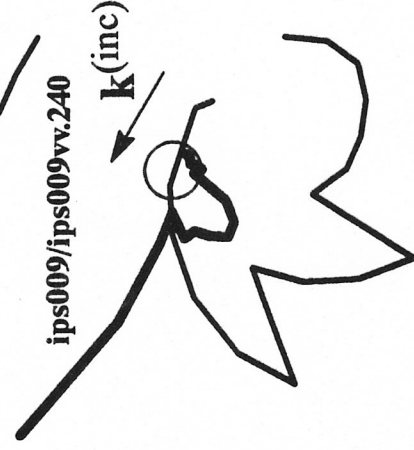
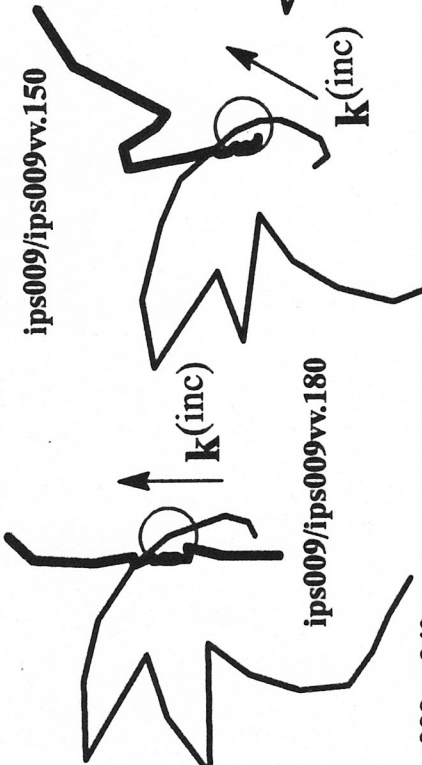
$$\frac{\psi_4^{(0)}}{\psi_1^{(0)}} = -0.4$$

MF9x5EA $r_0 = .495868D + 01$ It. = 15
 $B^{[14]} = .464231D + 03; \|\nabla B\| = .537194D + 05$

RECONSTRUCTION BY F_{DV} MINIMIZATION

AND LIMITED APERTURE IPSWICH DATA

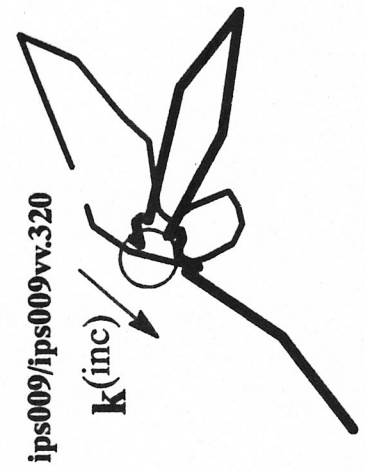
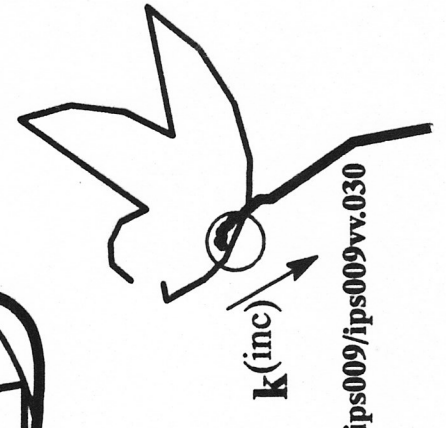
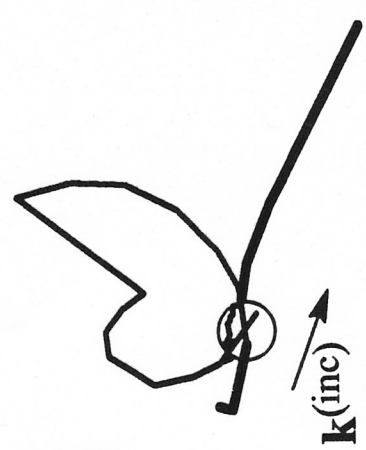
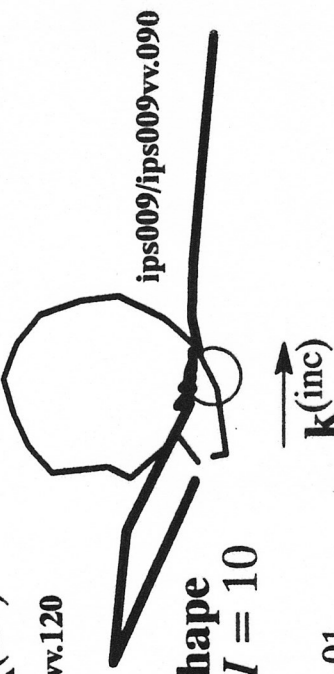
(π rad for each view)



ips009/ips009vv.120

reconstructed shape
 $L = 12; n_p = 63; I = 10$
Mr9x8CA

$r_0 = .409808D+01$
 $It. = 15$
 $B[12] = .449799D+03$
 $\|\nabla B\| = .172676D+05$



PART TWO

THE $X_1^{(L)}$ - LEAST SQUARES BOUNDARY COEFFICIENTS

THEM J1.1

I) $\forall \epsilon > 0$, \exists at least an approximation order $L[\epsilon]$ and a vector of coefficients $\mathbf{c}^{(L[\epsilon])}$ such that $B_{DV}^{(L[\epsilon])} < \epsilon^2$.

II) Select one such $L[\epsilon]$. Let $\varrho \gg r_{\max}$ and $C[\Gamma, \varrho, k]$ be a quantity, which depends on Γ , ϱ and k . Then the corresponding $\mathbf{c}^{(L[\epsilon])}$ is related to the far zone coefficients, $\{f_\lambda \mid \lambda \in \Lambda[L[\epsilon]]\}$ by the error bound

$$|f_\lambda - c_\lambda^{(L[\epsilon])}|^2 < \frac{k^2 \epsilon^2}{\pi} C[\Gamma, \varrho, k], \quad \forall \lambda \in \Lambda(L[\epsilon]).$$

REMARKS.

♠ In general, the determination of $\mathbf{c}^{(L)}$ is asymptotically ill posed, because $\mathbf{P}^{(L)}$ is asymptotically ill conditioned [RAMM, 1986].

♠ The dependence of $L[\epsilon]$ on ϵ has not yet been investigated.

DEF An obstacle is of RAYLEIGH class if $\sum_\lambda f_\lambda \nu_\lambda[\mathbf{x}]$ converges, $\forall \mathbf{x} \in \Gamma$.

THEM J1.2 [BARANTSEV *et al.*, 1971].

Ellipses, $r[\phi] = \frac{1}{\sqrt{1 - \epsilon^2 \cos^2 \phi}}$, of eccentricity $\epsilon < \frac{1}{\sqrt{2}}$ are RAYLEIGH obstacles and

$$f_\lambda \approx \frac{Cl^{-\gamma}}{\sqrt{2\pi l}} \left(\frac{ek}{2l} \right)^l \left(\frac{\epsilon}{\sqrt{1 - \epsilon^2}} \right)^l \text{ as } l \rightarrow \infty, \text{ where } \gamma > 0.$$

Proof : by asymptotics of $H_l^{(1)}$ and J_l , singular integral equations, and saddle point method.

... MORE ABOUT RAYLEIGH OBSTACLES ...

THEM J1.3 [MILLAR, 1973]. *On the boundary of a RAYLEIGH obstacle*

$$\lim_{L \rightarrow \infty} |f_\lambda - c_\lambda^{(L)}| = 0, \forall \lambda \in \Lambda(L).$$

DEF (ROUND obstacle). $\Omega \in \text{RAYLEIGHObstacleUniformNormalDerivative}$ class if $\sum_\lambda f_\lambda \partial_N v_\lambda[\mathbf{x}]$ converges uniformly to $\partial_N E_z^{(\text{sc})}|_\Gamma$, $\forall \mathbf{x} \in \Gamma$.

THEM J1.4. *Ellipses, such that $\varepsilon < \frac{1}{\sqrt{2}}$ are a class of ROUND obstacles.*

Proof: Denote $\xi := kr[\phi]$. Let $r[\phi] \geq 1$.

Since $\frac{dH_l^{(1)}(\xi)}{d\xi} f_\lambda \approx i \frac{Cl^{-\gamma}}{kr\pi} \left[\frac{\varepsilon}{r\sqrt{1-\varepsilon^2}} \right]^l$ as $l \rightarrow \infty$, then $\sum_\lambda f_\lambda \partial_N v_\lambda[\mathbf{x}]$ converges uniformly on Γ .

MOREOVER [KLEINMAN, ROACH, STRØM, 1984] the infinite dimensional matrix $\mathbf{L} := -\frac{i}{4} [\langle v_\lambda|_\Gamma, \partial_N v_\mu \rangle]$ is invertible and therefore

$$\sum_\lambda f_\lambda \partial_N v_\lambda|_\Gamma = \partial_N E_z^{(\text{sc})}|_\Gamma. \quad \blacksquare$$