Isometries, symmetry, teacher training and WIMS

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Isometries

Teaching geometry

Cycles in Italy

- Primaria: grade 1 (6 years) to grade 5.
- Secondaria di primo grado: grades 6, 7 and 8.
- Secondaria di secondo grado: grades 9 to 13.

Teacher training

- Primary: University teacher training degree “Scienze della formazione primaria” (5 years)
- Secondary: University degree (3 year + 2 year) and “Tirocinio formativo attivo” (1 year)

Why isometries?

The scope of geometry was spectacularly broadened by Klein in his Erlanger Programm *(Erlangen program) of 1872, which stressed the fact that, besides plane and solid Euclidean geometry, there are many other geometries equally worthy of attention.*

(H. S. M. Coxeter, *Introduction to geometry*, John Wiley & Sons Inc., second edition edition, 1969, p. ix)

Geometry or Geometries?

[. . .] Euclidean geometry is by no means the only possible geometry: other kinds are just as logical, almost as useful, and in some respect simpler. According to the famous Enlargen program *(Klein's inaugural address at the University of Erlangen in 1872), the criterion that distinguishes one geometry from another is the group of transformations under which the proposition remain true.*

(*ibid.*, p. 67) fet

Isometries and teacher training

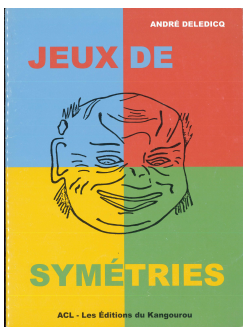
Prospective teachers

- I hate mathematics, I never understood mathematics, I do not want to have anything to do with mathematics
- I already know everything I need to know

In both cases we need to show them “something new” (possibly something likable).

Symmetry

Symétrie

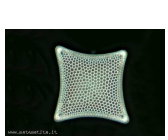
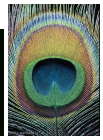


Beautiful images






Beautiful images


Beautiful images









Beautiful images







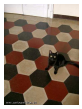
Beautiful images





Wallpaper patterns








Analogy

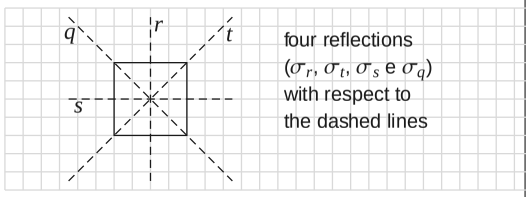
Difference

Deep mathematical concepts

- Groups
  - the tool to describe “symmetry” of a figure is its symmetry group i.e. the set of all isometries of the plane that leave the figure unchanged

Groups through images

Given a figure, you can find its symmetry group

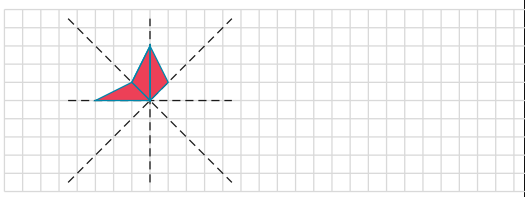


four reflections (σ_r, σ_t, σ_s e σ_q) with respect to the dashed lines

(Something is missing)

Groups and images

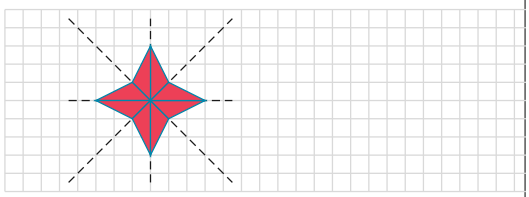
Given a symmetry group you can build images with that symmetry



The composition of two reflection with intersecting axes is a rotation

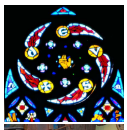
Groups and images

Given a symmetry group you can build images with that symmetry

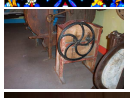


D4

Shaping an idea



Milano



Trento

Seoul

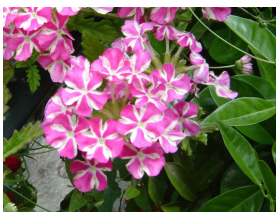
C5

Rosettes with flowers

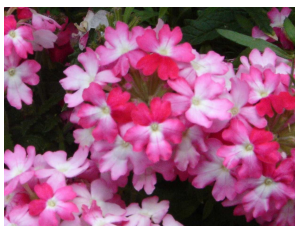
Falso gelsomino
Trachelospermum jasminoides
5. ($C5$)

Verbena
Verbena officinalis
$*5$. ($D5$)

Breaking symmetry



Breaking symmetry



Tools

matematita

Interuniversity Research Center for the Communication and Informal Learning of Mathematics

`http://www.matematita.it/`

- originates from the experience of promoting mathematics by four Italian universities: Milano, Milano-Bicocca, Pisa and Trento
- focus on informal learning as one of the main prerequisites to any subsequent more formal learning
- aims to identify the right form of contents and methods for this type of communication

matematita

The word "**mate**matita" resembles the word "**matematica**" which means "mathematics".

Also
"**mate**" = maths
"matita" = pencil

doing mathematics with the pencil

matematita: products&offers

`http://www.matematita.it/`

- training courses for pre-service and in-service teachers;
- problem-based mathematical laboratories in school (both in primary school and at a higher level);
- interactive exhibitions;
- web-based mathematical game contests;
- iconographic references on mathematical topics.

Publishing: books&CDrom

Il ritmo delle forme

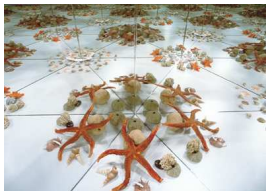
Publishing: books

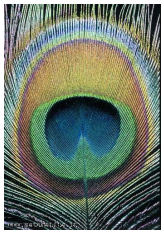
`http://www.quadernoaquadretti.it/`

A magazine for secondary school students

`http://www.xlatangente.it/`

Images for mathematics





Images for mathematics

http://www.matematita.it/materiale/

- Use images to communicate mathematical ideas;
- make **mate**matita's collection of images and animations available by creating an online website.

The website (~ 10 000 images, constantly evolving) is designed to be user-friendly while still ensuring a high level of scientific correctness alongside top quality relevant images. Each image has a presentation with a full (mathematical) description, possibly connecting with other images.

Interactive exhibitions

Symmetry, playing with mirrors

matemilano, mathematical explorations of the city

matetrentino, mathematical explorations of Trento and its surroundings

Transparent mathematics: minimal surfaces and soap bubbles

The exhibits

Isometries

Symmetry

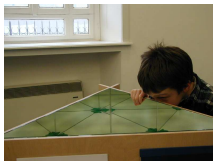
Tools
- matematita
- Putilishing
- Il ritmo delle forme
- Putilishing
- Images for mathematics
- Interactive
- Rosettes
- Wallpaper patterns
- Kaleido
- Simetria

WIMS

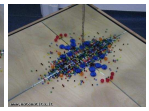
- interactive
  - "*interattività e libertà di sperimentazione della mostra (non certo le mostre cui sono abituata io in cui si deve solo guardare, qualche volta ascoltare, ma mai toccare!)*"[1]
- multilevel
  - propose problems that can be experienced at different levels, from different viewpoints

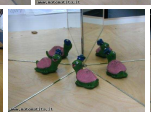
[1] "interactivity and freedom: not really the kind of exhibition I am familiar with, where you are only allowed to watch, sometimes to listen, but never to touch"

Interactive

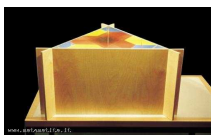


Rosettes





Wallpaper patterns



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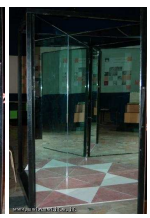
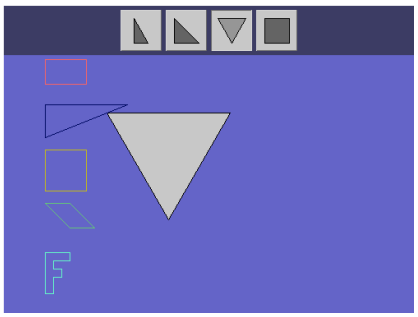
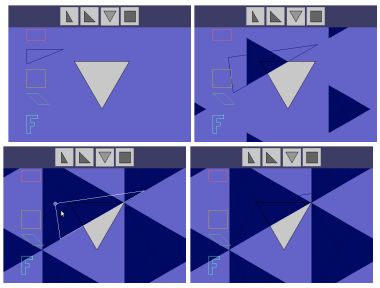


Kaleido



Kaleido



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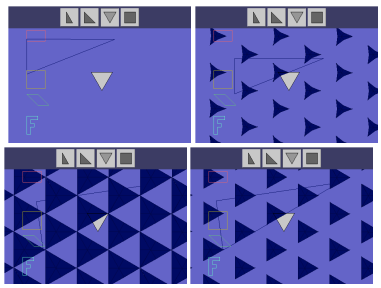

Simetria

WIMS

Issues

- what is an isometry? how do you deal with isometries?
- isometries and figures: apply an isometry to an image
  - converse problem
- symmetry
  - recognize symmetry
  - build symmetric figures