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Equilibrium Principal-Agent Contracts: Competition and R&D Incentives

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We analyze competition between firms engaged in R&D activities and market competition to study the choice of the incentive contracts for managers with hidden productivity. Oligopolistic screening requires extra effort/investment from the most productive managers: under additional assumptions on the hazard rate of the distribution of types we obtain no distortion in the middle rather than at the top. The equilibrium contracts are characterized by effort differentials between (any) two types always increasing with the number of firms, suggesting a positive relation between competition and high-powered incentives. An inverted U curve between competition and absolute investments can emerge for the most productive managers.

1. Introduction

A wide literature on contract theory has described how asymmetric information shapes the optimal contracts between a principal and an agent with private information. For instance, when the agent is the manager of a monopolistic firm and has private information on the productivity of effort, the optimal contract requires the first best effort for the most productive type and a downward distortion of effort for all the less productive types, with effort differentials associated with wage differentials in a way that insures incentive compatibility (see Stiglitz, 1977 or Baron and Myerson, 1982). However, when a firm is not a monopolist, but competes in the market with other firms, we can expect that the optimal contracts are affected by competition and the competitive prices are affected by the contracts implemented by all the firms. Although the analysis of equilibrium principal–agent contracts has been studied in models with perfect competition [see Rothschild and Stiglitz (1976), or more recently, Bisin and Gottardi (2006) and Pouyet et al. (2008)], there is limited work for the general case of imperfect competition. The notable exception of Martimort (1996) examines equilibrium contracts in a duopoly where the types of managers of the firms are perfectly correlated: this is a reasonable assumption in the presence of common aggregate shocks, but not in the presence of firm-specific shocks or asymmetric information on the productivity of the managers. We augment principal–agent models between firms competing in the market and their managers

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with idiosyncratic and uncorrelated shocks that give raise to equilibrium contracts that differ from those offered by a monopolistic firm. The aim is to investigate how entry in a market affects the equilibrium contracts and, in turn, how these contracts affect the endogenous market structure.

We consider firms that in a first stage choose contracts for their managers and in a second stage compete in the market. Each contract provides incentives to undertake R&D activities that reduce the marginal cost. At the time of competing *à la* Cournot in the last stage, the contracts or the cost-reducing activity become observable in our baseline model. The interesting interaction occurs at the initial contractual stage between the firms and their managers. The managers are *ex ante* homogenous but, after being matched with a firm, they differ in their productivities, which are identically and independently distributed. We consider as a benchmark case the one in which productivities can be observed by the firm's owner (but not by the rival), and then we consider the more realistic situation in which each productivity level is private information to each manager. The contracts are expressed in terms of effort/wage schedules. Our focus is on Bayesian Nash competition in contracts: these are chosen simultaneously, taking as given the contracts offered by the other firms.

The equilibrium principal–agent contracts require extra effort from the most productive managers compared to the equilibrium contracts emerging with symmetric information. This deviation from the traditional property of "no distortion at the top" emerges because of the strategic interactions between contracts for different types (due to strategic substitutability between efforts). In particular, the fact that all the competitors commit to distort downward the effort of their inefficient managers increases the marginal profitability of the effort of an efficient manager (likely to compete with inefficient ones), and vice versa. This enhances the polarization of the efforts required from different managers. Under additional assumptions on the hazard rate of the distribution of types (which must be positive enough) we actually obtain "no distortion in the middle": all the best (worst) types exert more (less) effort than under symmetric information.

Our main result on the relation between competition and incentives is the following: the effort differential between (any) two types of managers is always increasing in the number of firms, which may suggest a positive relation between competition and high-powered incentive schemes. Loosely speaking, competition enhances meritocracy: when the number of firms increases, each firm tends to differentiate more its contracts (i.e., spend relatively more on wages for the best managers), requiring a relatively higher effort from an efficient manager because this can lead to larger gains against less efficient rivals. Also the absolute effort levels can increase with the number of firms, but only for the most efficient managers: moreover, we show that in such a case an inverted U curve between competition and absolute investment in cost-reducing activities can emerge.

A wide industrial organization literature, started by Dasgupta and Stiglitz (1980), Tandon (1984), and Sutton (1991) and generalized in recent work by Vives (2008), has studied the impact of competition on deterministic cost-reduction activities, showing that an increase in the number of homogenous firms tends to reduce production, profits, and investment of each firm. Our model generalizes this framework with heterogeneity and uncertainty in firms' productivity and also asymmetric information between firms' owners and their managers engaged in the cost-reducing activities. Following the cited literature, we also consider the case of endogenous market structures, that emphasizes a positive relation between market size and relative (and possibly absolute) measures of effort. In particular, we find that an increase in the size of the market or a reduction in the fixed cost amplifies the effort differentials.

Only few works have examined principal–agent contracts in oligopoly. Martin (1993) has developed a first example of Cournot competition with asymmetric information between firms and managers on the cost-reduction technology, confirming the negative relation between entry and effort due to a scale effect on the profit and the absolute effort of each firm. The specification adopted led to constant cost targets for all firms, eliminating any strategic interaction between contracts (see also Bertoletti and Poletti, 1996).[1] Such a focus on the scale effect can be misleading, because the absolute investment/effort can be a bad measure of the strength of the incentive mechanisms under different market conditions. A more appropriate measure of this strength is the effort differential between managers of different types, which represents the divergence of efforts and compensations (relative to revenues or profits) between employees with different productivities: we focus mainly on this comparative measure and show that the effort differentials are always increasing in the level of competition. Part of the literature on contract theory (Ivaldi and Martimort, 1994; Stole, 1995; Martimort, 1996) has analyzed duopolies engaged in price discrimination, which generates a problem of common agency that is fundamentally different from our competitive interaction between two principal–agent hierarchies. Piccolo et al. (2008) have analyzed cost-plus contracts *à la* Laffont and Tirole (1986) in a Cournot duopoly with perfectly correlated shocks, which excludes any strategic interaction between contracts: the agency problem remains formally equivalent to that of a monopolist. Note that these works emphasize the relation between product substitutability and managerial incentives,[2] while we mainly focus on the relation between entry of firms in the market and incentives.

We should note that other works have identified instances of effort provision above the first best level for the most productive types in violation of the "no distortion at the top" convention, but from different sources such as externalities between multiple agents (Lockwood, 2000) or learning in dynamic environments with noisy signals (Jeitschko and Mirman, 2002). Cella and Etro (2010) analyze a two-types-two-firms environment where efforts can be either strategic substitutes or complements, competition can take different forms (patent races, competition in prices *à la* Hotelling, competition in quantities with complement goods and with investments in advertising), and allow for imperfect correlation across types. Finally, Piccolo (2011) has adopted a framework similar to ours to study the role of information sharing between competing vertical hierarchies.

Our results on the positive relation between number of firms and effort differentials may contribute to explain the weak but positive correlation between competition and incentive mechanisms found in many empirical studies, for instance in Hubbard and Palia (1995), Cuñat and Guadalupe (2005), and Bloom and Van Reenen (2007).[3] In particular, our results in case of endogenous entry of firms appear in line with the findings

1. On the relation between incentive contracts and competition see also Hermalin (1994), Schmidt (1997), and Etro (2011). Schmidt (1997) develops a model of moral hazard where a positive impact of competition on effort may emerge from a threat of liquidation associated with low effort; however, his model does not generate any feedback effect of the equilibrium contracts on competition. Here, we are interested in studying both how competition affects contracts and how contracts affect competition.

2. Piccolo et al. (2008) examine also the case of profit-target contracts, which leads again to equilibria with the traditional no distortion at the top and a downward distortion on the effort of the inefficient managers: the interesting aspect is that the effort of the inefficient managers may be nonmonotonic in the degree of product substitutability.

3. Note that our mechanism works under independent shocks but not in case of common shocks. Empirically, one could discriminate between the two by looking at cost shocks at the firm level and demand shocks at the sector level.

of Cuñat and Guadalupe (2005), who emphasize a positive impact of openness (which implies larger and more competitive markets) on the strength of the incentive mechanisms, especially for the top managers or, in general, for the highest paid managers. We also emphasize the possibility of an inverted U relation between the number of firms and the absolute investment in cost reductions, which is in line with the evidence on competition and innovation found by Aghion et al. (2005).

The paper is organized as follows. Section 2 analyzes the general model with multiple firms and a continuum of agents. Section 3 endogenizes the market structure. Section 4 extends the basic model in other directions. Section 5 concludes. All proofs are in the Appendix.

2. The Model

In this section we present a general model of Cournot competition with managers' types independently drawn from a distribution function known by everybody. Consider a market with inverse demand $p = a - X$, where $X = \sum_{j=1}^{n} x_j$ is the total quantity produced by n firms and a is a demand-size parameter. Production takes place at a constant marginal cost which can be reduced by the manager: for simplicity, we assume that effort e generates the marginal cost $c - \sqrt{e}$. Each firm hires a manager from a pool of (*ex ante* undistinguished) managers to reduce the costs in a first stage and maximize profits with the relevant market strategy (here the output level) in a second stage. In the first stage, the contracts are chosen independently by all firms, but in the second stage all the contracts and the cost-reducing activities become public knowledge and competition occurs under perfect information.[4]

In the second stage, the output strategy of each firm i is chosen to maximize profits:

$$\pi_i = \left(a - \sum_{j=1}^{n} x_j\right) x_i - (c - \sqrt{e_i})x_i - w_i, \tag{1}$$

where the investment (effort) of the manager e_i and the wage w_i have been already decided and the cost structure is known to everybody.

In the first stage, the contract between each firm and its manager can establish the size of the cost reduction, or equivalently the effort e, and the wage w.[5] A contract (e, w) determines the utility of the manager as:

$$u(w, e) = w - \theta e, \tag{2}$$

4. More formally, contract offers are made simultaneously and realized costs become observable (but not verifiable) by everybody at the end of the first stage. The complete timing of our game is as follows:

1.1. Nature draws the productivity of each manager;
1.2. All firms offer a menu of contracts (effort-wage schedules) to their managers;
1.3. Managers choose one of the contracts;
1.4. Efforts are undertaken and wages are paid;
1.5. Marginal costs become public.
2. Firms compete *à la* Cournot in the product market.

5. We assume that the managers choose quantities to maximize profits in the interest of the firms' owners because this implies zero cost for them—we introduce quantity commitments directly in the contracts in a later section. However, we maintain the assumption that contracts cannot be conditioned on the contracts of the other firms or on final profits or prices. We are thankful to Jacques Crémer for a discussion on the last point.

where θ represents the marginal cost of effort. Our focus will be on the case in which the productivities of the managers are different and uncorrelated, as realistic when there is heterogeneity between managers or when firms are hit by independent shocks. Suppose that each type θ is distributed on $[\theta_1, \theta_2] \subseteq \Re^+$ according to a cumulative distributive function $F(\theta)$ that is assumed twice differentiable, with density $f(\theta)$, and satisfying the monotone hazard rate property for which $h(\theta) \equiv F(\theta)/f(\theta)$ is increasing in θ.

First of all, note that the Cournot equilibrium of the last stage with n firms with observable efforts $e_1, e_2, \ldots, e_n$, implies the production of each firm:

$$x_i = \frac{a - c + n\sqrt{e_i} - \sum_{j\neq i} \sqrt{e_j}}{n+1}, \tag{3}$$

which generates profits $\pi_i = x_i^2 - w_i$.

As a benchmark case, let us consider symmetric information between each firm and its manager. This amounts to introduce uncertainty on the cost of the other firms in the oligopolistic competition. We can think of each firm i choosing a map of contracts $(e^i(\theta), w^i(\theta))$, with $(e, w) : [\theta_1, \theta_2] \to \Re_+^2$, that maximizes expected profits under a participation constraint for the manager, that is, $w^i(\theta) = \theta e^i(\theta)$. Therefore, the optimal contracts $(e^i(\theta), \theta e^i(\theta))$ maximize:

$$E(\pi_i) = E_\theta \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j\neq i} \sqrt{e^j(\theta_j)}}{n+1} \right)^2 - \theta e^i(\theta), \tag{4}$$

where the expectation $E_\theta(\cdot)$ of the gross profits is conditional on the type θ of the internal manager and it is taken over the types θ_j of all the managers of the other firms $j \neq i$. The maps of contracts of these firms $(e^j(\theta), w^j(\theta))$ are taken as given.[6] The first-order conditions with respect to $e^i(\theta)$ are:

$$E_\theta \left[\frac{n\left(a - c + n\sqrt{e^i(\theta)} - \sum_{j\neq i} \sqrt{e^j(\theta_j)}\right)}{(n+1)^2\sqrt{e^i(\theta)}} \right] = \theta.$$

Since the manager's type is known and the expectation is taken only over the types of the rivals, that are uncorrelated, this can be rearranged as:

$$\sqrt{e^i(\theta)} = \frac{n\left(a - c - \sum_{j\neq i} E\left[\sqrt{e^j(\theta_j)}\right]\right)}{(n+1)^2\theta - n^2}, \tag{5}$$

where the expectation is now an unconditional expectation over the others' efforts. We focus on parameter restrictions for which we have interior solutions for the effort levels, which requires the assumption:

$$\theta_1 > \frac{n^2}{(1+n)^2}. \tag{6}$$

6. One could specify in more detail the conjectures on the contracts chosen by the competitors for their managers. We avoid this because we will focus on symmetric equilibria. In general, this may lead one to neglect other equilibria with the associated conjectures. Nevertheless, we believe that the symmetric equilibria where all firms adopt the same effort and wage schedules are the most interesting.

Taking the expectation over θ on both sides, we have:

$$E\left[\sqrt{e^i(\theta)}\right] = \frac{a - c - \sum_{j \neq i} E\left[\sqrt{e^j(\theta_j)}\right]}{n} \cdot E\left[\frac{1}{\left(\frac{n+1}{n}\right)^2 \theta - 1}\right].$$

Solving out under the assumption of symmetry for which the firms adopt identical maps of contracts $(e^*(\theta), w^*(\theta))$, we obtain the average (square root of) effort:

$$E\left[\sqrt{e^*(\theta)}\right] = \frac{(a-c)\, S^*(n)}{n + (n-1)S^*(n)} \text{ with } S^*(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s)ds}{\left(\frac{n+1}{n}\right)^2 s - 1}, \tag{7}$$

where the function $S^*(n)$ is increasing in n. Substituting in (5), we finally have the equilibrium effort:

$$\sqrt{e^*(\theta)} = \frac{a - c}{\left[\left(\frac{n+1}{n}\right)^2 \theta - 1\right][n + (n-1)S^*(n)]}, \tag{8}$$

which is decreasing and convex in the manager's type and depending on the number of firms and on the entire distribution of θ. Note that for $n = 1$ we have the simple rule $\sqrt{e^*(\theta)} = (a - c)/(4\theta - 1)$.

2.1 Oligopolistic Screening

Let us assume now that the managers have private information on their types and contracts cannot be conditioned on them. After the contracts are signed, production choices are taken under perfect information as before. Our focus is on the initial stage in which contract choice takes place. Firm i chooses a map of contracts $(e^i(\theta), w^i(\theta))$ to solve a problem of maximization of the expected profits:

$$E(\pi_i) = \int_{\theta_1}^{\theta_2} \left[E_\theta \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} \sqrt{e^j(\theta_j)}}{n+1} \right)^2 - w^i(s) \right] f(\theta) d\theta, \tag{9}$$

under individual rationality and incentive compatibility constraints. The expectation operator $E_\theta(\cdot)$ is always taken over the types of the rivals, whose maps of contracts $(e^j(\theta_j), w^j(\theta_j))$ are taken as given.

As usual, the Revelation Principle (Dasgupta et al., 1979; Myerson, 1979) and the incentive compatibility constraint require that the effort schedule must be nonincreasing, $\partial e^i(\theta)/\partial\theta \leq 0$ and that truth-telling is always optimal, $\partial w^i(\theta)/\partial\theta = \theta\left(\partial e^i(\theta)/\partial\theta\right)$.[7] Solving the last differential equation for the wage schedule, and using the fact that the

7. Indeed any type θ must find it optimal to reveal its true type $\hat{\theta} = \theta$ to maximize the utility $w^i(\hat{\theta}) - \theta e^i(\hat{\theta})$, whose first-order condition implies exactly the truth-telling condition in the text.

individual rationality constraint must be binding on the least efficient type ($w^i(\theta_2) = \theta_2 e^i(\theta_2)$), we have the incentive compatibility constraint:

$$w^i(\theta) = \theta e^i(\theta) + \int_{\theta}^{\theta_2} e^i(s)ds. \tag{10}$$

Substituting this into (9) and integrating by parts, the optimal contract of firm i must solve the following problem:

$$\max_{e^i(\theta)} \int_{\theta_1}^{\theta_2} \left[E_\theta \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j\neq i} \sqrt{e^j(\theta_j)}}{n+1} \right)^2 - [\theta + h(\theta)]\, e^i(\theta) \right] df.$$

The first-order condition for pointwise maximization can be rearranged as:

$$\sqrt{e^i(\theta)} = \frac{n\left(a - c - \sum_{j\neq i} E\left[\sqrt{e^j(\theta_j)}\right]\right)}{(n+1)^2\,[\theta + h(\theta)] - n^2}. \tag{11}$$

Comparing this with its equivalent under symmetric information (5), one can verify that the optimal contract is chosen by each firm according to the usual principles. In particular, given the expectations on the contracts of the other firms $E[\sqrt{e^j(\theta_j)}]$, the effort choice is downward distorted for all types except the most efficient. Nevertheless, strategic interactions will induce new changes in the optimal contracts chosen in equilibrium.[8]

After imposing symmetry of the map of contracts (exploiting the absence of correlation across types), we can derive the average effort:

$$E\left[\sqrt{e(\theta)}\right] = \frac{(a-c)\,S(n)}{n + (n-1)S(n)} \quad \text{with} \quad S(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s)ds}{\left(\frac{n+1}{n}\right)^2 [s + h(s)] - 1}, \tag{12}$$

where $S(n) < S^*(n)$ is increasing in the number of firms. Substituting in (11), we can express the equilibrium effort function as follows:

$$\sqrt{e(\theta)} = \frac{a - c}{\left\{\left(\frac{n+1}{n}\right)^2 [\theta + h(\theta)] - 1\right\}[n + (n-1)S(n)]}. \tag{13}$$

This shows that, in general, the effort and the marginal cost do change with the type of managers and induce asymmetries in the market.

Our explicit characterization of the equilibrium contracts allows us to determine the equilibrium market structure, here summarized by the expected price:

$$E[p] = c + \frac{(a-c)\,[n - S(n)]}{(n+1)\,[n + (n-1)S(n)]}. \tag{14}$$

We can describe the properties of the equilibrium under asymmetric information as follows:

8. We are grateful to Patrick Rey for insightful discussions on this point.

Proposition 1: *Competition in contracts with multiple firms under asymmetric information is characterized by a map of contracts $(e(\theta), w(\theta))$ with efforts defined by (13) and decreasing in the type of manager, and with wages defined by $w(\theta) = \theta e(\theta) + \int_{\theta}^{\theta_2} e(s)ds$. The average effort is reduced and the expected price is increased by the presence of asymmetric information between firms and managers.*

The equilibrium effort (13) depends not only on the type of manager, but also on the entire distribution of types in a novel way through the $S(n)$ function. Of course, when $n = 1$ the optimal contract boils down to the traditional rule $\sqrt{e(\theta)} = (a - c)/[4(\theta + h(\theta)) - 1]$ with the "no distortion at the top" property: the most productive type exerts the first best effort independently from the distribution of θ. However, a deviation from this property emerges in equilibrium when there are more than one firm. In particular, since $h(\theta_1) = 0$ but $S(n) < S^*(n)$ we immediately derive from (13) and (8) that $e(\theta_1) > e^*(\theta_1)$: the most efficient managers are always required to exert more effort than in "first best" when they are competing in the market. This result is driven by commitment with substitutability between the strategies of the competing firms: cost reduction in the first stage for the most efficient manager is greater than with symmetric information because of the reduced investment from less efficient competing managers.[9]

The effort required from all the types different from the most efficient one must be lower, but the comparison with the "first best" contract is now more complex. Moreover, the number of firms affects in a substantial way the equilibrium contracts, creating a complex interdependence between these and the market structure. Multiple mechanisms are at work in influencing the impact of competition on the equilibrium contracts. First of all, we have a price channel: an increase in the number of firms strengthens competition, which tends to reduce the equilibrium price, which in turn leads to lower incentives to exert effort. Second, we have a profitability channel: an increase in the number of firms reduces the profits available to each firm, but it also increases the marginal profitability of effort, especially for more efficient types. The net impact of the last two effects on effort is positive and stronger for more efficient types—as can be seen from the term $[(n+1)/n]^2$ at the denominator of (13). Finally, we have an indirect strategic channel: an increase in the marginal profitability of effort for the other firms reduces the incentives to invest in cost reduction—as can be seen from the term $S(n)$ at the denominator of (13). The net impact of these channels is ambiguous in general, but we can expect that a positive impact of competition on effort could emerge only for the most efficient managers. The next section verifies this and the other properties of the equilibrium.

2.2 Competition and Incentives

The following proposition describes the impact of competition, measured by entry of firms, on the equilibrium contracts:

9. Other works have identified the optimality of effort provision above the first-best level for the most productive managers due to strategic reasons within a principal–agent hierarchy, not between competing hierarchies as here. In particular, an important work by Jeitschko and Mirman (2002) has analyzed a dynamic principal–agent problem with noisy signals, showing that the principal may require an initial extra effort (from a productive manager) to enhance the flow of information. In the presence of noisy signals, polarization of the contracts is aimed at extracting better information on the type of manager for the future contracts (as long as the mentioned mechanism prevails on the ratchet effect which requires increasing targets for the good managers). This signalling mechanism within a vertical relation is clearly distinct from our strategic mechanism in horizontal relations.

Proposition 2: *An increase in the number of firms: (1) increases the effort of all the most productive types $\theta \in [\theta_1, \tilde{\theta}(n))$, and decreases the effort of all types $\theta \in (\tilde{\theta}(n), \theta_2]$ for a cutoff $\tilde{\theta}(n) \in [\theta_1, \theta_2]$; and (2) always increases effort differentials: given any types $\theta_p < \theta_q$, the ratio between their equilibrium efforts $e(\theta_p)/e(\theta_q)$ is always finite and increasing in n.*

This proposition tells us that the absolute effort levels may increase when the number of firms goes up, but this can happen only for the most efficient types, whereas the effort levels of the least efficient types always decreases with n. This is exactly what can happen in a simple example with only two types (analyzed in detail by Cella and Etro, 2010), in which a shift from monopoly to duopoly can increase effort only for the most efficient types.[10] More generally, when a new firm enters the market, all the managers with productivity above a certain threshold (which depends on the number of firms) are required to exert more effort, and the others end up exerting less effort. Note that this is only a possibility result, in the sense that there may not be types more efficient than $\tilde{\theta}(n)$ or it may be that $\tilde{\theta}(n) < \theta_1$, in which case effort is decreasing in the number of firms for all types.[11]

In addition, the proposition has unambiguous implications for relative efforts, which provides a more accurate description of the relative spending of firms on different type of managers. It shows that effort differentials between more and less efficient managers increase always with competition. For instance, considering two types θ_p and θ_q with $\theta_p < \theta_q$, the equilibrium function (13) allows us to express the corresponding effort differential as:

$$\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{(n+1)^2\left[\theta_q + h(\theta_q)\right] - n^2}{(n+1)^2\left[\theta_p + h(\theta_p)\right] - n^2}, \tag{15}$$

which is always increasing in the number of firms. The intuition of this result is driven by commitment with substitutability between the strategies of the competing firms: when the number of firms increases, each firm tends to differentiate more its contracts, requiring a relatively higher effort from an efficient manager, because a commitment of this kind leads to larger gains against less efficient rivals. In other words, each firm spends a larger percentage of its revenues (or of its profits) in compensating efficient managers rather than inefficient ones. As long as we interpret this as the relevant measure of the power of the incentive schemes, this framework shows that more competition requires more high-powered incentive schemes. Our results may contribute to recover a theoretical motivation for the weak but positive correlation between entry and the strength of incentive mechanisms found in many empirical studies, for instance in Hubbard and Palia (1995) and Bloom and Van Reenen (2007). In particular, and contrary to the received literature, the positive relation between the number of competitors and the effort differentials across more and less productive managers can be seen as a rationale for more aggressive incentive mechanisms in case of stronger competition.

Another message of Proposition 2 is that the effort differentials reach a maximum level when we approach the perfectly competitive limit with infinite firms. In the case

10. This is in contrast with the negative relation between entry and cost-reducing activities which emerges in the models of Dasgupta and Stiglitz (1980), Tandon (1984), Vives (2008), and others, where there is no heterogeneity between firms.

11. As we will see below, for n large enough all effort levels must be decreasing in the number of firms.

above, the limit is:

$$\lim_{n\to\infty}\sqrt{\frac{e(\theta_p)}{e(\theta_q)}}=\frac{\theta_q+h(\theta_q)-1}{\theta_p+h(\theta_p)-1}.$$

Of course, this does not tell us much about the absolute effort levels when the number of firms is large and when it increases indefinitely. It turns out that the absolute efforts of all types must decrease when the number of firms is large enough, and they all approach the same limit when the number of firms tends to infinity. This is characterized in the following proposition together with the associated market structure:

Proposition 3: *When the number of firms tends to infinity, competition in contracts generates zero effort for all types of managers, the price equals the maximum marginal cost, and profits are zero.*

This result should not be surprising: the incentives to invest for price-taking firms depend on the size of their production (with or without asymmetric information), but in the limit of a Cournot model all firms produce an infinitesimal output and those incentives must vanish. At the competitive limit, there are neither informational rents nor extra effort for any type of managers, and hence asymmetric information has no bite.[12]

From the last propositions we can draw the following conclusions on the impact of competition (number of firms) on contracts. First and most important, more competition increases always the effort differentials between managers of different types. Second, more competition can increase the absolute level of effort for the most efficient types, but reduces it when the number of firms is large enough. Third, more competition reduces the absolute effort of the least efficient types. All this suggests that a nonmonotone effort function can arise for the most efficient types, and this can take an inverted U shape:[13] with effort maximized for an intermediate degree of competition.

The numerical example of Figure 1 helps to visualize this pattern under a uniform distribution. With such a distribution, $F(\theta)=(\theta-\theta_1)/(\theta_2-\theta_1)$ and $f(\theta)=(\theta_2-\theta_1)^{-1}$.[14] We assume $a-c=1$, $\theta_1=1$, and $\theta_2=10$ and plot the equilibrium effort of the most and least efficient types (respectively red vs. blue) for $n\in[1,20]$. One can verify that in correspondence of $n=1$ we have no distortion on the top and downward distortion at the bottom, the effort of the most productive manager is always above its equivalent under symmetric information, and the effort of the least productive is always below. Finally, an increase of the number of firms induces a reduction of the effort of the least

12. Puyet et al. (2008) already showed that under private values, equilibrium allocations in perfect competition are all efficient with and without asymmetric information, arguing that unobserved inefficiency in trade under asymetric information can only be attributed to imperfect competition. Our result reinforces their findings.

13. We cannot exclude different, but still nonmonotonic, shapes of the relation between effort and number of firms.

14. This implies:

$$S^*(n)=n^2\frac{\log\left[(n+1)^2\theta_2/n^2-1\right]-\log\left[(n+1)^2\theta_1/n^2-1\right]}{(n+1)^2(\theta_2-\theta_1)},$$

which, combined with (8), provides the equilibrium effort under symmetric information in solid lines. Analogously, we have:

$$S(n)=n^2\frac{\log\left[(n+1)^2(2\theta_2-\theta_1)/n^2-1\right]-\log\left[(n+1)^2\theta_1/n^2-1\right]}{2(n+1)^2(\theta_2-\theta_1)},$$

which, combined with (13) provides the equilibrium effort under asymmetric information reported in dashed lines.

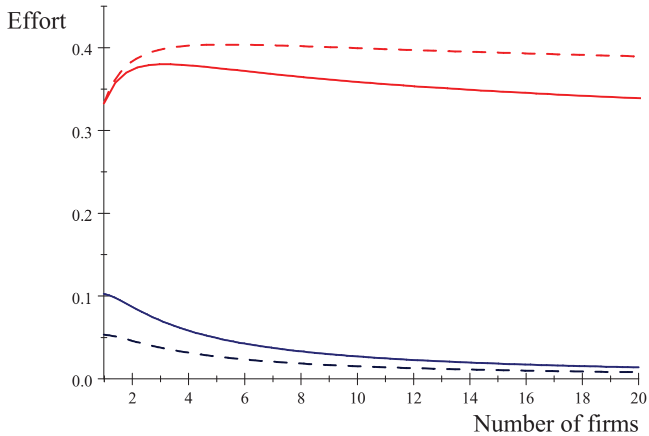

FIGURE 1. EFFORT UNDER (A)SYMMETRIC INFORMATION WITH SOLID (DASHED) LINES

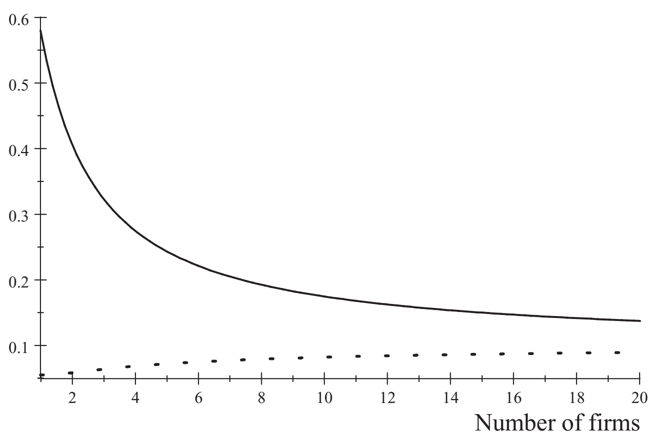

FIGURE 2. EXPECTED PRICE (SOLID) AND AVERAGE UNITARY COST (DOTS)

productive managers and an inverted U shape for the equilibrium effort of the most productive managers.

Therefore, the model can exhibit a bell-shaped relation between the number of firms and the investment of the most productive firms in cost-reducing activities, in line with the evidence on competition and innovation found by Aghion et al. (2005). Note that such an outcome cannot emerge in case of homogenous firms, as recently shown by Vives (2008) for a general class of models, but it emerges in the presence of heterogeneity between firms. For completeness, in Figure 2 we also report the equilibrium expected price and the average unitary cost (net of the cost reduction) for the case of asymmetric information (assuming $c = 0.1$). The latter increases because the average effort decreases with entry, but the former decreases with the number of firms because the direct impact of competition is stronger than the indirect impact on the cost-reducing activities.

2.3 Conditions for a Two-Way Distortion

The numerical example leads us to the general comparison of the equilibrium effort emerging with and without informational asymmetry. However, the relation between (13) and (8) is way more complex than in the case of a single principal–agent contract. In fact, although both functions are decreasing in the type of the manager, the behavior of their concavity is different in the presence of symmetric or asymmetric information. Although the function $\sqrt{e^*(\theta)}$ is always convex, the function $\sqrt{e(\theta)}$ is not necessarily so and its slope not necessarily larger in absolute value, meaning that multiple crossing between the two functions may arise (whereas in the case of monopolistic screening the two relations cross only once in correspondence of the most efficient type).

Nevertheless, introducing a stronger condition on the slope of the hazard rate we can avoid multiple crossings and obtain a simple result: the no distortion of the effort occurs for an intermediate type, with larger (lower) effort under asymmetric information for all types above (below) that intermediate type.[15] A comparison of (13) and (8) shows that the intermediate type $\hat{\theta}$ must satisfy the condition $e(\hat{\theta}) = e^*(\hat{\theta})$, which can be rewritten as:

$$\frac{\left(\frac{n+1}{n}\right)^2 h(\hat{\theta})}{\left(\frac{n+1}{n}\right)^2 \hat{\theta} - 1} = \frac{(n-1)\,[S^*(n) - S(n)]}{n + (n-1)S(n)}. \tag{16}$$

Single crossing of the two effort functions requires that $e(\theta) < e^*(\theta)$ for any $\theta > \hat{\theta}$. Developing this inequality from (13) and (8), we can rewrite the condition as follows:

$$\frac{\left(\frac{n+1}{n}\right)^2 h(\theta)}{\left(\frac{n+1}{n}\right)^2 \theta - 1} > \frac{(n-1)\,[S^*(n) - S(n)]}{n + (n-1)S(n)} = \frac{\left(\frac{n+1}{n}\right)^2 h(\hat{\theta})}{\left(\frac{n+1}{n}\right)^2 \hat{\theta} - 1},$$

where the last equality follows from (16). A sufficient condition for this is that the function on the left-hand side is always increasing in θ. Deriving the left-hand side with respect to θ, one can verify that this is equivalent to an additional assumption on the slope of the hazard rate:

$$h'(\theta) > \frac{\left(\frac{n+1}{n}\right)^2 h(\theta)}{\left(\frac{n+1}{n}\right)^2 \theta - 1}. \tag{17}$$

As long as the hazard rate is not only increasing, but increasing enough to satisfy (17), we can be sure that asymmetric information increases the effort of all the best managers and reduces the effort of all the worst managers with no distortion only for the intermediate type $\hat{\theta}$. Although the above condition may appear rather demanding, we can easily show that it is always satisfied by the uniform distribution. In such a case

15. A similar two-way distortion emerges in the model with one principal and many agents by Lockwood (2000) because of production externalities (the average effort increases the effectiveness of individual effort).

$h(\theta) = \theta - \theta_1$, therefore (17) reads as:

$$1 > \frac{\left(\dfrac{n+1}{n}\right)^2 (\theta - \theta_1)}{\left(\dfrac{n+1}{n}\right)^2 \theta - 1},$$

or $(n+1)^2 \theta_1 / n^2 - 1 > 0$, which is always satisfied under our assumption (6).

Of course, condition (17) is sufficient but not necessary for a "two-way distortion." The next lemma derives the necessary and sufficient condition for such a "well-behaved" case:

Lemma 1: *Define the positive roots of (16) as $\hat{\theta}_1 < \hat{\theta}_2 < \cdots < \hat{\theta}_z$. Asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$ and decreases the effort of all types $\theta \in (\hat{\theta}_1, \min(\hat{\theta}_2, \theta_2))$. Under the additional assumption:*

$$h'(\hat{\theta}) > \frac{(n-1)\,[S^*(n) - S(n)]}{n + (n-1) S(n)} \text{ for any } \hat{\theta}, \tag{18}$$

we have $z = 1$ (a single root).

The mathematical intuition for this lemma is simple. If the hazard rate is increasing fast enough, the equilibrium effort function under asymmetric information decreases significantly with the type so as to cross only once the equilibrium effort function under symmetric information. Note that the condition corresponds to the simple monotone hazard rate property ($h'(\theta) > 0$) in the baseline case of $n = 1$, but is more demanding with multiple firms. As a consequence, we have:

Proposition 4: *Under competition in contracts between nfirms, if and only if (18) holds, asymmetric information increases (decreases) the effort of all the most (least) efficient types compared to the equilibrium with symmetric information, without distortion only for an intermediate type $\hat{\theta} \in (\theta_1, \theta_2)$.*

In other words, competition in contracts with asymmetric information delivers "no distortion in the middle" and amplifies the differences between the efforts exerted by managers of different productivities. In Figure 3 we exemplify this result for the case of a uniform distribution with the same parameterization as above ($a - c = 1$, $\theta_1 = 1$, and $\theta_2 = 10$) and $n = 10$. The solid line represents the equilibrium effort under symmetric information, and the dashed line the one under asymmetric information, shown only in the interval of the most efficient types with $\theta \in (1, 1.3)$. The most efficient type $\theta_1 = 1$ exerts effort $\sqrt{e^*(1)} \simeq 0.3$ in the former case and $\sqrt{e(1)} \simeq 0.41$ in the latter, while the least efficient type $\theta_2 = 10$ exerts effort $\sqrt{e^*(10)} = 0.009$ under symmetric information and $\sqrt{e(10)} = 0.006$ under asymmetric information. We have no distortion of effort only for an intermediate type $\hat{\theta} \simeq 1.1$.

2.4 Endogenous Market Structures

In this section, we extend the model in a couple of directions aimed at providing a more realistic description of market competition. The first extension is to take into account imperfect observability of the contracts at the competition stage. Maintaining this assumption, we then endogenize the number of firms competing in the market.

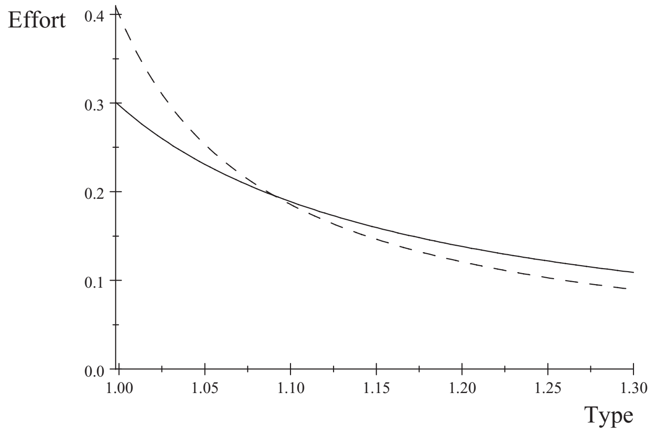

FIGURE 3. THE FUNCTIONS $\sqrt{e^*(\theta)}$ (SOLID LINE) AND $\sqrt{e\theta}$ (DASHED LINE)

One may think that the assumption of perfect information on contracts and costs affects substantially the contracts signed at the initial stage. However, in our model this is only a simplifying assumption without consequences on the nature of the equilibrium contracts. To verify this aspect, let us consider the case of unobservable contracts and costs at the time of competing in the market. In the last stage, all firms choose their strategies taking as given the expected strategies of the other firms (rather than the actual strategies), and, in the contractual stage, they choose the effort/wage schedules to maximize their net profits in function of the expected equilibrium strategies. Under our specification, we can show the following equivalence result:

Proposition 5: *The equilibrium contracts with symmetric and asymmetric information do not depend on whether these contracts are observable or not at the time of the competition in the market.*

Such an equivalence is due to the quadratic form of profits in our model. This implies that the marginal profits are a linear function of the expected efforts and equal to the expected marginal profits: as a consequence, firms adopt the same effort function on the basis of the expectation either of their equilibrium strategies (when contracts are observable) or of their average expected strategies (when contracts are not observable).[16]

Until now we have considered exogenous market structures in which the number of competitors was given. A more realistic situation emerges when entry requires a preliminary fixed investment and the number of firms is endogenized through an endogenous entry condition in the presence of a small entry cost. We now develop this analysis, which provides a generalization of the endogenous market structure approach of Dasgupta and Stiglitz (1980), Tandon (1984), Sutton (1991), and Vives (2008)[17] to the case of heterogeneity between firms and asymmetric information between their owners and their managers engaged in (preliminary) cost-reducing activities. Our earlier analysis with an exogenous market structure generated a positive correlation between

16. Of course, the ultimate equilibrium production levels differ depending on whether the contracts are observable or not, but the initial contracts, the effort exerted by managers of different types in equilibrium and even the expected production levels are not affected by contract observability, at least under our functional form assumptions.

17. See Etro (2009) for a survey of the theory of endogenous market structures with applications.

the number of firms and the relative effort levels, but it also suggested the possibility of a negative correlation with the absolute effort levels, at least for low-productivity managers and/or when the number of firms is high. As we will see, the first implication holds true when we endogenize the market structure, but the second implication can be questioned.

Let us focus on the last case examined, in which contracts are nonobservable.[18] In the initial stage entry occurs if nonnegative profits are expected, and, after that, the contractual and competitive stages are the same as above. Under asymmetric information, for a given n, the expected profits of each firm are given by:

$$E\left[\pi_i(n)\right] = \int_{\theta_1}^{\theta_2} \left[\left(\frac{a - c + n\sqrt{e^i(\theta)} - E_\theta\left[\sum_{j\neq i}\sqrt{e^j(\theta_j)}\right]}{n+1} \right)^2 - w^i(\theta) \right] f(\theta) d\theta.$$

Substituting the equilibrium efforts and wages we obtain:

$$E\left[\pi(n)\right] = \frac{(a-c)^2 Z(n)}{\left[n + (n-1)S(n)\right]^2} \quad \text{with} \quad Z(n) \equiv \int_{\theta_1}^{\theta_2} \frac{[s + h(s)]\, f(s) ds}{\left(\frac{n+1}{n}\right)^2 (s + h(s)) - 1}. \tag{19}$$

This implies that, given a fixed cost of entry K, the endogenous number of firms $\hat{n}$ satisfies $E\left[\pi(\hat{n})\right] = K$ and must be increasing in the relative size of the market $(a-c)/\sqrt{K}$. The endogenous entry condition allows us to rewrite the equilibrium effort (13) as:

$$\sqrt{e(\theta)} = \frac{\sqrt{\frac{K}{Z(\hat{n})}}}{\left(\frac{\hat{n}+1}{\hat{n}}\right)^2 [\theta + h(\theta)] - 1}, \tag{20}$$

with average effort:

$$E\left[\sqrt{e(\theta)}\right] = S(\hat{n})\sqrt{\frac{K}{Z(\hat{n})}}.$$

As usual, the absolute effort of all types tends to zero when the fixed cost tends to zero.

An increase of demand measured by a (for instance associated with the process of opening up to trade) or a reduction in fixed costs K are associated with an increase in the number of players and, again, with an ambiguous impact on the absolute effort: on one side this tends to increase because of the direct size effect, but on the other side it may tend to decrease for the indirect impact due to the larger number of firms.[19] In any case, the size effect implies that the set of (most productive) types for which the absolute effort increases is enlarged. Moreover, we can immediately derive an unambiguous conclusion on the effort differentials:

18. Similar qualitative results emerge when the contracts are observable, but the analysis is more complex.

19. Note that Tandon (1984) and Vives (2008) show that, in the absence of heterogeneity between firms (and of asymmetric information), the size effect always dominates and effort increases with an increase in the size of demand. However, their models of endogenous market structures differ from ours because they assume simultaneous investment and production choices, while we assume sequential choices.

Proposition 6: *When the number of firms competing in contracts is endogenous, an increase in demand or a reduction in the fixed cost amplifies the effort differentials.*

Therefore, our model is consistent with a positive relation between incentives and number of competitors both in relative terms (effort differentials) and in absolute terms for the most productive managers. These results appear in line with the findings of Cuñat and Guadalupe (2005), who emphasize a positive impact of openness (which implies larger and more competitive markets) on the strength of the incentive mechanisms, especially for the top managers or, in general, for the highest paid managers.

Our analysis opens space for further investigations on the role of asymmetric information in markets with endogenous structures. Etro (2011) has analyzed optimal unilateral screening contracts in markets with endogenous entry, showing that a firm would always gain from committing to aggressive incentive contracts (with extra effort required from all types) to limit entry and gain market shares over the competitors. Although that model, following the analysis of leadership with endogenous market structures (as in Etro, 2006 or Kováč et al., 2010), neglected contract competition, a similar outcome is likely to emerge also in the present context. Finally, one could investigate welfare analysis in our more complex environment.[20]

3. Extensions

Our general analysis can be employed to address a number of related applications. Here, we focus on issues concerning product differentiation and different contractual arrangements.

3.1 Product Differentiation

We can easily generalize the model to the case of product differentiation with imperfect substitutability between goods. Assume an inverse demand function for firm i given by $p_i = a - x_i + b\sum_{j\neq i} x_j$, where $b \in [0, 1]$ parameterizes substitutability. When $b = 1$ we are in the case of homogenous goods, when $b = 0$ we have independent monopolies producing nonsubstitutable goods.

The Cournot equilibrium with n firms with efforts $e_1, e_2, \ldots, e_n$, implies the production of each firm:

$$x_i = \frac{(2-b)(a-c) + [2 + b(n-2)]\sqrt{e_i} - b\sum_{j\neq i}\sqrt{e_j}}{(2-b)[2+b(n-1)]}.$$

Our methodology allows us to derive the equilibrium map of contracts under asymmetric information with the following effort:

$$\sqrt{e(\theta)} = \frac{(2-b)(a-c)}{\{A(n)[\theta + h(\theta)] - 1\}\{[n + (n-1)S(n)]b + 2(1-b)\}}, \tag{21}$$

20. We should also mention an important work by Creane and Jeitschko (2009) that studies the role of adverse selection for markets characterized by perfect or Cournot competition. It shows that the traditional market failure due to informational asymmetries tends to vanish under endogenous entry and to be replaced by limited entry with above-normal profits for the entrants.

where

$$S(n) = \int_{\theta_1}^{\theta_2} \frac{f(s)ds}{A(n)[s + h(s)] - 1} \quad \text{and} \quad A(n) \equiv \frac{(2-b)^2[2+b(n-1)]^2}{[2+b(n-2)]^2}.$$

This equilibrium can be compared with the one emerging under symmetric information, which allows us to obtain:

Proposition 7: *As long as firms produce imperfect substitutable goods, equilibrium contracts with asymmetric information are characterized by extra effort for the most efficient managers compared to the equilibrium contracts with symmetric information.*

Indeed, one can verify that independent goods ($b = 0$) lead to equilibrium contracts that are identical to the optimal monopolistic contracts, with no distortion at the top, but imperfect substitutability ($0 < b < 1$) leads to the same two-way distortion of the case with homogenous goods, with extra effort for the most efficient types. Similar results can be derived in case of price competition (see Cella and Etro, 2010), because the efforts of the firms are still strategic substitutes.[21]

Finally, note that, given two types $\theta_p > \theta_q$, the equilibrium effort differential generalizes to:

$$\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{A(n)\left[\theta_q + h(\theta_q)\right] - 1}{A(n)\left[\theta_p + h(\theta_p)\right] - 1}, \tag{22}$$

which is always increasing in the number of firms (except for $b = 0$) and also in the substitutability parameter b: an increase in competition associated with more firms or with higher substitutability between products is going to increase the effort differentials. Therefore, our qualitative results on the relation between competition and both absolute and relative efforts persist. Endogenizing the market structure we can derive again a positive correlation between number of firms and effort differentials and, possibly, also between number of firms and the absolute effort (at least for the most productive managers).

3.2 Effort-Output Contracts

A precommitment on cost reduction before competing in the market generates a typical tendency to overinvest in cost reductions to be more aggressive in the choice of output. Although this effect was present in our model, it is important to remark that it did not drive our qualitative results. To verify this, we now consider an alternative framework in which principals can write contracts specifying both effort and market strategies for their agents. Such an enlargement of the contractual options in the first stage allows firms to reduce their initial investment, but has the cost of giving up to discretionary strategies in the competition stage.[22] As we will show, the availability of such contracts

21. However, this is not the case anymore when goods are complements, as with b negative here or in few other cases in which efforts are strategic complements (for instance, when effort is exerted in general advertising for the market). In these cases equilibrium contracts distort downward all the effort levels compared to the symmetric information benchmark (see Cella and Etro, 2010).

22. Note that, even in this general specification of the contracts, we are still assuming some form of contractual incompleteness. The reason is that the contracts of the other firms are observable but not verifiable, therefore firms cannot write contracts contingent on the *ex post* realization of the contracts of the other firms (neither on profits or prices). If such an unrealistic form of complete contingent contracts was allowed, we would obtain more complex results. In the two-types-two-firm case we would have "no distortion at the top"

does not change our main results: the two-way distortion and the positive link between competition and effort differentials.

Reconsider the model with homogenous goods. Under symmetric information, the expected profits are:

$$E(\pi_i) = E_\theta \left(a - c + \sqrt{e^i(\theta)} - x^i(\theta) - \sum\nolimits_{j \neq i} x^j(\theta_j)\right) x^i(\theta) - w^i(\theta),$$

where the expectation is again conditional on the type θ of the internal manager and it is taken over the types θ_j of all the managers of the other firms $j \neq i$. The optimal mechanism for firm i is now given by a map of contracts $(e^i(\theta), x^i(\theta), w^i(\theta))$ with $(e, x, w) : [\theta_1, \theta_2] \to \Re^3_+$, which maximizes expected profits under a participation constraint $w^i(\theta) = \theta e^i(\theta)$. The optimality conditions are:

$$\sqrt{e^i(\theta)} = \frac{x^i(\theta)}{2\theta}, \tag{23}$$

$$x^i(\theta) = \frac{a - c + \sqrt{e^i(\theta)} - E\left[\sum\nolimits_{j \neq j} x^j(\theta_j)\right]}{2}. \tag{24}$$

From the second condition we can derive the expected value of total output and, rearrange the output of firm i with a manager of type θ as:

$$x^i(\theta) = \frac{a - c + n\sqrt{e^i(\theta)} - \sum\nolimits_{j \neq i} E\left[\sqrt{e^j(\theta_j)}\right]}{n+1}. \tag{25}$$

Substituting this in (23), and isolating $\sqrt{e^i(\theta)}$, we have:

$$\sqrt{e^i(\theta)} = \frac{a - c - \sum\nolimits_{j \neq i} E\left[\sqrt{e^j(\theta_j)}\right]}{n\left[\dfrac{2(n+1)}{n}\theta - 1\right]}, \tag{26}$$

whose expectation can be solved for the symmetric expected effort:

$$E\left[\sqrt{\tilde{e}(\theta)}\right] = \frac{(a-c)\,\tilde{S}^*(n)}{n + (n-1)\tilde{S}^*(n)} \quad \text{with } \tilde{S}^*(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s)ds}{\dfrac{2(n+1)}{n}s - 1}, \tag{27}$$

where $\tilde{S}^*(n)$ can be shown to be smaller than $S^*(n)$. This implies that the average effort is reduced when the firms can contractually commit to their market strategies. Of course, the lower effort levels tend to reduce production and increase profits. The intuition for these results relies on the fact that, under basic contract competition, firms tended to invest too much to commit to a higher production in the market: since the managers decided how much to produce without taking in consideration the impact on the rivals, this led to excessive investment *ex ante* and excessive production *ex post* from the point

for an efficient manager who happens to meet an efficient rival, an upward distortion of effort for an efficient manager who happens to meet an inefficient rival, and standard downward distortions of the effort for an inefficient manager. Details are available from the authors.

of view of the firms. Contrary to this, the effort-output contracts allow firms to soften competition.

The final equilibrium effort can be derived substituting (27) in (26):

$$\sqrt{\tilde{e}^*(\theta)} = \frac{a - c}{\left[\frac{2(n+1)}{n}\theta - 1\right]\left[n + (n-1)\tilde{S}^*(n)\right]}. \tag{28}$$

The introduction of asymmetric information determines the same qualitative results of our basic model. The first-order conditions for the optimal contract of firm i under the individual rationality and incentive compatibility contracts are:

$$\sqrt{e^i(\theta)} = \frac{x^i(\theta)}{2\left[\theta + h(\theta)\right]}, \tag{29}$$

and (25). The usual analysis allows us to derive the equilibrium effort as:

$$\sqrt{\tilde{e}(\theta)} = \frac{a - c}{\left[\frac{2(n+1)}{n}(\theta + h(\theta)) - 1\right]\left[n + (n-1)\tilde{S}(n)\right]}, \tag{30}$$

where we defined:

$$\tilde{S}(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s)}{\frac{2(n+1)}{n}(s + h(s)) - 1} ds < \tilde{S}^*(n).$$

A comparison of (28) and (30) leads to conclude with:

Proposition 8: *Under contractual commitment on the production schedules, asymmetric information induces extra effort for the most efficient managers compared to the equilibrium with symmetric information and reduces the average effort compared to the case without such a contractual commitment.*

All the qualitative results of the previous sections hold also in this case with immediate adaptations. In particular, given any two types $\theta_p > \theta_q$, the effort differential:

$$\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{2(n+1)\left[\theta_q + h(\theta_q)\right] - n}{2(n+1)\left[\theta_p + h(\theta_p)\right] - n}, \tag{31}$$

is still increasing in the number of firms. Moreover, one can verify that the effort differential for a given number of firms is higher compared to the baseline model. Also the extension to product differentiation confirms the same results as above. Finally, since effort-output contracts soften competition, the case of endogenous market structures may lead to a larger number of firms compared to the baseline case. Nevertheless, it is important to remind that welfare comparisons are not straightforward in this context because equilibria are typically characterized by a suboptimal number of firms engaged in suboptimal investments and production activities.

4. Conclusions

In this paper we have analyzed competition between firms in the choice of incentive contracts for their managers. We have shown that in an equilibrium with perfect information for each firm on its manager's productivity, contract competition tends to increase the effort of the efficient managers and to decrease that of the inefficient managers. With asymmetric information, the equilibrium screening contracts are typically characterized by no distortion in the middle, with efficient managers providing extra effort and with an additional downward distortion on the effort of the inefficient managers. In both cases, the relative effort required from an efficient type increases when the number of firms increases, but all absolute effort levels converge to zero when approaching the perfectly competitive limit. We also considered the case of endogenous market structures: in general, a larger market increases the effort differentials and tends to increase also the absolute investments of the most efficient firms. This implies that a positive correlation between the number of firms (as a proxy of competition) and the strength of the incentive schemes is perfectly consistent with standard principal-agent theory.

As shown in Cella and Etro (2010), our analysis applies to other related models, including those based on price competition and spatial competition. Future research could investigate other applications of this form of contract competition and the impact of exogenous shocks, for instance a mean-preserving spread on the distribution of the types, a change in the number of consumers of the market or other structural shocks. Another direction for future research may focus on type-dependent participation constraints,[23] a case in which the "no distortion at the top" property can fail also in a single-firm environment, and on competition between managers and between firms to hire the best managers. Finally, it would be important to verify the positive predictions of the model on the empirical side.

Appendix

Proof of Proposition 1. Deriving (13) with respect to θ and using the monotone hazard rate property it follows that effort decreases in the type for any n, which confirms that the neglected constraint $\partial e^i(\theta)/\partial\theta \leq 0$ was nonbinding. Since $S(n) \leq S^*(n)$, a comparison of the weighted average efforts in (7) and (12) shows that $E[\sqrt{e(\theta)}] < E[\sqrt{e^*(\theta)}]$. The expected total production is $E[X] = \{a - c + E[\sqrt{e(\theta)}]\}n/(n+1)$, therefore, the price is given by:

$$E\,[p] = \frac{a + nc - nE\left[\sqrt{e(\theta)}\right]}{n+1},$$

which provides the expression in the text after substituting for the average effort. Since the latter decreases with asymmetric information, the expected price must increase. □

Proof of Proposition 2. Deriving (13) with respect to n we obtain:

$$\frac{\partial\sqrt{e(\theta)}}{\partial n} \propto 1 + S(n) + (n-1)S'(n) - [\theta + h(\theta)]\left(1 + \frac{1}{n}\right)\cdot$$
$$\cdot\left[\frac{n-1}{n}(1 + S(n)) + \frac{2S(n)}{n^2} + \frac{n^2-1}{n}S'(n)\right],$$

23. See, for instance, Laffont and Martimort (2002, chapter 3.3), Maggi and Rodriguez-Clare (1995), and Biglaiser and Mezzetti (2000).

which is decreasing in θ and positive only for values of θ close enough to its lower limit $n^2/(1+n)^2$. Therefore, if the effort of some types increases with the number of firms, it must be for any type $\theta \in [\theta_1, \tilde{\theta}(n))$ where the cutoff $\tilde{\theta}(n)$ is such that $\partial\sqrt{e(\tilde{\theta})}/\partial n = 0$. Finally, from the equilibrium efforts for types $\theta_p < \theta_q$ one can derive:

$$\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{(n+1)^2\left[\theta_q + h\left(\theta_q\right)\right] - n^2}{(n+1)^2\left[\theta_p + h\left(\theta_p\right)\right] - n^2},$$

which is greater than one (because the virtual type is increasing in θ under the monotone hazard rate property) and it is increasing in n since:

$$\frac{\partial\sqrt{e(\theta_p)/e(\theta_q)}}{\partial n} = \frac{(n+1)\left[\theta_q + h(\theta_q) - \theta_p - h(\theta_p)\right]}{n^3\left[(n+1)^2\left[\theta_p + h(\theta_p)\right] - n^2\right]^2} > 0.$$

Its upper limit is:

$$\lim_{n\to\infty}\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{\theta_q + h(\theta_q) - 1}{\theta_p + h(\theta_p) - 1},$$

which is always finite. To verify this, note that the ratio between maximum and minimum effort tends to:

$$\sqrt{e(\theta_1)/e(\theta_2)} = \frac{\theta_2 + 1/f(\theta_2) - 1}{\theta_1 - 1},$$

since $F(\theta_1) = 1 - F(\theta_2) = 0$. □

Proof of Proposition 3. First of all, note that:

$$\lim_{n\to\infty} S(n) = \lim_{n\to\infty}\int_{\theta_1}^{\theta_2}\frac{f(s)}{\left(1+\frac{1}{n}\right)^2(s+h(s))-1}ds = \int_{\theta_1}^{\theta_2}\frac{f(s)ds}{s+h(s)-1} > 0,$$

which also shows that $\lim_{n\to\infty} S(n)/n = 0$. This implies:

$$\lim_{n\to\infty}\sqrt{e(\theta)} = \lim_{n\to\infty}\frac{a-c}{n\left\{\left(1+\frac{1}{n}\right)^2[\theta+h(\theta)]-1\right\}\left[1+S(n)-\frac{S(n)}{n}\right]} = 0,$$

for any θ. Zero effort by all firms implies an equilibrium independent from the type of managers with a limit price $\lim_{n\to\infty} p = c$. This trivially implies zero profits for all firms. □

Proof of Lemma 1. First of all, note that under our assumptions, the functions (13) and (8) are both continuous and decreasing on $[\theta_1, \theta_2]$. Since $S(n) \le S^*(n)$ and $F(\theta_1) = 0$, we know that asymmetric information increases the effort required from the most efficient type: $e(\theta_1) > e^*(\theta_1)$. Moreover, since $E[\sqrt{e(\theta)}] < E[\sqrt{e^*(\theta)}]$ from Proposition 1, continuity of the two equilibrium effort functions imply that they must cross at least once—otherwise effort would always be larger under asymmetric information, which would be a contradiction. Any common point of the two functions must satisfy $e(\theta) = e^*(\theta)$. Using (13) and (8), this provides:

$$\frac{(1+1/n)^2\left[\theta + \frac{F(\theta)}{f(\theta)}\right] - 1}{(1+1/n)^2\theta - 1} = \frac{n + (n-1)S^*(n)}{n + (n-1)S(n)}. \tag{A1}$$

Define $\hat{\theta}_i$ as the positive roots of this equation with $\theta_1 < \hat{\theta}_1 < \hat{\theta}_2 < \cdots \hat{\theta}_z$. It immediately follows that asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$ and $\theta \in (\hat{\theta}_j, \min(\hat{\theta}_{j+1}, \theta_2))$ with j even, and decreases the effort of all types $\theta \in (\hat{\theta}_j, \min(\hat{\theta}_{j+1}, \theta_2))$ with j odd.

We now verify the condition under which the two functions cross only once. The right-hand side of (A1) does not depend on θ. The left-hand side is equal to 1 for $\theta = \theta_1$, and its slope is proportional to:

$$h'(\theta) - \frac{h(\theta)(1+1/n)^2}{(1+1/n)^2\theta - 1}.$$

We can rewrite (A1) as:

$$\frac{(1+1/n)^2 h(\theta)}{(1+1/n)^2\theta - 1} = \frac{(n-1)\left[S^*(n) - S(n)\right]}{\left[n + (n-1)S(n)\right]} > 0.$$

Using this, the slope of the left-hand side of (A1) is positive if:

$$h'(\theta) > \frac{(1+1/n)^2 h(\theta)}{(1+1/n)^2\theta - 1} = \frac{(n-1)\left[S^*(n) - S(n)\right]}{\left[n + (n-1)S(n)\right]}. \tag{A2}$$

If this condition is satisfied for any $\hat{\theta}$, we must have a single root for (A1) and asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$ and decreases the effort of all types $\theta \in (\hat{\theta}_1, \theta_2)$. If the condition is not satisfied for one $\hat{\theta}$ we must have at least double crossing between the effort functions. □

Proof of Proposition 4. Immediate from Lemma 1. □

Proof of Proposition 5. In case of unobservable contracts and costs, at the market competition stage all firms choose their strategies taking as given the expected strategies of the other firms. The first-order conditions for firms $j = 1, 2, \ldots, n$, given by:

$$x_j = a - c + \sqrt{e_j} - E_\theta[X],$$

provide the expected total production:

$$E_\theta[X] = \frac{n(a-c) + E_\theta\left[\sum_{j=1}^{n} \sqrt{e_j}\right]}{n+1}.$$

This allows us to rewrite the equilibrium production of each firm as:

$$x_i = \frac{a - c + n\sqrt{e_i} - E_\theta\left[\sum_{j\neq i} \sqrt{e_j}\right]}{n+1},$$

which generates expected profits $\pi_i = x_i^2 - w_i$.

At the contractual stage, under symmetric information, firm i chooses a map of contracts to maximize for each type θ the following net profits:

$$\pi_i = \left(\frac{a - c + n\sqrt{e^i(\theta)} - E_\theta \left[\sum_{j \neq i} \sqrt{e^j(\theta_j)} \right]}{n+1} \right)^2 - w_i,$$

where the expectations of the effort functions are taken over the types of all the other firms, whose contracts are considered as given. Subject to the constraint $w_i = \theta e^i(\theta)$, the profit maximizing conditions are exactly the same as in (5), which under symmetry leads to the same equilibrium contracts as in (8).

At the contractual stage, under asymmetric information, the net expected profits of firm i are:

$$E(\pi_i) = \int_{\theta_1}^{\theta_2} \left[\left(\frac{a - c + n\sqrt{e^i(\theta)} - E_\theta \left[\sum_{j \neq i} \sqrt{e^j(\theta_j)} \right]}{n+1} \right)^2 - w^i(\theta) \right] f(\theta) d\theta,$$

and the optimality conditions correspond to (11), leading again to the same equilibrium contracts (13) as before. □

Proof of Proposition 6. The proof is immediate since, according to Proposition 2, the effort differentials are increasing in n, and (20) implies for any two types θ_p and θ_q with $\theta_p > \theta_q$, the following effort differential:

$$\sqrt{\frac{e(\theta_p)}{e(\theta_q)}} = \frac{(\hat{n}+1)^2 \left[\theta_q + h(\theta_q)\right] - \hat{n}^2}{(\hat{n}+1)^2 \left[\theta_p + h(\theta_p)\right] - \hat{n}^2},$$

where the endogenous number of firms $\hat{n}$ is decreasing in $(a - c)/\sqrt{K}$. Accordingly, the effort differential is increasing in $(a - c)/\sqrt{K}$.

□

Proof of Proposition 7. First of all, under symmetric information, the model generates the equilibrium effort:

$$\sqrt{e^*(\theta)} = \frac{(2-b)(a-c)}{[A(n)\theta - 1]\{[n + (n-1)S^*(n)]\, b + 2(1-b)\}}$$

$$\textit{with } S^*(n) = \int_{\theta_1}^{\theta_2} \frac{f(s)ds}{A(n)s - 1}.$$

Now, assume $\theta_1 + h(\theta_1) > A(n)^{-1}$ to insure the existence of an interior equilibrium under both symmetric and asymmetric information. Deriving $A(n)$ one can verify that:

$$A'(n) = -2b^2(2-b)^2 \left[\frac{2 + b(n-1)}{2 + b(n-2)}\right] < 0,$$

for any b. Therefore, it follows that $S(n)$ and $S^*(n)$ are increasing in the number of firms for any b and $S(n) \leq S^*(n)$ for any b. Accordingly, we can compare the equilibrium effort with and without asymmetric information for the most efficient type, for which $h(\theta_1) = 0$. For any $b > 0$ we have:

$$\sqrt{e(\theta_1)} = \frac{(2-b)(a-c)}{[A(n)\theta_1 - 1]\{[n + (n-1)S(n)]b + 2(1-b)\}}$$
$$\geq \sqrt{e^*(\theta_1)} = \frac{(2-b)(a-c)}{[A(n)\theta_1 - 1]\{[n + (n-1)S^*(n)]b + 2(1-b)\}},$$

since both denominators are increasing in, respectively, $S(n)$ and $S^*(n)$ but $S^*(n) \geq S(n)$. For $b = 0$, we have $A(n) = 4$ for any n and:

$$\sqrt{e(\theta_1)} = \frac{a-c}{4\theta_1 - 1} = \sqrt{e^*(\theta_1)},$$

as under a monopoly, therefore this is the case in which asymmetric information induces the same effort for the most efficient managers compared to the equilibrium with symmetric information.

□

Proof of Proposition 8. The proof of the first part is equivalent to the proof in Proposition 2. The average effort can be derived from (30) as:

$$E\left[\sqrt{\tilde{e}(\theta)}\right] = \frac{(a-c)\,\tilde{S}(n)}{n + (n-1)\tilde{S}(n)},$$

which differs from (12) only for $\tilde{S}(n)$ replacing $S(n)$. To verify that effort-output contracts reduce the average effort, it is enough to check that $\tilde{S}(n)$ is smaller than $S(n)$, but since:

$$\frac{2(n+1)}{n} > \frac{(n+1)^2}{n^2} \quad \Leftrightarrow \quad n > 1,$$

this is immediate for any n by direct comparison. □

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