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CONTRACTS**
Competition and R&D Incentives

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EQUILIBRIUM PRINCIPAL-AGENT CONTRACTS

Competition and R&D Incentives

Federico Etro and Michela Cella¹

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Abstract

We analyze competition between firms engaged in R&D activities in the choice of incentive contracts for managers with hidden productivities. The equilibrium screening contracts require extra effort/investment from the most productive managers compared to the first best contracts: under additional assumptions on the hazard rate of the distribution of types we obtain no-distortion in the middle rather than at the top. Moreover, the equilibrium contracts are characterized by effort differentials between (any) two types that are always increasing with the number of firms, suggesting a positive relation between competition and high-powered incentives. An inverted-U curve between competition and absolute investments can emerge for the most productive managers, especially when entry is endogenous. These results persist when contracts are not observable, when they include quantity precommitments, and when products are imperfect substitutes.

Key words: Principal-agent contracts, asymmetric information, endogenous market structures.

JEL Classification: D82, D 86, L13.

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1 Introduction

A wide literature on contract theory has described how asymmetric information shapes the optimal contracts between a principal and an agent with private information. For instance, when the agent is the manager of a monopolistic firm and has private information on the productivity of effort, the optimal contract requires the first best effort for the most productive type and a downward distortion of effort for all the less productive types, with the effort differentials associated with the wage differentials so as to insure incentive compatibility (see Stiglitz, 1977 or Baron and Myerson, 1982). However, when a firm is not a monopolist, but competes in the market with other firms, we can expect the optimal contracts of these firms to be affected by competition and the same equilibrium prices to be affected by the contracts implemented by all the firms. While the analysis of equilibrium principal-agent contracts has been studied in particular examples with perfect competition (see Rothschild and Stiglitz, 1976, on the insurance market), there is surprisingly little work for the general case of imperfect competition. The notable exception of Martimort (1996) examines equilibrium contracts in a duopoly where the types of managers of the two firms are perfectly correlated: this is a reasonable assumption in the presence of common demand shocks, but not in the presence of firm-specific shocks or private information. We introduce idiosyncratic and uncorrelated shocks that give raise to new forms of interaction between the contracts chosen by the firms and lead to equilibrium mechanisms that are quite different from those offered by a monopolistic principal. The aim of this paper is to characterize the equilibrium emerging from competition between multiple firms in the choice of contracts. This allows us to investigate how entry (competition) affects the equilibrium contracts and, in turn, how these contracts affect the market structure.

We consider firms that in the first stage choose contracts for their managers and in the second stage compete in the market. Each contract provides incentives to undertake R&D activities that reduce marginal cost.² At the time of competing *à la* Cournot in the last stage, the contracts or the cost reducing activity become observable in our baseline model (but we show that none of our results depends on the observability of contracts or on the nature of market competition). The interesting interaction occurs at the initial contractual stage between the firms and their managers. Agents differ in their productivities, that are indepen-

²The R&D activity of each firm does not exert spillovers on the other firms, but generates cost reductions that are exclusive for the same firm, for instance because of Intellectual Property Rights associated with the innovation. This is essential for the incentives to invest in R&D.

dently distributed. We consider as a benchmark case the one in which productivities can be observed by the firm’s owner (but not by the rival), and then we consider the case in which each productivity level is private information to the manager. The contracts are expressed in terms of effort/wage schedules - but we show that the nature of our results does not change when contracts can commit also to the subsequent market strategies, i.e. when contracts are expressed in terms of effort/wage/production schedules. Our focus is on (Bayesian) Nash competition in contracts: these are chosen simultaneously, taking as given the contracts offered by the other firms.

The equilibrium principal-agent contracts require extra effort from the most productive managers compared to the equilibrium contracts emerging with symmetric information: therefore, the classic property of “no-distortion at the top” disappears when we depart from a single principal-agent structure. The reason for this relies on the new interactions between contracts for different types: the fact that all the competitors distort downward the effort of their inefficient managers increases the marginal profitability of the effort of an efficient manager meeting inefficient ones, and *vice versa*. Under additional assumptions on the hazard rate of the distribution of types we actually obtain “no-distortion in the middle” rather than at the top: all the best (worst) types exert more (less) effort than under symmetric information.³

Our most important result on the relation between competition and incentives is the following: the effort differential between (any) two types of managers is always increasing in the number of firms, which suggests a positive relation between competition and high-powered incentive schemes. When the number of firms increases, each firm tends to differentiate more its contracts, requiring a relatively higher effort from an efficient manager because this can lead to larger gains against less efficient rivals. Also the absolute effort levels can increase with the number of firms, but only for the most efficient managers: moreover, we show that in such a case an inverted-U curve between competition and absolute investment in cost reducing activities can emerge.

A wide industrial organization literature, started by Dasgupta and Stiglitz (1980) and generalized in a recent work by Vives (2008), has studied the impact of competition on cost reducing activities, showing that an increase in the number of homogenous firms tends to reduce

³Similar results emerge under product differentiation between substitute goods. However, in the presence of complement goods some of the results change: in particular, all effort levels are now below the first best levels. See the companion paper Cella and Etro (2010) for a more general investigation of the conditions under which we have upward (downward) distortion at the top.

production, profits and investment of each firm. Our model generalizes this framework with heterogeneity between firms' productivity and asymmetric information between firms' owners and their managers engaged in the cost reducing activities. Following the cited literature, we also consider the case of endogenous market structures, that emphasize a positive relation between market size and relative (and possibly absolute) measures of effort.

Only few works have examined principal-agent contracts in oligopoly. Martin (1993) has developed a first example of Cournot competition with asymmetric information between firms and managers on the cost-reduction technology, confirming the negative relation between entry and effort due to a scale effect on the profit and the absolute effort of each firm. The specification adopted led to constant cost targets for all firms, eliminating any strategic interaction between contracts and any feedback effect of the equilibrium contracts on competition.⁴ Moreover, the focus on the scale effect can be highly misleading, because the absolute investment/effort can be a bad measure of the strength of the incentive mechanisms for managers of different firms under different market conditions. A more appropriate measure of this strength is the effort differential between managers of different types, which represents the divergence of efforts and compensations between employees with different productivities: in this paper we focus mainly on this comparative measure and show that the effort differentials are always increasing in the level of competition.

Part of the theoretical literature, for instance Ivaldi and Martimort (1994) and Stole (1995), has analyzed duopolies engaged in price discrimination, which generates a problem of common agency that is fundamentally different from our competitive interaction between two principal-agent hierarchies. As already mentioned, the major article analyzing competition between hierarchies is by Martimort (1996), who characterizes the equilibrium screening contracts in a duopoly with perfectly correlated types: as a consequence of perfect correlation, the equilibrium contracts are quite similar to those of a single principal-agent structure, with "no-distortion at the top" and downward output distortion below - and we also show that the effort differentials are in this case independent from the number of firms and the absolute effort levels decrease

⁴On the relation between incentive contracts and competition see also Hart (1983) and Hermalin (1994). Schmidt (1997) develops a model of moral hazard where a positive impact of competition on effort may emerge from a threat of liquidation associated with low effort; however, this model does not generate any feedback effect of the equilibrium contracts on competition. We are interested in studying both how competition affects contracts and how contracts affect competition.

with competition for all types.⁵ A recent work by Piccolo *et al.* (2008) has analyzed cost-plus contracts *à la* Laffont and Tirole (1986) in a Cournot duopoly again with perfectly correlated shocks, which excludes any strategic interaction between contracts: the agency problem remains formally equivalent to that of a monopolist. Notice that these works emphasize the relation between product substitutability and managerial incentives,⁶ while we focus on the relation between entry of firms in the market and incentives. Our companion paper Cella and Etro (2010) analyzes also other forms of competition as price competition *à la* Hotelling, Stackelberg competition in quantities and Cournot competition between a regulated firm and a private one.

Finally, our results on the positive relation between number of firms and effort differentials may contribute to explain the weak but positive correlation between competition and incentive mechanisms found in many empirical studies, for instance in Hubbard and Palia (1995), Cuñat and Guadalupe (2005) and Bloom and Van Reenen (2007). We also emphasize the possibility of an inverted-U relation between the number of firms and the absolute investment in cost reductions, which is in line with the evidence on competition and innovation found by Aghion *et al.* (2005).

The paper is organized as follows. Section 2 analyzes the model with multiple firms and a continuum of agents. Section 3 extends the basic model in different directions. Section 4 concludes. All proofs are in the Appendix.

2 THE MODEL

Consider a market with inverse demand $p = a - X$, where X is total quantity produced and a is a size parameter. Production requires a constant marginal cost which can be reduced by the manager: for simplicity, we assume that effort e generates the marginal cost $c - \sqrt{e}$. A firm hires a manager and delegates two operative tasks: reducing the costs and maximizing the profits with the relevant market strategy (here the output level). The contract between the firm and the manager can establish the size of the cost reduction, or equivalently the effort e , and the wage

⁵Nevertheless, Martimort (1996) already emphasized the role of strategic interactions and, most of all, provided an important comparison with the equilibrium contracts emerging under common agency.

⁶Notice that Piccolo *et al.* (2008) examine also the case of profit-target contracts, which leads again to equilibria with the traditional no-distortion at the top and a downward distortion on the effort of the inefficient managers: the interesting aspect is that the effort of the inefficient managers may be non-monotonic in the degree of product substitutability.

w .⁷ A contract (e, w) determines the utility of the manager as:

$$u(w, e) = w - \theta e \quad (1)$$

where θ represents the marginal cost of effort. A firm chooses its contract to maximize profits subject to the constraint that $u(w, e)$ is above the reservation utility, which is normalized to zero.

We assume that there are n firms and the managers' types are independently drawn from a distribution function known by everybody. Suppose that each type θ is distributed on $[\theta_1, \theta_2] \subseteq \mathfrak{R}^+$ according to a cumulative distributive function $F(\theta)$ that is assumed twice differentiable, with density $f(\theta)$, and satisfying the monotone hazard rate property for which $h(\theta) \equiv F(\theta)/f(\theta)$ is increasing in θ .

In a first stage, all firms simultaneously choose the contracts for their managers, and in the second stage, after the contracts and the final marginal costs are observable (an assumption relaxed later on), the managers compete *à la* Cournot.

The Cournot equilibrium of the last stage with n firms with observable efforts e_1, e_2, \dots, e_n , implies the production of each firm to be:

$$x_i = \frac{a - c + n\sqrt{e_i} - \sum_{j \neq i} \sqrt{e_j}}{n + 1} \quad (2)$$

which generates profits $\pi_i = x_i^2 - w_i$. We will now characterize the equilibria of the game starting from the benchmark case in which each firm chooses its own contract knowing the type of its manager, but not the type of managers of the rivals. The contracts are chosen simultaneously taking as given those offered by the other firm. After that, we will consider the case of genuine asymmetric information between each firm and its manager.

2.1 Equilibrium contracts with symmetric information

As a benchmark case, let us consider symmetric information between the firms and their managers. Firm i chooses a map of contracts that ensures participation $(e^i(\theta), w^i(\theta))$ with $e : [\theta_1, \theta_2] \rightarrow \mathfrak{R}_+$ and $w : [\theta_1, \theta_2] \rightarrow \mathfrak{R}_+$. The optimal map of contracts that ensures participation $(e^i(\theta), \theta e^i(\theta))$

⁷We assume that the managers choose quantities to maximize profits in the interest of the firms' owners because this implies no costs for them - we introduce quantity commitments directly in the contracts in a later section. However, we maintain the assumption that contracts cannot be conditioned on the contracts of the other firm or on final profits. We are thankful to Jacques Crémer for a discussion on the last point.

maximizes for each type θ the following net profits:

$$\pi_i = E \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} \sqrt{e^j(\theta_j)}}{n + 1} \right)^2 - \theta e^i(\theta) \quad (3)$$

where the expectation of the gross profits is taken over the types θ_j of all the other firms $j \neq i$, whose maps of contracts $(e^j(\theta), w^j(\theta))$ are taken as given. The first order condition with respect to $e^i(\theta)$ are:

$$E \left[\frac{n \left(a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} \sqrt{e^j(\theta_j)} \right)}{(n + 1)^2 \sqrt{e^i(\theta)}} \right] = \theta$$

Since the manager's type is known and the expectation is taken only over the types of the rivals, this can be rewritten as:

$$\sqrt{e^i(\theta)} = \frac{n \left(a - c - \sum_{j \neq i} E \left[\sqrt{e^j(\theta_j)} \right] \right)}{(n + 1)^2 \theta - n^2} \quad (4)$$

whose sign constraint requires the assumption $\theta_1 > n^2/(1 + n)^2$.

We focus on symmetric equilibria. Taking the expectation over θ on both sides, and solving out under the assumption that the firms adopt identical maps of contracts $(e^*(\theta), w^*(\theta))$, we obtain the average (square root of) effort:

$$E \left[\sqrt{e^*(\theta)} \right] = \frac{(a - c) S^*(n)}{n + (n - 1) S^*(n)} \text{ with } S^*(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s) ds}{\left(\frac{n+1}{n}\right)^2 s - 1} \quad (5)$$

where the function $S^*(n)$ is increasing in n . Substituting in (4), we finally have the equilibrium effort:

$$\sqrt{e^*(\theta)} = \frac{a - c}{\left[\left(\frac{n+1}{n}\right)^2 \theta - 1\right] [n + (n - 1) S^*(n)]} \quad (6)$$

which is decreasing and convex in the manager's type. Notice that when $n = 1$ this boils down to the monopoly solution $\sqrt{e^*(\theta)} = (a - c) (4\theta - 1)$, but when $n > 1$ it depends on the number of firms in a complex way.

2.2 Equilibrium oligopolistic screening

Let us move to the case of asymmetric information. Firm i chooses a map of contracts $(e^i(\theta), w^i(\theta))$ to solve a problem of maximization of the expected profits:

$$E(\pi_i) = \int_{\theta_1}^{\theta_2} \left[E \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} \sqrt{e^j(\theta_j)}}{n + 1} \right)^2 - w^i(s) \right] f(\theta) d\theta \quad (7)$$

under individual rationality and incentive compatibility constraints.⁸ The expectation operator is taken over the types of the rivals, whose maps of contracts $(e^j(\theta), w^j(\theta))$ are considered as given.

As usual, the Revelation Principle (Dasgupta *et al.*, 1979; Myerson, 1979) and the incentive compatibility constraint require that the effort schedule must be non-increasing, $\partial e^i(\theta)/\partial\theta \leq 0$ and that truth-telling is always optimal, $\partial w^i(\theta)/\partial\theta = \theta(\partial e^i(\theta)/\partial\theta)$. Solving the last differential equation for the wage schedule, and using the fact that the individual rationality constraint must be binding on the least efficient type ($w^i(\theta_2) = \theta_2 e^i(\theta_2)$), we have the incentive compatibility constraint:

$$w^i(\theta) = \theta e^i(\theta) + \int_{\theta}^{\theta_2} e^i(y) dy \quad (8)$$

Substituting this in (7) and integrating by parts, the optimal contract of firm i must solve the following problem:

$$\max_{e^i(\theta)} \int_{\theta_1}^{\theta_2} \left[E \left(\frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} \sqrt{e^j(\theta_j)}}{n + 1} \right)^2 - [\theta + h(\theta)] e^i(\theta) \right] df$$

The first order condition for pointwise maximization can be rearranged as:

$$\sqrt{e^i(\theta)} = \frac{n \left(a - c - \sum_{j \neq i} E \left[\sqrt{e^j(\theta_j)} \right] \right)}{(n + 1)^2 [\theta + h(\theta)] - n^2} \quad (9)$$

After imposing symmetry of the map of contracts (exploiting the absence of correlation across types), we can derive the average effort:

$$E \left[\sqrt{e(\theta)} \right] = \frac{(a - c) S(n)}{n + (n - 1)S(n)} \text{ with } S(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(s) ds}{\left(\frac{n+1}{n}\right)^2 [s + h(s)] - 1} \quad (10)$$

where $S(n) < S^*(n)$ is increasing in the number of firms.⁹ Substituting in (9), we can express the equilibrium effort function as follows:

$$\sqrt{e(\theta)} = \frac{a - c}{\left\{ \left(\frac{n+1}{n}\right)^2 [\theta + h(\theta)] - 1 \right\} [n + (n - 1)S(n)]} \quad (11)$$

⁸See Laffont and Tirole (1986, 1993) on the applications to a regulated firm, Maskin and Riley (1984) on price discrimination by a monopolist and Laffont and Martimort (2002) for a general survey.

⁹Indeed we have:

$$S'(n) = \frac{2(n + 1)}{n^3} \int_{\theta_1}^{\theta_2} \frac{(s + h(s))f(s) ds}{\left[\left(\frac{n+1}{n}\right)^2 (s + h(s)) - 1\right]^2} > 0$$

This shows that, in general, the effort and the marginal cost do change with the type of managers and induce asymmetries in the market: notice that the constant marginal cost obtained by Martin (1993) was highly dependent on his specific assumptions and eliminated crucial interactions between different contracts.

Our explicit characterization of the equilibrium contracts allows us to determine the equilibrium market structure, here summarized by the expected price, that can be derived as:

$$E[p] = c + \frac{(a - c)[n - S(n)]}{(n + 1)[n + (n - 1)S(n)]} \quad (12)$$

We can describe the properties of the equilibrium under asymmetric information as follows:

Proposition 1. *Competition in contracts with multiple firms under asymmetric information is characterized by a map of contracts $(e(\theta), w(\theta))$ with efforts defined by (11) and decreasing in the type of manager, and with wages defined by $w(\theta) = \theta e(\theta) + \int_{\theta}^{\theta_2} e(y) dy$. The average effort is reduced and the expected price is increased by the presence of asymmetric information between firms and managers.*

The crucial aspect of the equilibrium contracts is that they depend on the strategic interactions in a novel way. In particular, the equilibrium effort (11) depends not only on the type of manager, but also on the entire distribution of types in a novel way through the $S(n)$ function. Of course, when $n = 1$ the optimal contract boils down to the traditional rule $\sqrt{e(\theta)} = (a - c) / [4(\theta + h(\theta)) - 1]$ with the “no-distortion at the top” property: the most productive type exerts the first best effort independently from the distribution of θ . However, this well known property of optimal contracts disappears when there are more firms than one. In particular, since $h(\theta_1) = 0$ but $S(n) < S^*(n)$ we immediately derive from (11) and (6) that $e(\theta_1) > e^*(\theta_1)$: the most efficient managers are always required to exert more effort than in “first best” when they are competing in the market. The effort required from the less efficient types must be lower, but the comparison with the “first best” contract is now more complex. Moreover, the number of firms affects in a substantial way the equilibrium contracts creating a complex interdependence between these and the market structure.

Multiple mechanisms are at work in influencing the impact of competition on the equilibrium contracts. First of all, we have a price channel: an increase in the number of firms reduces the average effort, which tends to reduce the equilibrium price, which in turn leads to lower incentives

to exert effort. Second, we have a profitability channel: an increase in the number of firms reduces the profits available to each firm, but it also increases the marginal profitability of effort, especially for more efficient types. The net impact of the last two effects on effort is positive and stronger for more efficient types - as can be seen from the term $(\frac{n+1}{n})^2 [\theta + h(\theta)]$ at the denominator of (11). Finally, we have an indirect strategic channel: an increase in the marginal profitability of effort for the other firms reduces the incentives to invest in cost reduction - as can be seen from the term $S(n)$ at the denominator of (11). The net impact of these channels is ambiguous in general, but we can expect that a positive impact of competition on effort could emerge only for the most efficient managers. The next section verifies this and the other properties of the equilibrium.

2.3 Competition and incentives

The following proposition describes the impact of competition, measured by entry of firms, on the equilibrium contracts:

Proposition 2. *An increase in the number of firms: 1) increases the effort of all the most productive types $\theta \in [\theta_1, \tilde{\theta})$, and decreases the effort of all types $\theta \in (\tilde{\theta}, \theta_2]$ for a cut-off $\tilde{\theta} \in [\theta_1, \theta_2]$; and 2) always increases effort differentials: given any types $\theta_a < \theta_b$, the ratio between their equilibrium efforts $e(\theta_a)/e(\theta_b)$ is always finite and increasing in n .*

This proposition tells us that the absolute effort levels may increase when the number of firms goes up, but this can happen only for the most efficient types, while the effort levels of the less efficient types always decreases with n . When a new firm enters the market, all the managers with productivity above a certain threshold (if there are any) are required to exert more effort, and the others end up exerting less effort.

In addition, the proposition has unambiguous implications for relative efforts. It shows that effort differentials between more and less efficient managers increase always with competition. For instance, considering two types θ_a and θ_b with $\theta_a > \theta_b$, the equilibrium function (11) allows us to express the corresponding effort differential as:

$$\sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{(n+1)^2 [\theta_b + h(\theta_b)] - n^2}{(n+1)^2 [\theta_a + h(\theta_a)] - n^2} \quad (13)$$

which is always increasing in the number of firms. The intuition is simple: when the number of firms increases, each firm tends to differentiate more its contracts, requiring a relatively higher effort from an efficient

manager because this can lead to larger gains against less efficient rivals. As long as we interpret the effort differentials between managers with different productivities as the relevant measure of the power of the incentive schemes, this framework shows that more competition requires more high-powered incentive schemes. Our results may contribute to recover a theoretical motivation for the weak but positive correlation between entry and the strength of incentive mechanisms found in many empirical studies, for instance in Hubbard and Palia (1995) and Bloom and Van Reenen (2007). In particular, and contrary to the received literature, the positive relation between the number of competitors and the effort differentials across more and less productive managers can be seen as a rationale for more aggressive incentive mechanisms in case of stronger competition.

Another message of Proposition 2 is that the effort differentials reach a maximum level when we approach the perfectly competitive limit with infinite firms. In the case above, the limit is:

$$\lim_{n \rightarrow \infty} \sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{\theta_b + h(\theta_b) - 1}{\theta_a + h(\theta_a) - 1}$$

Of course, this does not tell us much about the absolute effort levels when the number of firms is large and when it increases indefinitely. It turns out that the absolute efforts of all types must decrease when the number of firms is large enough, and they all approach the same limit when the number of firms tends to infinity. This is characterized in the following proposition together with the associated market structure:

Proposition 3. *When the number of firms tends to infinity, competition in contracts generates zero effort for all types of managers, the price equals the maximum marginal cost and profits are zero.*

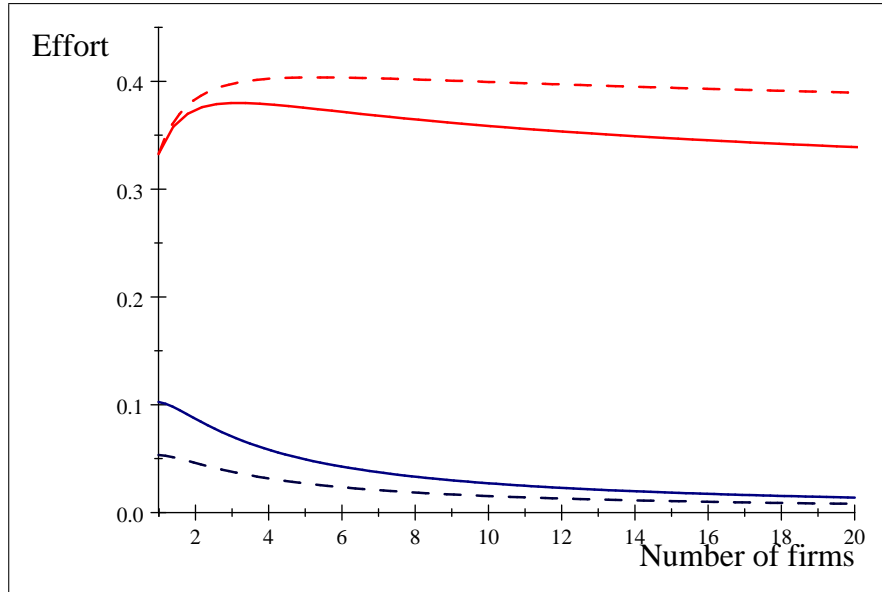


Fig. 1. Effort under (a)symmetric information with solid (dashed) lines.

This result should not be surprising: the incentives to invest for price taking firms depend on the size of their production (with or without asymmetric information), but in the limit of a Cournot model all firms produce an infinitesimal output and those incentives must vanish.¹⁰

From the last propositions we can draw the following conclusions on the impact of competition (number of firms) on contracts. First and most important, more competition increases always the effort differentials between managers of different types. Second, more competition can increase the absolute level effort for the most efficient types, but reduces it when the number of firms is large enough. Third, more competition reduces the absolute effort of the least efficient types. All this suggests that a non-monotone effort function can arise for the most efficient types, and this can take an inverted-U shape: with effort maximized for an intermediate degree of competition.

¹⁰We should remark that the extreme cases of monopoly and perfect competition, which have been the traditional focus of the literature, are quite similar from a qualitative point of view. In both cases, the presence of asymmetric information does not affect the contracts offered to the most efficient types (no-distortion on the top holds), while distortions concern only the inefficient types. These common features, however, are only peculiar of these extreme cases, and they break down when there are strategic interactions.

The numerical example of Fig 1 helps to visualize this pattern under a uniform distribution. With such a distribution, $F(\theta) = (\theta - \theta_1)/(\theta_2 - \theta_1)$ and $f(\theta) = (\theta_2 - \theta_1)^{-1}$, therefore we have:

$$S^*(n) = n^2 \frac{\log [(n+1)^2 \theta_2 / n^2 - 1] - \log [(n+1)^2 \theta_1 / n^2 - 1]}{(n+1)^2 (\theta_2 - \theta_1)}$$

which, combined with (6), provides the equilibrium effort under symmetric information in solid lines. Analogously, we have:

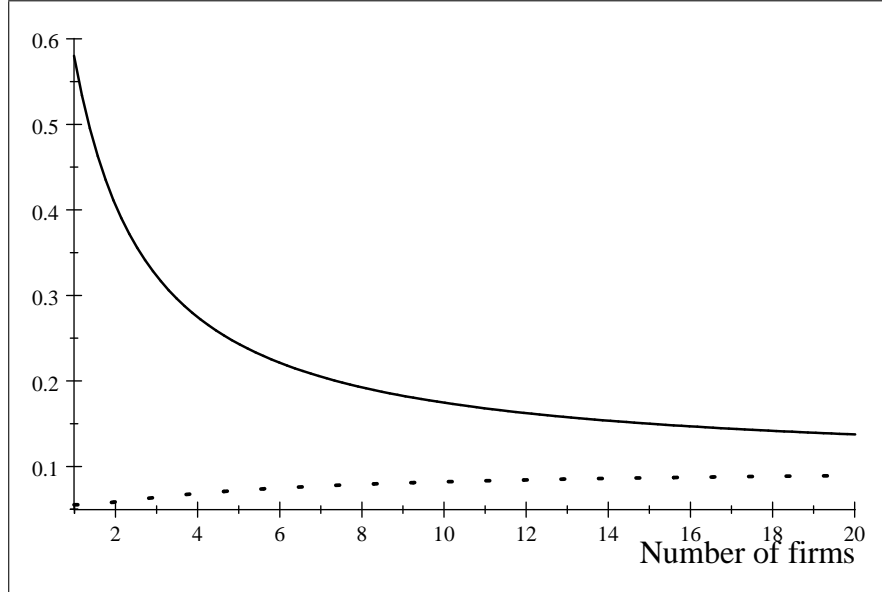
$$S(n) = n^2 \frac{\log [(n+1)^2 (2\theta_2 - \theta_1) / n^2 - 1] - \log [(n+1)^2 \theta_1 / n^2 - 1]}{2(n+1)^2 (\theta_2 - \theta_1)}$$

which, combined with (11) provides the equilibrium effort under asymmetric information reported in dashed lines. We assume $a - c = 1$, $\theta_1 = 1$ and $\theta_2 = 10$ and plot the equilibrium effort of the most and least efficient types (respectively red vs blue) for $n \in [1, 20]$. One can verify that in correspondence of $n = 1$ we have no distortion on the top and downward distortion at the bottom, the effort of the most productive manager is always above its equivalent under symmetric information, and the effort of the least productive is always below. Finally, an increase of the number of firms induces a reduction of the effort of the least productive managers and an inverted-U shape for the equilibrium effort of the most productive managers.

Therefore, the model can exhibit a bell shaped relation between the number of firms and the investment of the most productive firms in cost reducing activities, in line with the evidence on competition and innovation found by Aghion *et al.* (2005). Notice that such an outcome cannot emerge in case of homogenous firms, as recently shown by Vives (2008) for a general class of models, but it emerges in the presence of heterogeneity between firms.

For completeness, in Fig. 2 we also report the equilibrium expected price and the average marginal cost for the case of asymmetric information (assuming $c = 0.1$). The latter increases because the average effort decreases with entry, but the former decreases with the number of firms because the direct impact of competition is stronger than the indirect

impact on the cost reducing activities.



Expected price (solid) and average effort (dots)

2.4 Conditions for a two-way distortion

The numerical example leads us to the general comparison of the equilibrium effort emerging with and without informational asymmetry. However, the relation between (11) and (6) is way more complex than in the case of a single principal agent contract. In fact, while both functions are decreasing in the type of the manager, the behavior of their concavity is different in the presence of symmetric or asymmetric information. While the function $\sqrt{e^*(\theta)}$ is always convex, the function $\sqrt{e(\theta)}$ is not necessarily convex and its slope not necessarily larger in absolute value, meaning that multiple crossings between the two functions may arise (while in the case of monopolistic screening the two relations cross only once in correspondence of the most efficient type).

Nevertheless, introducing a stronger condition on the slope of the hazard rate we can avoid multiple crossings and obtain a simple result: the no-distortion of the effort occurs for an intermediate type, with larger (lower) effort under asymmetric information for all types above (below) that intermediate type.¹¹ A comparison of (11) and (6) shows that the

¹¹A similar two-way distortion emerges in the model with one principal and many agents by Lockwood (2000) because of production externalities (the average effort increases the effectiveness of individual effort).

intermediate type $\hat{\theta}$ must satisfy the condition $e(\hat{\theta}) = e^*(\hat{\theta})$, which can be rewritten as:

$$\frac{\left(\frac{n+1}{n}\right)^2 h(\hat{\theta})}{\left(\frac{n+1}{n}\right)^2 \hat{\theta} - 1} = \frac{(n-1)[S^*(n) - S(n)]}{n + (n-1)S(n)} \quad (14)$$

Single crossing of the two effort functions requires that $e(\theta) < e^*(\theta)$ for any $\theta > \hat{\theta}$. Developing this inequality from (11) and (6) we can rewrite the condition as follows:

$$\frac{\left(\frac{n+1}{n}\right)^2 h(\theta)}{\left(\frac{n+1}{n}\right)^2 \theta - 1} > \frac{(n-1)[S^*(n) - S(n)]}{n + (n-1)S(n)} = \frac{\left(\frac{n+1}{n}\right)^2 h(\hat{\theta})}{\left(\frac{n+1}{n}\right)^2 \hat{\theta} - 1}$$

where the last equality follows from (14). A sufficient condition for this is that the function on the left hand side is always increasing in θ . Deriving the left hand side with respect to θ , one can verify that this is equivalent to an additional assumption on the slope of the hazard rate:

$$h'(\theta) > \frac{\left(\frac{n+1}{n}\right)^2 h(\theta)}{\left(\frac{n+1}{n}\right)^2 \theta - 1} \quad (15)$$

As long as the hazard rate is not only increasing, but increasing enough to satisfy (15), we can be sure that asymmetric information increases the effort of all the best managers and reduces the effort of all the worst managers with no distortion only for the intermediate type $\hat{\theta}$. While the above condition may appear rather demanding, we can easily show that it is always satisfied by the uniform distribution. In such a case $h(\theta) = \theta - \theta_1$, therefore (15) reads as:

$$1 > \frac{\left(\frac{n+1}{n}\right)^2 (\theta - \theta_1)}{\left(\frac{n+1}{n}\right)^2 \theta - 1}$$

or:

$$\left(\frac{n+1}{n}\right)^2 \theta_1 - 1 > 0$$

which is always satisfied under our assumptions.

Of course, condition (15) is sufficient but not necessary for a “well-behaved” solution. The next lemma provides a characterization of the comparison between symmetric and asymmetric information, and derives the necessary and sufficient condition for a well behaved case:

Lemma 4. *Define the positive roots of (14) as $\hat{\theta}_1 < \hat{\theta}_2 < \dots < \hat{\theta}_z$. Asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$*

and decreases the effort of all types $\theta \in (\hat{\theta}_1, \min(\hat{\theta}_2, \theta_2))$. Under the additional assumption:

$$h'(\hat{\theta}) > \frac{(n-1)[S^*(n) - S(n)]}{n + (n-1)S(n)} \text{ for any } \hat{\theta} \quad (16)$$

we have $z = 1$ (a single root).

The intuition for this lemma is simple. If the hazard rate is increasing fast enough, the equilibrium effort under asymmetric information decreases significantly with the type so to cross only once the equilibrium effort under symmetric information. Notice that the condition corresponds to the simple monotone hazard rate property ($h'(\theta) > 0$) in the traditional case of $n = 1$, but is more demanding with multiple firms. As a consequence, we have:

Proposition 5. *Under competition in contracts between n firms, if and only if (16) holds, asymmetric information increases (decreases) the effort of all the most (least) efficient types compared to the equilibrium with symmetric information, without distortion only for an intermediate type $\hat{\theta} \in (\theta_1, \theta_2)$.*

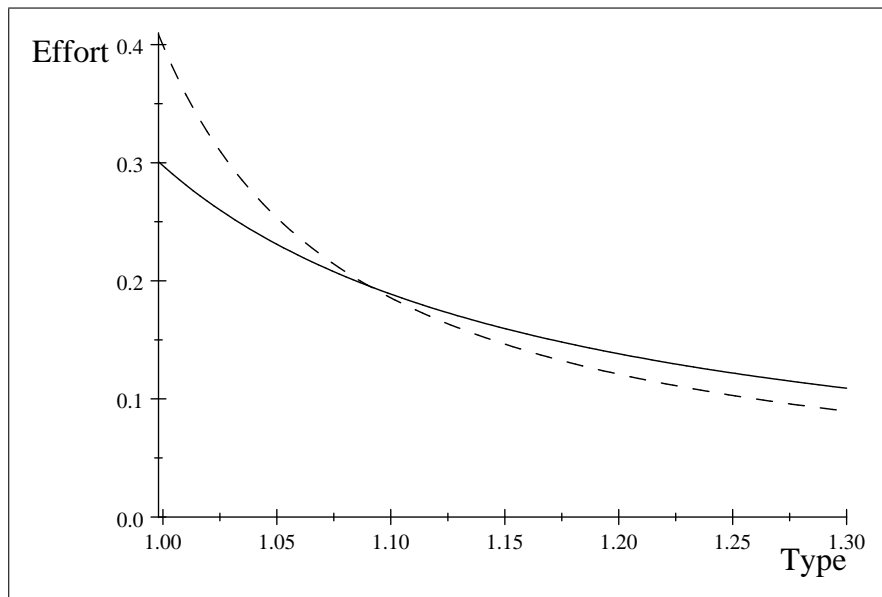


Fig. 3. The functions $\sqrt{e^*(\theta)}$ (solid line) and $\sqrt{e(\theta)}$ (dashed line).

In other words, competition in contracts with asymmetric information delivers “no distortion in the middle” and amplifies the differences between the efforts exerted by managers of different productivities. In Fig. 3 we exemplify this result for the case of a uniform distribution with the same parametrization as above ($a - c = 1$, $\theta_1 = 1$ and $\theta_2 = 10$) and $n = 10$. The solid line represents the equilibrium effort under symmetric information, and the dashed line the one under asymmetric information, shown for $\theta \in (1, 1.3)$. The most efficient type exerts effort $\sqrt{e^*(\theta_1)} \simeq 0.3$ in the former case and $\sqrt{e(\theta_1)} \simeq 0.41$ in the latter, while the least efficient type exerts $\sqrt{e^*(\theta_2)} = 0.009$ under symmetric information and $\sqrt{e(\theta_2)} = 0.006$ under asymmetric information. We have no distortion of effort only for the intermediate type $\hat{\theta} \simeq 1.1$.

When (16) is not satisfied we cannot exclude multiple crossing between the effort functions under symmetric and asymmetric information, with effort levels alternatively higher and lower in the two regimes, and therefore with “no distortion in multiple places”.

2.5 Unobservable contracts

One may think that the assumption of perfect information on contracts and costs at the time of competing in the market affects substantially the contracts signed at the initial stage. However, in our model this is only a simplifying assumption without consequences on the nature of the equilibrium contracts.

To verify this important aspect, let us consider the case of unobservable contracts and costs at the time of competing in the market. In the last stage, all firms choose their strategies taking as given the expected strategies of the other firms (rather than the actual strategies), and, in the contractual stage, they choose the effort/wage schedules to maximize their net profits in function of the expected equilibrium strategies. Under our specification, we can show the following equivalence result:

Proposition 6. *The equilibrium contracts with symmetric and asymmetric information do not depend on whether these contracts are observable or not at the time of the competition in the market.*

Such an equivalence is due to the quadratic form of profits in our model. This implies that the marginal profits are a linear function of the expected efforts and equal to the expected marginal profits: as a consequence, firms adopt the same effort function on the basis of the expectation either of their equilibrium strategies (when contracts are observable) or of their average expected strategies (when contracts are not observable).

Of course, the ultimate equilibrium production levels differ depending on whether the contracts are observable or not, but the initial contracts, the effort exerted by managers of different types in equilibrium and even the expected production levels are not affected by contract observability.

2.6 Endogenous market structures

Until now we have considered exogenous market structures in which the number of competitors was given. A more realistic situation emerges when entry requires a preliminary fixed investment and the number of firms is endogenized through an endogenous entry condition in the presence of a small entry cost. In this section we develop this analysis, which provides a generalization of the endogenous market structure approach of Dasgupta and Stiglitz (1980), Tandon (1984), Sutton (1991) and Vives (2008) to the case of heterogeneity between firms and asymmetric information between their owners and their managers engaged in (preliminary) cost reducing activities.

Our earlier analysis with an exogenous market structure generated a positive correlation between the number of firms and the relative effort levels, but it also suggested the possibility of a negative correlation with the absolute effort levels, at least for low productivity managers and/or when the number of firms is high. As we will see, the first implication holds true when we endogenize the market structure, but the second implication can be questioned.

Let us focus on the case in which contracts are non-observable, that in the main framework produced results equivalent to the observable contracts case.¹² In the initial stage entry occurs if non-negative profits are expected, and, after that, the contractual and competitive stages are the same as above. Under asymmetric information, for given n , the expected profits of each firm are given by:

$$E[\pi(n)] = \int_{\theta_1}^{\theta_2} \left[\left(\frac{a - c + n\sqrt{e^i(\theta)} - E\left[\sum_{j \neq i} \sqrt{e^j(\theta_j)}\right]}{n + 1} \right)^2 - w^i(\theta) \right] f(\theta) d\theta$$

Substituting the equilibrium efforts and wages we obtain:

$$E[\pi(n)] = \frac{(a - c)^2 Z(n)}{[n + (n - 1)S(n)]^2} \text{ with } Z(n) \equiv \int_{\theta_1}^{\theta_2} \frac{[\theta + h(\theta)] f(\theta) d\theta}{\left(\frac{n+1}{n}\right)^2 (\theta + h(\theta)) - 1} \quad (17)$$

This implies that, given a fixed cost of entry K , the endogenous number of firms \hat{n} satisfies $E[\pi_i(\hat{n})] = K$ and must be increasing in the relative

¹²Similar qualitative results emerge when the contracts are observable, but the analysis is more complex. Details are available from the authors.

size of the market $(a - c) / \sqrt{K}$. The endogenous entry condition allows us to rewrite the equilibrium effort (11) as:

$$\sqrt{e(\theta)} = \frac{\sqrt{\frac{K}{Z(\hat{n})}}}{\left(\frac{\hat{n}+1}{\hat{n}}\right)^2 [\theta + h(\theta)] - 1} \quad (18)$$

with average effort $E \left[\sqrt{e(\theta)} \right] = S(\hat{n}) \sqrt{K/Z(\hat{n})}$. As usual, the absolute effort of all types tends to zero when the fixed cost tends to zero.

An increase of market size measured by a (for instance associated with the process of opening up to trade) or a reduction in fixed costs K are associated with an increase in the number of players and, again, with an ambiguous impact on the absolute effort: on one side this tends to increase because of the direct size effect, but on the other side it may tend to decrease for the indirect impact due to the larger number of firms.¹³ In any case, the size effect implies that the set of (most productive) types for which the absolute effort increases is enlarged. Moreover, we can immediately derive an unambiguous conclusion on the effort differentials:

Proposition 7. *When the number of firms competing in contracts is endogenous, an increase in the size of the market or a reduction in the fixed cost amplifies the effort differentials.*

Therefore, our model is consistent with a positive relation between incentives and number of competitors both in relative terms (effort differentials) and in absolute terms for the most productive managers. These results appear in line with the findings of Cuñat and Guadalupe (2005), who emphasize a positive impact of openness (which implies larger and more competitive markets) on the strength of the incentive mechanisms, especially for the top managers or, in general, for the highest paid managers.

Our analysis opens space for further investigations on the role of asymmetric information in markets with endogenous structures. Etro (2010) has analyzed optimal unilateral screening contracts in markets with endogenous entry, showing that a firm would always gain from committing to aggressive incentive contracts (with extra effort required

¹³Notice that Tandon (1984) and Vives (2008) show that, in the absence of heterogeneity between firms (and of asymmetric information), the size effect always dominates and effort increases with an increase in the size of the market. However, their models of endogenous market structures differ from ours because they assume simultaneous investment and production choices, while we assume sequential choices.

from all types) to limit entry and gain market shares over the competitors.¹⁴ While that model, following the analysis of endogenous market structures in Etro (2006), neglected contract competition, a similar outcome is likely to emerge also in the present context.¹⁵

3 EXTENSIONS

Our general analysis can be employed to address a number of related applications. Here, we focus on issues concerning product differentiation, general contractual arrangements and aggregate shocks.

3.1 Product differentiation

We can easily generalize the model to the case of product differentiation with imperfect substitutability (or even complementarity) between goods. Assume an inverse demand function for firm i given by $p_i = a - x_i + b \sum_{j \neq i} x_j$ where $b \leq 1$ parametrizes substitutability. When $b = 1$ we are in the case of homogenous goods, when $b = 0$ we have independent monopolies producing non-substitutable goods, and when $b < 0$ we have complement goods.

The Cournot equilibrium with n firms with efforts e_1, e_2, \dots, e_n , implies the production of each firm:

$$x_i = \frac{(2-b)(a-c) + [2 + b(n-2)]\sqrt{e_i} - b \sum_{j \neq i} \sqrt{e_j}}{(2-b)[2 + b(n-1)]}$$

that sets a limit on the differentiation parameter, $b > -2/(n-1)$, to insure a positive production. Also this equilibrium generates the profits $\pi_i = x_i^2 - w_i$.

Our methodology allows us to derive the equilibrium map of contracts under asymmetric information with the following effort:

$$\sqrt{e(\theta)} = \frac{(2-b)(a-c)}{\{A(n)[\theta + h(\theta)] - 1\} \{[n + (n-1)S(n)]b + 2(1-b)\}} \quad (19)$$

where

$$S(n) = \int_{\theta_1}^{\theta_2} \frac{f(\theta)d\theta}{A(n)[\theta + h(\theta)] - 1} \quad \text{with} \quad A(n) \equiv \frac{(2-b)^2 [2 + b(n-1)]^2}{[2 + b(n-2)]^2}$$

¹⁴See also the companion paper Cella and Etro (2010) on Stackelberg competition in contracts with a leader and a single follower.

¹⁵We should mention an important work by Creane and Jeitschko (2009) that studies the role of adverse selection for markets characterized by perfect or Cournot competition and entry costs. It shows that the traditional market failure due to informational asymmetries tends to vanish under endogenous entry and to be replaced by limited entry with above-normal profits for the entrants.

This equilibrium can be compared with the one emerging under symmetric information, which implies:

$$\sqrt{e^*(\theta)} = \frac{(2-b)(a-c)}{[A(n)\theta - 1] \{[n + (n-1)S^*(n)]b + 2(1-b)\}}$$

with $S^*(n) = \int_{\theta_1}^{\theta_2} \frac{f(\theta)d\theta}{A(n)\theta - 1}$

A comparison of the two results allows us to obtain:

Proposition 8. *Under product differentiation, imperfect product substitutability induces extra effort for the most efficient managers compared to the equilibrium with symmetric information, while product complementarity induces a smaller effort for all types compared to the equilibrium with symmetric information.*

Indeed, one can verify that:

- 1) homogenous goods ($b = 1$) lead to the same formulas as before;
- 2) imperfect substitutability ($0 < b < 1$) leads to the same two-way distortion of the case with homogenous goods, with extra effort for the most efficient types;
- 3) independent goods ($b = 0$) lead to equilibrium contracts that are identical to the optimal monopolistic contracts;
- 4) complementarity between goods ($-2/(n-1) < b < 0$) leads to decrease the effort of all types compared to the case with symmetric information, with reduced effort also for the most efficient type.

Results 1)-3) generalize what we have found until now: the most efficient types exert more effort under asymmetric information. To the contrary, result 4) inverts what we have found in all the other cases, and is due to the strategic complementarity between the efforts exerted by different firms in case of complement goods. Our companion paper Cella and Etro (2010) has extended the model with profit functions that depend in a general way on the actions (here the efforts) of all the agents. It shows that, under asymmetric information, when the actions are strategic substitutes ($\partial^2 \pi_i / \partial e_i \partial e_j < 0$) the equilibrium contracts require always extra effort for the most efficient agents and a downward distortion for the least efficient agents, but when they are strategic complements ($\partial^2 \pi_i / \partial e_i \partial e_j > 0$), they require a downward distortion of effort for both kinds of agents. Our model of quantity competition with imperfectly substitute goods ($b > 0$) satisfies strategic substitutability of efforts, and the same occurs in standard models of price competition with substitute goods *à la* Bertrand or *à la* Hotelling (see Cella and

Etro, 2010). However, the case of quantity competition with complement goods ($b < 0$) satisfies strategic complementarity in efforts, and therefore leads to the opposite result.

Finally, notice that, given two types $\theta_a > \theta_b$, the equilibrium effort differential generalizes to:

$$\sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{A(n)\theta_b - 1}{A(n)\theta_a - 1} \quad (20)$$

which is always increasing in the number of firms (except for $b = 0$) and decreasing in the substitutability parameter b .

Under substitutability ($b > 0$), our qualitative results on the relation between competition and both absolute and relative efforts persist. Endogenizing the market structure we can derive again a positive correlation between number of firms and effort differentials and, possibly, also between number of firms and absolute effort (at least for the most productive managers).

Under complementarity ($b < 0$), the positive relation between number of firms and effort differentials remains. However, one can verify from (19) that also the absolute effort level is unambiguously increasing in the number of firms for all types. This is another case in which competition increases both relative and absolute efforts, but the intuition is now different: under our specification, the production of a larger number of complement goods increases the profitability of effort for each firm.¹⁶

3.2 Complete contracts

Until now we have implicitly assumed that the contracts were highly incomplete, in the sense that we excluded the possibility for the firm's owners to determine the market strategies under all the possible states of the world. When the contracts of the other firms are observable this is a binding assumption because firms would prefer to write down more complex contracts with their managers to constrain their operative activity. In this section we analyze exactly a form of contract competition where the contracts include not only a wage and an effort choice, but also an output level for each state of the world.¹⁷ This corresponds to

¹⁶This case has the undesirable feature that equilibrium profits can increase with the number of firms. For this reason, the model with complement goods does not allow one to endogenize the market structure.

¹⁷Notice that, even in this general specification of the contracts, we are still assuming some form of contractual incompleteness. The reason is that the contracts of the other firms are observable but not verifiable, therefore firms cannot write contracts contingent on the *ex post* realization of the contracts of the other firms. If such an unrealistic form of complete contingent contracts was allowed, we would obtain more

the assumption made by Martimort (1996) and Piccolo *et al.* (2008). As we will show, the availability of such a complete contract reduces the equilibrium efforts, but does not change the qualitative nature of our main result under strategic substitutability: competition increases the effort differentials.

Under symmetric information, the expected profits are:

$$E(\pi_i) = E \left(a - c + \sqrt{e_i} - \sum_{j=1}^n x_j \right) x_i - w_i$$

where the expectation is on the output levels of the rivals. A contract for firm i is given by a map of contracts $(e^i(\theta), x^i(\theta), w^i(\theta))$ with $(e, w, x) : [\theta_1, \theta_2] \rightarrow \mathbb{R}_+^3$. The optimal contract satisfies the optimality conditions:

$$\sqrt{e^i(\theta)} = \frac{E[x_i | \theta]}{2\theta} \quad (21)$$

$$x^i(\theta) = \frac{a - c + \sqrt{e^i(\theta)} - E \left[\sum_{j \neq i} x^j(\theta_j) \right]}{2} \quad (22)$$

$$w^i(\theta) = \theta e^i(\theta) \quad (23)$$

From the second condition we can derive the expected output level of a firm with a manager of type θ as:

$$E[x_i | \theta] = \frac{a - c + n\sqrt{e^i(\theta)} - \sum_{j \neq i} E \left[\sqrt{e^j(\theta_j)} \right]}{n + 1} \quad (24)$$

Substituting this in (21), and isolating for $\sqrt{e^i(\theta)}$, we have:

$$\sqrt{e^i(\theta)} = \frac{a - c - \sum_{j \neq i} E \left[\sqrt{e^j(\theta_j)} \right]}{n \left[\frac{2(n+1)}{n} \theta - 1 \right]} \quad (25)$$

whose expectation can be solved for the expected effort:

$$E \left[\sqrt{\tilde{e}(\theta)} \right] = \frac{(a - c) \tilde{S}^*(n)}{n + (n - 1) \tilde{S}^*(n)} \quad \text{with } \tilde{S}^*(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(\theta) d\theta}{\frac{2(n+1)}{n} \theta - 1} \quad (26)$$

where $\tilde{S}^*(n)$ can be shown to be smaller than $S^*(n)$. This implies that the average effort is reduced when the firms can contractually commit

complex results. In the two-types-two-firm case we would have “no distortion at the top” for an efficient manager who happens to meet an efficient rival, an upward distortion of effort for an efficient manager who happens to meet an inefficient rival, and standard downward distortions of the effort for an inefficient manager. Details are available from the authors.

to their market strategies. Of course, the lower effort levels tend to reduce production and increase profits. The intuition for these results relies on the fact that, under basic contract competition, firms tended to invest too much to commit to a higher production in the market: since the managers decided how much to produce without taking in consideration the impact on the rivals, this led to excessive investment *ex ante* and excessive production *ex post* from the point of view of the firms. The commitment possibility allows the firms to limit this tendency and reduce final production. In other words, complete contracts allow firms to soften competition.

The final equilibrium effort can be derived substituting (26) in (25):

$$\sqrt{\tilde{e}^*(\theta)} = \frac{a - c}{\left[\frac{2(n+1)}{n} \theta - 1 \right] \left[n + (n-1) \tilde{S}^*(n) \right]} \quad (27)$$

The introduction of asymmetric information determines the same qualitative results of our basic model. The first order conditions for the optimal contract of firm i under the individual rationality and incentive compatibility contracts are:

$$\sqrt{e^i(\theta)} = \frac{E[x_i | \theta]}{2[\theta + h(\theta)]} \quad (28)$$

and (24). The usual analysis allows us to derive the equilibrium effort as:

$$\sqrt{\tilde{e}(\theta)} = \frac{a - c}{\left[\frac{2(n+1)}{n} (\theta + h(\theta)) - 1 \right] \left[n + (n-1) \tilde{S}(n) \right]} \quad (29)$$

where we defined:

$$\tilde{S}(n) \equiv \int_{\theta_1}^{\theta_2} \frac{f(\theta)}{\frac{2(n+1)}{n} (\theta + h(\theta)) - 1} d\theta < \tilde{S}^*(n)$$

A comparison of (27) and (29) leads to conclude with:

Proposition 9. *Under contractual commitment on the production schedules, asymmetric information induces extra effort for the most efficient managers compared to the equilibrium with symmetric information and reduces the average effort compared to the case without such a contractual commitment.*

All the qualitative results of the previous sections hold also in this case with immediate adaptations. In particular, given any two types $\theta_a > \theta_b$, the effort differential:

$$\sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{2(n+1)\theta_b - n}{2(n+1)\theta_a - n} \quad (30)$$

is still increasing in the number of firms. Moreover, one can verify that the effort differential for a given number of firms is higher compared to the baseline model. Finally, since complete contracts soften competition, the case of endogenous market structures leads to a larger number of firms compared to the baseline case, and therefore to even higher effort differentials.

3.3 Aggregate shocks

While introducing imperfect correlation between types is quite complex, the case of perfect correlation, which corresponds to aggregate shocks on the cost function, can be easily analyzed. Consider the model with complete contracts of the previous section. In this set up, the introduction of perfect correlation between types leads to a similar framework as that in Martimort (1996), though extended to more than two firms.

The optimality conditions for any firm i are the same as before. However, under perfectly correlated shocks, we can impose the rational expectation assumption of symmetric contracts with $E[\sqrt{e^i(\theta)}] = E[\sqrt{e^j(\theta_j)}]$ for any i, j in (24). Accordingly, the expected production becomes:

$$E[x_i | \theta] = \frac{a - c + E[\sqrt{e(\theta)}]}{n + 1}$$

Under asymmetric information, combining the expected production with (28) allows one to derive the average effort and, consequently, the equilibrium effort function as:

$$\sqrt{e^{AS}(\theta)} = \frac{a - c}{[\theta + h(\theta)] [2(n + 1) - \tilde{S}]} \quad \text{with } \tilde{S} \equiv \int_{\theta_1}^{\theta_2} \frac{f(\theta)d\theta}{\theta + h(\theta)} \quad (31)$$

Notice that, contrary to the previous cases, the absolute effort level is now always decreasing in the number of firms for all types. Moreover, \tilde{S} is independent from n , therefore the “no-distortion at the top” property holds for any number of firms:

Proposition 10. *Under perfectly correlated shocks, asymmetric information reduces the equilibrium effort for all types, except the most efficient one, compared to the equilibrium with symmetric information.*

In this case, as in Martimort (1996) and Piccolo *et al.* (2008), there is no role for strategic interactions between different contracts, the “no-distortion at the top” property always holds and competition reduces the intensity of the absolute incentive mechanisms for all types. Moreover, one can verify that the effort differential is now independent from the

number of competitors. Given two types θ_a and θ_b with $\theta_a > \theta_b$, from (31) we can calculate the effort differential as:

$$\sqrt{\frac{e^{as}(\theta_a)}{e^{as}(\theta_b)}} = \frac{\theta_b + h(\theta_b)}{\theta_a + h(\theta_a)} \quad (32)$$

which is also independent from the number of firms. Therefore, a positive impact of competition on relative and absolute effort levels and on the power of the incentive mechanisms cannot emerge in the presence of aggregate shocks *à la* Martimort (1996). This impact requires heterogeneity between firms due to uncorrelated or, possibly, imperfectly correlated shocks.

While complex to analyze, intermediate situations with imperfect but positive correlation between types would lead to results qualitatively similar to our results under zero correlation. Finally, situations with a negative correlation between types would enhance the polarization of efforts. See the companion paper Cella and Etro (2010) for an analysis of imperfect correlation in the two-types-two-firms case.

4 CONCLUSION

In this paper we have analyzed competition between firms in the choice of incentive contracts for their managers. We have shown that in an equilibrium with perfect information for each firm on its manager's productivity, contract competition tends to increase the effort of the efficient managers and to decrease that of the inefficient managers. With asymmetric information, the equilibrium screening contracts are characterized by no-distortion in the middle, with efficient managers providing extra effort and with an additional downward distortion on the effort of the inefficient managers. In both cases, the relative effort required from an efficient types increases when the number of firms increases, but all absolute effort levels converge to zero when approaching the perfectly competitive limit. We also considered the case of endogenous market structures: in general a larger market increases the effort differentials and tends to increase also the absolute investments of the most efficient firms. This implies that a positive correlation between the number of firms (as a proxy of competition) and the strength of the incentive schemes is perfectly consistent with standard principal-agent theory.

As shown in the companion paper Cella and Etro (2010), our analysis applies to other related models, including those based on price competition and spatial competition, models of competition for the market and models of regulation in which one firm is aimed at maximizing welfare - in which case the pricing-incentive dichotomy of Laffont and Tirole

(1990, 1993) breaks down. Future research could investigate other applications of this form of contract competition, analyze its implications for the market structure in further details, and verify its positive predictions on the empirical side. A second direction for future research may focus on type-dependent participation constraints,¹⁸ a case in which the “no-distortion at the top” property can fail also in a single-firm environment. A last interesting extension could be about dynamic competition: two-period analysis of contracts without commitment have emphasized the emergence of pooling (rather than separating) contracts under both monopoly and perfect competition,¹⁹ but extensions to an oligopolistic framework remain entirely unexplored. Finally, we hope that this work will stimulate additional investigations on the relation between contract theory and industrial organization.

¹⁸See for instance Laffont and Martimort (2002, Ch. 3.3), Maggi and Rodriguez-Clare (1995) and Biglaiser and Mezzetti (2000).

¹⁹See for instance Laffont and Martimort (2002, Ch. 9.3) on the optimal monopolistic contracts and Etro (2004) on the equilibrium contracts in a two period insurance market with adverse selection.

Appendix

Proof of Proposition 1. Deriving (11) with respect to θ and using the monotone hazard rate property it follows that effort decreases in the type for any n , which confirms that the neglected constraint $\partial e^i(\theta)/\partial\theta \leq 0$ was non-binding. Since $S(n) \leq S^*(n)$ a comparison of the weighted average efforts in (5) and (10) shows that $E[\sqrt{e(\theta)}] < E[\sqrt{e^*(\theta)}]$. The expected total production is $E[X] = \left\{ a - c + E[\sqrt{e(\theta)}] \right\} n/(n+1)$, therefore the price is given by:

$$E[p] = \frac{a + nc - nE[\sqrt{e(\theta)}]}{n+1}$$

which provides the expression in the text after substituting for the average effort. Since the latter decreases with asymmetric information, the expected price must increase. \blacksquare

Proof of Proposition 2. Deriving (11) with respect to n we obtain:

$$\begin{aligned} \frac{\partial \sqrt{e(\theta)}}{\partial n} &\propto 1 + S(n) + (n-1)S'(n) - [\theta + h(\theta)] \left(1 + \frac{1}{n} \right) \\ &\cdot \left[\frac{n-1}{n}(1 + S(n)) + \frac{2S(n)}{n^2} + \frac{n^2-1}{n}S'(n) \right] \end{aligned}$$

which is decreasing in θ and positive only for values of θ close enough to its lower limit $n^2/(1+n)^2$. Therefore, if the effort of some types increases with the number of firms, it must be for any type $\theta \in [\theta_1, \tilde{\theta})$ where the cut-off $\tilde{\theta}$ is such that $\partial \sqrt{e(\tilde{\theta})}/\partial n = 0$. Finally, from the equilibrium efforts for types $\theta_a < \theta_b$ one can derive:

$$\sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{(n+1)^2 [\theta_b + h(\theta_b)] - n^2}{(n+1)^2 [\theta_a + h(\theta_a)] - n^2}$$

which is greater than one (because the virtual type is increasing in θ under the monotone hazard rate property) and it is increasing in n since:

$$\frac{\partial \sqrt{e(\theta_a)/e(\theta_b)}}{\partial n} = \frac{(n+1) [\theta_b + h(\theta_b) - \theta_a - h(\theta_a)]}{n^3 [(n+1)^2 [\theta_a + h(\theta_a)] - n^2]^2} > 0$$

Its upper limit is:

$$\lim_{n \rightarrow \infty} \sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{\theta_b + h(\theta_b) - 1}{\theta_a + h(\theta_a) - 1}$$

which is always finite. To verify this, notice that the ratio between maximum and minimum effort tends to:

$$\sqrt{e(\theta_1)/e(\theta_2)} = \frac{\theta_2 + 1/f(\theta_2) - 1}{\theta_1 - 1}$$

since $F(\theta_1) = 1 - F(\theta_2) = 0$. \blacksquare

Proof of Proposition 3. First of all, notice that:

$$\lim_{n \rightarrow \infty} S(n) = \lim_{n \rightarrow \infty} \int_{\theta_1}^{\theta_2} \frac{f(s)}{\left(1 + \frac{1}{n}\right)^2 (s + h(s)) - 1} ds = \int_{\theta_1}^{\theta_2} \frac{f(s) ds}{s + h(s) - 1} > 0$$

which also shows that $\lim_{n \rightarrow \infty} S(n)/n = 0$. This implies:

$$\lim_{n \rightarrow \infty} \sqrt{e(\theta)} = \lim_{n \rightarrow \infty} \frac{a - c}{n \left\{ \left(1 + \frac{1}{n}\right)^2 [\theta + h(\theta)] - 1 \right\} \left[1 + S(n) - \frac{S(n)}{n} \right]} = 0$$

for any θ . Zero effort by all firms implies an equilibrium independent from the type of managers with a limit price $\lim_{n \rightarrow \infty} p = c$. This trivially implies zero profits for all firms. \blacksquare

Proof of Lemma 4. First of all, notice that under our assumptions, the functions (11) and (6) are both continuous and decreasing on $[\theta_1, \theta_2]$. Since $S(n) \leq S^*(n)$ and $F(\theta_1) = 0$, we know that asymmetric information increases the effort required from the most efficient type: $e(\theta_1) > e^*(\theta_1)$. Moreover, since $E[\sqrt{e(\theta)}] < E[\sqrt{e^*(\theta)}]$ from Proposition 1, continuity of the two equilibrium effort functions imply that they must cross at least once - otherwise effort would always be larger under asymmetric information, which would be a contradiction. Any common point of the two functions must satisfy $e(\theta) = e^*(\theta)$. Using (11) and (6), this provides:

$$\frac{(1 + 1/n)^2 \left[\theta + \frac{F(\theta)}{f(\theta)} \right] - 1}{(1 + 1/n)^2 \theta - 1} = \frac{n + (n - 1)S^*(n)}{n + (n - 1)S(n)} \quad (33)$$

Define $\hat{\theta}_i$ as the positive roots of this equation with $\theta_1 < \hat{\theta}_1 < \hat{\theta}_2 < \dots < \hat{\theta}_z$. It immediately follows that asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$ and $\theta \in (\hat{\theta}_j, \min(\hat{\theta}_{j+1}, \theta_2))$ with j even, and decreases the effort of all types $\theta \in (\hat{\theta}_j, \min(\hat{\theta}_{j+1}, \theta_2))$ with j odd.

We now verify the condition under which the two functions cross only once. The right hand side of (33) does not depend on θ . The left hand side is equal to 1 for $\theta = \theta_1$, and its slope is proportional to:

$$h'(\theta) - \frac{h(\theta)(1 + 1/n)^2}{(1 + 1/n)^2 \theta - 1}$$

We can rewrite (33) as:

$$\frac{(1 + 1/n)^2 h(\theta)}{(1 + 1/n)^2 \theta - 1} = \frac{(n - 1) [S^*(n) - S(n)]}{[n + (n - 1)S(n)]} > 0$$

Using this, the slope of the left hand side of (33) is positive if:

$$h'(\theta) > \frac{(1 + 1/n)^2 h(\theta)}{(1 + 1/n)^2 \theta - 1} = \frac{(n - 1) [S^*(n) - S(n)]}{[n + (n - 1)S(n)]} \quad (34)$$

If this condition is satisfied for any $\hat{\theta}$, we must have a single root for (33) and asymmetric information increases the effort of all types $\theta \in (\theta_1, \hat{\theta}_1)$ and decreases the effort of all types $\theta \in (\hat{\theta}_1, \theta_2)$. If the condition is not satisfied for one $\hat{\theta}$ we must have at least two crossing between the effort functions. ◀

Proof of Proposition 5. Immediate from Lemma 4. ◀

Proof of Proposition 6. In case of unobservable contracts and costs, at the market competition stage all firms choose their strategies taking as given the expected strategies of the other firms. The first order conditions for firms $j = 1, 2, \dots, n$, given by:

$$x_j = a - c + \sqrt{e_j} - E[X]$$

provide the expected total production:

$$E[X] = \frac{n(a - c) + E \left[\sum_{j=1}^n \sqrt{e_j} \right]}{n + 1}$$

This allows us to rewrite the equilibrium production of each firm as:

$$x_i = \frac{a - c + n\sqrt{e_i} - E \left[\sum_{j \neq i} \sqrt{e_j} \right]}{n + 1}$$

which generates expected profits $\pi_i = x_i^2 - w_i$.

At the contractual stage, under symmetric information, firm i chooses a map of contracts to maximize for each type θ_i the following net profits:

$$\pi_i = \left(\frac{a - c + n\sqrt{e^i(\theta_i)} - E \left[\sum_{j \neq i} \sqrt{e^j(\theta_j)} \right]}{n + 1} \right)^2 - w_i$$

where the expectations of the effort functions are taken over the types of all the other firms, whose contracts are considered as given. Subject to the constraint

$w_i = \theta_i e^i(\theta_i)$, the profit maximizing conditions are exactly the same as in (4), which under symmetry leads to the same equilibrium contracts as in (6).

At the contractual stage, under asymmetric information, the net expected profits of firm i are:

$$E(\pi_i) = \int_{\theta_1}^{\theta_2} \left[\left(\frac{a - c + n\sqrt{e^i(\theta)} - E \left[\sum_{j \neq i} \sqrt{e^j(\theta_j)} \right]}{n + 1} \right)^2 - w^i(\theta) \right] f(\theta) d\theta$$

and the optimality conditions correspond to (9), leading again to the same equilibrium contracts (11) as before. ¶

Proof of Proposition 7. The proof is immediate since, according to Proposition 2, the effort differentials are increasing in n , and (18) implies for any two types θ_a and θ_b with $\theta_a > \theta_b$, the following effort differential:

$$\sqrt{\frac{e(\theta_a)}{e(\theta_b)}} = \frac{(\hat{n} + 1)^2 [\theta_b + h(\theta_b)] - \hat{n}^2}{(\hat{n} + 1)^2 [\theta_a + h(\theta_a)] - \hat{n}^2}$$

where the endogenous number of firms \hat{n} is decreasing in $(a - c) / \sqrt{K}$. Accordingly, the effort differential is increasing in $(a - c) / \sqrt{K}$. ¶

Proof of Proposition 8. First of all, assume $\theta_1 + h(\theta_1) > A(n)^{-1}$ to insure the existence of an interior equilibrium. Deriving $A(n)$ one can verify that:

$$A'(n) = -2b^2 (2 - b)^2 \left[\frac{2 + b(n - 1)}{2 + b(n - 2)} \right] < 0$$

for any b . Therefore, it follows that $S(n)$ and $S^*(n)$ are increasing in the number of firms for any b and $S(n) \leq S^*(n)$ for any b . Accordingly, we can compare the equilibrium effort with and without asymmetric information for the most efficient type, for which $h(\theta_1) = 0$. For any $b > 0$ we have:

$$\begin{aligned} \sqrt{e(\theta_1)} &= \frac{(2 - b)(a - c)}{[A(n)\theta_1 - 1] \{ [n + (n - 1)S(n)]b + 2(1 - b) \}} > \\ &> \sqrt{e^*(\theta_1)} = \frac{(2 - b)(a - c)}{[A(n)\theta_1 - 1] \{ [n + (n - 1)S^*(n)]b + 2(1 - b) \}} \end{aligned}$$

since both denominators are increasing in respectively $S(n)$ and $S^*(n)$ but $S^*(n) \geq S(n)$. For any $b < 0$ we have $\sqrt{e(\theta_1)} < \sqrt{e^*(\theta_1)}$ because the denominators are now decreasing in $S(n)$ and $S^*(n)$. Moreover, for any $\theta > \theta_1$ we also have $\sqrt{e(\theta)} < \sqrt{e^*(\theta)}$ because both terms at the denominator

of the effort functions are larger under asymmetric information compared to symmetric information:

$$\begin{aligned} A(n)(\theta + h(\theta)) - 1 &> A(n)\theta - 1 \\ [n + (n - 1)S(n)]b + 2(1 - b) &> [n + (n - 1)S^*(n)]b + 2(1 - b) \end{aligned}$$

Finally, for $b = 0$ we have $A(n) = 4$ for any n and:

$$\sqrt{e(\theta_1)} = \frac{a - c}{4\theta_1 - 1} = \sqrt{e^*(\theta_1)}$$

as under a monopoly, therefore this is the separating case in which asymmetric information induces the same effort for the most efficient managers compared to the equilibrium with symmetric information. ¶

Proof of Proposition 9. The proof of the first part is equivalent to the proof in Proposition 2. The average effort can be derived from (29) as:

$$E \left[\sqrt{\tilde{e}(\theta)} \right] = \frac{(a - c) \tilde{S}(n)}{n + (n - 1)\tilde{S}(n)}$$

which differs from (10) only for $\tilde{S}(n)$ replacing $S(n)$. To verify that complete contracts reduce the average effort, it is enough to check that $\tilde{S}(n)$ is smaller than $S(n)$, but since:

$$\frac{2(n + 1)}{n} > \frac{(n + 1)^2}{n^2} \Leftrightarrow n > 1$$

this is immediate for any n by direct comparison. ¶

Proof of Proposition 10. An analogous derivation to that the one in the text allows us to obtain the equilibrium effort under symmetric information as:

$$\sqrt{e^{*AS}(\theta)} = \frac{a - c}{\theta \left[2(n + 1) - \tilde{S} \right]}$$

which is clearly larger than $\sqrt{e^{AS}(\theta)}$ for any $\theta > \theta_1$, while $e^{AS}(\theta_1) = e^{*AS}(\theta_1)$. Deriving (31) with respect to n shows the last claim. ¶

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