DEPARTMENT OF ECONOMICS

UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

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No. 83 - November 2004

Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
http://dipeco.economia.unimib.it

Fiscal and Monetary Policy Interactions in a New Keynesian Model with Liquidity Constraints

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November 10, 2004

Abstract

This paper derives a New Keynesian dynamic general equilibrium model with liquidity constrained consumers and sticky prices. The model allows a role for both government spending and taxation in the DGE model. The model is then estimated using Euro area data. We demonstrate that there seems to be a significant role for rule-of-thumb consumer behaviour. Our model is then used to analyse the interaction between fiscal and monetary policies. We examine the extent to which fiscal policy (automatic stabilisers) assist or hinder monetary policy when the latter takes a standard forward-looking inflation targeting form. We also examine the extent to which inertia in fiscal policy and the presence of rule-of-thumb consumers affects output and inflation variability in the presence of such a monetary policy rule.

JEL Codes: E58, E62, E63

1 Introduction

Despite the existence of a vast literature on the robustness and optimality of monetary policy rules, relatively little attention has been given to the issue of monetary-fiscal policy interactions. A number of papers have examined the interdependence between fiscal and monetary policies using New Keynesian dynamic general equilibrium models[1], or game-theoretic models[2], but none of these models have been tested empirically, with the exception of Muscatelli et al. (2003). In this paper we estimate a small econometric model for the Euro area over the sample period 1970-1998, and analyse the performance of monetary rules in the presence of fiscal stabilizers. While the structural model used in this paper has many elements in common with other New Keynesian dynamic stochastic general equilibrium (DSGE) models, our analysis differs in many aspects. First, we extend some current DSGE models to include a wider range of fiscal policy transmission channels. Second, our model is estimated, in contrast to some attempts to calibrate or numerically simulate these models. Third, the focus of our paper is on the way in which inertial policy rules interact with inertia in the structural model due to the presence of non-optimising consumers and firms. Finally, we also examine the behavior of fiscal policies in a basic two-country version of our Euro-area model, in the presence of a monetary union.

Conventional New Keynesian DSGE models (as discussed for instance in Galí, 2003) typically provide a very limited role for fiscal policy. The standard forward-looking IS curve is based on the assumption of "Ricardian" forward-looking consumers, who have full access to complete financial markets. This assumption is contradicted by the empirical evidence on the permanent income hypothesis which supports the view that a significant proportion of consumers are non-Ricardian. Moreover, conventional DSGE models cannot rationalize the positive response of consumption to public expenditure shocks. To account for these effects, we adopt the innovation proposed by Galí *et al.* (2002), who assume that a fraction of households are constrained to consume out of current income. By doing so, we are also able to model the demand effect of other fiscal variables, i.e. taxes and transfers. On the

[1] See for example Leith and Wren-Lewis (2000), Schmitt-Grohé and Uribe (2001), Benigno and Woodford (2003) for an analysis of fiscal and monetary interactions in theoretical models. Perez and Hiebert (2002) and Zagaglia (2002) have experimented with DGE model simulations which include some fiscal closure rules.

[2] See Dixit and Lambertini (2000, 2001).

supply side of the economy, to our knowledge existing empirical New Keynesian DSGE models neglect fiscal distortions. In this paper we make a first attempt at estimating the empirical effect of the tax wedge on the Phillips curve in New Keynesian DSGE models.

We use our estimated model to undertake a number of dynamic simulations, examining the responses of the endogenous variables (including the policy instruments) to unanticipated structural and policy shocks.

Finally, we conduct some policy analysis with our estimated models. This allows us to consider whether the introduction of endogenous fiscal policy rules markedly changes the performance of the monetary policy rule. Earlier contributions (Muscatelli *et al.*, 2003) had found that countercyclical fiscal policy can be welfare-reducing in the presence of optimizing monetary policymakers. In contrast to this evidence, by introducing a role for taxation in the DSGE model, we find that automatic stabilizers based on taxation tend to be more efficient than those based on government spending. We also analyze the impact of inertia (persistence) in the fiscal rule and in the structural model on the performance of the monetary and fiscal policy rules, and find that inertial taxation rules tend to be more efficient than inertial government expenditure rules. Finally we confirm the results in Galí et al. (2003) that the presence of rule of thumb consumers tends to create more instability in the model (by increasing the variability of output and inflation following an inflation shock), but also find that automatic stabilizers based on taxation tend to offset the impact of rule-of-thumb consumers.

The rest of this paper is organized as follows. In the next section we briefly survey the existing literature. In Section 3, we outline the structure of our estimated model and the empirical methodology. In Section 4, we report our estimates and examine some dynamic simulations from our estimated models, whereas in Section 5 we examine the performance and interaction of the monetary and fiscal policy rules. In section 6 we present a two country model for monetary and fiscal policies interections. Section 7 concludes.

2 The Existing Literature

Much of the literature on fiscal-monetary policy interactions has focused on whether monetary and fiscal policy operate as strategic complements or substitutes. Dixit and Lambertini (2000, 2001) explore the interdependence between the fiscal authority and the central bank in a model where the latter has only partial control over inflation, which is also directly affected by the fiscal policy stance. They show that in equilibrium the two policy rules are complements when fiscal expansions have non-Keynesian (contractionary) effects on output and inflation. Buti, Roeger and in't Veld (2001) suggest that the specific form of interdependence between fiscal and monetary policies, i.e. the alternative between strategic substitutability and complementarity, should not necessarily be interpreted in terms of conflict or cooperation, and might be shock-dependent. In their model supply shocks unambiguously induce conflicting policies, whereas the opposite occurs for demand shocks.

Empirical contributions in this area are mainly based on panel data techniques and VAR analyses. Cross-sectional or panel data examine the relationship between fiscal and monetary policies over the cycle. Work by Mélitz (1997, 2000) and Wyplosz (1999) broadly supports the view that the two policies have acted as strategic substitutes over the last 2-3 decades. Von Hagen, Hughes-Hallett and Strauch (2001) find that the interdependence between the two policymakers is asymmetric: looser fiscal stances match monetary contractions, whereas monetary policies broadly accommodate fiscal expansions. Muscatelli *et al.* (2001) examine the interaction between fiscal and monetary policy instruments using conventional VAR and Bayesian VAR models for several G7 economies, and show that the fiscal shocks identified in the VAR have a significant impact[3]. They find that the result of strategic substitutability does not hold uniformly for all countries. Moreover, they report strong evidence that the linkage between fiscal and monetary policy has shifted post-1980, when fiscal and monetary policies became much more complementary. The main problem with this empirical literature is that without a structural model it is difficult to interpret the empirical correlations between the two policy variables. In the work of Mélitz (1997, 2000) and Wyplosz (1999) one cannot tell whether the correlation between the policy instruments over the cycle derives from systematic policy responses or from responses to structural or policy shocks. In the VARs estimated by Muscatelli *et al.* (2001) the focus is on the reaction of policy instruments to other policy shocks, but it is notoriously difficult to interpret implicit policy reaction functions in VARs especially if the 'true' underlying structural model is forward-looking. More recently, Muscatelli *et al.* (2003) examine the in-

[3] The number of contributions applying VAR techniques is still limited. This may be due to the critique in Mountford and Uhlig (2002) that true fiscal policy surprises may be difficult to detect in a VAR model.

teraction of monetary and fiscal policies using an estimated New Keynesian dynamic general equilibrium model for the US. In contrast to earlier work they show that the strategic complementarity or substitutability of fiscal and monetary policy depends crucially on the types of shocks hitting the economy, and on the assumptions made about the underlying structural model. The greater complementarity of fiscal and monetary policy seen in the 1990s compared to the 1980s was due to the changing nature of the underlying shocks.

Our focus in this paper is different. We estimate a New Keynesian DSGE model which, in contrast to our earlier work and other attempts to estimate structural New Keynesian models[4], allows for a richer range of transmission channels for fiscal policy, whilst still maintaining a model where the structural parameters are estimated using econometrics. This model is then used to conduct policy analysis to see how fiscal and monetary policy interact and what implications the degree of inertia in the structural model and in the policy rules has for monetary and fiscal policy design. The introduction of central bank independence in most of the industrialized economies has raised the issue of whether fiscal and monetary policies are properly co-ordinated. One motivation for this paper is to show that fiscal stabilizers, which can be shown to be counterproductive in standard DSGE models (e.g. Muscatelli *et al.*, 2003)[5] significantly improve welfare in an economy characterized by an important proportion of rule-of-thumb consumers. In particular, taxation rules based on automatic stabilisers can be shown to have a welfare-enhancing effect. Our results are complementary to those obtained using different frameworks by other researchers. Gordon and Leeper (2003), using a calibrated model for the US economy, find that fiscal stabilization policies tends to destabilize the business cycle because of their impact on debt service obligations. Jones (2002) uses an estimated stochastic growth model (without price stickiness) for the US to show that fiscal policy had limited stabilization effects in the post-war period.

[4] See Gali et al. (2001), Leith and Malley (2002), Smets and Wouters (2002).

[5] In Muscatelli et al. (2003) our fiscal rules are *estimated* and we do not examine alternative forms for these rules. In that paper we show that countercyclical fiscal policy can be welfare-reducing if fiscal and monetary policy rules are inertial and not co-ordinated. Our conjecture in that paper was that this surprising result was probably due to the interaction of highly inertial estimated monetary and fiscal policy rules. In this paper we study fiscal policy rules in a DSGE model which involves a richer range of fiscal channels.

3 A New-Keynesian Structural Model

We use a small forward-looking New Keynesian DSGE model, comprising a dynamic IS equation for output and a "New Keynesian Phillips Curve" specification for inflation.

3.1 Households

We assume two types of households. Households in the first group, i, benefit from full access to the capital markets and as such are free to optimize. The proportion of optimising consumers in the economy is given by $(1-\vartheta)$. Each optimizing consumer is assumed to maximize an intertemporal utility function given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{1}{1-\rho} (C_{t+s}^{oi}/H_{t+s}^{i})^{1-\rho} - \frac{\varepsilon_t^l}{1+\varphi} (N_{t+s}^{oi})^{1+\varphi} \right) \tag{1}$$

where C_t^o represents consumption of a basket of goods (to be defined below), H_t is an index of external habits and N_t^o is the level of employment. $\beta \in (0,1)$ represents the subjective rate of time preference, ρ the coefficient of relative risk aversion, φ the inverse of the elasticity of labour supply with respect to real wage and ε_t^l is a shock to labour supply. Finally, E_t denotes the expectation operator conditional on the time t information set. Following Smets and Wouters (2002) we assume that habits depend on past aggregate consumption, C^T:

$$H_{t+s}^{i} = \left(C_{t+s-1}^{T}\right)^{\lambda}. \tag{2}$$

Optimizing consumers maximize (1) subject to their intertemporal budget constraint, which is expressed in real terms as:

$$(1/r_t) a_{t+1}^{i} = a_t - C_t^{oi} + \frac{W_t}{P_t} N_t^{oi} + D_t^{i} + \left(G_t^{TRi} - T_t^{i}\right) \tag{3}$$

Accordingly, consumers hold their financial wealth (a_t) in the form of one-period state-contingent securities, which yield a return of r_t. The optimizing consumer's disposable income consists of labour income $w_t N_t^{oi}$ plus the dividends from the profits of the imperfectly competitive firms D_t^i, plus public transfers G_t^{TRi} minus personal taxes T_t^i, lump-sum by assumption.

As in Galí et al. (2002) we assume that a proportion ϑ of households follow a rule of thumb and consume out of current disposable income. This admittedly ad hoc assumption may be justified assuming myopia or limited participation to capital markets. We also assume that rule-of-thumb consumers supply a constant amount of labour[6], $\overline{N^{RT}}$. Thus, the consumption function of the representative rule-of-thumb consumer amounts to:

$$C_t^{RTj} = \bar{N}^{RT} \frac{W_t}{P_t} + \left(G_t^{TRj} - T_t^j \right). \tag{4}$$

Total consumption is given by a standard CES function of imperfectly substitutable varieties of consumption goods z:

$$C_t^i = \left[\int_0^1 \left(C_t^i(z) \right)^{\frac{\theta}{\theta-1}} dz \right]^{\frac{\theta}{\theta-1}} \tag{5}$$

where the constant elasticity of substitution, θ, between differentaited goods is assumed greater than one. Solving the *intratemporal* optimal allocation across each variety of the consumption goods leads to the following demand for good z:

$$C_t^i(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\theta} C_t^i \tag{6}$$

where $P_t(z)$ is the price of good z, and P_t is the consumption price index given by the aggregator:

$$P_t = \left[\int_0^1 \left(P_t(z) \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{7}$$

3.2 Firms

In the model economy there is a continuum of firms, indexed by $z \in [0, 1]$, acting as monopolistic competitors. Firms dispose the following simple Cobb-

[6] Galì et al. (2003) show that supplying a constant amount of labour is optimal when net taxes, $\left(G_t^{TR} - T_t \right)$, levied on rule-of-thumb consumers are always nil. This result would never obtain in our model, where taxes and transfers are explicitly modeled. Thus, for sake of simplicity we assume a constant labour supply. Since consumption cannot be negative, this implies that we impose a lower bound on $\left(G_t^{TR} - T_t \right)$ for any given level of the real wage.

Douglas function of labour for each consumption good variety z:

$$Y_t(z) = A_t(N_t(z))^{1-\alpha} \tag{8}$$

We introduce fiscal distortions by assuming that taxes on labour take the form of a uniform payroll tax[7]. Therefore, firms' demand for labour is defined as:

$$(1-\alpha)\, A_t(N_t(z))^{-\alpha} = \frac{W_t}{P_t} + t_t^{PR} \tag{9}$$

where t_t^{PR} is the tax rate per unit of employed labour, i.e. $t_t^{PR} = \frac{T_t^{PR}}{N_t}$, where T_t^{PR} is the total revenues from the payroll tax.

Turning next to the model of firms' pricing behavior, we consider a standard model of monopolistic competition with sticky prices, as set out in Galí, Gertler and López-Salido (2001), and Leith and Malley (2002)[8]. More precisely, sticky prices are incorporated into this model by assuming a Calvo pricing mechanism whereby only a given proportion of firms, defined as $(1-\xi)$, can adjust prices every period whereas the remainder supplies output on demand, at a constant price. A share γ of the adjusting firms is assumed to index prices to inflation in the previous period[9], while the remaininder, $(1-\gamma)$, set their prices optimally to maximize expected discounted real profits[10], with a discount factor β.

3.3 The IS and the Phillips curve

By log-linearizing the model around the deterministic steady state we are then able to derive a *hybrid* dynamic equation for output and the New Keynesian Phillips curve (see the Appendix for a proof)[11]. In what follows "hat-

[7] This implies that the optimizing consumer's choice between leisure and consumption is not affected.

[8] See also Erceg, Henderson and Levin (2000), and Sbordone (2002).

[9] This was pioneered by Galí and Gertler (1999). Similar backward-looking elements can be introduced to the NKPC equation by introducing indexation of all non-re-optimised prices (Christiano, Eichenbaum and Evans, 2001, and Woodford, 2002, chapter 3).

[10] A similar specification for the New Keynesian Phillips curve can be obtained by making the indexation process part of the optimisation process (see Smets and Wouters, 2002).

[11] We ignore investment and the external sector. Arguably, the open-economy considerations are less important to the USA, which is the focus of our analysis here. The extension of our modeling approach to the open economy is left to further work.

ted" lower-case variables represent percentage deviations from the steady state whereas "barred" variables denote steady-state values.

The log linearized equation for output reads as:

$$\widehat{y}_t = a_1 \left\{ \begin{array}{c} a_2 \left[a_3 E_t \left\{ \Delta \widehat{n}_{t+1} \right\} + a_4 \Delta \widehat{t}^{PR}_{t+1} \right] - a_5 E_t \left\{ \Delta \left(\widehat{G^{TR}_{t+1} - T_{t+1}} \right) \right\} + \\ + a_6 \frac{\overline{G}}{\overline{Y}} \widehat{g}_t + a_7 \left[\widehat{y}_{t-1} - \frac{\overline{G}}{\overline{Y}} \left(\widehat{g}_{t-1} \right) \right] - \left(\frac{\overline{C^o}}{\overline{C}} \right) \left(\frac{\overline{C}}{\overline{Y}} \frac{1}{\rho} \right) \widehat{r}_t + \widehat{y}_{t+1} - \frac{\overline{G}}{\overline{Y}} \widehat{g}_{t+1} \end{array} \right\} \tag{10}$$

where a_j, with $j = 1, ..., 7$, are function of structural parameters and steady state ratios. In particular, they are given by:

$$\begin{array}{ll} a_1 = \left[1 + a_7 \right]^{-1} & a_2 = \frac{\overline{N^{RT}}}{\overline{N}} \frac{\overline{N \left(\frac{W}{P} \right)}}{\overline{Y}} \\ a_3 = \left[\alpha - \frac{\overline{T^{TR}}}{\overline{N} \left(\frac{\overline{W}}{\overline{P}} + \frac{\overline{T^{TR}}}{\overline{N}} \right)} \right] \frac{\left(\frac{\overline{W}}{\overline{P}} + \frac{\overline{T^{TR}}}{\overline{N}} \right)}{\left(\frac{\overline{W}}{\overline{P}} \right)} & a_4 = \frac{\overline{T^{TR}}}{\overline{N \left(\frac{W}{P} \right)}} \\ a_5 = \vartheta \left(\frac{\overline{G^{TR} - T}}{\overline{Y}} \right) & a_6 = \left[1 + a_7 \right] \\ a_7 = \left(\frac{\overline{C^o}}{\overline{C}} \right) \left(\frac{\rho - 1}{\rho} \right) \lambda & \frac{\overline{C^o}}{\overline{C}} = 1 - \left(1 - \frac{\overline{G}}{\overline{Y}} \right)^{-1} \left[a_2 + a_5 \right] \end{array}$$

$\widehat{g}_t$ is government spending excluding government transfers $\widehat{G^{TR}}$. At first sight equation (10) looks very complex. In fact, by imposing no habit, $\lambda = 0$, and the absence of rule-of-thumb consumers, $\frac{\overline{N^{RT}}}{\overline{N}} = \vartheta = 0$, equation (10) collapses to a purely forward looking IS curve. Note that consumption habit introduces a link between current and past output (as in Carroll, 2000, Leith and Malley, 2002; Smets and Wouters, 2002). Moreover, the presence of non-optimizing consumers establishes a link between the demand for goods, net personal taxes, $\widehat{G^{TR} - T}$, and the real wage. Fiscal policy impacts on output in three ways. First, through the usual resource withdrawal effect of government consumption, $\widehat{g}_t$. Second, through the impact of net personal taxes $\widehat{G^{TR} - T}$ on the current disposable income of rule-of-thumb consumers. Third, through the impact of payroll taxes T^{PR}on the real wage of rule-of-thumb consumers[12]. Finally, rule-of-thumb consumers weaken the impact of interest rate policy on aggregate demand. As shown in Galì et al. (2003)

[12] From equations (4) and (9) it should be clear that, in each period, the equilibrium real wage is inversely related to employment and the payroll tax. In the Appendix we explain why the rate of change of these variables affects current output.

this may have important implications for the conduct of monetary policy. Indeed, our estimates confirm that rule-of-thumb consumers weaken the output response to interest rate changes.

It is important to note that whilst government spending impacts on the consumption behaviour of optimising consumers via the resource-withdrawal effect, taxation impacts through its effect on disposable income for rule-of-thumb consumers, and hence via the external habit (total consumption) variable. This ensures that government spending enters via a distributed lag in (10) which sum to zero, while personal and payroll taxes enter in differences, with coefficients of different size. As we shall see below, this drives some of the results of the model.

Log-linearization of the firms' optimal price, together with the assumptions about Calvo mechanism and indexation, leads to an expression for the inflation rate that reads as (see the Appendix for details):

$$\widehat{\pi}_t = \frac{\gamma\widehat{\pi}_{t-1} + \beta\xi E_t\widehat{\pi}_{t+1}}{\xi + \gamma(1-\xi(1-\beta))} + \frac{(1-\gamma)(1-\xi)(1-\gamma\xi)}{[\xi+\gamma(1-\xi(1-\beta))][1+(\alpha/(1-\alpha))\theta]}\widehat{s}_t \quad (11)$$

where $\widehat{s}_t$ is the percentage change from steady state of the labour cost share, which is given[13] by: $\widehat{s}_t = \frac{\overline{N\left(\frac{\overline{W}}{\overline{P}}\right)}}{\overline{N\left(\frac{W}{P}\right)}+\overline{T^{PR}}}\left(\widehat{w_t - p_t}\right) + \frac{\overline{T^{PR}}}{\overline{N\left(\frac{W}{P}\right)}+\overline{T^{PR}}}\left(\widehat{t}_t^{PR} - \widehat{n}_t\right) + \widehat{n}_t - \widehat{y}_t$

Equations (10) and (11) constitute our structural model. It is important to note that in estimating (11), we treat real wages and employment as exogenous. Other recent contributions (Leith and Malley, 2002, Smets and Wouters, 2002) estimate wage equations, and adding a wage equation would have enabled us to consider the possibility of sticky wage dynamics. However, this would have also added to the complexity of the model.

4 Empirical Results

4.1 Data and Scope of the Study

[13] Galí, Gertler and López-Salido (2001) specify (10) in terms of average real marginal cost (mc). Note that, in levels:

$$s_t = \frac{(1-\alpha)}{mc_t}$$

We now turn to the empirical results of the baseline model for the Euro area[14]. While we provide estimates of the structural parameters present in the IS curve (Eq.(10)), we use the estimates for the Phillips curve as reported in Galí *et al.* (2001, 2003). All data from the European Central Bank (see Fagan, Henry and Mestre (2001)), are quarterly time series over the sample period 1970(1)-1998(2). The data definitions used are reported in the Data Appendix.

To capture the spirit of the NK models as log-linearizations, the data are transformed so that the variables are expressed in deviations from the "steady state"[15]. Both real and nominal variables are de-trended using the Hodrick-Prescott filter with smooth parameter set to 1600. Note that as the inflation rate and interest rate always enter the model together, all the equations are 'balanced' in terms of the levels of integration of the dependent and explanatory variables.[16]

4.2 Estimation Methods

The New Keynesian model consists of equations that are non-linear in parameters. Following Hansen (1982) a model with rational expectations suggests some natural orthogonality restrictions that can be used in the generalized methods of moments (GMM) framework. Each equation estimated using GMM is of the form:

$$\mathbf{y}_{it} = \mathbf{f}_i(\boldsymbol{\theta}_i, \mathbf{z}_{it}) + \mathbf{u}_{it} \tag{12}$$

where for each equation i, $\mathbf{y}_{it}$ is the vector of dependent variables, $\boldsymbol{\theta}_i$ is the $(a_i \times 1)$ vector of unknown parameters to be estimated, and $\mathbf{z}_{it}$ is the $(k_i \times 1)$ vector of explanatory variables. The GMM approach is based on the fact that $\widetilde{\boldsymbol{\theta}}_i$, the true value of $\boldsymbol{\theta}_i$, has the property $E[\mathbf{h}_i(\ \widetilde{\boldsymbol{\theta}}_i, \mathbf{w}_{it})] = 0$, where $\mathbf{w}_{it} \equiv (\ \mathbf{y}'_{it}, \mathbf{z}'_{it}, \mathbf{x}'_{it})$, and $\mathbf{x}_{it}$ is an $(r_i \times 1)$ vector of instruments that are correlated with $\mathbf{z}_{it}$. GMM then chooses the estimate $\boldsymbol{\theta}_i$ so as to make the sample moment as close as possible to the population moment of zero. The validity of these instruments can be tested for each equation by using

[14] The estimation was carried out using RATS, version 5.

[15] Which is commonplace in this literature (see Smets and Wouters, 2002, Leith and Malley, 2002).

[16] The government spending data (G) is total government spending excluding transfers and interest payments, whilst we use employers' social security contributions as payroll taxes (T^*), and government transfers minus personal taxes as ($G^{TR} - T$).

Hansen's J-test, which is distributed as a $\chi^2(r_i - a_i)$ statistic under the null of valid orthogonality conditions.

GMM or IV estimation has been used by a number of authors to estimate NK models[17]. One problem is that the estimated IS and NKPC equations are highly nonlinear in parameters, and the rank condition for identification is not met unless a number of parameters in these two equations are fixed. To begin with, in estimating the output equation we impose that the steady-state ratios are given by their average values computed over the sub-sample period 1990(1)-1998(2).[18] Results are reported in Table 1.

Table 1: Steady State Values

$\frac{C}{Y}$	$\frac{G}{Y}$	$\frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{Y}}$	$\frac{\overline{T^{PR}}}{\overline{N\left(\frac{W}{P}\right)}}$	$\frac{\bar{T}}{\bar{Y}}$	$\frac{\overline{T^{PR}}}{\overline{N}\left(\frac{\overline{W}}{\overline{P}}+\frac{\overline{T^{PR}}}{\overline{N}}\right)}$	$\frac{\left(\frac{\overline{W}}{\overline{P}}+\frac{\overline{T^{PR}}}{\overline{N}}\right)}{\left(\frac{\overline{W}}{\overline{P}}\right)}$	$\frac{\overline{G^{TR}-T}}{\overline{Y}}$
0.62	0.16	0.503	0.353	0.121	0.260	1.353	-0.111

Furthermore, in order to increase the accuracy of the estimation we calibrate some structural parameters at values taken from other empirical studies. We impose the habit formation parameter on aggregate consumption to be equal to unity ($\lambda = 1$) and the coefficient of relative risk aversion to be equal to two ($\rho = 2$).

4.3 Model Estimates

Table 2 reports the estimated New Keynesian model using GMM over the full sample period. Standard errors for all the parameters estimates are reported in brackets. Our vector of instruments x_{it} include a constant plus de-trended output, government spending exculding government transfers, direct tax per worker, nominal exchange rate, wage rate, inflation rate and nominal interest rate.

[17] For instance, Galì, Gertler and Lopez-Salido (2001), Leith and Malley (2002), Kara and Nelson (2002), Muscatelli et al. (2003).

[18] $\overline{N\left(\frac{W}{P}\right)}/\overline{Y}$ is simply equal to the labour share in equilibrium, which we set equal to $(1-\alpha) = 0.6$. Furthermore, in computing the average value for $\frac{\overline{G^{TR}-T}}{\overline{Y}}$ we initially found a positive number (0.102), for this reason we decided to set to zero G^{TR} and obtain the numebr reported in the Table above.

In estimating the NK output equation we employed a two-step procedure. First of all, note that it is possible to write the IS curve so to have the first difference in expected real wage, $\Delta\hat{w}_{t+1}$, in place of $a_3 E_t \{\Delta\hat{n}_{t+1}\} + a_4 \Delta\hat{t}_{t+1}^{PR}$ and the first difference in personal taxes, $\Delta\hat{t}_{t+1}$, in place of $\Delta\left(\widehat{G_{t+1}^{TR} - T_{t+1}}\right)$. That being stated, one can estimate the reduced form of the IS equation(10) and obtain a point estimate for the fraction of rule-of-thumb consumers. Using six lags of the above instruments, the structural parameter ϑ turns out to be 0.505 with asymptotic standard error of 0.036.[19] More precisely, the estimated ϑ is computed by dividing the estimated coefficient on $\Delta\hat{t}_{t+1}$ by the coefficient on $\left\{\hat{y}_{t+1} - \frac{\overline{G}}{\overline{Y}}\hat{g}_{t+1}\right\}$ multiplied by -0.35.[20] In the second step of the estiamtion, we then re-estimate (10) having fixed the values of ρ and λ and the value ϑ from the first step and find structural estimates for the parameters $\overline{N^{RT}}/\overline{N}$ and $\overline{C^o}/\overline{C}$.

The overall fit for the estimated equation is good: the R^2 statistic for (10) is 0.83. The Hansen statistic for overidentifying restrictionts test is 59.39, which is distributed as a $\chi^2(41)$ under the null hypothesis of valid instruments. The null hypothesis of valid instruments is not rejected at the 5% significance level.

Our point estimates thus suggest that about 50% of consumers are rule-of-thumb consumers, whilst 65% of total consumption in steady state is given by optimising consumers. Rule-of-thumb consumers account for about 61% of total employment. Point estimates of the Calvo parameter suggests that about 84% of firms do not adjust their prices every period and of these about 30% simply index prices.

Table 2: Model Estimates

Parameter	λ	ρ	ϑ	$\frac{\overline{N^{RT}}}{\overline{N}}$	$\frac{\overline{C^o}}{\overline{C}}$	β	ξ	γ
Estimates	1.00 (−)	2.00 (−)	0.508 ()	0.617 (0.310)	0.658 (0.156)	0.923 (0.071)	0.843 (0.066)	0.307 (0.128)

Standard errors are reported in brackets.

[19]The standard error is been computed using the delta method.

[20]Note that from the first-step estimation one can also obtain point estimates for $\frac{\overline{N^{RT}}}{\overline{N}}$ and $\frac{\overline{C^o}}{\overline{C}}$, which are given by 0.604 and 0.778 respectively.

4.4 Dynamic and Stochastic Simulations

Having estimated our structural model, we now perform a number of dynamic simulation experiments to investigate the properties of this simple New Keynesian model[21] and the transmission mechanism of fiscal and monetary policies.

We focus on the dynamic model solution, shocking each structural equation and policy equation in turn, to simulate the effects of a structural or policy variable shock on the other endogenous variables in the model. This allows us to examine the properties of the model, and the response of output and inflation to policy and structural shocks. Essentially this involves simulating the model without any reference to actual data. The exogenous variables in the model i.e. government transfers (G^{TR}), the real wage $(\widehat{w-p})$ and employment ($\widehat{n}$), are simulated as follows: government transfers are simply assumed to be constant. We assume that nominal wages are indexed to inflation with a one-period lag[22], whilst employment is determined by the log-linearization of the production technology (8). To simulate the model, we close it by adding a Taylor rule for short-run nominal interest rate. In order to provide a baseline for an analysis of inertial rules below, we assume a very simple type of forward-looking non-inertial Taylor rule:

$$\widehat{i}_t = 1.5\left(E_t\widehat{\pi}_{t+1}\right) + 0.5\left(\widehat{y}_t\right) \tag{13}$$

Excluding inertia from this Taylor rule has the advantage of allowing us to focus on the simulation properties of the structural model and on the fiscal channels. The results of the dynamic model solution are shown in Figure 1, Figure 2 and Figure 3. These display the dynamic patterns of output, inflation and the real interest rate in response to a temporary shock to inflation equation. The initial shock is 1% and this then recedes with a 0.5 autoregressive parameter, and is set to zero after 4 quarters.

[21] The model is solved using Winsolve version 3.0 (see Pierse, 2000), which provides numerical solutions for linear and non-linear rational expectations models. We solve our model using the Stacked Newton method in Winsolve. In solving the models with structural shocks (and further below with policy shocks) these are treated as unanticipated by economic agents.

[22] The absence of a wage-setting equation is less problematic than might seem at first sight. If one looks at US data from the 1990s, one can see that real wages and employment were far less volatile around their trend during the 1990s. Thus the assumption that wages simply respond to lagged inflation is not a major departure from reality.

Figure1. Output Response to Inflation Shock

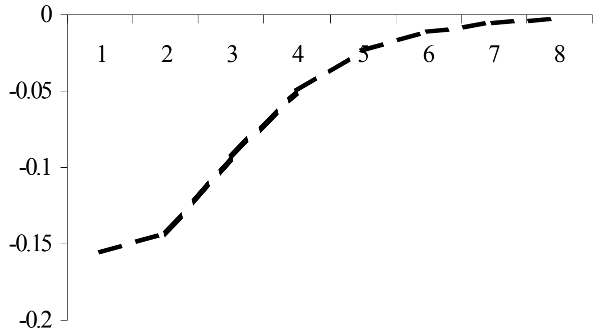


Figure2. Inflation Response to Inflation Shock

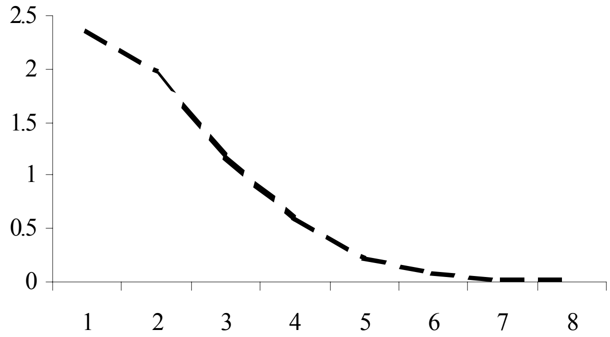
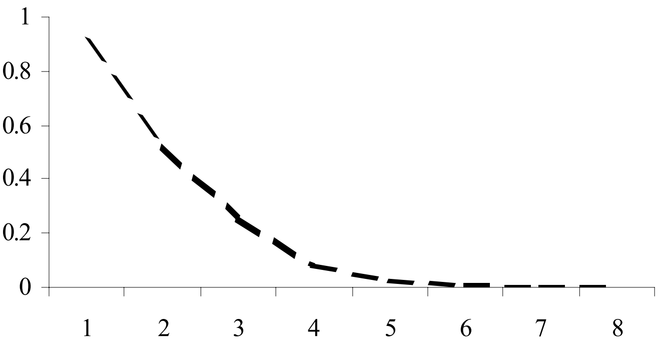


Figure3. Real Interest Rate Response to Inflation Shock



In the event of an inflation shock, the monetary authority responds by engineering a reduction in inflation and a persistent adjustment pattern in output. On impact, in fact, output jumps down to -0.15 and remains under steady state for about eight quarters. Such time patterns for $\hat{y}$ and $\hat{\pi}$ occur because of the persistent rise in the real interest rate, which remains above steady state for about six quarters.

5 Monetary and Fiscal Policy Interactions and Policy Design

5.1 Monetary and Fiscal Rules

Having examined the dynamic properties of our estimated model, we now turn to the issue of policy design. As noted above, the earlier literature on monetary-fiscal interactions focused exclusively on understanding whether monetary and fiscal policies have tended to act together over the cycle. A more important issue is whether fiscal policies, and in particular the automatic stabilizers considered here, actually assist or impede the efforts of an independent central bank which adopts a forward-looking inflation targeting rule. More precisely, how should automatic stabilizers be designed in order to ensure that monetary and fiscal policy act in concert, i.e. as strategic complements?

In an earlier paper, Muscatelli et al. (2003), we presented evidence that *estimated* fiscal policy rules for the US appeared to be welfare-reducing, which seemed to accord with the evidence (using different modeling approaches) in Gordon and Leeper (2003) and Jones (2002). From the point of view of a central bank adopting an optimal policy rule designed to minimize a standard quadratic loss function in deviations of output, inflation and changes in the policy instrument (the short run nominal interest rate), we are now able to re-examine the issue in a model where fiscal policy may play a more important role because rule-of-thumb consumers only indirectly react to the interest rate rule[23]. Furthermore, the current model considers some additional channels of transmission of fiscal policy: taxation effects on consumption through

[23]As shown in Galì et al. (2003), rule-of-thumb consumers are affected by interest rate changes only to the extent that the real wage adjusts following the new labour conditions determined by the optimising consumers' reaction to such interest rate changes

liquidity constrained consumers, and taxation wedge effects on inflation, as well as interaction effects due to the presence of rule-of-thumb consumers. In addition, instead of focusing on estimated fiscal rules we will consider a more systematic analysis of different rules for fiscal stabilizers.

5.1.1 Monetary Rule

Before turning to the issue of how one might design robust fiscal rules, let us turn first to monetary policy. In contrast to the numeros papers on the behaviour of the Federal Reserve and other central banks, the empirical literature on the European Central Bank's past behaviour seems instead at an initial stage, mainly due to its short history. The monetary policy rule for the nominal interest rate $\widehat{i}_t$ follows a form similar to the standard forward-looking Taylor rule specification which has become commonplace in the literature[24] (see Clarida, Galí and Gertler, 1998, 2000; Muscatelli *et al.* 2002; Giannoni and Woodford, 2002a,b):

$$\widehat{i}_t = \phi_1 E_t \widehat{\pi}_{t+q} + \phi_2 \widehat{y}_{t+s} + \phi_3 \widehat{i}_{t-1} \tag{14}$$

where the rule also allows for an interest-rate smoothing component if $\phi_3 \neq 0$.

In order to simulate monetary-fiscal policy interactions we use the estimates reported in Sauer and Sturm (2003), which provide us with a benchmark against which to assess the performance of different designs for automatic fiscal stabilizers in our structural model.

5.1.2 Fiscal Rules

We consider a simple backward-looking format for our fiscal policy rules (automatic stabilizers), following *inter alia* Van Den Noord (2000), Westaway (2003) and Andres and Domenech (2003). This captures the more realistic lagged response of fiscal policy to macroeconomic variables due to automatic stabilizers:

$$\widehat{g}_t = \delta_1 \widehat{g}_{t-1} - \delta_2 \widehat{y}_{t-1} \tag{15}$$

[24] The main difference is that we use a contemporaneous value of the output gap (see Muscatelli *et al.* 2002) as opposed to expected future values, as in Clarida, Gali and Gertler (1998, 2000). For a detailed discussion of these issues, see Giannoni and Woodford (2002a,b). For an alternative approach to modeling interest rate responses, involving nonlinearities in reaction functions, see Cukierman and Muscatelli (2001).

$$\widehat{\tau}_t = \varphi_1 \widehat{\tau}_{t-1} + \varphi_2 \widehat{y}_{t-1} \quad (16)$$

where $\widehat{\tau}_t$ is the vector of our two tax measures, personal taxes $\widehat{t}_t$ and payroll taxes, $\widehat{t}_t^{TR}$. Our taxation rule therefore imposes the same adjustment pattern on both taxes, and does not look at how a mix of tax measures might improve the design of policy[25]. The importance of the taxation policy mix is considered further below. Note that we do not allow for any feedback of policy to budget deficits or debt accumulation[26]. Recall that our models are estimated using detrended data and focus on stabilization over the cycle rather than the shifts in fiscal regimes which often accompany the correction of deficits, or debt-correction strategies. Our fiscal rules are largely capturing automatic stabilizers through the autoregressive and the output gap terms.

For our baseline case, we set $\delta_1 = \varphi_1 = 0.6$, $\delta_2 = \varphi_2 = 0.5$. A coefficient of 0.5 on output is consistent with the empirical evidence in Van Den Noord (2000) and adopted in studies on fiscal stabilization (e.g. Westaway, 2003). We allow for an element of inertia as empirical estimates of fiscal policy rules suggest an important role for an autoregressive term.

5.2 Government spending rules versus Taxation Rules

We now perform some dynamic simulation with our model, closing it by adding the estimated monetary policy rule and the taxation and government spending rules in (16) and (15). Rather than assuming a particular form of welfare loss function, in what follows we consider how the introduction of a fiscal policy rule impacts on output and inflation variability (variance

[25] Andres and Domenech (2003) provide an analysis of how different tax measures might impact on output and inflation variability.

[26] See for instance Bohn (1988) and Taylor (2000a,b). The lack of a debt or deficit stabilization term raises the issue of whether our fiscal rules imply a sustainable path for government debt. Given that we are not conducting historical simulations with our estimated models this not a problem, especially for small structural shocks. Obviously where one wishes to conduct historical or counterfactual simulations (see Muscatelli et al. 2003), then one would need to check whether the implied path for government debt is sustainable, and closely tracks that observed during the historical period analyzed. In this paper we will focus instead on dynamic simulations following small shocks and the issue of debt sustainability is less relevant, providing that we are considering sufficiently small shocks. Our fiscal rules are close in spirit to those of Taylor (2000a, b), who finds that countercyclical fiscal policy is almost entirely characterized by the working of automatic stabilizers.

frontiers) when it is combined with a monetary policy rule such as (14). Conducting welfare analysis with a NK model such as ours is complex, because of the presence of heterogeneous consumers (optimisers and rule-of-thumb consumers)[27], but computing variance frontiers allows a certain ranking of policy rules, where it is apparent that one rule dominates the other in terms of reducing both output and inflation variability.

To construct the variance frontiers we apply a monetary policy rule where we keep fixed parameters ϕ_2 and ϕ_3 and we allow ϕ_1 to vary[28]. We then compute the standard deviation of output and inflation in dynamic simulations following a shock to the Phillips Curve, and report these "variance frontiers" in the figures which follow. The results shown below do not seem to be too sensitive to small changes in the values of the model parameters, in the sense of reversing the rank of the various policy rules, and we shall return to this point below. Figure 4 shows the variance frontiers when the model is simulated following a temporary 1% inflation shock, combining the forward-looking monetary policy rule with the fiscal policy rules in four scenarios:

1. where fiscal policy is kept exogenously fixed, i.e. the automatic stabilizers (15) and (16) are kept switched off (labelled "*None*");
2. where only the government spending rule is switched on (labelled "G");
3. where only the taxation feedback rule is switched on (labelled "T");
4. where both rules are switched on (labelled "*Both*").

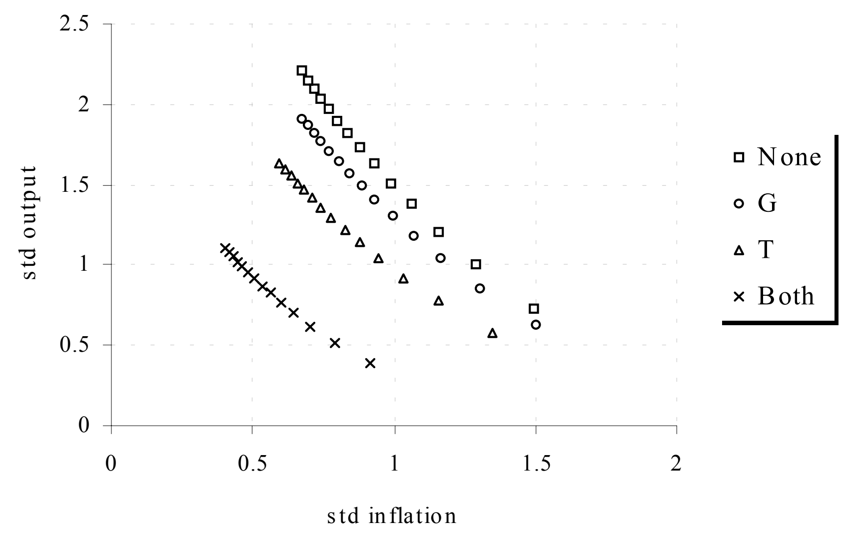
There are three points to note about these results. The first is that, in contrast with Muscatelli et al. (2003), automatic stabilizers are no longer welfare-reducing. In particular, countercyclical taxation policy seems able to reduce the variance of both output and inflation. The second point to note is that also government spending does not have an unambiguous welfare-enhancing effect: introducing a feedback rule for government spending tends

[27]See for instance Benigno and Woodford (2003). We are currently considering the extension of our modeling framework to include some welfare analysis.

[28]The variance frontiers are plotted for values of ϕ_1 which vary between between 0.2 and 1.5. The reason for focusing on higher values of ϕ_1 compared to the estimated value is that it is often argued that estimated monetary policy rules tend to underestimate the response of the central bank to shifts in expected inflation (and conversely overestimate the degree of inertia) because central banks do not continuously change their monetary stance.

to shift the variance frontier, although less than in the previous case, towards the origin lowering the variability of both output inflation. The explanation for this result lies in the different way in which government spending and taxation operates in the model: government spending varies the profile of output but its impact is ultimately reversed, as the distributed lag effect sums to zero. In contrast, taxation has an impact through both the wedge (a level effect) and through the IS curve (in difference terms), and this is not reversed because of its impact on external habits.Third, introducing both automatic stabilizers is still preferable to having none. In this case the shift in variance frontier westwards is even more visible suggesting that a combination of both automatic stabilizers have a much greater impact on the variance frontier.

Figure 4. Variance Frontiers and Monetary-Fiscal Interaction



To investigate the relative importance of personal taxes relative to payroll taxes in stabilizing output and inflation, we repeated the above experiment using only personal taxes and then using only payroll taxes. In general we found that most of the stabilization effect comes from payroll taxes through their impact on the wedge, especially for cases where ϕ_1 is high. The intuition for this is straightforward: following an adverse shock to the Phillips curve, output falls and as payroll taxes fall, they stabilise both inflation (through the wedge effect) and output (through the disposable income of rule-of-thumb consumers). In contrast personal taxes act only through the IS curve and hence stabilise output at the expense of inflation stability. Only where ϕ_1 is low, so that the monetary authority reacts less forcefully to the inflation shock, do personal taxes help to stabilise output and inflation. In other words, payroll taxes are generally more complementary to monetary policy in this model.

6 Fiscal Policy and EMU: A Two-Country Version of the Euro Model

Given the positive results obtained in the previous sections, the natural question is whether one could find a role for fiscal policy in a two-country version of the model where shocks and fiscal responses are not perfectly symmetric, and where fiscal policy is delegated to national authorities but there is a single European Central Bank. Analyzing monetary-fiscal policy interactions in two-country model would require a full paper in itself, and here we can only begin to highlight some of the issues that one might address.

We then consider a modified version of this model, which introduces the debt-channel as an additional channel of transmission for fiscal policy. Whilst retaining the assumption that some consumers follow a ROT behavior, here we introduce the assumption that optimizing consumers have Blanchard (1985)-type finite horizons with a constant probability of death as in Leith and Wren-Lewis (2004). This removes Ricardian equivalence, and allows debt-financed fiscal policy to impact, through wealth effects, on the consumption of optimizing consumers. The introduction of a wealth effect also introduces a channel of interaction between monetary and fiscal policy, as interest-rate changes will impact on aggregate demand through the government budget constraint.

In this version of the model all individuals do not expect to live forever and face a constant probability of death in each period, ϖ. However, as before there are two types of consumers. A proportion ϑ of consumers follow a rule of thumb.

The optimizing consumers, making up a proportion $(1-\vartheta)$, now behave differently because of the presence of a finite horizon. Each optimizing consumer i in cohort s, is assumed to maximize an intertemporal utility function given by:

$$E_t \sum_{t=0}^{\infty} \beta^t (1-\varpi) \left\{ \frac{1}{1-\rho} C_{s,t}^{oi} - \frac{\varepsilon^l}{1+\varphi} \left(N_{s,t}^{oi} \right)^{(1+\varphi)} \right\} \tag{17}$$

where the notation is the same as above.

The optimizing consumers intertemporal budget constraint, is now expressed as:

$$\frac{B_{s,t}^{oi}}{(1+r_t)} \leq (1-\varpi)^{-1} B_{s,t-1}^{oi} + D_{s,t}^{oi} + \frac{W_t}{P_t} N_{s,t}^{oi} - C_{s,t}^{oi} + \left(G_t^{TRi} - T_t^i \right) \tag{18}$$

where $B_{s,t}^{oi}$ denotes the debt stock and the other variables are defined as before. Again, we assume that government debt is indexed. For an comparison of cases where government debt are indexed and non-indexed in a model with Blanchard-type consumers, see Leith and Wren-Lewis (2004). In this version of the model we assume the same behavior on the part of firms as in the one-country model.

In the case of the two-country model, stability requires the expenditure and taxation rules to also respond to the debt stock $\widehat{b}_t$, that is:

$$\widehat{g}_t = \delta_1 \widehat{g}_{t-1} - \delta_2 \widehat{y}_{t-1} - \delta_3 \widehat{b}_t \tag{19}$$

$$\widehat{\tau}_t = \varphi_1 \widehat{\tau}_{t-1} + \varphi_2 \widehat{y}_{t-1} + \varphi_3 \widehat{b}_t \tag{20}$$

For our baseline case, we set $\delta_1 = \varphi_1 = 0.6$, $\delta_2 = \varphi_2 = 0.5$. A coefficient of 0.5 on output is consistent with the empirical evidence in Van den Noord (2000) and adopted in studies on fiscal stabilization (e.g. Westaway, 2003), and are broadly consistent with the correlations for US fiscal data over the cycle (cf. Gordon and Leeper, 2003). We allow for an element of inertia as empirical estimates of fiscal policy rules on quarterly data suggest an important role for an autoregressive term.

The government is assumed to finance its deficits using indexed bonds. The debt dynamics are given by a log-linearised version of the standard government budget constraint (where $\widehat{r_t}$ is the real interest rate, and $\widehat{g_t^{TR}}$ are government transfers which are kept constant during our simulations):

$$\widehat{b_t} = (1+\overline{r})\, \widehat{b_{t-1}} + \overline{r}\widehat{r}_t + \overline{\left(\frac{G^{TR}}{B}\right)} \widehat{g_t^{TR}} + \overline{\left(\frac{G}{B}\right)} \widehat{g}_t - \overline{\left(\frac{T^{PR}}{B}\right)} \widehat{t}_t^{PR} - \overline{\left(\frac{T}{B}\right)} \widehat{t}_t \tag{21}$$

Our models are simulated under forward-looking (model-consistent) expectations, where consumers take into account the policy rules and the government budget constraint.

In order to make our results comparable with those in the previous sections, we then take the simplest possible case and assume that the two countries are entirely symmetric in terms of structure, so that each has the same structural parameters as estimated on the Euro-wide data. The detailed model is outlined in the Appendix.

The model can be parameterized using the same structural parameter and steady state values as the single Euro-area model. The only caveat is that the assumed price elasticity of demand θ is quite large, as it is set at 4, and this implies a rather large relative demand effect within EMU. However, for most of the shocks considered here the relative depreciation/appreciation is quite small, so the relative demand effect will not dominate the results.

The other point to note is that equilibrium in asset markets implies that the sum of domestic and foreign bonds held by consumers in both country equals the joint supply of bonds provided by each fiscal authority. In simulating the model one could focus on equilibria where, given the absence of default risk and exchange risk, the debt of each fiscal authority grows or declines over time. To focus instead on a more realistic steady state, which embodies the type of constraint envisaged in the Maastricht criteria and the Stability and Growth Pact, we assume that the fiscal rule for each country follows not only a feedback on the output gap and an autoregressive parameter, but also has a feedback on deviations of debt from steady state (with feedback parameters $\delta_3 = \varphi_3 = 0.5$). This implies that following any shock, each country will seek to restore its initial level of debt. Given that our model is in deviations from equilibrium, this is equivalent to the fiscal authorities targeting a given level of the debt-income ratio.

6.1 Fiscal and Monetary Interactions in a Two-Country Model

In considering asymmetric shocks, we focus on demand and supply shocks on one of the two EMU countries. The reason for not considering pure asymmetric shocks (shocks of equal and opposite sign on each EMU country) is that, given the identical structure of the two countries, and that the ECB is assumed to target EMU average outcomes, monetary policy will not react to such shocks, and there will not be any fiscal-monetary policy interactions. Instead we focus on temporary shocks to the IS curve and Phillips curve of one of the two EMU countries, using the same format for the shocks as we have used before.

We plot the outcomes for output in the two countries when the fiscal rules are active and are absent in Figures 5 and 6 following a demand shock to country 1, and in Figures 7 and 8 following a supply shock to country 1. We focus solely on output, as the impact of fiscal policies through demand on inflation are quite small given the coefficients on outputs and payroll taxes in the estimated Phillips curve, and any benefits from fiscal policy will accrue largely through output stabilization. This was also apparent from the earlier single-country simulations following a supply shock.

Figure 5. Country 1 Output Response to Demand Shock

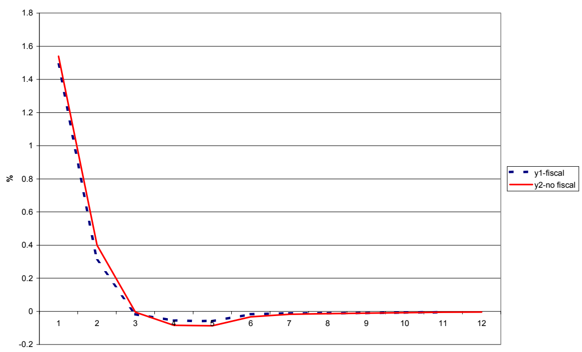


Figure 6. Country 2 Output Response to Demand Shock

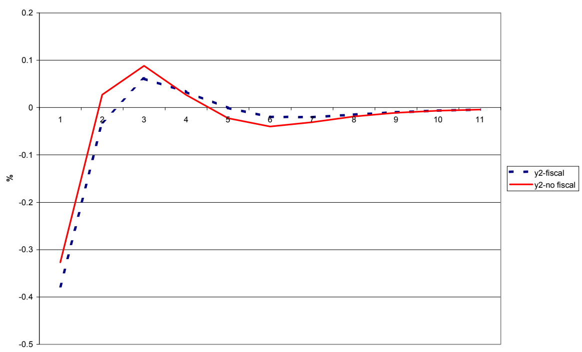


Figure 7. Country 1 - Output Response to Supply Shock

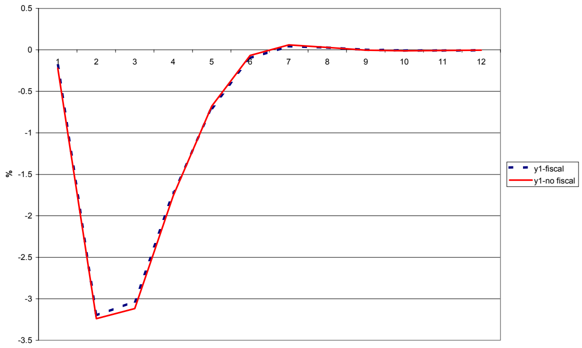
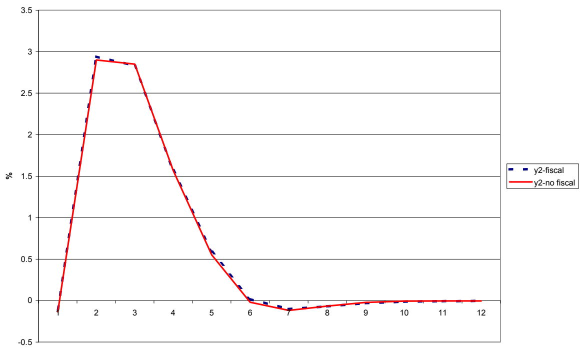


Figure 8. Country 2 Output Response to Supply Shock



Turning first to the demand shock, we see that there is a reduction in output volatility, albeit a small one, whilst in the case of country 2 the initial impact of the fiscal policy is to cause a greater deviation in output from equilibrium, although convergence is slightly improved. The reasons why in a two-country setting the value added from fiscal policy is less than might be expected is that we are not considering a pure asymmetric shock, when the two countries' fiscal policies would be acting in concert and monetary policy remains inactive. In the single country shock considered here, the monetary authority reacts to the demand shock by raising interest rates, thus causing output to fall in country 2. Thus, the two fiscal policies will be acting against each other. In addition, the presence of a feedback term on debt implies that the increase in interest rates will increase debt finance and will partially constrain fiscal policy in both countries. As noted by Leith and

Wren-Lewis (2004), varying the feedback term on debt in the fiscal rule can have a significant impact on the output dynamics in a model with Blanchard-type consumers.

Turning next to the supply shock in Figures 19 and 20, we can see that fiscal policy has little or no effect on the output dynamics following the shock. The main reason for this is that the path of output is dominated by the relative price effect, as country 1's competitiveness is eroded and country 2's competitiveness is improved.

7 Conclusions

This paper has provided a first attempt to model monetary-fiscal interactions in a New Keynesian context, in which we have allowed for a much richer role for fiscal policy compared to recent contributions to this literature. This represents the first attempt, to our knowledge, to estimate a NK model which incorporates liquidity-constrained consumers on Euro area data, and hence the impact of both government spending and taxation on the New Keynesian IS and Phillips Curve.

Having estimated this DGE model, we have conducted some preliminary analysis of the interactions between fiscal and monetary policy in such a model, to provide some understanding of the way in which different macroeconomic policy instruments interact over the business cycle.

The key conclusions which emerge from our policy analysis is that automatic stabilizers based on taxation policy seem to combine more efficiently with forward-looking inertial monetary policy rules than feedback government spending rules. This seems to be largely due to the way in which taxation (both personal and payroll taxes) enter the model, through the role played by rule-of-thumb consumers, whose consumption depends on current disposable income, but whose behaviour impacts on optimising consumers because of the presence of external habits. This causes the taxation effects to enter in difference terms in the IS curve. Interestingly, it also follows that inertia in fiscal rules may be more beneficial in taxation rules than in government spending rules, and in particular that payroll taxes, which act both through the tax wedge in the Phillips curve and through the diposable income of rule-of-thumb consumers, are the most effective fiscal stabilisation instrument.

In the two-country model it becomes apparent that automatic stabilizers may, in certain circumstances, offset each other in ways that may limit the effectiveness of fiscal policy. There is little doubt that fiscal stabilizers may cope reasonably well with the case of a pure asymmetric shock, when monetary policy is effectively inoperative. However, the interactions between fiscal and monetary policy where there is an asymmetric shock which impacts differentially on the two countries might hamper the efficacy of fiscal policy. In these cases, the precise design of the feedback rules and the automatic stabilizers becomes important and this should be the subject of further research. In particular, looking at optimally designed simple rules should improve the performance of fiscal policy against the benchmarks analyzed here.

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A Appendix

A.1 Derivation of IS and Phillips Curve

We begin with the definition of total demand and total consumption:

$$Y_t = C_t + G_t \tag{22}$$

$$C_t = C_t^{RT} + C_t^{O} \tag{23}$$

where C_t^{RT} defines the amount of consumption by rule-of-thumb consumers and C_t^{O} defines the amount of consumption by optimizing consumers. This is akin to Galí at al. (2002).

From equation (4), aggregate demand from rule-of-thumb consumers amounts to:

$$C_t^{RT} = \bar{N}^{RT}\frac{W_t}{P_t} + \vartheta\left(G_t^{TR} - T_t\right) \tag{24}$$

where ϑ defines the proportion of rule-of-thumb consumers. (we assume that $G_t^{TR} - T_t$ is uniformly spread across consumers).

We first turn to the behavior of optimizing consumers.

From equations (1), (2), (3), assuming that all consumers' preferences and their initial holdings of financial wealth are identical, the problem can be solved as a dynamic optimization problem and we can aggregate across consumers to obtain the following intertemporal aggregate Euler condition:

$$\frac{(C_t^i/H_t^i)^{-\rho}}{H_t^i} = E\left\{\beta\frac{(C_{t+1}^i/H_{t+1}^i)^{-\rho}}{H_{t+1}^i}R_t\frac{P_t}{P_{t+1}}\right\} \tag{25}$$

Taking logs we obtain a first order approximation, where we also omit $\ln\beta$ as we are interested in deviations from steady state:

$$\widehat{c_t^o} = -\left(\frac{1-\rho}{\rho}\right)\lambda\left(\widehat{c_{t-1}^T} - \widehat{c_t^T}\right) - \left(\frac{1}{\rho}\right)(\widehat{r_t}) + \widehat{c_{t+1}^o} \tag{26}$$

where c_{t-1}^T, c_t^T define the logs of total consumption.

Then, using the equilibrium condition for goods markets, given that we ignore investment and the external sector, we can loglinearise equation (17) in the main text

$$Y_t = C_t + G_t \tag{27}$$

to obtain:

$$y_t = \frac{\overline{C}}{\overline{\overline{Y}}}\widehat{c_t^T} + \frac{\overline{G}}{\overline{\overline{Y}}}\widehat{g_t} \tag{28}$$

where:

$$c_t^T = \frac{\overline{C^{RT}}}{\overline{C}}\widehat{c_t^{RT}} + \frac{\overline{C^o}}{\overline{C}}\widehat{c_t^O} \tag{29}$$

where $\widehat{c_t^{RT}}$ defines the log of total consumption by rule-of-thumb consumers:

$$\widehat{c_t^{RT}} = \frac{\overline{N^{RT}}\,\overline{\left(\frac{W}{P}\right)}}{\overline{C^{RT}}}(\widehat{w_t - p_t}) + \vartheta\left(\frac{\overline{G_t^{TR} - T_t}}{\overline{C^{RT}}}\right)\left(\widehat{G_t^{TR} - T_t}\right) \tag{30}$$

therefore

$$\widehat{y_t} = \frac{\overline{N^{RT}}\overline{\left(\frac{W}{P}\right)}}{\overline{Y}}(\widehat{w_t - p_t}) + \vartheta\left(\frac{\overline{G_t^{TR} - T_t}}{\overline{Y}}\right)\left(\widehat{G_t^{TR} - T_t}\right) + \left(\frac{\overline{C^o}}{\overline{Y}}\right)\widehat{c_t^O} + \frac{\overline{G}}{\overline{Y}}\widehat{g}_t \tag{31}$$

Substituting for $\widehat{c_t^O}$, we obtain

$$\begin{aligned}
\widehat{y_t} \quad = \quad & \frac{\overline{N^{RT}}}{\overline{N}}\frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{Y}}(\widehat{w_t - p_t}) + \vartheta\left(\frac{\overline{G^{TR} - T}}{\overline{Y}}\right)\left(\widehat{G_t^{TR} - T_t}\right) + \frac{\overline{G}}{\overline{Y}}\widehat{g}_t + \qquad (32) \\
+ \quad & \left(\frac{\overline{C^o}}{\overline{C}}\right)\left\{-\left(\frac{1-\rho}{\rho}\right)\lambda\left[y_{t-1} - y_t - \frac{\overline{G}}{\overline{Y}}(g_{t-1} - g_t)\right] - \left(\frac{\overline{C}}{\overline{Y}}\frac{1}{\rho}\right)r_t\right\} + y_{t+1} - \frac{\overline{G}}{\overline{Y}}g_{t+1} \\
& -\frac{\overline{N^{RT}}}{\overline{N}}\frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{Y}}(\widehat{w_{t+1} - p_{t+1}}) - \vartheta\left(\frac{\overline{G^{TR} - T}}{\overline{Y}}\right)\left(\widehat{G_{t+1}^{TR} - T_{t+1}}\right)
\end{aligned}$$

Bearing in mind that

$$\frac{\overline{C^{RT}}}{\overline{C}} = \left(1 - \frac{\overline{G}}{\overline{Y}}\right)^{-1}\left[\frac{\overline{N^{RT}}}{\overline{N}}\frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{Y}} + \frac{\vartheta\left(\overline{G^{TR} - T}\right)}{\overline{Y}}\right] \tag{33}$$

we get:

$$\frac{\overline{C^o}}{\overline{C}} = 1 - \frac{\overline{C^{RT}}}{\overline{C}} \tag{34}$$

To complete the model we want to introduce distortionary taxes. We assume that taxes take the form of a payroll tax, $t_t^{PR} = \frac{\overline{T}^{PR}}{\overline{N}}$ where $\overline{T}^{PR}$ are the total revenues from the payroll tax. Essentially the payroll tax is divided equally between the labour force. This means that the optimizing consumer's choice between leisure and consumption is not affected. Next, we define

$$\overline{MPL} = \frac{\overline{W}}{\overline{P}} + \frac{\overline{T^{PR}}}{\overline{N}}$$

The above expression is approximated by

$$\widehat{mpl} = \frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{NMPL}}\left(\widehat{w - p}\right) + \frac{\frac{\overline{T^{PR}}}{\overline{N}}}{\overline{MPL}}\left(\widehat{t}^{PR}\right) \tag{35}$$

where $\widehat{t^*_{PR}} = \widehat{t^*} - \widehat{n}$.

Then bearing in mind that

$$\ln(MPL) = \ln(1-\alpha) - \alpha \ln(N)$$

and ignoring $\ln(1-\alpha)$ because we are interested in deviations from steady state, we get

$$\left\{\widehat{n}\left[-\alpha + \frac{\overline{T^{PR}}}{\overline{NMPL}}\right]\right\}\frac{\overline{MPL}}{\overline{\left(\frac{W}{P}\right)}} - \left(\frac{\overline{T^{PR}}}{\overline{N\frac{W}{P}}}\right)\left(\widehat{t^{PR}}\right) = \left(\widehat{w-p}\right)$$

we can then substitute for $\left(\widehat{w-p}\right)$ into(32) to obtain equation (10).

The derivation of the Phillips Curve for the model structure set out in the main text is outlined in detailed in Galí et al. (2001) and Leith and Malley (2002), and will not be reproduced here for reasons of space. The introduction of the payroll tax, however, changes the definition of the percentage change from steady state of the labour cost share, $\widehat{s}_t$. Substituting for $\left(\widehat{w-p}\right)$ with $\widehat{mpl} = \frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{NMPL}}\left(\widehat{w-p}\right) + \frac{\frac{\overline{T^{PR}}}{\overline{N}}}{\overline{MPL}}\left(\widehat{t^{PR}} - \widehat{n}\right)$ into the expression for $\widehat{s}_t$, we obtain: $\widehat{s}_t = \frac{\overline{N\left(\frac{W}{P}\right)}}{\overline{NMPL}}\left(\widehat{w-p}\right) + \frac{\frac{\overline{T^{PR}}}{\overline{N}}}{\overline{MPL}}\left(\widehat{t^{PR}} - \widehat{n}\right) + \widehat{n}_t - \widehat{y}_t$. This yields our modified version of the Phillips Curve including the tax wedge (11).

A.2 A Two-country New Keynesian Model

We now extend the model to account for open economy features, assuming that two countries (Domestic, d, and Foreign, f) form a monetary union. Total consumption is still defined as in (5), but only a proportion n^* of them is produced in the Home economy. Domestic consumers can now hold their wealth in domestic (B^d) or foreign (B^f) bonds, denominated in the same currency, and earning the same nominal return, i_t. The Home price index is therefore defined as:

$$P_D = \left[\int_0^{n^*}\left(P_t^d(z)\right)^{1-\theta}dz + \int_{n^*}^1\left(\overline{e}P_t^f(z)\right)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \tag{36}$$

where $P_t^f(z)$ defines the foreign currency price of good (z) and $\overline{e}$ is the fixed nominal exchange rate, normalized at 1[29]. Home consumer's demand for

[29] Under this assumption $P_d = P_f$.

product z is defined as:

$$C_t^{i,j,d}(z) = \left[\frac{P_t^d(z)}{P_{D,t}}\right]^{-\theta} C_t^{i,j,d} \tag{37}$$

Correspondingly, for the foreign consumer

$$C_t^{i,j,f}(z) = \left[\frac{P_t^f(z)}{P_{F,t}}\right]^{-\theta} C_t^{i,j,f} \tag{38}$$

As in Leith and Wren Lewis (2004) we assume that PPP holds for the aggregate price level, and therefore world demand for product z is given by:

$$y(z)_t = \left[\frac{P_t(z)}{P_t}\right]^{-\theta} \left(C_t^d + G_t^d + C_t^f + G_t^f\right)$$

The log-linearised two-country model is then given by:

$$\widehat{y}_t^d = \begin{array}{c} \frac{a_1}{2}\left\{a_2\left[E_t\left\{\Delta\widehat{n}_{t+1}^d\right\} + E_t\left\{\Delta\widehat{n}_{t+1}^f\right\}\right] + a_3\left[\Delta\widehat{t}_{t+1}^{d,*} + \left(\Delta\widehat{t}_{t+1}^{f,*}\right)\right]\right\} \\ -\frac{a_4}{2}\left[E_t\left\{\Delta\left(\widehat{G_{t+1}^{d,TR} - T_{t+1}^d}\right)\right\} + E_t\left\{\Delta\left(\widehat{G_{t+1}^{f,TR} - T_{t+1}^f}\right)\right\}\right] + \\ +\left(\frac{\overline{C^o}}{\overline{C}}\right)\left\{\frac{a_6}{2}\left[\widehat{b_t^d + b_t^f}\right] - \left(\frac{\overline{C}}{\overline{Y}}\frac{1}{\rho}\right)\widehat{r}_t\right\} + \widehat{y}_{t+1}^d \\ -\frac{\overline{G}}{2\overline{Y}}\Delta\widehat{g^d}_{t+1} - \theta\left\{\left[\widehat{p}_t^d - \widehat{p}_t^f\right] - \left[E_t\left\{\widehat{p}_{t+1}^d\right\} - E_t\left\{\widehat{p}_{t+1}^f\right\}\right]\right\} \end{array} \tag{39}$$

$$\widehat{y}_t^f = \begin{array}{c} \frac{a_1}{2}\left\{a_2\left[E_t\left\{\Delta\widehat{n^d}_{t+1}\right\} + E_t\left\{\Delta\widehat{n}_{t+1}^f\right\}\right] + a_3\left[\Delta\widehat{t}_{t+1}^{d,*} + \left(\Delta\widehat{t}_{t+1}^{f,*}\right)\right]\right\} \\ -\frac{a_4}{2}\left[E_t\left\{\Delta\left(\widehat{G_{t+1}^{d,TR} - T_{t+1}^d}\right)\right\} + E_t\left\{\Delta\left(\widehat{G_{t+1}^{f,TR} - T_{t+1}^f}\right)\right\}\right] + \\ +\left(\frac{\overline{C^o}}{\overline{C}}\right)\left\{\frac{a_6}{2}\left[\widehat{b_t^d + b_t^f}\right] - \left(\frac{\overline{C}}{\overline{Y}}\frac{1}{\rho}\right)\widehat{r}_t\right\} + \widehat{y^f}_{t+1} \\ -\frac{\overline{G}}{2\overline{Y}}\Delta\widehat{g^f}_{t+1} + \theta\left\{\left[\widehat{p}_t^d - \widehat{p}_t^f\right] - \left[E_t\left\{\widehat{p}_{t+1}^d\right\} - E_t\left\{\widehat{p}_{t+1}^f\right\}\right]\right\} \end{array} \tag{40}$$

$$\widehat{\pi^d}_t = \frac{\gamma\widehat{\pi}_{t-1}^d + \beta\xi E_t\widehat{\pi}_{t+1}^d}{\xi + \gamma(1-\xi(1-\beta))} + \frac{(1-\gamma)(1-\xi)(1-\gamma\xi)}{[\xi+\gamma(1-\xi(1-\beta))][1+(\alpha/(1-\alpha))\theta]}\widehat{s}_t^d \tag{41}$$

$$\widehat{\pi}_t^f = \frac{\gamma\widehat{\pi}_{t-1}^f + \beta\xi E_t\widehat{\pi}_{t+1}^f}{\xi + \gamma(1-\xi(1-\beta))} + \frac{(1-\gamma)(1-\xi)(1-\gamma\xi)}{[\xi+\gamma(1-\xi(1-\beta))][1+(\alpha/(1-\alpha))\theta]}\widehat{s}_t^f \tag{42}$$

$$\widehat{s}_t^d = \frac{\overline{N\left(\frac{W}{\overline{P}}\right)}}{\overline{\overline{N\left(\frac{W}{P}\right)} + \overline{T^*}}}\left(\widehat{w}_t{}^d - \widehat{p}^d\right) + \frac{\overline{T^*}}{\overline{N\left(\frac{W}{P}\right)} + \overline{T^*}}\left(\widehat{t^*}^d - \widehat{n}^d\right) + \widehat{n}_t^d - \widehat{y}_t^d \tag{43}$$

$$\widehat{s}_t^f = \frac{\overline{N\left(\frac{W}{\overline{P}}\right)}}{\overline{\overline{N\left(\frac{W}{P}\right)} + \overline{T^*}}}\left(\widehat{w}_t{}^f - \widehat{p}^f\right) + \frac{\overline{T^*}}{\overline{N\left(\frac{W}{P}\right)} + \overline{T^*}}\left(\widehat{t^*}^f - \widehat{n}^f\right) + \widehat{n}_t^f - \widehat{y}_t^f \tag{44}$$

$$\widehat{b}_t{}^d = (1+\overline{r})\,\widehat{b_{t-1}}^d + \overline{r}\widehat{r}_t + \overline{\left(\frac{G^{TR}}{B}\right)}\widehat{g_t^{d,TR}} + \overline{\left(\frac{G}{B}\right)}\widehat{g}_t{}^d - \overline{\left(\frac{T^*}{B}\right)}\widehat{t_t^*}^d - \overline{\left(\frac{T}{B}\right)}\widehat{t}_t{}^d \tag{45}$$

$$\widehat{b}_t{}^f = (1+\overline{r})\,\widehat{b_{t-1}}^f + \overline{r}\widehat{r}_t + \overline{\left(\frac{G^{TR}}{B}\right)}\widehat{g_t^{f,TR}} + \overline{\left(\frac{G}{B}\right)}\widehat{g}_t{}^f - \overline{\left(\frac{T^*}{B}\right)}\widehat{t_t^*}^f - \overline{\left(\frac{T}{B}\right)}\widehat{t}_t{}^f \tag{46}$$