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Rule-of-thumb Consumers, Consumption Habits and the Taylor Principle

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Abstract

We show that the combination rule-of-thumb consumers and consumption habits dramatically affects the dynamic performance of DSGE models, resurrecting Bilbiie's (2008) inverted Taylor principle. Another original contribution of the paper is the analysis of optimal operational simple rules when RT households and habit formation in consumption are taken into account. We are able to show that the higher the share of RT consumers the more important for the optimal monetary policy is the stabilization of the wage gap, the variable that drives consumption volatility for RT consumers. The combination of consumption habits and RT consumers affect the dynamic performance of the model under the optimal simple rule. Even a relatively small share of RT consumers is sufficient to generate a substantial increase in volatility. When the share of RT consumers is sufficiently large to require an inversion of the Taylor principle to preserve dynamic stability, optimal monetary policy is forced to generate some "unconventional" impulse-response functions. For instance, a favourable productivity shock is followed by an increase in inflation and by a positive output gap.

JEL classification: E52.

Keywords: Rule of Thumb Consumers, DSGE, Determinacy, Limited Asset Market Participation, Taylor Principle, Optimal Simple Rule

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1 Introduction

The standard New-Keynesian framework is characterized by optimizing agents (households and firms), and by a number of nominal and real frictions in goods, labor and financial markets. A remarkable strand of this literature has focused on the properties that simple and implementable interest rate rules must fulfill in order to guarantee the uniqueness of the rational expectations equilibrium and to maximize the social welfare (see Woodford(2003), Schmitt-Grohè, Uribe(2004 and 2007))

Following a seminal contribution by Mankiw (2000), who introduced the notion of heterogeneous consumers (savers and spenders), a second strand of New Keynesian literature emphasizes the role of rule-of-thumb consumers (RT consumers henceforth) which fully consume their current income and do not participate to financial markets (Galì et al. 2004, 2007). De Graeve et al. (2010) introduce RT consumers to model financial risk premia. Empirical research cannot reject the RT consumers hypothesis. Estimated structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989) More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain estimates around 35%. Erceg, Guerrieri and Gust (2006) calibrate the share of RT consumers to 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets (Bilbiie, Meier and Müller, 2008). In fact, increasing regulation in the aftermath of the 2008 crisis (OECD 2009) is likely to increase the share of liquidity constrained households.

The RT consumers hypothesis bears important implications for model dynamics. Bilbiie (2008) shows that, in a world of flexible nominal wages, a low elasticity of labor supply combined with a sufficiently large share of non Ricardian agents leads to an equilibrium where an interest rate policy based on the Taylor principle cannot ensure model determinacy. The intuition behind this result is as follows. In standard models based on optimizing consumers, satisfying the Taylor principle generates a substitution effect from current to future consumption that is sufficient to rule out sunspot equilibria. By contrast, RT consumers generate a "Keynesian multiplier" effect on demand shocks that raises profits which are entirely appropriated by Ricardian agents. If the share of RT consumers is sufficiently large, this wealth effect dominates the substitution effect induced by the interest rate rule based on the Taylor principle. As a consequence, the standard monetary policy rule cannot pin down the optimizing consumers' choice to a unique equilibrium path.

Recent contributions downplay this conclusion. Colciago (2007) introduces nominal wage stickiness, finding that even a mild degree of wage stickiness dampens the Keynesian multiplier effect generated by RT consumers and restores the standard Taylor Principle even for a very large share of RT consumers. Ascari et al.(2010) show that the optimal monetary policy is almost unaffected by the presence of RT consumers as long as nominal wages are sticky.

In the paper we reconsider the issue and show that, just like wage stickiness undermines the wealth effect outlined in Bilbiie, other frictions may weaken the substitution effect induced by a policy that follows the Taylor principle. In fact, this happens when consumption habits enter the utility function. In addition, consumption habits affect the marginal rate of substitution between consumption and leisure, leading to a more rigid labor supply curve. Our simulations show that the combination of consumption habits and RT consumers has dramatic implications for model determinacy, resurrecting Bilbiie's inverted Taylor principle.

Another original contribution of the paper is the analysis of optimal operational simple rules when RT households and habit formation in consumption are taken into account. We are able to show that the higher the share of RT consumers the more important for the optimal monetary policy is the stabilization of the wage gap, the variable that drives consumption volatility for RT consumers. The combination of consumption habits and RT consumers affect the dynamic performance of the model under the optimal simple rule. Even a relatively small share of RT consumers is sufficient to generate a substantial increase in volatility. When the share of RT consumers is sufficiently large to require an inversion of the Taylor principle to preserve dynamic stability, optimal monetary policy is forced to generate some "unconventional" impulse-response functions. For instance, a favourable productivity shock is followed by an increase in inflation and by a positive output gap.

The remainder of the paper is organized as follows: In the first section we present and describe the model, in the second section we analyze the model stability properties under different specifications of labor markets. The third and fourth section describe the optimal policy problem and its implications. Section five concludes.

2 The Model

We consider a cashless small-scale New Keynesian model augmented for rule-of-thumb (RT) or non Ricardian consumers. We assume a continuum of households indexed by $i \in [0,1]$. As in Galì et al (2004) and (2007), households in the interval $[0,\theta]$ cannot access financial markets. The rest of the interval $(\theta,1]$ is composed by standard Ricardian households who have access to a full set of state contingent securities. The key distinction between the two groups concerns intertemporal optimization. Ricardian consumers' choices take into account future utility when choosing consumption and portfolio composition. Rule-of-Thumb consumers spend their whole income every period, thus they do not hold any wealth.

2.1 Households preferences

All households share the same utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(C_t^i - bC_{t-1}^i\right) - \frac{\psi_l}{1+\phi_l} (h_t^i)^{1+\phi_t} \right\} \tag{1}$$

where $i : o, rt$ stands for household type, C_t^i represents total individual consumption b denotes consumption internal habits and h_t^i denotes individual labour supply.

2.1.1 Consumption Bundles

C_t^i is a standard consumption bundle

$$C_t^i = \left[\int_0^1 c(z)_t^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where η represents the price elasticity of demand for the individual goods.

$$P_t = \left(\int_0^1 p(z)_t^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

is the aggregate consumption price index.

2.2 Firms

Goods are indexed by $z \in [0,1]$. Good z is produced by a monopolist with the following technology:

$$y_t(z) = h_t(z)$$

Where $h_t(z)$ is the composite labor input used by each firm z defined as follows:

$$h_t(z) = \left(\int_0^1 \left(h_t^j(z) \right)^{\frac{\alpha_w - 1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_w - 1}} \tag{3}$$

where the parameter $\alpha_w > 1$ is the intratemporal elasticity of substitution between labor inputs. For any given level of its labor demand $h_t(z)$, the optimal allocation of across labor inputs implies

$$h_t^j(z) = \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d(z) \tag{4}$$

where $W_t = \left(\int_0^1 \left(W_t^j \right)^{1-\alpha_w} dj \right)^{1/(1-\alpha_w)}$ is the standard wage index. Firm *z's* nominal total production cost is given by

$$TC_t(z) = W_t h_t(z)$$

The real marginal costs are:

$$mc_t = w_t \tag{5}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage and P_t is the consumption price index.

2.2.1 Sticky Prices

Price stickiness is based on the Calvo mechanism. In each period firm z faces a probability $1-\lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $(1+\pi_{t-1}) = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t(z) = (1+\pi_{t-1})^{\gamma_p} p_{t-1}(z) \tag{6}$$

where $\gamma_p \in [0,1]$ represents the degree of price indexation.

All the $1-\lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$ the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t+s$ will read as $\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p}$ where $\Pi_{t,t+s-1} = (1+\pi_t)\ldots(1+\pi_{t+s-1}) = \frac{P_{t+s-1}}{P_{t-1}}$.
$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} \left(\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p} - P_{t+s}mc_{t+s}\right) y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}}\right)^{-\eta} \tag{7}$$

where Y_t^d is aggregate demand and λ_t is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\eta)\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{1-\eta} \widetilde{P}_t^{-\eta} (P_{t+s})^{\eta} + \\ +\eta\widetilde{P}_t^{-\eta-1} P_{t+s}^{\eta+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{-\eta} \end{array}\right] = 0 \tag{8}$$

2.3 Ricardian Households

Ricardian households maximize 1 subject to the following period budget constraint:

$$P_{t+1}B_{t+1} = R_tB_t + P_tA_{j,t} + P_tD_t - P_tC_t + h_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} dj \tag{9}$$

Where B_t is a riskless bond, $A_{j,t}$ and D_t are respectively the net cash flow from participating in state-contingent securities at time t and firm dividends.

The solution for the optimizing household problem is standard. The Euler equation is

$$\lambda_t^o = \beta E_t \lambda_{t+1}^o \frac{R_t}{\pi_{t+1}} \tag{10}$$

where

$$\frac{1}{C_t^o - bC_{t-1}^o} - \frac{\beta b}{C_{t+1}^o - bC_t^o} = \lambda_t^o \tag{11}$$

2.4 Rule-of-Thumb Households

As pointed out above, RT consumers neither save or borrow, in each period they entirely consume their labor income.

$$C_t^{rt} = h_t^d \int_0^1 \frac{W_t^j}{P_t} \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} dj \tag{12}$$

2.5 Labor market

There is a continuum of differentiated labor inputs indexed by $j \in [0,1]$. Each labor market j is monopolistically competitive and there is a union j which sets the nominal wage, W_t^j, subject to (**??**). Each household i supplies all labour types at the given wage rates [1] and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d dj \tag{13}$$

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time, h_t. Individual labor income is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d dj \tag{14}$$

We posit that the union objective function is a weighted average $(1-\theta, \theta)$ of the utility functions of the two households types. This, in turn, implies that with flexible wages where $\frac{\alpha_w}{(\alpha_w - 1)}$ represents the

$$w_t = \frac{W_t}{P_t} = \frac{\alpha_w}{\alpha_w - 1} \frac{\psi_l h_t^{\phi_l}}{\left[(1-\theta) U' \left(C_t^o - bC_{t-1}^o \right) + \theta U' \left(C_t^{rt} - bC_{t-1}^{rt} \right) \right]} \tag{15}$$

wage markup over the average marginal rate of substitution.

Determinacy analysis in section (3) below will take perfect competition in the labor market as a benchmark. In that case the individual labor supplies of the two groups will differ:

[1] Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

$$w_t = \frac{\psi_l \left(h_t^i\right)^{\phi_l}}{\left[U'\left(C_t^i - bC_{t-1}^i\right)\right]} \tag{16}$$

Note that when habits are absent, $b = 0$, the labour supply of RT consumers is constant: $h_t^{rt} = \psi_l^{-\frac{1}{1+\psi_l}}$.

2.5.1 Sticky wages

In each period a union faces a constant probability $1 - \lambda_w$ of being able to reoptimize the nominal wage. Unions that cannot reoptimize simply index their wages to lagged inflation:

$$W_t^j = W_{t-1}^j \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} = W_{t-1}^j \left(\pi_{t-1}\right)^{\gamma_w}$$

where γ_w stands for the degree of wage indexation. Just like firms, when choosing the current wage, $\widetilde{W}_t$, the optimizing union will anticipate that in the future it might not be able to reoptimize. In this case, the wage at the generic period $t + s$ will read as (in real terms)

$$w_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \tag{17}$$

Following Colciago(2008), the representative union objective function is defined as

$$L^u = \sum_{s=0}^{\infty} \left(\beta\lambda_w\right)^s \left\{\left[\left(1-\theta\right) U^o(C_{t+s}^o) + \theta U^{rt}(C_{t+s}^{rt})\right] - U(h_{t+s})\right\} \tag{18}$$

Where U_s^o, U_s^{rt} are defined as in (1). Thus the wage-setting decision maximizes a weighted average of the two household types conditional to the probability that the wage cannot be reoptimized in the future. The relevant constraints are (13), (**??**), (12), (17).

The union's first-order condition is:

$$\sum_{s=0}^{\infty} \left(\beta\lambda_w\right)^s \left[\left(1-\theta\right)\lambda_{t+s}^o + \theta\lambda_{t+s}^{rt}\right] h_{t+s}^d \left(w_{t+s}\right)^{\alpha_w} \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}}\right)^{-\alpha_w} \cdot \tag{19}$$

$$\cdot \left[\widetilde{w}_t \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}}\right) - \frac{\alpha_w}{\left(\alpha_w - 1\right)} \frac{\psi_l h_{t+s}^{\phi_l}}{\left[\left(1-\theta\right)\lambda_{t+s}^o + \theta\lambda_{t+s}^{rt}\right]}\right] = 0$$

where $\lambda_{t+s}^{rt} = \frac{1}{C_t^o - bC_{t-1}^o} - \frac{\beta b}{C_{t+1}^o - bC_t^o}$. It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

2.6 Monetary Policy

We assume a monetary authority follows a rule of the type:

$$r_t = (\pi_t)^{\varphi_\pi} \tag{20}$$

where $r_t = R_t - 1$ is the net nominal interest rate

2.7 Aggregation

Aggregate consumption C_t is a weighted average of the respective variable for each household type, thus

$$C_t = \int_0^1 C_t^i(j)\ dj = \int_0^\theta C_t^{rt}(j)\ dj + \int_\theta^1 C_t^o(j)\ dj = \theta C_t^{rt} + (1-\theta)C_t^o \tag{21}$$

Aggregating budget constraints for each sector, after few manipulations we get the aggregate resource constraint as

$$Y_t = C_t$$

2.8 Steady State

As in Ascari et al.(2010) and Bilbiie (2008), we need to make the assumption of an efficient steady state in order to study the welfare properties of the economy represented by this model.

We therefore assume that at the steady state firms are taxed by the Government by a constant employment tax, τ, and then receive the money back through lump-sum transfer, $T = \tau \frac{W}{P} h$. In this case steady-state firms profits are:

$$D = Y - (1-\tau)\, h \frac{W}{P} - T$$

The efficient steady state is characterized by perfect competition and zero profits. If this is the case, it follows that $C^{rt} = C^o = C$ and all households have the same marginal rate of substitution between labour and consumption (MRS). Under the above assumption, the equilibrium wage at the steady state is give by

$$w = \frac{1}{(1-\tau)\left(1+\mu_p\right)} MPL = (1+\mu_w)\, MRS$$

where $\mu_w = \frac{\alpha_w}{\alpha_w - 1}$ and $\mu_p = \frac{\eta}{\eta - 1}$ are the markups in labour and good markets respectively.

Since $MPL = MRS = 1$ must hold at the efficient steady state, we need that

$$\tau = 1 - \frac{1}{\left(1+\mu_p\right)(1+\mu_w)}$$

The resulting value of τ will lead to zero steady state profits and to equilibria in goods and labour markets equivalent to those under perfect competition in both markets.

3 Stability Analysis

Given the model size, determinacy analysis requires numerical methods.

Parameters are calibrated following Christiano et al. (2005), technology process is modeled as in Schmitt-Grohe, Uribe(2007):

TABLE 1		
Parameter	Value	Description
b	0.7	degree of habit persistence
β	0.99	subjective discount factor
λ_p	0.6	price stickiness
λ_w	0.64	wage stickiness
γ_p	1	indexation on prices
γ_w	1	indexation on wages
φ_l	1	preference parameter
$\frac{\eta}{(\eta-1)}$	1.2	price mark-up
$\frac{\alpha_w}{(\alpha_w-1)}$	1.2	wage mark-up
ρ_a	0.8556	shock persistence
σ_a	1	shock std. deviation

Our model encompasses previous contributions that investigated the impact of RT consumers on the effectiveness of the Taylor principle, $\varphi_\pi > 1$ in (20), under different labor market structures. To facilitate comparison we first discuss the case of a perfectly competitive labor market, as in Bilbiie (2008). Then we introduce monopolistic competition under flexible wages. Finally, we consider the sticky-wage models of Colciago(2006) and Ascari, Colciago, Rossi (2010).

3.1 Competitive labor market

The white areas in Panel a of Figure 1 define the determinacy regions that obtains for different combinations of θ, φ_π when the labor market is competitive and consumption habits are absent. If the share of RT consumers exceeds a threshold value $\theta = 0.48$ determinacy requires an inversion of the Taylor principle: $\varphi_\pi < 1$. This broadly coincides with Bilbiie (2008) who has shown that, for a sufficiently large share of RT consumers the Taylor principle cannot rule out sunspot equilibria. The intuition behind this result is as follows. Suppose that firms form an arbitrary expectation of future price increases and therefore choose to raise the current price. The simultaneous (real) interest rate response induces a substitution effect in the consumption decisions of Ricardian households: C_t^o is such that $E_t\left\{\Delta C_{t+1}^o\right\} > 0$ (see equations 10, 11 for $b = 0$). If all consumers were ricardian, this would allow a unique $\Delta C_t^o < 0$ consistent with convergence to steady state, thus generating in t a negative output gap sufficient to rule out the initial price increase as a possible equilibrium. By contrast, in this model ricardian agents can react to the real interest rate surge by choosing $\Delta C_t^o > 0$, because RT consumers induce a "Keynesian multiplier" effect that raises profits which are entirely appropriated by ricardian agents. If this wealth effect is sufficiently strong, i.e. the share of RT consumers is sufficiently large, the choice of C_t^o such that $E_t\left\{\Delta C_{t+1}^o\right\} > 0, \Delta C_t^o > 0$ may be consistent with the rational expectation of future return to steady state. In this case C_t^o confirms the increases in current and expected inflation.

In Panel b of Figure 1 we show that determinacy regions remain almost identical when habits affect consumption utility. In fact habits substantially modify both the substitution and the wealth effects discussed above. To understand this, look at the log-linearized versions of conditions 10, 11 and of 16 subject to 12.

$$\left(\frac{1+b+\beta b^2}{(1-\beta b)(1-b)}\right)c_t^o = \left\{\begin{array}{c}\left(\frac{b}{(1-\beta b)(1-b)}\right)c_{t-1}^o - \left(\frac{\beta b}{(1-\beta b)(1-b)}\right)c_{t+2}^o + \\ +\left(\frac{1+\beta b+\beta b^2}{(1-\beta b)(1-b)}\right)c_{t+1}^o + \hat{\pi}_{t+1}^e - \hat{R}_t\end{array}\right\} \tag{22}$$

$$\hat{h}_t^{rt} = \frac{\left\{\begin{array}{c}\frac{\beta b}{(1-\beta b)(1-b)}c_{t+1}^{rt} - \left[\frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} - 1\right]\hat{w}_t + \\ +\frac{b}{(1-\beta b)(1-b)}c_{t-1}^{rt}\end{array}\right\}}{\phi_h + \frac{(1+\beta b^2)}{(1-\beta b)(1-b)}} \tag{23}$$

From (22) it is easy to see that consumption habits reduce the sensitivity of ricardian consumers to real interest rate changes, weakening the substitution effect that is crucial to obtain determinacy under the Taylor principle. Equation 23 shows instead that habits weaken the wealth effect induced by RT consumers' choices. In fact, when $b = 0$ the labour supply of RT consumers is constant and their consumption decisions are driven by the wage rate which increases if $\Delta c_t^o > 0$. If $b > 0$, then $\hat{h}_t^{rt}$ negatively correlates with the wage rate. This happens because habits induce RT consumers to behave in a forward-looking manner, taking into account that an increase in their current income will also raise next-period habits with adverse effects on future utility. Consumption habits therefore reverse the standard labor supply reaction to a wage rate increase when consumers are non-Ricardian. As a result the Keynesian multiplier effect generated by RT consumers is now weaker.

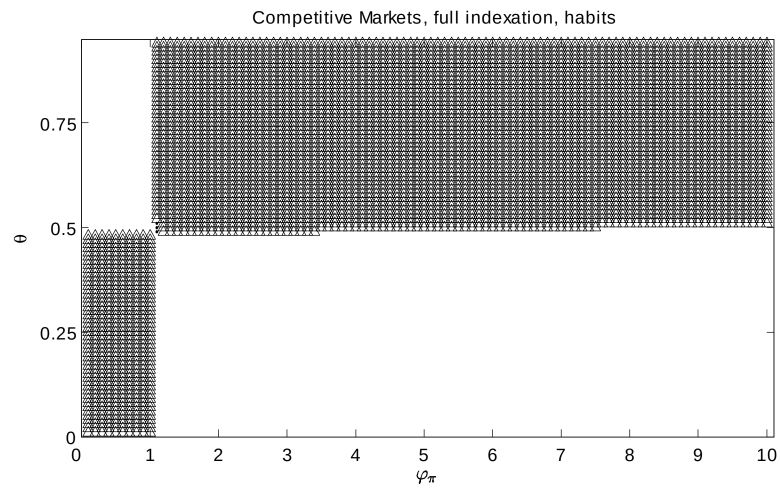

Figure 1a: Determinacy Area

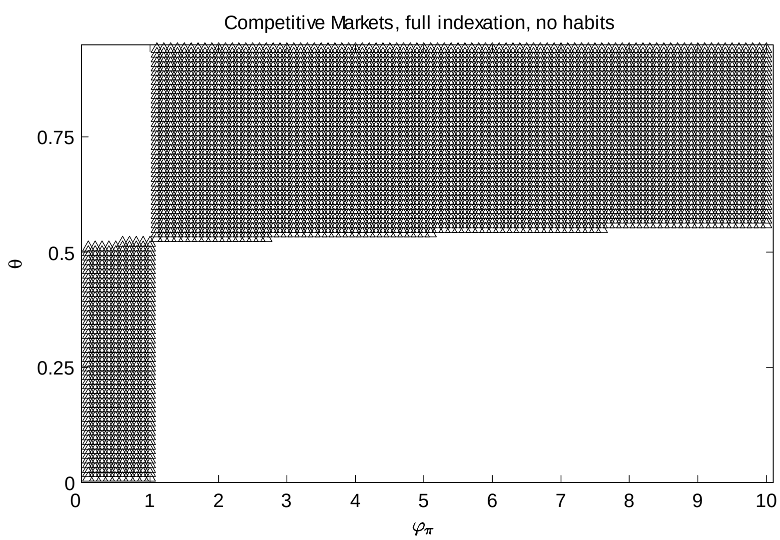

Figure 1b: Determinacy Area

3.2 Monopolistic wage setters

Consider now a monopolistically competitive labour market. Condition (15) characterizes labor market equilibrium if nominal wages are flexible. Determinacy regions for $b = 0$, $b > 0$ are reported in Figure 2, panels a and b respectively . From a comparison between panels a of Figures 1 and 2 we see that, relative to the case of no habits and perfectly competitive labor market, monopolistic competition lowers the share of RT consumers that requires an inversion of the Taylor principle. This happens because the labor supplies of the two households groups coincide and consumption choices of ricardian households directly affect the labor supply of RT consumers. As a result, the Keynesian multiplier effect induced by RT consumers is unambiguously stronger than under perfect competition

$$h_t = h_t^{rt} = \frac{\left\{ \begin{array}{c} \left(1 - \frac{\theta\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}\right) w_t - \frac{(1-\theta)\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} c_t^o + \frac{(1-\theta)\beta b}{(1-\beta b)(1-b)} c_{t+1}^o + \\ + \frac{\theta \beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} + \frac{(1-\theta) b}{(1-\beta b)(1-b)} c_{t-1}^o + \frac{\theta b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \end{array} \right\}}{\left(\phi_h + \frac{\theta(1+\beta b^2)}{(1-\beta b)(1-b)}\right)} \tag{24}$$

given the expectations

$$w_t = \left(\varphi_h + \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}\right) y_t$$

Introducing habits in the monopolistic competition model dramatically lowers the threshold value of θ that triggers an inversion of the Taylor principle. Relative to the perfect competition-cum-habit case, this happens because habits weaken the substitution effect triggered by real interest rate changes, but no longer induces the negative response of RT labor supply to a real wage increase.

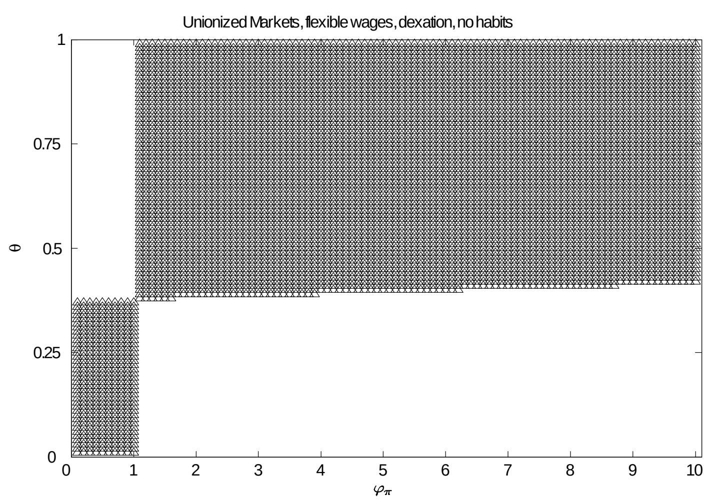

Figure 2b: Determinacy Area

3.2.1 Sticky wages

Colciago (2006) and Ascari, Colciago, Rossi (2010) show that, in a model without habits, wage stickiness is enough to wipe out the wealth effect identified in Bilbiie (2008), thus restoring the effectiveness of the Taylor principle (Figure 3, panel a). The intuition behind this result is very simple. Sticky wages dampen the real wage response to an aggregate demand increase and unambiguously limit the Keynesian multiplier effect of RT consumers. Panel b of Figure 3 shows that wage stickiness plays a much lesser role once consumption habits are introduced. Under our parameter calibrations, determinacy requires an inversion of the Taylor principle when the share of Rule-of-thumb consumers reaches 42%. As pointed out above, habits play their crucial role by weakening the substitution effect associated to real interest rate movements.

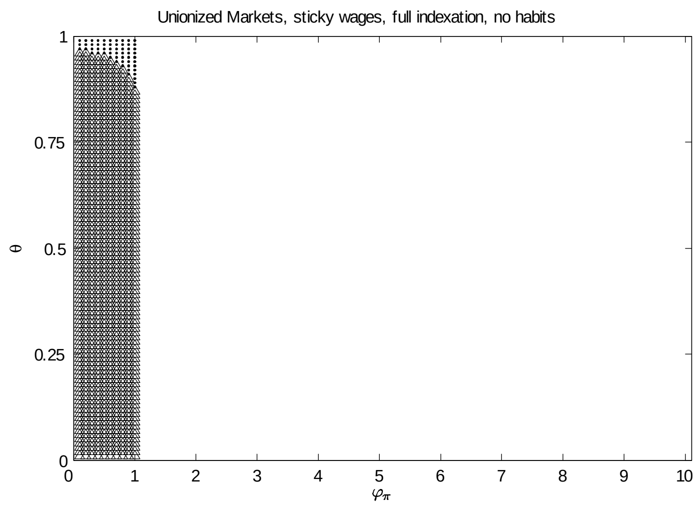

Figure 3a: Determinacy Area

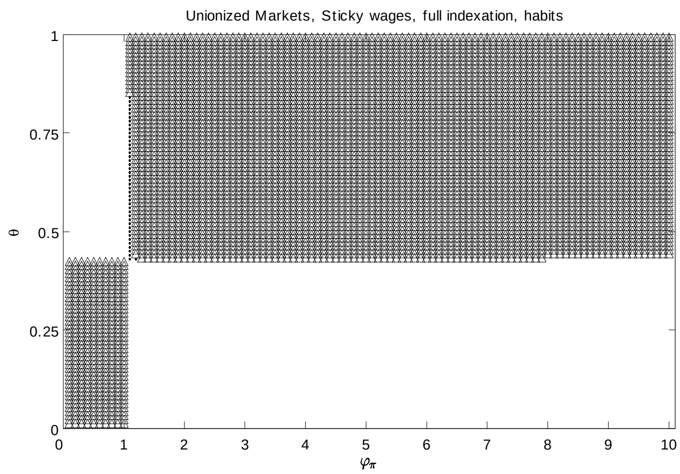

Figure 3b: Determinacy Area

4 Optimal Simple Implementable Rule

In this section we turn to the analysis of the optimal simple monetary policy rule as the one in (20) , given the determinacy constraints of the model. Our interest here is to identify a policy space that minimizes deviations from socially efficient outcomes. To this end we first identify the solution to the social planner problem.

4.1 Social Planner Problem

It should be noted from the outset that the two household groups have symmetrical preferences, but have different access to financial markets. As a result, from the social planner perspective, the consumption and worked hours responses to shocks should be identical for the two groups. In addition, the social planner faces an intertemporal problem due to internal habit formation.

The social planner problem can be summarized as:

$$\max_{c_t^o, c_t^{rt}, h_t^o, h_t^{rt}} E_t \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \theta \left(\log\left(c_t^{rt} - b c_{t-1}^{rt}\right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{rt}\right)^{1+\phi_l} \right) + \\ + (1-\theta) \left(\log\left(c_t^{o} - b c_{t-1}^{o}\right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{o}\right)^{1+\phi_l} \right) \end{array} \right]$$

subject to the following constraints which represent the composition of aggregate consumption and labour supply, the aggregate resource constraint in which in equilibrium total output must be equal to total consumption and the firms' production function:

$$\theta c_t^{rt} + (1-\theta) c_t^o = c_t$$

$$\theta h_t^{rt} + (1-\theta) h_t^o = h_t$$
$$y_t = c_t$$
$$y_t = a_t h_t$$

Since the social planner choices are taken in an perfectly competitive environment and we do not allow for capital accumulation, we have that $c_t^o = c_t^{rt} = c_t$ and the resulting Lagrangian is given by:

$$\max_{c_t, h_t} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \log(c_t - bc_{t-1}) - \frac{\psi_l}{1+\phi_l}(h_t)^{1+\phi_l} + \\ -\lambda_t [c_t - a_t h_t] \end{array} \right\}$$

The first order conditions to the social planner optimization problem are the following

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= 0 : \frac{1}{(c_t - bc_{t-1})} - \frac{\beta b}{(c_{t+1} - bc_t)} = \lambda_t \\ \frac{\partial \mathcal{L}}{\partial h_t} &= 0 : \psi (h_t)^{\phi_l} = \lambda_t a_t \end{aligned}$$

in loglinear terms

$$\frac{\beta b}{(1-\beta b)(1-b)} c_{t+1} - \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} c_t + \frac{b}{(1-\beta b)(1-b)} c_{t-1} = \lambda_t$$

$$\lambda_t = \phi h_t - a_t$$

which yields

$$\left(\phi + \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}\right) y_t^* = \frac{\beta b}{(1-\beta b)(1-b)} y_{t+1}^* + \frac{b}{(1-\beta b)(1-b)} y_{t-1}^* + (\phi+1) a_t$$

The efficient level of output y_t^* which would have been set by a benevolent social planner is therefore the result of an intertemporal choice, it depends on past and future level of output and it is a decreasing function of habit persistence (the more we consume today, the less utility we will have tomorrow) and a function of the technological process.

The social planner finally set the efficient wage equal to the marginal productivity of labour,i.e.

$$w_t^* = a_t \tag{25}$$

In figure 4 we show the efficient output dynamics in response to a technology shock. The "hump-shaped" response is due to the habit formation in households' utility function. Since the RT consumers have the same preferences as optimizing households, their presence does not affect the social planner optimal behavior.

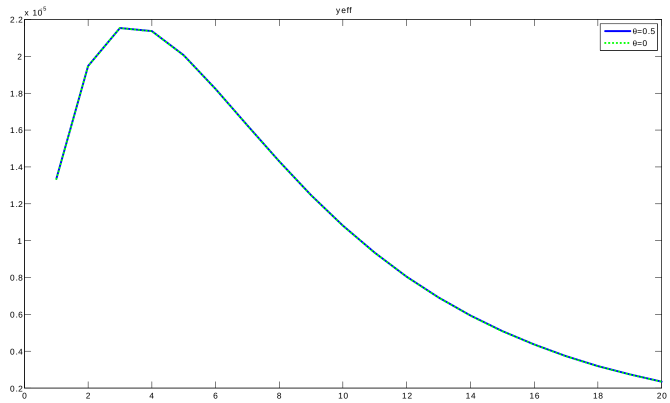

Figure 4: Social Planner Response to a Technology Shock

4.2 The central bank welfare function

We assume, as in Bilbiie(2008) and Ascari et al. (2010), that the central bank maximizes an average of the two groups of households utility functions weighted for their relative size. The period welfare function is therefore given by:

$$W_t = \theta\left[U\left(x_t^{rt}\right) - V(h_t^{rt})\right] + (1-\theta)\left[U\left(x_t^{o}\right) - V(h_t^{o})\right] \tag{26}$$

where $x_t^i = c_t^i - bc_{t-1}^i$. Moreover, given the unionized structure of the labour market, we have that $h_t^o = h_t^{rt} = h_t$ and the welfare function reads as

$$W_t = \theta U\left(x_t^{rt}\right) + (1-\theta)\, U(x_t^o) - V(h_t) \tag{27}$$

We derive the central bank loss function as a second order approximation to (27) around the efficient steady state. For sake of simplicity, we restrict our analysis to the no indexation case, i.e. $\gamma_p = \gamma_w = 0$.[2]. The derived loss function takes the following form[3]:

$$\mathcal{L} = -\frac{1}{2}\frac{(1-\beta b)}{(1-b)}\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}(1-\theta)\,\frac{(1-b)}{(1-\beta b)}\left(x_t^o\right)^2 + \theta\frac{(1-b)}{(1-\beta b)}\left(x_t^{rt}\right)^2 + \\ +\phi y_t^2 + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t\right)^2 - 2\left(1+\phi\right) y_t a_t\end{array}\right] + t.i.p. + O\left(||\xi||^3\right) \tag{28}$$

where $\kappa_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$, $\kappa_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w}$ and π_t^w represent the real wage inflation. All the variables in (28) represent deviations from the efficient steady state. Rewriting the loss function in terms of deviation of the variables from

[2] Simulations show that indexation plays no role in determining the optimal policy Proof available upon request.

[3] Derivations are available in appendix

the efficient levels resulting from the social planner solution it yields

$$\mathcal{L} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left[\begin{array}{c} \left(\phi + \frac{1}{(1-\beta b)(1-b)}\right)\left(y_t^{gap}\right)^2 + \left(2\phi + \frac{2}{(1-\beta b)(1-b)}\right) y_t^{gap} y_t^{*} + \\ + \frac{1}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_t^{gap}\right)^2 + \frac{b^2}{(1-\beta b)(1-b)}\left(y_{t-1}^{gap}\right)^2 + \\ + \frac{2b^2}{(1-\beta b)(1-b)} y_{t-1}^{gap} y_{t-1}^{*} + \frac{b^2}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_{t-1}^{gap}\right)^2 + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^{*} + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^{*} y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)} w_t^{gap} w_{t-1}^{gap} + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t\right)^2 - 2\left(1+\phi\right) a_t y_t^{gap} \end{array} \right] + tip \quad (29)$$

The central banker problem consists in finding the interest rate response to inflation which minimizes the welfare loss function subject to the behavior of households, firms and social planner.

We study the optimal responses to a technology shock a_t searching for the coefficient on inflation which minimizes function (29) in the interval [-5, 5][4].

$$a_t = \rho_a a_{t-1} + \varepsilon_t$$

It is worth to notice that in (29) the higher the share of RT consumers the more important is the wage gap stabilization for the optimal monetary policy. When $\theta = 0$, RT consumers do not matter, (29) real wage gap stabilization is not an objective. The reason why wage gap stabilization is so important is that this variable drives consumption volatility for RT consumers.

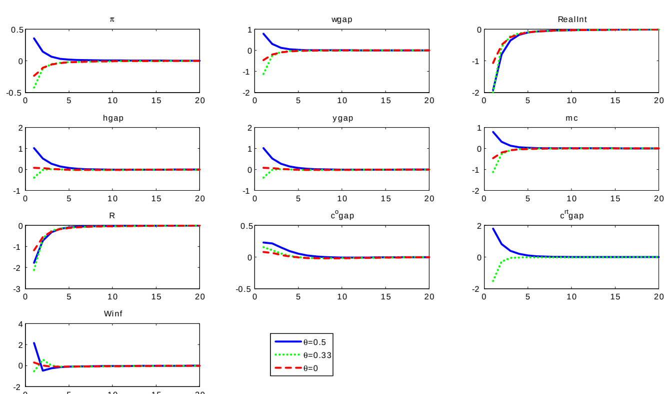

Figure 5: Responses to a Technology Shock

Figure 5 displays the dynamic responses to a technology shock when the optimal policy is implemented. Gap variables represent the deviations between

[4] The restriction on the interval [-5,5] is driven by the idea that rules characterized by stronger interest rate reaction to changes in inflation are unlikely to be implemented in practice (see for further examples Ascari et al.(2010) and Schmitt-Grohe, Uribe(2004,2007) .

the variable responses and the efficient responses resulting from the welfare optimization problem of a benevolent social planner, in a non-distorted economy.

It is clear that the presence of RT consumers in the economy significantly affects the dynamic responses to a technology shock. When only optimizing agents are present in the model (red dashed lines), the response of both nominal and real interest rate allows the policy maker to minimize both price and wage dispersion ensuring a volatility in output and consumption close to zero. Introducing a small share of RT consumers (green dotted line) which is still compatible with the Taylor principle, we observe that the fall in wage bills affect RT consumption which decreases together with the output gap. This, in turn, lowers inflation. The central banker will therefore decrease the interest rate in order to dampen inflation volatility.

Things change when we allow for a share of RT consumer which is big enough to require an inverted Taylor principle. Now, in order to obtain the dynamic stability of the economy, the central banker does not try to contract the consumption of optimizing agents when inflation increases. The Keynesian multiplier effect generated by RT consumers weakens the central banker ability to stabilize the economy. The system is characterized by a positive output gap and by an increased gap in both RT and Ricardians' consumption. This happens because the real interest rate still responds negatively to the shock, due to the inversion of the Taylor principle. As discussed in section 3.2, habit formation in a unionized labour market dramatically increases the wage elasticity to output movements. The increase in output generated by a positive productivity shock, increases labour demand and wages. The latter responds more strongly when habits are allowed and pushes up RT consumption generating a multiplicative effect on output. The output gap is therefore markedly higher. Notice that dynamics of the real interest rate under the inverted Taylor principle is quite similar to the one characterizing the economy where the share of RT is small enough to guarantee stability under the Taylor principle.

The importance of habit persistence in magnifying the response of RT consumers and therefore the implemented monetary policy is visible by contrast in figure 6. Here we display the model's responses to the same technology shock when habit formation is not present in the households utility function. As in Ascari et al., RT consumers no longer play a significant role in determining the economy's optimal response.

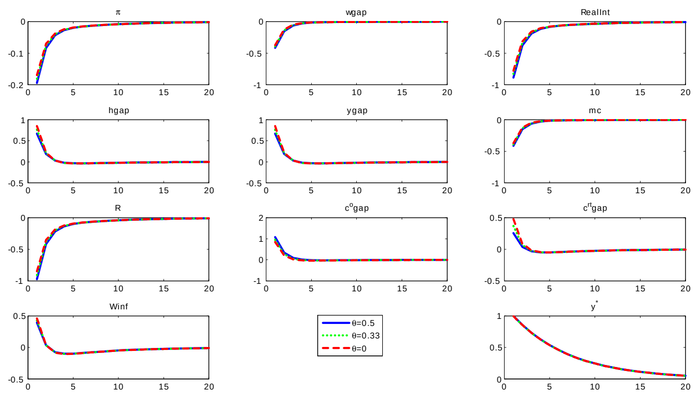

Figure 6: Responses to a Technology Shock

5 Conclusion

We have studied the interactions between consumption habits and RT households for what concerns both the stability of a New-Keynesian model and the optimal setup of a simple and implementable monetary rule. It emerged that when habits are taken into account, the presence of a share of financially constrained consumers cannot be ignored by the policy maker. A sufficiently large share of RT consumers requires an inversion of the Taylor principle. In addition, RT consumers affect the dynamic performance of the model under the optimal monetary policy even when the share of RT consumers is limited.

Further research will focus on a deeper analysis of the optimal policy. We are going to check the robustness of the results to different policy rules. We will also investigate how fiscal policy may contribute to stabilization. Finallly, our analysis will be extended to a medium scale new-Keynesian model accounting for capital accumulation and additional real rigidities, as in Christiano et al. (2005).

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6 Appendix

6.1 Loglinearized equilibrium conditions

6.1.1 Unionized Labour Market

The stability analysis is implemented using a linearized version of the model presented above. Lower case letters from now on denote the log of the corresponding variable or their log deviations from the steady state.

Aggregate consumption is defined by:

$$\hat{c}_t = (1-\theta)\frac{c^o}{c}\hat{c}_t^o + \theta\frac{c^{rt}}{c}\hat{c}_t^{rt} \tag{30}$$

Marginal costs are given by

$$\widehat{mc}_t = \hat{w}_t \tag{31}$$

Production function is given by

$$\hat{y}_t = \hat{h}_t \tag{32}$$

Aggregate resource constraint

$$\hat{y} = \hat{c}_t \tag{33}$$

RT consumption

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t \tag{34}$$

Euler equation

$$\hat{\lambda}_t^o = \hat{\lambda}_{t+1}^o + \hat{R}_t - \hat{\pi}_{t+1} \tag{35}$$

Households marginal utility of consumption

$$\lambda_t^o = \frac{\beta b}{(1-\beta b)(1-b)}c_{t+1}^o - \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}c_t^o + \frac{b}{(1-\beta b)(1-b)}c_{t-1}^o \tag{36}$$

$$\lambda_t^{rt} = \frac{\beta b}{(1-\beta b)(1-b)}c_{t+1}^{rt} - \frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}c_t^{rt} + \frac{b}{(1-\beta b)(1-b)}c_{t-1}^{rt} \tag{37}$$

Phillips Curve

$$\frac{\varepsilon_p}{1-\varepsilon_p}\left(\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1}\right) = (1-\beta\varepsilon_p)\widehat{mc}_t + \beta\varepsilon_p\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) + \beta\frac{\varepsilon_p^2}{1-\varepsilon_p}\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) \tag{38}$$

Taylor Rule

$$R_t = \varphi_\pi\pi_t + \varphi_y y_t \tag{39}$$

Wage Setting

$$\begin{bmatrix} \left(\frac{1}{1-\varepsilon_w} + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\right)\hat{w}_t - \beta\frac{\varepsilon_w}{1-\varepsilon_w}\hat{w}_{t+1} + \\ -\left(\beta\varepsilon_w + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\right)\hat{\pi}_{t+1} + \\ +\left(\beta\varepsilon_w\gamma_w + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\gamma_w + \frac{\varepsilon_w}{1-\varepsilon_w}\right)\hat{\pi}_t + \\ -\frac{\varepsilon_w}{1-\varepsilon_w}\hat{w}_{t-1} - \frac{\varepsilon_w}{1-\varepsilon_w}\gamma_w\hat{\pi}_{t-1} \end{bmatrix} = (1-\beta\varepsilon_w)\varphi\hat{h}_t - (1-\beta\varepsilon_w)\hat{\psi}_t \tag{40}$$

6.1.2 Competitive Labour Market

Under perfectly competitive markets, $\hat{h}_t \neq \hat{h}_t^o \neq h_t^{rt}$ and in detail

$$\hat{h}_t = \theta \frac{h^{rt}}{h} \hat{h}_t^{rt} + (1-\theta) \frac{h^o}{h} \hat{h}_t^o$$

where

$$\phi_h \hat{h}_t^o = \lambda_t^o + \hat{w}_t$$
$$\phi_h \hat{h}_t^{rt} = \lambda_t^{rt} + \hat{w}_t$$

6.2 Welfare-based Loss Function (Internal Habits)

We derive the welfare-based Loss function following step-by-step the method used in Ascari et al.

Households' utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(X_{i,t}\right) - \frac{\psi_l}{1+\phi_l} \left(l_{i,t}\right)^{1+\phi_t} \right\}$$

or

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \frac{\left(X_{i,t}\right)^{1-\sigma}}{1-\sigma} - \frac{\psi_l}{1+\phi_l} \left(L_{i,t}\right)^{1+\phi_t} \right\}$$

where

$$X_t^i = C_{i,t} - bC_{i,t-1}$$

$$W_t = \theta \left[U\left(X_{R,t}\right) - V(L_{R,t})\right] + (1-\theta)\left[U\left(X_{O,t}\right) - V(L_{O,t})\right] \tag{41}$$

since $L_{o,t} = L_{r,t} = L_t$

$$W_t = \theta U\left(X_{R,t}\right) + (1-\theta) U\left(X_{O,t}\right) - V\left(L_t\right) = \tag{42}$$
$$= U\left(X_t\right) - V\left(L_t\right) \tag{43}$$

remember that

$$\frac{Y_{i,t} - Y_i}{Y_i} = y_{i,t} + \frac{1}{2} y_{i,t}^2 + O[2]$$

A second order approximation of $\theta U\left(X_{R,t}\right)$ delivers

$$\theta U\left(X_{R,t}\right) \simeq \theta \left[U(X_R) + U_{X_R}(X_{R,t} - X_R) + \frac{U_{X_R X_R}}{2}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta U\left(X_{R,t}\right) \simeq \theta \left[U\left(X_R\right) + U_{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[U_{X_R X_R}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta \ln\left(X_{R,t}\right) \simeq \theta \left[\ln\left(X_R\right) + \frac{1}{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[-\frac{1}{X_R^2}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta \ln \left(X_{R,t}\right) - \theta \ln \left(X_R\right) \simeq \theta \frac{\left(X_{R,t} - X_R\right)}{X_R} - \frac{\theta}{2} \frac{\left(X_{R,t} - X_R\right)^2}{X_R^2}$$

with crra

$$U_{X_{i,t}} = \frac{\left(X_{i,t}\right)^{1-\sigma}}{1-\sigma}$$

$$\theta U\left(X_{R,t}\right) \simeq \theta \left[U\left(X_R\right) + U_{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[U_{X_R X_R}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta \left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta U_{X_R}\left(x_{R,t} + \frac{1}{2}x_{R,t}^2\right) X_R + \frac{\theta}{2} U_{X_R X_R} X_R^2 \left[x_{R,t}^2\right]$$

$$\theta \left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta X_R^{1-\sigma}\left(x_{R,t} + \frac{1-\sigma}{2}x_{R,t}^2\right)$$

for Ricardians

$$(1-\theta)\ln\left(X_{O,t}\right) \simeq (1-\theta)\left[\ln\left(X_O\right) + \frac{1}{X_O}\left(X_{O,t} - X_o\right)\right] + \frac{(1-\theta)}{2}\left[-\frac{1}{X_R^2}\left(X_{R,t} - X_R\right)^2\right]$$

$$(1-\theta)\ln\left(X_{O,t}\right) - (1-\theta)\ln\left(X_O\right) \simeq (1-\theta)\frac{\left(X_{O,t} - X_O\right)}{X_O} - \frac{(1-\theta)}{2}\frac{\left(X_{O,t} - X_O\right)^2}{X_O^2}$$

$$(1-\theta)\ln\left(X_{O,t}\right) - (1-\theta)\ln\left(X_O\right) \simeq (1-\theta)\frac{\left(X_{O,t} - X_O\right)}{X_O} - \frac{(1-\theta)}{2}\frac{\left(X_{O,t} - X_O\right)^2}{X_O^2}$$

or in crra

$$(1-\theta)\left[U\left(X_{O,t}\right) - U\left(X_O\right)\right] \simeq (1-\theta) X_O^{1-\sigma}\left(x_{O,t} + \frac{1-\sigma}{2}x_{O,t}^2\right)$$

recalling that

$$X_{i,t} = C_{i,t} - bC_{i,t-1}$$

we have that it can be rewritten in terms of consumption as:

$$x_{i,t} = \frac{1}{1-b}c_{i,t} - \frac{b}{1-b}c_{i,t-1} = \frac{1}{1-b}\left(c_{i,t} - bc_{i,t-1}\right)$$

and therefore

$$x_{i,t}^2 = \frac{1}{(1-b)^2}\left(c_{i,t} - bc_{i,t-1}\right)^2$$

$X_{i,t}$ can be approximated to second order by

$$\frac{X_{i,t} - X_i}{X_i} = x_{i,t} + \frac{1}{2}x_{i,t}^2$$

so that

$$x_{i,t}+\frac{1}{2}x_{i,t}^{2}=\frac{1}{1-b}\left(c_{i,t}+\frac{1}{2}c_{i,t}^{2}\right)-\frac{b}{1-b}\left(c_{i,t-1}+\frac{1}{2}c_{i,t-1}^{2}\right)$$

$$x_{i,t}=\frac{1}{1-b}\left(c_{i,t}+\frac{1}{2}c_{i,t}^{2}\right)-\frac{b}{1-b}\left(c_{i,t-1}+\frac{1}{2}c_{i,t-1}^{2}\right)-\frac{1}{2}x_{i,t}^{2}$$

the following equations

$$(1-\theta)\left[U\left(X_{O,t}\right)-U\left(X_{O}\right)\right]\simeq(1-\theta)\,X_{O}^{1-\sigma}\left(x_{O,t}+\frac{1-\sigma}{2}x_{O,t}^{2}\right)$$

$$\theta\left[U\left(X_{R,t}\right)-U\left(X_{R}\right)\right]\simeq\theta X_{R}^{1-\sigma}\left(x_{R,t}+\frac{1-\sigma}{2}x_{R,t}^{2}\right)$$

become

$$(1-\theta)\left[U\left(X_{O,t}\right)-U\left(X_{O}\right)\right]\simeq(1-\theta)\,X_{O}^{1-\sigma}\left(\begin{array}{c}\frac{1}{1-b}\left(c_{O,t}+\frac{1}{2}c_{O,t}^{2}\right)+\\-\frac{b}{1-b}\left(c_{O,t-1}+\frac{1}{2}c_{O,t-1}^{2}\right)-\frac{1}{2}x_{O,t}^{2}+\frac{1-\sigma}{2}x_{O,t}^{2}\end{array}\right)$$

$$(1-\theta)\left[U\left(X_{O,t}\right)-U\left(X_{O}\right)\right]\simeq(1-\theta)\,X_{O}^{1-\sigma}\left(\begin{array}{c}\frac{1}{1-b}\left(c_{O,t}+\frac{1}{2}c_{O,t}^{2}\right)+\\-\frac{b}{1-b}\left(c_{O,t-1}+\frac{1}{2}c_{O,t-1}^{2}\right)-\frac{\sigma}{2}x_{O,t}^{2}\end{array}\right)$$

$$\theta\left[U\left(X_{R,t}\right)-U\left(X_{R}\right)\right]\simeq\theta X_{R}^{1-\sigma}\left(\begin{array}{c}\frac{1}{1-b}\left(c_{R,t}+\frac{1}{2}c_{R,t}^{2}\right)+\\-\frac{b}{1-b}\left(c_{R,t-1}+\frac{1}{2}c_{R,t-1}^{2}\right)-\frac{\sigma}{2}x_{R,t}^{2}\end{array}\right)$$

Approximation of $U(L_t)$ delivers

$$U(L_t)=\frac{\psi_l}{1+\phi_l}(L_t)^{1+\phi_t}$$

$$U(L_t)=U(L)+U_L(L_t-L)+\frac{U_{LL}}{2}(L_t-L)^2$$

$$\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l}\simeq\frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l}+\psi_l(L)^{\phi_l}\left(L_t-L\right)+\frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}\left(L_t-L\right)^2$$

$$\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l}-\frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l}\simeq\psi_l(L)^{\phi_l}\left(L_t-L\right)+\frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}\left(L_t-L\right)^2$$

$$\begin{aligned}\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l}-\frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l}\quad&\simeq\quad\psi_l(L)^{\phi_l}L_t-\psi_l\phi_l(L)^{\phi_l}L_t+\\&+\left(\frac{\psi_l\phi_l}{2}-\psi_l\right)(L)^{\phi_l+1}+\frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}L_t^2\end{aligned}$$

$$\begin{aligned}\frac{1}{1+\phi_l}(L_{i,t})^{1+\phi_l}-\frac{1}{1+\phi_l}(L)^{1+\phi_l}\quad&\simeq\quad\left(1-\phi_l\right)(L)^{\phi_l}L_t+\frac{\phi_l}{2}(L)^{\phi_l-1}L_t^2+\\&+\left(\frac{\phi_l}{2}-1\right)(L)^{\phi_l+1}\end{aligned}$$

$$U(L_t) - U(L) = \psi_l (L)^{\phi_l} l_t L + \frac{1}{2} \psi_l (L)^{\phi_l} \phi_l l_t^2 L$$

$$U(L_t) - U(L) = U_L L l_t + \frac{1}{2} U_L L \phi_l l_t^2$$

$$U(L_t) - U(L) = U_L L \left(l_t + \frac{\phi_l}{2} l_t^2 \right)$$

$$U(L_t) - U(L) = U_L L l_t + \frac{U_{LL}}{2} L^2 l_t^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l (L)^{\phi_l} \phi_l}{2} L l_t^2$$

$$U(L_t) - U(L) = U_L L l_t + U_L L \frac{\phi_l}{2} l_t^2$$

$$U(L_t) = \frac{\psi_l}{1+\phi_l} (L_t)^{1+\phi_t}$$

$$U(L_t) = U(L) + U_L (L_t - L) + \frac{U_{LL}}{2} (L_t - L)^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l \phi_l}{2} (L)^{\phi_l - 1} l_t^2 L^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l \phi_l}{2} (L)^{\phi_l} l_t^2 L$$

$$U(L_t) = U(L) + U_L (L_t - L) + \frac{\psi_l \phi_l}{2} (L)^{\phi_l} (L_t^2 + L^2 - 2 L L_t)$$

$$U(L_t) = U(L) + U_L \frac{(L_t - L)}{L} L + \frac{U_{LL}}{2} \frac{(L_t - L)^2}{L^2} L^2$$

$$U(L_t) - U(L) = U_L L l_t + U_L L \frac{1}{2} l_t^2 + \frac{\phi_l}{2} U_L L \left(l_t^2 \right)$$

$$U(L_t) - U(L) = U_L L \left(l_t + \frac{1+\phi_l}{2} l_t^2 \right)$$

Summing all the terms

$$\begin{aligned} W_t - W \quad = \quad & (1-\theta) X_O^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ - \frac{b}{1-b} \left(c_{O,t-1} + \frac{1}{2} c_{O,t-1}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \end{array} \right) + \\ & + \theta X_R^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) + \\ - \frac{b}{1-b} \left(c_{R,t-1} + \frac{1}{2} c_{R,t-1}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \end{array} \right) + \\ & - U_L L \left(l_t + \frac{1+\phi_l}{2} l_t^2 \right) \end{aligned}$$

or, given that steady state consumption and hours worked level are identical for the two groups of agents

$$W_t - W = (1-\theta) U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \left(l_t + \frac{1+\phi_l}{2} l_t^2\right)$$

$$W_t - W = (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \left(l_t + \frac{1+\phi_l}{2} l_t^2\right)$$

From the economy production function we know that

$$l_t = y_t + d_{w,t} + d_{p,t} - a_t$$

where $d_{w,t} = \log \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj$ is the log of the wage dispersion and $d_{p,t} = \log \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\theta_p} di$ is the log of the price dispersion. Both terms are of second order and therefore they cannot be neglected in a second order approximation. Notice that

$$l_t^2 = \left(\hat{y}_t + d_{w,t} + d_{p,t} - a_t\right)^2 = y_t^2 + a_t^2 - 2y_t a_t$$

thus

$$W_t - W = (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}\left(y_t^2 + a_t^2 - 2y_t a_t\right) \end{pmatrix} + tip$$

or

$$W_t - W = (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2}y_t^2 + \\ +\frac{1+\phi}{2}a_t^2 - (1+\phi)\, y_t a_t \end{pmatrix} + tip$$

$$W_t - W = (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \end{pmatrix} + tip$$

$$W_t - W = (1-\theta) U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \end{pmatrix} + tip$$

$$W_t - W = \sum_{t=0}^{\infty} (1-\theta) U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\ +\theta U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\ -U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \end{pmatrix} + tip$$

$$W_t - W = (1-\beta b) \sum_{t=0}^{\infty} \left[(1-\theta) \frac{1}{1-b} U_X X \left(\left(c_{O,t} + \frac{1}{2}c_{O,t}^2 \right) - (1-b)\frac{\sigma}{2}x_{O,t}^2 \right) \right] + \\ + (1-\beta b) \sum_{t=0}^{\infty} \left[(\theta) \frac{1}{1-b} U_X X \left(\left(c_{R,t} + \frac{1}{2}c_{R,t}^2 \right) - (1-b)\frac{\sigma}{2}x_{R,t}^2 \right) \right] + \\ - \sum_{t=0}^{\infty} \left[U_L L \left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \right) \right] + tip$$

$$W_t - W = \frac{(1-\theta)(1-\beta b)}{1-b} U_X X \sum_{t=0}^{\infty} \left[\left(\left(c_{O,t} + \frac{1}{2}c_{O,t}^2 \right) - (1-b)\frac{\sigma}{2}x_{O,t}^2 \right) \right] + \\ + \frac{\theta(1-\beta b)}{1-b} U_X X \sum_{t=0}^{\infty} \left[\left(\left(c_{R,t} + \frac{1}{2}c_{R,t}^2 \right) - (1-b)\frac{\sigma}{2}x_{R,t}^2 \right) \right] + \\ -U_L L \sum_{t=0}^{\infty} \left[\left(y_t + d_{w,t} + d_{p,t} + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \right) \right] + tip$$

$$W_t - W = U_X X (1-\theta) \sum_{t=0}^{\infty} \left[\left(\frac{(1-\beta b)}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \right) \right] + $$
$$+ U_X X \theta \sum_{t=0}^{\infty} \left[\left(\frac{(1-\beta b)}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \right) \right] + $$
$$- U_L L \sum_{t=0}^{\infty} \left[\left(y_t + d_{w,t} + d_{p,t} + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) \right] + tip$$

since $U_X X = U_L L = U_C C$ and $MRS = MPL = 1$ at the efficient steady state

$$W_t - W = X^{1-\sigma} (1-\theta) \sum_{t=0}^{\infty} \left[\left(\frac{(1-\beta b)}{(1-b)} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \right) \right] + $$
$$+ X^{1-\sigma} \theta \sum_{t=0}^{\infty} \left[\left(\frac{(1-\beta b)}{(1-b)} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \right) \right] + $$
$$- \frac{(1-\beta b)}{(1-b)} X^{1-\sigma} \sum_{t=0}^{\infty} \left[\left(\begin{array}{c} y_t + d_{w,t} + d_{p,t} + \\ + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \end{array} \right) \right] + tip$$

$$\frac{W_t - W}{U_C C} = \frac{(1-\beta b)}{(1-b)} \left(\begin{array}{c} (1-\theta) \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ + \theta \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) \end{array} \right) + $$
$$- (1-\theta) \frac{\sigma}{2} x_{O,t}^2 - \theta \frac{\sigma}{2} x_{R,t}^2 + $$
$$- \frac{(1-\beta b)}{(1-b)} \left(y_t + d_{w,t} + d_{p,t} + \frac{1+\phi}{2} y_t^2 - (1+\phi) y_t a_t \right) + tip$$

$$\frac{W_t - W}{U_C C} = - \left(\frac{\phi}{2} \frac{(1-\beta b)}{(1-b)} \right) y_t^2 - (1-\theta) \frac{\sigma}{2} x_{O,t}^2 - \theta \frac{\sigma}{2} x_{R,t}^2 + $$
$$- \frac{(1-\beta b)}{(1-b)} (d_{w,t} + d_{p,t}) + \frac{(1-\beta b)}{(1-b)} (1+\phi) y_t a_t + tip$$

$$\frac{W_t - W}{U_C C} = - \frac{(1-\beta b)}{(1-b)} \left[\begin{array}{c} \frac{\phi}{2} y_t^2 + \theta \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{O,t}^2 + \\ + (1-\theta) \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{R,t}^2 + (d_{w,t} + d_{p,t}) + \\ - (1+\phi) y_t a_t \end{array} \right] + tip$$

$$\frac{W_t - W}{U_C C} = - \frac{(1-\beta b)}{(1-b)} \left[\begin{array}{c} \frac{\phi}{2} y_t^2 + (1-\theta) \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{O,t}^2 + \\ + \theta \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{R,t}^2 + \frac{\alpha_w}{\kappa_w} \left(\pi_t^w \right)^2 + \\ + \frac{\eta}{\kappa_p} \left(\pi_t^p \right)^2 - (1+\phi) y_t a_t \end{array} \right] + tip$$

$$\frac{W_t - W}{U_C C} = - \frac{(1-\beta b)}{(1-b)} \left[\begin{array}{c} \frac{\phi}{2} y_t^2 + (1-\theta) \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{O,t}^2 + \\ + \theta \frac{(1-b)}{(1-\beta b)} \frac{\sigma}{2} x_{R,t}^2 + \frac{1}{2} \frac{\alpha_w}{\kappa_w} \left(\pi_t^w \right)^2 + \\ + \frac{1}{2} \frac{\eta}{\kappa_p} \left(\pi_t^p \right)^2 - (1+\phi) y_t a_t \end{array} \right] + tip$$

substituting $x_{i,t}$ with its definition in terms of output and rearranging we obtain

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\left[\begin{array}{c} \phi y_t^2 + \frac{1}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_t^2 + \frac{\theta}{(1-\theta)} w_t^2 + \\ -\frac{2\theta}{(1-\theta)} w_t a_t \end{array}\right) + \\ + \frac{b^2}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_{t-1}^2 + \frac{\theta}{(1-\theta)} w_{t-1}^2 + \\ -\frac{2\theta}{(1-\theta)} w_{t-1} a_{t-1} \end{array}\right) + \\ -\frac{2b}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_t y_{t-1} + \frac{\theta}{(1-\theta)} w_t w_{t-1} + \\ -\frac{\theta}{(1-\theta)} w_t a_{t-1} - \frac{\theta}{(1-\theta)} a_t w_{t-1} \end{array}\right) + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2 - 2\left(1+\phi\right) y_t a_t \end{array}\right] + tip$$

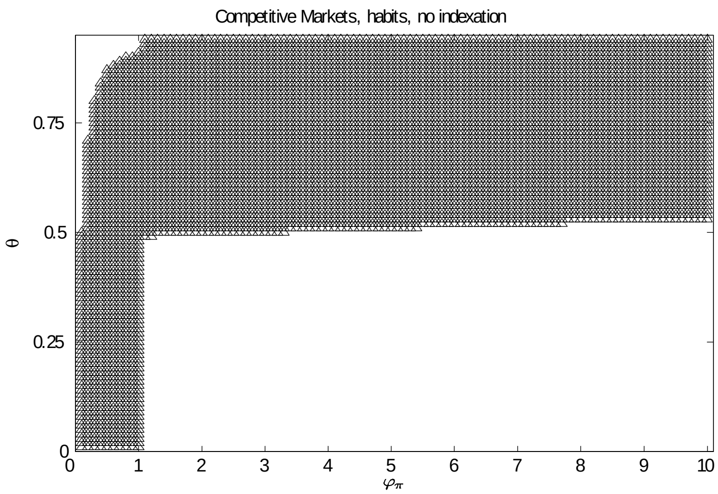
which can be rewritten in terms of gap variables as

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\left[\begin{array}{c} \left(\phi + \frac{1}{(1-\beta b)(1-b)}\right)\left(y_t^{gap}\right)^2 + \\ + \left(2\phi + \frac{2}{(1-\beta b)(1-b)}\right) y_t^{gap} y_t^* + \\ + \frac{1}{(1-\beta b)(1-b)} \frac{\theta}{(1-\theta)} \left(w_t^{gap}\right)^2 + \\ + \frac{b^2}{(1-\beta b)(1-b)}\left(y_{t-1}^{gap}\right)^2 + \frac{b^2}{(1-\beta b)(1-b)} 2 y_{t-1}^{gap} y_{t-1}^* + \\ + \frac{b^2}{(1-\beta b)(1-b)} \frac{\theta}{(1-\theta)}\left(w_{t-1}^{gap}\right)^2 + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^* + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^* y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)} \frac{\theta}{(1-\theta)} w_t^{gap} w_{t-1}^{gap} + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2 - 2\left(1+\phi\right) a_t y_t^{gap} \end{array}\right] + tip$$

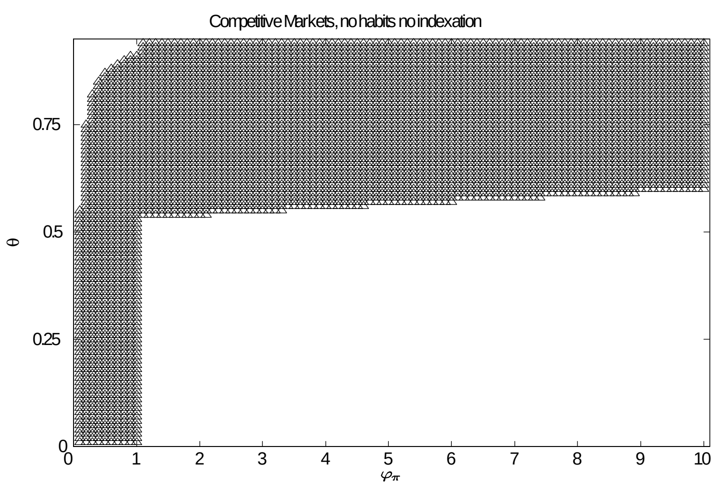
6.3 Robustness

Determinacy with no indexation on prices and wages ($\gamma_p = 0, \gamma_w = 0$)

6.3.1 Competitive Labour Markets

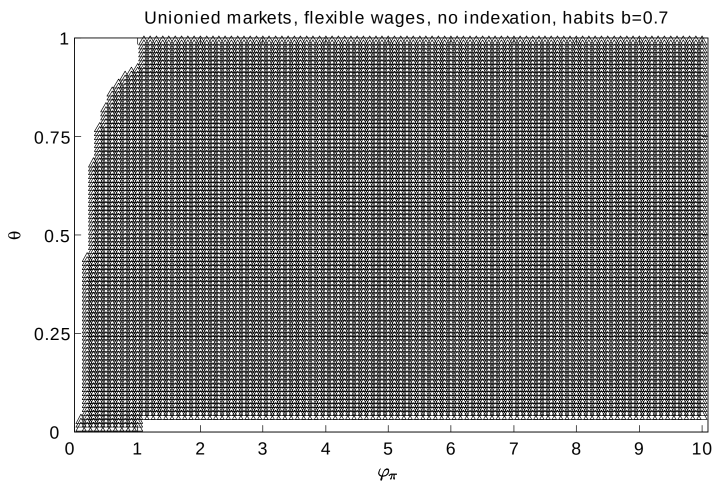


Habits in Consumption

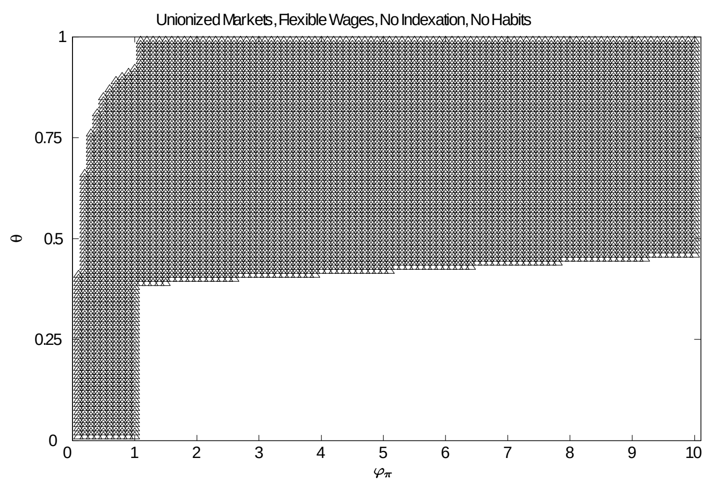


No Habits in Consumption

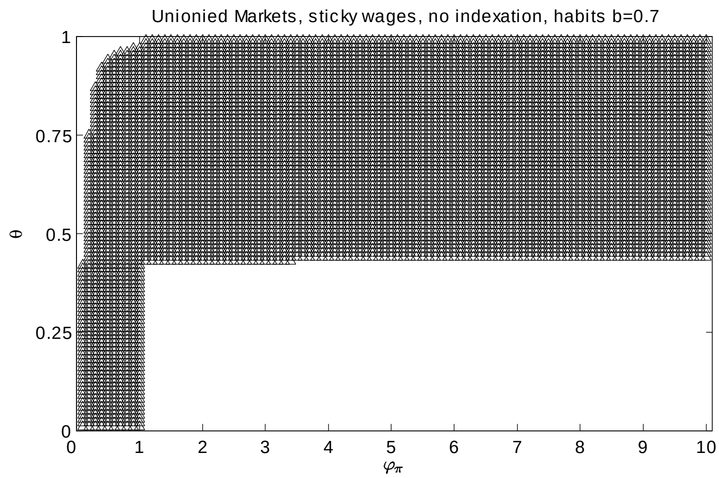
6.3.2 Unionize Labour Markets



Flexible Wages, Habits in Consumption



Flexible Wages, no Habits in Consumption



Sticky Wages, Habits in Consumption

Sticky Wages, no Habits in Consumption