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THE NUMBER-SPACE INTERACTION AT THE PERCEPTUAL AND THE REPRESENTATIONAL LEVEL

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SUMMARY

Chapter 1. INTRODUCTION

1. Background

Current cognitive models postulate a numerical spatial representation where numbers are conceived as a variable distribution of activation along a mental number line (Dehaene, 1992; Dehaene, Piazza, Pinel, & Cohen, 2003). The most convincing evidence in support of a spatial organization of numbers is the SNARC effect (Spatial-Numerical-Association of Response Codes).

The SNARC effect describes the observation that, across different tasks and experimental settings, small numbers seem to be associated with the leftward space while large numbers are associated with the rightward space. Although the SNARC effect seems unaffected by handedness or hemispheric dominance (Dehaene, Bossini, & Giraux, 1993; see Wood, Willmes, Nuerk, & Fischer, 2008 for a meta-analysis), recent data suggest that the SNARC effect, and with it the spatial orientation of the mental number line, seems correlated with reading and writing direction, as suggested by a weaker SNARC effect in Iranian participants (Dehaene et al., 1993) and a temporal advantage in processing pairs of ascending numbers in right-to-left direction in Arabic participants (Fischer, Shaki, & Cruise, 2009; Shaki, Fischer, & Petrusic, 2009).

In the last years, the numerical influence on visual-spatial task has been explored by means of both bisection and reproduction paradigms.

In a first line bisection study exploring the relationship between numbers and space, Fischer (2001) observed a bias toward the larger flanker number, regardless of its left or right location. Fischer (2001) suggested that the bisection performance indicated activation of a numerical representation with spatial properties. In particular, the bias toward the larger digit was interpreted according to the MNL hypothesis, as if perception of two numbers automatically evoked a corresponding segment within this representation.

de Hevia and colleagues (2006) replicated and extended the larger-digit effect in the bisection of lines and of unfilled spaces flanked by different number. The systematic misplacement of the subjective midpoint toward an irrelevant larger number strongly suggests that numbers impact on the visuo-spatial processing by inducing a so-called ‘cognitive illusion’ of length (de Hevia et al., 2006). According to this assumption, processing of large-magnitude numbers brings about an illusory expansion of space and processing of small-magnitude numbers brings about an illusory compression of space (de Hevia, Girelli, Bricolo, & Vallar, 2008; de Hevia et al., 2006). Further support to this interpretation came from a length reproduction task (de Hevia, Girelli et al., 2008) that required participants to reproduce the length of a spatial extension delimited by two digits. They reported an underestimation of the spatial length delimited by small magnitude numbers, and an overestimation of the spatial length when delimited by large magnitude numbers, according to which expansion and compression of a spatial extension would take place as a function of the represented numerical magnitude.

Finally, important for the current study, the spatial representation of numbers is intimately related with action (for recent review see Andres, Olivier, & Badets, 2008). The motor system is not limited to action control and its role in semantic processing has been progressively acknowledged. Recently, the “sensorimotor” theories of cognition proposed that the motor system contributes to high-level cognitive processes, such as the building up of a numerical representation. So far, the interactions between the motor system and the cognitive process have mainly described from semantic to sensory-motor process. However, the functional link will be establish also once the sensory-motor will be proven to shape semantics (Badets & Pesenti, 2010). Indeed sensorimotor experience is assumed to play an important role in language and number representation (Glenberg & Kaschak, 2002).

Evidence for the influence of the numerical information on specific motor components has been found by means of various and original tasks. Automatic activation of the numerical magnitude code has been proposed to account for the numerical influence on motor tasks such as i) lateralized movements or unimanual responses, and ii) manual gestures such as grasping and different grip aperture. In the following, this categorisation will be further elaborated.

First, SNARC-like spatial numerical associations were observed: leftward movements and/or left-lateralized responses were elicited consistently faster by relatively small numbers whereas rightward movements and/or right-lateralized responses were elicited faster by relatively large numbers (Fischer, 2003; Fischer, Warlop, Hill, & Fias, 2004; Fischer & Miller, 2008; Ishihara et al., 2006; Schwarz & Keus, 2004; Schwarz & Müller, 2006; Song & Nakayama, 2008). These findings support the strong link between different magnitude domains such as numerical magnitude and motor planning.

Second, the automatic processing of numerical information impact on manual gestures and grasping movements showing a temporal advantage in case of a compatibility between numerical magnitude and grip posture/shape (Andres, Davare, Pesenti, Olivier, & Seron, 2004; Andres, Ostry, Nicol, & Paus, 2008; Badets, Andres, Di Luca, & Pesenti, 2007; Lindemann, Abolafia, Girardi, & Bekkering, 2007; Moretto & di Pellegrino, 2008).

Overall, this empirical evidence suggests an association between numbers and space that determine action. According to the ATOM theory (Walsh, 2003), the association between number, space and action might be due to the activation of a common representation of magnitude, given that an action has temporal and as well as spatial components (Wood & Fischer, 2008). Furthermore, this common representation of magnitudes would arise from a partial overlapping of neuroanatomical structures subserving numbers, space and time processing whose integration is required for goal-related action.

2. Aims and summary of the research

The present thesis contributes to this line of research exploring the MNL and 'cognitive illusion' hypotheses by means of visuo-spatial and motor tasks.

The general question we address is to what extent and at which stage the spatial mapping of numbers in the representational space (MNL) and the interaction between numerical and physical magnitude ('cognitive illusion') interferes with the motor system.

Evidence providing support to the mental number line effect on action but is not lacking but several questions are still unresolved. Critically, the studies reported so far in the literature bring forward many methodological problems. To overcome these limitations, in the current investigation original experimental paradigms will be developed and adopted.

In the following section are the abstracts of the different studies here reported.

- Chapter 5. **NUMBER CAN MOVE OUR HAND: A SPATIAL REPRESENTATION EFFECT IN DIGIT HANWRITING**

The present study investigated the effect of numbers on an overlearned complex motor plan that does not require explicit lateralized movements or strict spatial constraints: spontaneous handwriting. In particular, we investigated whether the spatial mapping of numbers interferes with the motor planning involved in writing. To this aim participants' spontaneous handwriting of single digits (Experiment 1) and letters (Experiment 2) was recorded with a digitizing tablet. We show that the writing of numbers is characterised by a spatial dislocation of the digits as a function of their magnitude, i.e., small numbers were written leftwards compare to large numbers. In contrast, the writing of letters showed a null or marginal effect with respect to their dislocation on the writing area. These findings show that the automatic mapping of numbers into space interacts with action planning by modulating specific motor parameters in spontaneous handwriting

- Chapter 6. **HOW SPATIAL ORIENTATION OF THE MENTAL NUMBER LINE AUTOMATICALLY INFLUENCES ELEMENTARY PHYSICAL MOVEMENTS**

Part one. THE SPATIAL FEATURES OF THE MENTAL NUMBER LINE INFLUENCES PHYSICAL MOVEMENTS

The current study aimed at investigating the influence of the mental number line representation on simple motor movements such as the manual connection of two points. To this aim we manipulated the direction of the connection movement between two points flanked by different numerical values. The results indicated that the maximum velocity of the connection movement was modulated by the movement direction with an advantage for movement compatible with the reading / writing direction (Experiment 3). Critically, we also observed that the maximum velocity was driven by relative position of the irrelevant numbers, with movements from the smaller to the larger number being faster irrespective of the movement direction (Experiment 4). Taken together, these data provides further evidence for the impact of the spatial attributes of the mental number line on motor movements – in the present case, even against a dominant spatial bias such as reading direction.

Part two. HOW NUMBERS CAN SPEED UP MANUAL ACTION

In the present study we evaluated the impact of numerical information on motor output by exploiting the evidence that the speed reached by the manual connection of two distinct points is correlated with their physical distance (Cooke, Brown, & Cunningham, 1989). We reason that if irrelevant numbers may induce a mis-perception of the distance between two points to be connected, this should be reflected on the movement speed. In particular, according to the mental number line hypothesis we found a speed difference in the manual connection of two numerically close numbers and two numerically distant numbers, placed at equal physical distance (Experiment 6). The representational length effect found suggests that symbolic distance impact on speed movement in the same way as physical distance. Besides, this effect suggests that numbers impact not only on motor planning but also on motor execution.

- Chapter 7. **MAPPING NUMBERS ONTO SPACE: FURTHER SUPPORT FOR THE COGNITIVE ILLUSION HYPOTHESIS**

The present study explored, by means of a reproduction tasks, whether numerical processing modulates the mental representation of a horizontal and bidimensional spatial extension. In particular, this study tested the ‘cognitive illusion’ hypothesis in three experiments exploiting a digitized tablet to record the reproduction performance. Magnitude effects emerge not only in the reproduction of horizontal extensions (Experiment 7), but also in the reproduction of circles enclosing irrelevant digits (Experiment 8), suggesting that numbers are also mapped onto a non-linear space. Moreover, presenting numerical and spatial information sequentially only marginally modulates this effect (Experiment 9).

Chapter 2. THE ASSOCIATION BETWEEN NUMBER AND SPACE

1. On the spatial mapping of numbers

1.1. The 'number sense'

Human mathematical competence emerges from the integration of two distinct representational systems, a verbal system of number word and a non-symbolic representation of approximate quantities[1] (for a recent review see Cantlon et al., 2009; see also Lemer, Dehaene, Spelke, & Cohen, 2003).

The first system allows individuals to precisely keep track of small numbers of objects and to represent information on their continuous quantitative properties. This system arises from the language competence, which enables humans to overtake other species in arithmetic, and to develop symbol systems that support exact calculation and higher mathematics.

On the other hand, the approximate numerical system is an evolutionary precursor of the human arithmetic abilities that exists in animals and supports representation of the number of discrete objects or events along an analogical mental continuum. This analogical representation, mediating what has been called 'number sense', implies the spatial mapping of numbers and suggests a strong connection between numerical and spatial processes.

The link between mathematical abilities and spatial skills has been put forward for a long time. Over a century ago, Galton introduced the concept of a spatial numerical representation. According to his early investigation over 15% of normal adults reported to mentally represent numbers in some visuo-spatial forms (Galton, 1880a, 1880b; Seron, Pesenti, Noël, Deloche, & Cornet, 1992). This early observation already suggested that the integration of a numerical representation into visuo-spatial coordinates is not a rare phenomenon. The reported spatial layouts were predominantly oriented from left to right, were mostly

[1] In the Cantlon et al.'s paper (Cantlon, Platt, & Brannon, 2009) this system has been called ANS (Approximate Number System).

automatically activated, were stable in time and emerged in childhood. Galton's study on imagery suggested that the internal representation of numbers might evoke a stable, linear and spatially oriented representation.

The intuition that numerical representation may be spatially organized has received a growing amount of experimental evidence (discussed in paragraph 1.2.), which is formally interpreted in favour of the ' mental number line'. This numerical representation formalized within the 'Triple Code Model' (Dehaene, 1992) is one of the three codes in which numbers can be mentally represented. Besides the visual Arabic code (in which numbers are represented as digit strings) and the verbal code (in which numbers are represented as sequences of words), this model postulates an analogical code that represents numbers as variable distributions of local activation along a mental number line.

1.1.1. Evidence supporting the approximate number system

The existence of an approximate number system is supported by converging evidence from cognitive development, comparative cognition, cross-cultural cognition and neurobiology (for more details see par. 3).

The universality of the ANS is well represented by the fact that the Weber-Fechner law[2] predicts adequately numerical performance across species, developmental ages and human cultures. Indeed, when monkeys and college students are tested in the same numerosity comparison and addition tasks, the Weber-Fechner law similarly predicts their respective performance (Cantlon & Brannon, 2006; Cantlon & Brannon, 2007). Furthermore, the behavioural signatures of the ANS emerge within the first year of human life (Lipton & Spelke, 2003; Xu & Spelke, 2000). Besides, Amazonian people with very limited verbal counting systems show a ratio-dependent performance when comparing relative numerosities, despite the lack of an exact appreciation of large numerical quantities (Pica, Lemer, Izard, & Dehaene, 2004).

The evidence for similar numerical processes, from a diverse set of methods, populations and species, makes a strong case that the basis of numerical representation is a primitive cognitive and neural system.

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[2] The Weber-Fechner law states that the difference in intensity needed to discriminate two stimuli is proportional to their objective intensities. Under the Weber-Fechner law, discrimination performance is modulated by the ratio (min/max) of the intensities rather than their absolute difference.

Two competing models attempted to explain the origin of an ANS system. First, the Mode-Control Model (Meck & Church, 1983) postulates an accumulator mechanism serially incremented by a constant amount for each object or event (the same mechanism would be used for counting and time). The second model invokes a parallel summation mechanism (Dehaene, 2008) in which objects in a set are detected in parallel and passed to a summation stage that accumulates signals across the object detection stage, ultimately indexing the total number of objects in the set. Relevant, besides the opposite mechanism in detection stage (serially vs. parallel) in their accumulation stages, both models postulate that discrete numerical value is analogically translated into a continuous subjective representation of numerical value or analogical magnitude representation. This analogue magnitude representation is thought to be fast, inaccurate, and preverbal; it is assumed to be a prerequisite developmental stage to the build-up of slower, but exact, verbal algorithms that are the basis of our abstract computational cognitive skills (Feigenson, Dehaene, & Spelke, 2004).

1.2. Evidence supporting the spatial representation of numbers

1.2.1. The distance, size and SNARC effects

The ‘Triple Code Model’ (Dehaene, 1992) proposes that the numerical magnitude is coded into an analogical representation, which is conceived as an oriented mental number line.

Detailed chronometric studies of number processing can therefore reveal the nature and attributes of our cognitive representation of numbers. In particular, two semantic effects, the distance and the size effect, characterising the performance in basic numerical tasks have been interpreted as evidence for a representational continuum compressed towards the right (Dehaene, 1992; Dehaene et al., 2003).

The distance effect shows that participants required more time to discriminate two number magnitudes close together, whereas they answer faster when two numbers are increasingly different. This widely replicated effect suggests that numbers are mapped onto some internal continuum with distinct entries for each number magnitude (for a review see Dehaene, 1997). Instead, the size effect refers to the phenomenon according to which, at a constant distance, it takes longer to process a larger number than a small one.

More specifically, at least in western cultures, the MNL seems to be oriented along a left to right direction. The most convincing evidence for this hypothesis comes from the SNARC effect (Spatial-Numerical-Association of Response Codes), a stimulus–response association where small digits are associated with left-sided responses and large digits with right-sided responses (Dehaene et al., 1993).

Dehaene et al. (1993) found that in a parity judgment task (odd/even) participants were faster in responding to relatively small numbers with the left hand and to relatively large numbers with the right hand, which suggests that people represent number magnitude spatially, with lower values represented on the left and higher values on the right. This effect was originally reported in two-digit number comparison (Dehaene, Dupoux, & Mehler, 1990), then extended to one digit number, clarifying that the effect is modulated by the relative magnitude, it does not depend on handedness or hemispheric dominance, and it seems relatively independent of hands crossing (Dehaene et al., 1993; but see also Wood, Nuerk, & Willmes, 2006). Furthermore, the SNARC effect seems rather notation-independent since several studies obtained SNARC-like effects both with Arabic digits or written number words (e.g., Dehaene et al., 1993; Fias, 2001; Nuerk, Iversen, & Willmes, 2004). In particular, the slopes of the SNARC functions had similar magnitudes, although sometimes they tended to be smaller for number words (Nuerk, Wood, & Willmes, 2005), in agreement with the idea that the spatial association reflects access to an abstract representation of number magnitude.

Although the SNARC effect has been primarily investigated in parity tasks, and to a lesser extent in magnitude comparison, the effect is clearly not task specific. For example, a SNARC effect was also reported in a phoneme monitoring task that simply required participants to indicate whether the name corresponding to a visually presented digit contained a sound or not.

Some of the tasks reviewed above required no explicit number-related information to be performed. However, despite the fact that number magnitude was not needed, the numbers had to be processed to some degree. The SNARC effect, however, has also been obtained in studies where the visually presented numbers were completely irrelevant. For instance, using digits as a background upon which oriented lines or triangles were superimposed for classification, Fias et al. (2001) found that participants' manual responses were influenced by the spatial-numerical association evoked by the background. This is a strong argument in favour of an automatic spatial coding of numbers. Furthermore, in Fischer et al.'s study (2003) adopting a visual-spatial attention allocation task, digits served merely as a fixation point but did nevertheless influence speed of target detection. The fact that the SNARC effect emerges when numerical information is not required for correct performance, and may even interfere with performing the task, suggests that a high degree of automaticity is involved in the processes that give access to the magnitude representation and its spatial association.

Recently, the SNARC effect has been reported with pedal (Schwarz & Müller, 2006) and saccadic responses (Fischer, Warlop, Hill, & Fias, 2004; Schwarz & Keus, 2004), suggesting that this phenomenon does not simply reflect an overlearned motor association between numbers and manual responses, and that it does not only characterize the effectors associated to writing (i.e., hands) or reading (i.e., eyes).

Critically, it is conceivable that our mental representation uses spatial codes that are richer than strictly unidimensional, and, consequently, that mapping is not limited to the horizontal direction. Indeed, Ito and Hatta (2004) extended the investigation of the SNARC effect by adopting a parity judgement task with a vertical arrangement of the response keys,. Results indicate that participants answered more quickly to digits of greater numerical magnitude when they had to press the upper key, rather than the lower key. A similar vertical SNARC was firstly reported by Dehaene (1997) and then replicated by Schwarz and Keus (Schwarz & Keus, 2004) strengthening the view that our magnitude representation resembles a number map, rather than a number line.

In summary, the mental number appears as an abstract, analogical and logarithmic representation automatically activated and characterised by a left-to-right direction. In the following section, I will review the evidence showing that the numerical spatial representation should not be considered as fixed and unchangeable, since the spatial number coding is largely determined by various numerical (e.g., range of stimuli) and spatial (e.g., imagery instructions) parameters specific to the task.

1.2.2. The modulation of spatial compatibility: chronometric, cultural and context differences

The mapping of numbers onto specific spatial locations is a dynamic and flexible process modulated by individual and context-dependent features.

Critically, the spatial orientation of the mental number line appears to correlate with the ***reading and writing scanning direction**,* as suggested by a weaker SNARC effect in Iranian participants, who write and read from right to left (Dehaene et al., 1993), and a temporal advantage in processing pairs of numbers along a right to left direction in Arabic readers (Zebian, 2005). In this latter study, the authors found that monolingual Arabic speakers processed two numbers more easily when the larger number was placed to the left of the smaller number, compared to a display with the larger number on the right. This effect decreased for a group of bilingual Arabic-English speakers. These results prompted the proposal that directional scanning habits may be partially responsible for the SNARC effect.

However, a recent study reported a vertical SNARC effect in Japanese participants: larger numbers were associated with the top button whereas smaller numbers were responded to faster with the bottom button (Ito & Hatta, 2004). Given that Japanese is written from top to bottom, this finding reveals a dissociation of the SNARC effect and the writing direction, casting doubt on a tight relation between reading/writing habits and the SNARC effect. Clearly, more systematic studies from cultures with different notation systems are needed to clarify exactly which notational properties shape the direction and strength of the SNARC effect.

Besides, the SNARC effect is modulated by the relative magnitude rather than absolute magnitude of numbers, suggesting context-dependent interpretation of numerical information. Indeed, the ***relative magnitude*** (range of stimuli) changed the association between number and manual responses so that, in a standard number classification task, the number 4 was preferentially associated to the left-hand or to the right-hand response depending on the range of stimulus digits displayed in the experimental (i.e., 0-5 or 4-9; Dehaene et al., 1993; Experiment 3).

Besides, the spatial mapping of a given number can be reversed as a result of ***imagery instructions*** (Bächtold, Baumüller, & Brugger, 1998). The authors demonstrated not only that the spatial coordinate systemof the response but also the internal representation of the numerical information is important. They instructed participants to think of the digits as either lengths on a ruler or times on an analogue clock-face. The results confirmed that the SNARC effect was modulated by the task demands: when subjects imagined numbers displayed on a ruler, smaller numbers were answered faster with the left-hand than with the right hand and vice versa for larger numbers. Critically, the number-space compatibility effect was reversed when subjects were asked to imagine numbers on a clock-face with smaller numbers they responded faster with the right-hand while larger numbers responded faster with the left hand. These results strongly suggest that the spatial features and orientation of the mental number line are flexible and can be shifted within a given cognitive situation.

Furthermore, the SNARC effect was modulated by the difficulty of the task and, in particular, by the different ***response speed***. Average reaction time (RT) had an impact on the size of the SNARC effect: in general, longer RTs were needed for this effect to emerge and its size increased as a function of the response speed (cf. SNARC and Simon effects in Mapelli, Rusconi, & Umiltà, 2003).

Finally, Proctor and Cho (2006) have recently proposed an alternative explanation for the SNARC effect that does not explicitly refer to an internal representational continuum. According to their polarity correspondence principle, bipolar dimensions undergo a verbally mediated spatial coding, and polarity correspondence (unmarked vs. marked) sufficient to produce mapping effects. In particular, it is assumed that number magnitude (small [¡] and large [+]) and responses (left [¡] and right [+]) are coded on a bipolar dimension and corresponding polarities induce faster response selection.

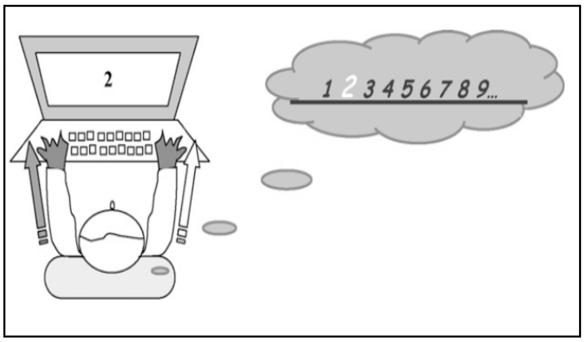

Fig. 1. The picture shows a typical experimental setting in a numerical task. In the example, the small number ("2") elicited a faster answer when participant answered with the left hand compared to when he answered with the right hand (from de Hevia, Vallar, & Girelli, 2008).

1.2.3. The analogy between number and other ordinal sequence

It is worth considering whether spatial associations are exclusively numerical or whether they can occur with non-numerical stimuli that are sequentially ordered (e.g., letters of the alphabet, days of the week, months of the year). An initial study (Dehaene et al., 1993; Experiment 4) found no reliable associations between letters and space when participants classified letters as vowels or consonants. However, many behavioural and neuroimaging studies have later on demonstrated analogies in the processing of numerical and non-numerical ordered information. For instance, a SNARC effect has been reported for letters of the alphabet, days of week, months of the year (Gevers, Reynvoet, & Fias, 2003, 2004) but also for arbitrary ordered newly acquired information (Previtali, de Hevia, & Girelli, 2010). Even more critically, some typical properties of numerical processing, such as the distance effect and the size effect hold for non-numerical magnitude dimensions. A reliable distance effect was found when participants compared font size (Cohen Kadosh et al., 2005; Pinel, Piazza, Le Bihan, & Dehaene, 2004), time duration (Dormal, Seron, & Pesenti, 2006), and pitch height (Rusconi, Kwan, Giordano, Umiltà, & Butterworth, 2006). The size effect was also observed in the comparison of line length and angle amplitude (Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003).

Moreover, in line with the proposal of a shared magnitude code underlying different quantity dimensions (Walsh, 2003), numerical and non-numerical quantities have also been demonstrated to interact with each other. In the numerical Stroop paradigm, participants take more time to judge either the numerical size or the physical size of an Arabic digit when these two dimensions are incongruent (Kadosh, Henik, & Rubinsten, 2007; Schwarz & Ischebeck, 2003). Such mutual interference suggests that a common magnitude code (i.e., the general mental representation of quantity) is involved in the processing of both numerical magnitude and physical size; hence the incompatible magnitude of one task-irrelevant dimension would interfere with the judgment on the other magnitude dimension. In addition to the psychophysical evidence mentioned above, recent neuroimaging studies show that the processing of different quantity dimensions involves very similar, if not identical, brain regions. It is well-documented that the intraparietal sulcus (IPS) plays a crucial role in processing numerical magnitude (for a review see Brannon, 2006). The activation of this area is also associated with the processing of non-numerical magnitudes. For instance, comparing font size and luminance activated largely overlapping areas in the inferior parietal cortex, particularly along the IPS (Cohen Kadosh et al., 2005; Pinel et al., 2004). Similarly, Fias et al. (2003) found common activation of the left posterior IPS associated with making comparisons of lines, angles, and numbers.

Given that both numbers and ordered sequences can elicit similar behavioural effects and shared to some extent neural activation, one could argue that it is the ordinal property and not the quantitative property of numbers that is spatially coded.

Overall, these results suggested that any ordered sequences might be preferentially mapped onto a spatial mental representation (e.g. Previtali et al., 2010; Van Opstal, Fias, Peigneux, & Verguts, 2009) although behavioural (Turconi, Campbell, & Seron, 2006; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006) and processing difference (Badets, Andres, Di Luca, & Pesenti, 2007; Turconi, Jemel, Rossion, & Seron, 2004) applied to numerical and non numerical information. In fact, spatial mapping may occur for different ordered sequences (not only numbers) although the intrinsic relevance and, consequently, the access to it are likely to differ for numerical, alphabetical, and other ordered sequences.

1.2.4. Neuropsychological evidence for the number-space association

Neuropsychological studies further confirm the close link between visuo-spatial processing and basic number processing. A particular example is the Gerstmann syndrome, typically associated with damage to the angular gyrus of the left hemisphere. This syndrome is characterized by the co-occurrence of finger agnosia, left-right disorientation, agraphia and acalculia (Gerstmann, 1940) and it has recently attracted renewed attention as an instance of the number-space-finger functional association.

More recently, new insights on the number-space link, came from the investigation of brain-damaged patients with neglect. This deficit consists of an inability to explore the contro-lesional side of the visual space, resulting in a deficit in reporting stimuli presented in that portion of space (Bisiach & Vallar, 2000). For instance, typically, when neglect patients have to bisect a linear segment positioned in front of them, they systematically displaced the midpoint towards the right, as if they ignored the leftmost part of the segment.

In order to explore the spatial features of the mental number line, Zorzi et al. (2002) required neglect patients to mentally bisect a numerical interval by estimating the midpoint between two extreme values (e.g., “which is the middle number between 3 and 7?”). Indeed, their results showed that this task yielded a systematic representation-based midpoint shift towards the right. For instance, their patients named 6, instead of 5, as the middle number between 3 and 7. The authors interpreted these results as if the patients neglected the left side of their mental number line shifting towards the right the midpoint of the numerical interval. Thus, by analogy to what has been observed for physical lines, neglect patients appeared to shift the objective midpoint of a representational spatial continuum towards the ipsilesional space (cf. Doricchi, Guariglia, Gasparini, & Tomaiuolo, 2005).

To further explore the impact of representational neglect on number processing, Vuilleumier et al. (Vuilleumier, Ortigue, & Brugger, 2004) required neglect patients to compare numbers to a fixed reference. The patients were selectively slow in classifying numbers just smaller than the reference, indicating difficulties in orienting attention towards the left side of their mental number line. Interestingly, when asked to imagine whether the presented target number was earlier or later than 6 o’clock, the patients showed the reverse effect: a selective slow down of answers to numbers larger than 6, thereby further confirming the dynamic and representational nature of the association between numbers and space. These results support the hypothesis that the number comparison task invokes an internal spatial magnitude representation, oriented from left-to-right. In patients with left spatial neglect, the left side of such a representation may be not available for numerical processing. Alternatively, the neglect produces a deficit in accessing an intact mental number line, rather than a distortion in the representation of that line (Vuilleumier et al., 2004).

2. On the number-space interaction

2.1. Behavioural evidence for cross-magnitude interactions

The number-space association may be also looked at as an instance of the cross-magnitudes interaction due to the representational and/or processing similarities between numerical and non-numerical magnitude (Fias et al., 2003; Pinel et al., 2004). Indeed, many behavioural effects hold for different magnitudes, as for example the semantic congruity effect, according to which for identical stimuli, the speed of comparison depends on the direction of the comparisons and the size of the stimuli. For example, people are faster to report that a mouse is smaller than a squirrel than to answer that a squirrel is larger than a mouse (see figure 2). The 'semantic congruity effect' emerges whenever people compare two things along a single dimension (e.g. judging animal size, brightness, line length). In particular, this effect has been observed in number-size comparison (Girelli, Lucangeli, & Butterworth, 2000; Henik & Tzelgov, 1982; Kaufmann et al., 2005; Rubinsten, Henik, Berger, & Shahar-Shalev, 2002), in numerosity-length comparison (Dormal & Pesenti, 2007), as well as in numerosity-size comparisons (Hurewitz, Gelman, & Schnitzer, 2006).

A further signature of cross-magnitude interaction is the size congruency effect that results from the competition between numerical and physical dimensions in comparative judgments. In particular, in the numerical comparison task, participants are faster to select the larger digit when the irrelevant physical dimension is congruent with the numerical size such as in 3 7, compared to when the two dimensions are incongruent such as in 3 7 (Girelli et al., 2000).

These examples of cross-magnitude interaction suggest that quantitative judgments of continuous dimensions imply a common mental code for quantitative representation and/or a common mental comparison process for judging their magnitude. The notion of a generalized magnitude system for representing number and non-numerical quantities has been recently proposed by Walsh (2003). By summarising the behavioural and neurobiological evidence that time, space and numbers share common processing mechanisms, the author proposed that these dimensions may be associated to guide goal-directed action. In fact, to this end, the spatial location of an object is calculated crucially thanks to quantitative computations such as 'how far', 'how many' and 'how long'. Accordingly, mental magnitudes would be linked to one another through the computational demands of the motor control system with the parietal lobes providing neural support for their integration (Walsh, 2003).

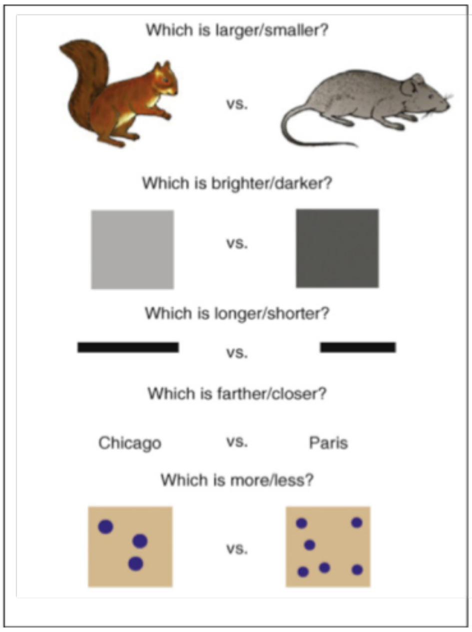

Fig. 2. The picture shows examples of quantitative comparison that elicit a semantic congruity effect (from Cantlon et al., 2009).

2.2. The cognitive illusion hypothesis

In the last few years, the numerical influence on visual-spatial task has been explored by means of both bisection and reproduction paradigms.

In the first number bisection study, Fischer (2001) required participants to perform two bisection tasks. The stimuli were long strings of identical digits (Fischer, 2001, Experiment 1) or lines with single identical digits as flankers (Fischer, 2001, Experiment 2). The author observed two different results. First, the bisection performance was biased to the left or to the right accordingly to the size of the digit, with a left bias for strings made of digits 1 or 2 (e.g. 11111111111) and with a right bias for strings made of digits 8 or 9 (e.g.

9999999999). This finding was interpreted, in analogy with the SNARC effects, as depending on an automatic activation of a spatial response code associated to number magnitude. Second, line bisection was biased toward the larger flanker number, regardless of its left or right location. Fischer (2001) suggested that the bisection performance indicated activation of a numerical representation with spatial properties. In particular, the bias towards the larger digit was interpreted according to the MNL hypothesis, as if perception of two numbers automatically evoked a corresponding segment within this representation.

de Hevia and colleagues (2006) further explored the influence of numerical information on the bisection performance and, although they did not replicate the effect induced by absolute magnitude in digit strings[3], they replicated and extended the larger-digit effect in the bisection of lines and of unfilled spaces flanked by a different number. In a series of experiments they reported a systematic bias towards the numerically larger digit in the bisection performance, despite numbers being irrelevant to the performance.

This systematic misplacement of the subjective midpoint towards an irrelevant larger number strongly suggests that numbers impact on the visuo-spatial processing by inducing a so-called 'cognitive illusion' of length (de Hevia et al., 2006). According to this assumption, processing of large-magnitude numbers brings about an illusory expansion of space and processing of small-magnitude numbers brings about an illusory compression of space (de Hevia, Girelli et al., 2008; de Hevia et al., 2006).

In order to further support the 'cognitive illusion' hypothesis, a length reproduction task was adopted (de Hevia, Girelli et al., 2008). Participants were required to reproduce the length of a spatial extension delimited by two digits by extending or compressing a horizontal segment. The flanked digits were identical (in Experiment 1) and different (in Experiment 2) in order to evaluate the influence of both absolute and relative magnitude information (i.e., the numerical distance), respectively. They reported an underestimation of space length delimited by small magnitude numbers, and an overestimation of space length when delimited by large magnitude numbers. These results suggested that processing of the numerical magnitude conveyed by an

[3] Please note that, the two studies differ as to the methods of presentation (individual stimuli vs. multiple stimuli per page in Fischer's study) and the numbers of digits including in the strings (odd/even vs. only odd in Fischer's study). Furthermore, the same result was reported by Calabria and Rossetti (2005, Experiment 2)

Arabic digit modulate the mental representation of a spatial extension. In particular, numbers can induce a mis-estimation of length. This effect confirmed the cognitive illusion hypothesis, according to which expansion and compression of a spatial extension takes place as a function of the represented numerical magnitude.

Overall, these findings are consistent with the view that numbers are spatially coded, as suggested by Dehaene's models (Dehaene, 1992; Dehaene et al., 2003) but they also suggest that the interaction between numbers and space is more complex and go behind the mental number line hypothesis. Indeed, the authors proposed two different aspects of the numbers-space association: *the magnitude space* and *the numerical space*. The magnitude space is conceptualised as an analogical representation of magnitude and this abstract numerical code may be internally represented in the same way as other continuous dimension (Moyer & Landauer, 1967). Whereas, the ordered numerical space corresponds to a visuo-spatial representation where digits are strategically organised along an oriented mental number line, similarly to what may occur for representing any other type of over learned ordered information (Gevers et al., 2003, 2004; Previtali et al., 2010).

3. Neural evidence in magnitude processing

The prediction of parietal interactions between number processing and visuo-motor behaviour is in line with the most influencial neuro-cognitive model of numerical cognition (Dehaene & Cohen, 1995). According to this model, the semantic numerical representation is supported bilaterally by the **inferior parietal cortex** of the brain. Evidence for such bilateral parietal localization of number meaning has been provided by several studies.

Recently, the contribution of the parietal cortex, specifically of the **angular gyrus** and the inferior parietal lobule (**IPL**) (Göbel, Walsh, & Rushworth, 2001; Göbel, Calabria, Farnè, & Rossetti, 2006), in number representation has been suggested.

For example, in one of the early studies investigating transcranical magnetic stimulation (TMS) effect on number processing, a temporary interference with the spatial representations of mental number line when stimulating the **angular gyrus** was shown (Göbel et al., 2001). Göbel and colleagues (2001) showed that stimulation of the left and right parietal cortices leads to defective performance in number comparison tasks. Furthermore, a null effect was found with the stimulation of the supramarginal gyrus supporting the hypothesis that the angular gyrus may be the cerebral substrate mediating the MNL representation.

Several studies highlighted the role of the intraparietal sulcus (**IPS**)[4] in the spatial numerical representation exploiting different behavioural effect as an index of common activation of the number and space representations (i.e., SNARC and distance effects).

In the study by Fias, Lauwereyns and Lammertyn (2001) the SNARC effect was used to identify the neuro-anatomical substrate of the number/space interaction. Participants were required to respond to the orientation, colour, or shape of visually presented stimuli while ignoring simultaneously visible digits. Orientation is known to activate parietal cortical areas (i.e., the dorsal visual stream), whereas colour and shape are preferentially processed in the ventral cortical areas (i.e., the ventral visual stream). Nevertheless, the presence of task-irrelevant digits only during orientation decisions was sufficient to induce a SNARC effect suggesting a common parietal activation for number magnitude and orientation.

In other studies the distance effect was used as a marker for localizing the cerebral correlates of magnitude representation (Pinel, Dehaene, Rivière, & LeBihan, 2001; Pinel et al., 2004). For instance, Pinel and colleagues (2004) observed a common activation in the parietal cortex in judgements of approximate brightness, size or numerical value of two Arabic numerals that were visually presented simultaneously. All three tasks (brightness, size and number) activated a broad swath of the cortex along the IPS relative to baseline, and each of these three tasks evoked distance-related activations in this area. More importantly, activation associated with each task varied along adjacent segments of the IPS and only partially overlapped. Anterior portions of the horizontal segment of the IPS responded more strongly during numerical comparisons than during either of the other two types of comparisons. The authors argued that this pattern of activity reflected a distributed but overlapping organization of quantitative processing in the IPS. Furthermore, the

[4] The intraparietal sulcus is a deep fissure in the parietal lobe, dividing the superior and inferior parietal lobules.

horizontal IPS seems sensitive to the numerical distance effect, whit the numerical comparisons of closer digits leading to a higher degree of parietal activation than that of digits farther (Pinel et al., 2001).

Also Walsh (2003) argued that the role of IPS reflects the common need for space, time, and other quantity information for sensorimotor transformations, suggesting that the IPL is the neural substrate for a generalized magnitude system for action. Indeed, recent brain imaging studies have shown that judgements on numerical magnitude and physical size are both associated with increased activity in the intraparietal sulcus (**IPS**) and **premotor cortex** (Cohen Kadosh et al., 2005; Fias et al., 2003; Pinel et al., 2004). Dehaene and colleagues (Dehaene, Dehaene-Lambertz, & Cohen, 1998) reported that the left IPS is specifically active when two stimuli have to be quantitatively compared suggesting a common cerebral representation of quantitative processing of non-symbolic stimuli.

Overall, these data point to the relevant role of the bilateral parietal areas in distributed and overlapping neural coding of magnitude processing (for review see Cantlon et al., 2009; Hubbard, Piazza, Pinel, & Dehaene, 2005).

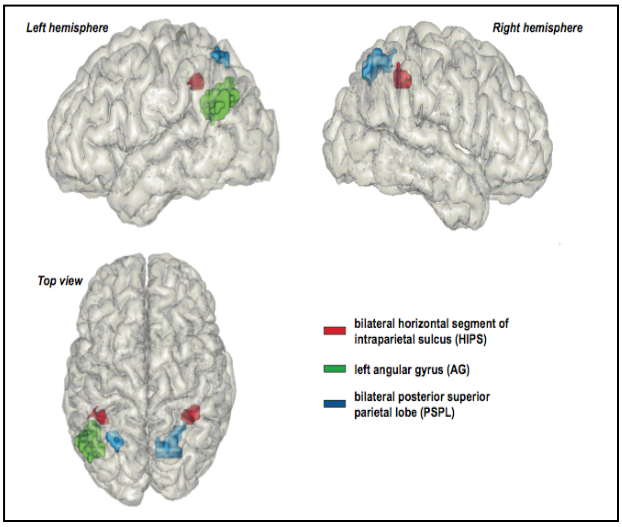

Fig. 3. The picture shows three-dimensional representation of the parietal regions of interest. The left angular gyrus (AG) is also often activated in neuroimaging studies of number processing. This region is left-lateralised and located posterior and inferior to the horizontal intraparietal sulcus (HIPS) (from Dehaene et al., 2003).

Chapter 3. NUMBERS IMPACT ON ACTION

1. A review of studies regarding numerical impact on action

The results obtained with the classical paradigm such as the bimanual classification tasks (SNARC, for review see Wood et al., 2008) and the bisection task with numerical flankers (cf. de Hevia et al., 2006; Fischer, 2001) may be also interpreted as an instance of the effect that numbers exert on action. These studies investigated the behavioural effect of the spatial mapping of numbers and the interaction between numerical magnitude and space. Although the impact on action was not their main goal, these works clearly show that numbers influence to some extent motor tasks. Therefore, they could be considered among the evidence for a relevant interaction between spatial numerical representation and motor output (see Chapter 2).

Recently, the interaction between numbers and action-related processes has developed into a new and most investigated topic in numerical cognition. Indeed, evidence for the influence of numerical information on specific motor components has been found by means of various and original tasks.

This chapter will review these studies classifying them according to the task and to the movement required (see table 1) and, in doing so, will pinpoint the still debated questions relative to the number-action association.

Automatic activation of the numerical magnitude code has been proposed to account for the numerical influence on motor tasks such as i) lateralized movements or unimanual responses, and ii) manual gestures such as grasping and different grip aperture. In the following, this categorisation will be further elaborated.

1.1. Numbers effect on lateralized movements or unimanual responses

Several studies aimed to investigate a sort of SNARC effect in motor-related tasks such as i) lateralized or unimanual pointing (Fischer, 2003; Ishihara et al., 2006; Song & Nakayama, 2008), ii) tasks requiring motor responses with effectors different from hands (i.e., pedal, Schwarz & Müller, 2006) and eyes responses (Fischer et al., 2004; Schwarz & Keus, 2004), and iii) classical SNARC paradigm with different observed variables (Fischer & Miller, 2008; Vierck & Kiesel, 2010).

As regards the first category, a SNARC-like effect was found by means of a unimanual odd-even pointing task with lateralized target areas (Fischer, 2003). Participants moved their arm from a central starting location to a left or to a right target area, depending on the parity of a centrally presented digit. Numbers yielded faster movements towards the left if they were small, and faster movements towards the right if they were large (Fischer, 2003).

Further support for an association between number magnitudes and the external space comes from the study by Fischer et al. (2003). In their experiment, subjects were required to press a response key with their preferred hand as quickly as possible when a visual target appeared on either the left or the right side of the computer screen, followed by the presentation of a digit in central fixation. They showed that the digit presentation shifted covert attention to the left or right side of visual working space with shorter RTs for the left target (compared to the right target) when a small digit had been shown and vice versa when a large digit had been shown.

Besides, the impact of numerical information on motor planning has been strengthened by the finding of a congruity effect between spatial location of a target number and its magnitude in a Go/No-Go parity judgement task with pointing as a motor response (Ishihara et al., 2006). Numbers were presented along a left-right axis and participants were asked to point to the target by moving their finger from the starting position only when the target was an odd number. In this study, left-sided movements were initiated faster towards small digits compared to larger digits and right-sided movements were initiated faster towards large digits compared to smaller digits.

Furthermore, Song and Nakayama (2008), using a lateralised manual pointing task, reported a systematic deviation of the hand trajectory as a function of number magnitude. Subjects were required to classify centrally presented numbers by their magnitude (smaller or larger than ‘five’) by reaching a square located to the left or to the right, respectively. They observed that reaching trajectories were systematically shifted in position according to numerical differences between the target and the number ‘five’.

In the second category of motor task, similar results were found with effectors not associated with writing suggesting that the SNARC effect does not simply reflect an over learned motor association between numbers and hands. Indeed, in a parity classification task responses to small digits initiated faster to the left and responses to large digits initiated faster to the right also when executed with saccadic eye movements (Fischer et al., 2004; cf. Schwarz & Keus, 2004) and pedal response (Schwarz & Müller, 2006).

Moreover, in the study of Schwarz & Keus (2004), similar results were described with saccadic movement in both the horizontal and the vertical axes. In this later condition, the authors observed that eye movements to the lower response location started earlier with small than with large numbers, whereas eye movements to the upper response location started earlier with large than with small numbers.

Concerning the third category, Fischer and Miller (2008) investigated whether the influence of numerical magnitude on action extends to irrelevant parameters of the motor response, such as the relative force of a key-press response in a parity or magnitude judgement task. In both conditions, the numerical size had little or no effect on the response force parameter, suggesting that the numerical impact was limited to motor planning.

In line with this, number magnitude was reported to not affect the relevant response force (Vierck & Kiesel, 2010) in a parity judgment task with lateralised responses. Indeed in this study, the authors required participants to respond to the parity of a single digit by executing a weak or forceful key press. Response selection was faster when small digits required a weak response and large digits required a forceful response than when this mapping was reversed. This temporal advantage suggested a compatibility effect between number magnitude and intensity. However, the numerical effect was limited on the initiation of the motor response and was not extended to execution level. Indeed, there is no effect on response execution because the response force intensity was not modulated by number magnitude.

Overall, in all the studies mentioned above, SNARC-like spatial numerical associations were observed: leftward movements and/or left-lateralized responses were elicited consistently faster by relatively small numbers whereas rightward movements and/or right-lateralized responses were elicited faster by relatively large numbers. These findings support the strong link between different magnitude domains such as numerical magnitude and motor planning. However, only few studies explored the numerical impact on motor execution and only a single one reported a positive result with significant effect (Song & Nakayama, 2008). These discordant findings raise the question regarding which levels of action are modulated by numbers and only additional studies will further clarify this point.

1.2. Numerical magnitude-action congruency effects

Recently, a series of studies shed light on the interaction between numbers and action-related processes, reporting a systematic effect of magnitude information on grasping movements (Andres, Davare, Pesenti, Olivier, & Seron, 2004; Andres, Ostry, Nicol, & Paus, 2008; Lindemann, Abolafia, Girardi, & Bekkering, 2007; Moretto & di Pellegrino, 2008), on grasping estimation (Badets et al., 2007), and on prehension action judgement (Chiou, Chang, Tzeng, & Wu, 2009).

In a parity classification task, the starting time of the movement was modulated by the numerical magnitude: grip closure was initiated faster in response to a small digit while grip opening was initiated faster in response to a large digit (Andres, Davare, Pesenti, Olivier, & Seron, 2004).

Further support for a functional connection between numerical processing and goal-directed hand action is provided by studies associating precision (i.e., using the finger and thumb) and power grips (i.e., using the whole hand) to magnitude information: during the former, action was initiated faster in response to small digits, whereas during the latter action was initiated faster in response to large digits, whether (Lindemann, Abolafia, Girardi, & Bekkering, 2007) or not (Moretto & di Pellegrino, 2008) reaching movements were required.

In addition, in the Lindemann and colleagues (2007) analysis of the grasping kinematics, it was revealed that, in the reaching period, grip aperture was larger in the context of responding to large numbers than to small numbers, irrespective of the selected grip.

The compatibility effect was reported in grasping movements involving the two fingers (i.e., opening and closure grips) as well as in grips involving whole hand or two fingers (i.e., precision and power grips). It is worth noting that the movements are not the same because they can or cannot involve the whole hand (power grip). This compatibility effect was interpreted as the link between number and action.

Alternatively, the outcome of studies that contrasted precision and power grip movements could also be interpreted as compatibility between numerical magnitude and numerosity (i.e., number of fingers involved in the movement) but further investigation is needed to disambiguate this interpretation.

Another relevant and debated question is which stage of action is modulated by the activation of a numerical representation. In order to evaluate this issue, in the study of Andres and colleagues (2008) participants were required to reach and grasp a block using a precision grip and place it either forward or backward as a function of the parity of the digit printed on the visible face of the object to grasp. The kinematic of grip aperture (i.e., the measured distance between the finger and thumb) was recorded throughout the duration of the movement. In the first stage of reaching, grip aperture was found to be larger as a function of the presentation of a large rather than a small digit. The numerical influence on grip was limited only to the initial phase of the movement, strengthening the hypothesis that numbers magnitude interacts with object spatial information during motor planning, i.e., initiation and selection of action (see next paragraph for the discussion regarding opening questions).

In the studies so far described, participants were required to make a movement. In contrast, the following research tested a possible dissociation in numerical effect between perception vs. estimation of grasping and between the latter vs. grasping execution exploring the number/action interaction in different tasks.

First, a similar interaction was also found when grip action was not executed but only judged as adequate for a specific affordance, i.e., judging if a visually presented rod could be grasped between the thumb and the index finger (Badets et al., 2007). Preceding irrelevant numbers were shown to influence the perceived length of the rod, with small number inducing an overestimation of one’s own grip, and large numbers inducing an underestimation of the grip. Indeed, the authors showed that the numerical information calibrates the judgement of action even when no actual action is required.

Finally, in the study of Chiou and colleagues (2009), photographs of a graspable object with a superimposed Arabic digit were presented in a dual-task paradigm: a parity judgement task was performed closely in time with either an action judgment (e.g., pinch vs. clutch) or a perceptual judgment for the depicted object (e.g., colour or size). When parity and action judgement were performed close in time, the compatibility effect between the numerical magnitude and the appropriate action for the object was demonstrated in both manual and vocal responses. In contrast, such a compatibility effect was absent when participants performed parity and perceptual judgement.

Overall, the reviewed literature supports that the automatic processing of numerical information impacts on manual gestures and grasping movements showing a temporal advantage in case of compatibility between numerical magnitude and grip posture/shape.

This empirical evidence suggests an association between numbers and space that determine action. According to the ATOM theory (Walsh, 2003), the association between number, space and action might be due to the activation of a common representation of magnitude, given that an action has temporal as well as spatial components (Wood & Fischer, 2008). Furthermore, this common representation of magnitudes arises from partial overlapping of neuroanatomical structures subserving numbers, space and time processing whose integration is required for goal-related action.

2. Open questions

The literature so far reviewed brings to light many open questions. The most relevant for this project is which specific stage of action is modulated by the numerical magnitude information.

There is now accumulating evidence supporting the notion of an influence of number magnitude on motor planning. However a possible influence of number magnitude on motor execution is still under debate. In fact, in the saccadic and pointing responses referred to above, movement amplitude was usually not modulated by number magnitude (Fischer, 2003; Fischer et al., 2004)[5]. Comparably, in the study by Andres and colleagues (2008), relevant digits influenced grip aperture only in the initial phase of the grasping movement suggesting that the number magnitude influenced selection and/or action initiation, rather than action execution itself. Furthermore, in line with this, number magnitude had little or no effect both on the irrelevant response force of a key-press (Fischer & Miller, 2008) as well as on the relevant response force (Vierck & Kiesel, 2010). Once again, in the parity classification with two different grasping (Andres et al., 2004) numbers impact only on the motor planning, i.e., with a temporal advantage for starting the opening or closure grip movement compatible with the magnitude, although the movement amplitude was not modulated by the numbers.

In contrast, to our knowledge, only two studies reported that the numerical information clearly impacts on motor execution (Lindemann et al., 2007; Song & Nakayama, 2008).

Therefore, further investigation is needed to clarify at which processing stage numerical information is integrated with other magnitude information supporting the role of the motor system in the organization of semantic knowledge. According to theories, known as “embodied” or “sensimotor” theories of cognition, the motor system is not simple and exclusively dedicated to the control of action. Thus, the motor system may contribute critically to conceptual processes such as those involved in number representation (Andres et al., 2008).

A further debated question concerned the possible dissociation in the numerical magnitude/type of movement congruency effect emerged in different tasks with or without explicit involvement of grasping. In particular, it might be that the numerical interference on action will be limited to conditions when participants execute or estimate a movement but it will be absent in case of perceptive judgement.

Badets and colleagues (2007) have shown that when a perceptual match was required without the grasping movement, the numerical information had no effect on perceptual judgment but only in grasping estimation. This finding showed that the magnitude/affordance interaction was not due to a simple perceptual

[5] In the Fischer’s study, the numerical magnitude was not limited to motor planning but the effect found on execution was not observed in all of two motor variables recorded (cf. Fischer, 2003).

effect. The authors proposed an anatomo-functional interpretation of this interaction in dissociation between two different visual systems. It has been argued that goal-directed actions imply the dorsal and ventral streams (for a review, see Goodale & Milner, 2004; Milner & Goodale, 1995). The ventral stream that terminated in the infero-temporal cortex was in charge of object recognition such as the size and orientation of objects. Instead, the dorsal stream, terminated in the posterior parietal cortex, was responsible for object-directed action. In Badets and colleagues' study (2007) a different engagement of visual streams was reported as a function of task demands. When participants were required to make an action judgement this would mainly engage the dorsal stream, whereas when participants were required to give a perceptual judgement, without the grasping movement, the ventral stream areas was engaged. Their findings showed the following dissociations: number magnitude influenced the action judgement (i.e., the dorsal stream) but not the perception judgement (i.e., the ventral stream). Interestingly, this study showed the opposite dissociation found in behavioural experiments of visual illusion paradigms. A similar dissociation between magnitude influence on perception and action has found by Chiou and colleagues (2009).

A further question regards whether the magnitude effects reported on action arise from a spatial representation of number in a dichotomic or continuous way.

The results described above imply that the spatial representation of number (i.e., the Mental Number Line, Dehaene, 1992) is tightly linked to motor response, according to Walsh's theory (Walsh, 2003) supporting the IPL role for a generalized magnitude system for action. In fact, number magnitude effects can be observed in several motor-related tasks such as bisection (Calabria & Rossetti, 2005; Fischer, 2001), manual pointing (Fischer, 2003), oculo-motor tasks (Fischer et al., 2004), and grasping (e.g., Andres et al., 2004). Various evidence on the SNARC effect showed that smaller and larger numbers are mapped to the left and to right sides of space, respectively, but very few specify whether this is a dichotomic categorization (polarity correspondence principle in Proctor & Cho, 2006) or a one-to-one mapping between digits and discrete points in space. The Ishihara et al.'s study (2006) investigated whether the link between spatial and number information processing may arise from a *continuous mapping* between space and number representations. In this Go/No-Go parity judgment task, participants were required to point to the numerical target presenting along a left-right axis. Results show that left-sided movements were initiated faster towards small digits compared to larger digits and right-sided movements were initiated faster towards large digits compared to smaller digits. Critically, this mapping seemed to occur in a continuous, rather than dichotomic way (Ishihara et al., 2006; see also Song & Nakayama, 2008). The meaning of numbers is indeed spatially coded, and the mental number line seems a useful metaphor to capture this surprising fact. However, this metaphor should not be taken literally, rather, spatial associations are attached to numbers as part of our strategic use of knowledge and skills, and as a result these associations are highly task-dependent. Further evidence of this flexibility of spatial associations challenges the appropriateness of the number line metaphor. For example, the existence of vertical as well as horizontal spatial associations (Ito & Hatta, 2004; Schwarz & Keus, 2004) suggests that the magnitude representation is richer than a strictly horizontal unidimensional code, such as a number line, and is better characterized as a number map. In particular, the numerical magnitude has a two-dimensional internal representation, much like an internal number map. Alternatively, these findings are also consistent with the assumption of two functionally independent number representations or even with a single representation that can be adaptively reoriented according to the specific task demands.

Finally, a further open issue is whether the numerical effect on motor task may be induced but ordinal rather than magnitude information, thus extending to other non-numerical ordinal sequences. The numerical effects so far reviewed are mainly determined by the spatial mapping of numbers into a representational continuum. However, numbers have been proved not to be the only dimension that is spatially organised. Indeed, it has been shown that various magnitudes related effects (such as distance, and SNARC effects) observed with numbers are also observed with non-numerical ordered sequences (e.g. months of the year and letters of the alphabet; Gevers et al., 2003). Order can thus produce the same effect as magnitude, with items occurring at the beginning or end of a given sequence behaving like small or large numbers respectively. These results suggest that a spatial mental representation is not unique to magnitude information (Gevers et al., 2003, 2004; Previtali et al., 2010).

Thus, it is expected that the numerical impact on action may occur for different ordered sequences such as the letter of the alphabet. In Badets et al.'s (2007) and in Fischer's (2003) studies, alphabetical stimuli does not have the same effect than numerical stimuli. The relevance of ordered information is likely to differ for numerical and alphabetical sequences. Numbers are indeed the most overlearned and overused ordered sequence, and the only sequence where order holds cardinal meaning.

The spatial representation of numbers and its impact on action has recently received increasing attention but the increasing evidence favouring this interaction leaves open a series of questions that will be addressed in the series of studies reported in this thesis.

In the following chapter I will introduce the main rationale of my experimental studies.

Table 1. Studies of numerical impact on action

Authors	Type of movement	Task	General effect found or hypothesized	Planning	Execution
Fischer, 2003	Pointing	Parity classification	SNARC effect with temporal advantage V SNARC effect with shifted amplitude of movement X	V	V X
Ishihara et al., 2006	Pointing	Go/no-Go parity classification	SNARC effect with temporal advantage V SNARC effect with amplitude of movement X Numerical magnitude/pressure congruency effect X	V	X
Song & Nakayama, 2008	Pointing	Number comparison	SNARC effect with shifted trajectories V	V	V
Fischer et al., 2004	Saccadic response	Parity classification	SNARC effect with temporal advantage V SNARC effect with amplitude of movement X	V	X
Schwarz and Keus, 2004	Saccadic response	Parity classification	SNARC effect (horizontal and vertical) with temporal advantage V	V	not indagated
Schwarz & Müller, 2006	Pedal response	Parity classification	SNARC effect with temporal advantage V	V	not indagated
Rico Fischer & Miller, 2008	Key-press response	Parity classification and number comparison	SNARC effect with temporal advantage V Numerical magnitude/force congruency effect X	V	X
Vierchk & Kiesel, 2010	Key-press response (weak vs. forcefull)	Parity classification	Numerical magnitude/response intensity congruency effect with temporal advantage V Numerical magnitude/force congruency effect X	V	X
Andres et al., 2004	Grasping (closure vs. opening grip)	Parity classification	Numerical magnitude/grasping congruency effect with temporal advantage V	V	X
Lindemann et al., 2007	Grasping (precision vs. power grip) with reaching	Parity classification	Numerical magnitude/graspingcongruency effect with temporal advantage V Numerical magnitude/amplitude grip congruency effect V	V	V
Moretto & di Pellegrino, 2008	Grasping (precision vs. power grip) without reaching	Parity classification and perceptive judgment	Numerical magnitude/graspingcongruency effect with temporal advantage V	V	X

(continued)

Authors	Type of movement	Task	General effect found or hypothesized	Planning	Execution
Andres et al., 2008	Reaching and grasping (precision grip)	Parity classification	Numerical magnitude / grip aperture congruency effect **V**	**V**	**X**
Badets et al., 2007	Not movement but grasping estimation	Judgement of affordance	Numerical magnitude / grip aperture congruency effect **V**	**V**	**X**
Chiou et al., 2009	Key-press response and vocal response	Dual-task: parity classification and action (pinch vs. clutch)/perceptive judgement (color or object size)	Numerical magnitude / appropriate action congruency effect **V**	**V**	**not indagated**

Chapter 4. EXPERIMENTAL PART

The present study brings the work on parietal involvement in visuo-motor control and in number processing together and provides behavioural evidence for their interaction. Specifically, it shows that visuo-motor processing can reflect the congruency between a representation symbolic and an action space, just as number processing can be sensitive to spatial response requirements.

Overall, the specific questions addressed may be summarise as follow:

(i) Does the spatial mapping of numbers in the representational space interfere with the motor planning involved in an overlearned complex motor plan, such as spontaneous handwriting? If this is so, to what extent a) it is modulated by the task requirements? and b) it reflects the processing of magnitude or ordinal information?

(ii) Does the the processing of numbers influence motor planning and to what extent this effect extends to motor execution? How far the spatial features of the numerical representation modulate the execution of simple manual movements?

(iii) Finally, to what extent processing of magnitude information interfere with spatial processing? Does the implicit processing of numbers induce misperception of space information, whether in the horizontal and the bidimensional dimensions?

Chapter 5. NUMBER CAN MOVE OUR HAND: A SPATIAL REPRESENTATION EFFECT IN DIGIT HANDWRITING[6]

1. Introduction

The interaction between numbers and action-related processes is currently one of the most investigated topics in numerical cognition. Indeed, number magnitude influences motor outcomes during tasks requiring either oculomotor or unimanual responses: a SNARC effect has been reported with eye movements during a parity classification task, with responses to small digits initiated faster to the left and those to large digits to the right (Fischer et al., 2004), as well as with an odd-even pointing task, where centrally presented digits yielded faster movements towards the left or towards the right as a function of their magnitude (Fischer, 2003). However, in both studies movement amplitude was not modulated by number magnitude, suggesting that numerical information seems to influence motor planning rather than motor execution (Fischer, 2003; Fischer et al., 2004). Furthermore, the impact of numerical information on motor planning has been strengthened by the finding of a congruity effect between spatial location of a target number and its magnitude in a Go / No Go parity judgment task with pointing as motor response (Ishihara et al., 2006). In this study, left-sided movements were initiated faster towards small digits compared to larger digits and right-sided movements were initiated faster towards large digits compared to smaller digits. Critically, this mapping seemed to occur in a continuous, rather than dichotomic, way (Ishihara et al., 2006).

Recently, a series of studies shed light on the interaction between numbers and action-related processes, reporting a systematic effect of magnitude information on grasping movements (Andres et al., 2004; Lindemann et al., 2007; Moretto & di Pellegrino, 2008), on grasping estimation (Badets et al., 2007), and on prehension action judgment (Chiou et al., 2009).

[6] The present study is published in Experimental Brain Research, 2010, 205(4), pp.479-487 (the article is available in the present link http://www.springerlink.com/content/t3x761x8jk66m4xh/).

Critically, Fischer and Miller (2008) investigated whether the influence of numerical magnitude on action extends to irrelevant parameters of the motor response, such as the relative force of a key-press response in a parity or magnitude judgment task. In both conditions, the numerical size had little or no effect on the response force parameter, suggesting that the influence of number on the initiation and selection of action (Andres et al., 2008) does not extent to action execution (Fischer & Miller, 2008).

Finally, Song and Nakayama (2008), using a lateralised manual pointing task, reported a systematic deviation of hand trajectory related to number magnitude. Subjects were required to classify central numbers by their magnitude (smaller or larger than 'five') by reaching a square located to the left or right, respectively. They observed that reaching trajectories were systematically shifted in position according to numerical differences between the target and the number 'five'.

Overall, the reviewed literature supports one or the other of two phenomena. First, the automatic[7] activation of a spatial numerical representation influences the planning of lateralized movements across different conditions (i.e., eyes, pointing), suggestive of a spontaneous association between left-sided responses and small-quantities vs. right-sided responses and large quantities (Fischer et al., 2004; Ishihara et al., 2006). Second, the automatic processing of numerical information has an impact on planning manual-gestures (Andres et al., 2004; Badets et al., 2007; Chiou et al., 2009; Lindemann et al., 2007; Moretto & di Pellegrino, 2008), suggestive of a generalized magnitude code for action-related purposes, as proposed by the ATOM theory (Walsh, 2003).

The present study contributes to this line of research by investigating, for the first time, the effects of number on an overlearned complex motor plan that does not require explicit lateralized movements or strict spatial constrains: spontaneous handwriting. Handwriting is a complex multicomponential motor skill that, at a first stage, requires the control of hand placement by the movement of the arm. The trajectory of this movement modulates the specific location in the writing plane where writing occurs.

[7] In line with all the relevant literature within this context, access to numerical information has been said to be automatic by virtue of being autonomous; that is, it begins and runs to completion without intention (Zbrodoff & Logan, 1986).

Our research investigates whether the motor parameters involved in the handwriting of numbers are modulated by the numbers' magnitude. The general question we address is whether the spatial mapping of numbers in the representational space interferes with the motor planning involved in numbers handwriting, determining a spatial dislocation of the written output as a function of their magnitude.

First, we hypothesised that the automatic mapping of numbers on an oriented spatial representation induces a spatial dislocation in writing, with small numbers being written more towards the left relative to large numbers.

Second, we investigated whether this phenomenon may be modulated by task requirements and, thus, by the level of processing requested. Several studies have recently shown that magnitude information influences performance even (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996) or more so (Priftis, Zorzi, Meneghello, Marenzi, & Umiltà 2006) when it is irrelevant to the task. This effect is mainly reflected in the chronometric data by the speeding up of the responses compatible with the irrelevant numbers (e.g. SNARC effect). In particular, unimanual or bimanual lateralized responses to numbers, whether relevant or irrelevant to the task, are speeded up when compatible with the relative position of the numbers along the mental number line.

To this aim, the present study investigates the effect of numerical magnitude on the action planning involved in handwriting of Arabic digits in three different tasks: simple copy, writing by dictation and written naming. These tasks share the final executive components required in handwriting but differ in their input processing. In particular, in the simple copy task the activation of an internal representation is not necessarily required, since the stimulus to be produced is always available (Margolin, 1984). On the contrary, writing, whether by dictation or in written naming, implies the retrieval of an internal representation of the target, although the two tasks may differ in the extent to which they imply semantic mediation (Cipolotti & Butterworth, 1995; Seron & Noël, 1995). Our prediction was that the impact of numerical information on motor planning would be maximized when the number to be produced must be internally generated (e.g., written naming and writing to dictation), but limited or absent when the stimulus is visually available (e.g., copy from a model).

Third, we tested whether the observed spatial dislocation in numbers handwriting reflects an overlearned motor pattern associated with counting. Thus, stimuli in all tasks were presented in both random and ordered sequences: in the former the stimuli sequence was randomly produced (e.g., "nine, two, four, …"), and in the latter the stimuli consisted of the conventional counting sequence (e.g., "one, two, three, …"). If the spatial dislocation in written production is induced by the automatic activation of a spatial representation, the effect of number should appear for both ordered and random sequences.

Finally, we investigated whether other non-numerical sequences, such as the letters of the alphabet, may produce spatial dislocation effects during handwriting. Since the SNARC effect emerges with non-numerical ordinal sequences, it has been recently suggested that a spatial mental representation is not unique to magnitude information (Gevers et al., 2003, 2004; Previtali et al., 2010). However, although spatial mapping may occur for different ordered sequences, its intrinsic relevance is likely to differ for numerical, alphabetical and other verbal sequences (e.g., months). Numbers are indeed the most overlearned and overused ordered sequence, and the only sequence where order holds cardinal meaning. For this reason, we hypothesized the spatial dislocation to be systematic only in numbers writing.

2. Experiment 1. Writing of numbers

In order to test the hypothesis that the automatic mapping of numbers on the *mental number line* may induce a spatial dislocation in writing, subjects were presented with three different tasks all requiring written production of Arabic numbers: i) copy from a model (hereafter, copy task), ii) writing to dictation, and iii) written naming of numerosities. While simple copy does not require semantic processing, writing to dictation and written naming vary in their semantic requests. Although writing Arabic digits to dictation may, in principle, be accomplished bypassing semantics (Cipolotti & Butterworth, 1995; Dehaene & Cohen, 1995; Deloche & Seron, 1987), the evidence suggests that access to magnitude information occurs automatically during transcoding (e.g. Dehaene & Akhavein, 1995). Finally, in written naming access to magnitude information is obligatory, since the task requires associating an Arabic number to a specific non-symbolic numerosity.

2.1. Material and method

Subjects. Sixteen right-handed undergraduate students (8 females) from the University of Milano-Bicocca participated in this study. The mean age was 24.9 years (range 20 - 38). All participants had normal or corrected to normal vision, and were naive to the purpose of the experiment.

Apparatus. Participants were individually tested in a quite and dark room. They sat about 45 cm away from the screen and the centre of the digitizing tablet was aligned with the mid-saggital plane of the participant's trunk. Stimuli were presented on a white background on Samsung Sync Master 7535 Monitor (resolution of 1024 x 768 pixels, 35.8° x 29.1°). Spontaneous writing, performed with an electromagnetic pen, was registered by means of a digitizing tablet ("Wacom Intuos 2", format A3, 420 X 297 mm, frequency of 50 Hertz, accuracy 0.41 mm) allowing recording of all relevant information relative to the spatial parameters of written production.

Stimuli and Procedure. In each task (copy, writing to dictation, and written naming) the numbers from 1 to 9 were presented in two sequences, random and ordered. In the random sequence, the numbers from 1 to 9 were presented in casual order. In the ordered sequence, the written or spoken word "count" instructed the subject to write down the counting sequence from 1 to 9 at a given rate determined by an alert sound. In both sequences, subjects had to write down the target stimulus as soon as the alert sound (lasting 1000 ms) was presented.

In the random sequence of the copy task, Arabic digits were presented against a white background, in black Triplex Sans Light font 3 cm wide and 5 cm high (visual angle: 3,8° x 6,3°). Stimulus presentation was terminated by the experimenter who pressed a key on the keyboard as soon as the participant stopped writing. In the ordered sequence, the word "count" was presented for 3000 ms, followed by nine sounds in sequence, one every 3000 ms.

In the writing to dictation task, verbal numerals and the word "count" were recorded with a female voice and normalized to the same tonality with the software Audacity (a free software, Audacity Developer

Team, 2007). Stimulus duration was 1000 ms, and the inter-stimulus interval was controlled as for the copy task.

In the written naming task, arrays of black dots (diameter: 0.5 cm, visual angle: 0.6°) varying in numerosity from one to nine were visually presented. In each trial, the dots were randomly positioned within an imaginary central square (5 X 5 cm, visual angle 6.3° X 6.3°) resulting in different configurations. The ordered sequence was presented as in the copy task. Note that, although production of the ordered sequences is identical across the tasks, we do not exclude differences in processing induced by the different contexts.

For each task, every digit was presented 5 times for each sequence type, for a total of 90 experimental trials. Each task was preceded by three practice sequences, two of which were ordered. Practice trials were repeated if subjects appeared uncertain about the procedure and discarded from all subsequent analyses. Subjects, who hold the pen with their dominant hand over the entire task, did not receive any visual feedback of their own graphic marks. Instructions required participants to move their hand to the starting position (centrally positioned on the inferior border of the tablet) after each trial. No other constraints were given in the instructions.

In order to familiarise participants to the script and to the apparatus, the first task for all the subjects was the copy task. The order of presentation of writing to dictation and written naming was counterbalanced across participants.

2.2. Data analysis

Four subjects' data were excluded from the analysis of the copy task because of a very high proportion of inaccurate recording (missing trials over 50%). For all remaining subjects, in all tasks, errors and ambiguous symbols were identified and not further analysed (42, 0.97% - in the copy task; 36, 0.83% - in the writing to dictation task; and 23, 0.53% - in the written naming task).

For all accepted trials, the smallest inscribing rectangle containing each digit written by each subject in each trial was computed. The horizontal coordinate of the rectangle barycentre (X centre), which is its distance from the left border of the drawing area (X=0) expressed in mm, was the dependent variable (see Figure 1). The mean value of the X centre for each digit was submitted to two repeated-measures analyses of variance. The first one explored the copy task with two within-subject factors: number size (with pairs of numbers 1-2, 3-4, 6-7, 8-9 merged together) and sequence (random, ordered).

In the second ANOVA, the performance in the two experimental tasks was analyzed with task (writing to dictation, written naming), number size (1-2, 3-4, 6-7, 8-9) and sequence (random, ordered) as within-subject factors.

Whenever necessary, the Huynh-Feldt correction for repeated measures analysis (Huynh & Feldt, 1970, 1976) was used to correct for violations of the sphericity assumption. The epsilon (ε) used to reduce the degrees of freedom, the original degrees of freedom, and the corrected *p*-values are reported.

Finally, the influence of the numerical information on writing performance was further investigated by means of a regression analysis for repeated measures data (Lorch & Myers, 1990). Regression equations (x = B *NS +A) were calculated using numerical magnitude (NS) as the predictor variable for each subject, for each task and for each type of sequence. The slope coefficients (B) obtained from the regression were then contrasted to zero with a T-test analysis (see Fias et al., 1996; Lorch & Myers, 1990).

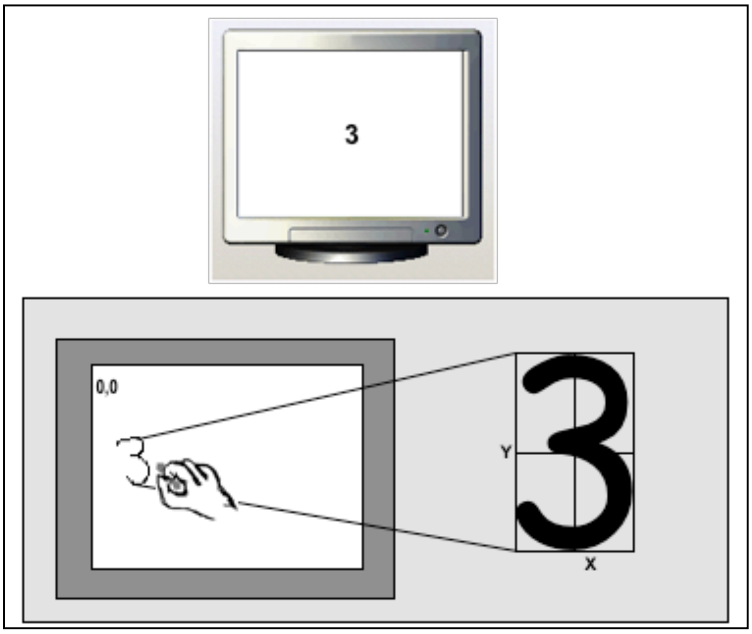

Fig. 1. Dependent variable (X centre): the X and Y coordinates determining the centre of rectangle including the reproduced digit. The horizontal coordinate of X centre is the distance from the left border of the drawing area (X coordinate of value 0) and the rectangle's centre. The value in pixels was converted to mm (1 pixel = 0.41mm).

2.2. Results

In the copy task ANOVA, the main effect of number size was marginally significant ($F[3,33] = 3.22$, $\varepsilon = .72$, $p = .054$), although planned comparisons[8] did not reveal significant differences between consecutive sizes (all $p > .1$) (see Figure 2a). The effect of sequence was also marginally significant ($F[1,11] = 3.74$, $p = .08$), while the interaction number size X sequence was not significant ($F[3,33] = 2.53$, $\varepsilon = .43$, $p > .1$).

In the other two experimental tasks the effects of task and sequence were not significant ($F[1,15] = 2.06$, $p > .2$; $F[1,15] = 1.54$, $p > .2$). The main effect of number size was significant ($F[3,45] = 9.45$, $\varepsilon = .66$, $p < .001$) (see Figure 2b). Planned comparisons revealed the following: 1-2 vs. 3-4, $p < .05$, 3-4 vs. 6-7, $p < .01$, 6-7 vs. 8-9, n.s. Only the number size X sequence interaction was marginally significant ($F[3,45] = 2.43$, $p = .08$; all others interaction $p > .2$). Two separate ANOVAs for the two sequences, replicated the number size effect: both in random ($F[3,45] = 5.33$, $p < .01$)and in ordered sequences ($F[3,45] = 10.47$, $\varepsilon = .58$, $p < .001$].

In relation to the regression analyses, in the copy task, the regression line is described by the following equation:

ordered sequence: $X_c = 1.83\ NS + 309.7$;

random sequence: $X_c = 0.36\ NS + 233.31$, where X_c is the X centre and NS is the number size.

The t-test on the slope coefficient was marginally significant in the ordered sequence only ($t(11) = 2.12$, $SD = 4.16$, $p = .06$, random sequence: $t(11) = 0.90$, $SD = 1.57$, $p = .39$), reflecting the contribution of number magnitude to the observed variable (with a positive slope observed in 11 participants).

In the writing to dictation task the regression analysis revealed the following equation:

ordered sequence: $X_c = 0.81\ NS + 227.56$;

random sequence: $X_c = 0.81\ NS + 229.78$.

The contribution of number magnitude resulted significantin both sequences: in random, $t(15) = 2.14$, $SD = 2.10$, $p < .05$ (with a positive slope observed in 14 participants), and in ordered, $t(15) = 4.14$, $SD = 1.09$, $p < .001$, (with a positive slope observed in 15 participants).

[8] The Bonferroni correction was applied to all planned comparisons.

This holds true also for the written naming task, where the regression line is described by the following equation:

ordered sequence: $X_c = 1.11 \text{ NS} + 230.36$;

random sequence: $X_c = 0.28 \text{ NS} + 234.67$.

The T-test on the slope coefficients was significant in both random, $t(15) = 2.54$, $SD = .61$, $p < .05$ (with a positive slope observed in 11 participants) and ordered sequences $t(15) = 2.87$, $SD = 2.15$, $p < .01$ (with a positive slope observed in 14 participants).

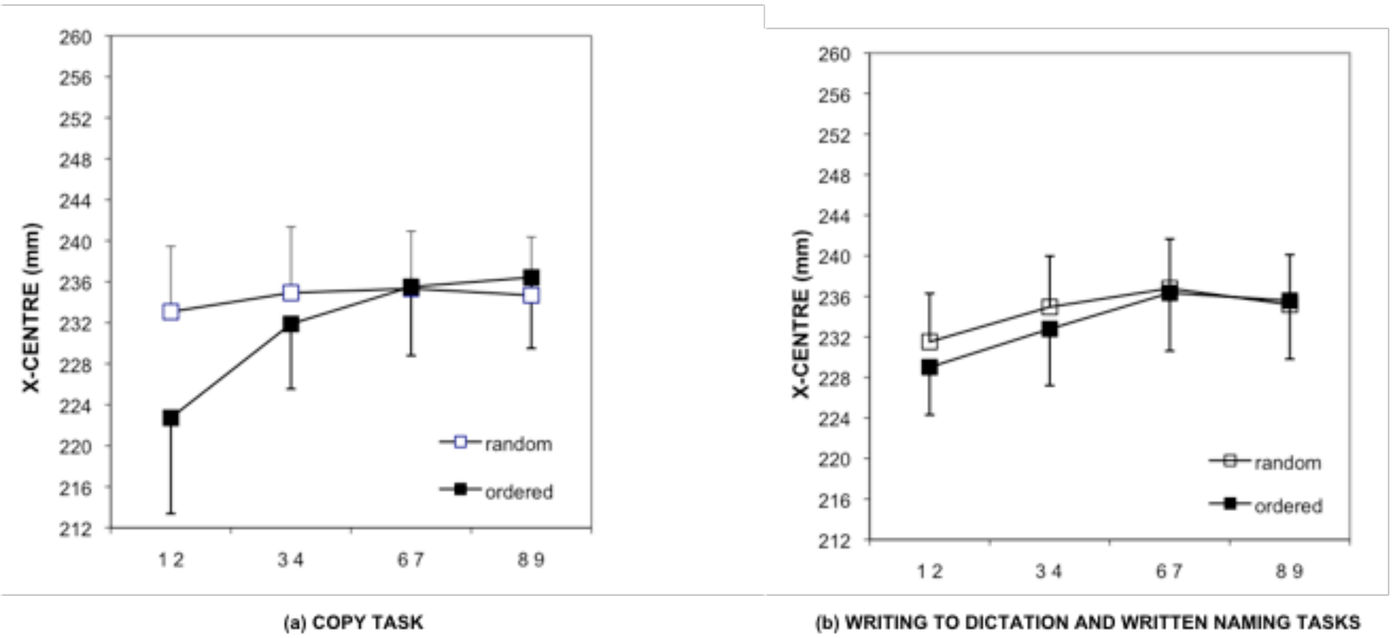

Fig. 2. Experiment 1 (*writing of numbers*). Subjects' average X centre and ± SE bars of written production as a function of number size (1-2, -34, 6-7, 8-9) and sequence (random, ordered) in a) copy task and b) in writing to dictation and written naming tasks.

2.3. Discussion

Three main points may be drawn from the results of Experiment 1. First, overall written performance was characterised by a spatial dislocation of digits along the horizontal axis as a function of their magnitude: small digits were produced more to the left relative to larger ones. Second, the spatial dislocation was present in both ordered and random sequences; however, it was different across the tasks. In written naming and writing to dictation, the magnitude effect emerged in all sequences, while in the copy task the spatial dislocation was marginal and limited to ordered sequences. In particular, the left to right spatial dislocation of number handwriting as a function of the digit magnitude is consistent with their spatial mapping on a mental number line, as if the activation of this representation had an effect on the arm movements responsible for the action endpoint, i.e., the specific location in the horizontal plane where handwriting takes place. Moreover, as reflected in the regression analyses, the relative position of the different digits was dislocated along the left to right horizontal direction (within a rather central area of the tablet) so that smaller numbers were written to the left of the larger ones. Critically, when writing was more than a simple copy, this occurred even when numbers had to be written in a random sequence, with no explicit reference to their intrinsic order.

These results are in line with the hypothesis that number processing interacts with action planning and/or execution. However, whether this effect is generated by the ordinal or the cardinal value of the numbers remains to be established. Indeed, non-numerical ordered sequences (e.g., letters, months, and days of the weeks) have been shown to produce magnitude-related effects such as the SNARC and the distance effects (Gevers et al., 2003, 2004), with the first elements in the sequence behaving like small numbers and the last ones behaving like large numbers. To investigate whether the spatial dislocation in written performance was simply induced by access to order information, Experiment 2 seeks to replicate the findings from Experiment 1 using the letters of the alphabet as stimuli.

3. Experiment 2. Writing of letters

Experiment 2 was similar to Experiment 1, except for the stimuli, which were the first nine letters of the Italian alphabet. Copy and writing to dictation were identical to Experiment 1. In addition, we presented participants with a task that required transcoding of letters so that the allograph to be produced must be internally activated. If the spatial dislocation in written performance derives from the activation of a quantity-related spatial continuum, the effects should not extend to non-numerical sequences, such as alphabetical characters. On the other hand, if the activation of ordinal information is responsible for the observed dislocation, the effects should emerge in the written performance of any ordered sequence.

3.1. Material and method

Subjects. Sixteen right-handed undergraduate students (13 females and 3 males) from the University of Milano-Bicocca participated in this study; none of them participated in Experiment 1. The mean age was 25 (range 20 - 44). All participants had normal or corrected to normal vision, and were naive about the hypotheses of the experiment.

Stimuli and Procedure. The first nine letters of the Italian alphabet were employed for the three tasks: i) copy from a model, ii) writing to dictation, and iii) transcoding from lower case to upper case letter. All tasks required written production of upper case letters. The written stimuli were presented centrally against a white background, in black and bold Arial font of 60-point size. In the copy task, stimuli occupied a 2 x 2 cm (2.5° x 2.5° of visual angle) non-visible frame while in the transcoding task stimuli occupied a 1.3 x 2 cm non-visible frame (1.7° X 2.5° visual angle). The auditory stimuli were the names of the first nine letters of the alphabet and the word “go” acted as starting signal for ordered sequences. All were registered with a female voice equated for tonality by software Audacity. Each stimulus lasted 1000 ms.

Experimental procedure, presentation time, and order of the tasks were identical to Experiment 1.

3.2. Data analysis

Inspection of errors and inaccurate recordings led to the exclusion of 72 trials (1.7%) from all further analyses. The smallest inscribing rectangle for each digit written by each subject in each trial was calculated. The horizontal coordinate of the barycentre (X-centre), which is the distance from the left border of the drawing area, was used as the dependent variable (see Figure 1). The mean value of the X-centre for each letter was submitted to two repeated-measures analyses of variance. The first one explored the effects of position and sequence in the copy task with two within-subjects factors: letter position (with pairs of letters A-B, C-D, F-G, H-I merged together) and sequence (random, ordered). A second ANOVA was carried out for the two experimental tasks, with task (writing to dictation, transcoding), letter position (A-B, C-D, F-G, H-I) and sequence (random, ordered) as within-subjects factors.

Finally, as in Experiment 1, the influence of letter position on writing performance was further investigated by means of a regression analysis for repeated measures data (Lorch & Myers, 1990).

3.3. Results

In the copy task (see Figure 3a), a marginally significant effect of letter position was found ($F[3,45] = 3.7$, $\varepsilon = .45$, $p = .06$). Planned comparison revealed the following: A-B vs. C-D, $p < .05$, C-D vs. F-G, n.s., F-G vs. H-I, n.s. The sequence effect was not significant ($F[1,15] < 1$, n.s.), nor the interaction letter position x sequence ($F[3,45] = 2.03$, $\varepsilon = .39$, $p > .2$).

In the analysis of variance for the two other experimental tasks none of the main effects were significant (all, $p > .2$). Task X letter position was the only interaction marginally significant ($F[3,45] = 2.77$, $\varepsilon = .77$, $p = .07$). Two separated one-way ANOVAs for the two tasks showed a null effect of letter position (writing to dictation task: $F[3,45] = 2.55$, $\varepsilon = .46$, $p > .1$; transcoding task: $F[3,45] = 1.68$, $\varepsilon = .48$, $p > .2$). None of the other interactions reached significance (task X sequence, $F[1,15] < 1$, n.s.; letter position X sequence, $F[3,45] = 1.36$, $\varepsilon = .41$, $p > .3$; task X letter position X sequence $F[3,45] < 1$, n.s.).

The T-tests on the slope coefficients for each subject, task, and sequence did not reveal any effect of letter position on performance (all $p > .1$)[9].

[9] In the copy task, the regression line is described by the following equations (in brackets T values against 0 and number of participants with a positive slope [N]): ordered sequence: $X_c = 2.53$ LP + 231.48 where X_c is the X centre and LP is the letter position (T-test(15) = 1.57, SD = 14.31, p = .14 [N=12]); random sequence: $X_c = 0.15$ LP + 243.77 (T-test(15) = 0.69, SD = 1.99, p = .50 [N=10]). In the writing to dictation, the regression line is described by the following equations: ordered sequence: $X_c = 2.38$ LP + 232.26 (T-test(15) = 1.42, SD = 14.9, p = .18 [N=12]); random sequence: $X_c = 0.49$ LP + 242.61 (T-test(15) = 1.73, SD = 2.53, p = .1 [N=9]). In the transcoding task, the regression line is described by the following equations: ordered sequence: $X_c = 2.05$ LP + 235.65 (T-test (15) = 1.29, SD = 14.14, p = .22 [N=11]); random sequence: $X_c = 0.05$ LP + 244.38 (T-test(15) = 0.48, SD = 0.98, p = .64 [N=8]).

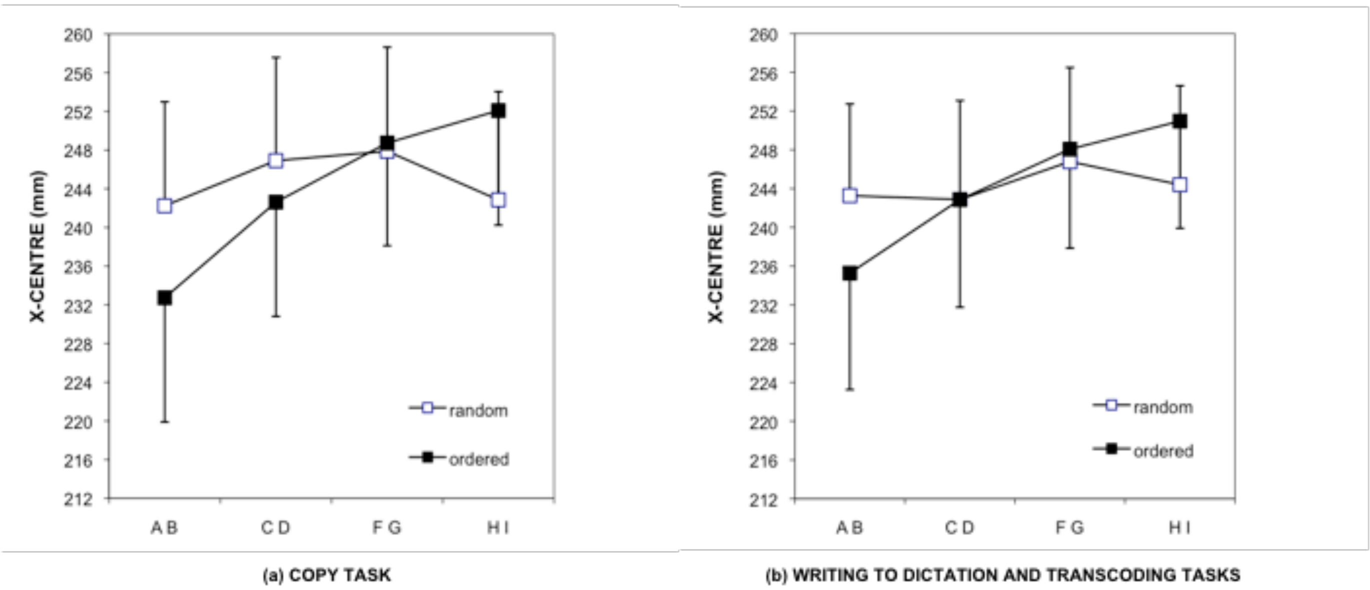

Fig. 3 Experiment 2 (*writing of letters*). Subjects' average X centre and ± SE bars of written production as a function of letters alphabetical position (AB, CD, FG, HI) and sequence (random, ordered) in a) copy tasks and b) writing to dictation and transcoding from lower case to upper case tasks.

3.4. Discussion

Results of Experiment 2 are rather clear. When the writing output consists of letters, their relative position in the alphabet had a null or marginal effect on the movement endpoint. In fact, spatial dislocation of writing as a function of letter position in the alphabet was minimal and limited to the copy task. Although spatial coding seems to occur in the representation of any ordered information (Gevers et al., 2003, 2004) it seems reasonable to suggest that order is intrinsically relevant only for numbers. Accordingly, in writing performance, where no explicit lateralized movements or strict spatial constraints are imposed (cf. Fischer et al., 2003), the obligatory activation of a spatially oriented representation takes place and interacts with action planning only when numbers are involved.

4. General discussion

The aim of the present study was to explore whether the automatic mapping of numbers in the representational space exerts an influence on the motor planning involved in numbers handwriting. In particular, we show that the automatic association between numbers and their relative position along an oriented spatial continuum induces spatial dislocations of the writing movement endpoints as a function of their magnitude. In particular, we measured the relative position on the horizontal axis of each single digit handwritten on a digitizing tablet, under different task conditions. The results show that, across different tasks, numbers writing is characterised by a spatial dislocation of the digits as a function of their magnitude: small digits are produced leftwards relative to larger ones. Critically, this holds true not only for the ordered sequences but also for the random ones, where the overlearned left-to-right sensory-motor association evoked by a counting sequence is minimised. However, the regression analyses indicated that this applies only when writing is more than a simple copy, since in the copy task the spatial dislocation of written production emerged only for the ordered sequences. This result is in line with our prediction according to which the influence of a representational spatial mapping is maximised in tasks that require a deeper, although not necessarily semantic, processing of numbers. Indeed, in both writing to dictation and written naming, the target digit must be retrieved from memory and its graphemic representation must be activated, while copying may be easily accomplished by merely reproducing a visual model (Margolin, 1984). Moreover, in the copy task the written stimulus was available and centrally displayed and we may not exclude that it served as a visual cue for guiding the arm movement endpoints. This partially applies also to written naming although, in this case, the stimulus was visually more complex (patterns of dots vs. digits), and visually different from what was required to be produced. In the writing to dictation task, no visual cues were provided, and thus writing performance was free from visuo-spatial constraints.

Finally, we intended to verify whether the magnitude/motor interaction responsible for the spatial dislocation in number writing extends to non-numerical ordered sequences. Order has been shown to produce several magnitude-related effects, such as the SNARC and the distance effects (Gevers et al., 2003, 2004), and on these grounds it has been suggested that any ordered sequences may be preferentially mapped onto a spatial mental representation (e.g., Previtali et al., 2010; Van Opstal et al., 2009). In the present study, numbers and letters writing are differently modulated by the implicit and irrelevant mapping of the ordinal trials in the representational space. In particular, the impact of a spatial mental representation on the motor planning associated to handwriting is limited to numerical sequences. Indeed, a dislocation in writing letters occurs marginally only in the copy task and only when trials are produced in ordered sequences. These results add to existing evidence pointing to behavioural (Turconi et al., 2006; Zorzi et al., 2006) and processing differences (Badets et al., 2007; Turconi et al., 2004) of ordinal and magnitude information. However, our results do not reveal whether the absence of the position effect in letters writing derived from a null or a weak mapping of letters on the representation space. In fact, spatial mapping may occur for different ordered sequences (not only numbers) although the intrinsic relevance and, consequently the access to it, are likely to differ for numerical, alphabetical and other ordered sequences.

Overall, these findings extend previous evidence of an influence of numerical magnitude on motor outcomes, as shown by magnitude-space congruency effects in various tasks (Andres et al., 2004; Badets et al., 2007; Fischer, 2003; Fischer et al., 2004; Ishihara et al., 2006). However, the novelty of the present study consists in showing that numerical information may impact on an overlearned complex motor plan, such as spontaneous handwriting. Yet, at present, we can only speculate if the processing of magnitude information exerts its influence on handwriting movement endpoints at the level of action planning or of motor execution (Andres et al., 2008; Fischer & Miller, 2008). In fact, spontaneous handwriting does not allow us to disentangle between planning and execution of the arm movements implied in written production. Moreover, in contrast with paradigms adopting forced lateralised motor responses (Fischer, 2003; Fischer et al., 2004; Ishihara et al., 2006), the spatial compatibility between magnitude and motor action is not dichotomical (left/right) but continuous along the horizontal axis. This implies that the location of handwriting is not wrong *per se* but slightly modulated by the relative position of the number along a representational continuum. On this ground, we did not expect subjects to self-correct or adjust their motor plans during execution (cf. Andres et al., 2008).

Finally, our results fit with the ATOM theory according to which the impact of magnitude information on motor action arises within the parietal cortex (Walsh, 2003). The reported compatibility effect between spatial mapping of numbers in the representational space and their spatial dislocation in the horizontal plane reflects the interaction between number representation and writing movements. The involvement of the intra-parietal sulcus in supporting the visuo-spatial representation of numbers is well-documented and universally accepted (Dehaene et al., 2003; Hubbard et al., 2005). Moreover, both neuropsychological and neuroimaging data point to the extensive involvement of the left inferior parietal lobe in subserving writing performance (Delazer, Lochy, Jenner, Domahs, & Benke, 2002; Harrington, Farias, Davis, & Buonocore, 2007). Thus, the present study adds to the large body of evidence pointing to the parietal cortex as the crucial brain district where multi-dimensional magnitude information is integrated to subserve action (for a review, Bueti & Walsh, 2009).

Chapter 6. HOW SPATIAL ORIENTATION OF THE MENTAL NUMBER LINE AUTOMATICALLY INFLUENCES ELEMENTARY PHYSICAL MOVEMENTS

1. Introduction

It is widely acknowledged that numbers are spatially coded into an analogical representation conceived as an oriented mental number line. Empirical support for the mental number line comes from systematic effects observed in numerical comparison tasks. For example the time to compare two numbers decreases according to their numerical distance and to their size, in the same way as our sensory discrimination depends on the ratio between the stimuli (Cantlon et al., 2009). The ratio-dependent performance in number discrimination has been attributed to the psychophysical properties of the analogical numerical representation where numerical differences are instantiated as spatial distances. Furthermore, small numbers are typically associated with the leftward space and large numbers with the rightward space, as reflected in the SNARC effect (Spatial-Numerical-Association of Response Codes, Dehaene et al., 1993; for a review Wood et al., 2008), consisting in a temporal advantage for lateralised responses congruent to the magnitude of the numerical stimuli.

Relevant to the current study, activation of this numerical representation has been called into question to account for the impact of numerical magnitude on motor tasks. The studies relevant to this line of research may be categorised by the motor response required by the task: i) lateralized or unimanual response or ii) manual-gestures as grasping.

In the first category, a temporal advantage for starting leftwards movement to answer small numbers and rightwards movements to answer large numbers is repeatedly reported (Fischer, 2003; Fischer et al., 2004; Ishihara et al., 2006; Schwarz & Keus, 2004; Schwarz & Müller, 2006). For example, in a pointing study participants were required to move their hands from a central location to the left or to the right target area according to the parity of a centrally presented number (Fischer, 2003). As expected, leftwards movements were initiated faster when classifying small digits and rightwards movements were initiated faster when classifying large digits.

In the second category of studies, the influence of numbers on motion emerges as compatible between numerical magnitude and grip posture in grasping movement (Andres et al., 2004), in grasping estimation (Badets et al., 2007) or in different types of grip (i.e., power vs. precision grip, Lindemann et al., 2007; Moretto & di Pellegrino, 2008). For example, in Andres et al.'s study (2004), participants were required to classify a number as even or odd by performing a closure or an opening grip. Their results showed that the grip closure was initiated faster in response to small numbers and grip opening was initiated faster in response to large numbers.

Overall, this evidence has been interpreted as reflecting the influence that numerical information exerts on action, and specifically, on motor planning rather than on motor execution, although this latter has been, so far, less systematically considered (Andres et al., 2004; Badets et al., 2007; Ishihara et al., 2006; Lindemann et al., 2007; Moretto & di Pellegrino, 2008; Schwarz & Keus, 2004; Schwarz & Müller, 2006). Single positive evidence is represented by the study of Andres and colleagues (2008), where irrelevant digits printed on visual objects were found to influence grip aperture only in the initial phase of movement, suggesting an influence of numbers on selection and/or action initiation rather than action execution itself.

Moreover, when motor execution resulted unaffected by magnitude information, response modality (Fischer & Miller, 2008; Vierck & Kiesel, 2010), or task procedure (Fischer, 2003) appear problematic. For example, in Vierck and Kiesel's study (2010, see also Fischer & Miller, 2008), the key-press response made the movement duration too short to capture magnitude-related variability in the peak force.

On the other hand, strict spatial constrains imposed to response execution, in saccadic or pointing movements, prevent movement amplitude to vary as a function of number magnitude (Fischer, 2003; Fischer et al., 2004).

However, when varying the dependent variable in a pointing task, the impact of numbers on motor execution was testified by a systematic lateralised shift in hand movement trajectories as a function of number magnitude (Song & Nakayama, 2008).

Overall, these studies provide little evidence for a significant number modulation on motor execution, this effect being absent or limited to lateralised goal-directed pointing.

PART ONE
THE SPATIAL FEATURES OF THE MENTAL NUMBER LINE INFLUENCES PHYSICAL MOVEMENTS

This first study aims at providing further evidence corroborating the influence of number magnitude on motor execution. Specifically, the influence of the mental number line representation on simple movements will be evaluated in a simple manual task requiring the connection of two points flanked by irrelevant digits. Therefore, we manipulated the direction of the connection movement (i.e., left-to-right vs. right-to-left) and the position of the numerically larger flanker to further investigate the interaction of the induced MNL representations (i.e., ascending from small to large, reflected by the position of larger flanker) and the congruency between movement direction and reading / writing direction.

The hypotheses of the current study are the following:

(i) We expect to find the effect of physical distance between the to-be-connected points indicating faster movements as the physical distance between the points increases.

(ii) Driven by possibly overpowering influences of reading/writing direction from left to right we expect connection movements along this direction to be executed with a higher maximum velocity than movements from right to left.

(iii) We expect the effect of movement direction to be modulated by its congruency with the mental number line representation induced by the two irrelevant flankers. Accordingly the relative position of the flanker numbers and the required movements may be congruent between each other when small-left/large-right flankers are connected with a left-to-right movement or when large-left/small-right flankers are connected with a right-to-left movement. In other words, congruency with the MNL direction occur when the movement connection goes from the smaller to the larger digits, irrespective of their relative position. Following the congruency between induced number line representation and movement direction connection movements from right to left should be facilitated in this case while left to right movements should suffer from the incongruence between movement direction and induced mental number line representation.

Hypotheses (ii) and (iii) go beyond these previous findings as they suggest that the spatial direction of elementary movements is driven by elementary cognitive representations such as writing direction or the number line.

2. Experiment 3. Connection of points delimited by numbers

2.1. Material and method

Participants. Twenty right-handed students of the University of Tuebingen (17 women, *M* = 20.8 years, *SD* = 1.2, range: 19-24 years) were tested in individual sessions, as partial fulfilment of course requirements. All participants reported normal or corrected-to-normal vision and were naive as to the purpose of the study.

Task, Design, Stimuli and Procedure. The task employed required participants to connect two points (diameter 1 mm, black against grey background) displayed on a tablet-pc by means of an electromagnetic pen (Hp Pavilion tx2000, format tablet 260 x 160 mm, resolution of 1280 x 800 pixels providing a spatial accuracy of pen movements of less than 0.2 mm). Physical distance between the two points was either 12 cm or 18 cm with the two points presented equidistantly to the left and right of the centre of the screen. However, the vertical position of the two points was varied between 3 positions on the screen to avoid participants creating reference frames based on e.g., the position of the points as compared to the screen borders. As cued at the beginning of each trial participants had to either connect the left point with the right point (indicated by the word “links” [left] cueing the connecting movement to be started at the left point) or the right point with the left point (as indicated by the written cue “rechts” [right]). Besides each of the two to-be-connected points a digital flanker was presented (i.e., numbers 1, 2, 3, 7, 8, 9; displayed in font Courier New 27 bold; see Figure 1 for an illustration). Please note that the digital flankers were irrelevant to the task just requiring participants to connect the two points. Before each trial the electronic pen had to be placed centrally at the lower end of the tablet-pc to ensure a standardized starting point for each participant. Then the two points together with the flankers and the cue appeared on the screen. When connecting the two points participants received visual feedback about their movement as the path of their movement remained visible on the screen. An inter-stimulus-interval of 500 ms was used initiated when participants lifted the electronic pen off the tablet-pc. The tablet-pc was located centrally in front of the participants with a viewing distance of approximately 40 cm.

Taken together, the experimental design was 2 x 2 x 2 x 2 within-participant design with the following factors manipulated: physical distance between the points (i.e., 12 cm vs. 18 cm), movement direction (i.e., from left to right vs. from right to left), position of the numerically larger flanker (i.e., left vs. right) and the numerical distance between flankers (i.e., close as in flanker pairs 1_2, 2_3, 7_8, 8_9 vs. far as in pairs 1_8, 1_9, 2_8, 2_9). Each physical distance was presented 12 times for each direction movement, each position of larger digit, and for each numerical distance resulting in a total of 192 experimental trials. These critical trials were preceded by six practice trials (with numerical flankers not used in the experimental trials). Trial order was randomized for each participant individually. Verbal instructions focused on both speed and accuracy of the connection movement.

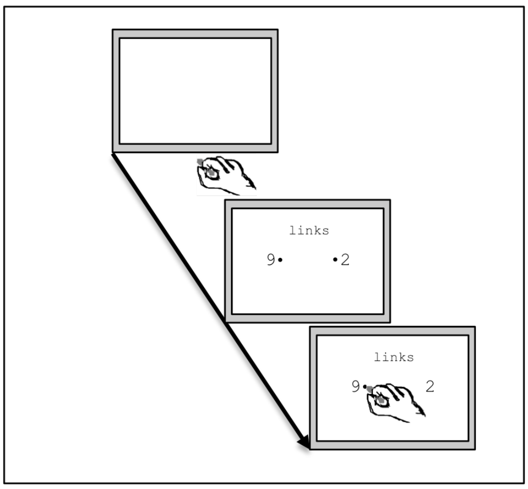

Fig. 1 Example of procedure and stimuli of Experiment 3. Before each trial the electronic pen had to be placed centrally at the lower end of the tablet-pc. Then the two points together with the flankers and the cue appeared on the screen. An inter-stimulus-interval of 500 ms was used initiated when participants lifted the electronic pen off the tablet-pc.

2.2. Data analysis

For each subject trials with an obvious problem with the pen, i.e., incorrect registration of a short movement of only a few pixels, were identified by means of visual scanning and eliminated from further analysis resulting in a 0.42% loss of the data. For all accepted trials, the dependent variable was computed: the maximum velocity of the connection movement (henceforth referred to as v_{max}). The mean values for each condition were submitted to a repeated-measures analysis of variance (ANOVA) discerning the within-participant factorial design of the study: physical distance between points (i.e., 12 cm vs. 18 cm), movement direction (i.e., from left to right vs. from right to left), position of the larger flanker (i.e., left vs. right) and the numerical distance between flankers (i.e., close vs. far).

2.3. Results

The main effect of physical distance was significant [$F(1, 19) = 85.54$, MSE = .28, $p< .001$, $\eta^2_p = .82$] indicating that the maximum velocity was higher when the physical distance between the two points was larger (12cm: v_{max} = 38.2 cm/s, 18 cm: v_{max} = 49.2 cm/s). Moreover, the effect of movement direction was also significant [$F(1, 19) = 5.53$, MSE = .13, $p< .05$, $\eta^2_p = .23$], with connection movements from left to right (corresponding to German reading direction) being faster ($v_{max =}$ 44.6 cm/s) as compared to movements from right to left (v_{max} = 42.8 cm/s). The main effects of both position of the larger flanker as well as numerical distance was not significant [both $F(1, 19) < 1$]. Furthermore, the interaction of movement direction and position of the larger digit was significant [$F(1, 19) = 5.47$, MSE = .01, $p < .05$, $\eta^2_p = .22$]: the main effect of movement direction was modulated by the position of the larger number. Inspection of the marginal means revealed that a movement from left to right was faster than a movement from right to left when the larger of the two flankers was presented at the right of the to-be-connected points (+2.42 cm/s). Contrarily, connection movements from right to left were faster than movements from left to right (+1.30 cm/s) when the larger number was presented at the left of the two points. No other main effect nor any other interaction was significant [all $F(1, 19) < 1$, all $p > .2$].

To evaluate whether this interaction effect may be driven by a non-numerical sequential ordering principle instead of by induction of a spatial mental number line representation we replicated the experiment with flanking letters instead of numbers.

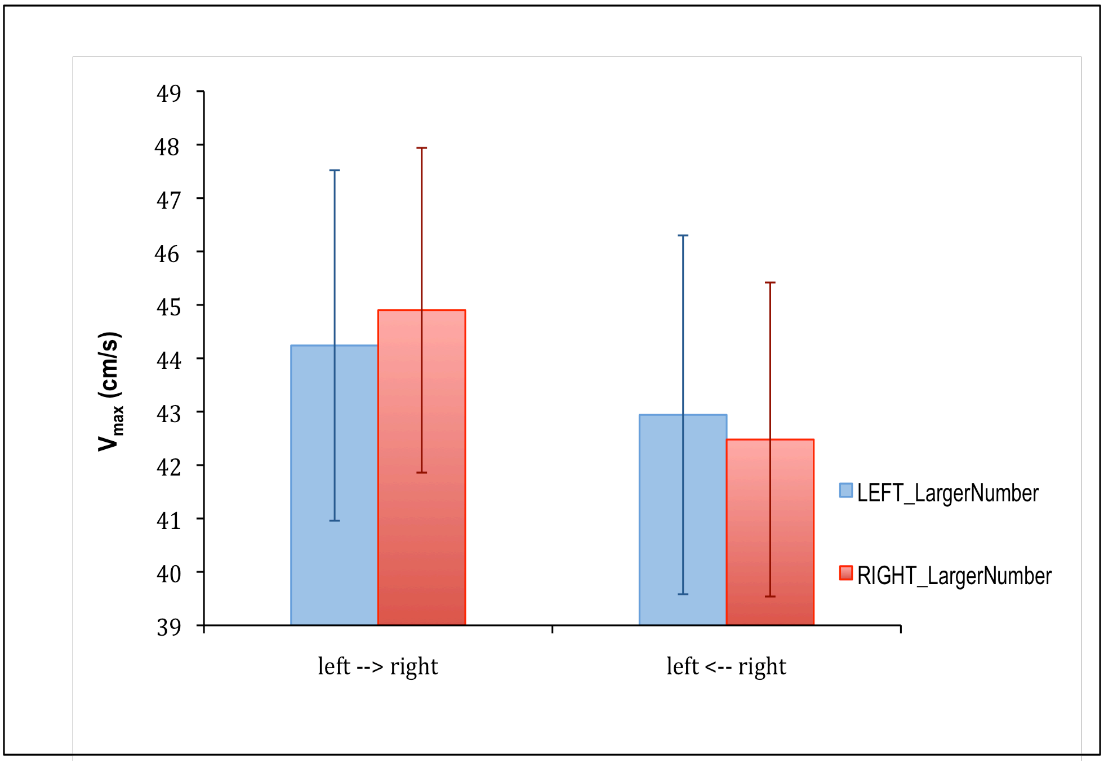

Fig. 2. Participants' average v_{max} separated for movement direction (from left to right, from right to left) and position of the larger flanker (left or right). Error bars indicate 1 standard error of the mean (SEM).

3. Experiment 4. Connection of points delimited by letters

3.1. Material and method

Participants. The same participants took part in this experiment. The order of the tasks was counterbalanced between the subjects.

Task, Design, Stimuli and Procedure. Experimental procedure and presentation time were identical. The experimental design was 2 X 2 X 2 X 2 within-participant design with the following factors manipulated: physical distance between the points (i.e., 12 cm vs. 18 cm), movement direction (i.e., from left to right vs.

from right to left), alphabetical position of the last letters as flankers (i.e., left vs. right) and the alphabetical position between flankers (i.e. close as in flanker pairs A_B, B_C, R_U, U_Z vs. far as in pairs A_Y, A_Z, B_Y, B_Z). The letters were printed in font Arial 27 no bold.

3.2. Data analysis

For each subject trials with an evident problem with the pen were identified by means of visual scanning and eliminated from further analysis resulting in a 0.2% loss of the data. For all accepted trials, the mean values of the observed variable for each condition were submitted to a repeated-measures analysis of variance with the within-participant factor as well as of Experiment 1: physical distance between points (i.e. 12 cm vs. 18 cm), movement direction (i.e. from left to right vs. from right to left), alphabetical position of the last letter as flankers (i.e., left vs. right) and the alphabetical position between flankers (i.e. close vs. far).

3.3. Results

The main effect of physical distance was significant [$F(1, 19) = 95.91$, MSE = .24, $p < .001$, $\eta^2_p = .84$] indicating that the maximum velocity was higher when the physical distance between the two points was larger (12cm: V_{max} = 38.2 cm/s, 18 cm: V_{max} = 48.9 cm/s). Moreover, the effect of movement direction was also significant [$F(1, 19) = 5.26$, MSE = .13, $p < .05$, $\eta^2_p = .22$], with connection movements from left to right being faster (V_{max} = 44.5 cm/s) as compared to movements from right to left (V_{max} = 42.6 cm/s). The main effects of alphabetical position of the last letter as well as alphabetical position between flankers was not significant [both $F(1, 19) < 1$, $p > .42$]. No other interaction was significant [all $F(1, 19) < 1$, all $p > .14$] except for the interaction of physical distance X alphabetical position of the last letter [$F(1, 19) = 9.91$, MSE = .01, $p < .01$, $\eta^2_p = .34$].

4. General discussion

The current study aimed at investigating possible directional influences of the mental number line representation on the execution of a simple motor movement such as the one required to connect two points. In general, we hypothesized that the maximum velocity of the connection movement should increase (i) as the physical distance between the to-be-connected points increase and (ii) should be higher for movements along the main reading / writing direction than for the reversed connection movements. (iii) More specifically we assumed the maximum velocity of the connection movements to be modulated by the location of the numerically larger flanker. In line with an ascending mental number line representation movements from left to right should be faster when the larger of the two flankers was presented at the right point, whereas movements from right to left should be facilitated by the larger flanker being presented at the left.

The results were consistent with all three research hypotheses. First, the maximum velocity of the connection movement increased as the physical distance between the to-be-connected points increased. Second, as has been hypothesized, maximum velocity was modulated by movement direction with connection movements from left to right and thus in line with the reading / writing direction of our participants having been faster than movements antidromic to reading / writing direction.

Finally, and most importantly, we found the latter effect to be modulated by number line related numerical representations. The interaction of movement direction with the position of the numerically larger flanker indicated that connection movements were reliably influenced by the irrelevant magnitude information given by the numerical flankers. In particular, when the connection movement required was directed from left to right, maximum velocity of the connection movements was higher for trials with the larger of the two flankers on the right side than trials with the large numerical flanker on the left side. Contrarily, when the requested movement went from right to left, maximum velocity was higher for trials with the larger flanker on the left side as compared to trials with the larger flanker on the right (see Figure 2). Thereby, the current data suggest that in a simple manual connection of two points the velocity is modulated by the automatic processing of numerical information irrelevant for the task.

In line with recent studies (Lindemann et al., 2007; Song & Nakayama, 2008), our data further clarify the numerical impact in motor execution. As a consequence, our results provide further evidence that the impact of number magnitude is not limited to motor planning but generalize to motor execution. The absence of modulation by number in some studies (Fischer, 2003; Fischer et al., 2004) may be due to methodological differences regarding task requirements, but not by a general lack of the influence of spatial representations (reading/writing direction, spatial number line). Indeed, in the saccadic and pointing responses referred to in the introduction (Fischer, 2003; Fischer et al., 2004) participants' movements were limited to going from the starting point (e.g., the initial fixation point) to a target whereas in the current study participants had to move the electronic pen to the starting point before starting the actually required movement. Thereby, the movement pattern observed in the current study may be less influenced by experimental constraints as it measured a more complete movement from selecting the starting point until choosing at what point the movement has to be stopped. Probably, these differences may account for the fact that movement amplitude was usually not modulated by number magnitude (Fischer, 2003; Fischer et al., 2004).

Importantly, this pattern of results was found only with numerical flankers and was not replicated with letters suggesting the effect to reflect a specific influence of number magnitude information rather than being based on purely ordinal information (Badets et al., 2007; Turconi et al., 2004).

So, the effects of irrelevant number magnitude information on motor execution again imply that internal representations of external distances may be coded similarly as internal representations of number magnitude, time, and so on.

Finally, this study extends previous evidence on the relative flexibility of the spatial representation of numbers (Bächtold et al., 1998; Dehaene et al., 1993) emphasizing the importance of the context for an interaction between numbers and space. So far, this flexibility has been mainly demonstrated by the modulation of the SNARC effect yielded by lateralized responses in different numerical tasks. The novelty of the present study consists of introducing a non-numerical task (i.e., connecting two points vs. parity judgement / magnitude comparison) without any dichotomized or lateralized responses and with movement direction as an independent variable. In conclusion, while the SNARC effect may be determined by the representational context, our results indicate that the interaction of number magnitude and action is modulated by the motor context, i.e., movements from smaller to larger flankers (and thus following the ascending order of the MNL) were facilitated (higher v_{max}) as compared to movements from the larger to the smaller flankers irrespective of the conventional reading / writing direction.

In our view, these findings are also important in a more general way: task-irrelevant cognitive representations such as numbers or reading/writing direction do not only influence motor planning, but also motor execution. Numerical representations exert this influence even on a trial-by-trial basis because the position of the larger number was changed from trial to trial. Therefore, this study underlines the powerfulness of these cognitive representations. Even the most elementary motor movements can be altered by the spatial direction of the mental number line although neither spatial associations of numbers nor the numbers itself were relevant in any respect.

PART TWO
HOW NUMBERS CAN SPEED UP MANUAL ACTION

In the second part of our study, we further investigate the numerical impact on action by simplifying the experimental paradigm used in Experiment 3-4 and by testing a more sensible index of motor execution used as dependent variable, i.e., the peak of speed.

This paradigm has at least two main advantages: 1) it requires rapid and easy goal-directed movements characterised by a relatively low individual motor variability, and 2) it allows us to directly evaluate a motor execution index, i.e., the peak of speed.

Before making a stroke, we expect participants to estimate the physical distance between points for deciding the movement amplitude. The key prediction in our study arises from the Fitts's law, that formalises the link between movement time, amplitude, spatial errors and target width (Fitts, 1954; for a review see Plamondon & Alimi, 1997). Critically for the present study, it has been shown that the kinematic properties of a simple rapid movement are correlated with the physical distance between its starting and final point. Accordingly, Cooke and colleague (1989) by exploiting a visually guided, step-tracking task found that the peak of speed reached by connecting manually two points is correlated with their physical distance.

On these grounds, we reason that if the spatial representation of numbers holds psychometric features similar to a perceptual linear extension (Doricchi et al., 2005), a symbolic distance, such as the one running between two numbers, could impact on speed movement in the same way as a physical distance. If this is so, at a fixed physical distance, the connection of two distinct points should be modulated by the symbolic distance represented by irrelevant flanker numbers. Accordingly, numerically close flankers should be connected as if they were physically closer, i.e., evoking a smaller portion of mental number line and inducing a lower peak of speed, compared to numerically far flankers, i.e., evoking a larger portion of the mental number line and inducing higher peak of speed.

Following this rationale, in the present study we hypothesised that the speed reached by manual connection of two distinct points is correlated either i) with their physical distance (Experiment 5) or ii) with the symbolic distance conveyed by two delimiting flanker numbers (Experiment 6).

5. Experiment 5.

In order to test the paradigm and to extend the evidence that the speed reached by the manual connection of two points is correlated with their physical distance (cf. Cooke et al., 1989), this variable was systematically manipulated in this experiment where we will introduce the best-index of peak of velocity (see below 5.2.).

5.1. Material and method

*Participants***.** Twenty (15 females, mean age of 22 years, range: 19-41) right-handed undergraduate students from the University of Milano-Bicocca underwent individual 10 min session. All students gave informed consent and were naive about the purpose of the experiment.

Apparatus. Participants were individually tested in a quiet and dark room. Their distance from the screen was about 30 cm. Stimuli were presented on a green screen on a tablet-pc (Hp Pavilion tx2000, tablet format: 260 x 160 mm, resolution: 1280 x 800 pixels, visual angle: 40.9° x 28.1°). The tablet-pc was centrally located on the mid-saggital plane of the participants. Participants were required to connect, by means of an electromagnetic pen and moving from left to right, two points (diameter 1 mm, 0.2°) displayed on the tablet screen (see Fig.1).

Stimuli and Procedure. Participants were required to connect two points. The physical distance between them was 6, 12 and 18 cm. To increase stimuli variability we manipulated the location of the leftward point: fixed on the left side of the tablet-pc or variable with the two points equally distant from the centre of the screen. The electromagnetic pen allows recording of all relevant information relative to the motor output.

Each physical distance was presented 10 times for each starting point, for a total of 60 experimental trials. The task was preceded by ten practice trials. Subjects received visual feedback of their own graphic marks. Instructions required participants to return their hand to a central position after each trial.

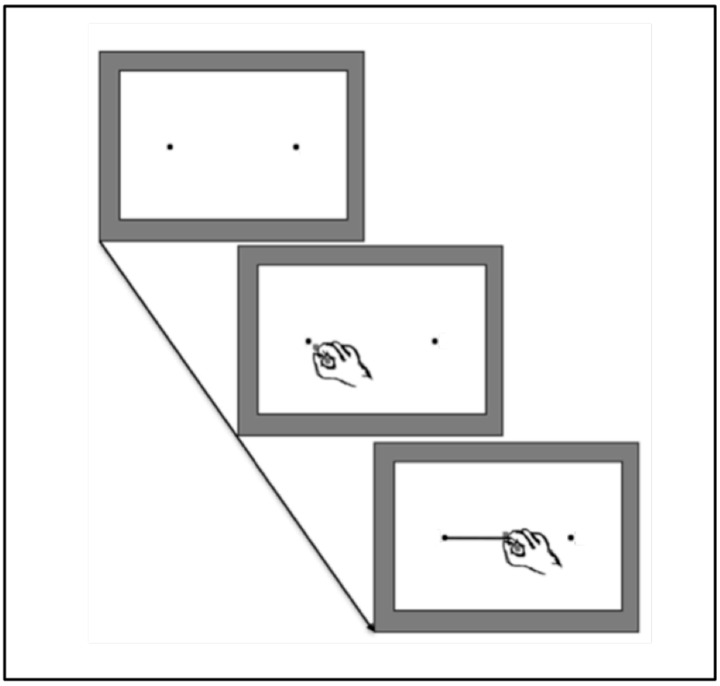

Figure 1. Example of procedure and stimuli of Experiment 5. Before each trial the electronic pen had to be placed centrally at the lower end of the tablet-pc. An inter-stimulus-interval of 500 ms was used initiated when participants lifted the pen off the tablet-pc.

5.2. Data analysis

The observed variable was the amplitude of the Gaussian curve that fits better the speed of each connecting movement, as detailed in the following. The recording output consists in the X and Y coordinates of the pen position on the tablet as a function of time. These data were first converted into values of speed (i.e., modulus of velocity) as a function of time. Eventually, speed data for each trial were fitted with a Gaussian function:

$$v(t) = \frac{A}{\sigma\sqrt{2\pi}} exp\left[-\frac{(t-t_o)^2}{2\sigma^2}\right] + v_0$$

with A, t_o, σ and v_o as fitting parameters. Here A represents the distance between the points to be connected by the subject, t_o is the time at which the maximum speed is reached, σ is the standard deviation for the Gaussian distribution and v_o is a velocity baseline. After fitting the single curves (thus obtaining different values for every fitting parameter and for every $trial_i$: A_i, t_{oi}, σ_i and v_{oi}) every data of speed as a function of time was shifted in time by t_{oi}, and in intensity by v_{oi}, then data from similar (repeated) trials were averaged. The resulting data were again fitted by a Gaussian curve, which amplitude represented the observed variable, that is a sort of peak of speed averaged over similar trials from a single subject (see Figure 2).

5.3. Results

10% of the overall data was not included in the analysis because of low index of goodness of Gaussian fit. For all accepted trials, the mean amplitude of the Gaussian distribution for each physical distance was computed and submitted to repeated-measures analysis of variance with two factors (**physical distance** between the points X **position of the starting point**[10]).

[10] Hereafter, the position of the starting point was called only position.

The main effect of physical distance was significant ($F[2,38] = 14.03$, $\varepsilon = .69$, $p < .001$, $\eta^2_p = .43$). Planned comparisons[11] revealed the following: small vs. medium, $p < .01$, medium vs. large, $p = .07$. The effect of position and the interaction physical distance X position were not significant ($F[1,19] = 1.52$, $p > .2$; $F[2,38] = 1.50$, $\varepsilon = .56$, $p > .2$). Results from Experiment 5 show that the peak of speed reached in the manual connection of two points increases as a function of their physical distance. In the following experiment, we explore the impact of symbolic distance on motor execution by manipulating the numerical difference represented by irrelevant flankers digit delimiting a fixed physical distance.

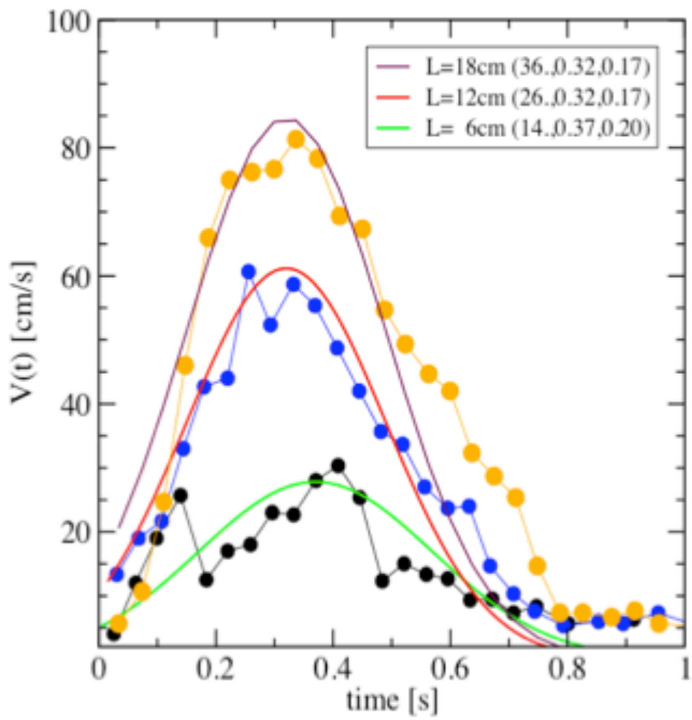

Figure 2. The graph shows the index of peak calculated. The 3 different colors represented the different levels of physical distance variable. Each dot is the module of velocity. The line represented the Gaussian function that fitted by data and this is an example of the maximum amplitude of distribution taken as index of peak speed. In the inset, we report the value of fitting parameters calculated (A, t_0, and σ).

[11] The Bonferroni correction was applied to all planned comparisons.

6. Experiment 6

6.1. Material and method

*Participants***.** Thirty (23 females, mean age=23, range 19-30) right-handed undergraduate students from the University of Milano-Bicocca underwent an individual 15 min session. None of them participated in experiment 3.

Stimuli and Procedure. The method was the same as in Experiment 5 except that the to-be-connected points were flanked by two irrelevant Arabic numbers (see Figure 3). This allows us to vary the represented numerical distance at a fixed physical distance. Indeed, numbers were either close (numerical distance "1", i.e., 1--2, 8---9) or numerically far (numerical distances "6", "7", "8", i.e., 2---8, 1---8, 2---9, 1---9). Each level of numerical distance was presented 20 times for each physical distance (6, 12 and 18 cm) for a total of 120 experimental trials. Stimuli were presented only centred on the screen. The task was preceded by ten practice trials.

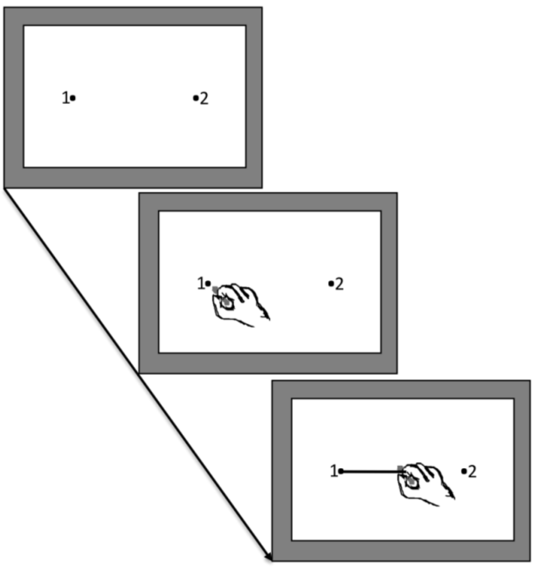

Figure 3. Example of procedure and stimuli of Experiment 6 (the same of Experiment 5 with numerical flankers).

6.2. Results

17.6% of the overall data was not included in the analysis because of low index of goodness of Gaussian fit. For all accepted trials, the mean amplitude of the Gaussian distribution for each physical distance was computed and submitted to a repeated-measures analysis of variance with two factors (**physical distance** between the points X **numerical distance**).

The main effect of physical distance was significant ($F[2,58] = 27.69$, $\varepsilon = .62$, $p < .001$, $\eta^2_p = .49$). Planned comparisons revealed the following: small vs. medium, $p < .01$, medium vs. large, $p < .01$. Critically, the effect of numerical distance was significant ($F[1,29] = 3.99$, $p = .05$, $\eta^2_p = .12$, see Figure 4) while the interaction physical distance X numerical distance was not significant ($F[2,58] < 1$, n.s.).

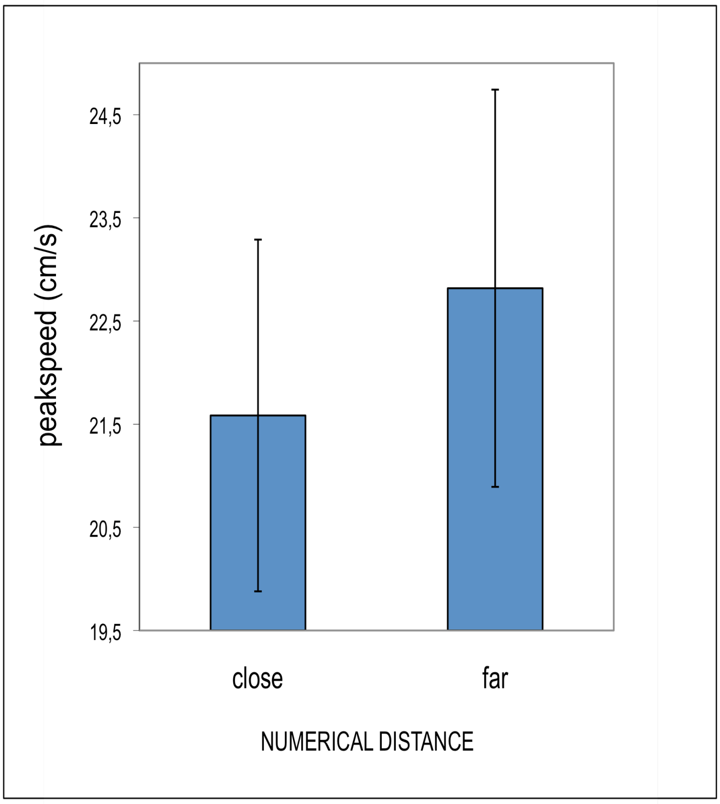

Figure 4. Experiment 6 (manual connection of two points delimited by different numbers). Subjects' average peak of speed (+/- 1SE bars) as a function of numerical distance (close vs. far).

7. General discussion

The present study shows that symbolic distances act as physical distances by modulating a kinematic parameter of a single and unidirectional movement. This result has both theoretical and practical implications.

The impact of the numerical distance between flankers on a simple unidirectional hand movement strictly mimics the influence that a perceptual physical length exerts on motion: the peak of speed of the junction movement increases as a function of either the physical or the numerical covered length.

This representational length effect strongly suggests that the numerical spatial continuum, conceived as a mental number line (Dehaene, 1992), partially holds psychometric features similar to a perceptual linear extension. Although analogies between the representational and the physical space have been previously reported (de Hevia & Spelke, 2010; Zorzi et al., 2002) the present study shows, for the first time, that activation of a symbolic distance at the representational level modulates directly a motor act as if the physical extension defining the movement amplitude was misperceived. In other words, the effect of the irrelevant flankers on the kinematic of the junction movement shows that numerically close numbers were perceived physically closer, i.e., delimiting a shorter extension, with numerically far numbers perceived as physically far apart, i.e., delimiting a longer extension. Since numerical flankers were irrelevant to the task, our results add to the existing evidence of an automatic mapping of numbers along the representational continuum (Dehaene et al., 1993; Fias, 2001; Fias et al., 1996; Fias et al., 2001; Girelli et al., 2000). Critically, the novelty of the current result is that automatic activation does not imply only access to absolute (Fischer, 2001) or coarse relative magnitude information (small vs. larger digit, de Hevia et al., 2006; Dehaene et al., 1993; Fias et al., 1996) but at least to a dichotomic, i.e., close-far, numerical distance. Moreover, the specific kinematic parameter we extracted from the participants performance, i.e., the peak of speed, clearly shows that numbers' influence is not limited to movement preparation but extend to movement execution, strengthening the scanty evidence in this direction (Lindemann et al., 2007; Song & Nakayama, 2008).

Overall, the experimental manipulations introduced in Experiment 6 resulted successful in inducing a representational length effect that in Experiment 3 we failed to observe. This latter result may be accounted for by many factors. On the one hand, the null effect of the representational distance in Exp 3 may simply reflect limited statistical power (please note the small size effect found in Experiment 6, i.e.,η^2_p = .12). On the other hand, despite the identical experimental paradigm, the task requirements in Experiment 3 and 6 were different. In the experiment 3, participants were required to connect two points by two different directional movements (from left to right vs. from right to left). Besides, the relative position of larger number was manipulated. It is plausible to assume that in this experimental condition, participants were likely to pay attention to the congruency between position of the flanked numbers and the mental number line representation. Critically, the most relevant information was not the numerical distance between the flankers but the position of the larger number that, likely, was simply elaborated in a coarse dichotomicway (small/large) masking any detailed numerical information needed to yield a distance effect.

Finally, compared to typical tasks where the number-space association is explored looking for compatibility effects in dichotomic responses, e.g., left-right button press (Wood et al., 2008), lateralized pointing task (Fischer, 2003; Ishihara et al., 2006; Song & Nakayama, 2008), grasping movements (Andres et al., 2004; Lindemann et al., 2007; Moretto & di Pellegrino, 2008), the present study provides direct empirical evidence for a numeric representational continuum by means of a single, unidirectional and ecological hand-movement, such as the manual connection of two points. The use of a very basic and ecological tasks for investigating high level cognitive mechanisms and representations has methodological and theoretical advantages. On the one hand, a simple motor task such as the one here reported may be effective to test children, as well as elderly subjects or neurological patients, since it requires little attentional and cognitive resources. On the other hand, the influence of symbolic irrelevant information on a very basic motor task highlights the salience of our representational world in modulating our interaction with the external world. To this regard this study, along with many others, reveals the primacy of numbers in our environment.

Chapter 7. MAPPING NUMBERS ONTO SPACE: FURTHER SUPPORT FOR THE COGNITIVE ILLUSION HYPOTHESIS

1. Introduction

Numbers and space are related to one another in ways suggestive of a mental number line. The MNL is an analogical continuum where each number was represented as variable distributions of activation along a left-to-right oriented mental number line (Dehaene, 1992). The most important evidence for a spatially oriented numbers representation comes from the SNARC (Spatial-Numerical Association of Response code) effect, an association between spatial and numerical information, whereby small numbers are associated with the left side of space and larger number with the right. For example, Dehaene et al.(1993)reported that when participants were required to make parity judgments were faster to respond to small numbers with the left hand then with the right hand and were faster to respond to larger numbers with the right hand then with the left hand. Although the SNARC effect has been primarily investigated with the parity task, the effect is not limited to this task. Indeed, the SNARC effect has also been obtained in studies where the visually presented numbers were completely irrelevant (Fias, 2001; Fias et al., 1996). This is a strong argument in favour of automatic spatial coding.

Further support for a spatial mapping of numbers along a left-to-right axis arises from recent investigation in spatial attention. The presentation of small numbers speeded subsequent detection of peripheral stimuli in the left visual field, while the presentation of larger numbers speeded detection in the right visual field (Fischer et al., 2003). Also in parity tasks with lateralized saccadic/pedal response, a SNARC-like effect with a temporal advantage was found in case of compatibility between numerical magnitude and side of response, in the same way described above (Fischer et al., 2004; Schwarz & Müller, 2006).

The SNARC effect showed that smaller and larger numbers are mapped to the left and right sides of space, but it does not specify whether this is a dichotomic or continuous mapping. According to Proctor and Cho (2006), the SNARC may be alternatively explained. Their polarity correspondence principle suggested that stimulus and response alternatives are coded as positive and negative polarity along several dimension, and polarity correspondence was sufficient to produce mapping effects. They concluded that RTs were shorter when stimulus and response attributes have the same polarity than when the attributes have different polarity (see also Nuerk et al., 2004). Contrarily, the Ishihara and study further support to continuous mapping of numbers. In a Go/No-Go parity judgement task, participants were required to point to the numerical target presented along a left-right axis. Results show that left-sided movements were initiated faster towards small digits compared to larger digits and right-sided movements were initiated faster towards large digits compared to smaller digits. Critically, this mapping seemed to occur in a continuous, rather than dichotomic way (Ishihara et al., 2006; see also Song & Nakayama, 2008).

Besides, the spatial representation of numbers was flexible and modulated by context. For example an inverted SNARC effect was induced by the imagery instruction in the Bächtold et al.'s study (1998). When participants were asked to imagine a number inside a rule before making a comparison task, parity judgement, a SNARC effect was observed. However, when participants were asked to imagine a clock face, a reversed SNARC was reported. This suggests that the mental number line representing the spatial positional codes is flexible and task-dependent. According to that, a vertical SNARC (Ito & Hatta, 2004; Schwarz & Keus, 2004) was found and the range of stimuli was able to change this representation (Dehaene et al., 1993).

Recently, the interaction between number and space has been further investigated by means of the numerical influence in visuo-spatial task such as the bisection and reproduction task (de Hevia, Girelli et al., 2008; de Hevia et al., 2006; Fischer, 2001).

In Fischer's investigation (2001), participants were required to make a bisection of strings formed by uniform digits (e.g., 11111111111) and lines delimited by different flanked numbers (e.g., 1-----2). A bias towards the lateralized spatial code activated by number forming strings was observed, i.e., towards left in case of small-magnitude numbers and towards right in case of large-magnitude numbers (for different results see Calabria & Rossetti, 2005; de Hevia et al., 2006). This finding was interpreted as the consequence of an automatic activation of a spatial response code with strong analogy with the SNARC effect.

Second, line bisection was biased toward the larger flanker number, regardless of its position. Fischer interpreted this larger number effect according to the MNL hypothesis, suggesting that the perception of two numbers automatically evokes a corresponding segment of its representation.

Relevant for this study, de Hevia and colleagues (2006) further investigated the spatial proprieties of numerical information by means of bisection paradigm. Participants were required to bisect lines and unfilled space flanked by different numbers. They replicated the large number effect. However, the numerical distance between numerical flankers did not modulate performance, even when numerical processing was explicitly required, suggesting a categorical processing of magnitude (i.e., small/large). Since the bisection task did not requirelateralized responses, these findings cannot be accounted forin terms of MNL representation. The authors proposed that numbers impact visuo-spatial representations by inducing a 'cognitive illusion' of length; according to this, processing of large-magnitude numbers brings about an illusory expansion of space and processing of small-magnitude numbers brings about an illusory compression of space (de Hevia et al., 2006).

Relevant for this investigation, a length reproduction task was adopted to further test the cognitive illusion hypothesis (de Hevia, Girelli et al., 2008). Participants were required to reproduce the length of a spatial extension delimited by two digits by extending or compressing a horizontal segment visually presented. The flanked digits were identical (Exp.1) and different (Exp.2) in order to evaluate the absolute magnitude and the numerical distance, respectively. Their finding supported the hypothesis that visuo-spatial resources are involved in the representation of numerical magnitude. According to the cognitive illusion hypothesis, results showed an underestimation of space delimited by small numbers, and an overestimation of space delimited by large numbers.

In the current study we aimed at providing further evidence corroborating the cognitive illusion hypothesis. To this aim, three experiments were performed exploiting a digitizing tablet to record the reproduction performance.

Overall, we explored, by means of a horizontal length and circle reproduction tasks, whether numerical processing modulates the mental representation of a horizontal as well as a bidimensional spatial extension.

First, we expected to replicate the interaction between physical and symbolic magnitude (i.e., numerical information) with a digitizing tablet that did not impose any spatial constraints or limit to reproduction movement. Participants reproduced the extension of the empty space delimited by two uncircled numbers. Following de Hevia et al (2008) we expected an underestimation of the length delimited by circles including small magnitude number, while an overestimation of length delimited by circle including large magnitude number.

Second, we investigated whether the numerical influence on perceived extension is limited to the horizontal axis or, more critically, to any bidimensional space. To this aim, participants were required to reproduce circles of various dimensions enclosing irrelevant digits. Indeed, we expected to find an interaction between number and space in both - unidirectional and bidirectional - extensions.

Finally, we investigated if the simultaneous presentation of numerical and spatial information is necessary for the numerical influence to act on spatial processing.

2. Experiment 7. Length reproduction of a virtual space delimited by identical numbers

2.1. Material and method

Participants. Twenty-seven right-handed (16 females and 11 males) from the University of Milano-Bicocca participated in this study as unpaid volunteers. The mean age was 23 (range 20-27). All participants had a normal or corrected-to-normal vision, and were naïve about the hypotheses of the Experiment.

Apparatus. Participants were individually tested in a quiet and dark room. They sat about 70 cm away from the screen and the centre of the digitizing tablet was aligned with the mid-saggital plane of the participant's trunk. Stimuli were presented on a white background on Samsung Sync Master 7535 Monitor (resolution of 640 x 480 pixels; 27.5° x 18.1°). The reproduction, performed with an electromagnetic pen, was registered by means of a digitizing tablet ("Wacom Intuos 2", format A3, 420 X 297 mm, frequency of 50 Hertz, accuracy 0.41 mm) allowing recording of all relevant information relative to the spatial parameters of reproduction movement.

Stimuli and Procedure. Stimuli consisted of pairs of digits '1 1', '2 2' ('small' magnitude), '8 8', '9 9', ('large' magnitude), and letters 'X X', 'Y Y' ('neutral' condition) black-printed on a white screen (640 x 480 pixels; 27.5° x 18.1°) with 32 point Courier font[12]. Each digit was 14 pixel wide, resulting in a 0.57° horizontal visual angle. Each digit occupied the centre of a circumference, whose diameter was 44 pixels long, resulting in a 1.4° visual angle. The horizontal gaps separating encircled symbols were short (139 pixels; 5.9°), medium (225 pixels; 9.6°), or long (309 pixels; 13.2°). Each pair of encircled digits was presented five times for each length for a total of 90 stimuli. The instructions requested subjects to reproduce the extension of the empty space delimited by the two circles, writing on a digitizing tablet (see Figure 1).

Trials began with a central fixation point (500 ms). After 200 ms blank, a pair of identical symbols was shown for 1000 ms (see Figure 2). Before the experimental task, 15 training trials were presented. Only during the practice session, did participants have a visual feedback of their writing. They were demanded to return to a central position after each trial. Stimuli were presented in a random fixed order. The entire experiment lasted approximately 30 min.

[1] In order to minimize the effect of symbol physical differences on the perception of virtual extensions, a font that represents each character of the same length was used. The neutral condition was different by de Hevia et al.'s study because we chose symbols like numbers (letters) without other means, symmetric and unusual in Italian language.

Fig. 1. The picture shows a participant during the reproduction task.

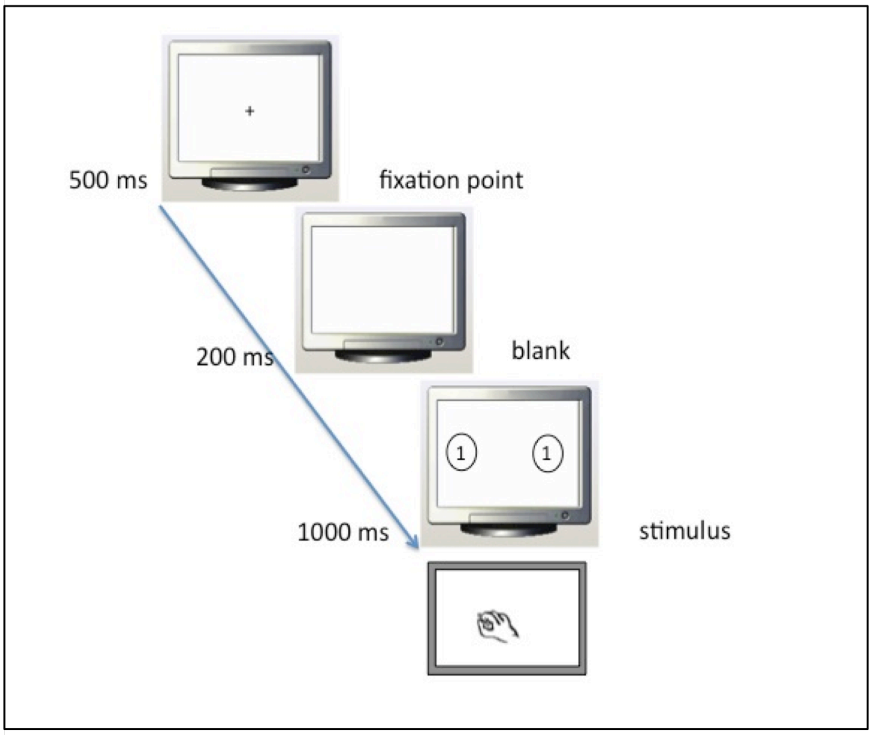

Fig. 2. Experiment 7: *Length reproduction of a virtual space delimited by identical numbers.* Examples of experimental stimuli and procedure

2.2. Data analysis

The difference between the subjects' reproduction performance and the objective extension between the two circles was calculated: positive values corresponded to overestimation, negative values to underestimation (see Figure 3). Adjustment values were submitted to a repeated-measures analysis of variance with two within-subjects factors (*numerical magnitude:* small, large, neutral condition); *gap length*: (short, medium, large).

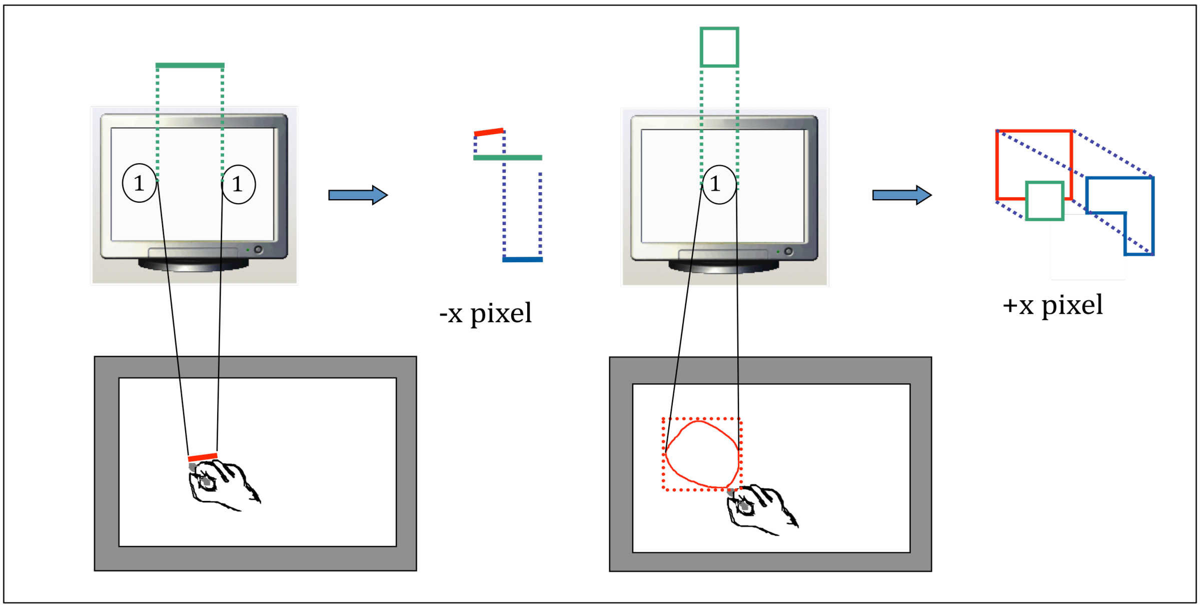

Fig. 3. Exp. 7 (on the left). The objective length of the virtual space (green line); length of the line reproduced by participants (red line). The difference between the subjects' reproduction performance and the objective extension (blue line) was calculated as observed variable.

Exp.8 (on the right). The objective area of the circle with digit (green rectangle); area of the figure reproduced by participants (red rectangle). The difference between the subjects' reproduction performance and the objective extension (blue figure).

2.3. Results

Figures 4 shows that, in the context of a global underestimation, small-magnitude numbers (mean = -13.39, SE = 5.51) produced a further underestimation, and large-magnitude (mean = -10.30, SE = 5.77) numbers produced overestimation. When the gap length increases, so does the underestimation error.

Indeed, the analysis of variance revealed significant main effects of *gap length* ($F_{2, 52}$=117.50, ε = .65, p< .001, η^2_p = .82)[13], and *numerical magnitude* ($F_{2, 52}$=5.05, p= .01, η^2_p = .16). There was no significant interaction *magnitude x gap length* ($F_{4, 104}$ <1, n.s.).

Paired t test analyses showed that performance with smaller number was significantly different from performance with larger number, t test one-tail (26) = -2.37, p = .013, and that the performance with larger numbers was significantly different from performance with neutral symbols, t (26) = 2.96, p < .01. Smaller numbers and neutral symbols did not significantly different from each other, t (26) = -.02, p = .98.

The reproduction performance of space delimited by the digits "1 1" was comparable to the reproduction of stimuli delimited by the digits "2 2" (paired t test, t (26) = - .98, p = .34). This consistency was also present both for large magnitude digits (paired t test between digits "8 8" and "9 9", t (26) = .58, p = .56) and neutral condition (paired t test, t < 1, ns).

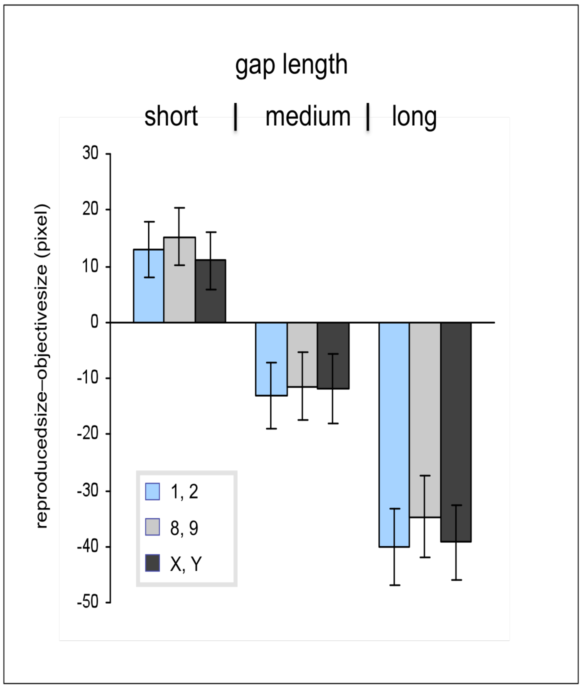

Fig. 4. Experiment 7: *Length reproduction of a virtual space delimited by identical numbers.* Mean error (SE) in the reproduction of virtual space by symbol [small-magnitude number ('1 1' and '2 2'), large-magnitude numbers ('8 8' and '9 9') and neutral condition ('X X' and 'Y Y')]; gap length (short, medium, long).

[13] Whenever necessary, the Huynh-Feldt correction for repeated measures analysis (Huynh & Feldt, 1970, 1976) was used to correct for violations of the sphericity assumption. The epsilon (ε) used to reduce the degrees of freedom, the original degrees of freedom, and the corrected *p*-values are reported.

3. Experiment 8. Reproduction of circles enclosing irrelevant digits

3.1. Material and method

Participants. The same students of Experiment 7 participated in this study.

Stimuli and Procedure. The Stimuli consisted of circles which included the digits ‘1 ’, ‘2 ’ (‘small’ magnitude), ‘8’, ‘ 9’, (‘large’ magnitude), letter X, Y (‘neutral’ condition) and empty circles (‘empty neutral condition’)[14] blank-printed on a white screen (640 x 480 pixels; 27.5° x 18.1°) with 32 point Courier font. Each digit was 14 pixels wide, resulting in a 0.57° horizontal visual angle, at a viewing distance of 70 cm. Each digit occupied ever the centre of the circumference, whose diameter was short (55 pixels; 2.4°), medium (83 pixels; 3.6°), or long (111 pixels, 4.8°). Each digit, letter and empty circle was presented five times for each diameter for a total of 105 stimuli. The dimension of numbers and letters was a proportion of 1/3 on circle. The instructions requested subjects to reproduce the circle matching as much as possible the dimension of the model.

Trials began with presentation in a central position of number, letter or * for empty circle (1000 ms). After stimuli were shown for 2000 ms. Inter-trial interval was 1000 ms. Before the experimental task, 21 training trials were presented, all empty circles. Only during the practical session participants have feedback on screen on writing. They were demanded of to return to a central position after each trial. Stimuli were presented in a random fixed order. The entire experiment lasted approximately 30 min.

3.2. Data analysis

The difference between the subjects’ reproduction performance and the objective area (area of rectangular included the circle) was calculated: positive values corresponded to overestimation, negative values to underestimation (see Figure 3). Adjustment values were submitted to a repeated-measures analysis of variance with two within-subjects factors (*magnitude:* small, large, neutral; *circle size:* short, medium, large).

[14] In this experiment we added a second neutral condition to control the perceptive effect due to different reproduction of empty or not empty circle. The raison of choise of symbol X and Y was the same as Experiment 7.

3.3. Results

Figure 5 shows that, in the context of a global underestimation, small-magnitude numbers (mean = -1377.59, SE = 561.51) produced a larger underestimation, and large-magnitude numbers (mean = -1070.18, SE = 612.13) reduced the underestimation. When the diameter length increases, so does the underestimation error.

The analysis of variance revealed significant main effects of *magnitude* ($F_{2, 52}$=3.37, $p < .05$, η^2_p = .11), and of *circle size* ($F_{2, 52}$=26.98, ε = .54, p<.001, η^2_p= .51). There was no significant interaction *magnitude x circle size* ($F_{4, 104}$ =1.35, ε = .74, n.s.). Paired t test analyses showed that the performance with smaller number was significantly different from performance with larger numbers, t test one-tail (26) = -2.61, p = .015, and that the performance with larger numbers was marginally different from performance with neutral symbols (X and Y), t (26) = 1.79, p = .09. Smaller numbers and neutral symbols did not significantly differ from each other, t(26) = -.87, p = .39. The performance with empty circles ('empty neutral condition') did not differ from all conditions (all p > .47).

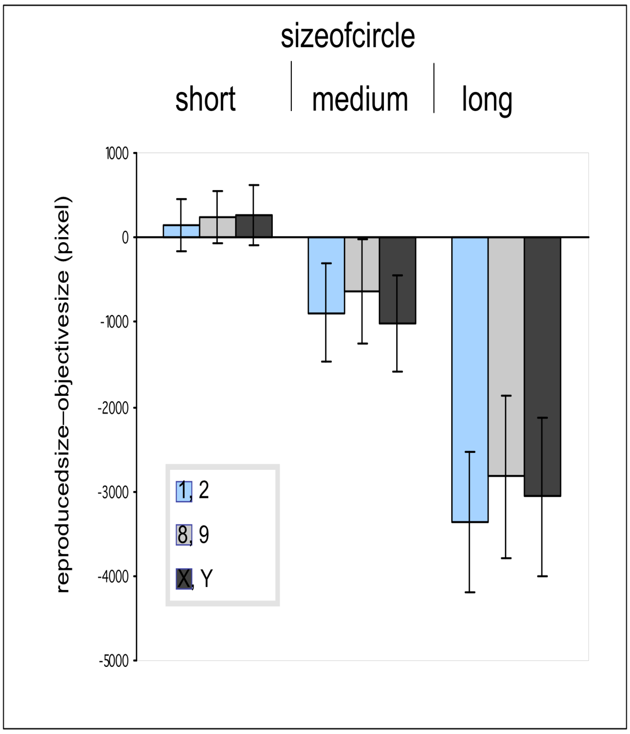

Fig. 5. Experiment 8: *Reproduction of circles enclosing irrelevant digits.* Mean error (SE) in the reproduction of circle, which included the digits, letters and empty [small-magnitude number ('1' and '2'), large-magnitude numbers ('8' and '9'), neutral condition ('X', 'Y', empty circle)]; dimensions circle (short, medium, large).

4. Discussion Exp 7 & 8

Overall, in both experiments the objective extension (gap length and circle area) modulated the reproduction of performance. Indeed, when the physical length/diameter increases, so does the overall underestimation.

More relevant for this investigation, we replicated and extended de Hevia and colleagues' findings (2008): the objective space was underestimated with smaller numbers, while overestimation took place whit larger numbers. This result suggests that numerical information was semantically and automatically processed.

Besides, the numerical effect emerged not only in the reproduction of horizontal extensions but also in the reproduction of circles enclosing irrelevant digits. This suggests that numbers modulated reproduction in the horizontal extension but also in bidimensional directions suggesting that numbers are also mapped onto a non-linear space.

5. Experiment 9. Circle reproduction with sequential numerical and spatial information

5.1. Material and method

Participants. Twenty-six right-handed (17 females and 9 males) from the University of Milano-Bicocca participated in this study. The mean age was 23 (range 18-38). All participants had a normal or corrected-to-normal vision, and were naïve about the hypotheses of the Experiment.

Stimuli and Procedure. Stimuli consisted of numbers or letters presented in central position and circles empty presented after it. The digits was '1 ', '2 ' ('small' magnitude), '8', ' 9', ('large' magnitude), letter X, Y ('neutral' condition) blank-printed on a white screen (1024 x 768 pixels) with 32 point Courier font. Each digit was 20 pixels wide, resulting in a 0.31° horizontal visual angle, at a viewing distance of 70 cm. Each number and letter was presented ten times for 3 different circle dimensions for a total of 180 stimuli. The small circle had a diameter of 3 cm (55 pixels), the medium circle with a diameter of 4,5 cm (83 pixels) and the large circle with a diameter of 6 cm (111 pixels). The dimension of numbers and letters was a proportion of 1/3 on the circle. The instructions requested subjects to reproduce the circle matching as much as possible the dimensions of the model.

Trials began with a central fixation point (500 ms). After the digits were presented for 1500 ms, the empty circles were shown for 1500 ms. Before the experimental task, 21 training trials were presented. During the practical session, participants had feedback on screen on writing. The participants were required to return to a central position after each trial. Stimuli were presented in a random fixed order. The entire experiment lasted approximately 30 min.

The difference between the subjects' reproduction performance and the objective area (area of rectangle included the circle) was calculated: positive values corresponded to overestimation, negative values to underestimation.

5.2. Data analysis

Adjustment values were submitted to a repeated-measures analysis of variance with two within-subjects factors (*magnitude*: small, large, neutral; *dimension* circle: short, medium, large) and a further t-test analyse for factor *number*.

5.3. Results

Figure 6 shows that, in the context of a global underestimation, small-magnitude numbers (mean = -4498.97, SE = 904.15) produced a larger underestimation, and large-magnitude numbers (mean = -4207.72, SE = 936.12) reduced underestimation. When the diameter length increases, so does the underestimation error.

The analysis of variance revealed a nearly significant main effect of *magnitude* ($F_{2, 50}$=3.36, p = .06, η^2_p= .11), and a significant main effect of *circle size* ($F_{2, 50}$=79.88, ε = .60, p< .001, η^2_p = .76). There was no significant interaction *magnitude x circle size* ($F_{4, 100}<1$, n.s.). Paired t test analyses showed that the performance with smaller number was significantly different from performance with larger numbers, t test one-tail (25) = 2.24, p = .03, and that the performance with larger numbers was different from performance with neutral symbols (X and Y), t(25) = 2.44, p = .04. Smaller numbers and neutral symbols did not significantly differ from each other, t(25) = .71, p = .96.

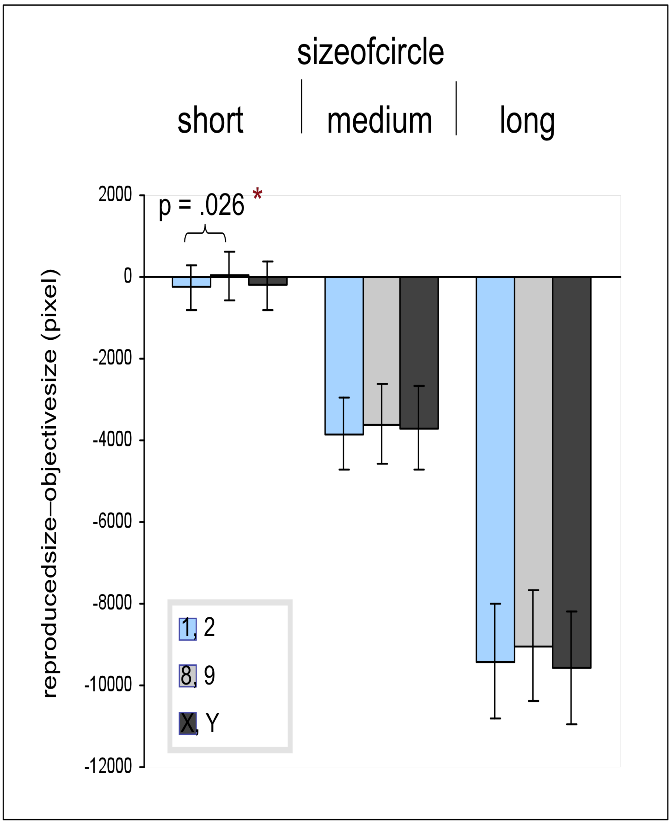

Fig. 6. Experiment 9: *Circle reproduction with presenting numerical and spatial information in sequence.* Mean error (SE) in the reproduction of circle, which included the digits and letters [small-magnitude number ('1' and '2 2'), large-magnitude numbers ('8' and '9'), neutral condition ('X', 'Y')]; dimensions circle (short, medium, large). The only significant planned comparison between small and large magnitude number was reported with relative p-value.

*t test one tailed

6. General discussion

The current study aimed at investigating the numerical influence on the representation of perceived extensions by means of a reproduction task.

In the context of a global underestimation, small-magnitude numbers produced a larger underestimation, and large-magnitude numbers produced overestimation. Underestimation errors increased as a function of virtual space length (exp. 7) and of circle size (exp. 8 and 9). The present results further provide support to the hypothesis that numbers induce a cognitive illusion of length, according to which the elaboration of magnitude information brings about an expansion or compression of the mental representation of horizontal and bidimensional spatial extensions.

The present results are consistent with the view that the processing of a number involves its conversion to an analogue quantity code. This representation may be internally represented in the same way as other continuous dimension and elaborated by shared mechanism ("neural recycling hypothesis", Dehaene, 2005) or contribute to a general representation of magnitude (Walsh, 2003) or, alternatively, be separately represented but supported by partial overlapping neural network in the parietal lobes (Pinel et al., 2004).

According to the “neural recycling hypothesis” it can be concluded that the cognitive and neural systems mediating the magnitude representation are largely co-extensive with those mediating other types of quantitative judgements, thereby calling into question the notion of a domain-specific system devoted to numerical processing. The “neural recycling hypothesis” (Dehaene, 2005) proposed that the evolution of numbers was supported by neuronal network originally devoted to specific pre-existing cognitive systems reconverted later in the mathematical system. Both hypotheses are in line with the evidence of a interaction between numerical and physical size at the representational levels, as reported in Stroop-like task (Girelli et al., 2000; Henik & Tzelgov, 1982).

The cognitive illusion, as an instance of the many interactions between number and space, cannot be working only at the representational level but appears to extend to the perceptual level. So far, this issue has been rarely investigated. For example, Badets and colleagues (2007) reported that the magnitude/affordance interaction was not due to a simple perceptual effect. The current study contributes to clarify this point testing the sequentially presentation of numerical and spatial information. The sequential presentation of circles and numbers did not or only marginally modulate the interaction between these magnitudes (i.e., number and space). The results showed a nearly significant main effect of numerical information according to the cognitive illusion suggesting that this effect is not only limited to the perceptual level but involved the representational level. To support that, I will discuss the following two points.

First, despite the p-value, the effect of the numerical magnitude shows overlapping effect size in all experiment. Second, in the last experiment, the planned comparison[15] revealed that the only significant difference between small and large magnitude number is reported with the smaller circles (see Figure 6). It seems that the numerical magnitude modulated the reproduction performance just in case of increased accuracy, when the relevance of the circle size is less. It seems that only when the strongly perceptive effect of underestimation of largest size did not work, this pick out and allow the numbers/space integration.

To sum up, the evidence here reported provides further support for a fundamental relationship between the numerical and the spatial domains. The effect that numbers exert on the representation of a spatial extent, as observed in the present experiments, may indicate that spatial resources are involved in the representation of numerical magnitude. The strictly unidimensional concepts such as a number line might be an incomplete description of how we represent numerical magnitude and should be replaced with higher dimensional concepts, such as an internal number map.

[15] The Bonferroni correction was applied to all planned comparison

Chapter 8. CONCLUSION

The current investigation contributes to our understanding of the interaction between numerical information and motor action by developing original experimental paradigms that proved useful to detect the modulation of number processing on motor planning and execution.

Overall the findings provide support to the view that the processing of magnitude information exerts its influence on action planning and motor execution (cf. Andres et al., 2008; Fischer & Miller, 2008).

We found a significant representational length effect in the manual connection of two points. The impact of the numerical distance between flankers on a simple unidirectional hand movement strictly mimics the influence that a perceptual physical length exerts on motion: the peak of speed of the junction movement increases as a function of either the physical or the numerical covered length. This representational length effect strongly suggests that the numerical spatial continuum, conceived as a mental number line (Dehaene, 1992), partially holds psychometric features similar to a perceptual linear extension. Although analogies between the representational and the physical space have been previously reported (de Hevia & Spelke, 2010; Zorzi et al., 2002) this evidence shows that activation of a symbolic distance at the representational level modulates directly a motor act as if the physical extension defining the movement amplitude was misperceived. In other words, the effect of the irrelevant flankers on the kinematic of the junction movement shows that numerically close numbers were perceived physically closer, i.e., delimiting a shorter extension, with numerically far numbers perceived as physically far apart, i.e., delimiting a longer extension. Since numerical flankers were irrelevant to the task, our results add to the existing evidence of an automatic mapping of numbers along the representational continuum (Dehaene et al., 1993; Fias, 2001; Fias et al., 1996; Fias et al., 2001; Girelli et al., 2000). Critically, the novelty of the current result is that automatic activation does not imply only access to absolute (Fischer, 2001) or coarse relative magnitude information (small vs. larger digit, de Hevia et al., 2006; Dehaene et al., 1993; Fias et al., 1996) but at least to a dichotomic, i.e., close-far, numerical distance. Moreover, the specific kinematic parameter we extracted from the participants performance, i.e., the peak of speed, clearly shows that numbers' influence is not limited to movement preparation but extend to movement execution, strengthening the scanty evidence in this direction (Lindemann et al., 2007; Song & Nakayama, 2008).

The present results are consistent with the view that the processing of a number involves its conversion to an analogue quantity code. This representation may be internally represented in the same way as other continuous dimension and elaborated by shared mechanism ("neural recycling hypothesis", Dehaene, 2005) or contribute to a general representation of magnitude (Walsh, 2003) or, alternatively, be separately represented but supported by partial overlapping neural network in the parietal lobes (Pinel et al., 2004).

According to the "neural recycling hypothesis" it can be concluded that the cognitive and neural systems mediating the magnitude representation are largely co-extensive with those mediating other types of quantitative judgements, thereby calling into question the notion of a domain-specific system devoted to numerical processing. The "neural recycling hypothesis" (Dehaene, 2005) proposed that the evolution of numbers was supported by neuronal network originally devoted to specific pre-existing cognitive systems reconverted later in the mathematical system. Both hypotheses are in line with the evidence of a interaction between numerical and physical size at the representational levels, as reported in Stroop-like tasks (Girelli et al., 2000; Henik & Tzelgov, 1982).

Furthermore, this example of cross magnitude interaction (i.e., the representational length effect in manual connection of points) suggests that quantitative judgments of continuous dimensions imply a common mental code for quantitative representation and/or a common mental comparison process for judging their magnitude. The hypothesis of a generalized magnitude system for representing number and non-numerical quantities has been recently proposed by Walsh (2003). By summarising the behavioural and neurobiological evidence that time, space and number share common processing mechanisms, the author proposed that these dimensions may be associated to guide goal-directed action. In fact, to this end, the spatial location of an object is calculated crucially thanks to quantitative computations such as 'how far', 'how many' and 'how long'. Accordingly, mental magnitudes would be linked to one another through the computational demands of the motor control system with the parietal lobes providing neural support for their integration (Walsh 2003).

A further related question addressed in this dissertation was to clarify at which level the magnitude information exert its influence on spatial processing, that is at the perceptual or the response levels (cf. Badets, Andres et al., 2007).

To this aim we exploit the cognitive illusion induced by numbers in visuo-spatial tasks, i.e., a mis-estimation of the perceived space, as an example of the number/space interaction. Accordingly, in last study it has been show not only that numbers induced underestimation or overestimation of an horizontal extensions as well as of a non linear-space, but that this may occur both at the perceptual and at the representation levels, since mis-estimation occurs even if when two symbolic and physical magnitudes were not - simultaneous presented.

However, although promising, only future studies may definitely establish to what extent numerical information influence spatial processing at the perceptual or at the representational level and to clarify role of the sensorimotor experience in modulating the interaction between number and action. Recently, the "sensorimotor" theories of cognition has suggested that the motor system contributes to high-level cognitive processes, such as the numerical representation (for recent review see, Andres et al., 2008). To date, the interactions between the motor system and the cognitive process have been mainly described in terms of the semantic influence onto sensory-motor process. However, the functional link should be reflected also in the other direction, thus showing the influence of the sensory-motor system to semantic (Badets & Pesenti, 2010).

The results from second study (chapter 6 – part one) may be interpreted within this theoretical framework. In particular, the interaction of movement direction with the position of the numerically larger flankers shows that when the connection movement required was directed form left to right, maximum velocity of the connection movement was higher for trials with the larger of the two flankers on the right side than trials with the large numerical flanker on the left side. In contrast, when the requested movement went from right to left, maximum velocity was higher for trials with the larger flanker on the left side as compared to trials with the larger flanker on the right. This study extends previous evidence on the relative flexibility of the spatial representation of numbers (Bächtold et al., 1998; Dehaene et al., 1993) emphasizing the importance of the context for an interaction between numbers and space. So far, this flexibility has been mainly demonstrated by the modulation of the SNARC effect yielded by lateralized responses in different numerical task. While the SNARC effect may be determined by the representational context, our result indicate that the interaction of number magnitude and action is modulated by the motor context, i.e., movements from smaller to larger flankers (and thus following the ascending order of the MNL) were facilitated (higher v_{max}) as compared to movements from the larger to the smaller flankers irrespective of the conventional reading / writing direction.

To conclude, the present results are consistent with the view that the processing of a number involves its conversion to an analogue quantity code. The involvement of the intra-parietal sulcus in supporting the visuo-spatial representation of numbers is well-documented and universally accepted (Dehaene et al., 2003; Hubbard et al., 2005). The present investigation provide further behavioural evidence pointing to the parietal cortex as the crucial brain district where multi-dimensional magnitude information is integrated to subserve action (for a review, Bueti & Walsh, 2009).

Acknowledgements

Chi è maestro nell'arte di vivere distingue poco
fra il suo lavoro e il suo tempo libero,
fra la sua mente e il suo corpo,
la sua educazione e la sua ricreazione,
il suo amore e la sua religione.
Con difficoltà sa cos'è cosa.
Persegue semplicemente la sua visione dell'eccellenza
in qualunque cosa egli faccia,
lasciando agli altri decidere se stia lavorando o giocando.
Lui pensa sempre di fare entrambe le cose insieme.
(Zen)

A master in the art of living cannot make a clear distinction
between work and free time,
between mind and body,
education and recreation,
love and religion.
It is difficult to now what is what.
They simply pursue their vision of excellence
in whatever they do,
leaving others to decide whether they are working or playing.
They always believe they are doing both together.
(Zen)

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I hope no to forget anybody because I'm printing now!

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