DEPARTMENT OF ECONOMICS

UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

# Delegation Games with Full Commitment

Paolo Balduzzi

No. 70 - April 2004

Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
http://dipeco.economia.unimib.it

# Delegation Games with Full Commitment*

Paolo Balduzzi†
Edinburgh School of Economics
University of Milan-Bicocca

April 2004

**Abstract**

In this paper we present and solve some bargaining games a la Rubinstein, where the subjects can delegate the negotiating process to agents. Delegation is a possible commitment tactic. Its aim is to provide the delegating party with a higher bargaining power. When both the parties delegate, uncertainty arises about the final distribution of the payoffs and multiple equilibria are possible. The seller loses his usual first mover's advantage. When we allow for delegation costs, the range of multiple equilibria shrinks. The final outcome of the game may be now inefficient for the principals and a prisoners' dilemma may arise.

JEL classification: C72; C78

*This paper is part of my Ph.D. research. I thank Jozsef Sakovics, John H. Moore and Jonathan Thomas for their suggestions and supervision. I also thank seminar participants at the SGP Peeble Conference 2003, Edinburgh, Milan-Bicocca and Scottish Economic Society Conference 2004 for their comments. All mistakes are my responsability.

†Contacts: paolo.balduzzi@ed.ac.uk; +44.(0)131.6506955.

# 1 Introduction

Bargaining games have found lots of applications in economic theory. In particular, economic and political behaviours are often modelled as alternating offers situations. Moreover, in real life, we observe that most of bargaining games are played by agents acting for other people. Principals often delegate the bargaining process to agents for different reasons (saving time, higher search ability, better skills, etc.). For instance, house owners delegate the search of tenants to estate agents, savers usually leave their money administration to professional traders, a premier delegates some of his duties to his ministers, many football players, singers, TV showmen hire agents.

One of the main questions in negotiation models is about the source of bargaining power and, above all, about the devices to increase it. Schelling (*The strategy of conflict*, 1960) was probably the first author emphasizing the role of delegation as a strategic device. In particular, he dealt with delegation as a commitment tactic, through which a player could bind himself to a strategy and force the other to believe his threats. Subsequent studies have formalized the intuition that one-sided delegation is very effective.

In this paper we present some models of bargaining with bilateral delegation. The aim of this work is to present, define, solve and comment different delegation games among the following four subjects: a seller, a buyer, the seller's intermediary, and finally the buyer's intermediary. Our model is developed in the well-known alternating offers framework a la Rubinstein (1982). The structure of the game implies that the agents' profits depend upon both principals' proposals. The first consequence is that the initial Rubinstein game among principals becomes a typical Nash demand game. We show that, when delegation is costless, principals always delegate, either as a dominant strategy or as a Nash reply. Multiple equilibria arise and one of the two parties is always better off. This means that bilateral delegation can provide effective gains to at most only one of the two principals (the "winner" of the game). As regards the agents, they are always paid their reservation wage. Assuming that the seller has the usual first mover advantage, delegation is more likely to be profitable to the buyer. Indeed, he has less to lose by switching from the direct bargaining game to the delegation one.

These results are no longer true when we allow for costs, being them exogenous or endogenous. These costs reduce the range of possible equilibria arising. In some of these ranges, the delegation game can be characterized as a prisoner's dilemma, that is, the principals may decide to delegate even if they both lose with respect to a direct bargaining situation. The presence of costs also reduces the set of equilibria emerging. When delegation is costly for the agents, they are both paid at least their reservation wage. Nevertheless, I may have a strategic first mover advantage and he is able to gain more than J.

The paper is organized as follows. After a review of the literature on this topic (section 2), in section 3 we develop a baseline model of two-sided delegated bargaining without renegotiation. Then, we enrich the model by introducing exogenous delegation costs (section 4) and endogenous opportunities (section

5). The players' best strategies are shown to depend on the level of these costs and opportunities. Finally, in section 6 we discuss the limits of our model, suggest some possible developments and draw our conclusions.

## 2 Review of the literature

The study of bargaining has been developed through two main formal approaches: an axiomatic one and a strategic one. The former is based on the idea that a bargaining solution should satisfy some reasonable conditions (axioms). Its first development is due to Nash (1950; 1953). The latter relies on the fact that outcomes are the results of interdependent players' choices (Nash, 1951). The widest used framework in this case is due to Rubinstein (1982), who exploits the idea that bargaining is actually a process of alternating offers. Elements like the degree of impatience influence the final outcome of the game. Very good expositions of these models and of their limits are provided by Sutton (1986) and Osborne and Rubinstein (1990).

One of the main question in negotiation models is about the source of bargaining power and, above all, about the devices to increase it. Schelling (1956, 1960) is probably the first author emphasizing the role of delegation as a strategic device. In particular, he deals with delegation as a commitment tactic, through which a player can bind himself to a strategy and force the other to believe his threats. As he pointed out:

> *"if the buyer can accept an irrevocable commitment, in a way that is unambiguously visible to the seller, he can squeeze the range of indeterminacy down to the point most favorable to him"(Schelling, 1956; p. 283).*

The effectiveness of this tactic is clearly linked to its *irrevocability, credibility* and *transparency*: if a delegation contract can be renegotiated, if an intermediary can be convinced that he will be better off by accepting a different proposal, or if the characteristics of the delegation contract cannot be effectively communicated to the other party, then delegation turns out to be much less powerful than expected.

Subsequent research has formalized this simple but strong idea in different ways. As in the rest of the literature about bargaining, contributions follow either an axiomatic or a strategic approach. As explained in the introduction, our model is developed in an alternating offers framework.

The literature in this field is not very wide. Table 1 summarizes it and the rest of the section is devoted to its exposition. Our approach is to discriminate the different contributions according to the following characteristics: degree of the commitment, framework, number of agents, level of information and application to particular problems. A few of less relevant (for our purposes) papers are also illustrated at the end of this section.

Jones (1989) and Burtraw (1992) model two-sided delegation games in an axiomatic framework. These papers are both based on the idea that the way the subjects can increase their bargaining power is through misrepresentation of their preferences. The former develops a game, where players (principals) bargain over the division of two private goods and only differ in their individual tastes. They both can hire agents to bargain on their behalf, and the choice of an agent corresponds to the choice of his individual taste parameters. Principals non-cooperatively choose the type of their agents. The agents' bargaining process is solved by a Nash solution. Two kinds of equilibria emerge. The first is a self representation one (all the subjects have the same individual taste parameter), where no gains from delegation arise. The second is given by the general case when principals differ in tastes. Conclusions about the welfare properties of this latter equilibrium are similar to ours: situations with inefficient outcomes arise (both parties lose; i.e.: a prisoner's dilemma) along with situations where only one party is better off. As long as delegation is costless, inefficiency never emerges in our model.

Burtraw (1992) links the choice of an intermediary to risk aversion considerations, which is the most natural way to misrepresent one party's preferences. Delegating to a risk-neutral agent is undoubtedly effective (as well as, we can reasonably guess, delegating to less impatient agents in a Rubinstein framework). This idea was not actually new but had been developed only in one-side delegation models. His contribution differs from Jones' one to the extent that the former explicitly relies on risk-aversion differences among players. As in our model, multiple equilibria arise. Unfortunately, the author is not interested in the welfare implications of his results.

| Table 1: Review of the literature on bargaining and delegation | |
|---|---|
| **Topic** | **Author** |
| 1) Commitment and bargaining: | |
| a) Full commitment | Schelling (1956; 1960) |
| b) Partial commitment | Muthoo (1996) |
| 2) Delegation: | |
| a) In an axiomatic framework | Jones (1989) |
| | Burtraw (1992) |
| $b_1$) In a strategic framework | Fershtman, Judd and Kalai (1991) |
| | Muthoo (1999) |
| | Polo and Tedeschi (2000) |
| $b_2$) In a Rubinstein framework | Bester and Sakovics (2001) |
| 3) Delegation and renegotiation: | |
| a) Bilateral delegation | Polo and Tedeschi (2000) |
| | Muthoo (1999) |
| b) One-side delegation | Bester and Sakovics (2001) |
| 4) Incomplete information | Katz (1991) |
| | Fershtman and Kalai (1997) |
| | Corts and Neher (2001) |
| 5) Applications | Vickers (1985) |
| | Fershtman and Judd (1987) |

Papers developing general models (two-sided delegation, general utility functions and compensation schemes) in a non-cooperative framework are provided by Fershtman, Judd and Kalai (1991), Polo and Tedeschi (2000) and Muthoo (1996; 1999).

Fershtman, Judd and Kalai (1991) show that strategic delegation introduces some cooperative elements in a typical non-cooperative game. They do not specify any particular form for the players' utility functions or compensation schemes (but they stress the importance of weak monotonicity for the latter) and show that principals may coordinate through their agents to obtain efficient outcomes in equilibrium. In our model, each agent gains at least his reservation price (or exactly his reservation price in the baseline model) as a result of individual (and not collusive) rationality constraints. Moreover, delegating may emerge as Nash equilibrium but never allows for a Pareto improvement in the principals' outcome.

Polo and Tedeschi (2000) generalize Fershtman, Judd and Kalai (1991) and consider non separable utilities. As a result, they obtain multiple equilibria, where all the individually rational allocations are in the set of equilibria (that is, not only the Pareto efficient ones). This multiplicity is ruled out by relaxing the assumption of full commitment (i.e., allowing for renegotiation). We obtain a multiplicity problem as well. The range of possible equilibria dramatically shrinks when we introduce costs.

Muthoo (1996) introduces costly revocable commitments, showing that higher revocation costs lead to a stronger bargaining position. In this model, he does not explain how commitment is reached. This drawback is overcome by Muthoo (1999), where the author extends his previous general model to delegation. Both the principals can hire a negotiator, who is characterized by a positive number $k$. This number influences both the cost of revoking the partial commitment and the agent's wage. Principals choose $k$ maximizing their own payoff functions. According to different shapes of the agents' wage function, we can observe either equilibria without delegation or equilibria with delegation. In the latter case, the unique Nash Equilibrium is Pareto inefficient. As far as this model allows for renegotiation, it is not directly comparable to ours.

These latest models analyze the delegation game as a strategic one. Nevertheless, they fail to explicitly consider the time-consuming process of negotiations (and possibly of renegotiation). Bester and Sakovics (2001) have developed a delegated bargaining game in an alternating offer framework. Our model is different to the extent that we allow for two-sided delegation (instead of one-sided) but not for renegotiation. A first result by Bester and Sakovics is that, when full commitment is possible (as in a Schelling-type situation), the delegating player (specifically, the seller) can gain the full pie, whereas his intermediary and the buyer cannot have anything. This conclusion is strictly based on the particular compensation scheme used, that is a fixed payment from I to S whenever trade occurs. In the same framework, we show that adding the possibility of two-sided delegation turns the initial situation into a typical Nash demand game. Their main result is that renegotiation does not fully wipe out the positive effects of delegation: this is still profitable when the cost of renegotiation is high and the one of delegation is low. These costs are explicitly represented by the time which is consumed to reach an agreement between parties.

An interesting feature of delegating games is that, by allowing for the presence of additional players, inefficiency may arise. This inefficiency can be realized in two ways: lower total profits and possibility of disagreement. Only the first type of inefficiency is usually discussed. The possibility of disagreements, or stalemates, has been analyzed by Crawford (1982) and it is based on the idea that irrevocability (of commitments) and uncertainty (of the principal's choice) matter. Even if the possibility of impasse in our model is not ruled out, this result never emerges in equilibrium.

Despite most of the contributions focus on generic models, strategic delegation has found natural application in oligopoly sequential games, as, for example, in Vickers (1985) and Fershtman and Judd (1987). The first shows that, in a typical "predation game", delegating to aggressive managers (maximizing market share) provides more profitable outcomes for the incumbent. A similar argument is discussed in Fershtman and Judd (1987): they show that, in oligopolies, the owner has an incentive to distort his management utility function from profit-maximization.

As already highlighted, delegation is meant to be effective as long as contracts are observable. Katz (1991) confirms this point of view, showing that most of the strategic advantages given by delegation may be lost with incomplete information. Corts and Neher (2001) challenge this result, pointing out that, despite incompleteness, Schelling's intuition is still correct with multilateral delegation and decentralized ownership. Also according to Fershtman and Kalai (1997) there are still benefits from delegation. These benefits depend on the type of delegation and the probability of observability. In our model we assume complete information to keep the analysis as simple as possible.

Other contributions are not directly related to this paper. Segendorff (1998a, 1998b) develops delegation games for the provision of a public good. Two populations (nations) delegate to agents (politicians) the negotiation process about the provision of a public and a private good. The direct bargaining situation (called autarchy) leads to an inefficiently small amount of the public one. This benchmark is compared with two delegation games, which differ according to the level (weak or strong) of authority given to the agents (i.e., the institutional set). A weak delegation game always provides a Pareto outcome whereas a strong one makes at least one of the two parties worse off. The latter result also emerges when delegation of power is determined endogenously. Rubinstein and Wolinsky (1987) deal with the presence of agents, but they do not present them as a strategic device of the principals. Gains in this model are obtained as the intermediary reduces the search costs, expressed in terms of time consumption. Finally, some experimental evidence about outcomes from delegation games is provided by Schotter, Zheng and Snyder (2000). In an attempt to explain some real world inconsistencies of face-to-face bargaining behavior, as compared to laboratory experiments, they argue that a possible explanation may be the existence of intermediaries. By allowing for agents in their experiments, they can show how inefficiency arises (and how it is worsened by introducing for renegotiation) in the form of a longer time to find an agreement and higher opportunity costs.

## 3 A simple model of delegated bargaining

In this section, we develop a baseline case, where the subjects bargain about the sale of an indivisible good. Both principals can hire an intermediary: I for the seller S and J for the buyer B. The valuation of the good is 0 to S, I and J and 1 to B. Every player has the same discount rate $\delta$. Finally, we assume there is no possibility of renegotiation between a party and his intermediary, that is, delegation acts as full commitment. Bester and Sakovics (2001) have already shown that, when only one party can fully commit himself (specifically, S), he can gain the entire pie by signing a contract with I such that S will be paid a fixed amount $f$ whenever trade will occur between I and B[1].

When both S and B can fully commit themselves to a strategy by hiring an intermediary, we expect delegation to be less effective. We are interested in understanding when, why and how, bilateral delegation is anyhow sustainable as equilibrium strategy.

[1]It is easy to show that the same is true when only B can delegate: he will get the entire pie, whereas S and I will gain nothing.

In this first version of the model, delegation is completely costless. To make the exposition as clear as possible, we distinguish three stages:

(1) *The delegation stage*: in the first stage, the principals decide whether to delegate or not. If a principal decides to delegate, he makes a take-it-or-leave-it offer to his own agent about the level of a fixed compensation scheme ($f_S$ or $f_B$, with $f_i \in [0,1]$, $i = B, S$)[2]. If he decides to do not delegate, he does not hire any agent. These decisions (hiring an agent and level of offers) are not made public until the bargaining stage begins.

(2) *The agents' stage*: each agent decides whether to accept or not his principal's offer on the basis of his own reservation wage and outside options. We assume in this section that there are no outside options and that, once an agent makes his decision (accept or reject), he is then committed to it[3]. As delegation is not costly, an agent's rejection and a principal's choice of not delegating have the same effect on the final outcome.

(3) *The bargaining stage:* in this stage, all the information is made public (specifically, presence of intermediaries, levels of $f_S$ and $f_B$). The parties bargain over the price of the good according to an alternating offers model a la Rubinstein. A priori, there are three possible bargaining situations: bilateral delegation (I with J), one-side delegation (I with B or S with J), direct bargaining (S with B). The selling party (S or I) always makes the first offer.

Compensation schemes and profit functions in case of bilateral delegation are set as follows:

- I pays $f_S$ to S whenever trade occurs and gains the selling price of the good;
- S gains $f_S$ whenever trade occurs;
- B pays $f_B$ to J whenever trade occurs and gains the value of the good (which is equal to 1);
- J pays the selling price to I and gains $f_B$ whenever trade occurs,

where $f_S$ and $f_B$ are the fixed amounts decided in the delegation stage and are not renegotiable.

The agents' incentive to find an agreement in the bargaining stage is embodied by the assumption that their payoffs depend on the actual realization of the trade.

[2]In lemma 1, we demonstrate the optimality of fixed compensation schemes.

[3]This means that if an agent realizes *ex-post* that the compensation scheme he accepted would provide him a negative payoff, in the bargaining stage no acceptable offer will be made and no agreement will be reached. This perpetual disagreement (impasse) will imply no gains for all the players.

The extensive form of this game is presented in Figure 1. As illustrated above, the players' strategies give rise to four possible cases: I) bilateral delegation $(D, D)$ when both S and B delegate *and* both the agents accept; II) and III) one-side delegation $(D, ND)$ and $(ND, D)$ when only one principal delegates *or* only one agent accepts; IV) direct bargaining $(ND, ND)$ when both the principals do not delegate *or* both the agent reject the offer[4].

Figure 1: The delegation game

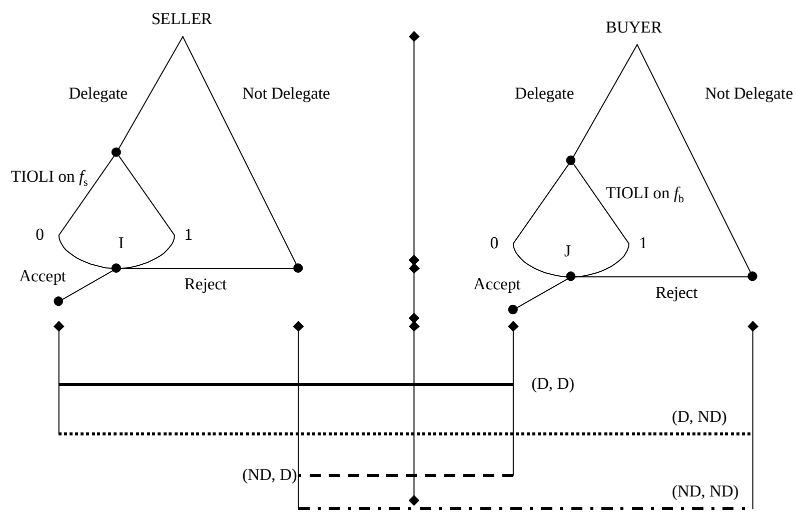



Table (1) summarizes the principals' payoff matrix at the initial delegation stage:

| Seller - Buyer | D | ND |
|---|---|---|
| D | $f_S; 1 - f_B$ | $f_S; \frac{\delta}{1+\delta}(1 - f_S)$ |
| ND | $\frac{f_B}{1+\delta}; 1 - f_B$ | $\frac{1}{1+\delta}; \frac{\delta}{1+\delta}$ |

(1)

When both the principals play $ND$, they gain the usual payoffs from direct bargaining. We call this situation the *default* one. When only one party delegates, the payoffs can be obtained following Bester and Sakovics (2001). The payoffs from bilateral delegation can be worked out by the subsequent stages. Before taking any decision, the principals must therefore look at the consequences of two-sided delegation in stages (2) and (3).

***The bargaining stage.*** The agents' choice in stage (2) is made on the basis of their possible gains in stage (3). If they reject, they gain their reservation wage, that is 0; if just one of them accepts and the other rejects, the latter gain the reservation wage and the former the gains according to Bester and

[4]For sake of completeness, $(ND, ND)$ can be a case also when only one principal decides to delegate but his offer is rejected.

Sakovics (2001). Finally, they can both accept. In this case, assuming stationary preferences, in each sub-period the bargaining parties (I and J) face the following situation:

$$\begin{cases} p_J - f_S = \delta \left( p_I - f_S \right) \\ -p_I + f_B = \delta \left( -p_J + f_B \right) \end{cases} \tag{2}$$

which means that, for each player, the gains from accepting the other party's offer today must equal the gains from rejecting it and make an offer tomorrow. In addition, since these gains must be non negative, the solutions of the system above are:

$$p_I = \frac{f_B + \delta f_S}{1+\delta} \qquad p_J = \frac{\delta f_B + f_S}{1+\delta}$$

As I moves first and as the equilibrium is reached in the first period, $p_I$ is accepted by J (i.e., $p_I$ is the selling price). The payoffs for each player are: $\Pi_I = p_I - f_S = \dfrac{f_B - f_S}{1+\delta}$; $\Pi_J = f_B - p_I = \delta \dfrac{f_B - f_S}{1+\delta}$; $\Pi_S = f_S$; $\Pi_B = 1 - f_B$. The payoff matrix for all the players in the four possible situations of Figure 1 is given by Table (3):

| Seller - Buyer | $\Pi_S$ | $\Pi_B$ | $\Pi_I$ | $\Pi_J$ |
|---|---|---|---|---|
| $(D, D)$ | $f_S$ | $1 - f_B$ | $\frac{f_B - f_S}{1+\delta}$ | $\delta \frac{f_B - f_S}{1+\delta}$ |
| $(D, ND)$ | $f_S$ | $\frac{\delta}{1+\delta}(1 - f_S)$ | $\frac{1 - f_S}{1+\delta}$ | $0$ |
| $(ND, D)$ | $\frac{f_B}{1+\delta}$ | $1 - f_B$ | $0$ | $\frac{\delta}{1+\delta} f_B$ |
| $(ND, ND)$ | $\frac{1}{1+\delta}$ | $\frac{\delta}{1+\delta}$ | $0$ | $0$ |

(3)

As the players' payoffs are now clear, we introduce lemma 1, which explains the rationality behind the use of fixed compensation schemes.

**Lemma 1** *In bilateral delegation games, a fixed compensation scheme is profit maximizing for a principal and also the best response to the other principal's strategy.*

***Proof.*** *A principal can not extract the entire surplus by his compensation scheme as the other principal is delegating as well. Nevertheless, he can still maximize his profit by leaving the minimum to the agents. He can do this through a fixed payment compensation scheme. Since the difference* $(f_B - f_S)$ *constitutes the net gain the agents can share, both B and S wish to minimize it. For this reason, when a principal is playing this strategy, the other principal's best response is to ask for a fixed payment as well. As long as this difference is non negative, each agent has no incentive to reject any offer. Then, as the agent's payoffs (good or money) are conditional to an agreement, this is finally reached.* ■

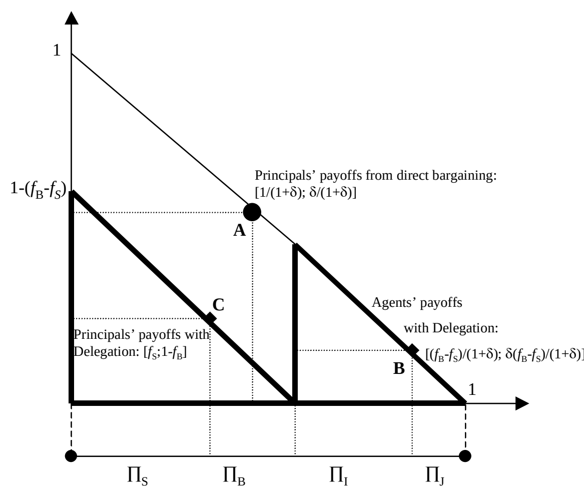
***The agents' stage.*** Consider the choice faced by the agents. Take for instance agent I. If he rejects, he gains 0; if he accepts, he may gain $\frac{(f_B - f_S)}{1+\delta}$

or $\frac{(1-f_S)}{1+\delta}$ if he bargains with J or B respectively. So, he will accept any compensation $f_S$ providing him a non negative profit. For bilateral delegation to be a possible equilibrium, the following rationality (participation) constraints, respectively for I and J, must be satisfied:

$$\begin{cases} \dfrac{f_B - f_S}{1+\delta} \geq 0 \\ \delta \dfrac{f_B - f_S}{1+\delta} \geq 0 \end{cases} \tag{4}$$

The difference $(f_B - f_S)$ cannot be negative: in the bargaining stage, the agents will realize they will gain negative payoffs in case of agreement. No compatible offers will be made and no agreement will be struck, leading all the players to the impasse point. Figure 2 compares two situations, that is bargaining with or without delegation. $A$ is the equilibrium of a typical Rubinstein game between principals: the position of this point only depends upon $\delta$. In the second situation (equilibria $B$ and $C$), two games are played: a Rubinstein one between agents, sharing a pie of $(f_B - f_S)$, and a Nash demand game between principals. $B$ is the solution of the former game whereas $C$ is the solution of the latter one.

Figure 2: Direct Vs Delegated bargaining




There is obviously a conflict between S and B, as S wants to maximize $f_S$ and B wants to minimize $f_B$.

**Lemma 2** *Equilibria with bilateral delegation are supported only when* $f_B = f_S = f^*$. *This strategy gives the following profits:* $\Pi_I = 0$; $\Pi_J = 0$; $\Pi_S = f^*$ *and* $\Pi_B = 1 - f^*$.

***Proof.*** *From (4) we know that* $f_B - f_S \geq 0$*. Suppose* $f_B > f_S$*. Then either the buyer would have an incentive to reduce* $f_B$ *or the seller to raise* $f_S$*. Therefore, in equilibrium equality must hold. Finally, if* $f_B = f_S$*,* $\Pi_I = \Pi_J = 0, \Pi_S = f^*$ *and* $\Pi_B = 1 - f^*$ *from Table (3).* ■

As long as the agents do not have any outside option, they are not able to extract any extra-profit from the delegation contract. This result is actually not striking, as all the bargaining power was given to the principals.

We can now focus on the principals delegation stage.

***The delegation stage.*** This is illustrated in Table (5), which updates and expands Table (1):

| | | | Buyer | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | D | | | | | ND |
| | | | 0 | $f_1$ | $f_2$ | ... | 1 | |
| Seller | D | 0 | $0;1$ | $0;1-f_1$ | $0;1-f_2$ | | $0;0$ | $0;\frac{\delta}{1+\delta}$ |
| | | $f_1$ | $0;0$ | $f_1;1-f_1$ | $f_1;1-f_2$ | | $f_1;0$ | $f_1;(1-f_1)\frac{\delta}{1+\delta}$ |
| | | $f_2$ | $0;0$ | $0;0$ | $f_2;1-f_2$ | | $f_2;0$ | $f_2;(1-f_2)\frac{\delta}{1+\delta}$ |
| | | ... | | | | | | |
| | | 1 | $0;0$ | $0;0$ | $0;0$ | | $1;0$ | $1;0$ |
| | ND | | $0;1$ | $\frac{f_1}{1+\delta};1-f_1$ | $\frac{f_2}{1+\delta};1-f_2$ | | $\frac{1}{1+\delta};0$ | $\frac{1}{1+\delta};\frac{\delta}{1+\delta}$ |

(5)

When a player decides to delegate, he sets a generic value $f$ such that $0 \leq f \leq 1$. For simplicity, in Table (5) we consider only two intermediate values of $f$ (with $f_1 < f_2$). This payoff matrix is set according to known results: payoffs for $(ND, ND)$, for instance, are simply given by the usual Rubinstein model of direct bargaining, whereas gains for $(ND, D)$ and $(D, ND)$ are valuated for the particular $f$. Our own model provides the gains for $(D, D)$ and for the impasse points $(f_S > f_B)$. The choice between delegating and not delegating constitutes a simple game between the two principals and the final outcome will depend on the level of $f$. The principals' best responses are as follows:

- If S plays $D$ with a non negative $f_S$, then B always plays $D$ setting $f_B = f_S = f^*$.
- If S plays $D$ with $f_S = 1$, then B either plays $D$ setting $f_B = f_S = 1$ or plays $ND$.
- If S plays $ND$ then B always plays $D$ setting $f_B = 0$.
- If B plays $D$ with a strictly positive $f_B$, then S always plays $D$ setting $f_S = f_B = f^*$.
- If B plays $D$ with a $f_B = 0$, then S either plays $D$ setting $f_S = f_B = 0$ or plays $ND$.

- If B plays $ND$ then S always plays $D$ setting $f_S = 1$.

Three cases emerge. They are presented in Proposition 1, which also summarizes the results of this first section.

**Proposition 1** *A two-sided delegation game is characterized as a typical Nash demand game. For extreme values of the compensation schemes, one-sided delegation equilibria are also supported. In particular, for $0 < f < 1$ there's a continuum of NEPS (Nash Equilibria with Pure Strategy) with bilateral delegation $(D, D)$; for $f = 0$ there are two NEPS: $(D, D)$ and $(ND, D)$; finally, for $f = 1$ there are two NEPS: $(D, D)$ and $(D, ND)$.*

**Proof.** Proposition 1 directly follows from the discussion above and from the solution of the game in (5). ■

Some discussion about these results is worthy. First of all, the presence of additional players (the agents) does not affect the size of the pie, that is, the total available profit. Of course, this result is strongly dependent on the no costs assumption.

Then, with regard to distributional issues, we first define the winner of the game.

**Definition 1** *A principal is the winner of the delegation game when his payoff from delegating is bigger than his payoff from direct bargaining.*

When a principal expects the other to obtain the entire pie ($f = 0$ or $f = 1$), he has no strategic reason to look for an agent, as his payoff is going to be exactly the same. According to our definition, the buyer is winning from bilateral delegation as long as $f < \frac{1}{1+\delta}$; on the contrary, the seller is winning for higher values of $f$. There are more equilibria where the buyer is the winner for any $0 < \delta < 1$. That is, delegation fully destroys any first mover advantage for the seller. Indeed, this advantage is typical of time consuming games whereas the delegation one is characterised as a Nash demand game.

In the next sections, we will discuss the consequences of relaxing an important assumption, that is, that delegation is costless. First, we consider exogenous costly delegation for the players. Then, we analyze the effects of endogenous costly delegation. We expect the range of bilateral delegation equilibria to shrink or even disappear.

# 4 Exogenous costly delegation

We want to understand the robustness of this multiplicity result. In particular, we analyze the effects of introducing in the model some costs for the players. First, we assume costs to be exogenous and we distinguish between principals' and agents' costs.

## 4.1 Costly delegation for principals

Let us suppose that, simply because a principal employs an intermediary, he suffers a strictly positive fixed cost $c$ independently from the fact that trade occurs or not. In particular, $c$ is paid by the principal only if he decides to delegate. For instance, $c$ may be the time cost of looking for an intermediary.

For simplicity, we assume $c_S = c_B = c$ to be the principals' common cost of delegation. How will the equilibria of the previous section be affected? First of all, we update the payment scheme in case of bilateral delegation:

- I pays $f_S$ to S whenever trade occurs and gains the selling price of the good;
- S pays $c$ when delegating and gains $f_S$ whenever trade occurs;
- B pays $c$ when delegating and $f_B$ to J whenever trade occurs and gains the value of the good ( which is equal to 1);
- J pays the selling price to I and gains $f_B$ whenever trade occurs

The range of equilibria in the previous section is modified as stated in proposition 2.

**Proposition 2** *In a two-sided delegation game with exogenous delegation costs $c$ for the principals, the following equilibria emerge: $(D,D)$, $(D,ND)$ and $(D,ND)$ for $0 \leq c \leq \frac{\delta}{(1+\delta)^2}$; $(D,ND)$ and $(D,ND)$ for $\frac{\delta}{(1+\delta)^2} < c \leq \frac{\delta}{1+\delta}$; $(ND,D)$ for $\frac{\delta}{1+\delta} < c \leq \frac{1}{1+\delta}$; and $(ND,ND)$ for $c > \frac{1}{1+\delta}$. The presence of costs is a source of inefficiency. In particular, in some intervals bilateral delegation equilibria are characterized as prisoner's dilemmas.*

**Proof.** The rest of this subsection is devoted to proving these results. ■

We notice that nothing changes for I and J. The level of $c$ does not affect their reservation wage or strategy and the rationality constraints observed in (4) apply unchanged. Therefore we should only update the payoffs matrix in (5) and analyze the new delegation stage between B and S. Again, with bilateral delegation the condition $f_S = f_B = f^*$ must be satisfied.

| | | | Buyer | | | | |
|---|---|---|---|---|---|---|---|
| | | | D | | | | ND |
| | | | 0 | $f_1$ | ... | 1 | |
| Seller | D | 0 | $-c; 1-c$ | $-c; 1-f_1-c$ | | $-c; -c$ | $-c; \frac{\delta}{1+\delta}$ |
| | | $f_1$ | $-c; -c$ | $f_1-c; 1-f_1-c$ | | $f_1-c; -c$ | $f_1-c; (1-f_1)\frac{\delta}{1+\delta}$ |
| | | ... | | | | | |
| | | 1 | $-c; -c$ | $-c; -c$ | | $1-c; -c$ | $1-c; 0$ |
| | ND | | $0; 1-c$ | $\frac{f_1}{1+\delta}; 1-f_1-c$ | | $\frac{1}{1+\delta}; -c$ | $\frac{1}{1+\delta}; \frac{\delta}{1+\delta}$ |

(6)

The players' best responses are now functions of $c$:

- If S plays $D$ then $\begin{cases} \text{B plays } D \text{ if he expects} & f_S < 1 - c(1+\delta) \\ \text{B plays } ND \text{ if he expects} & f_S > 1 - c(1+\delta) \end{cases}$

- If S plays $ND$ then $\begin{cases} \text{B plays } D \text{ setting } f = 0 \text{ if} & c < \frac{1}{1+\delta} \\ \text{B plays } ND \text{ if} & c > \frac{1}{1+\delta} \end{cases}$

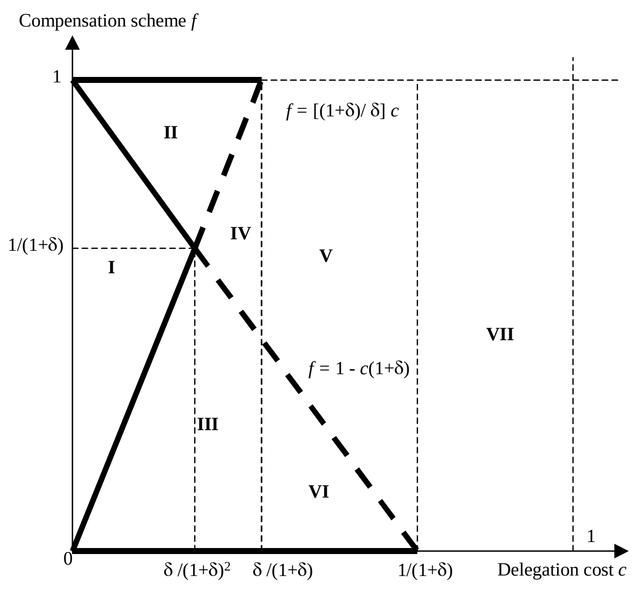
- If B plays $D$ then $\begin{cases} \text{S plays } D \text{ if he expects} & f_B > c\left(\frac{1+\delta}{\delta}\right) \\ \text{S plays } ND \text{ if he expects} & f_B < c\left(\frac{1+\delta}{\delta}\right) \end{cases}$

- If B plays $ND$ then $\begin{cases} \text{S plays } D \text{ setting } f = 1 \text{ if} & c < \frac{\delta}{1+\delta} \\ \text{S plays } ND \text{ if} & c > \frac{\delta}{1+\delta} \end{cases}$

We can still apply the same logic we used to solve (5). There are 7 possible combinations of $f$ and $c$ arising. The simplest way to study and analyze them is to look at graph 1.

We start focusing on intervals where $(D, D)$ is a solution. It can be shown (possibly, in Appendix), that any $f^*$ in case I can be sustained as equilibrium compensation scheme. In other words, bilateral delegation is sustained whenever $c\left(\frac{1+\delta}{\delta}\right) < f^* < 1 - c(1+\delta)$. Of course, this interval is shrinking as $c$ grows and eventually collapses to 0 when $c = \frac{\delta}{(1+\delta)^2}$. In this point, $f^* = \frac{1}{1+\delta}$. For higher value of $c : 1 - c(1+\delta) < c\left(\frac{1+\delta}{\delta}\right)$ and therefore $(D, D)$ is no longer a possible equilibrium. On the contrary, $(ND, ND)$ is the unique solution for any $c > \frac{1}{1+\delta}$: delegation is too costly for the principals and even one side delegation equilibria are ruled out.

Graph 1: Equilibria with principals' costs




In cases (I), (II), (III), and (IV) one-side delegation equilibria arise as well. In these equilibria, the delegating party obviously plays his profit maximizing level of $f$, that is $f_S = 1$ or $f_B = 0$. All the uncertainty disappears once $c$ is big enough to rule out multiplicity: for $\frac{\delta}{1+\delta} < c < \frac{1}{1+\delta}$, $ND$ is a dominant strategy for the seller, no matter the expected value of $f_B$. This situation is obviously exploited by the buyer, who plays his profit maximising $f_B = 0$. Finally, when $c > \frac{1}{1+\delta}$, both the players prefer $ND$.

### 4.1.1 Welfare implications: distributional and efficiency issues

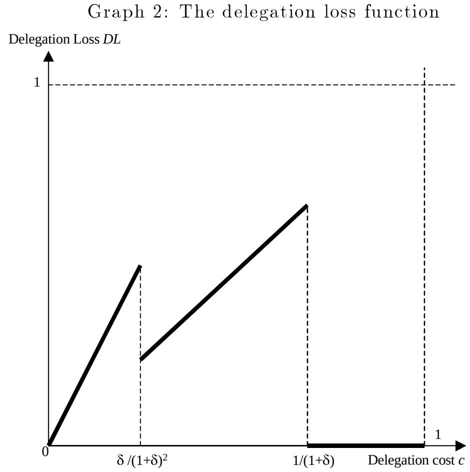
When it is costly, delegation is with no doubt inefficient, as it shrinks the size of the pie available to the principals. Nevertheless, $D$ is still played in some intervals.

To see the extent to which delegation is inefficient, we can define the delegation loss $DL$ as the difference between the size of the non-delegating equilibrium total outcome (i.e., 1) and the delegating equilibria one ($1 - c$ or $1 - 2c$ with one-side and bilateral delegation respectively). This difference clearly depends on the level of the costs $c$. Just for simplicity and for expositional purposes, we focus on the worst possible case and assume bilateral delegation equilibria to be played when one-side delegation ones are possible as well. When both the players delegate, a total loss of $2c$ is produced. For central values of the delegation costs, only one-side delegation equilibria are sustainable, therefore the loss is equal to $c$. Finally, for larger $c$, no principal delegates and there are no losses.

Graph 2 shows the non monotonic behavior of the delegation loss function, which is defined by (7):

$$DL = \begin{cases} 2c & \text{for} \quad c < \frac{\delta}{(1+\delta)^2} \\ c & \text{for} \quad \frac{\delta}{(1+\delta)^2} < c < \frac{1}{1+\delta} \\ 0 & \text{for} \quad c > \frac{1}{1+\delta} \end{cases} \tag{7}$$

Graph 2: The delegation loss function

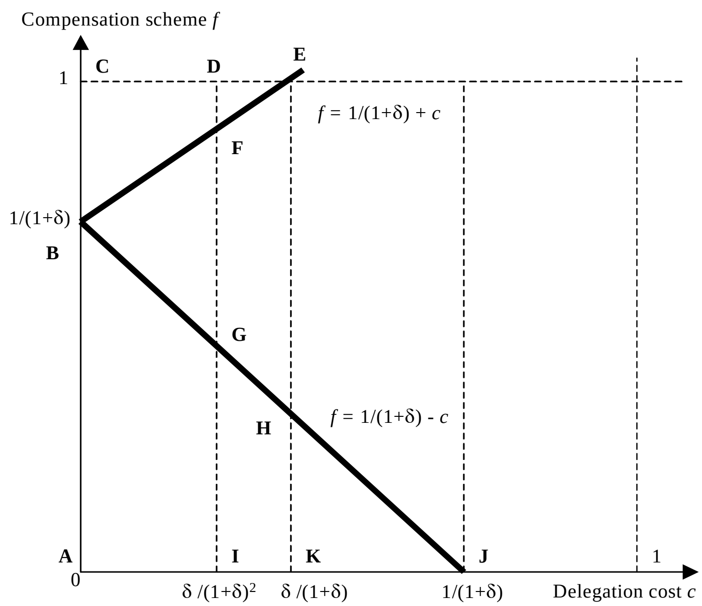



(8)

It is also interesting to understand who is the winner of the delegation game. In the basic model, for instance, S is the winner whenever $f > \frac{1}{1+\delta}$. Now the discussion is a little bit more complex, as payoffs depend both on $c$ and $f$. Graph 3 helps us to understand the situation.

The two tick straight lines are the functions $f = \frac{1}{1+\delta} - c$ and $f = \frac{1}{1+\delta} + c$. The big triangle $\overset{\triangle}{BCE}$ represents the gains from delegation for the seller[5]. The big triangle $\overset{\triangle}{ABJ}$ gives the profits from delegation for the buyer. We start focusing on the interval $0 < c < \frac{\delta}{(1+\delta)^2}$ and $(D, D)$ equilibria. S is the winner[6] whenever $f - c > \frac{1}{1+\delta}$, that is whenever $f > \frac{1}{1+\delta} + c$ (area $BCDF$). B is the winner if $1 - f - c > \frac{\delta}{1+\delta}$, that is, $f < \frac{1}{1+\delta} - c$ (area $ABGI$). In the triangle $\overset{\triangle}{BFG}$, that is when $\frac{1}{1+\delta} - c < f < \frac{1}{1+\delta} + c$ both the players are losing from delegation. This means that for some values of $c$ and $f$, the delegation game is similar to a prisoner's dilemma: both the players would be better off by not delegating but strategic behaviors force them towards a Pareto dominated equilibrium.

Graph 3: The winner of the game (1)




For completeness, we should note that the game played along the vertical axis is simply the case we studied in the previous section, that is, costless delegation ($c = 0$). As $c$ grows, the "both losing" space enlarges.

[5] We simply call gains from delegation the difference between the player's outcome when he delegates and his outcome from direct bargaining.

[6] See Definition 4.

When $\frac{\delta}{(1+\delta)^2} < c < \frac{\delta}{1+\delta}$, only one-side delegation equilibria are possible: if $(D, ND)$ is played, we are in $\overset{\triangle}{DEF}$; if $(D, ND)$ is played, we are in $GIKH$. As these equilibria are ex ante equally possible, the delegation game is similar to a chicken (or hawk-dove) game. Then, for $\frac{\delta}{1+\delta} < c < \frac{1}{1+\delta}$, only $(D, ND)$ is played and we are in the little triangle $\overset{\triangle}{HKJ}$: the buyer is the only possible winner. The existence of this area is quite surprising. For some values of $c$, the usually weakest party (that is, the second mover) is stronger. We recall that we had a similar kind of asymmetry for $c = 0$. We can dare to give a new interpretation to Schelling's intuition that "weakness is often strength". This is not true just when a player can bind himself to a strategy. It also matches with the following (common sense) statement: weakness is strength when the starting payoff is lower (i.e., the player has less to lose).

Finally, for higher values of $c$, nobody finds delegation profitable.

## 4.2 Costly delegation for agents

Delegation may be costly for agents, too. We call their common cost $y$. This is known ex ante by all the players and it is suffered only once the agent accepts the delegation contract. We may think of $y$ as the job effort. Or it can be the time spent to arrange meetings with the counterpart. I and J will accept a compensation scheme only if it will provide them with at least this value $y$. This cost is not sunk until the agent accepts a contract. In fact, if it were sunk before, a principal could leave all the burden upon the agent and the problem would be the same as in the basic model.

The range of equilibria shrinks as $y$ grows. For high values of the agents' costs, delegation never occurs. A detailed characterization of the possible equilibria and corresponding relevant intervals of $y$ is provided by Proposition 3. The rest of the subsection is then devoted to its explanation.

**Proposition 3** *In a two-sided delegation game with symmetric exogenous delegation costs y for the agents, the following equilibria emerge:* $(D, D)$, $(D, ND)$ *and* $(ND, D)$ *for* $0 \leq y \leq \frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$; $(D, ND)$ *and* $(ND, D)$ *for* $\frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)} < y \leq \frac{\delta}{(1+\delta)^2}$; *and* $(ND, ND)$ *for* $y > \frac{\delta}{(1+\delta)^2}$. *For some range of y, delegation is played even if it provides the principals with a lower payoff than in the direct bargaining case.*

**Proof.** The rest of the subsection is devoted to proving these results. ∎

When delegation is costly, there is still room for agreements between principals and intermediaries. The size of this room is determined by the level of these costs. The agents can force the principals to give up some of their gains to hire them. Agent I is also able to gain more than his own cost $y$.

### 4.2.1 Existence of equilibria with two-sided delegation

When both the principals delegate, the agents' rationality constraints are now given by (9) which updates (4):

$$\begin{cases} \dfrac{f_B - f_S}{1+\delta} \geq y \\ \delta \dfrac{f_B - f_S}{1+\delta} \geq y \end{cases} \tag{9}$$

Both conditions must be satisfied simultaneously[7]. B and S cannot set the difference $(f_B - f_S)$ equal to 0 any longer; the most they can do is set it such that:

$$f_B - f_S = \left(\frac{1+\delta}{\delta}\right) y \geq (1+\delta)\, y \qquad \forall \delta \in [0,1] \tag{10}$$

**Lemma 3** *In any bilateral delegation equilibrium, the players' payoffs will be:* $\Pi_I = \dfrac{y}{\delta}$; $\Pi_J = y$; $\Pi_S = f_S$ *and* $\Pi_B = 1 - f_B$.

***Proof.*** *The logic of Lemma 2 still applies.* ■

From the agents' point of view, there is an important difference. Agent I can gain more than the level $y$ of his costs. This is a consequence of I being the first mover in the bargaining stage with J. As both the constraints in (9) must be satisfied, $(f_B - f_S)$ must be set so that it satisfies the stricter of the constraints, which is the second mover's one.

The principals play the following game: if S delegates he gains $f_S$; if he does not delegate but B delegates he gets $\frac{f_B}{1+\delta}$. If B delegates he gains $1 - f_B$; if he does not delegate but S delegates he gets $\frac{\delta}{1+\delta}(1 - f_S)$.

Taking into account (10), bilateral delegation equilibria will be sustained whenever:

$$\begin{cases} f_S > \dfrac{f_B}{1+\delta} \\ 1 - f_B > \dfrac{\delta}{1+\delta}(1 - f_S) \\ f_B - f_S = \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \quad \text{that is} \quad \begin{cases} f_B < f_S(1+\delta) \\ f_B < \dfrac{1}{1+\delta} + \dfrac{\delta}{1+\delta} f_S \\ f_B = f_S + \left(\dfrac{1+\delta}{\delta}\right) y \end{cases} \tag{11}$$

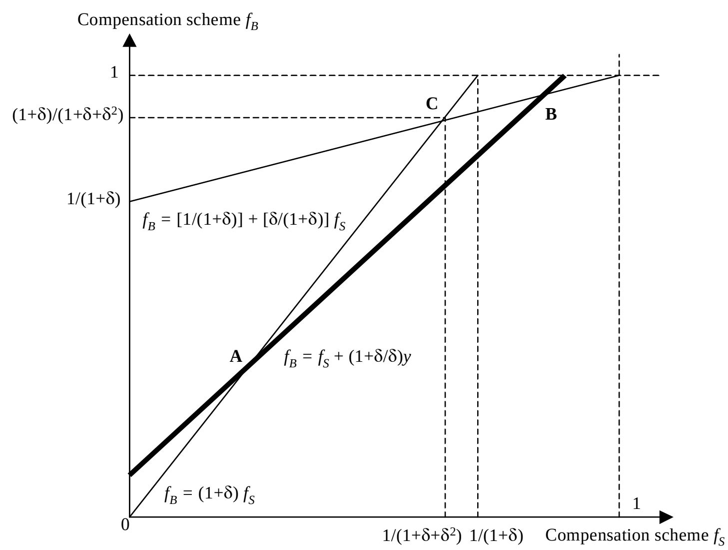
The two inequalities in (11) respectively represent the rationality constraints for the seller and the buyer to play bilateral delegation. The last function (equality) is the agents' participation constraint (10). We depict all these functions in graph 4, in order to illustrate the range of possible bilateral equilibria in terms of compensation schemes.

The inequalities determine a possible area $\overset{\triangle}{ABC}$ where bilateral delegation is an equilibrium. As the agents' participation constraint is binding (see Lemma

[7] Suppose the difference $(f_B - f_S)$ is set such that J will not be able to gain any positive profit in the bargaining stage. Then, as already argued above, this choice will lead the game to the impasse point, where everybody has a zero payoff.

7), the range of equilibria reduces to the segment $\overline{AB}$. The presence of $y$ influences the position of the line $f_B = f_S + \left(\frac{1+\delta}{\delta}\right) y$. Indeed, when $y = 0$, we know that any $f_S = f_B = f^*$ can be sustained as equilibrium (and indeed the segment $\overline{AB}$ sustains every $f \in [0, 1]$). For higher values of $y$, this set of equilibria shrinks (the thick straight line shifts upwards) and eventually collapses to a singleton (point $C$) when $y = \frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$. In this case: $f_S = \Pi_S = \frac{1}{1+\delta+\delta^2}$ and $f_B = \frac{1+\delta}{1+\delta+\delta^2}$ ($\Pi_B = \frac{\delta^2}{1+\delta+\delta^2}$).

Graph 4: Equilibria with exogenous agents' costs




For intermediate values of $y$, any compensation scheme between $A$ and $B$ is sustainable as bilateral delegation equilibrium. In terms of $f_S$, the range of equilibria is given by: $\left(\frac{1+\delta}{\delta^2}\right) y \leq f_S \leq 1 - \frac{(1+\delta)^2}{\delta} y$. In terms of $f_B$, this corresponds to: $\left(\frac{1+\delta}{\delta}\right)^2 y \leq f_B \leq 1 - (1+\delta) y$

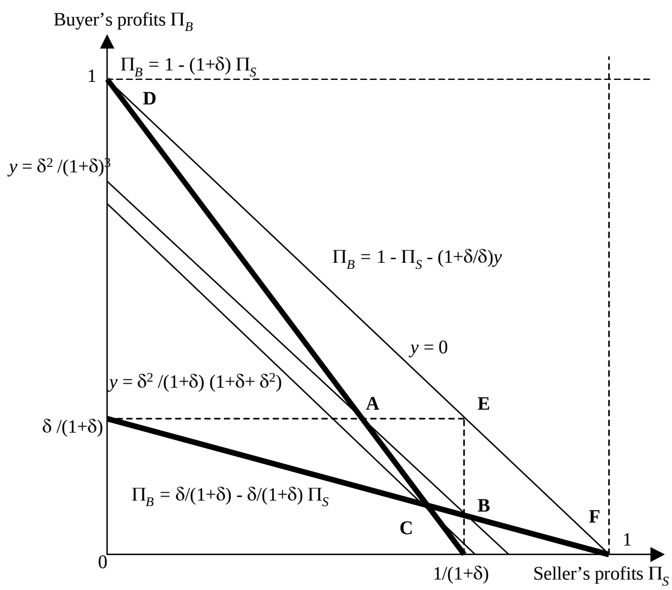
**Welfare implications of bilateral delegation** We wish to obtain some insights about the welfare implications of these equilibria. In graph 5 we present the same information as in graph 4, but with respect to the principals' profits[8]. The presence of a strictly positive cost for the agents is always inefficient from the principals' point of view. Indeed, both of them must give up a share of the entire pie when hiring an agent. Though, the distribution of payoffs is such that, for some values of $y$, at least one of the two parties is better off.

The thin straight lines represent the agents' participation constraints. In this case, higher values of $y$ imply a downward shift of the line. In graph 5 we depict three of these lines. The highest is for $y = 0$. The middle one is for $y = \frac{\delta^2}{(1+\delta)^3}$. The lowest one is for $y = \frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$. Given definition

[8] We recall that $\Pi_S = f_S$ and $\Pi_B = 1 - f_B$.

4, we can state that for $0 \leq y \leq \frac{\delta^2}{(1+\delta)^3}$, the buyer is the winner whenever his payoff is bigger than $\frac{\delta}{1+\delta}$ (any compensation scheme on the segment $\overline{DE}$). On the contrary, the seller is the winner whenever $f_S > \frac{1}{1+\delta}$ (segment $\overline{EF}$). Nevertheless, equilibria where a prisoner's dilemma arises are possible. In these equilibria $1 - f_B < \frac{\delta}{1+\delta}$ and $f_S < \frac{1}{1+\delta}$. This interval is represented by any segment similar to $\overline{AB}$, that is, any portion of agents' participation constraint line in $\overset{\triangle}{ABE}$. For $\frac{\delta^2}{(1+\delta)^3} \leq y < \frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$, both the principals still delegate even if they are both worse off. Finally, for higher values of $y$, bilateral delegation never occurs.

Graph 5: The winner of the game (2)




Again, with symmetric intermediaries' costs, the delegation game is characterized (at least for some intervals of $y$) as a typical prisoner's dilemma: B and S would be both better off by not delegating. As in the previous subsection, these equilibria arise as delegation is a dominant strategy in the interval. The difference with the case of symmetric principals' costs is that now the loss is not a deadweight one, as the intermediaries are gaining and sharing it.

### 4.2.2 Existence of equilibria with one-sided delegation

For some values of $f_B, f_S$ and $y$, equilibria with one-sided delegation are as well possible. If we go back to (11), it's clear that, say, S will prefer $ND$ whenever he expects B to play $f_B > f_S(1+\delta)$. Likewise, B will prefer $ND$ whenever he expects S to play $f_S < \frac{(1+\delta)f_B - 1}{\delta}$. We have the following cases:

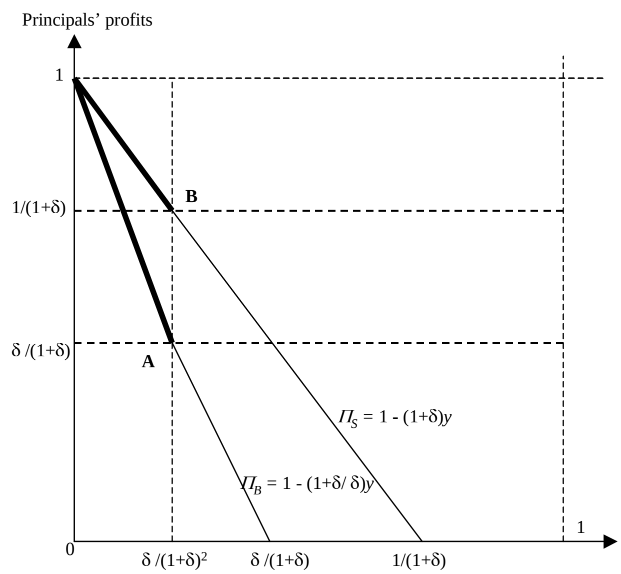
(a) Only the buyer delegates; for this three players case, the agent J's participation constraint is $\frac{\delta}{(1+\delta)} f_B \geq y$ and the buyer's rationality constraint is $1 - f_B \geq \frac{\delta}{(1+\delta)}$. Therefore, we have: $\Pi_S = \frac{y}{\delta}$; $\Pi_J = y$; $\Pi_B = 1 - f_B = 1 - \left(\frac{1+\delta}{\delta}\right) y$ if $y \leq \frac{\delta}{(1+\delta)^2}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_J = y$; $\Pi_B = \frac{\delta}{1+\delta}$ if $y > \frac{\delta}{(1+\delta)^2}$. Suppose S wants to deviate and to play $D$. He would gain $\frac{f_B}{(1+\delta)}$ evaluated in $f_B = \left(\frac{1+\delta}{\delta}\right) y$, that is $\frac{y}{\delta}$. As in the baseline model, for extreme values of $f_B$, the seller is indifferent whether to delegate or not.

(b) Only the seller delegates; it can be easily shown that now the agent I's participation constraint is $\frac{\delta}{(1+\delta)}(1 - f_S) \geq y$. The resulting payoffs are $\Pi_S = f_S = 1 - (1+\delta) y$; $\Pi_I = y$; $\Pi_B = \delta y$ if $y \leq \frac{\delta}{(1+\delta)^2}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_I = y$; $\Pi_B = \frac{\delta}{1+\delta}$ if $y > \frac{\delta}{(1+\delta)^2}$. Suppose B wants to deviate and to play $D$. He would gain $\frac{\delta}{(1+\delta)} (1 - f_S)$ evaluated in $f_S = 1 - (1+\delta) y$, that is $\delta y$. Again, for extreme values of $f_S$, the buyer is indifferent whether to delegate or not.

We recall from the previous subsection that gains from one-sided delegation equilibria were not evenly distributed between the principals. This is not true in this case: one-sided delegation equilibria are sustained in the same interval for both the principals. The previous asymmetry is now offset by an additional one. The weight of $y$ is different whether this cost is sustained by the first mover or by the second one. The buyer has to give up to more as he has to compensate a second mover agent with a strictly positive delegation cost.

Graph 6 shows how the level of $y$ influences the principals' payoffs:

Graph 6: One -side delegation equilibria



The upper downward sloping line represents the seller's payoffs. S will delegate as long as these are higher than his payoffs from direct bargaining, that is $\frac{1}{1+\delta}$. The lower downward sloping line represents the buyer's payoffs. B will delegate as long as these are higher than his payoffs from direct bargaining, that is $\frac{\delta}{1+\delta}$. It is clearly shown by the graph that this happens for the same value of $y$, that is $\frac{\delta}{(1+\delta)^2}$.

For higher values of $y$, as already stated in proposition 6, delegation never occurs.

## 4.3 A comparison between principals' and agents' costs

In the basic model without costs, we were concerned by the presence of multiple equilibria. These are still possible with positive levels of costs but their range is dramatically reduced even for very low levels of $c$ and $y$. Nevertheless, the impact of $c$ and $y$ is different. In (12) we compare the threshold values of these costs.

Agents' costs are more restrictive about the possibility of bilateral delegation equilibria. A first possible explanation is that, from the principals' point of view, a total agents' cost of $2y$ is actually paid as $y + \frac{y}{\delta}(> 2y)$ by the principals. Nevertheless, this is not fully satisfactory. For $\delta = 1$ the impact of the two costs is still different ($c = \frac{1}{4}$ and $y = \frac{1}{6}$ with bilateral delegation equilibria), even if the total cost is now exactly equal to $2y$. This first explanation only highlights the fact that the cost $c$ is paid in a Nash demand game and, by construction, is independent of time. On the other hand, $y$ is supported by individuals playing a Rubinstein game, where payoffs (and costs) are affected by elements like the degree of impatience ($\delta$).

| | Equilibrium | | |
|---|---|---|---|
| | $(D, D)$ | $(D, ND)$ | $(ND, D)$ |
| Max $c$ | $\frac{\delta}{(1+\delta)^2}$ | $\frac{\delta}{1+\delta}$ | $\frac{1}{1+\delta}$ |
| Max $y$ | $\frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$ | $\frac{\delta}{(1+\delta)^2}$ | $\frac{\delta}{(1+\delta)^2}$ |

(12)

Another possibility is the following. From the principals' point of view, $c$ is an *individual* cost. Suppose S has a positive $c$ and B has not costs: only the seller's strategy will be affected. As regards $y$, it affects *both* the principals. Both of them have to support a share of this cost to be sure the agents will accept their offers. In addition, suppose I has no costs and J has a positive one. As the only relevant constraint is the most binding one, the principals strategy would still be the same as in the case with symmetric agents' cost[9].

## 4.4 A general model of exogenous costly delegation

In this subsection, we present a general model where delegation is costly both for the agents and the principals. We will only present the results and the

[9] In the opposite case, when only I has a positive delegation cost, the maximum $y$ supporting bilateral delegation equilibria is $\frac{\delta}{(1+\delta)(1+\delta+\delta^2)}$ (which is still equal to $1/6$ when $\delta = 1$).

main intuitions, as the way to solve the game has been already discussed in the previous subsections.

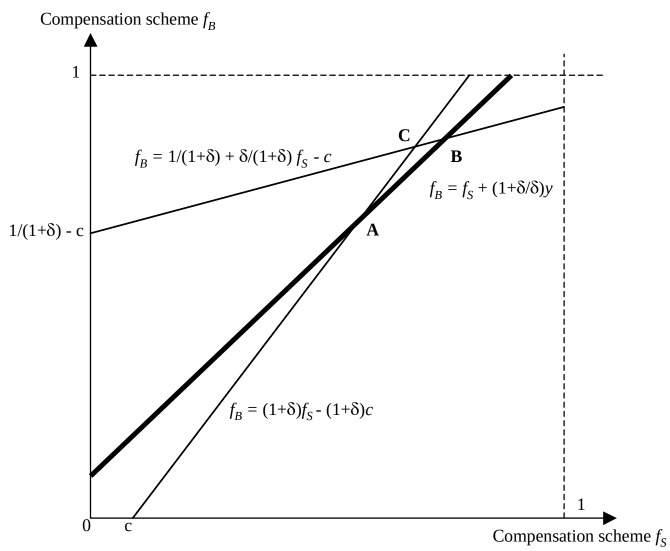
### 4.4.1 Existence of equilibria with bilateral delegation

The conditions for the existence of bilateral delegation equilibria are given by a modification of (11):

$$\begin{cases} f_S - c > \dfrac{f_B}{1+\delta} \\ 1 - f_B - c > \dfrac{\delta}{1+\delta}\left(1 - f_S\right) \\ f_B - f_S = \left(\frac{1+\delta}{\delta}\right) y \end{cases} \tag{13}$$

Graph 7 shows how the range of multiple equilibria quickly shrinks.

Graph 7: Bilateral delegation equilibria with exogenous costs

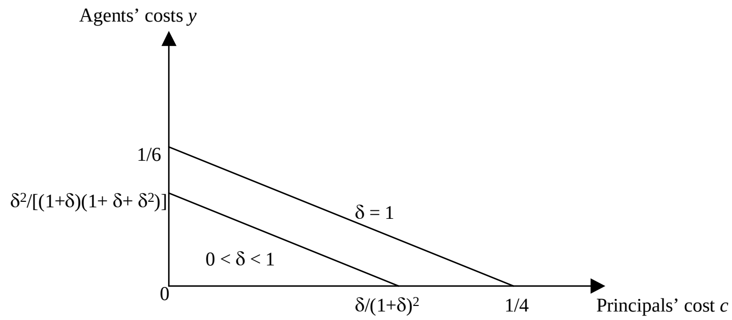



From a geometric point of view, the presence of $c$ influences the position (intercept point with the axes) of the principals' constraints. In particular, positive costs imply a downward shift of these lines. With regard to $y$, it moves the agents' constraint line upwards. From (13), we can work out a relation between the values of $c$ and $y$ such that bilateral delegation equilibria are possible. In particular, we have that:

$$y \leq \frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)} - c\frac{\delta(1+\delta)}{1+\delta+\delta^2} \text{ or } c \leq \frac{\delta}{(1+\delta)^2} - y\frac{1+\delta+\delta^2}{\delta(1+\delta)} \tag{14}$$

This relation is graphically shown in Graph 8 for different values of $\delta$. The higher the degree of impatience, the bigger the range of values of $c$ and $y$ which can support bilateral delegation equilibria. As we should expect from the previous subsection, for $c = 0$, the maximum level of exogenous costs supporting bilateral delegation equilibria is $y = \frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$.

Graph 8: Costs, impatience and equilibria (1)




On the contrary, when $y = 0$, the maximum level of exogenous principal costs turns out to be $c = \frac{\delta}{(1+\delta)^2}$.

# 5 Delegation with endogenous opportunities

We now introduce a different kind of cost, that is an endogenous quantity $x$ which can be consumed by the agents only once the delegation process is over. This additional pie of size $x$ is shrinking according to the agent's own discount factor (we still assume that all the subjects have the same $\delta$). That is, its actual size depends on the particular time the players reach an agreement. We will refer to $x$ both as a cost, given that it influences the compensation scheme an agent will require, and as an opportunity, because it is a source of additional income. We can interpret $x$ as a wage from a different job: an agent can decide to do just this job and gain $x$, or to accept the delegation contract and then go back to what remains of $x$.

Proposition 4 characterizes the set of equilibria of this game.

**Proposition 4** *In a two-sided delegation game with endogenous delegation costs $x$ for both agents, the following equilibria emerge: bilateral delegation equilibria for $0 \leq x \leq \frac{\delta^2}{(1+\delta)}$; $(ND, D)$ equilibria for $0 < x \leq \frac{\delta}{1+\delta}$; $(D, ND)$ equilibria for $0 < x \leq \frac{1}{1+\delta}$; and direct bargaining equilibria for $x > \frac{1}{1+\delta}$.*

**Proof.** The rest of the section will provide the proof for this proposition. ■

## 5.1 Equilibria with bilateral delegation

When both the principals delegate, I and J will be engaged in the following process, which is a modification of the condition in (2):

$$\begin{cases} p_J - f_S + x = \delta\,(p_I - f_S + x) \\ f_B - p_I + x = \delta\,(f_B - p_J + x) \end{cases}$$

As usual, this system of simultaneous equations means that the gains from accepting the other party's offer today must equal the gains from rejecting it and make an offer tomorrow. The first equation refers to I and the second to J. Taking I, for instance, his profits are determined by the selling price he gains from J, by the compensation scheme he must pay to S and by the opportunity $x$. As an agreement between I and J is struck in the first period, the relevant solution of the system above is given by I's first offer, that is:

$$p_I = \frac{f_B + \delta f_S}{1+\delta} + x\frac{1-\delta}{1+\delta}$$

The selling price $p_I$ is now corrected by the presence of an additional term, that is $x\frac{1-\delta}{1+\delta}$. The presence of positive endogenous costs positively influences the equilibrium selling price. The way this influence works is more clear when we look at the payoffs for each of the players:

$$\begin{cases} \Pi_I = p_I - f_S + x = \dfrac{f_B - f_S + 2x}{1+\delta} \\ \Pi_J = f_B - p_I + x = \delta\Pi_I = \delta\dfrac{f_B - f_S + 2x}{1+\delta} \\ \Pi_S = f_S \\ \Pi_B = 1 - f_B \end{cases}$$

We can express the agents' payoffs also in a different way, highlighting the weight of this additional opportunity:

$$\begin{cases} \Pi_I = \dfrac{f_B - f_S}{1+\delta} + 2x\dfrac{1}{1+\delta} \\ \Pi_J = \delta\dfrac{f_B - f_S}{1+\delta} + 2x\dfrac{\delta}{1+\delta} \end{cases}$$

The total additional opportunity ($2x$) is split between the agents according to the usual Rubinstein's rule. The agents do not simply bargain across the share of pie left by the principals ($f_B - f_S$). They also consider their additional opportunity, which is external to the bargaining problem. In other words, an agent's payoff is influenced also by the other agent's opportunity. I's payoff is not only augmented by $x$; it is actually augmented by a quantity $2x\frac{1}{1+\delta} > x$. This means that I, as first mover, is also able to obtain some of J's endogenous opportunity. Looking back at the selling price $p_I$, in the bargaining process it seems that I gives up to a share $\frac{\delta}{1+\delta}$ of his $x$ and gains a share $\frac{1}{1+\delta}$ of J's share (this can be more easily seen by allowing for different costs, say, $x_I$ and $x_J$). Eventually, I has two bigger shares ($2x\frac{1}{1+\delta}$) and J two smaller ones ($2x\frac{\delta}{1+\delta}$).

We know that $x$ plays the role of the agents' reservation wage as well; indeed, they may decide to refuse the delegation contract and to consume directly the quantity $x$. The following agents' rationality constraints must be simultaneously satisfied[10]:

$$\begin{cases} \dfrac{f_B - f_S + 2x}{1+\delta} \geq x \\ \delta \dfrac{f_B - f_S + 2x}{1+\delta} \geq x \end{cases}$$

that is,

$$f_B - f_S \geq x \frac{1-\delta}{\delta} \qquad \forall \delta \in [0,1] \tag{15}$$

The condition in (15) is different from the one in (10), when we introduced exogenous agents' costs. This is due to the fact that $x$ is not only a positive reservation wage but also a source of additional profit within the bargaining process (i.e., it appears also on the left hand side of the inequality). In equilibrium, the principals fix the compensation schemes such that the equality holds:

**Lemma 4** *In any bilateral delegation equilibrium, the players' payoffs will be:* $\Pi_I = \dfrac{x}{\delta}$; $\Pi_J = x$; $\Pi_S = f_S$ *and* $\Pi_B = 1 - f_B$. *The range of the principals' payoffs is as follows:* $\frac{1-\delta}{\delta^2} x \leq \Pi_S \leq 1 - \frac{(1-\delta)(1+\delta)}{\delta} x$; $(1-\delta) x \leq \Pi_B \leq 1 - \frac{(1-\delta)(1+\delta)}{\delta^2} x$.

**Proof.** The logic of Lemma 2 still applies to the first part of Lemma 4. As regards the second part, we just note that the game is solved through the system in (16):

$$\begin{cases} f_S > \dfrac{1}{1+\delta} f_B + \dfrac{1}{1+\delta} x \\ 1 - f_B > \dfrac{\delta}{1+\delta} (1 - f_S + x) \\ f_B = f_S + x \dfrac{(1-\delta)}{\delta} \end{cases} \tag{16}$$

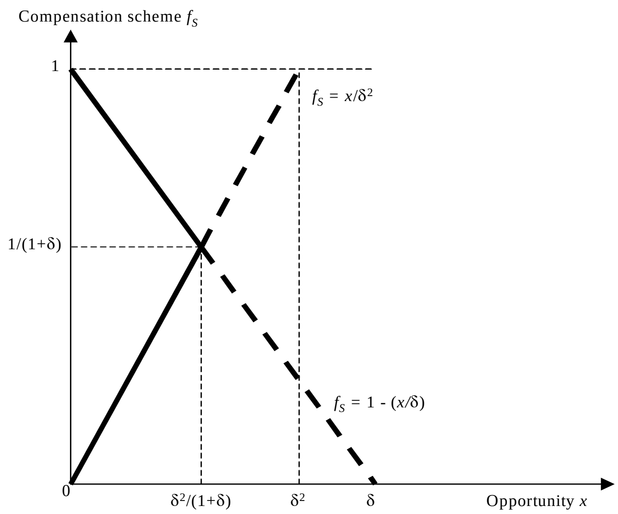
The two inequalities determine the range of values of $f_S$ and $f_B$ supporting bilateral delegation equilibria. They require the principals' profits from bilateral delegation (left hand side) to be higher than profits from one-side delegation (right hand side). The particular nature of $x$ is such that a principal can enjoy some of the other principal's agent opportunity even if the former is not delegating (right hand side of the inequalities). The final equality is the agents' participation constraint (which is binding) and directly comes from (15). We

[10] Again, if the difference $(f_B - f_S)$ is set such that J will not be able to gain any positive profit in the bargaining stage, then the game will move towards an impasse and everybody will get a zero payoff.

obtain equilibria with bilateral delegation for $x \leq \frac{\delta^2}{(1+\delta)}$ and $\frac{x}{\delta^2} \leq f_S \leq 1 - \frac{x}{\delta}$ (or, alternatively, $\frac{1+\delta-\delta^2}{\delta^2} x \leq f_B \leq 1 - x$). ■

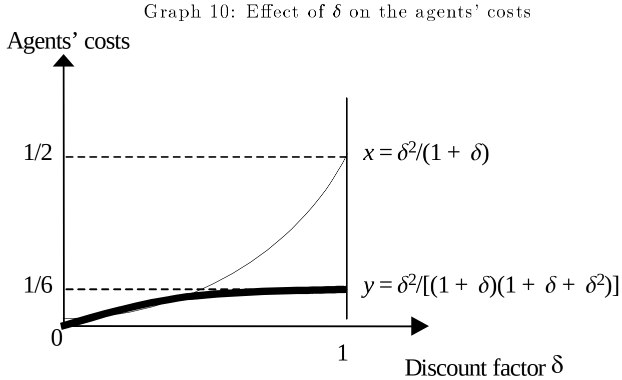
Graph 9 illustrates this interval in terms of $x$ and $f_S$.

Graph 9: Equilibria with endogenous opportunities




The limit value of $x$ is bigger than the one with exogenous costs $(y)$ for any strictly positive $\delta$. Graph 10 compares these two different costs. In the relevant interval, that is for $\delta \in [0, 1]$, $y$ is concave while $x$ is convex. When the subjects are very patient ($\delta$ close to 1), sensible levels of $x$ can still support bilateral delegation equilibria. On the contrary, these are supported only for small values of $y$. The difference gets smaller the more the individuals become more impatient.

Graph 10: Effect of $\delta$ on the agents' costs




This impact is different as, with additional opportunities of profit $(x)$, delegation is "less damaging" than with exogenous costs from the principals' point of view. In addition, for $x = \frac{\delta^2}{(1+\delta)}$, the payoffs of the principals are the following: $\Pi_S = \frac{1}{1+\delta}$ and $\Pi_B = \frac{1+\delta-\delta^2}{1+\delta+\delta^2}$, which are bigger (or equal) to their payoffs from direct bargaining (for any possible $\delta$). The level of $x$ positively influence the principals' payoffs and even for high values of these costs the principals are never worse off with respect to direct bargaining.

## 5.2 Equilibria with one-sided delegation

Equilibria with one-sided delegation are possible as well. We have the following cases:

(a) When only S delegates, B and I solve the following game:

$$\begin{cases} p_B - f_S + x = \delta\left(p_I - f_S + x\right) \\ -p_I + 1 = \delta\left(-p_B + 1\right) \end{cases}$$

The first equation refers to the usual seller's agent problem. The second equation explains the buyer's decision: he can accept the selling price proposed by I today and gain the value of the good, or reject it and propose a different selling price tomorrow. The relevant solution of this system is $p_I = \frac{1+\delta f_S}{1+\delta} - x\frac{\delta}{1+\delta}$ and consequently:

$$\begin{cases} \Pi_I = p_I - f + x = \dfrac{1+x-f_S}{1+\delta} \\ \Pi_B = \delta \Pi_I \end{cases}$$

If S wants I to accept the job, he must design $f_S$ such that I's profit from accepting the offer (left hand side of the inequality in 17) is at least equal to the profit he obtain by refusing it:

$$\Pi_I = p_I - f_S + x \geq x \tag{17}$$

that is:

$$f_S \leq 1 - \delta x$$

In equilibrium the equality holds. As $\Pi_S = f_S$, then S will prefer delegation to non delegation as long as:

$$1 - \delta x \geq \frac{1}{1+\delta}$$

that is:

$$x \leq \frac{1}{1+\delta}$$

So we have the following payoffs: $\Pi_S = f_S = 1 - \delta x$; $\Pi_I = x$ and $\Pi_B = \delta x$ if $x \leq \frac{1}{1+\delta}$; $\Pi_S = \frac{1}{1+\delta}$; $\Pi_J = x$; $\Pi_B = \frac{\delta}{1+\delta}$ if $x > \frac{1}{1+\delta}$. Is it profitable for $B$ to play $ND$ in this case? Is really $(D, ND)$ an equilibrium? We must compare the following payoffs: $\delta x$, gained by $B$ in the (putative) equilibrium $(D, ND)$, and the payoff from deviating and playing $D$ when $S$ is delegating with $f_S = 1 - \delta x$. In the latter case, taking into account that the agents' rationality constraint must be satisfied, we work out $\Pi_B = \frac{\delta^2+\delta-1}{\delta}x$. By comparison, $\delta x \geq \frac{\delta^2+\delta-1}{\delta}x$ for any possible $x$ and $\delta$. Therefore $(D, ND)$ is sustainable as equilibrium in the interval. There is also another case, that is what happen when $S$ does not delegate because $x$ is too high. This is equivalent to studying the case when only $B$ delegates, which is shown in (b).

(b) We solve a similar three subjects game when only B delegates. We have: $\Pi_S = \frac{x}{\delta}$; $\Pi_J = x$ and $\Pi_B = 1 - f_B = 1 - \frac{x}{\delta}$ if $x \leq \frac{\delta}{1+\delta}$ and $\Pi_S = \frac{1}{1+\delta}$; $\Pi_I = x$; $\Pi_B = \frac{\delta}{1+\delta}$ if $x > \frac{\delta}{1+\delta}$. We apply the same logic as before and we compare $\frac{x}{\delta}$ (from the putative one-sided delegation equilibrium) and $x$ (from $S$ deviating to $D$). Again, $\frac{x}{\delta} \geq x$ for any possible $x$ and $\delta$. Therefore $(ND, D)$ is sustainable as equilibrium in the interval.

One-sided delegation equilibria are therefore characterized as follows: $(D, ND)$ and $(ND, D)$ for $x \leq \frac{\delta}{1+\delta}$; $(D, ND)$ for $\frac{\delta}{1+\delta} < x \leq \frac{1}{1+\delta}$; $(ND, ND)$ for $x > \frac{1}{1+\delta}$. The results of Proposition 8 directly follow.

The presence of agent's endogenous opportunities is actually a benefit for the principals: no prisoner's dilemma games emerge. Actually, from the players' point of view, bilateral delegation equilibria weakly dominate direct bargaining ones, that is, every player is at least better off.

It is therefore very interesting to analyse a model with endogenous and exogenous agents' costs and understand which is the net effect.

## 5.3 A general model of costly delegation for agents

We present a general model where agents face both exogenous ($y$) and endogenous ($x$) costs. We will only present the results and the main intuitions, as the way to solve the game has been already discussed in the previous subsections.

### 5.3.1 Existence of equilibria with bilateral delegation

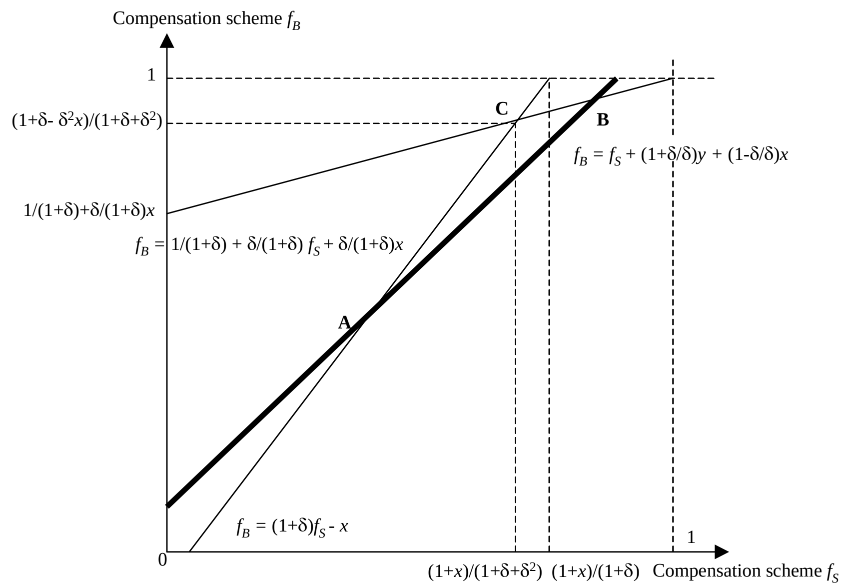
The conditions for the existence of bilateral delegation equilibria are given by:

$$\begin{cases} f_S > \dfrac{f_B + x}{1+\delta} \\ 1 - f_B > \dfrac{\delta}{1+\delta}(1 - f_S + x) \\ \delta\dfrac{f_B - f_S + 2x}{1+\delta} = x + y \end{cases} \tag{18}$$

which updates (11). The only difference is in the agents' participation constraint (the equality). We consider the joint presence of $y$ as positive reservation wage and of $x$ as both a positive reservation wage and a source of additional profit within the bargaining process.

From a geometric point of view, the presence of $y$ influences only the position of the agents' constraint line (the thick one in graph 11) while the presence of $x$ influences all the three conditions. The range of multiple equilibria shrinks the higher the values of $x$ and $y$. The range of values of $f_S$ and $f_B$ sustaining bilateral delegation equilibria are found as the intersections between the agents' constraint and both of the principals' line. These values are the following: $\frac{1}{\delta^2}x+\frac{1+\delta}{\delta^2}y \leq f_S \leq 1-\frac{1}{\delta}x-\frac{(1+\delta)^2}{\delta}y$ and $\frac{1+\delta-\delta^2}{\delta^2}x+\frac{(1+\delta)^2}{\delta^2}y \leq f_B \leq 1-x-(1+\delta)\,y$. They simply mix the range values we found with only endogenous or exogenous costs.

Graph 11: Equilibria with agents' costs

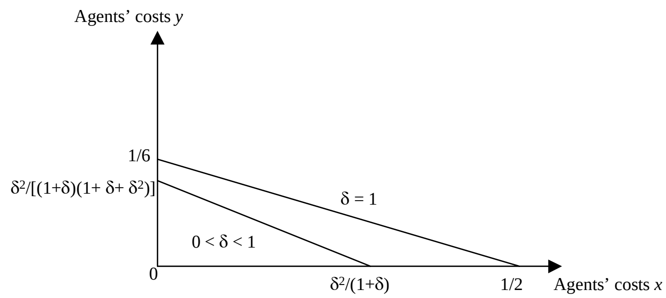



From (18), we can work out a relation between the values of $x$ and $y$ such that bilateral delegation equilibria are possible. We find that:

$$y=\frac{\delta^2}{(1+\delta)\left(1+\delta+\delta^2\right)}-x\frac{1}{\left(1+\delta+\delta^2\right)} \text{ or } x=\frac{\delta^2}{(1+\delta)}-y\left(1+\delta+\delta^2\right)$$

The first of these relations is graphically shown in Graph 12 for different values of $\delta$. The higher $\delta$, the bigger the range of values of $x$ and $y$ which can support bilateral delegation equilibria. The intercepts with the axes are already known results: when $x=0$, the maximum level of exogenous costs supporting bilateral delegation equilibria is $y=\frac{\delta^2}{(1+\delta)(1+\delta+\delta^2)}$. When $y=0$, the maximum level of endogenous costs is $x = \frac{\delta^2}{1+\delta}$.

Graph 12: Costs, impatience and equilibria (2)




# 6 Conclusions

In this paper we present some models of bilateral delegation. Delegation is introduced as a device to increase one party's bargaining power and we are interested in understanding why, when and how it is sustainable as an equilibrium strategy. Indeed, previous studies showed how one-sided delegation is very effective. With bilateral delegation the analysis is more complicated.

There are four subjects involved in the games: a seller S, a buyer B, and their agents, I and J respectively. These agents are hired by the principals through take-it-or-leave-it offers about the level of some compensation schemes, and then play a Rubinstein game. The structure of the game implies that the agents' profits depend upon both principals' proposals. The first consequence is that the initial Rubinstein game among principals becomes a typical Nash demand game. When delegation is costless, multiplicity arises and uncertainty about the identity of the winner is present. From a positive point of view, this confirms that when delegation is available, the players decide to use it. When a party delegates, the other always replies by delegating as well (a part from extreme cases). This is true in a lot of real life situations: workers' unions often negotiate with firms' unions; foreign ministers deals with others foreign ministers, a divorcing couple usually leaves the process to two lawyers. Of course, in many cases delegation is better explained by different reasons. The possibility of saving time, for instance. Or the need of particular skills, i.e.: deep legal knowledge. Still, are we sure that a divorcing lawyer would not hire his own lawyer? To stress the importance of the strategic power of delegation, we assume that all the players have the same degree of impatience (which is the only distinctive element of the game).

As one-sided delegation is very powerful, bilateral delegation provides a way of compensating bargaining powers. This compensation is even more important, as it offsets the typical first mover's advantage of Rubinstein games. Delegation is more likely to be profitable to the buyer. Indeed, he has less to lose by switching from the direct bargaining game to the delegation one.

Unfortunately, the model can not indicate any particular equilibrium to be played. In order to reduce the range of possible equilibria arising, we introduce different kinds of delegation costs. A common result is the possibility of prisoner's dilemma type games. The principals may decide to delegate even if they are both worse off with respect to a direct bargaining situation. As usual, the problem relies in the fact that the parties do not collaborate and in the fact that the hiring choice is unknown before the bargaining stage. The relative importance of these two arguments depends on the particular situation. In the workers-firms relation, for instance, it is well known that the parties will use a delegate. The main problem is that the conflict between parties is usually too strong to induce cooperation towards an efficient agreement. In private bargaining cases (i.e., the sale of houses), the contrary is true: the identity of the other party is likely to be unknown until the end.

When delegation costs are exogenous, agents' costs are more restrictive than principals' ones. This is due to the fact that the latter are time independent and supported on individual basis.

Finally, when we consider endogenous costs, inefficiency concerns are less obvious. These costs are also a source of income and the more the players are patient, the wider the range of bilateral delegation equilibria.

Further developments are possible and, to some extent, necessary. As we note in the introduction, we fail to consider important issues. A more realistic model should consider the presence of incomplete information: compensation schemes or the details of the delegation contracts are not always public. Moreover, the assumption of full bargaining power for the principals is quite extreme. A redistribution of power would lead to different payment schemes and therefore to different equilibrium payoffs.

## References


[1] Bester, H. and Sakovics, J. (2001). **Delegated Bargaining and Renegotiation**, *Journal of Economic Behavior and Organization*, 45, pp. 459-473.

[2] Burtraw, D. (1992). **Strategic delegation in bargaining**, *Economic Letters*, 38, pp. 181-185.

[3] Corts, K. S. and Neher, D. V. (2001). **Credible Delegation**, *Forthcoming in European Economic Review.*

[4] Crawford, V. P. (1982). **A Theory of Disagreement in Bargaining**, *Econometrica*, 50 (3), pp. 607-638.

[5] Fershtman, C.and Judd, K. L. (1987). **Equilibrium Incentives in Oligopoly**, *The American Economic Review*, 77 (5), pp. 927-940.

[6] Fershtman, C. and Kalai, E. (1997). **Unobserved Delegation**, *International Economic Review*, 38 (4), pp. 763-774.

[7] Fershtman, C., Judd, K. L. and Kalai, E. (1991). **Observable Contracts: Strategic Delegation and Cooperation**, *International Economic Review*, 32 (3), pp. 551-559.

[8] Jones, S. R. G. (1989). **Have your lawyer called my lawyer. Bilateral delegation in bargaining situations**, *Journal of Economic Behavior and Organization*, 11, pp. 159-174.

[9] Katz, M. L. (1991). **Game-playing agents: unobservable contracts as precommitments**, *RAND Journal of Economics*, 22 (3), pp. 307-328.

[10] Muthoo, A. (1996). **A Bargaining Model Based on the Commitment Tactic**, *Journal of Economic Theory*, 69, pp. 134-152.

[11] Muthoo, A. (1999). **Bargaining Theory with Applications**, Cambridge University Press, Cambridge.

[12] Nash, J. F. (1950). **The Bargaining Problem**, *Econometrica*, 18, pp. 155-162.

[13] Nash, J. F. (1951). **Non-Cooperative Games**, *The Annals of Mathematics*, 54 (2), pp. 286-295.

[14] Nash, J. F. (1953). **Two-Person Cooperatives Games**, *Econometrica*, 21, pp. 128-140.

[15] Osborne, M. J. and Rubinstein, A. (1990). **Bargaining and Markets**, Academic Press, San Diego, California.

[16] Polo, M. and Tedeschi, P.(2000). **Delegation Games and Side-Contracting**, *Research in Economics*, 54 (1), pp. 101-116.

[17] Rubinstein, A. (1982). **Perfect equilibrium in a bargaining model**, *Econometrica*, 50, pp. 97-109.

[18] Rubinstein, A. and Wolinsky, A. (1987). **Middlemen**, *The Quarterly Journal of Economics*, 102 (3), pp. 581-594.

[19] Schelling, T. C. (1956). **An Essay on Bargaining**, *The American Economic Review*, 46 (3), pp. 281-306.

[20] Schelling, T. C. (1960). **The Strategy of Conflict**, Harvard.University Press, Cambridge, Massachusetts.

[21] Schotter, A., Zheng, W. and Snyder, B. (2000). **Bargaining Through Agents: An Experimental Study of Delegation and Commitment**, *Games and Economic Behavior*, 30, pp. 248-292.

[22] Segendorff, B. (1998a). **Delegation and Threat in Bargaining**, *Games and Economic Behavior*, 23, pp. 266-283.

[23] Segendorff, B. (1998b). **Delegation of Bargaining and Power**, *Working Paper Series in Economics and Finance No. 248*, Stockholm School of Economics, Stockholm.

[24] Sutton, J. (1986). **Non-Cooperative Bargaining Theory: An Introduction**, *Review of Economic Studies*, 53 (5), pp. 709-724.

[25] Vickers, J. (1985). **Delegation and the Theory of the Firm**, *The Economic Journal*, Issue Supplement: Conference Papers, 95, pp. 138-147.