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Designing monetary and Fiscal policy rules in a New Keynesian model with rule-of-thumb consumers

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Designing monetary and fiscal policy rules in a New Keynesian model with rule-of-thumb consumers.[*]

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Abstract

This paper develops a small New Keynesian model augmented with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC henceforth) as in Gali' et al. (2004, 2007). The paper focuses on the consequences for the design of monetary and fiscal rules, of the bifurcation generated by the presence of ROTC on the demand side of the economy, in the absence of Ricardian equivalence. We find that, when fiscal policy follows a balanced budget rule, the amount of ROTC determines whether an active and/or a passive monetary policy in the sense of Leeper (1991) guarantees determinacy. When short run public debt assets are introduced, the amount of ROTC determines whether equilibrium determinacy requires a mix of active (passive) monetary policy and a passive (active) fiscal policy or a mix where policies are both active or passive. This set of equilibria has the potential to explain the empirical evidence on the U.S. postwar data on monetary and fiscal policy interactions.

JEL classification: E32; E62; H30.

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1 Introduction

The analysis of the properties of macro-policy rules has been one of the central themes of the recent literature on monetary and fiscal policy. This field of research has shown that simple rules seem to explain relatively well the observed policy choices as well as their role in different macroeconomic episodes. While this point of view is widely shared, most of the literature makes convenient assumptions, i.e. a fiscal policy which implies *Ricardian* equivalence, that allows monetary and fiscal policy rules to be studied separately. However, these assumptions are often questionable, and therefore it has been argued that the resulting conclusions of this approach could be misleading. Leith and Wren-Lewis (2000), Linnemann (2006), Davig and Leeper (2006) and Schmitt-Grohe' and Uribe (2007) are some of the recent works that point out how the assumptions regarding the interactions between monetary and fiscal policy are of crucial importance in understanding macro-policy rules. In particular, a common point of all these works is that, when, for any reason, Ricardian equivalence does not hold, fiscal policy cannot be recursively separated from the rest of the model and the equilibrium dynamics are determined by the interactions between monetary and fiscal policy.

In this paper we augment a standard New Keynesian (NK) model with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC) as in Galí et al. (2004, 2007). These consumers, who are not allowed to participate in financial markets, i.e. they cannot hold public debt in order to smooth consumption over time, but consume their available income in each period, stand next to standard forward looking agents (OPTC). From this, and independently of the tax instrument adopted, lump-sum taxes or proportional income taxation, the presence of ROTC implies a clear departure from Ricardian equivalence: both types of consumers pay the burden of public debt but only the optimisers benefit from it. Hence public debt becomes net wealth, therefore a relevant state variable which has to be taken into account for the equilibrium dynamics of the system.

Moreover, as stressed in the literature (Galí et al.; 2004, Di Bartolomeo and Rossi; 2007, Colciago; 2008, Bilbiie, 2008), the introduction of a set of ROTC can drastically change the determinacy conditions of an otherwise standard NK model. On this subject the main contribution can be found in Bilbiie (2008). The author shows that in a NK model with no capital accumulation, a Walrasian labour market and no fiscal policy, the presence of a share of ROTC may generate a bifurcation on the demand side of the economy which has dramatic consequences on the conduct of monetary policy. In particular, with a small share of ROTC, the traditional results on equilibrium determinacy hold: necessary and sufficient condition for determinacy is to have, using Leeper's (1991) definition, an active monetary policy, whereby nominal interest rate is adjusted such that the real rate increases in response to positive inflation. However, when the share of ROTC is above a specified threshold, determinacy requires a passive monetary policy, whereby nominal interest rate is adjusted such that the real rate decreases in response to positive inflation.

The basic intuition for this result is that when the monetary authority increases the interest rate, the system experiences downward pressure on wages. This, combined with a sticky price environment, implies an increase in profits which are held only by the optimiser consumers (OPTC

henceforth), i.e. each unit of increase in profits translates in more than a unit in OPTC wealth. With a high share of ROTC, the raise in OPTC wealth caused by the increase in profits may generate an increase in total demand, putting, via the Phillips curve, upward pressure on prices. A monetary authority wishing to stabilise the price level may therefore need to cut the real interest rate in the face of an inflationary shock.

The main contribution of this paper is to study the effects of the bifurcation generated by the presence of ROTC on the interactions between monetary and (a non *Ricardian*) fiscal policy.

To this end we conduct several exercises. First, we study the interactions between monetary and fiscal policy, assuming that monetary policy adopts a contemporaneous interest rate rule which is a function only of the inflation rate, i.e. a Taylor rule as in Clarida et al. (2000), and fiscal policy sets the income tax rate in every period in order to generate enough revenues to pay a level of public spending and service the long run level of public debt, without releasing short run public debt assets.

This type of fiscal rule, commonly known as *balanced budget rule,* has been studied in detail by Schmitt-Grohe and Uribe (1997) in a Real Business Cycle model with capital accumulation, and by Linnemann (2006) in a NK model with a contemporaneous monetary rule and no capital accumulation. While both works stress the destabilising role of such a fiscal rule, given the NK elements of our model, we use Linnemann's (2006) results as a benchmark for ours.

The author finds that with a balanced budget rule, an active monetary policy rule that reacts "*too strongly*" to inflation leads easily to the possibility of self fulfilling expectations, i.e. indeterminacy. In other words, in Linnemann's (2006) model, monetary policy has an upper limit in its active strength, and this upper limit is tighter the higher the long run level of public debt.

This result is a direct consequence of the distortive nature of fiscal policy and its interaction with monetary policy. If monetary policy increases "*too much*" the real interest rate in order to contrast higher inflation expectations, the burden of the service of public debt increases, therefore forcing fiscal policy to increase taxation in order to collect extra revenues. This increase in taxation feeds back on the endogenous variables of the model, inflation and output, via the supply side of the economy, the Phillips curve, generating a positive wedge between tax rate and current inflation, which could make the initial expectations of higher inflation self fulfilling, generating endogenous sunspots fluctuations. In our paper we show that even with a small share of ROTC, the upper bound on monetary policy which characterise Linnemann's (2006) results, gets looser, in turn helping to reestablish the validity of the Taylor principle for a wider range of policy parameters values. This is because a small proportion of ROTC strengthens the validity of the Taylor principle or, in other words, it increases the sensitivity of aggregate demand to interest rate movements. Hence monetary policy can reduce output to the desired level to contrast inflation with lower movements in interest rates, therefore generating a weaker fiscal response, avoiding sunspot fluctuations.

Furthermore, we find that, when the share of ROTC is above a specified threshold similar to the one found by Bilbiie (2008), both a strongly passive or a strongly active monetary policy can lead to equilibrium determinacy. As described above, a passive monetary policy, through its effect

on aggregate profits and financial portfolio, can reduce aggregate demand and, *ceteris paribus*, decreases the cost of servicing the public debt, avoiding the perverse effect of an increase in the tax rate on current inflation. On the other hand, a strong active monetary policy can expand aggregate demand. While higher output can have a destabilising effect on inflation stabilisation, it increases, *ceteris paribus*, government revenues, potentially implying a decrease in the tax rate and this, via the Phillips curve, can act as stabilisation device on inflation, leading to determinacy.

Next we assume a more general fiscal policy rule in which the fiscal authority is allowed to release short run public debt assets in order to balance its budget. This type of fiscal policy, jointly with a traditional interest rate type of monetary rule, allows us to analyse the equilibrium dynamics of our model under the active/passive logic of Leeper (1991).

The traditional benchmark results of this field of research are the following: a) an active monetary policy delivers a unique rational expectation equilibrium if and only if fiscal policy adopts a passive tax policy role, i.e. it raises tax revenues when public debt rises. However, if fiscal policy does not adopt a tax policy which implies public debt stabilisation- active fiscal policy- monetary policy has to abandon the Taylor principle, embracing a passive role. A passive/passive policy mix delivers indeterminacy while an active/active policy mix implies instability, i.e. no solution. b)the first type of regime (active monetary/passive fiscal) is more likely to deliver low inflation and a sustainable path for public debt. c) periods of passive monetary policy can substantially alter the propagation mechanism of the shocks to the fundamentals, Lubik and Schorfeide (2004).

However, as pointed by Favero and Monacelli (2005) and by Davig and Leeper (2006), the active/passive policy logic is not able to capture the macro-evidence of the US post-war data on monetary and fiscal policy regimes. Indeed the empirical investigations in these papers show long periods of policy regime mixes, i.e. both policies active or both passive, which are incompatible with the traditional results of the literature on monetary and fiscal policy interactions. While Favero and Monacelli (2005) remain completely agnostic on a possible theoretical explanation of their findings, Davig and Leeper (2006) explain the unconventional policy mixes resulting from the data with the introduction of macro-policy switches. They show that a standard New Keynesian model, where in each period macro policies have a probability of switching from active to passive and this probability is taken into account by the agents, is able to deliver a unique rational expectation equilibrium for any policy combination.

The results we present in this paper could be considered as complementary to the ones of Davig and Leeper (2006). In particular we find that when the share of ROTC is below the threshold previously described, determinacy requires either an active monetary policy jointly with a passive fiscal one or *viceversa*. When instead the share of ROTC is above the threshold, determinacy requires for monetary and fiscal policy to be both either active or passive.

Intuitively, this result is driven by the consequences of a share of ROTC on the demand side of the economy. Suppose, for example, that our system is affected by a large share of ROTC so that we are above the threshold previously described. When fiscal policy adopts a debt stabilisation policy, i.e. passive fiscal policy, monetary authority is free to stabilise inflation. As shown by Bilbiie

(2008) and Leith and von Thadden (2008) this is ensured by a passive monetary policy. If instead fiscal policy follows an active role, monetary policy has to abandon the inflation stabilisation policy, adopting an active role.

The remainder of the paper proceeds as follows: section 2 derives the model, section 3 outlines the results, section 4 conducts some robustness analysis with a more general specification of the monetary policy rules and different fiscal arrangements, and section 5 concludes.

2 The model

The economy consists of two types of households, a continuum of firms producing differentiated goods in a monopolistic competitive-sticky price environment, a perfectly competitive labour market, a central bank in charge of monetary policy and a government in charge of fiscal policy.

The totality of households is normalised to unity. Of this, a fraction $(1 - \lambda)$, with $\lambda \leq 1$, behaves in a traditional forward-looking, optimising way. Hence they maximise their (infinite) lifetime utility, hold profits coming from the monopolistic nature of the goods market, and participate in perfect and complete financial markets. We define the remaining λ households as rule-of-thumb consumers (ROTC) as in Galí *et al.* (2004, 2007). They care only for their current disposable income and they hold no financial assets nor any profit shares. For these consumers all their wealth is represented by their after tax wages and therefore they cannot smooth consumption over time. Variables with the suffix o and r indicate OPTC and ROTC respectively. A variable without time index identifies its steady state value.

2.1 Optimisers

The (lifetime) OPTC utility function has a standard form and it simply includes consumption and labour

$$U_t^o = E_0 \sum_{t=0}^{+\infty} \beta^t u^o \left(C_t^o, N_t^o \right) \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, E_t is the rational expectations operator, $u^o (\cdot, \cdot)$ represents instantaneous utility. We assume, in line with most of the literature, that $\frac{du^o}{dC_t^o} > 0$ and $\frac{du^o}{dN_t^o} < 0$. The shape of u^o is[1]

$$u^o \left(C_t^o, N_t^o \right) = \log C_t^o - \theta \frac{(N_t^o)^{1+\eta}}{1+\eta} \tag{2}$$

where C_t^o is the level of consumption of the OPTC, N_t^o is the OPTC labour supply. The parameter θ, with $\theta \in (0, \infty)$ indicates how leisure is valued relative to consumption. The parameter $\eta > 0$ is the inverse of the Frisch elasticity of labour supply and represents the risk aversion to variations in leisure.

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[1]We assume this shape of the utility function in order to make our results comparable with the existing literature on ROTC, i.e. Bilbiie et al.(2004), Gali' et al.(2007), Bilbiie(2008), Leith and von Thadden(2008).

The nominal OPTC flow budget constraint is

$$\int_0^1 P_t(j) C_t^o(j)\, dj + R_t^{-1}\frac{B_{t+1}}{1-\lambda} + \frac{E_t(Q_{t,t+1}V_{t+1})}{1-\lambda} = \left(W_t N_t^o + \frac{D_t}{1-\lambda}\right)(1-\tau_t) + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t S^o$$

(3)

where $P_t(j)$ is the price level of the variety of good j, W_t is the nominal wage, D_t are the nominal profits coming from the monopolistic competitive structure of the goods market, B_{t+1} is the nominal payoff of the one period risk-less bond purchased at time t, R_t is the gross nominal return on bonds purchased in period t, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoff and V_t is nominal payoff of a state-contingent asset portfolio.[2] The government is assumed to pay a level of public spending, G_t and the service of debt, levying a proportional income tax, τ_t. S^o is a steady state transfer such that at steady state the two types of agents have the same level of consumption and supply the same amount of labour.[3]

OPTC must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. This, combined with the CES Dixit-Stiglitz aggregator, results in a demand function for any single good that is downward sloping in the current price of the specific j good.

$$C_t^o(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t^o$$

where the price index is found by

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$$

at the optimum we have

$$\int_0^1 P_t(j) C_t^o(j)\, dj = P_t C_t^o$$

(4)

where the parameter ε represents the elasticity of substitution among goods and it is a measure of the market power held by each firm.

The budget constraint can be therefore rewritten as

$$P_t C_t^o + R_t^{-1}\frac{B_{t+1}}{1-\lambda} + \frac{E_t(Q_{t,t+1}V_{t+1})}{1-\lambda} = \left(W_t N_t^o + \frac{D_t}{1-\lambda}\right)(1-\tau_t) + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t S^o$$

(5)

Next the OPTC have to decide their labour supply and their intertemporal consumption allocation. This problem involves maximising the utility (1) subject to the budget constraint (5). The

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[2] Note that given all the financial assets are held by the OPTC, $V_{t+1} = (1-\lambda)V_{t+1}^o$. Therefore $V_{t+1}^o = \frac{V_{t+1}}{1-\lambda}$. The same thing holds for bonds and profits.

[3] The assumption of steady state homogeneity across consumer types is present in most of the literature on ROTC, see for example Gali' et al. (2007) Bilbiie (2008) and Colciago (2008). As in Gali' et al. (2007), the steady state transfer simplifies greatly the algebra but is innocuous for our results.

first order condition for the intertemporal consumption allocation is

$$\beta\left(\frac{C_t^o}{C_{t+1}^o}\right)\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}$$

taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\left[\left(\frac{C_t^o}{C_{t+1}^o}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = 1 \tag{6}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is implied by the non arbitrage condition. This expression is the familiar *Euler equation* for consumption. It describes the desire to smooth consumption over time once the opportunity cost implied by the real interest rate has been taken into account. The first order condition with respect to labour states that the *marginal rate of substitution* between labour and consumption must be equal to the after tax real wage

$$\theta\left(N_t^o\right)^\eta C_t^o = \frac{W_t}{P_t}\left(1 - \tau_t\right)$$

From the last expression one can see that taxation distorts the leisure-consumption choice. Any change in the tax rate has a direct effect on real wage and therefore on the marginal rate of substitution between consumption and labour.

2.2   Rule of Thumb Consumers

The ROTC utility function is represented by a single period expression. In particular, following Galí *et al.* (2004, 2007), it is assumed that the shape of the *instantaneous* utility is the same for the two types of consumer. Therefore

$$U_t^r = \log C_t^r - \theta\frac{\left(N_t^r\right)^{1+\eta}}{1+\eta} \tag{7}$$

As stressed above, the ROTC do not participate in financial markets and do not hold profits. Their budget constraint can be expressed as follows

$$\int_0^1 P_t\left(j\right)C_t^r\left(j\right)dj = W_tN_t^r\left(1 - \tau_t\right) - P_tS^r \tag{8}$$

Where $C_t^r\left(j\right)$ and N_t^r are the level of consumption of each j product and the labour supply of the ROTC. Furthermore, it is assumed that similarly to the behaviour of the OPTC, the ROTC exploit any relative price differences in creating their consumption basket. Hence, at the optimum

$$P_tC_t^r = \int_0^1 P_t\left(j\right)C_t^r\left(j\right)dj \tag{9}$$

On the consumption side the ROTC are forced to consume all their income in each period, therefore consumption can easily be inferred by combining (8) with (9). The first order condition

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for the optimal supply of labour implies

$$\theta \left(N_t^r \right)^\eta C_t^r = \frac{W_t}{P_t} \left(1 - \tau_t \right) \tag{10}$$

The last two expressions state the ROTC "*hand to mouth*" attitude towards consumption. This means that they consume in every period all their resources which, as previously stated, are equal to their after tax income. The optimal supply of labour takes the same analytical form as that of the OPTC.

2.3 Firms

The firms' problem is completely standard and therefore could be skipped by some readers without loss of continuity.

In this economy, firms are assumed to possess an identical production technology. This production function is linear in labour and can be written as

$$Y_t \left(j \right) = N_t \left(j \right) \tag{11}$$

Furthermore, it is worth noting that each firm faces the following demand function

$$Y_t \left(j \right) = \left(\frac{P_t \left(j \right)}{P_t} \right)^{-\varepsilon} Y_t \tag{12}$$

where

$$Y_t = \left[\int_0^1 Y_t \left(j \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{13}$$

Following the NK literature it is assumed that prices are sticky. We model this feature of the economy following Calvo (1983). In each period there is a (randomly selected) set of firms, $(1 - \alpha)$ with $\alpha < 1$, who reset their price optimally, while the remaining α keep their prices fixed. When a firm is allowed to reset its prices, it takes into account the expected future stream of profits discounted for the probability of not resetting its prices. In particular the maximisation problem of a price setter can be written in real terms as

$$\max_{P_t^* (j)} E_t \sum_{i=0}^{+\infty} \alpha^i q_{t,t+i} \left(\left(\frac{P_t^* \left(j \right)}{P_{t+i}} \right) Y_{t+i} \left(j \right) - mc_{t+i} Y_{t+i} \left(j \right) \right) \tag{14}$$

Where $q_{t,t+1} = \beta \left(\frac{C_t^o}{C_{t+1}^o} \right)$ is the real stochastic discount factor and $mc_t = W_t / P_t$ represents the real marginal costs. The first order condition with respect to $P_t^* (j)$ is

$$\frac{P_t^* \left(j \right)}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o} \right) \left(mc_{t+i} \left(P_{t+i} \right)^\varepsilon Y_{t+i} \right)}{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o} \right) \left(P_{t+i} \right)^\varepsilon P_{t+i}^{-1} Y_{t+i}} \tag{15}$$

while the price level follows

$$P_t^{(1-\varepsilon)} = \left[(1 - \alpha) P_t^{*(1-\varepsilon)} + \alpha P_{t-1}^{(1-\varepsilon)} \right] \tag{16}$$

2.4 Aggregation rules and market clearing condition

The aggregate expressions for consumption and labour are simply the weighted average of the single consumer type variables. Therefore aggregate consumption follows

$$C_t = \lambda C_t^r + (1 - \lambda) C_t^o \tag{17}$$

and aggregate labour

$$N_t = \lambda N_t^r + (1 - \lambda) N_t^o \tag{18}$$

In the absence of capital accumulation, everything produced must be consumed in the same period. Furthermore each product j can be purchased by the private sector or by the government as

$$Y_t(j) = C_t(j) + G_t(j) \tag{19}$$

In aggregate, given the price dispersion implied by Calvo price setting

$$Y_t s_t = N_t \tag{20}$$

where $s_t = \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj$. Given our assumption of zero steady state inflation, fluctuations of s_t around the steady state are of second-order importance[4], and therefore can be ignored in the present analysis which employs a linearised framework, i.e. first order approximation around the non-stochastic steady state. In equilibrium total demand is equal to total supply. Hence

$$Y_t = C_t + G_t \tag{21}$$

2.5 The Government

The government uses income tax revenues, $P_t \tau_t Y_t$ to finance a stream of public spending, $P_t G_t$[5], and the service of public debt. Therefore the government budget constraint can be expressed as

$$R_t^{-1} B_{t+1} = B_t - P_t \tau_t Y_t + P_t G_t \tag{22}$$

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[4]A detailed discussion of this can be found in Woodford (2003).

[5]As the private sector, the government exploits any price differences in the market to form its consumption basket G_t. This jointly with a CES aggregator gives the following downward sloping demand function for each single public spending good. $G_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} G_t$

where $P_t G_t - \tau_t P_t Y_t$ is the primary deficit. The government budget constraint can be expressed in real terms as

$$R_t^{-1} b_{t+1} = \frac{b_t}{\pi_t} - \tau_t Y_t + G_t \tag{23}$$

where $b_{t+1} = \frac{B_{t+1}}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$.

2.6 Monetary Policy

Monetary policy sets the nominal interest rate, R_t, in every period. Following the literature on monetary policy, for example Clarida *et al.* (2000), we approximate monetary policy by a simple Taylor rule of the type

$$R_t = R \left(\pi_t \right)^{\phi_\pi} \tag{24}$$

Where $R = \frac{1}{\beta}$ is the steady state interest rate. The single policy parameter ϕ_π in (24) is the Taylor coefficient, as discussed in the literature on interest rate rules inspired by Taylor (1993). Accordingly, following Leeper (1991), monetary policy is called *active* (or *passive)* if the nominal interest rate, R_t, rises more (or less) than one-for-one with the current inflation rate, i.e. if $\phi_\pi > 1$ ($\phi_\pi < 1$).

2.7 Fiscal Policy

Regarding fiscal policy, we assume a government revenue rule of the type

$$Y_t \tau_t = \delta_0 + \delta_1 \frac{\tau Y}{b} (b_t - b) + \delta_2 \tau \left(Y_t - Y \right) \tag{25}$$

where $\delta_0 = (1 - \beta) b + G$ and δ_1 and δ_2 are policy parameters identifying the relative weight given to debt stabilisation and output stabilisation. This fiscal rule has the characteristic of being steady state neutral (at steady state the fiscal rule collapses to $\tau = \frac{(1-\beta)b}{Y} + \frac{G}{Y}$ which is equal to $\tau = \delta_0/Y$).

Unlike monetary policy, there is no widely accepted specification for fiscal policy. The rule we assume is similar to the one considered in Linnemann (2006), Davig and Leeper (2006, 2007) and Schmitt-Grohe and Uribe (2007). This type of rule has two main advantages. The first is that it allows the study of the interactions between monetary and fiscal policy under the logic of Leeper (1991).[6] Second is that these rules are receiving particular attention from an empirical point of view, given their ability to capture many stylised fiscal facts of US postwar data.[7]

Several special cases of fiscal policy will be specified and discussed in detail below. One prominent example is a fiscal policy which follows a balanced budget rule, i.e. no short run public debt fluctuations, in the fashion of Schmitt-Grohe and Uribe (1997) and Linnemann (2006). In this case fiscal policy has to collect enough revenues to repay the cost of public debt and a level of government spending. This type of fiscal policy can be described by simply imposing in the government budget

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[6] Following the definition of Leeper (1991), we call the fiscal rule (25) *passive* if $\delta_1 > \left(\frac{1}{\beta} - 1 \right)$, i.e positive fiscal response to increase in public debt from its steady state value, while it is *active* in the opposite case of $\delta_1 < \left(\frac{1}{\beta} - 1 \right)$.

[7] See inter alia Perotti (2007).

constraint (23) that $b_t = b \ \forall t$ as

$$\tau_t Y_t = G_t + b \left(\frac{1}{\pi_t} - \frac{1}{R_t} \right) \tag{26}$$

2.8 Equilibrium

The non linear structural equations of the model are log-linearised around the non stochastic steady state.[8] Furthermore, we present the model in terms of aggregate variables. These equations are: the New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\left(\frac{1}{\gamma_c} + \eta \right) \widehat{Y}_t - \frac{(1-\gamma_c)}{\gamma_c} \widehat{G}_t + \frac{\tau}{1-\tau} \widehat{\tau}_t \right) \tag{27}$$

where $\gamma_c = \frac{C}{Y}$ is the steady state consumption to output ratio. The dynamic IS curve augmented for the presence of ROTC

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \Theta \left(\frac{1-\gamma_c}{\gamma_c} \right) \left(\widehat{G}_{t+1} - \widehat{G}_t \right) - \Theta \left(\widehat{R}_t - E_t \pi_{t+1} \right) \tag{28}$$

where we define Θ as the elasticity of the demand side of the economy to changes in real interest rate. This parameter is defined as $\Theta = \left(\frac{1}{\gamma_c} - \eta \frac{\lambda}{1-\lambda} \right)^{-1}$. The market clearing condition,

$$\widehat{Y}_t = \gamma_c \widehat{C}_t + (1-\gamma_c) \widehat{G}_t \tag{29}$$

the monetary policy rule,

$$\widehat{R}_t = \phi_\pi \pi_t \tag{30}$$

and the fiscal policy, described by the government budget constraint and the tax rule when public debt is allowed to fluctuate along the business cycle as

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta} \left(\widehat{b}_t - \pi_t - \frac{\tau}{\gamma_b} \left(\widehat{Y}_t + \widehat{\tau}_t \right) + \frac{G}{b} \left(\widehat{G}_t \right) \right) \tag{31}$$

and

$$\widehat{\tau}_t = \delta_1 \widehat{b}_t + (\delta_2 - 1) \widehat{Y}_t \tag{32}$$

where γ_b represents the steady state public debt to GDP ratio and it is equal to $\frac{b}{Y}$. In the case of balanced budget rule, fiscal policy is or simply described by the government budget constraint where $\widehat{b}_t = 0 \ \forall t$ as

$$0 = \widehat{R}_t + \frac{1}{\beta} \left(-\pi_t - \frac{\tau}{\gamma_b} \left(\widehat{Y}_t + \widehat{\tau}_t \right) + \frac{G}{b} \left(\widehat{G}_t \right) \right) \tag{33}$$

A few points are worth stressing.

Firstly, this model displays a clear departure from the so called *Ricardian* equivalence of fiscal

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[8] Algebrical details are provided in Appendix A.

policy. Both types of consumer pay the burden of public debt, but only the optimisers benefit from it, holding public debt assets. Therefore public debt is net wealth and, independently of how it is financed, it implies a wealth transfer from the ROTC to the OPTC. Moreover, fiscal policy levies a proportional income tax, which distorts the marginal rate of substitution between consumption and leisure. This feeds back directly into the NKPC via the labour supply, i.e. a higher tax rate induces OPTC to substitute leisure from the future to the present, lowering labour supply, increasing the firms' real marginal cost, and thus generating a positive wedge between the tax rate and inflation. These properties of the model, together with the non neutral effects of monetary policy due to sticky prices, imply that: a) the government budget constraint cannot be separated from the rest of the model, i.e. government debt turns into a relevant state variable which needs to be accounted in the analysis of local equilibrium dynamics, b) that equilibrium dynamics are driven by a genuine interaction of monetary and fiscal policy.

Secondly, the presence of ROTC dramatically affects the dynamic IS equation (28), i.e. the demand side of the economy, via Θ. This parameter is linked in a non-linear way to λ, the share of ROTC, and to η, the inverse of the Frisch elasticity of labour. Both the size and the sign of Θ can potentially alter the transmission mechanism and local determinacy properties of the model. The intuition for this result is as follows. Assume the monetary authority suddenly increases the real interest rate. This increase shifts downward the consumption of the optimisers, through the usual intertemporal Euler equation channel. This, *ceteris paribus*, generates a reduction in labour demand and therefore in nominal wages. The reduction in wages lowers firms marginal costs. Consequently prices fall, via the NKPC. Due to the *Walrasian* structure of the labour market and to the Calvo price mechanism, nominal wages decrease more than prices, implying as a result lower real wages.

Furthermore, the form of the utility function, i.e. log-consumption, together with the assumption of no capital accumulation and the shape of the tax structure, causes the ROTC to supply labour inelastically[9] and therefore to pass through their consumption any change in real wage. This is not all. The asymmetric decrease in wages and prices, i.e real wages decrease more than real prices, generates an increase in profits. Note that the OPTC hold all the financial activities present in the system, i.e. profits share and public debt bonds. In particular they hold $(1-\lambda)^{-1}$ of total firms share. If, for example, profits increase by one unit, dividend income of asset holders (OPTC) increases by $\frac{1}{1-\lambda} > 1$ units. The same thing is true for public debt bonds: a unit of increase in the real return of public debt generates a $\frac{1}{1-\lambda} > 1$ increase in the optimisers' wealth.[10] These financial effects work in the opposite direction relative to the traditional intertemporal Euler equation: while the latter imply a contractionary effect of higher real interest rate, the opposite is true for the former.

As argued by Bilbiie et al. (2004) and Bilbiie (2008), the sign of Θ determines which of these two channels prevails. Of course, the sign of Θ depends on the share of ROTC, i.e. the higher λ, the

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[9]Although this assumption simplifies the algebra and the economic mechanism behind our results, it does not drive them. This is shown when other types of fiscal arrangment are introduced.

[10]Note that these effects of interest rate movements on financial portfolio would be irrelevant if $\lambda = 0$, i.e. no ROTC.

higher the financial channel of interest rate, and on the elasticity of labour supply (of the OPTC), i.e. the higher η, the higher the sensibility of real wage to interest rate movements.[11] A necessary condition for $\Theta > 0$ is

$$\lambda < \frac{1}{(1 + \eta\gamma_c)} \tag{34}$$

Figure 1 sketches the sign of Θ for a given value of γ_c in the $(\lambda - \eta)$ space. As one can see, Θ remains positive for combinations of high values of the Frisch elasticity of labour supply, i.e. low η, and high shares of ROTC, i.e high λ, or *viceversa*. The reason is now understood: when the share of ROTC is low (or the total labour supply is inelastic), the intertemporal Euler equation transmission channel prevails on the financial one: an increase in the real interest rate decreases the economic activity. Furthermore inside the parameter values where Θ is positive an increase in the share of ROTC increases the sensitivity of aggregate demand to interest rate movements, i.e. lower real wages imply lower consumption for the ROTC and the traditional intertemporal effect prevails on the financial one for the optimisers. This ceases to be true when $\Theta < 0$: an increase in the real rate could potentially expand aggregate demand.[12]

It is quite intuitive that these effects have dramatic consequences on the equilibrium dynamics: as discussed in Bilbiie (2008), a monetary economy with a share of ROTC that displays a negative Θ requires, for the RE equilibrium to be unique, the monetary policy to abandon the Taylor principle adopting a passive monetary rule.

Here we explore the consequences of the sign of Θ on the RE equilibrium determinacy in a model where, due to the presence of a distortive fiscal policy, equilibrium dynamics are driven by a genuine interaction of monetary and fiscal policy.

2.9 Determinacy

Given the focus of the paper on the equilibrium dynamics of the model we assume that non fundamental shocks hit the economy. We further assume that government spending is always at its steady state level, i.e. $G_t = G \quad \forall t$.

We combine the log-linearised dynamic equations presented in 2.8 in order to obtain a system of difference equations describing the equilibrium dynamics of our economy. After some algebra, we can reduce this system to one involving three variables[13] as

$$AE_t \{x_{t+1}\} = B \{x_t\} \tag{35}$$

where $x_t \equiv \left(\widehat{y}_t, \ \pi_t, \ \widehat{b}_t \right)'$ and $A = \begin{bmatrix} 1 & -\Theta & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

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[11] High sensitivity of real wage to interest rate movements enhances the financial effects described.

[12] Bilbiie (2008) refers to this as the "*inverted aggregate demand logic*". We use the same terminology in section 5.

[13] In the case of balanced budget fiscal rule, the system can be reduced to one involving only two variables. This will be discussed in 3.1.

$$
B = \begin{bmatrix} 1 & -\Theta\phi_\pi & 0 \\ -\kappa\left(\frac{1}{\gamma_c} + \eta + \frac{\tau}{1-\tau}\left(\delta_2 - 1\right)\right) & 1 & -\kappa\left(\frac{\tau}{1-\tau}\delta_1\right) \\ -\frac{1}{\beta}\left(\frac{\tau Y}{b}\delta_2\right) & \phi_\pi - \frac{1}{\beta} & \frac{1}{\beta}\left(1 - \frac{\tau}{\gamma_b}\delta_1\right) \end{bmatrix}
$$

In order to study the determinacy of the system we need to analyse the eigenvalues of $J = A^{-1}B$. Given that the x vector displays two non-predetermined variables (inflation and output) and one predetermined (public debt), determinacy requires the J matrix[14] to have two eigenvalues outside the unit circle and one inside the unit circle. Alternatively if more than one eigenvalue of J lie inside the unit circle, the system is locally undetermined: from any initial value of the stock of public debt there exists a continuum of equilibrium paths converging to the steady state, and the possibility of sunspots fluctuations arises. If instead there are no eigenvalues inside the unit circle, there is no solution to (35) that converges to the steady state.[15]

2.10 Calibration

The model is calibrated to a quarterly frequency.[16] We assume the elasticity of substitution among goods, ε, is equal to 6. This implies a steady state markup of 20%, which is in line with most of the macro literature. The discount factor β has been fixed at 0.99. As a consequence, the real annual interest rate is 4%. θ, the parameter of relative disutility of labour to consumption, has been chosen to obtain an average steady state labour supply of 1/3. The steady state ratio between private consumption and total output, γ_c, is 0.75. This value implies a steady state ratio of government spending over output of 25%, which is in line with the level of public consumption in most industrialised countries, see Galí et al.(2007). As in most of the NK literature, we assume that prices remain unchanged on average for one year. Therefore α, the parameter ruling the degree of price stickiness, is fixed at 0.75. When not differently specified, these parameters are kept at their baseline values throughout the determinacy exercise. Next we turn to the parameters for which some sensitivity analysis is conducted, by examining a range of values in addition to their baseline settings. Given the aim of the paper, the model has been solved with several pairs of λ, the share of ROTC and η, the inverse Frisch elasticity of labour supply, depending whether we want to study a situation where Θ is positive or negative.[17]

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[14]For the sake of clarification

$$
J = \begin{bmatrix} 1 + \Theta\Psi & \Theta\left(\phi_\pi - \frac{1}{\beta}\right) & \Theta\Lambda \\ -\Psi & \frac{1}{\beta} & -\Lambda \\ -\frac{\delta_2\tau}{\beta\gamma_b} & \left(\phi_\pi - \frac{1}{\beta}\right) & \frac{1 - \frac{\tau\delta_1}{\gamma_b}}{\beta} \end{bmatrix}
$$

With $\chi = \frac{1}{\gamma_c} + \eta + \frac{\tau}{1-\tau}\left(\delta_2 - 1\right)$, $\Lambda = \frac{\delta_1\kappa\tau}{\beta(1-\tau)}$ and $\Psi = \frac{\chi\kappa}{\beta}$

[15]Unless the initial level of the public debt stock is at its steady state value, in which $x_t = 0$ for all t is the only non explosive solution.

[16]We insert this paragraph on calibration before presenting the analytical results. This is because in the section where we present the analytical results, we use simple numerical examples based on the calibration presented here, in order to generate the economic intuitions behind our results.

[17]In particular we allow η to vary in a range between 1 and 4 and λ between 0.05 and 0.5. These values are consistent with most empirical literature.

In the case of a balanced budget fiscal policy rule, the determinacy has been studied for different values of γ_b, the steady state level of public debt to GDP ratio, while in the case of general fiscal rules, we fix $\gamma_b = 2.4$, a value which implies an annual steady state ratio of public debt to output equal to 60% and a steady state level of taxation of $27, 4\%$ of total output. The determinacy, and consequently the calibration exercise, has been studied with different values of δ_2, the fiscal policy parameter of the output gap. A value of $\delta_2 = 0$ implies a policy rule very similar to the one studied by Leeper (1991), and describes a situation in which the total government revenues do not respond to output fluctuations. We furthermore define a countercyclical (procyclical) fiscal policy in terms of output if $\delta_2 > 0$ $(\delta_2 < 0)$. Similarly, in order to describe the active-passive policy mix, the determinacy conditions is analysed for a broad range of policy parameters[18], ϕ_π and δ_1.

3 Results

3.1 Balanced Budget Rule

As a first step in analysing the interaction between monetary and fiscal policy with a share of ROTC, we study the equilibrium dynamics of the model in the case where the government has to balance its budget in every period without accessing to short run public debt assets. Such a fiscal policy implies that the tax rate is fixed in every period to satisfy[19]

$$\widehat{\tau}_t = \frac{\gamma_b}{\tau} \left(\widehat{R}_t - \frac{1}{\beta} \widehat{\pi}_t \right) - \widehat{Y}_t \tag{36}$$

Thus it is assumed there is a historical inherited stock of real public debt, on which interest has to be paid by the government, but this stock never changes because the tax rate is adjusted appropriately. With a balanced rule of this type the dynamic system can be written as

$$E_t \{x_{t+1}\} = J^{br} \{x_t\} \tag{37}$$

where $x_t = \{\widehat{Y}_t, \pi_t\}$, $J^{br} = \begin{bmatrix} 1 + \frac{\Lambda_1 \Theta}{\beta} & \Theta \phi_\pi - \frac{\Theta(1+(1-\phi_\pi \beta)\Psi)}{\beta} \\ -\frac{\Lambda_1}{\beta} & \frac{(1+(1-\phi_\pi \beta)\Psi)}{\beta} \end{bmatrix}$, with $k = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$, $\Psi = \frac{k\gamma_b \varepsilon}{(\varepsilon-1)}$ and $\Lambda_1 = k \left(\frac{1}{\gamma_c} + \eta - \frac{\tau}{1-\tau} \right)$.

In line with most of the macroeconomic literature we impose that $\phi_\pi > 0$. We further assume that $\tau < \left(1 + \frac{1}{\frac{1}{\gamma_c}+\eta} \right)^{-1}$. This implies that $\Lambda_1 > 0$.

The restriction on τ greatly simplifies the algebra and it is mild in empirical terms. Consider for example a standard parametrisation where $\gamma_c = 0.75$ and $\eta = 1$. The assumption on τ implies that the tax rate has to be smaller than 70%. With $\gamma_c = 0.75$ and $\eta = 4$, the restriction implies that τ has to be smaller than $84, 7\%$. Given that both variables are non-predetermined, determinacy requires both eigenvalues of J^{br} lying, in absolute values, outside the unit circle. As previously stated the sign

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[18] In particular we allow $\phi_\pi \in (0, 6)$ and $\delta_1 \in (-0.5, 2)$.

[19] We continue to assume that $G_t = G \ \forall t$.

of the elasticity of demand to the real interest rate, Θ, changes markedly the dynamic properties of the model. Let us first assume $\Theta > 0$. In this case[20], necessary and sufficient conditions for determinacy require

$$\text{If } \Lambda_1 > 2\frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi > 1 \tag{38}$$

$$\text{else if } \Lambda_1 < \frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \min\{\xi_1, \xi_2\} \tag{39}$$

$$\text{else if } \frac{\Psi\beta}{\Theta} < \Lambda_1 < 2\frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \xi_2 \tag{40}$$

With $\xi_1 = \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}$ and $\xi_2 = \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Theta\Lambda_1}$. (38) represents the case with no or very low level of steady state public debt, or high values of Θ. As in any standard New Keynesian sticky price model, the only condition for equilibrium determinacy is to have an active monetary policy, i.e. $\phi_\pi > 1$. Two main reasons drive this result. First, with $\Theta > 0$, the effect of an interest rate change on the economy follows the standard "*Taylor principle*" logic: a higher interest rate generates a contraction in aggregate demand and, through the NKPC, downward pressure on inflation. For any given level of $\phi_\pi > 1$, this contraction of aggregate demand is positively correlated with Θ. Therefore the higher Θ, the easier it is for monetary policy to keep inflation under control. Second, for values of the steady state ratio of public debt to output, γ_b, close to zero, the feedback of monetary policy on the government budget constraint is very limited. The tax rate moves only to balance changes in output and this movement does not imply any major feedback on the endogenous variables of the model.[21]

This stops being partly true when (39) or (40) are verified. As in the previous case monetary policy has to adopt an active role, but this is now constrained by some upper bounds which are functions of the structural parameters of the model. They depend, among other things, on the long run level of debt, the share of ROTC, the Frisch elasticity of labour supply and the degree of price stickiness. Note that when $\frac{\Psi\beta}{\Theta} < \Lambda_1 < 2\frac{\Psi\beta}{\Theta}$, ξ_1 is not binding, meanwhile, when $\Lambda_1 < \frac{\Psi\beta}{\Theta}$, ξ_1 is more likely to bind than ξ_2 for standard parameter values. For example, suppose that $\alpha = 0.75$, $\beta = 0.99$, $\gamma_b = 2.4$, $\eta = 1$, $\lambda = 0.3$. This set of parameters implies $\Lambda_1 = 0.1679$, $\Psi = 0.28$, $\Theta = 1.17$, $\xi_1 = 3.02$ and $\xi_2 = 12.407$. With instead $\gamma_b = 3$ (all other parameters constant), $\xi_1 = 2.04$ and $\xi_2 = 9.20$.

These upper bounds are directly generated by the distortive nature of fiscal policy. Let for instance assume agents suddenly expect higher inflation. Monetary policy adopting an active role increases the real interest rate so as to decrease current aggregate demand and thus stabilise inflation via the NKPC. The magnitude of the effect of an interest rate increase on aggregate output via the IS equation depends in a non-trivial way on Θ, i.e the higher Θ, the more sensitive the aggregate demand on monetary policy. On the other hand a higher interest rate feeds back on the government

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[20]In Appendix B we provide formal proof of this determinacy results.

[21]The distortive nature of fiscal policy implies a Laffer curve in the government revenues. However the peak of the Laffer curve happens for steady state tax rate values which are far above to the ones assumed in this analysis. For a detailed discussion see Schmitt-Grohe and Uribe (1997).

budget constraint, generating an increase in the cost of the service of public debt and therefore an upward pressure on the tax rate. Note that the higher the level of steady state public debt, the higher the tax rate increase for each increase in interest rate. Furthermore, the contractionary effect of monetary policy on output implies a decrease in the government revenues tax base, which causes a further increase in the tax rate. Moreover each increase in the tax rate feeds back positively, via the NKPC, on current inflation. This positive wedge can, for high levels of public debt or, *ceteris paribus*, for high responses of monetary policy to inflation, neutralise the initial attempt of monetary policy to stabilise inflation via a reduction of output, making the initial expectations of higher inflation self-fulfilling.

Figure (2) displays determinacy analysis in the $(\phi_\pi - \gamma_b)$ space for different parameter combinations of the share of ROTC, λ, and the inverse Frisch elasticity of labour supply, η. With low levels of public debt, the only condition for determinacy is to have $\phi_\pi > 1$, i.e. the Taylor principle. Furthermore, the constraints on the monetary policy parameter are less likely to bind the higher is the share of ROTC or the lower is the Frisch elasticity on labour supply (high η). The reason is now well understood. Within the parameter values where $\Theta > 0$, a high share of ROTC, or a more elastic aggregate labour supply, increases the effect of an interest rate changes on aggregate demand, preventing the fiscal policy feedback on the supply side of the economy to generate self-fulfilling expectations. When, for example, $\eta = 3$ and $\lambda = 0.3$ the only condition to obtain determinacy is $\phi_\pi > 1$.

We now turn to study of the determinacy properties of the model when $\Theta < 0$. Necessary and sufficient conditions for determinacy require

$$0 < \phi_\pi < \min\{1, \xi_1\} \ \cup \ \phi_\pi > \max\{1, \xi_2\} \tag{41}$$

In this case there are two determinacy spaces. In the first one, monetary policy has to adopt a passive role, i.e. $\phi_\pi < 1$. This conduct may have an upper limit represented by ξ_1. In the other determinacy space monetary policy needs to adopt an active role, with a potential downward limit represented by ξ_2. As before, both ξ_1 and ξ_2 depend crucially on the structural parameters of the model. In particular, ξ_1 is increasing in λ, η and γ_b, while ξ_2 is increasing in λ and η, and decreasing[22] in γ_b. We start by explaining the first determinacy space, i.e. $\phi_\pi < 1$. Suppose, for example agents suddenly expect higher inflation. The monetary authority, adopting a passive role, cuts the real interest rate. This cut contracts aggregate demand through a decrease in the financial activities held by the optimiser consumers. A decrease in the real rate and in output have opposite effects on the government budget constraint: a lower real interest rate, cutting the cost of the service of public debt, implies a decrease in the tax rate, while a decrease in output, lowering the government revenue base, implies an opposite effect. As stressed above an increase in the tax rate could have, through the supply side of the economy, a destabilising effects on inflation. Therefore if the combination of monetary policy and output increases the tax rate, the initial expectations

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[22] For example with $\varphi = 3$, $\lambda = 0.35$ and $\gamma_b = 2$, $\xi_1 = 0.17$ and $\xi_2 = 1.93$, while with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 2$, $\xi_1 = 0.62$ and $\eta_2 = 6.81$. Finally with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 3$, $\xi_1 = 0.71$ and $\xi_2 = 5.19$.

could be self-fulfilling. This situation is more likely to happen with low values of γ_b or high values of ϕ_π, i.e. lower monetary feed back on fiscal policy, low values of λ and η, i.e. higher sensitivity of aggregate demand[23] to interest rate movements.

The second area of determinacy requires monetary policy to be active with a lower bound represented by ξ_2. As before, let us assume agents suddenly expect higher inflation. The monetary authority would increase the real rate which, given its effect on financial assets, expands aggregate demand. This expansion feeds back on the government budget constraint generating an increase in the tax base and therefore a reduction of the tax rate. A reduction of the tax rate can stabilise inflation through the NKPC. However, a higher interest rate feeds back on fiscal policy causing an increase in tax rate and this, *ceteris paribus*, puts upward pressure on prices. It is therefore important for determinacy that the effect of output on fiscal policy overcompensates the monetary one. When this happens the decrease in the tax rate stabilises current inflation contrasting the initial expectations of higher inflation. As stressed before, the higher λ and η, the lower the sensitivity of aggregate demand to monetary policy, and therefore more likely that, for each increase in the real interest rate, tax rate increases, raising the possibility of sunspot fluctuations.

Figure 3 displays determinacy in the $(\phi_\pi - \gamma_b)$ space for different values of λ and η. As stressed above, increasing these two parameters, (when Θ is negative) lowers the sensitivity of aggregate demand to interest rate movements. This increases the possibility of determinacy when monetary policy adopts a passive role, while the opposite is true when monetary policy is active.

This simple exercise helps us to motivate and explain the importance of inserting fiscal policy when analysing the effects on the equilibrium determinacy of a share of ROTC. First of all, when $\Theta > 0$, a balanced budget rule delivers determinacy for parameter values which are consistent with the empirical evidence. This result is a clear departure from Linnemann (2006). Linnemann finds that in a standard NK model with a balanced budget fiscal rule, an active monetary policy could, through its feedback on fiscal policy and the feedback of fiscal policy on aggregate supply, easily lead to indeterminacy even for low positive values of long run public debt. The differences of our results stem from an increased sensitivity of aggregate demand to monetary policy due to the presence of ROTC.

Similarly, when $\Theta < 0$, the presence of balanced budget fiscal rule, through its feedback on the endogenous variables of the model, helps to reestablish the Taylor principle within realistic monetary policy responses to inflation fluctuations. This result extends the ones found by Bilbiie (2008) in the absence of fiscal policy, where a contemporaneous active monetary policy rule could deliver determinacy only for implausibly high levels of the monetary parameter ϕ_π.

3.2 Endogenous Debt

Here we study a more general version of monetary/fiscal policy mix. Short run public debt fluctuations are allowed and fiscal policy can be represented by the government budget constraint (31) and by the tax rate rule (32). Note that, differently from the previous case of a balanced budget

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[23]Note that with $\lambda = 0.35$ and $\varphi = 3$, $\Theta = -3.54$, while with $\lambda = 0.5$ and $\varphi = 4$, $\Theta = -0.37$.

rule, here the tax rate is a policy instrument which can be discretionally set according to δ_1 and δ_2. Given the lack of a straightforward and intuitive analytical result for the determinacy analysis of this exercise, we rely on numerical solutions.

Figure 4 displays the determinacy analysis in the $(\phi_\pi - \delta_1)$ space for different values of the fiscal parameter on output, δ_2, when Θ is positive, i.e. $\eta = 1$ and $\lambda = 0.3$.

As previously described, when $\Theta > 0$ the monetary policy effects on the system follow the common wisdom. Hence the presence of ROTC does not alter Leeper's (1991) logic. In other words equilibrium determinacy is guaranteed by an active (passive) monetary policy, $\phi_\pi > 1$, $(\phi_\pi < 1)$ and a passive (active) fiscal policy, $\delta_1 > \left(\frac{1}{\beta} - 1\right)$, $\left(\delta_1 < \left(\frac{1}{\beta} - 1\right)\right)$.[24] When both policies are passive, the system displays an infinite number of solutions and the possibility of endogenous sunspot fluctuations arises. When both policies are active there is no solution to (35). Furthermore it is interesting to note how the fiscal parameter on output is not relevant for the equilibrium determinacy.

The intuition for these results goes as follows. Similarly to the exercise conducted in the case of a balanced budget rule, suppose agents suddenly expect higher inflation. Monetary policy, following an active role, raises the real interest rate. Higher interest rate increases the cost of the service of public debt. A fiscal policy which follows a passive role increases government revenues raising the tax rate. The combined effect of a higher interest rate and a higher tax rate, reduces disposable real wages, potentially lowering consumption of the ROTC and that of the OPTC. On the other hand, both lower real wages, through an increase in profit share, and higher return on public debt, generate an increase in the financial portfolio of the optimisers. For low values of λ, these positive financial effects of monetary policy on the optimisers' wealth are overcompensated by the traditional Euler equation channel: an increase in real interest rate reduces aggregate demand. Via the NKPC, this reduction in aggregate demand puts downward pressure on current inflation[25]: initial expectations of higher inflation are not self-fulfilled and the combination of monetary and fiscal policy stabilises both the price level and the public debt.

Consider instead a passive/passive policy mix. The monetary authority would respond to the initial expectations of higher inflation by cutting the real interest rate. This would feed back on public debt generating a tax rate cut. The combined effect of lower interest rate and lower taxation would increase consumption of both type of consumer, expanding aggregate demand and in turn putting upward pressure on prices. The initial expectations of higher inflation are self-fulfilled, so generating local indeterminacy.

The active/active policy mix generates a perverse path for inflation and public debt which in turn leads the system to be unstable.

Figure (5) displays the determinacy analysis in the $(\phi_\pi - \delta_1)$ space for different values of the fiscal parameter on output, δ_2, when Θ is negative, i.e. $\eta = 3$ and $\lambda = 0.5$. A necessary condition

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[24]Note that this definition of fiscal policy is a simplifying approximation, given that it refers to an environment where the Ricardian equivalence holds. For a detailed discussion see Leith and Wren-Lewis(2000). For our benchmark parameterisation when $\Theta > 0$, a stable public debt dynamics requires $\delta_1 > 0.011$, which is very close to $\frac{1}{\beta} - 1$.

[25]Note that higher taxation *per se* puts upward pressure on prices via (27). However this effect is overcompensated by the decrease in aggregate demand.

for determinacy is that monetary and fiscal policy are both either active or passive. As previously described, the reason for these results is that when $\Theta < 0$, the financial effects of an interest rate change overturn the traditional transmission mechanism of monetary policy on aggregate demand. As a consequence, an active monetary policy ($\phi_\pi > 1$), through an increase in the return of the optimiser consumers' financial activities has the potential to expand aggregate demand. The intuition for these results goes as follows. Let us assume that both monetary and fiscal policy adopt an active rule ($\phi_\pi > 1$, $\delta_1 < \frac{1}{\beta} - 1$). Let us further assume that agents suddenly expect higher public debt. Fiscal policy reacts to this, cutting the tax rate (active fiscal policy) and therefore generating an explosive path for public debt. Moreover the tax rate cut feeds back on aggregate demand, generating, *ceteris paribus,* an increase in output and a downward pressure on inflation via the NKPC. Monetary policy through an active rule expands further output, i.e. $\Theta < 0$, causing, via the NKPC an explosive path on inflation, which in turn deflates the cost of public debt, implying a stable RE equilibrium.

Let us assume now that both policies are passive ($\phi_\pi < 1$, $\delta_1 > \frac{1}{\beta} - 1$) and that agents suddenly expect higher inflation. Monetary policy contrasts these expectations by cutting the real interest rate (passive rule). A lower interest rate lowers current output, putting downward pressure on prices via the NKPC, and therefore stabilising inflation. At the same time, monetary policy has two opposite effects on fiscal policy. On one hand, a lower interest rate implies, via a reduction in the optimiser consumers' wealth, a reduction in aggregate demand and therefore of the tax base, while on the other hand, a lower interest rate implies a lower cost of the service of public debt. These effects imply an important role for the equilibrium determinacy, of δ_2, the fiscal policy parameter on output. In particular the determinacy region increases when δ_2 decreases. This is due to the fact that if fiscal policy reacts to a decrease in output with a further cut in the tax rate ($\delta_2 > 0$), it could potentially fail to generate enough revenues to balance its budget, causing an explosive path for public debt and therefore generating indeterminacy. This destabilising situation can be partially avoided with a procyclical, in terms of output, fiscal rule, i.e. $\delta_2 < 0$. Note that within this monetary/fiscal policy mix determinacy requires a stronger fiscal policy, i.e. high δ_1, the closer ϕ_π is to unity, i.e. constant real interest rate.

With $\Theta < 0$, a policy mix of active monetary ($\phi_\pi > 1$) passive fiscal ($\delta_1 > \frac{1}{\beta} - 1$) policy generates indeterminacy. Monetary policy responds to higher inflation expectations, increasing the real interest rate. This generates an increase in the cost of the service of public debt and therefore an increase in the tax rate. However as previously described, the initial increase in the interest rate would expand aggregate demand, putting further pressure on current inflation and making the initial expectation self-fulfilling. Similarly, a policy mix of passive monetary policy ($\phi_\pi < 1$) and active fiscal policy ($\delta_1 < \frac{1}{\beta} - 1$) generates the stabilisation of inflation and the destabilisation of public debt, generating indeterminacy.

4 Robustness

4.1 General Monetary Policy Rules

We extend the determinacy analysis for a more general class of monetary policy rules of the type

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right) \ \ with \ i = -1, 0, 1 \tag{42}$$

where ρ identifies the nominal interest rate smoothing parameter, while ϕ_Y is the output parameter on monetary policy. In particular, when $i = -1$, (42) reduces to a backward looking rule. When $i = 0$ it corresponds to a contemporaneous rule, and when $i = 1$ it becomes a forward looking rule. Figure (6) reports the determinacy analysis in the $(\phi_\pi - \delta_1)$ space when $\Theta > 0$ with λ and η calibrated as in the previous section and $\delta_2 = 0$. Scrolling down the figure changes i (contemporaneous, backward-looking and forward-looking), while scrolling the figure from left to right changes the parameter values on ρ and ϕ_Y. Obviously, with i, ϕ_Y and ρ equal to zero, (42) collapses[26] to (30). For what concerns the first two top rows, i.e. contemporaneous rule and backward-looking rule, the adoption of more general monetary rules does not change the equilibrium dynamics of the model. In other words, the presence of a response in output ($\phi_Y > 0$) or the persistence of the interest rate ($\rho > 0$) does not alter, or does so only marginally, the logic of Leeper (1991). As in the previous section, in order to have a unique RE equilibrium when $\Theta > 0$, monetary policy has to be active (passive) and fiscal policy has to be passive (active).

With a forward-looking monetary policy rule (last row), this stops being true.

Forward-looking monetary rules where originally proposed by Bernanke and Woodford (1997), and estimated by Clarida et al. (1999, 2000). As noted by Bernanke and Woodford (1997) and Bullard and Mitra (2002), this type of rule can change markedly the equilibrium conditions of a standard monetary sticky price model respect its contemporaneous counterpart. In particular, when monetary policy is active, equilibrium determinacy imposes an upper limit on ϕ_π, which in turn depends on ρ and ϕ_Y. In other words, when in (42) $i = 1$, there is an upper bound to the size of the response to expected inflation that must be satisfied. If that upper bound is overshot, the equilibrium becomes indeterminate. Galí et al. (2004) find a similar result in a monetary model with capital accumulation and ROTC.

Here, while there are no changes in the case of active fiscal policy/passive monetary policy, in the case of active monetary/passive fiscal policy, the upper limit on ϕ_π depends, other than on ϕ_Y and ρ, in a non monotonic way on δ_1, the response of fiscal policy to public debt fluctuations. The upper limit on the monetary policy response to inflation expectations is present only for high responses of the tax rate to public debt. When fiscal policy reacts too strongly to public debt fluctuations, the implied increase in the tax rate feeds back on the supply side of the system via the NKPC, generating a destabilising effect on the attempt of monetary policy to contain inflation expectations and hence, causing indeterminacy. With an increase of interest rate inertia, via ρ or an

---

[26] For the sake of clarification, note that the case represented in the top left quadrant in figure (6) is the same as the one at the left bottom in figure (4).

increase in monetary response to output, via ϕ_Y, the upper limit on ϕ_π disappears. These results are consistent with the ones presented in Galí et al. (2004).

Figure (7) reports the same exercise when $\Theta < 0$ ($\eta = 3$ and $\lambda = 0.5$). As in the previous section, in this case the logic of Leeper (1991) is reversed: necessary conditions for a unique RE equilibrium are that monetary and fiscal policy are both either active or passive. There are a few notable results. First, when monetary policy reacts to output, i.e. positive ϕ_Y, and independently of the timing and the policy inertia of the monetary rule (42), the only policy mix that ensures equilibrium determinacy requires both monetary and fiscal policy to be active. In other words, if monetary policy cares about output stabilisation, it cannot adopt a passive rule in terms of inflation[27], i.e. $\phi_\pi < 1$. The reason for this result is that when $\Theta < 0$, a passive monetary policy would contract aggregate demand while a positive ϕ_Y would have the opposite effect, generating indeterminacy.

Secondly, with a forward-looking monetary rule, the upper limit on ϕ_π, which characterised the case where $\Theta > 0$, disappears. When fiscal policy adopts a destabilising public debt policy, monetary policy has to inflate the system through an active policy. The explosive path of both inflation and public debt overshoot the importance of the fiscal feedback on the supply side of the economy, leading to determinacy.

4.2 Different fiscal arrangements: the case of lump sum taxation.

One might rightly wonder if the results thus far presented depend on the particular specification of fiscal policy or instead are robust to a different specification of fiscal policy, i.e. lump sum taxation. This represents a natural extension of the analysis under several points of view. First of all, lump sum taxation maintains the distortive nature of fiscal policy: both types of consumers pay the burden of public debt but only the OPTC hold public debt assets. Therefore the government budget constraint cannot be separated from the rest of the model, and public debt remains an important state variable which has to be taken into account in the dynamics of the model. Secondly, despite the shape of the utility function (log-consumption), with lump sum taxation, ROTC do not supply labour inelastically.

The model with lump sum taxation is very similar to the one presented in the literature, as in Bilbiie et al. (2004), Galí et al. (2008). We therefore relegate to the appendix the standard derivation of the model, while here we only present the log-linearised equilibrium equations. When not differently specified we maintain the same notation and the same calibration as in the case of income taxation. As before, we assume public spending to be always at its steady state value. The equilibrium can be described by this set of equations. The NKPC

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\left(\frac{1}{\gamma_c} + \eta\right)\widehat{Y}_t\right) \tag{43}$$

---

[27]Precisely, when $\phi_Y > 0$, an active monetary policy has to be defined as $\phi_\pi > 1 - \frac{(1-\beta)\phi_Y}{k(1+\varphi)}$. Given our baseline parameterisation, $\frac{(1-\beta)\phi_Y}{k\left(\frac{1}{\gamma_c}+\varphi\right)} = 0.024$, and it is therefore ignored in the present analysis.

the government budget constraint

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{b}_t - \pi_t - \frac{\tau^{ls}}{b}\widehat{\tau}_t^{ls}\right) \tag{44}$$

where τ^{ls} identifies the steady state value of lump sum taxes, the monetary policy

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right) \ \ with \ i = -1, 0, 1 \tag{45}$$

the fiscal policy rule[28]

$$\widehat{\tau}_t^{ls} = \delta_1\widehat{b}_t + \delta_2\widehat{Y}_t \tag{46}$$

and aggregate demand

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \Gamma_c^{-1}\Theta^{ls}(1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) + \Gamma_\tau\Gamma_c^{-1}\Theta^{ls}\left(E_t\widehat{\tau}_{t+1}^{ls} - \widehat{\tau}_t^{ls}\right) \tag{47}$$

where $\Gamma_c = \left(1 - \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}\right)$, $\Gamma_n = \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}$, $\Theta^{ls} = (1/\gamma_c - \Gamma_n/\Gamma_c)^{-1}$ and $\Gamma_\tau = \frac{\lambda\tau^{ls}\eta}{Y(1+\eta\mu\gamma_c)}$.

A few things are worth noticing. The distortive nature of fiscal policy is represented by the feedback of taxation on the endogenous variables of the model via (47), the demand side of the economy. The sign and size of this effect depend crucially on the share of ROTC[29] and the elasticity of labour supply. Similarly, the sign of the elasticity of aggregate demand to interest rate movements depends on Θ^{ls} while its size depends on Γ_c. Despite a less straightforward analytical expression, the economic intuition for the sign of Θ^{ls} is the same to Θ, introduced in the income taxation environment. A necessary condition for $\Theta^{ls} > 0$ is

$$\lambda < \frac{1 + \gamma_c\mu\eta}{1 + \gamma_c + \eta + \gamma_c\eta} \tag{48}$$

Figure (8) sketches in the $(\lambda - \eta)$ space the sign of Θ^{ls}. As in the case with income taxation, Θ^{ls} remains positive for high values of the Frisch elasticity of labour supply, i.e. low η, and the higher, the higher the share of ROTC, i.e. high λ. We repeat the exercise conducted in 3.1 assuming that fiscal policy balances its budget in every period without accessing short term public debt asset but has to repay the interest on the long term public debt. The behavior of fiscal policy can be represented in log-linear form as

$$\widehat{\tau}_t^{ls} = \frac{\gamma_b Y}{\tau^{ls}}\left(\widehat{R}_t - \frac{1}{\beta}\widehat{\pi}_t\right) \tag{49}$$

While we assume that monetary policy implements (45) with $i = \rho = \phi_Y = 0$, i.e. contemporaneous rule. The dynamic system can be written as

$$E_t\{x_{t+1}\} = J^{ls}\{x_t\}$$

---

[28] For the sake of semplicity from now on we ignore the term on output in the fiscal rule, i.e. $\delta_2 = 0$.

[29] Note that in the limiting case of no ROTC, $\Gamma_n = \Gamma_\tau = 0$ and $\Gamma_c = 1$.

where, as before, $x_t = \left\{\widehat{Y}_t, \pi_t\right\}$ and $J^{ls} = \begin{bmatrix} 1 + \frac{\Gamma_c^{-1}\Theta^{ls}\chi(\beta-\Upsilon+\beta\Upsilon\phi_\pi)}{\beta^2} & \frac{\Gamma_c^{-1}\Theta^{ls}(\beta+(\beta-1)\Upsilon)(\phi_\pi\beta-1)}{\beta^2} \\ -\frac{\chi}{\beta} & \frac{1}{\beta} \end{bmatrix}$

with $\Upsilon = \frac{\lambda\eta\gamma_b}{(1+\eta\mu\gamma_c)} > 0$ and $\chi_1 = k\left(\frac{1}{\gamma_c} + \eta\right) > 0$. We continue to impose $\phi_\pi > 0$. As in the case with income taxation, determinacy requires J^{ls} to have both eigenvalues outside the unit circle. When $\Theta^{ls} > 0$, necessary and sufficient condition for determinacy is

$$\phi_\pi > 1 \tag{50}$$

The upper bound on active monetary policy that is present in the case of income taxation, disappear in the case of lump sum taxation. In other words, a monetary policy which follows the Taylor principle, i.e. $\phi_\pi > 1$, always delivers determinacy when only lump sum taxes are available and fiscal policy follows (49). This is due to the lack of direct feedback of a tax change on the supply side of the economy together with the ordinary effect of an interest rate change on the demand side of the economy. A graphical inspection of determinacy can be found in figure (9).

When $\Theta^{ls} < 0$, necessary and sufficient conditions for determinacy require

$$0 < \phi_\pi < \min\left(1, \xi_3\right) \cup \phi_\pi > \max\left(1, \xi_4\right) \tag{51}$$

where $\xi_3 = \frac{\beta^2-\beta+\Upsilon\Gamma_c^{-1}\chi_1\Theta^{ls}}{\beta\Gamma_c^{-1}\chi_1\Theta^{ls}(1+\Upsilon)}$ and $\xi_4 = \frac{-2\beta-2\beta^2-\beta\Gamma_c^{-1}\chi_1\Theta^{ls}+2\Gamma_c^{-1}\Upsilon\chi_1\Theta^{ls}}{\beta\Gamma_c^{-1}\chi_1\Theta^{ls}(1+2\Upsilon)}$. As in the case with income taxation, when $\Theta^{ls} < 0$, there are two determinacy areas, one in which monetary policy follows an active rule and one in which monetary policy follows a passive one. Both ξ_3 and ξ_4 are functions of the structural parameters of the model. Despite a less intuitive expression, ξ_3 and ξ_4 have the same interpretation and the same behaviour of ξ_1 and ξ_2, respectively. Figure (10) displays the determinacy results in the case of Θ^{ls}. The economic intuition for these results is very similar to the analogous case with income taxation. We therefore refer to paragraph 3.1 for a detailed discussion.

Finally, we repeat the exercise conducted in (4.1) for the case of lump sum taxes. Fiscal policy is allowed to release short run public debt assets and it balances its budget following[30] (46). The analysis can therefore be conducted under the active/passive logic of Leeper(1991).

Figures (11) and (12) report the determinacy analysis with lump sum taxation with positive ($\eta = 1$ and $\lambda = 0.3$) and negative ($\eta = 3$ and $\lambda = 0.5$) Θ^{ls}. In both cases there are no markable differences with the income taxation scenario and the policy mix which guarantees determinacy is mainly driven by the sign of Θ^{ls}.

While, as detailed in Bilbiie et al. (2004), different fiscal arrangements imply important consequences for the transmission mechanism of macro policies, in particular for fiscal policy, they do not cause important changes for the equilibrium dynamics.

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[30]For semplicity we fix $\delta_2 = 0$ throughout this exercise.

5 Concluding Remarks

The introduction of ROTC has dramatic consequences for the equilibrium dynamics of a standard NK model. While most of the literature focuses only on the monetary policy aspect of these consequences, for example Galí et al.(2004) and Bilbiie (2008), we concentrate on the effects of a share of ROTC on fiscal policy and its interaction with monetary policy. In doing so, we analyse a broad range of monetary and fiscal policy rules. To this end this paper contributes to enrich the theoretical literature on macro-policy rules and has the potential to explain the U.S. postwar empirical evidence on monetary and fiscal policy regimes.

We summarise our results as follow.

1) When the share of ROTC and the elasticity of labour supply guarantee that the elasticity of demand follows the common wisdom, i.e. negative relation between interest rate and aggregate demand, monetary policy adopts a contemporaneous interest rate rule and fiscal policy balances its budget constraint without releasing short run public debt, an active monetary policy rule is necessary but not sufficient condition for determinacy. In other words, a monetary policy which respects the Taylor principle and reacts 'too strongly' against inflation might lead to indeterminacy for high levels of long run public debt. This upper bound on monetary policy gets tighter the higher the level of public debt and tends to disappear with an increase of the share of ROTC and a more elastic labour supply.

2) When the combination of ROTC and the elasticity of labour supply inverts the elasticity of aggregate demand to the interest rate and monetary and fiscal policy rules follow from 1), equilibrium determinacy can be guaranteed by both an active and a passive monetary policy rule. While a passive monetary rule leads to a unique RE equilibrium with an upper bound which is increasing in the share of ROTC, labour supply elasticity and long run public debt, an active monetary rule leads to determinacy with a lower bound which in turn is increasing in the share of ROTC and the elasticity of labour supply and decreasing in the long run level of public debt.

3) When fiscal policy is allowed to realise short run public debt and it follows a tax revenue rule as in (32), equilibrium determinacy requires, following the definition of Leeper (1991), to have an active (passive) monetary rule together with a passive (active) fiscal rule when the aggregate demand responds negatively to increases in real interest rate or both monetary and fiscal policy simultaneously active or passive when the aggregate demand logic is inverted.

4) Results 1, 2 and 3 survive to different specifications of monetary rules (contemporaneous, forward-looking, backward-looking) and different specifications of fiscal arrangements (income tax, lump sum tax).

We interpret our results in several directions. Results 1 and 2 represent a clear extension on fiscal balanced budget rule. While for many reasons this type of fiscal policy might not be a wise policy choice, i.e. it increases business cycle fluctuations and generates significant welfare losses (Barro, 1979; Lucas and Stokey, 1983;), it creates incentives for illegal and non-transparent economic activity (Alesina and Perotti, 1996) and it may lead to indeterminacy, thus inducing belief-driven aggregate instability and endogenous sunspot fluctuations (Schmitt-Grohe and Uribe,

1997; Linnemann, 2005), its analysis represents a recurrent theme of debate in many countries. The present work does not deal with business cycle fluctuations nor with welfare analysis, but only with the determinacy properties of a balanced budget fiscal policy. To this respect, we find that, within reasonable parameter values, an active monetary policy together with a moderate level of ROTC, i.e usual aggregate demand logic, guarantees determinacy with a balanced fiscal policy rule. This result can be compared to that of Linnemann (2005). The author finds that in a similar model, although with no ROTC, an active monetary policy with a balanced budget fiscal policy can easily lead to indeterminacy. The difference in our results are crucially driven by the presence of ROTC.

On the other hand, result 2 can be seen as an extension of Bilbiie (2008). He shows that when the share of ROTC, or *ceteris paribus,* the elasticity of labour supply, imply an inverted aggregate demand logic, monetary policy has to be passive in order to guarantee a unique RE equilibrium. In this paper we argue that when the aggregate demand logic is inverted and fiscal policy follows a balanced budget rule, an active monetary policy can realistically lead to determinacy.

Favero and Monacelli (2005) and Davig and Leeper (2006) find that in the US postwar macro policy regimes, alongside with periods in which monetary and fiscal policy respect the active/ passive logic of Leeper (1991), there are periods in which monetary and fiscal policy are both active or passive. This evidence cannot generally be explained with a traditional Real Business Cycle or NK model. We show in result 3, that within a reasonable parameters region, our model[31] has the potential to explain this empirical evidence.

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[31] A similar result in a continuous time NK model with ROTC and lump sum taxation is obtained by Leith and von Thadden (2007).

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Appendix A

Steady State

This section describes the steady state of the model with income taxation. A few points are worth stressing. First of all, we impose, through a transfer, that the two agents have the same level of consumption and supply the same level of labour at steady state. Hence the heterogeneity between the two consumers is only along the business cycle. Price are normalised to unity and we fix $\frac{G}{Y} = 1 - \gamma_c$. The OPTC budget constraint is

$$C^o = \left(WN^o + \frac{D}{1-\lambda}\right)(1-\tau) + \left(1-R^{-1}\right)\frac{B}{1-\lambda} + S^o \tag{52}$$

Where S^o is the OPTC transfer. The steady state ROTC budget constraint is

$$C^r = (WN^r)(1-\tau) + S^r \tag{53}$$

where S^r is the ROTC transfer. Furthermore we need to impose

$$(1-\lambda)S^o + \lambda S^r = 0 \tag{54}$$

From the steady state Euler equation it is possible to find the steady state interest rate

$$\frac{1}{\beta} = R$$

While the steady state profits follow

$$D = (1-W)Y \tag{55}$$

Homogeneity requires

$$C^o = \left(WN^o + \frac{D}{1-\lambda}\right)(1-\tau) + (1-\beta)\frac{B}{1-\lambda} + S^o = C^r = WN^r(1-\tau) + S^r \tag{56}$$

Therefore

$$S^o = -\frac{\lambda}{1-\lambda}\left(D(1-\tau) + (1-\beta)B\right) \tag{57}$$

and

$$S^r = -\frac{(1-\lambda)}{\lambda}S^o \tag{58}$$

From the firm's marginal cost

$$W = \frac{\varepsilon - 1}{\varepsilon} = \frac{1}{\mu}$$

While the steady state government budget constraint can be written as

$$\tau Y = (1-\beta)b + G \tag{59}$$

Given that $\frac{C}{Y} = \gamma_c$, $\frac{G}{Y} = (1 - \gamma_c)$ and that $\frac{b}{Y} = \gamma_b$ we can rewrite the last equation as

$$\tau = (1 - \beta)\gamma_b + (1 - \gamma_c) \tag{60}$$

Combining the fact that at steady state $Y = N$ with the steady state optimal labour supply it yields

$$\theta \gamma_c (N)^{\eta+1} = W(1 - \tau) \tag{61}$$

After rearranging, the latter yields the steady state level of labour supply

$$Y = N = \left(\frac{W(1 - \tau)}{\theta \gamma_c}\right)^{\frac{1}{\eta+1}} \tag{62}$$

Consequently

$$
\begin{aligned}
G &= (1 - \gamma_c)Y \\
C &= \gamma_c Y
\end{aligned}
$$

These equations give us to have a full description of the steady state variables.

Log Linearisation

This section presents a log-linearised version of the model with income taxation around the non stochastic steady state. Henceforth, all the upper hat variables identify the variable percentage deviation from its steady state value (i.e. $\widehat{X}_t = \log\left(\frac{X_t}{X}\right)$). While $\pi_t = \log P_t - \log P_{t-1}$ identifies the inflation rate.

The log linearisation of the OPTC Euler equation and optimal supply of labour are

$$\widehat{C}_t^o = E_t\widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{63}$$

$$\widehat{C}_t^o + \eta\widehat{N}_t^o = \widehat{w}_t - \frac{\tau}{1 - \tau}\widehat{\tau}_t \tag{64}$$

where $\widehat{w}_t = \widehat{W}_t - \widehat{P}_t$. The ROTC consumption and labour supply follow

$$\widehat{C}_t^r = \widehat{w}_t + \widehat{N}_t^r - \frac{\tau}{1 - \tau}\widehat{\tau}_t \tag{65}$$

$$\eta\widehat{N}_t^r + \widehat{C}_t^r = \widehat{W}_t - \widehat{P}_t - \frac{\tau}{1 - \tau}\widehat{\tau}_t \tag{66}$$

Log linearising the optimal price for a setter firm (15) and the evolution of prices in (16) around a zero steady state inflation yields to the traditional New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t\pi_{t+1} + \kappa(\widehat{mc}_t) \tag{67}$$

Where $\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$. The log linearisation of the aggregation rules for consumption and labour

yield

$$\widehat{C}_t = \lambda \widehat{C}_t^r + (1 - \lambda) \widehat{C}_t^o \tag{68}$$

$$\widehat{N}_t = \lambda \widehat{N}_t^r + (1 - \lambda) \widehat{N}_t^o \tag{69}$$

while the market clearing condition follows

$$\widehat{Y}_t = \gamma_c \widehat{C}_t \tag{70}$$

where $\gamma_c = \frac{C}{Y}$. Furthermore from the production function (11)

$$\widehat{Y}_t = \widehat{N}_t \tag{71}$$

The log linearisation of the monetary and fiscal rule yields

$$\widehat{R}_t = \phi \pi_t \tag{72}$$

$$\widehat{\tau}_t = \delta_1 \widehat{b}_t + (\delta_2 - 1)\widehat{Y}_t$$

.Finally, a log linearisation of the government budget constraint can be written as

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta} \left(\widehat{b}_t - \pi_t + \frac{1 - \gamma_c}{b} \widehat{G}_t - \frac{\tau Y}{b} \left(\widehat{Y}_t + \widehat{\tau}_t \right) \right) \tag{73}$$

Equilibrium

This section presents the equilibrium of the model. Further analysis is simplified by rewriting the model as a function of aggregate variables only. First, combining (65) with (66), we obtain

$$\widehat{N}_t^r = 0 \tag{74}$$

and

$$\widehat{C}_t^r = \widehat{w}_t - \frac{\tau}{1 - \tau} \widehat{\tau}_t \tag{75}$$

From the last two expressions one can see that the introduction of distortive taxation is completely internalised in the ROTC consumption, while their labour supply remains constant at the steady state level[32]. Therefore changes in the tax rate over the business cycle do not have any effect on the ROTC labour supply.

Combining the last expression with the optimal labour supply of the OPTC yields

$$\widehat{C}_t^o + \eta \widehat{N}_t^o = \widehat{C}_t^r \tag{76}$$

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[32] For the ROTC the substitution effect on the labour supply is equal to the income effect.

Furthermore, combining (69) with (74) it is possible to rewrite the total supply of labour as

$$\widehat{N}_t = (1 - \lambda)\,\widehat{N}_t^o \tag{77}$$

Therefore aggregate labour fluctuations are just a function of changes in OPTC labour supply. Moreover, plugging these results into the equation for total consumption yields

$$\widehat{C}_t = \lambda\left[\widehat{C}_t^o + \frac{\eta}{1-\lambda}\widehat{N}_t\right] + (1 - \lambda)\,\widehat{C}_t^o$$

Simplifying gives

$$\widehat{C}_t = \widehat{C}_t^o + \eta\frac{\lambda}{1-\lambda}\widehat{N}_t \tag{78}$$

From the latter we can rewrite the Euler equation in terms of aggregate consumption as

$$\widehat{C}_t = E_t\left(\widehat{C}_{t+1}\right) - \left(\widehat{R}_t - E_t\pi_{t+1}\right) - \eta\frac{\lambda}{1-\lambda}E_t\Delta\widehat{N}_{t+1} \tag{79}$$

To have the full picture it is necessary to substitute in (79) the market clearing condition and the production function. Substituting in the latter the market clearing condition and the production function one can obtain the dynamic IS equation presented in the main text.

On the supply side, using the market clearing condition and the definition of real marginal cost, we can express the New Keynesian Phillips Curve (NKPC) in terms of aggregate variables as follows

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\left(\left(\frac{1}{\gamma_c} + \eta\right)\widehat{Y}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t\right) \tag{80}$$

Model with Lump-sum taxes

This model shares with its income taxation counterpart the shape of the utility function, the production sector, the aggregation and the monetary policy rules. The optimiser budget constraint is

$$P_tC_t^o + R_t^{-1}\frac{B_{t+1}}{1-\lambda} + \frac{E_t\left(Q_{t,t+1}V_{t+1}\right)}{1-\lambda} = \left(W_tN_t^o + \frac{D_t}{1-\lambda}\right) + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t\tau_t^{ls} - P_tS^o \tag{81}$$

Where τ_t^{ls} identifies the level (common to the two types of consumer) of lump sum taxes.

The optimisers first order conditions are

$$\beta R_t E_t\left[\left(\frac{C_t^o}{C_{t+1}^o}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = 1 \tag{82}$$

Where as before $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is implied by the non arbitrage condition. This expression is the familiar *Euler equation* for consumption. The first order condition with respect to labour states that the *marginal rate of substitution* between labour and consumption must be equal to the real wage

32

$$\theta \left(N_t^o\right)^\eta C_t^o = \frac{W_t}{P_t} \tag{83}$$

The budget constraint for the ROTC is

$$P_t C_t^r = W_t N_t^r - \tau_t^{ls} - S^o \tag{84}$$

The ROTC first order condition is

$$\theta \left(N_t^r\right)^\eta C_t^r = \frac{W_t}{P_t} \tag{85}$$

while the optimum level of consumption is directly derived from (84).

The government budget constraint is

$$R_t^{-1} b_{t+1} = \frac{b_t}{\pi_t} - \tau_t^{ls} + G_t \tag{86}$$

Steady state

This section sketches the steady state for the model with lump-sum taxation.

$$C^o = WN^o + \frac{D}{1-\lambda} + \left(1 - R^{-1}\right)\frac{B}{1-\lambda} - \tau^{ls} + S^o \tag{87}$$

Where S^o is the OPTC transfer. The steady state ROTC budget constraint is

$$C^r = (WN^r) - \tau^{ls} + S^r \tag{88}$$

where S^r is the ROTC transfer. Furthermore we need to impose

$$(1-\lambda) S^o + \lambda S^r = 0 \tag{89}$$

From the steady state Euler equation it is possible to find the steady state interest rate

$$\frac{1}{\beta} = R$$

While the steady state profits follow

$$D = (1 - W) Y \tag{90}$$

Homogeneity requires

$$C^o = \left(WN^o + \frac{D}{1-\lambda}\right) + (1-\beta)\frac{B}{1-\lambda} + S^o = C^r = WN^r + S^r \tag{91}$$

Therefore

$$S^o = -\frac{\lambda}{1-\lambda}\left(D + (1-\beta)B\right) \tag{92}$$

and

$$S^r = -\frac{(1-\lambda)}{\lambda}S^o \tag{93}$$

The steady state government budget constraint can be written as

$$\tau^{ls} = (1-\beta)\,b + G \tag{94}$$

Given that $\frac{C}{Y} = \gamma_c$, $\frac{G}{Y} = (1-\gamma_c)$ and that $\frac{b}{Y} = \gamma_b$ we can rewrite the last equation as

$$\frac{\tau^{ls}}{Y} = (1-\beta)\,\gamma_b + (1-\gamma_c) \tag{95}$$

Combining the fact that at steady state $Y = N$ with the steady state optimal labour supply it yields

$$\theta\gamma_c\,(N)^{\eta+1} = W \tag{96}$$

After rearranging, the latter yields the steady state level of labour supply

$$Y = N = \left(\frac{W}{\theta\gamma_c}\right)^{\frac{1}{\eta+1}} \tag{97}$$

Consequently

$$G = (1-\gamma_c)\,Y \tag{98}$$

$$C = \gamma_c Y \tag{99}$$

Log-linearisation and equilibrium

This paragraph derives the log-linearisation of the demand side of the economy with lump sum taxes. Log linearisation of the first order conditions for both types of consumers yields

$$\widehat{C}_t^o = E_t\widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{100}$$

$$\widehat{C}_t^o + \eta\widehat{N}_t^o = \widehat{w}_t \tag{101}$$

$$\widehat{C}_t^r = \left(\frac{1}{\mu\gamma_c}\right)\left(\widehat{N}_t^r + \widehat{w}_t\right) - \frac{\tau^{ls}}{C}\left(\widehat{\tau}_t^{ls}\right) \tag{102}$$

where $W = \frac{1}{\mu}$ and $\frac{N}{C} = \frac{1}{\gamma_c}$.

$$\widehat{C}_t^r + \eta\widehat{N}_t^r = \widehat{w}_t \tag{103}$$

From the aggregation rules

$$\widehat{C}_t = \lambda\widehat{C}_t^r + (1-\lambda)\,\widehat{C}_t^o \tag{104}$$

$$\widehat{N}_t = \lambda\widehat{N}_t^r + (1-\lambda)\,\widehat{N}_t^o \tag{105}$$

The market clearing conditions are

$$\widehat{Y}_t = \gamma_c \widehat{C}_t \tag{106}$$

$$\widehat{Y}_t = \widehat{N}_t \tag{107}$$

Therefore the total labour supply follows

$$\widehat{C}_t + \eta \widehat{N}_t = \widehat{w}_t \tag{108}$$

Plugging the aggregation rules and the labour supply into (102) it yields

$$\widehat{C}_t^r = \left(\frac{(1+\eta)\,\eta}{1+\eta\mu\gamma_c}\right)\widehat{N}_t + \frac{1+\eta}{1+\eta\mu\gamma_c}\widehat{C}_t - \frac{\mu\tau^{ls}\eta}{Y\left(1+\eta\mu\gamma_c\right)}\left(\widehat{\tau}_t^{ls}\right) \tag{109}$$

Substituting the aggregation rule into the optimisers' Euler equation one obtains

$$\widehat{C}_t - \lambda\widehat{C}_t^r = E_t\widehat{C}_{t+1} - E_t\widehat{C}_{t+1}^r - (1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{110}$$

Using (109) we can write the aggregate demand as

$$\Gamma_c\widehat{C}_t = \Gamma_c E_t\widehat{C}_{t+1} - \Gamma_n E_t\Delta\widehat{N}_{t+1} + \Gamma_T E_t\Delta\widehat{\tau}_{t+1}^{ls} - (1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{111}$$

where $\Gamma_c = \left(1 - \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}\right)$, $\Gamma_n = \frac{\lambda(1+\eta)\eta}{1+\eta\mu\gamma_c}$, $\Gamma_T = \frac{\lambda\tau^{ls}\eta}{Y(1+\eta\mu\gamma_c)}$ and $\Delta\widehat{X}_t = \widehat{X}_t - \widehat{X}_{t-1}$. Finally, using the market clearing condition it yields

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \Gamma_c^{-1}\Theta^{ls}\left(1-\lambda\right)\left(\widehat{R}_t - E_t\pi_{t+1}\right) + \Gamma_\tau\Gamma_c^{-1}\Theta^{ls}\left(E_t\widehat{\tau}_{t+1}^{ls} - \widehat{\tau}_t^{ls}\right) \tag{112}$$

as in the main text.

The log-linearisation of the government budget constraint is

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{b}_t - \pi_t - \frac{\tau^{ls}}{b}\widehat{\tau}_t^{ls}\right) \tag{113}$$

5.1   Analytical determinacy analysis: the case of a balanced budget rule

5.1.1   Case with income taxation

After some algebra we can write the model with a balanced budget rule as

$$E_t x_{t+1} = J^{br} x_t \tag{114}$$

where x is defined in the main text and

$$J^{br} = \begin{bmatrix} 1 + \frac{\Lambda_1\Theta}{\beta} & \Theta\phi_\pi - \frac{\Theta(1+(1-\phi_\pi\beta)\Psi)}{\beta} \\ -\frac{\Lambda_1}{\beta} & \frac{(1+(1-\phi_\pi\beta)\Psi)}{\beta} \end{bmatrix}$$

Where $\Lambda_1 = k\left(\frac{1-\tau}{\gamma_c} + (1-\tau)\eta - \tau\right)$, $\Psi = \frac{k\gamma_b}{w} \geq 0$. We assume that $\tau < \left(1 + \frac{\gamma_c}{1+\gamma_c\eta}\right)^{-1}$. As explained in the main text, the latter assumption implies that $\Lambda_1 > 0$. Given that the x vector contains two jump variables, determinacy requires that both eigenvalues of J^{br} lye outside the unit circle. Determinant and trace of J^{br} are respectively $Det(J) = \frac{1+\Lambda_1\Theta\phi_\pi+\Psi-\phi_\pi\beta\Psi}{\beta}$ and $Tr(J) = \frac{1+\beta+\Lambda_1\Theta+\Psi-\phi_\pi\beta\Psi}{\beta}$. We start from the case where $\Theta > 0$. Following Woodford (2003, appendix C), every determinate equilibrium satisfies either criterion I with

$$(\text{I.a}): \qquad Det(J) > 1 \iff \frac{1+\Lambda_1\Theta\phi_\pi+\Psi-\phi_\pi\beta\Psi}{\beta} > 1$$

$$(\text{I.b}): \qquad Det(J) - Tr(J) > -1 \iff \Lambda_1\Theta(\phi_\pi - 1) > 0$$

$$(\text{I.c}): \qquad Det(J) + Tr(J) > -1 \iff \frac{2+\beta+\Lambda_1\Theta+\Lambda_1\Theta\phi_\pi+2\Psi-2\beta\Psi\phi_\pi}{\beta} > -1$$

or criterion II as

$$(\text{II.a}): \qquad Det(J) - Tr(J) < -1 \iff \Lambda_1\Theta(\phi_\pi - 1) < 0$$

$$(\text{II.b}): \qquad Det(J) + Tr(J) < -1 \iff \frac{2+\beta+\Lambda_1\Theta+\Lambda_1\Theta\phi_\pi+2\Psi-2\beta\Psi\phi_\pi}{\beta} < -1$$

We want to express the determinacy conditions in terms of the monetary policy parameter ϕ_π. (I.a) implies that

$$\begin{aligned}
\text{if } \Lambda_1 &< \frac{\Psi\beta}{\Theta} \implies \phi_\pi < \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1} \\
\text{elseif } \Lambda_1 &> \frac{\Psi\beta}{\Theta} \implies \phi_\pi > \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}
\end{aligned}$$

while (I.b) implies

$$\phi_\pi > 1$$

and (I.c)

$$\begin{aligned}
\text{if } \Lambda_1 &< 2\frac{\Psi\beta}{\Theta} \implies \phi_\pi < \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} \\
\text{elseif } \Lambda_1 &> 2\frac{\Psi\beta}{\Theta} \implies \phi_\pi > \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}
\end{aligned}$$

36

Putting things together criterion I implies

$$
\begin{aligned}
\text{if } \Lambda_1 \; &> \; 2\frac{\Psi\beta}{\Theta} \implies \phi > 1 \\
\text{elseif } \frac{\Psi\beta}{\Theta} \; &< \; \Lambda_1 < 2\frac{\Psi\beta}{\Theta} \implies 1 < \phi_\pi < \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta} \\
\text{elseif } \Lambda_1 \; &< \; \frac{\Psi\beta}{\Theta} \implies 1 < \phi_\pi < \min\left\{ \frac{1 + \Psi - \beta}{\beta\Psi - \Theta\Lambda_1}, \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta} \right\}
\end{aligned}
$$

Criterion II can be ruled out due to sign restrictions.[33]∎

Let now turn to the case when $\Theta < 0$. (I.b) implies $\phi_\pi < 1$, while (I.a) is verified when

$$
\phi_\pi < \frac{1 + \Psi - \beta}{\beta\Psi - \Theta\Lambda_1} \tag{115}
$$

and (I.c)

$$
\phi_\pi < \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta} \; \cap \Lambda_1 \in \left(0, -2\frac{(1 + \beta + \Psi)}{\Theta}\right) \tag{116}
$$

Therefore when $\Theta < 0$ criterion I implies[34]

$$
\phi_\pi < \min\left\{1, \frac{1 + \Psi - \beta}{\beta\Psi - \Theta\Lambda_1}\right\} \tag{117}
$$

When $\Theta < 0$, criterion II cannot be ruled out, therefore (II.a) implies

$$
\phi_\pi > 1 \tag{118}
$$

and (II.b)

$$
\phi_\pi > \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta} \tag{119}
$$

This yields, for criterion II, to

$$
\phi_\pi > \max\left\{1, \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta}\right\} \tag{120}
$$

Therefore, putting things together, when $\Theta < 0$ there are two determinacy spaces

$$
0 < \phi_\pi < \min\left\{1, \frac{1 + \Psi - \beta}{\beta\Psi - \Theta\Lambda_1}\right\} \; \cup \; \phi_\pi > \max\left\{1, \frac{2 + 2\beta + 2\Psi + \Lambda_1\Theta}{2\beta\Psi - \Lambda_1\Theta}\right\} \quad \blacksquare \tag{121}
$$

---

[33]In fact (II.a) implies $\phi_\pi < 1$, while (II.b) requires $\phi_\pi > \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} \cap \Lambda_1 < 2\frac{\Psi\beta}{\Theta}$. Note that if $\Lambda_1 < 2\frac{\Psi\beta}{\Theta}$, $\frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}$ is greater than one.

[34]The condition $\Lambda_1 \in \left(0, -2\frac{(1+\beta+\Psi)}{\Theta}\right)$ is always verified within standard parametrisations. This also imples that $\frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} > 1$. Hence (I.c) is not binding for standard parametrisation.

5.1.2 Case with lump sum taxation

The relevant matrix is

$$
J^{ls} = \begin{bmatrix} 1 + \frac{\Gamma_c^{-1}\Theta^{ls}\chi(\beta-\Upsilon+\beta\Upsilon\phi_\pi)}{\beta^2} & \frac{\Gamma_c^{-1}\Theta^{ls}(\beta+(\beta-1)\Upsilon)(\phi_\pi\beta-1)}{\beta^2} \\ -\frac{\chi}{\beta} & \frac{1}{\beta} \end{bmatrix}
$$

Note that $\Upsilon = \frac{\lambda\eta\gamma_b}{(1+\eta\mu\gamma_c)} \geq 0$ and $\chi = k\left(\frac{1}{\gamma_c}+\eta\right) > 0$. Given that the x vector contains two jump variables, determinacy requires that both eigenvalues of J^{ls} lye outside the unit circle. Determinant and trace of J^{ls} are respectively $Det\left(J^{ls}\right) = \frac{1}{\beta} + \frac{\Gamma_c^{-1}\chi\Theta^{ls}(\beta\phi_\pi(1+\Upsilon)-\Upsilon)}{\beta^2}$ and $Tr\left(J^{ls}\right) = 1 + \frac{\Gamma_c^{-1}\Theta^{ls}\chi(\beta-\Upsilon+\beta\Upsilon\phi_\pi)}{\beta^2} + \frac{1}{\beta}$. In the same fashion adopted in the case of labour income taxation, we follow Woodford (2003, appendix C). Every determinate equilibrium satisfies either criterion I

$$
\begin{aligned}
\text{(I.a):} \quad & Det\left(J^{ls}\right) > 1 \\
\text{(I.b):} \quad & Det\left(J^{ls}\right) - Tr\left(J^{ls}\right) > -1 \\
\text{(I.c):} \quad & Det\left(J^{ls}\right) + Tr\left(J^{ls}\right) > -1
\end{aligned}
$$

or criterion II

$$
\begin{aligned}
\text{(II.a):} \quad & Det\left(J^{ls}\right) - Tr\left(J^{ls}\right) < -1 \\
\text{(II.b):} \quad & Det\left(J^{ls}\right) + Tr\left(J^{ls}\right) < -1
\end{aligned}
$$

Let start from criterion I when $\Theta^{ls} > 0$. It is easy to show that (I.b) is verified if and only if $\phi_\pi > 1$. Furthermore, if (I.b) holds, (I.a) and (I.c) are verified as well.

We can rule out criterion II due to sign restrictions.[35] ∎

Now we turn to study the determinacy conditions when $\Theta^{ls} < 0$. Let start with criterion I. (I.a) implies

$$
\phi_\pi < \xi_3 \tag{122}
$$

With $\xi_3 = \frac{\beta^2-\beta+\Upsilon\Gamma_c^{-1}\chi\Theta^{ls}}{\beta\Gamma_c^{-1}\chi\Theta^{ls}(1+\Upsilon)}$. (I.b) implies

$$
\phi_\pi < 1 \tag{123}
$$

while (I.c) implies

$$
\phi_\pi < \xi_4 \tag{124}
$$

where $\xi_4 = \frac{-2\beta-2\beta^2-\beta\Gamma_c^{-1}\chi_1\Theta^{ls}+2\Gamma_c^{-1}\Upsilon\chi_1\Theta^{ls}}{\beta\Gamma_c^{-1}\chi_1\Theta^{ls}(1+2\Upsilon)}$.

---

[35] As in the analogous case with labour income taxation, (II.a) requires $\phi_\pi < 1$ while (II.b) implies $\phi_\pi > 1$.

Let now analyse criterion II. (II.a) implies

$$\phi_\pi > 1 \tag{125}$$

while (II.b) implies

$$\phi_\pi > \xi_4$$

Summing up the results: when $\Theta^{ls} < 0$ necessary and sufficient conditions for determinacy require

$$0 < \phi_\pi < \min\left(1, \xi_3, \xi_4\right) \cup \phi_\pi > \max\left(1, \xi_4\right) \quad \blacksquare \tag{126}$$

Figures
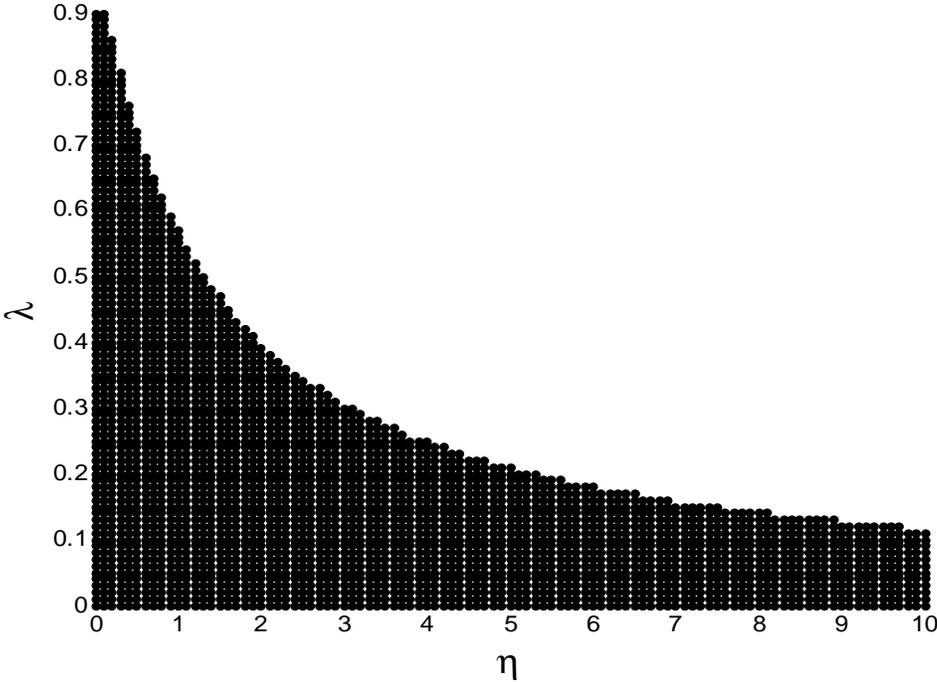
Figure 1: Sign of Θ. Black spots, $\Theta > 0$, white area $\Theta < 0$.
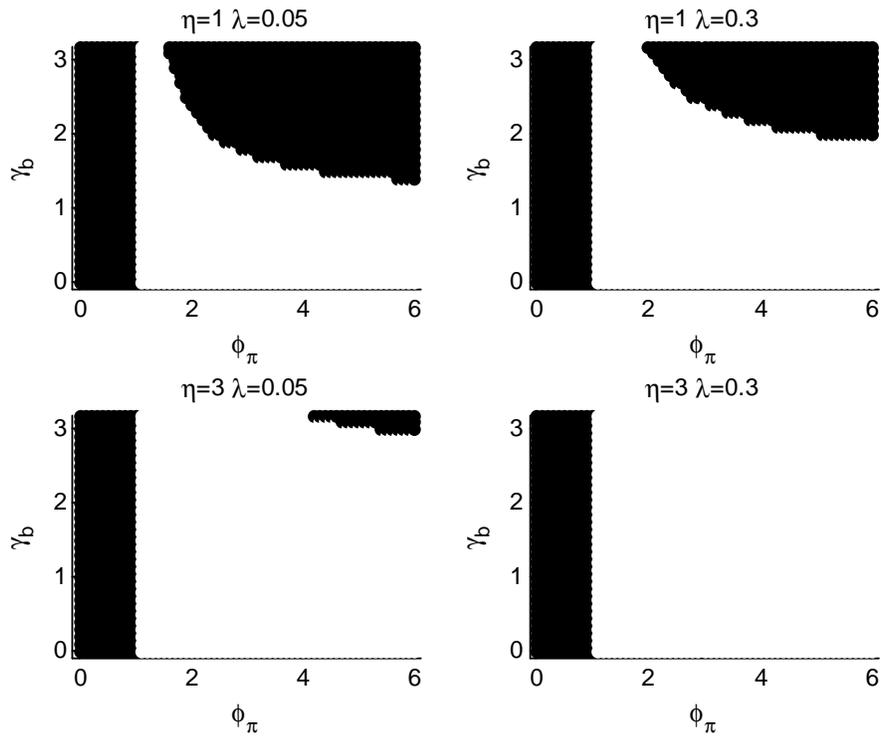
Figure 2: Determinacy analysis with a balanced budget fiscal policy, positive Θ. White area, determinacy. Black area, indeterminacy.
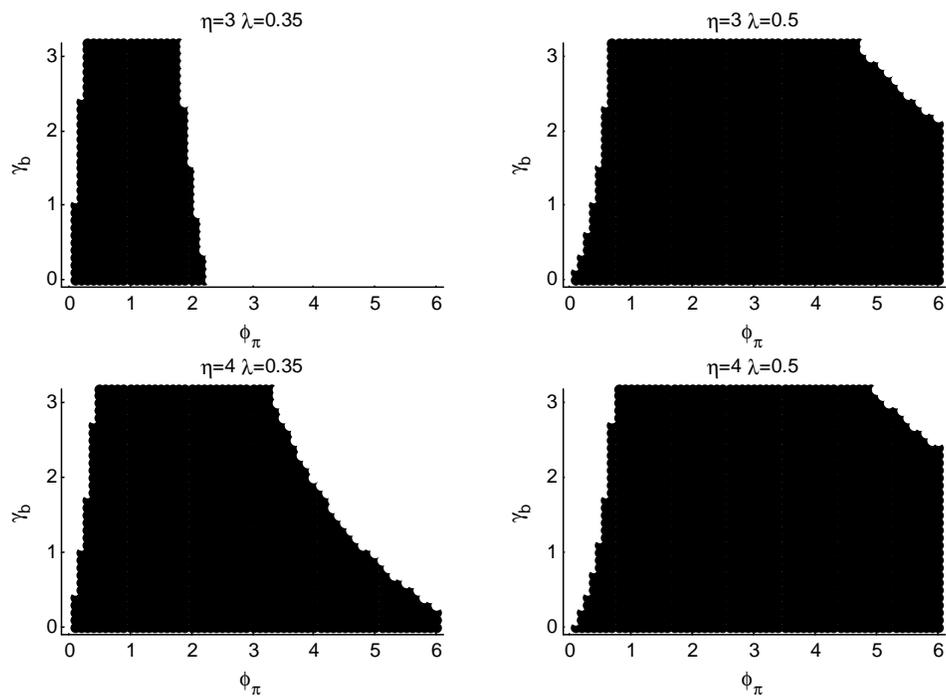
Figure 3: Determinacy analysis with a balanced budget fiscal policy, negative Θ. White area, determinacy. Black area, indeterminacy.
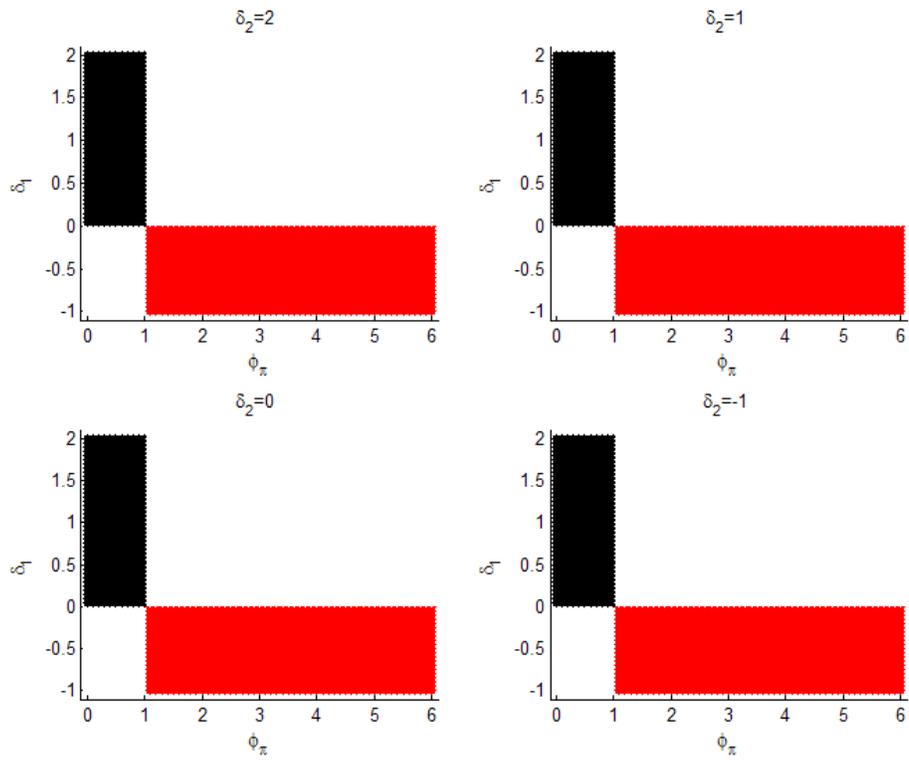
Figure 4: Determinacy area with contemporaneous monetary rule and a fiscal rule of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t + (\delta_2 - 1)\widehat{Y}_t$ and positive Θ ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.
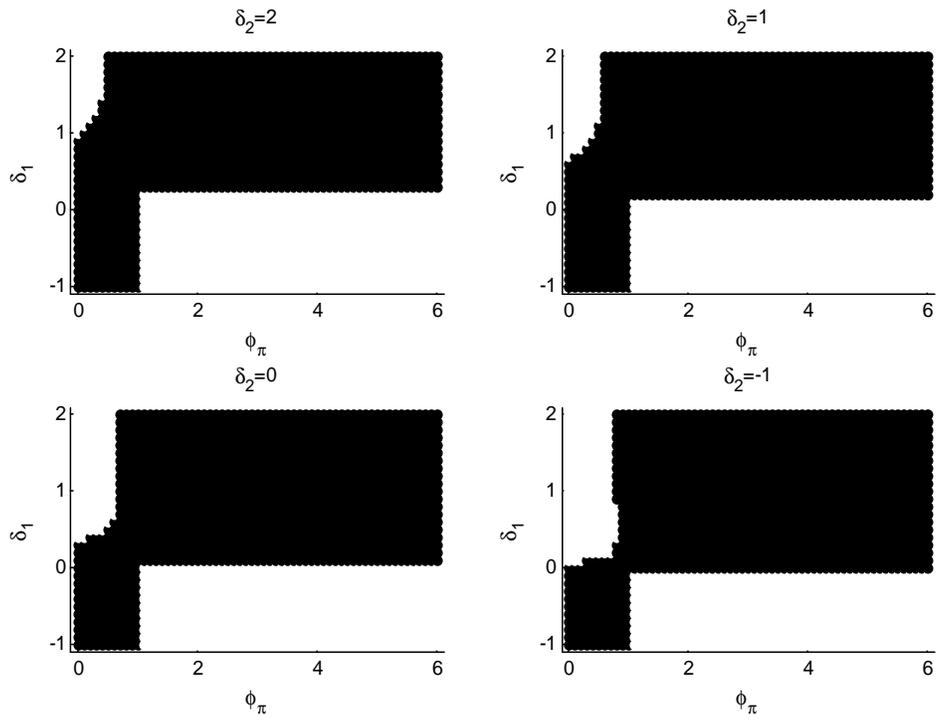
Figure 5: Determinacy area with contemporaneous monetary rule and a fiscal rule of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t + (\delta_2 - 1)\widehat{Y}_t$ and positive Θ ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.
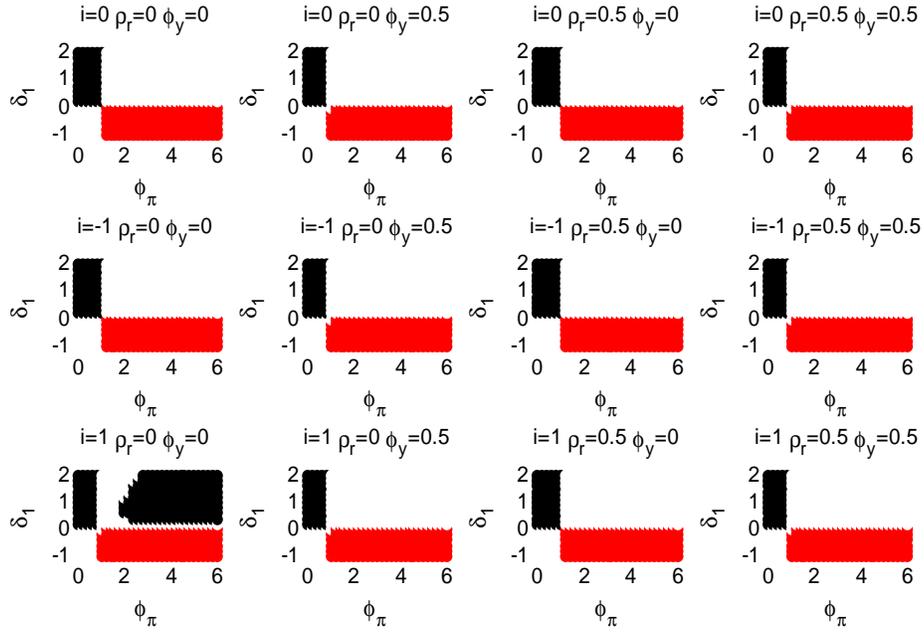
Figure 6: Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$ and a fiscal rule of the type $\widehat{\tau}_t = \delta_1\widehat{b}_t - \widehat{Y}_t$ and positive Θ ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.
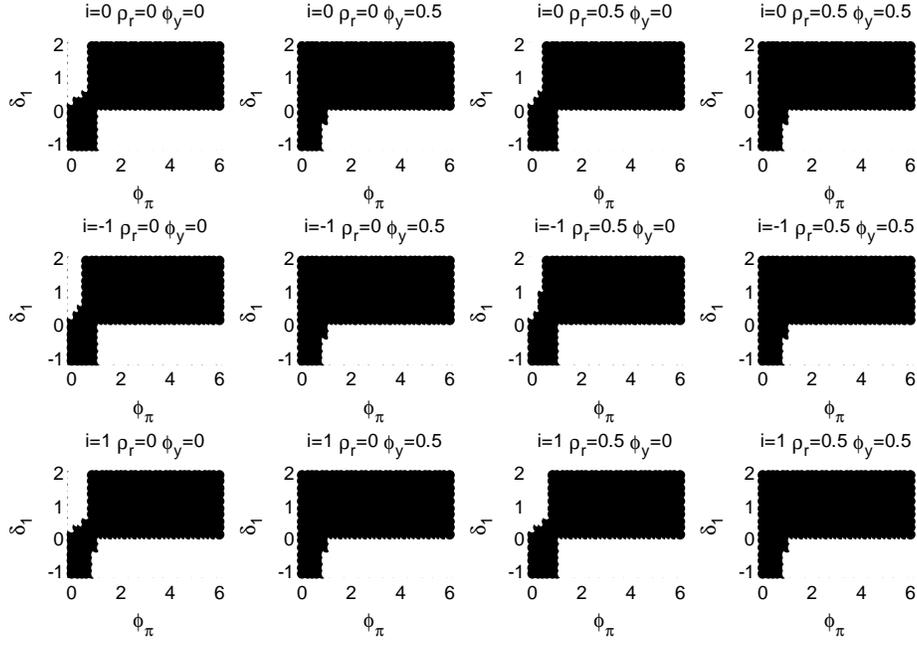
Figure 7: Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$ and a fiscal rule of the type $\widehat{\tau}_t = \delta_1\widehat{b}_t - \widehat{Y}_t$ and negative Θ ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.
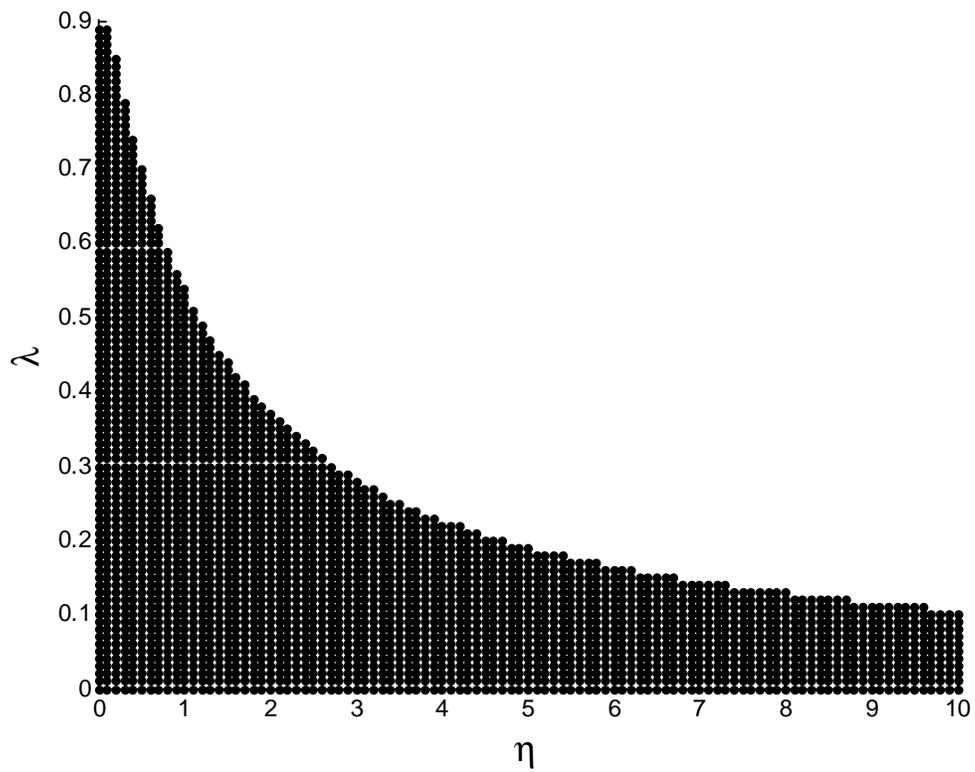
Figure 8: Sign of Θ^{ls}. Black spots, $\Theta^{ls} > 0$, white area $\Theta^{ls} < 0$.
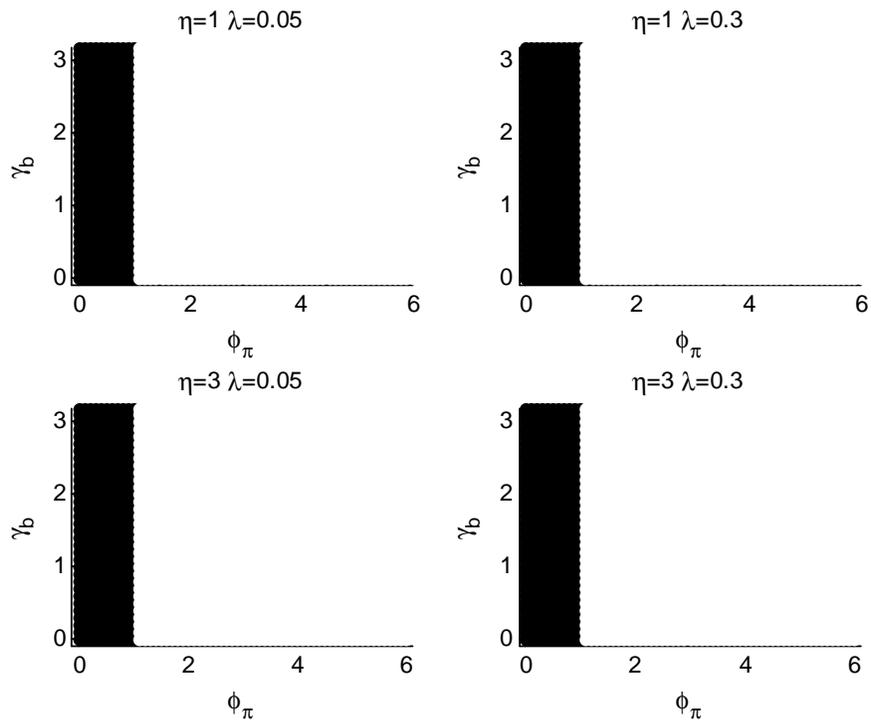
Figure 9: Determinacy analysis with a balanced budget fiscal policy, positive Θ^{ls}. White area, determinacy. Black area, indeterminacy.
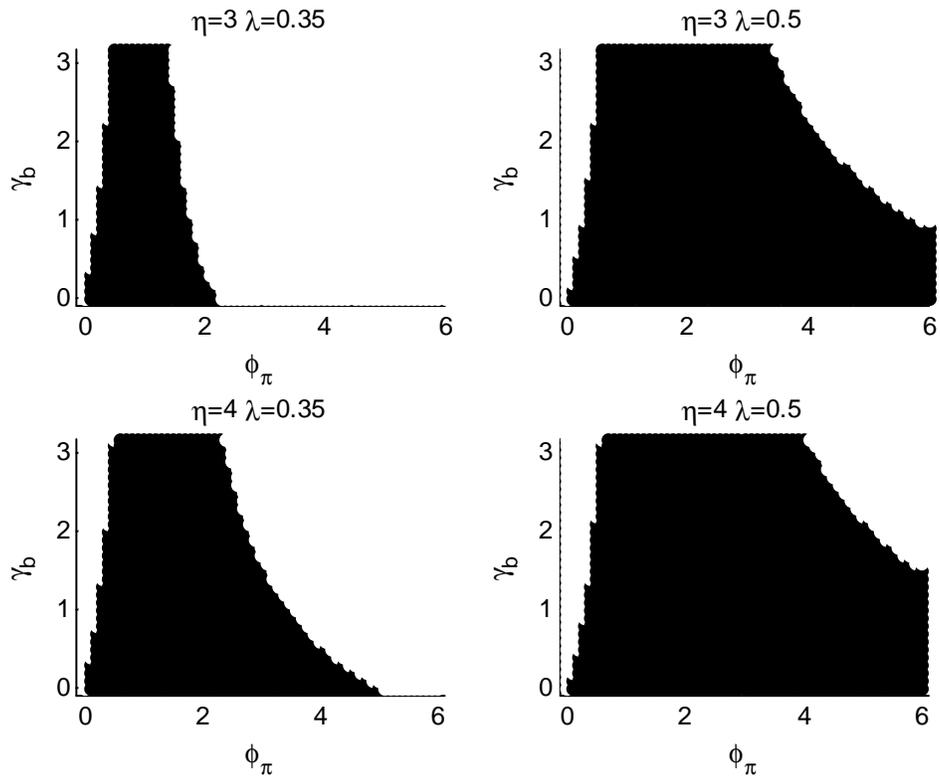
Figure 10: Determinacy analysis with a balanced budget fiscal policy, negative Θ^{ls}. White area, determinacy. Black area, indeterminacy.
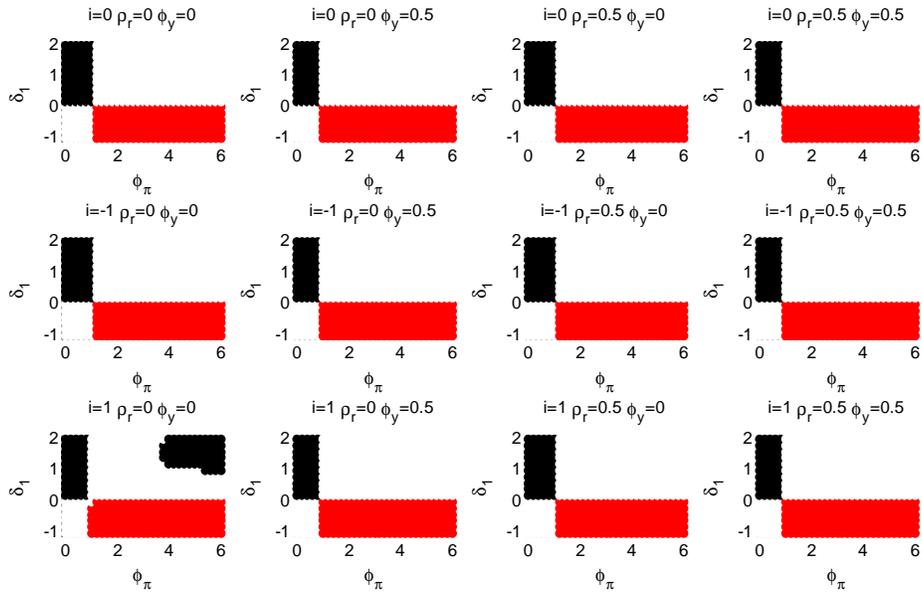
Figure 11: Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$, a fiscal rule of the type $\widehat{\tau}_t^{ls} = \delta_1\widehat{b}_t$ and positive Θ^{ls} ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.
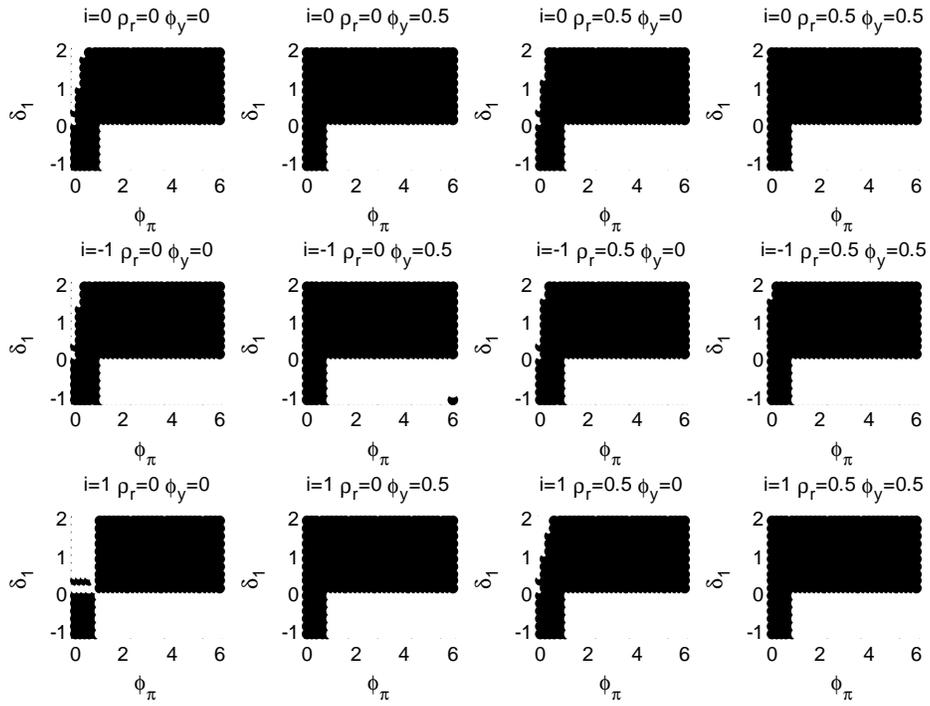
Figure 12: Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$, a fiscal rule of the type $\widehat{\tau}_t^{ls} = \delta_1\widehat{b}_t$ and negative Θ^{ls} ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.