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Estimation and calibration of a Dynamic Variance Gamma model using Vix data

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Abstract

The aim of this paper is to investigate the ability of the Dynamic Variance Gamma model, recently proposed by Bellini and Mercuri (2010), to evaluate option prices on the S&P500 index. We also provide a simple relation between the Dynamic Variance Gamma model and the Vix index. We use this result to build a maximum likelihood estimation procedure and to calibrate the model on option data.

Keywords: Variance-Gamma distribution; Stochastic Volatility model; Vix index; Maximum Likelihood Estimation; Calibration

JEL classification codes: C00; C63; C65; G12; G13

1 Introduction

In this work, we investigate the ability of the Dynamic Variance Gamma model, recently proposed by Bellini and Mercuri (2010), to reproduce the option prices on the S&P500 index (SPX). We also propose a simple historical estimation procedure based on the SPX index and the Vix index.

The Dynamic Variance Gamma model (DVG henceforth) is a discrete time stochastic volatility model where the logreturns follow a conditional Variance Gamma distribution (VG henceforth). The VG distribution belongs to the class of normal variance-mean mixture and corresponds to the case of a Gamma mixing density. Madan and Seneta (1990) have shown the strong ability of the VG to reproduce the stylized fact of the financial logreturns and empirical tests seem to justify the VG option pricing model (see Lam et al. (2002) for example).
The aim of the DVG model is to generalize a model with i.i.d. VG innovations. For this reason, the DVG model has been built to capture a time varying conditional distribution by means of a time varying mixing density. To achieve this result, the shape parameter of the mixing density follows an affine Garch with Gamma innovations. Moreover, the model captures a time varying conditional

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variance and a time dynamics of higher order moments.
Another feature of the DVG model is that it is possible to obtain a recursive procedure for the characteristic function of the logprice at maturity and hence a semi-analitycal option pricing formula based on inverse Fourier transform as in Heston (1993) and Carr and Madan (1999).

The first aim of this paper is to provide a simple historical estimation procedure for the DVG model. As mentioned before, the model is a stochastic volatility model and the construction of a maximum likelihood function requires extensive computational methods, since the variance dynamics is not observable from SPX logreturns. Our idea is to extrapolate this information from the Vix data. We find a linear relation between the variance dynamics and the current Vix value (a similar result has been provided by Zang and Zhu (2006) under the assumption that the logreturns follow the Heston model (1993)).
The Vix index was introduced by the Chicago Board Options Exchange (CBOE) in 1993 and it was originally designed to measure the market's expectation of 30-day volatility implied by at-the money S&P100 Index (OEX) option prices. In 2003, CBOE together with Goldmann Sachs has substantially modified the Vix index. The OEX has been replaced by the SPX and a new methodology of evaluating the Vix has been based on an option portfolio (see the CBOE's white paper (2003) for details)

In order to assess the capability of the DVG model to capture the option market's behavior, we follow two approaches applied in financial literature.
The first approach is to calibrate the parameters using only option prices (see for example Bakshi et al (1996)); since the DVG model allows a semi-analytical procedure for option pricing formula this approach is easy to implement. However, as observed by Christoffersen and Jacobs (2004), the results could depend on the choice of the objective function and they could be unstable over time.
The second methodology is a "mixed" approach; the idea is to estimate the parameters using underlying time series and the available option prices in the market. As observed by Menn and Rachev (2009), this method tries to obtain more stable estimations and to reproduce the behavior of option markets at the same time.

The paper is organized as follows. In Section 2, we review some features and results of the DVG model. In Section 3, we construct a maximum likelihood estimation procedure using the Vix data and we compare its results to a model with i.i.d. VG innovations. In Section 4, The DVG option pricing model is tested on 2009 daily closing European option prices on the S&P500 index and compared with the VG model.

2 Dynamic Variance Gamma model

The aim of this section is to review the main features and the main results of option pricing for the DVG model. We consider a discrete-time market composed by a risky asset and a free-risk bond. The dynamics of riskless security is given by

$$B_t = B_{t-1} \exp(r)$$

where r is the risk-free rate, for simplicity assumed constant.
The stock price dynamics is specified by

$$S_t = S_{t-1} \exp(Y_t)$$

and the logreturns Y_t follow a DVG model defined as:

$$\begin{cases} Y_t = r + \lambda V_t + \sigma \sqrt{V_t} Z_t \\ V_t | F_{t-1} \sim \Gamma(a h_t, 1) \\ Z_t \sim N(0,1) \ \ i.i.d. \\ h_t = \alpha_0 + \alpha_1 V_{t-1} + \beta_1 h_{t-1} \end{cases} \tag{1}$$

with a, α_0, α_1, $\beta_1 > 0$.
By posing

$$a = \frac{1}{\left(\sigma^2 + \lambda^2\right)}$$

the h_t process is the variance dynamics.

As observed by Bellini and Mercuri (2010), the basic idea of the model is to make the conditional distribution of logreturns time-varying and this result is achieved by imposing that the shape parameter of the Gamma mixing density evolves according to a predictable process h_t. Therefore the model is able to capture the time varying conditional variance exactly as in Garch models, moreover it allows a time dynamics for conditional skewness and kurtosis in a simple way. The first four conditional moments are given by:

$$\begin{array}{l} E_{t-1}(Y_t) = r + a\lambda h_t \\ Var_{t-1}(Y_t) = a\left(\lambda^2 + \sigma^2\right) h_t \\ Skew_{t-1}(Y_t) = \frac{\left(2\lambda^2 + 3\sigma^2\right)\lambda}{\sqrt{a\left(\lambda^2+\sigma^2\right)^3 h_t}} \\ kurt_{t-1}(Y_t) = 3\left\{1 + \frac{2\lambda^4 + \sigma^4 + 4\sigma^2\lambda}{a\left(\lambda^2+\sigma^2\right)^2 h_t}\right\} \end{array} \tag{2}$$

We can see that the asymmetry has the same sign of the parameter λ and the kurtosis is always greater than 3 and it is a decreasing function of the h_t. When $\alpha_0 = \alpha_1 = 0$ and $\beta_1 = 1$ we obtain a model with i.i.d. Variance Gamma logreturns and when $\sigma = 0$ we recover an affine Gamma Garch model that has been studied by Bellini and Mercuri (2007)

As Heston and Nandi (2000), Bellini and Mercuri (2010) showed that the DVG process has a recursive procedure for the conditional m.g.f. of the terminal log-price of stock asset:

$$E_t\left(S_T^c\right) = S_t^c \exp\left(A(t;T,c) + B(t;T,c) h_{t+1}\right)$$

where the time-dependent coefficients $A(t;T,c)$ and $B\left(t;T,c\right)$ follow the recursions:

$$\begin{cases} A(t;T,c) = cr + A(t+1;T,c) + \alpha_0 B\left(t+1;T,c\right) \\ B\left(t;T,c\right) = \left[\beta_1 B\left(t+1;T,c\right) - a\log\left[1-\left(c\lambda + \alpha_1 B\left(t+1;T,c\right) + \frac{c^2\sigma^2}{2}\right)\right]\right] \end{cases}$$

with terminal conditions

$$\begin{cases} A(t;T,c) = 0 \\ B(t;T,c) = 0 \end{cases}$$

We conclude this section reviewing the issue of option pricing. The DVG model is an incomplete market, therefore it is necessary to identify an equivalent martingale measure and the standard way of choosing it is based on the conditional Esscher transform proposed by Buhlmann et al. (1996).
Under the new measure, the logreturns dynamics is given by

$$\begin{cases} S_t = S_{t-1}\exp\left(Y_t\right) \\ Y_t = r - \frac{\sigma_Q^2}{2}V_t + \sigma_Q\sqrt{V_t}Z_t \\ V_t|\,F_{t-1} \sim \Gamma\left(ah_t, 1\right) \\ h_t = \alpha_0 + \alpha_1 V_{t-1} + \beta_1 h_{t-1} \\ \sigma_Q^2 = -\frac{8\sigma^4}{\sigma^4 - 4\lambda^2 - 8\sigma^2} \end{cases} \tag{3}$$

(see Bellini and Mercuri (2010) for details).
In order to identify the martingale measure we need five parameters $(a, \alpha_0, \alpha_1, \beta_1, \sigma_Q)$ and the first four are the same under real measure.

3 Historical estimation procedure and comparison with the Variance Gamma model

The purpose of this section is to deal with the issue of historical estimation for the DVG model and we provide a comparison with the VG model based on SPX and VIX time series.
In order to build a maximum likelihood procedure, we have to address two problems. First, the VG density has not got a simple analytical form. However, as reported in Bellini and Mercuri (2010), it is well approximated by a finite mixture of normals. Indeed the VG belongs to the class of normal variance mean mixture and the key idea of approximating its density is based on the Gauss-Laguerre quadrature of the following integral:

$$\begin{aligned} f_{t-1}\left(y_t\right) &= \int_0^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2 s}}\exp\left(-\frac{\left(y_t - r - \lambda s\right)^2}{2\sigma^2 s}\right)\frac{s^{ah_t-1}}{\Gamma\left(ah_t\right)}e^{-s}ds \\ &\simeq \sum_{i=1}^{M}\varphi\left(y_t, r+\lambda x_i, \sigma\sqrt{x_i}\right)\frac{u_i^{ah_t-1}}{\Gamma\left(ah_t\right)}w\left(x_i\right) \end{aligned}$$

where $\varphi(y, \mu, \sigma)$ is a normal density with mean μ and variance σ^2, x_i are the roots of the Laguerre polynomials $L_n(x)$ and the weights $w(x_i)$ are calculated as

$$w(x_i) = \frac{u_i}{(n+1)^2 L_{n+1}^2(x_i)}$$

(see Abromowitz and Stegun 1972).
As shown in the figure 1, the approximation seems to work very well in practice.

Insert fig. 1 here

The second problem arises from the fact that the DVG model is a stochastic volatility model, hence a maximum likelihood procedure based exclusively on the time series of logreturns seems to be difficult. For this reason, we provide a relation between the h_t process and the Vix index.
The methodology of computing the Vix index is based on the replication strategy of variance swaps proposed by Demeterfi et al. (1999). Indeed the current Vix value is related to a portfolio composed by out-of-the money put/call options on the S&P500 index. Although the Vix index depends on the available option quotes on the market, it is reasonable to assume that the strike prices are continuously distributed from 0 to $+\infty$. We neglect the discretization error and the Vix formula is definitively given by:

$$\begin{aligned} \left(\frac{Vix_t}{100}\right)^2 &= \frac{2e^{r(T-t)}}{T-t}\left[\int_0^{S^*}\frac{1}{K^2}P(S_t,K)\,dK + \int_{S^*}^{+\infty}\frac{1}{K^2}C(S_t,K)\,dK\right] = \\ &= \frac{2e^{r(T-t)}}{T-t}\left(E_t^Q\left(\frac{S_T-S^*}{S^*} - \ln\left(\frac{S_T}{S^*}\right)\right)\right). \end{aligned} \tag{4}$$

where K are the strike prices and S^* is the forward price of the S&P500 index.
When the SPX logreturns follow a DVG model, the equation (4) becomes:

$$\left(\frac{Vix_t}{100}\right)^2 = \frac{\sigma_Q^2 e^{r(T-t)}}{T-t}\left(A_t + B_t h_{t+1}\right) \tag{5}$$

where

$$\begin{aligned} A_t &= a\alpha_0\left(\frac{(T-t) - 1 - (\alpha_1 a + \beta)\left(\frac{1-(\alpha_1 a+\beta)^{(T-t)-1}}{1-(\alpha_1 a+\beta)}\right)}{1-(\alpha_1 a+\beta)}\right) \\ B_t &= a\frac{1-(\alpha_1 a+\beta)^{T-t}}{1-(\alpha_1 a+\beta)} \end{aligned}$$

(see the appendix for a complete derivation of the formula).
In conclusion the log-likelihood function is given by

$$L(\lambda,\sigma,a,\alpha_0,\alpha_1,\beta) = \sum_{t=1}^{T}\ln\left(\sum_{i=1}^{M}\varphi\left(y_t, r+\lambda x_i, \sigma\sqrt{x_i}\right)\frac{u_i^{ah_t-1}}{\Gamma(ah_t)}w(x_i)\right)$$

We obtain the parameters $\lambda, \sigma, a, \alpha_0, \alpha_1, \beta$ by solving the optimization problem as defined:

$$\max_{\lambda,\sigma,a,\alpha_0,\alpha_1,\beta} L(\lambda, \sigma, a, \alpha_0, \alpha_1, \beta)$$

with constrains

$$\begin{cases} \left(\frac{Vix_t}{100}\right)^2 = \frac{\sigma_Q^2 e^{r(T-t)}}{T-t}\left(A_t + B_t h_{t+1}\right) \\ \sigma_Q^2 = -\frac{8\sigma^4}{\sigma^4 - 4\lambda^2 - 8\sigma^2} \end{cases}$$

The dataset is composed by 1575 SPX logreturns and 1575 observations of the Vix index ranging from 02/01/2004 to 06/04/2010. The SPX logreturns have been corrected by the dividend yield computed by Bloomberg and we have used the free risk rate extracted by Bloomberg's C079 curve. The optimization problem have been solved by using the Matlab function `FMINCON` and the results have been reported in the following table:

Insert tab. 1 here

In both cases, the risk premium is positive, moreover for the DVG model the parameters α_1 and β are different from zero, therefore a portion of the previous h_t and previous V_t will propagate to future variance. To evaluate the improvement of the DVG model with respect to the VG model we have also performed a likelihood ratio test that supports the idea of a time varying conditional distribution for logreturns.

4 Calibration and Comparison

In the preceding section we saw that the DVG model is an improvement compared to a model with i.i.d. Variance Gamma innovations when we consider a historical estimation. In this section we investigate the capability of this model to fit the option market prices and, as we did for the historical procedure, we compare its performance with the VG model.

We use SPX European options quoted on CBOE. The dataset is composed by closing prices collected every Wednesday from 01/01/2007 to 12/31/2009. We include only options with time to maturity more than 10 days and less than 100 and moneyness ranging from 0.9 to 1.10. All options have to obey the Merton's constraints and the condition of the "convex in strike". Starting initially with 5799 price quotes we end up with 2009 remaining option prices. On each Wednesday, we have approximately 16 option prices

In order to estimate the DVG model from option prices we follow the calibration and the "mixed" approach, both procedures have been implemented in Matlab environment. In the first case, the parameters have been found by minimizing the (dollar) root mean squared error (RMSE) defined as:

$$\$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(C_i^{theo} - C_i^{mkt}\right)^2} \tag{6}$$

and the current value of the process h_t has been extrapolated by the Vix index using the preceding result (5) . In the "mixed" approach, the parameters of the process h_t have been estimated from the time series of the S&P500 index and the Vix index by maximum likelihood estimation while the parameters of the logreturns Y_t dynamics have been obtained by calibration.
As is customary in empirical works, we split the dataset into in-sample and out-of-sample data. We estimate the model in-sample data and we measure its performances in out-of-sample. To check the performance of the fit we adopt the RMSE.

The following tables refer respectively to the in-sample estimation of the DVG model obtained by calibration and "mixed" procedures

Insert tab. 2, 3 here

In the in-sample analysis, we used only the Wednesdays reported in tables 2 and 3. For the "mixed" approach, we perform the maximum likelihood estimation using the preceding 1000 observations of the SPX index and the Vix index. With regards to the out-of-sample analysis, we report the RMSE in the following figures:

Insert fig. 2, 3, 4 here

The best result is achieved by the DVG model with calibration (DVG1). Indeed its RMSE is less than that obtained by "mixed" procedure (DVG2) in the 63.26% of cases while, compared to the VG model (VG), the percentage goes up to the 77.51%. The second best performance is obtained by the "mixed" approach. Indeed DVG2 improves VG in the 63.94% of cases.
In order to further analyze and compare the pricing error, we report the average RMSE for different levels of moneyness and maturity in the following table:

Insert tab. 4 here.

On the one hand the table 4 seems to confirm the preceding result between DVG1 and VG. On the other, the comparison between DVG2 and VG deserves more attention. Since in the first two years we see that DVG2 has got smaller pricing error than VG but, in 2009, we observe a different situation. Moreover if we look to options with maturity less than 30 days the VG model obtain better performance than the DVG model with both procedures. Nevertheless in general the idea of a model with the time varying conditional distribution seems to be supported by option markets.

References

[1] Abramowitz, M., Stegun I. A. (1972). "Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables" Dover Publications

[2] Bakshi, G., Cao C., Chen Z. (1997) "Empirical performance of alternative option pricing models" Journal of Finance 52 pp. 2003 - 2049

[3] Bellini, F. Mercuri, L. (2007) “Option Pricing in Garch Models” *Working paper n.124*, University of Milano – Bicocca

[4] Bellini, F. Mercuri, L. (2010) “Option Pricing in a dynamic Variance Gamma model” accepted for publication in Journal of Financial Decision Making.

[5] Carr, P., Madan, D.B., (1999). "Option valuation using the fast Fourier transform". Journal of Computational Finance 2 (4). pp. 61–73.

[6] Chicago Board Options Exchange, (2003), Vix-CBOE Volatility Index, http://www.cboe.com/micro/vix/vixwhite.pdf.

[7] Christoffersen, P., Jacobs, K. (2004) "The importance of loss function in option pricing" Journal of Financial Economics Volume 72, Issue 2, pp. 291-318

[8] Christoffersen, P., Heston, S.L., Jacobs, C., (2006). "Option valuation with conditional skewness". Journal of Econometrics 131, 253–284.

[9] Demeterfi, K., Derman, E., Kamal, M. and Zou, J. (1999). "A guide to volatility and variance swaps". Journal of Derivatives, 6, 385-407.

[10] Heston, S., (1993). "A closed-form solution for option with stochastic volatility with applications to bond and currency options". Review of Financial Studies 6, pp. 327–343.

[11] Heston, S.L. Nandi, S. (2000) "A closed form option pricing model" Review of financial studies 13,3, pp. 585-562.

[12] Lam, K., Chang, E. and Lee, M. C. (2002) "An empirical test of the Variance Gamma option pricing model" Pacific-Basin Finance Journal, 10(3), pp.267-285

[13] Madan, D. B. and Seneta, E. (1987) "Chebyshev Polynomial Approximations and Characteristic Function Estimation" Journal of the Royal Statistical Society. Series B (Methodological), Vol. 49, No. 2 (1987), pp. 163-169

[14] Madan, D. B. and Seneta, E. (1990). "The variance-gamma (V. G.) model for share market returns". J. Business 63, pp. 511–524.

[15] Menn, C., Rachev, S.T. (2009) "Smoothly truncated stable distributions, Garch-models, and option pricing" Mathematical methods in Operation Research 69 pp. 411-438

[16] Zhang, J. E., Zhu, Y (2006) "Vix Futures" The Journal of Futures Markets 26, 6, pp. 521-531.

5 Appendix

Vix and the Dynamic Variance Gamma model.

The formula of the Vix index is

$$\begin{aligned}\left(\frac{Vix_t}{100}\right)^2 &= \frac{2e^{r(T-t)}}{T-t}\left[\int_0^{S^*}\frac{1}{K^2}P\left(S_t,K\right)dK+\int_{S^*}^{+\infty}\frac{1}{K^2}C\left(S_t,K\right)dK\right]= \\ &= \frac{2e^{r(T-t)}}{T-t}\left(E_t^Q\left(\frac{S_T-S^*}{S^*}-\ln\left(\frac{S_T}{S^*}\right)\right)\right)\end{aligned}$$

where $C\left(S_t,K\right)$ and $P\left(S_t,K\right)$ are respectively the prices of out-of-the money call and put options, S^* is the forward price of the SPX Index. Therefore the Vix is given by

$$\left(\frac{Vix_t}{100}\right)^2 = -\frac{2e^{r(T-t)}}{T-t}E_t^Q\left(\ln\left(\frac{S_T}{S^*}\right)\right) \tag{7}$$

Under the assumption that the underlying process follows a DVG model, the (7) becomes

$$\left(\frac{Vix_t}{100}\right)^2 = \frac{e^{r(T-t)}}{T-t}\sigma_Q^2E_t^Q\left(\sum_{d=t+1}^{T}V_d\right) \tag{8}$$

We assume that the conditional expected value in (8) has the following form

$$E_l^Q\left(\sum_{d=t+1}^{T}V_d\right) = \sum_{d=t+1}^{l}V_d+A_l+B_lh_{l+1} \tag{9}$$

we suppose the relation holds at time $l+1$ and by the iteration law of the conditional expected value we obtain

$$\begin{aligned}E_l^Q\left(\sum_{d=t+1}^{T}V_d\right) &= \sum_{d=t+1}^{l}V_d+E_l^Q\left(V_{l+1}+A_{l+1}+B_{l+1}h_{l+2}\right)= \\ &= \sum_{d=t+1}^{l}V_d+A_{l+1}+\alpha_0B_{l+1}+\left[a+\left(\alpha_1a+\beta\right)B_{t+1}\right]h_{t+1}\end{aligned} \tag{10}$$

by comparison the assumption (9) with (10) we obtain the following recursive system for the coefficients A_{l+1} and B_{l+1}

$$\begin{cases} A_l = A_{l+1}+\alpha_0B_{l+1} \\ B_l = a+\left(\alpha_1a+\beta\right)B_{t+1}\end{cases} \tag{11}$$

with final condition

$$\begin{cases} A_T = 0 \\ B_T = 0\end{cases}$$

We solve analytically the system (11) and we have:

$$
\begin{aligned}
A_t &= a\alpha_0 \left(\frac{(T-t) - 1 - (\alpha_1 a + \beta)\left(\frac{1-(\alpha_1 a+\beta)^{(T-t)-1}}{1-(\alpha_1 a+\beta)}\right)}{1-(\alpha_1 a+\beta)} \right) \\
B_t &= a\frac{1-(\alpha_1 a+\beta)^{T-t}}{1-(\alpha_1 a+\beta)}
\end{aligned}
$$

then

$$
\begin{aligned}
\left(\frac{Vix_t}{100}\right)^2 &= \frac{e^{r(T-t)}\sigma_Q^2 a}{T-t}\left[\alpha_0 \left(\frac{(T-t) - 1 - (\alpha_1 a + \beta)\left(\frac{1-(\alpha_1 a+\beta)^{(T-t)-1}}{1-(\alpha_1 a+\beta)}\right)}{1-(\alpha_1 a+\beta)} \right) + \right. \\
&\quad \left. + \frac{1-(\alpha_1 a+\beta)^{T-t}}{1-(\alpha_1 a+\beta)} h_{t+1} \right]
\end{aligned}
$$

	VG model	DVG model
a	2,24 (4,56E-03)	39,07 (4,19)
λ	4,73E-03 (1,06E-05)	3,34E-03 (1,12E-03)
σ	5,75E-04 (5,06E-06)	1,06E-03 (2,08E-03)
α_0	-	2,94E-07 (2,52E-02)
α_1	-	1,24E-02 (5,32E-03)
β	-	0,79 (0,35)
Logl	4846,93	4933,14
LR statistic	172,41	
p value	3,82E-37	

Tab.1 Estimated parameters by maximum likelihood procedure

	a	lambda_Q	sigma_Q	alpha_0	alpha_1	beta_1	RMSE
17/01/07	192,46	-4,05E-06	2,85E-03	5,48E-04	3,28E-03	3,25E-01	2,02
18/04/07	51,12	-2,28E-05	6,76E-03	1,48E-03	1,63E-02	3,05E-05	2,32
15/08/07	75,91	-3,48E-04	2,64E-02	3,39E-04	8,51E-03	1,59E-01	15,99
12/12/07	152,83	-1,45E-05	5,39E-03	7,75E-04	3,46E-03	4,46E-01	4,80
16/04/08	422,13	-2,94E-06	2,43E-03	2,79E-02	3,11E-04	1,13E-02	7,50
13/08/08	2,53	-6,57E-05	1,15E-02	5,42E-02	2,98E-01	6,75E-07	2,18
17/12/08	0,05	-2,87E-02	2,39E-01	1,90E-01	8,18E-01	6,16E-02	7,85
15/04/09	1,09	-4,15E-04	2,88E-02	3,26E-02	8,72E-01	6,64E-07	3,49
12/08/09	0,87	-5,66E-04	3,36E-02	1,90E-02	1,00E-01	2,97E-03	2,73
16/12/09	60,46	-2,69E-04	2,32E-02	5,63E-04	1,33E-02	4,05E-06	2,62

Tab. 2 Estimated parameters of the DVG model by calibration procedure using in-sample data

	a	lambda_Q	sigma_Q	alpha_0	alpha_1	beta_1	RMSE
17/01/07	203,67	-6,04E-06	3,48E-03	5,30E-04	3,55E-03	2,46E-01	2,16
18/04/07	127,54	-2,91E-05	7,63E-03	9,60E-04	3,20E-03	2,41E-01	2,46
15/08/07	154,00	-1,01E-12	1,42E-06	1,44E-03	3,60E-03	4,43E-01	16,14
12/12/07	120,04	-4,64E-11	9,64E-06	1,41E-03	3,45E-03	5,69E-01	4,91
16/04/08	48,78	-1,73E-03	5,89E-02	3,30E-04	7,29E-03	1,07E-01	7,74
13/08/08	0,97	-4,39E-06	2,96E-03	3,30E-04	3,46E-01	9,70E-07	5,66
17/12/08	2,86	-6,35E-05	1,13E-02	3,30E-04	3,55E-01	9,70E-07	8,07
15/04/09	0,02	-2,55E-09	7,14E-05	1,48E-16	8,28E-01	1,29E-04	17,08
12/08/09	1,12	-3,34E-05	8,18E-03	1,48E-16	8,97E-01	7,19E-05	3,27
16/12/09	2,17E-04	-1,43E-03	5,34E-02	1,48E-16	1,55E-02	6,81E-06	4,96

Tab. 3 Estimated parameters of the DVG model by mixed procedure using in-sample data

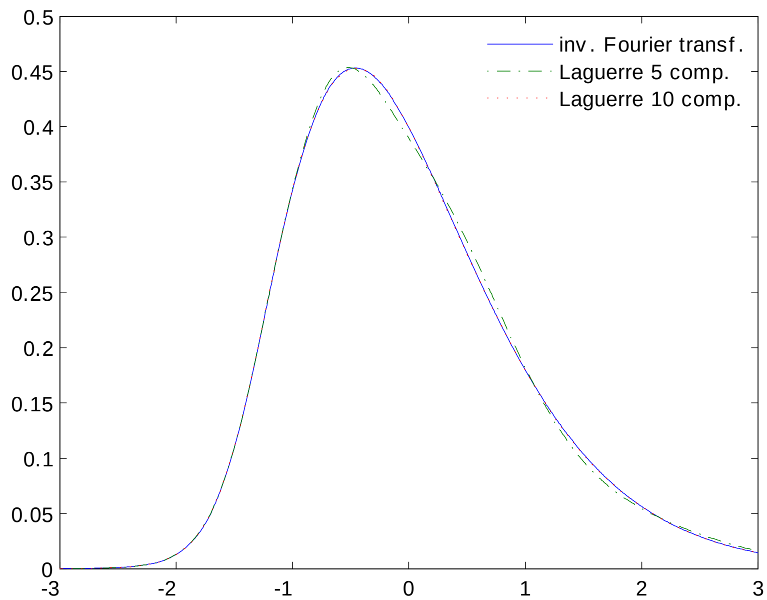

Fig.1 Comparison between the Gauss-Laguerre approximation densities and the inverse Fourier transform. We choose $r = 0$, $\lambda = 1.30$, $a = 0.28$ and $\sigma = 0.72$

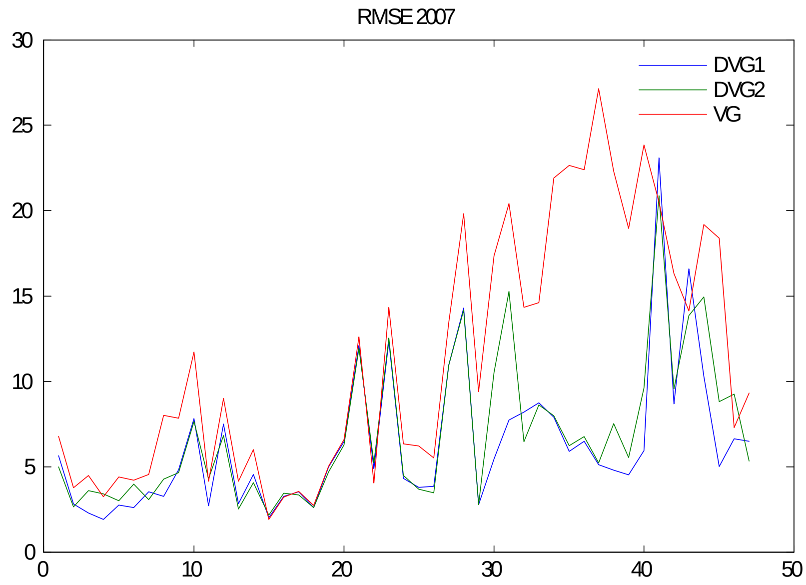

Fig.2 Out-of-sample root Mean Squared Error

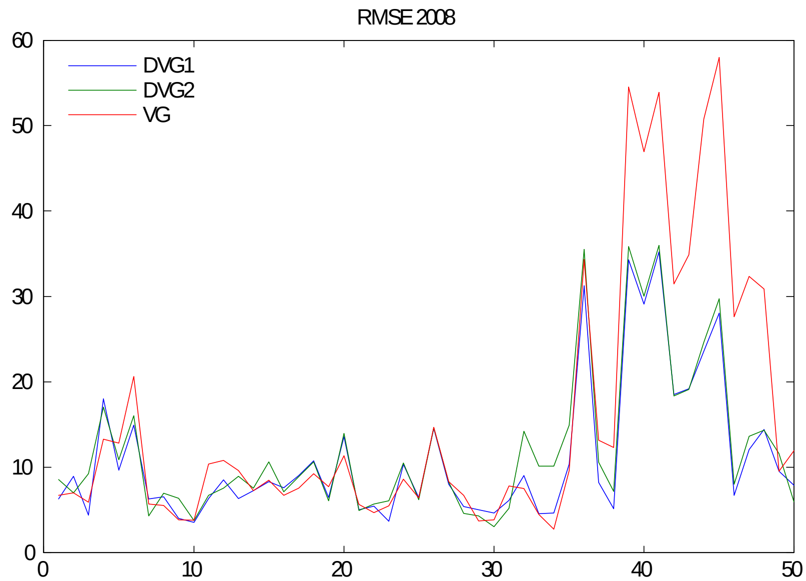

Fig.3 Out-of-sample root Mean Squared Error

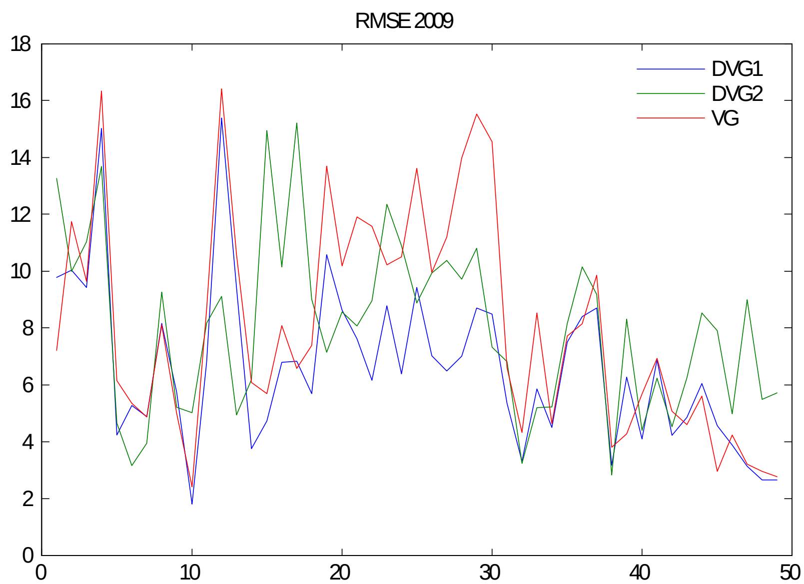

Fig.4 Out-of-sample root Mean Squared Error

	Moneyness K/S			Time to Maturity			Tot.
VG	0.9-0.95	0.95-1.05	1.05-1.1	0-30	31-60	61-100	
2007	10,65	13,76	15,11	9,25	12,25	14,43	13,25
2008	21,59	11,67	17,39	9,29	20,65	16,47	17,04
2009	8,01	7,91	9,89	3,04	6,05	18,89	8,25
DVG1	0.9-0.95	0.95-1.05	1.05-1.1	0-30	31-60	61-100	
2007	9,88	6,23	4,82	3,92	8,27	7,31	7,24
2008	15,63	8,04	9,00	6,43	12,98	12,19	11,65
2009	6,61	5,88	8,35	4,01	5,18	8,67	6,59
DVG2	0.9-0.95	0.95-1.05	1.05-1.1	0-30	31-60	61-100	
2007	9,93	6,14	7,09	4,58	8,17	7,88	7,61
2008	16,27	10,05	9,50	6,83	13,35	13,92	12,71
2009	8,48	8,75	6,27	5,58	5,58	11,24	8,33

Tab. 4 Out-of-sample root Mean Squared Error for Moneyness and Time to Maturity