Classification of Fractal Antenna Radiation Patterns by the Spectrum Enhancement Algorithm

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Spectrum enhancement. Let g denote a scalar function which is at least continuous on the surface $\mathcal{T}$ of a torus. Let $\mathbf{u} \equiv \{u_1, u_2\}$ denote the spatial frequency vector and $G[\mathbf{u}]$ the (distribution-valued) FOURIER transform of g. The spectrum enhancement (*SE*) algorithm, which was introduced a few years ago [e.g., **??**] consists of suitable transformations of the function $H^{(p)}[\mathbf{u}] := |\mathbf{u}|^{2\beta} \frac{|G[\mathbf{u}]|^2}{|a_{0,0}|^2} + \delta[\mathbf{u}]$, where δ is the DIRAC measure, $a_{0,0}$ is the FOURIER amplitude at the origin such that $\mathcal{F}(g)[\mathbf{0}] = a_{0,0}\delta[\mathbf{u}]$, $\beta \in \mathbf{R}^+$ is the *enhancement order* and $\beta = 2p$. It has been recently shown that *SE* is related to FOURIER transforms of derivatives of $g[.]$, which are of integer order whenever $\beta \in \mathbf{N}$ [**??**] and of fractional order otherwise ($\beta \notin \mathbf{N}$) [**??**]. Herewith, the role played by the enhancement order β in the fractal analysis of $g[.]$ will be described.

Fractal radiation patterns. Array antennas exhibiting fractal radiation patterns are synthesized by means of generalized WEIERSTRASS functions [**??**]. For example, in one spatial dimension (x_1) one has

$$f[x_1] := \sum_{m=1}^{\infty} \eta^{(D-2)m} h[\eta^m x_1]$$

where $1 < D < 2$, $\eta > 1$ and $h[.]$ is a suitably chosen bounded, periodic function of one variable. It can be shown that the box-counting (B) fractal dimension ($\dim_B$) of the graph of $f[.]$ (graph$[f]$) satisfies $\dim_B[\text{graph}[f]] = D$. Two-dimensional fractal radiation patterns can be synthesized in a similar way e.g., by separation of variables.

Recognition of fractal radiation patterns. Let simulated or measured radiation patterns be available on the whole surface of the unit sphere or part thereof. Let the pattern $\dim_B$ be unknown or assume the pattern has to be classified by fractal dimension. The *SE* algorithm can be used to extract features from said patterns and estimate their $\dim_B$. Moreover, patterns can be classified by a procedure which includes supervised training. Some preliminary classification and recognition results will be provided, based on numerically synthesized fractal radiation patterns.

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