A MICROECONOMIC RECURSIVE MODEL OF HUMAN CAPITAL, INCOME AND WEALTH DETERMINATION: SPECIFICATION AND ESTIMATION

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KEY WORDS: Human capital, Income, Wealth, Recursive model, Dagum model

1. Introduction

In a period of time, e.g., a year, it can be considered as given the economic space (ES), e.g., a country, and its institutional (I) and technological (T) structures, their functioning (including the levels of transparency, independency, honesty and accountability of their decision makers, such as parliamentarians, judges, corporate executive officers and trade union leaders), and the socioeconomic infrastructure (SEI), where the latter includes research and development (R&D), information technology, school and health systems, drinkable water supply, sewage, energy supply, labor and capital markets, and highway system. They are an integral part of what we call the national structures, which condition the outcome of the economic processes of production, distribution and expenditures. The economic units or economic agents involved in these economic processes are the sets of households (**A**), firms (**F**) and political decision makers, in brief, the government (**G**), and their respective relevant subsets of economic units, such as, trade unions, and associations of manufacturers, bankers and farmers. They are the active units (Perroux, 1975, Dagum, 1973, 1978, 1998, 1994a, 1994b) that operate within a national structure that evolves in time. Their behaviors are conditioned by the economic spaces and sets of active units **A**, **F** and **G** of the rest of the world.

For a given national structure, the modes of action and interaction of the elementary units belonging to the sets **A** and **F** bring about the microeconomic foundation of macroeconomic behavior (known in philosophy of science as *methodological individualism*). On the other hand, the modes of action and interaction of the members of **G** and of the relevant subsets of **A** and **F** bring about the macroeconomic foundation of microeconomic behavior (known in philosophy of science as *methodological holism*). Hence, the economic processes (economic dynamics) and the process of structural changes (structural dynamics) are the outcome of micro-macro foundations of macro-micro behavior (Dagum, 1995).

This study deals with the microeconomic behavior of the set of households

$$\mathbf{A} = \{\mathbf{a}_1, ..., \mathbf{a}_i, ..., \mathbf{a}_n\} \qquad (1)$$

belonging to an economic space endowed with a national structure {**I, T**, SEI}. It purports to explain the process of formation of the levels of 17 endogenous socioeconomic variables. Among them we mention, years of schooling of the household head (**H**) and spouse (**S**) if present, years of full-time and not-full-time work of **H** and **S**, job status and occupation of **H** and **S**, and total wealth, total debt, human capital and income of the households.

Some important predetermined (exogenous and lagged endogenous) variables were not included because of lack of information in the sample survey used to fit the specified model. Among them we mention the lagged values of income and wealth of the households, and particularly, the following variables of the parents of **H** and **S**: income and wealth at the time of birth and at the time of marriage of **H** and **S**, years of schooling and place of residence at the time **H** and **S** completed their high school.

This research is organized as follows: Section 2 specifies the structural form of a recursive model and derives its short and long term multiplier matrices; Section 3 makes a brief presentation of a new method proposed by Dagum and Slottje (2000) to estimate the household human capital; Section 4 presents the model specification and estimation. From the estimated model (20) Section 5 derives and analyzes the short and long term multiplier matrices and the intermediate causal effects; Section 6 fits the Dagum model to the household distribution of total wealth, total debt, human capital and income; Section 7 presents the conclusions.

2. The structural Form of a Recursive Model and Its Short and Long Term Multipliers Matrices

This section specifies and analyzes Dagum recursive model (1994a, 1994b). It mainly purports to explain the household human capital, income and wealth determination and their distributions, and to derive the model short and long term multiplier matrices.

The *factual referent* (Dagum, 1995) of this research is the set **A** of households introduced in (1). Given the microeconomic base of Dagum model we take the discrete set **A**, i.e., the set formed by the singleton subsets $\{\mathbf{a}_i\}$ of **A**, **i = 1, …, n**, as a base to build the sigma algebra $\mathbf{B} = \mathbf{P(A)}$, which is, in this case, the power set of **A**, hence, **(A, B)** is our measurable space. Introducing a probability metric we obtain our basic probability space **(BPS)**:

$$\mathbf{BPS = (A, B, P_b)} \qquad (2)$$

The sigma algebra **B** of **A** contains as members, the empty set ∅, the set of household **A**, and all possible subsets of **A**, including the singleton sets $\{\mathbf{a}_i\}$, **i=1,…,n**; $\mathbf{P_b}$ is the probability of observing the event **b**, such that, $\mathbf{b \in B}$, $\mathbf{b \subset A}$, $\mathbf{P_b \geq 0}$, $\mathbf{P(\emptyset)=0}$, and $\mathbf{P(A)=1}$.

To build a model we have to start with the specification of the **r**-order vector **x** of explanatory (exogenous and lagged endogenous) variables, and the **m**-order vector **z** of endogenous (explained) variables. In Dagum model

(1994a), **r**=7 and **m**=17, hence, it is a recursive model of 17 equations. The 17-th, i.e., the last one, is a specification of Dagum income generating function (Dagum, 1978, 1980, 1994a, 1999b) $\mathbf{z}_{17} = \mathbf{y} = \varphi(\mathbf{h}, \mathbf{k})$ which determines the level and the distribution of income **y** as a function of human capital **h** and total wealth **k**. It plays a central role in linking the functional to the personal distribution of income, and offers a sound base to substantiate the micro-macro foundations of income distribution.

The (**r+m**)-order vector of explanatory and endogenous variables maps the BPS (2) into the following induced probability space

$$(\mathbf{x}, \mathbf{z}): \quad (\mathbf{A}, \mathbf{B}, \mathbf{P_b}) \to (\mathbf{R}^{+}_{\mathbf{r+m}}, \mathsf{B}, \mathbf{P}_\beta) \qquad (3)$$

where (**x, z**) maps: (i) the set of households **A** into the non-negative orthant $\mathbf{R}^{+}_{\mathbf{r+m}}$; (ii) the sigma algebra **B** into the Borel set B; and (iii) the probability measure $\mathbf{P_b}$ into $\mathbf{P}_\beta$, such that, $\mathbf{b}\in\mathbf{B}$, $\mathbf{b}\subset\mathbf{A}$, $\beta\in\mathsf{B}$, $\beta\subset\mathbf{R}^{+}_{\mathbf{r+m}}$, and $\beta=\beta(\mathbf{b})$, hence, $\mathbf{P}_\beta = \mathbf{P_b}$.

The induced probability space (3) determines the content of the methodology of science that is relevant to the subject of inquiry, i.e., probability theory, stochastic processes and statistical inference, hence, econometrics. Therefore, the object of inquiry is not, *per se*, the elementary members $\mathbf{a_i}$ of **A**, but a set of relevant qualitative and quantitative information associated with each household $\mathbf{a_i}$, **i = 1, ..., i, ..., n**. Once specified the (**r+m**)-order vector (**x, z**), to the **i**-th household $\mathbf{a_i}$ corresponds a vector of **r+m** qualitative and quantitative observations which constitutes a sample realization of the **i**-th household.

Hence, to each $\mathbf{a_i}$ corresponds the (**r+m**)-order vector

$$(\mathbf{x},\mathbf{z})(\mathbf{a_i}) = (\mathbf{x_{i1}}, \dots, \mathbf{x_{ir}}, \mathbf{z_{i1}}, \dots, \mathbf{z_{im}}),\ \mathbf{i = 1, \dots, n}; \quad (4)$$

such that, if **n** is the population size, as in a census, we have the population data of the vector (**x, z**), where each $\mathbf{a_i}$ has the constant weight **1/n**; if **n** is the sample size, we have a sample survey of **n** households, as in the national sample surveys of income and wealth distributions, where observation $\mathbf{a_i}$ represents $\mathbf{p_i>0}$ households, and $\mathbf{\Sigma p_i=1}$. The $\mathbf{p_i}$, **i=1, ..., n**, are determined by the statistical design of the experiment. In general, it is a mixture of stratified and random samples purporting to have representative subsamples of the socioeconomic and geographic attributes retained for inquiry, such as gender of the household head, his/her job status, years of schooling and region of residence.

From the (**r+m**)-order vector (**x,z**) we build the recursive model

$$\mathbf{B\,z + \Gamma\,x = u}, \qquad (5)$$

such that, by definition of recursive model, **B** is an **m x m** triangular matrix; **Γ** is an **m x r** matrix of structural parameters, and **u** is an **m**-order vector of independent random variables with zero mean and constant variance. Model (5) establishes the pattern of causation among the endogenous variables $\mathbf{z_i}$, **i = 1, …, m**. Being a recursive model we derive the following equivalent form of (5):

$$\mathbf{z = (I - B)\,z - \Gamma\,x + u}. \qquad (6)$$

Being **B** a triangular matrix, it is non singular and the reduced form of model (5) is:

$$\mathbf{z = -B^{-1}\Gamma x + B^{-1}u = \Pi x + v}; \qquad (7)$$
$$\mathbf{\Pi = -B^{-1}\Gamma}, \qquad \mathbf{v = B^{-1}u}$$

The short and long term multiplier matrices can be deduced from the structural and the reduced forms of the model respectively. Hence, it follows from (6) that the short term (ST) multiplier matrices (direct causal effects) of vectors **z**, **x** and **u** on the vector **z** are,

$$\mathbf{ST_{zz}} = \left.\frac{\mathbf{dz}}{\mathbf{dz}}\right|_{\mathbf{ST}} = \mathbf{I - B}, \qquad (8.1)$$

$$\mathbf{ST_{zx}} = \left.\frac{\mathbf{dz}}{\mathbf{dx}}\right|_{\mathbf{ST}} = -\Gamma, \qquad (8.2)$$

$$\mathbf{ST_{zu}} = \left.\frac{\mathbf{dz}}{\mathbf{du}}\right|_{\mathbf{ST}} = \mathbf{I}. \qquad (8.3)$$

If follows from the reduced form (7) that the long term (LT) multiplier matrices (total causal effects) of vectors **z**, **x** and **u** on the vector **z** are,

$$\mathbf{LT_{zz}} = \left.\frac{\mathbf{dz}}{\mathbf{dz}}\right|_{\mathbf{LT}} = 0, \qquad (9.1)$$

$$\mathbf{LT_{zx}} = \left.\frac{\mathbf{dz}}{\mathbf{dx}}\right|_{\mathbf{LT}} = -\mathbf{B}^{-1}\Gamma = \Pi, \qquad (9.2)$$

$$\mathbf{LT_{zu}} = \left.\frac{\mathbf{dz}}{\mathbf{du}}\right|_{\mathbf{LT}} = \mathbf{B}^{-1}. \qquad (9.3)$$

Therefore, the short term multipliers of the variables $\mathbf{z_i}$, $\mathbf{x_i}$ and $\mathbf{u_i}$ on $\mathbf{z_i}$ are, respectively,

$$\mathbf{ST}_{\mathbf{z_j z_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{z_i}}\right|_{\mathbf{ST}} = \begin{cases} -\beta_{\mathbf{ji}}, & \mathbf{if} \quad \mathbf{j < i}, \\ 0, & \mathbf{if} \quad \mathbf{j \geq i}, \end{cases} \qquad (10.1)$$

because of the triangular structure of **B**, where $\mathbf{B}=(\beta_{\mathbf{ji}})$;

$$\mathbf{ST}_{\mathbf{z_j x_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{x_i}}\right|_{\mathbf{ST}} = -\gamma_{\mathbf{ji}},\ \mathbf{j=1, ..., m;\ i=1, ..., r}; \qquad (10.2)$$

and $\Gamma=(\gamma_{\mathbf{ji}})$;

$$\mathbf{ST}_{\mathbf{z_j u_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{u_i}}\right|_{\mathbf{ST}} = \begin{cases} 1, & \mathbf{if} \quad \mathbf{j = i}, \\ 0, & \mathbf{if} \quad \mathbf{j \neq i}, \end{cases} \qquad (10.3)$$

because of the independence assumption between pairs of random variables.

The corresponding long term multipliers of the variables $\mathbf{z_i}$, $\mathbf{x_i}$ and $\mathbf{u_i}$ on $\mathbf{z_i}$ are, respectively:

$$\mathbf{LT}_{\mathbf{z_j z_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{z_i}}\right|_{\mathbf{LT}} = 0, \qquad \forall \mathbf{i}, \mathbf{j}; \qquad (11.1)$$

$$\mathbf{LT}_{\mathbf{z_j x_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{x_i}}\right|_{\mathbf{LT}} = \pi_{\mathbf{ji}} = -\beta^{-1}_{\mathbf{j.}}\gamma_{\mathbf{i.}}, \qquad (11.2)$$

where $\pi_{\mathbf{ji}}$ is the negative value of the product of the **j**-th row of $\mathbf{B^{-1}}$ and the **i**-th column of **Γ**;

$$\mathbf{LT}_{\mathbf{z_j u_i}} = \left.\frac{\partial \mathbf{z_j}}{\partial \mathbf{u_i}}\right|_{\mathbf{LT}} = \begin{cases} \beta^{-1}_{\mathbf{ji}}, & \mathbf{i < j} \\ 1, & \mathbf{i = j} \\ 0, & \mathbf{i > j} \end{cases}, \qquad (11.3)$$

where $\beta^{-1}_{\mathbf{ji}}$ stands for the **j**-th row and **i**-th column entry of $\mathbf{B^{-1}}$, which is also a triangular matrix.

The indirect causal effects (IC) of vectors **z, x** and **u** on the vector **z** are by definition equal to the difference between their corresponding long and short term multipliers. Hence,

$$\mathbf{IC_{zz}} = \mathbf{LT_{zz}} - \mathbf{ST_{zz}} = \mathbf{B} - \mathbf{I}\,; \qquad (12.1)$$

$$\mathbf{IC_{zx}} = \mathbf{LT_{zx}} - \mathbf{ST_{zx}} =$$

$$= -\mathbf{B}^{-1}\Gamma + \Gamma = (\mathbf{B}^{-1} - \mathbf{I})(-\Gamma) \quad ; \quad (12.2)$$

$$= (\mathbf{LT_{zu}} - \mathbf{ST_{zu}})\mathbf{ST_{zx}} = \mathbf{IC_{zu}}\mathbf{ST_{zx}}$$

$$\mathbf{IC_{zu}} = \mathbf{LT_{zu}} - \mathbf{ST_{zu}} = \mathbf{B}^{-1} - \mathbf{I} \qquad (12.3)$$

When the components of **z, x** and **u** are presented in standard score form (zero mean and unit variance), the matrices **B** and **Γ** become correlation matrices.

3. A Note on Human Capital Method of Estimation

The recursive model (5) is specified and fitted to the data collected in the 1983 U.S. Federal Reserve Board (FRB) Sample Survey of Income and Wealth Distributions. The variables specified in (5), where **H** stands for household head and **S** for spouse, and their corresponding code numbers (see Avery and Elliehausen, 1985), are:

a) Exogenous variables and code numbers:

$\mathbf{x_1}$ = age of the household head (H); Code: B4503(H);
$\mathbf{x_2}$ = gender of H; Code: B3126(H);
$\mathbf{x_3}$ = race of H; Code: B3111(H);
$\mathbf{x_4}$ = region of residence; Code: B3117;
$\mathbf{x_5}$ = marital status of H; Code: B3112(H);
$\mathbf{x_6}$ = age of the spouse (S); Code: B3130(S);
$\mathbf{x_7}$ = gender of S; Code: B3129(S).

b)Endogenous variables and code numbers:

$\mathbf{z_1}$ = years of schooling of H; Code: B4505(H);
$\mathbf{z_2}$ = years of schooling of S; Code: B4605(S);
$\mathbf{z_3}$ = number of children; Code: B3101;
$\mathbf{z_4}$ = years of full-time work of H; Code: B4516(H);
$\mathbf{z_5}$ = years of not full-time work of H; Code: B4517(H);
$\mathbf{z_6}$ = years of full-time work of S; Code: B4616(S);
$\mathbf{z_7}$ = years of not full-time work of S; Code: B4617(S);
$\mathbf{z_8}$ = job status of H; Code: B4511(H);
$\mathbf{z_9}$ = occupation of H; Code: B4535(H);
$\mathbf{z_{10}}$ = industry of H; Code: B4539(H);
$\mathbf{z_{11}}$ = job status of S; Code: B4611(S);
$\mathbf{z_{12}}$ = occupation of S; Code: B4635(S);
$\mathbf{z_{13}}$ = industry of S; Code: B4639(S);
$\mathbf{z_{14}}$ = **k** = household total wealth; Code: B3305;
$\mathbf{z_{15}}$ = **d** = household total debt; Code: B3320;
$\mathbf{z_{16}}$ = **h** = household human capital (this is a latent variable);
$\mathbf{z_{17}}$ = **y** = household income; Code: B3201.

The endogenous variable $\mathbf{z_{16}}$ = **h** = Human Capital (HC) is a *latent*, hence, a *non observable* variable. The estimated value of HC for each household in the sample survey is obtained applying a new method of estimation developed by Dagum and Slottje (2000) that combines the latent variable method (Dagum and Vittadini, 1996, Vittadini and Lovaglio, 2001, Vittadini et al., 2003) with the actuarial mathematical approach to estimate, in U.S. dollars of 1983, (*i*) HC for each household in the 1983 U.S. Sample Survey; (*ii*) the average HC by age of the household head; and (*iii*) the 1983 average and total HC of the U.S. households.

For all the exogenous variables and all but the HC endogenous variables listed above, the 1983 U.S. FRB Sample Survey provides the statistical qualitative and quantitative information to estimate the specified recursive model. To estimate the equations corresponding to the six qualitative endogenous variables $\mathbf{z_8}$ to $\mathbf{z_{13}}$ we apply the logit transformation. To estimate the latent variable HC, i.e., $\mathbf{z_{16}=h}$, we specify **H** as a linear function of 11 indicators (qualitative and quantitative observed variables), i.e.,

$$\mathbf{H_i = L(X_{1i}, X_{4i}, X_{5i}, X_{7i}, Z_{1i}, Z_{2i}, Z_{3i}, Z_{4i}, Z_{6i}, Z_{14i}, Z_{15i}) + u_i} \qquad (13)$$

where the capital letters **H**, **X** and **Z** stand for the standardized (zero mean and unit variance) forms of their corresponding exogenous and endogenous variables listed above.

The linear eq. (13) is a particular case of a system of structural equations (path analysis), that is a linear system relating the latent vectors ξ and η, and two measurement models linking each LV vector to the observed vectors **x** and **y**. In the general case, we have,

$$\mathbf{B}\eta + \Gamma\xi = \zeta, \qquad (14)$$

$$\mathbf{y} = \Lambda_y\eta + \varepsilon, \qquad (15)$$

$$\mathbf{x} = \Lambda_x\xi + \delta, \qquad (16)$$

where **B**, **Γ**, Λ_y and Λ_x are matrices of unknown coefficients to be estimated; **B** is triangular, hence, non-singular; ζ, ε and δ are uncorrelated random vectors among themselves and with respect to the vectors ξ and η of latent exogenous and endogenous variables, respectively.

The first comprehensive method to estimate the system (14)-(16) was LISREL proposed by Joreskog (1970, 1977). It is obtained minimizing a loss function, in general, the log-likelihood, stated as a function of the distance between the model expected and observed covariance matrices.

Several objections raised to LISREL such as, the lack of identifiability of the equations (Joreskog, 1981, p. 73) and the lack of accuracy of the assumption of multivariate normal distribution of the observed variables (Olsson, 1979; Wold, 1982), stimulated the proposition of alternative approaches, such as:

(i) The Partial Least Squares (PLS) method of parameter estimation proposed by Wold (1982), which does not make the assumption of normality;

(ii) The Regression Component Decomposition (RCD) applied to factor analysis, proposed by Schonemann and Steiger (1976);

(iii) The RCD method was extended by Haagen and Vittadini (1991) for application to the reduced form of the structural model (14) and the measurement models (15) and (16);

(iv) Incorporating *a priori* information about the parameters and making use of the assumption of non-correlation among LVs, Haagen and Vittadini (1998) extended further the RCD, proposing the Restricted Regression Component Decomposition (RRCD);

(v) Starting from the path diagram and the properties of PLS, Vittadini and Lovaglio (2001) estimated the LVs as functions of linear transformations of the observed variables;

(vi) After solving the problem of the uniqueness of the solution of the structural model when the observed variables are quantitative, Vittadini (1999) extended the method to the case of having a mixture of quantitative and qualitative observed variables. This is the case that applies to the standardized form **Z** of the latent variable HC in (14). Hence, to estimate **Z**, we make use of the ALSOS MORALS (De Leeuw et al., 1976) method of parameter estimation in its PRINCALS version (De Leeuw and van Rijckevorel, 1980), extended by Vittadini (1999), i.e., the RRCD-PRINCALS method.

Once eq. (13) is estimated, to pass from $\mathbf{H_i}$ in (13) to $\mathbf{h^*(i)}$ in an accounting monetary value, where **i** stands for the **i**-th economic unit, we apply the following transformation

$$\mathbf{h^*(i) = \exp H_i}. \qquad (17)$$

The average value of $\mathbf{h^*(i)}$ is

$$\mathbf{Av(h^*)} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{h(i)p(i)}, \qquad \mathbf{p(i)} = \mathbf{f(i)} \Big/ \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{f(i)}, \quad (18)$$

where **n** is the sample size and **f(i)** is the weight attached to the **i**-th sample observation, because they are not purely random.

Eq. (17) obeys the assumption that, to absolute increments of the standardized variables $\mathbf{H_i}$ correspond relative increments of an accounting monetary value $\mathbf{h^*(i)}$ of HC subject to the initial condition that when $\mathbf{H_i} \to \mathbf{0}$, $\mathbf{h^*(i)} \to \mathbf{1}$. Hence, $\mathbf{dH_i = dh^*(i) / h^*(i)}$, and applying the initial condition we obtain (17).

Working with the average household earnings by age of the household head, and applying actuarial mathematics to the cross-section average earning data by age, Dagum and Slottje (2000) estimated the average household HC by age of the household head and the 1983 U.S. average household HC, **Av(h)**, at the 6% and 8% discount rate. Hence,

$$\mathbf{h(i) = h^*(i)\ Av(h) / Av(h^*)}, \qquad \mathbf{i = 1, ..., n}, \quad (19)$$

give us the dollar estimate of the **i**-th household HC, **i =1, …, n**.

4. Model Specification and Estimation

Using the magnetic tape of the 1983 FRB Sample Survey, Avery and Elliehausen (1985) Technical Manual and Codebook and the vector estimate of the HC latent variable (19), we present the estimation of the recursive model (5). For the quantitative endogenous variables, including HC, we apply ordinary least squares (OLS), beacuase for recursive models it can be proved (Wold, 1953) that the OLS estimators are consistent. The Student-t values are given in parenthesis, and it is also given the adjusted $\mathbf{R^2}$ and F value. For the qualitative endogenous variables $\mathbf{z_8}$ to $\mathbf{z_{13}}$ we apply the logit approach to the parameter estimation, and present the χ^2 values and concordant coefficients. The estimated model is:

$\mathbf{z_{1i}}$ = 12.63 –0.068 x_1 + 0.446 x_3 + 0.333 x_4 (20)
(56.13) (-23.78) (12.15) (7.56)
F=257 Adj. R^2=0.183 (Pr > F)<0.0001

$\mathbf{z_{2i}}$=12.032–0.004x_1–0.171x_2+0.072x_4-3.875x_5+0.333z_1
(37.99) (-2.13) (-1.56) (2.56) (-130.37) (31.46)
F=7109 Adj. R^2=0.912 (Pr > F)<0.0001

$\mathbf{z_{3i}}$ =5.677–0.022x_1–0.158x_3+0.037x_4-0.585 x_5 -0.012z_1
(41.83)(-17.24) (-10.14) (2.05) (-42.45) (-1.76)
F=438 Adj. R^2=0.389 (Pr > F)<0.0001

$\mathbf{z_{4i}}$ = -26.481+0.622 x_1 +11.577 x_2 + 0.235x_3 +0.455 x_4
(-22.49) (70.47) (22.97) (2.08) (3.39)
-1.666 x_5 - 0.092z_2
(-11.73) (-2.35)
F=969 Adj. R^2=0.628 (Pr > F)<0.0001

$\mathbf{z_{5i}}$ = -9.161+0.140 x_1 +1.061 x_2 -0.224x_3 + 0.309 x_5
(-12.82) (18.51) (4.42) (-3.69) (5.45)
+0.089z_1- 0.173z_4
(3.61) (-19.85)
F=90 Adj. R^2=0.121 (Pr > F)<0.0001

$\mathbf{z_{6i}}$ = -0.600 -0.507 x_3 -0.474x_5 +0.151 x_6 +1.191 x_7
(-0.30) (-4.28) (-2.88) (11.62) (1.29)
+0.445z_2- 0.630z_3+0.018z_4
(11.83) (-5.37) (1.25)
F=115 Adj. R^2=0.195 (Pr > F)<0.0001

$\mathbf{z_{7i}}$ =-1.375+0.233x_4-0.416x_5+0.091x_6+0.238z_2-0.077z_6
(-2.21) (2.24) (-3.07) (12.79) (7.14) (-4.65)
F=108 Adj. R^2=0.283 (Pr > F)<0.0001

$\mathbf{z_{8i}}$ = -5.718 (1) -2.360 (2) -1.893 (3)+0.092 x_1+0.179x_5
χ^2 (344.9) (74.1) (48.1) (605.5) (31.3)
Pr>χ^2: <.0001 <.0001 <.0001 <.0001 <.0001
-0.143z_1 -0.104z_3- 0.064z_4
χ^2 (154.1) (10.5) (247.0)
Pr>χ^2: <.0001 <.0012 <.0001
χ^2(5)=1582 Prob<.0001 Concordant=77.9%

$\mathbf{z_{9i}}$=2.374(1) + 3.932(2) + 4.941(3)- 0.032x_1- 0.231x_3
χ^2 (21.1) (57.2) (89.4) (150.8) (64.6)
Pr>χ^2:<.0001 <.0001 <.0001 <.0001 <.0001
-0.528z_8
χ^2 (18.7)
Pr>χ^2:<.0001
χ^2(3)=242 Prob<.0001 Concordant=62.4%

$\mathbf{z_{10i}}$ = -0.871 (1) +1.337 (2) +2.730 (3) +3.069 (4)
χ^2 (24.6) (62.6) (242.6) (300.5)
Pr>χ^2: <.0001 <.0001 <.0001 <.0001
-0.273 x_2 -0.088 z_1 -0.337z_9
χ^2 (27.2) (40.8) (78.2)
Pr>χ^2: <.0001 <.0001 <.0001
χ^2(3)=239 Prob<.0001 Concordant=60.4%

$\mathbf{z_{11i}}$ = -2.336(1)+0.343 (2)+0.943(3) +0.272 x_5 -0.031x_6
χ^2 (130.9) (3.2) (23.8) (24.7) (125.1)
Pr>χ^2: <.0001 <.07 <.0001 <.0001 <.0001
-0.127z_3+0.115z_6
χ^2 (18.3) (632.2)
Pr>χ^2:<.0001 <.0001
χ^2(4)=805 Prob<.0001 Concordant=74.3%

$\mathbf{z_{12i}}$ = 5.972(1) +6.812 (2) +9.235 (3) -0.016 x_6 -0.518z_2
χ^2 (165.2) (210.4) (330.1) (9.5) (281.4)
Pr>χ^2:<.0001 <.0001 <.0001 <.0020 <.0001
+0.081 z_3 -0.368 z_9
χ^2 (3.2) (36.2)
Pr>χ^2: <.0724 <.0001
χ^2(4)=528 Prob<.0001 Concordant=78.1%

$\mathbf{z_{13i}}$ = -3.306 (1) +0.166 (2) +0.845 (3) +1.029 (4)
χ^2 (145.3) (0.6) (16.2) (23.9)
Pr>χ^2: <.0001 <.43 <.0001 <.0001
+0.080 x_3 -0.383 z_{12}
χ^2 (4.2) (65.3)
Pr>χ^2:<.0400 <.0001

$\chi^2(2)=66$ Prob<.0001 Concordant=49.8%

$\mathbf{z_{14i}} = \mathbf{k} = 3443735.0 + 1262.0\ x_1 + 47066.0\ x_2 + 11697.0 x_3$
(39.66) (3.06) (2.67) (3.32)
$-2746299.0\ x_4 + 13470.0\ z_1 + 6559.7 z_2 + 4923.2\ z_4$
(-404.10) (11.51) (6.06) (9.23)
$+64218.0\ z_8 + 123448.0\ z_9 + 43966.0\ z_{11}$
(12.32) (26.98) (9.29)
$+459086.0\ z_{12} + 223216.0\ z_{13}$
(33.06) (19.87)
F=44356 Adj. R^2=0.992 (Pr > F)<0.0001

$\mathbf{z_{15i}} = \mathbf{d} = 409154.0 - 197128.0\ x_4 + 2586.1\ z_1 + 880.0\ z_2$
(41.11) (-181.57) (4.12) (1.26)
$+3269.1 z_9 - 2425.5 z_{10} + 4301.5 z_{11} + 2403.1 z_{12} + 0.036 z_{14}$
(2.34) (-2.27) (1.45) (1.86) (100.61)
F=5616 Adj. R^2=0.975 (Pr > F)<0.0001

$\mathbf{z_{16i}} = \mathbf{h} = 227182.0 - 965.3\ x_1 - 261810.0\ x_4 + 44600.0\ x_5$
(11.09) (-3.55) (-115.97) (9.89)
$+24861.0\ z_1 + 25999.0\ z_2 + 11174.0\ z_3$
(30.43) (25.79) (7.56)
$+1124.4\ z_4 - 769.5\ z_6 + 0.029\ z_{14} + 0.621\ z_{15}$
(3.7) (-3.35) (48.78) (87.29)
F=3554 Adj. R^2=0.921 (Pr > F)<0.0001

$\mathbf{z_{17i}} = \mathbf{y} = 0.571\ z_{14} + 0.300\ z_{16}$
(50.35) (26.44)
F=2367 Adj. R^2=0.536 (Pr > F)<0.0001,
where **i** = 1, 2, ..., 4103.

It follows from the above estimated model that it presents an excellent goodness of fit. In effect, in the equations $\mathbf{z_1}$ to $\mathbf{z_7}$ and $\mathbf{z_{14}}$ to $\mathbf{z_{17}}$, whose endogenous variables are quantitative, the F tests show that the probability of having F values greater than or equal to the corresponding observed F values is less than 0.0001, i.e., (Pr>F)<0.0001. Furthermore, almost all the regression coefficients present Student-t much greater than two in absolute value. Similarly, the remainder fitted equations $\mathbf{z_8}$ to $\mathbf{z_{13}}$, whose endogenous variables are qualitative, present also excellent goodness of fit. In effect, testing the global null hypothesis, the probability that the χ^2 be greater than or equal to the χ^2 of each of these equations is less than 0.0001, i.e., (Pr>χ^2)<0.0001. Moreover, among these 6 equations with qualitative endogenous variables, the regression coefficients of all but two explanatory variables reject the null hypothesis of not being statistically significant at the 0.001 significant level; one rejects the null hypothesis at the 0.05, and the other at the 10% significant levels.

5. Estimation of the Short and Long Term Multiplier Matrices and the Intermediate Causal Effects

The estimated model is given in (20). It presents the estimate of matrix **B** introduced in (5). The subdiagonal matrix $\mathbf{I} - \hat{\mathbf{B}}$ gives the estimated short term multiplier $\mathbf{ST_{zz}}$ derived in (8.1). It follows from (8.1), (10.1) and (20) that:

(i) $\partial \mathbf{z}_2 / \partial \mathbf{z}_1 = 0.333$, i.e., the short term multiplier of $\mathbf{z_1}$ (years of schooling of **H**) on $\mathbf{z_2}$ (years of schooling of **S**) is equal to 0.333, hence, in the short term, for each year of additional schooling of **H**, the marginal increase of the years of schooling of **S** is one third of a year, which shows a positive correlation between the years of schooling of **H** and **S**;

(ii) for total wealth $\mathbf{z_{14}} = \mathbf{k}$, $\partial \mathbf{z}_{14} / \partial \mathbf{z}_1 = \$13470$, i.e., in the short term, an increase of one year of schooling of **H**, keeping constant the remainder explanatory variables, increases **k** by \$13470; instead, $\partial \mathbf{z}_{14} / \partial \mathbf{z}_2 = \$6560$, i.e., in the short term, one year increase of schooling of **S** increases total wealth **k** by \$6560, almost half the impact on **k** of one additional year of schooling of **H**;

(iii) $\partial \mathbf{z}_{14} / \partial \mathbf{z}_4 = \$4923$, i.e., the estimated short term multiplier on **k** of one additional year of full-time work of **H** is equal to \$4923;

(iv) $\partial \mathbf{z}_{16} / \partial \mathbf{z}_1 = \$24861$ and $\partial \mathbf{z}_{16} / \partial \mathbf{z}_2 = \$25999$, i.e., the marginal increases of $\mathbf{z_{16}} = \mathbf{h}$, resulting from the increase of one year of schooling of **H** and **S** are \$24861 and \$25999, respectively. Hence, the contribution of one year of schooling of **S** to the household human capital $\mathbf{z_{16}}$ is 4.6% higher than that of **H** which seems reasonable given the important roles of the spouse (**S**) in the family HC formation and as a supporting member of the household head (**H**);

(v) $\partial \mathbf{z}_{16} / \partial \mathbf{z}_4 = \$1124$, i.e., in the short term, one year increase of full-time work of **H** increases $\mathbf{z_{16}}$ by \$1124;

(vi) the estimated equation of human capital in (20) shows that, $\partial \mathbf{z}_{16} / \partial \mathbf{z}_{14} = \$0.029$, i.e., in the short term, one dollar increase of total wealth $\mathbf{z_{14}}$ increases **h** by \$0.029, while $\partial \mathbf{z}_{16} / \partial \mathbf{z}_{15} = \$0.621$, i.e., in the short term, one dollar increase of total debt $\mathbf{z_{15}}$=**d** increases **h** by \$0.621, which is consistent with the economic units policy of indebtedness to further accumulate HC, moreover, **d** requires a shorter time lag than **k** to have an impact on **h**;

(vii) the income generating function $\mathbf{z_{17}} = \mathbf{y}$ shows that the short term multipliers, i.e., one dollar increase of **k** and **h** increases **y** by $\partial \mathbf{z}_{17} / \partial \mathbf{z}_{14} = \$0.571$ and $\partial \mathbf{z}_{17} / \partial \mathbf{z}_{16} = \$0.30$, respectively.

Besides offering quantitative estimates of the short term multipliers, the results explicitly considered above show an evident consistency with received economic theory.

The estimated matrix $-\hat{\mathbf{\Gamma}}$ in (20) gives the short term multiplier matrix $\mathbf{ST_{zx}}$ of vector **x** on vector **z**. It is of theoretical and factual interest to observe that, according to (8.2) and (10.2), $\partial \mathbf{z}_1 / \partial \mathbf{x}_1 = -0.068$ and $\partial \mathbf{z}_2 / \partial \mathbf{x}_1 = -0.004$, i.e., one year increase of the age of **H** determines a marginal decrease of 0.068 and 0.004 years of schooling of **H** and **S**, respectively. These results validate the statistical evidences of the U.S. and many other countries that show an increasing trend of the population average years of schooling, hence, for a cross-section data, the years of schooling of **H** and **S** are decreasing functions of the age of **H**. This conclusion is also consistent with the estimated short term multiplier of the age of **H** on the human capital, i.e., $\partial \mathbf{z}_{16} / \partial \mathbf{x}_1 = -\$965$. Instead, $\partial \mathbf{z}_{14} / \partial \mathbf{x}_1 = \$1262$, i.e., **k** increases with the age of **H**.

It follows from (9.2) and (20) that the estimated long term multiplier matrix $\mathbf{LT_{zx}}$ is equal to $\hat{\mathbf{\Pi}} = -\hat{\mathbf{B}}^{-1}\hat{\mathbf{\Gamma}}$. It can be verified that the long term multiplier of the age of **H** on the years of schooling of **H** and **S** are negative, i.e., $\partial \mathbf{z}_1 / \partial \mathbf{x}_1 = -0.068$ and $\partial \mathbf{z}_2 / \partial \mathbf{x}_1 = -0.027$. Being (5) a recursive model, the structural and reduced form of the first equation, i.e., $\mathbf{z_1}$, are identical, hence, the long and short term multipliers of vector **x** on the endogenous variables $\mathbf{z_1}$ present the same values. Instead, the estimated long term multiplier of $\mathbf{x_1}$ on $\mathbf{z_2}$ is more than five times its corresponding short term multiplier. On the other hand, according to (9.2) and (11.2), $\partial \mathbf{z}_{14} / \partial \mathbf{x}_1 = \$12974$, $\partial \mathbf{z}_{15} / \partial \mathbf{x}_1 = \$89.55$ and $\partial \mathbf{z}_{17} / \partial \mathbf{x}_1 = \$6670$, while $\partial \mathbf{z}_{16} / \partial \mathbf{x}_1 = -\$2461$, i.e., in the long term, one year increase of the age of **H** increases total wealth, total debt and income, and decreases human capital by the amount given above. It can be also verified that, in the long term, $\mathbf{z_{14}}, \mathbf{z_{15}}, \mathbf{z_{16}}$ and $\mathbf{z_{17}}$ increase per year of increase of the age of **S**.

It follows from (9.3) and (20) that the estimated long term multiplier matrix $\mathbf{LT_{zu}}$ is equal to $-\hat{\mathbf{B}}^{-1}$. Its entries give the long term impacts on **z** of the purely random effects and unanticipated innovations, i.e., the impacts that arise from chance events and unforeseen causes.

Finally, the estimated matrix $\mathbf{IC_{zx}}$ of the indirect causal effects derived in (12.2), i.e., the adding impact to the short term multiplier matrix $\mathbf{ST_{zx}}$ that gives the long term multiplier matrix $\mathbf{LT_{zx}}$, is given by $\hat{\mathbf{B}}^{-1} - \mathrm{I}$.

6. The Size Distribution of the Household Total Wealth, Total Debt, Human Capital and Income

The estimated causal structure (20) of model (5) determines the level of the 17 endogenous variables specified in (20), being the last four: $\mathbf{z_{14}}$=**k** (total wealth), $\mathbf{z_{15}}$=**d** (total debt), $\mathbf{z_{16}}$=**h** (human capital) and $\mathbf{z_{17}}$=**y** (income). Dagum model (1977, 1990, 1999a, 2001) is fitted to these observed distributions, where **k, d, h** and **y** are obtained from the 1983 U.S. FRB sample survey, and the "observed" vector of the household latent variable **h** is obtained from (19), applying Dagum and Slottje (2000) *latent variable-actuarial method* of estimation.

Dagum three-parameter (type I) model is:

$$\mathbf{F}(\nu) = \begin{cases} (1+\lambda\nu^{-\delta})^{-\beta}, & \nu > 0, \ (\beta,\lambda) > 0, \ \delta > 1; \\ 0, & \nu \le 0; \end{cases} \quad (21)$$

Dagum four-parameter model type II (when $0 < \alpha < 1$) is:

$$\mathbf{F}(\nu) = \begin{cases} \alpha + (1-\alpha)(1+\lambda\nu^{-\delta})^{-\beta}, & \nu \ge 0, \ (\beta,\lambda) > 0, \ \delta > 1; \\ 0, & \nu < 0; \end{cases} \quad (22)$$

and Dagum four-parameter model type III (when $\alpha < 0$) is:

$$\mathbf{F}(\nu) = \begin{cases} \alpha + (1-\alpha)(1+\lambda\nu^{-\delta})^{-\beta} = [1+\lambda(\nu-\nu_0)^{-\delta}]^{-\beta}, \\ \nu > \nu_0 > 0, \ (\beta,\lambda) > 0, \ \delta > 1; \\ 0, \quad \nu \le \nu_0 \end{cases} \quad (23)$$

Table 1 presents the estimated values of the parameters, the sum of squared errors **(SSE)** of the probability density function **(PDF)** and the cumulative distribution function **(CDF)**, the number **m** of class intervals, the estimated variance of the residuals ($\mathbf{s^2}$), the Kolmogorov-Smirnov (**K-S**) statistic, the Gini ratio, and the observed and estimated median of those distributions. Since there are high frequencies of null total wealth and null total debt, α is positive and statistically significant, hence Dagum type II model (22) is fitted. For human capital and income, α is negative and statistically significant, therefore Dagum type III model (23) is fitted. Being $\beta\delta < 1$ for the distribution of **k, d** and **h**, the corresponding fitted distributions of these variables are zeromodals, instead, for the fitted income distribution, $\beta\delta = 1.06 > 1$, hence, it is unimodal. For α**<0**, as in the fitted distributions of **h** and **y**, the solution of the equation $\mathbf{F}(\nu) = 0$ for the variable ν gives us the origin $\nu_0 > 0$ of ν. In our case, $\mathbf{h}_0 = 1.1326$ and $\mathbf{y}_0 = 0.1517$ in \$10000, hence, $\mathbf{h}_0 = \$11326$ and $\mathbf{y}_0 = \$1517$. The fitted model of the four variables **k**, **d**, **h** and **y** present excellent goodness of fit as can be substantiated from the estimated values of ***SSE*** **(*PDF*)**, ***SSE*** **(*CDF*)**, **K-S** statistic and the percentage discrepancy between the estimated and observed medians of the distributions. These indicators are important proxies to evaluate the goodness of fit of the distributions of **k**, **d**, **h** and **y**, given that the Dagum model belongs to the heavy tail class of models. Although, according to conventional statistical inference, the sample size **n** = 4103 is a large sample, for the heavy tail distributions under inquiry, a sample of size **n** = 4103 is at most of a moderate size. In effect, the estimated values of δ given in Table 1, tell us that, being $2 < \delta \le 3$, the distributions of **k**, **h** and **y** have finite variances and the moments of order **r** ≥ **3** are infinite. For total debt **d**, being δ **= 3.85**, the fitted distribution have finite moments up to **r = 3**.

Taking the K-S statistic as a proxy for the goodness of fit (it is a proxy because we are considering the observed and fitted distributions which are not independent), its asymptotic critical values at the 0.10, 0.05 and 0.01 significance levels are equal to 0.019, 0.021 and 0.025, respectively. Hence, we conclude that the goodness of fit of the distributions of **d**, **h** and **y** are accepted even at the 10% significance level, while the wealth distribution **k** is accepted at the 1% significance level.

7. Conclusion

This research specifies and estimates a recursive model of 17 equations. Among the corresponding 17 endogenous variables, of which 11 are quantitatives and 6 qualitatives, we specify the variables: years of schooling, years of full-time work and job status of the

household head (**H**) and spouse (**S**), and the household total wealth, total debt, human capital and income.
The equations of model (5) with quantitative endogenous variables are estimated by OLS, which gives consistent estimators. When the endogenous variables are qualitatives, their corresponding equations are estimated applying the logit transformation. Being the human capital a latent variable, we apply the new *latent variable-actuarial approach* developed by Dagum and Slottje (2000) to estimate each household human capital in U.S. dollars. This quantitative estimation of the households HC is used to estimate the human capital equation $\mathbf{z_{16}}$.
The estimation of 16 out of the 17 equations of the model presents an exceptional goodness of fit as can be verified by the **F** values of the fitted equations having quantitative endogenous variables, and the χ^2 values corresponding to the global null hypothesis of the fitted qualitative endogenous variables. In effect, for these 16 fitted equations, **p**<0.0001, i.e., the **p**-value of having an **F**-value for the estimated equations with quantitative endogenous variables, and a χ^2-value for those with qualitative endogenous variables greater than or equal to the observed **F** and χ^2 values, respectively, are less than 0.0001. On the other hand, the job status of **S**, i.e., the equation for $\mathbf{z_{11}}$ accepts the goodness of fit at the 7.5% significant level.
The estimated short and long term multiplier matrices present estimates whose signs are in agreement with received economic theory. Moreover, their corresponding values show the quantitative impacts on the vector of endogenous variables, which have rich policy implications.
This study concludes with the fitting of the Dagum model to the size distribution of the observed household total wealth, total debt, human capital and income.
The specified and estimated recursive model presents important econometric and statistical issues to be retained for further research. Among them, the re-specification of the model to include important causal variables not available in the 1983 U.S. FRB sample survey, such as years of schooling of the parents of **H** and **S**, their lagged wealth as well as the household lagged income and wealth. This will require a marginal change to the household sample survey questionnaires. Furthermore, its policy implications for growth, development and less unequal human capital, income and wealth distributions are issues worthy of further research. They can be easily spelled out from Dagum recursive model (Dagum 1994a,b) presented in (5) and its estimation given in (20).

ESTIMATES	DISTRIBUTIONS			
	Total wealth k	Total debt d	Human Capital H	Income y
α	0.0269	0.2035	-0.0762	-0.0386
$1-\alpha$	0.9731	0.7965	1.0762	1.0386
β	0.1909	0.0768	0.3410	0.3715
λ in \$10E3	1867.70	2092.90	3134.81	32.1705
δ	2.5029	3.8506	2.2929	2.8578
$\beta\delta$	0.4778	0.2957	0.7819	1.0617
ν_0 in \$10E3	0	0	1.1326	0.1517
SSE (PDF)	0.0014	0.0002	0.00049	0.0011
SSE (CDF)	0.0078	0.0007	0.00075	0.0007
M	42	35	47	45
s^2=SSE/(m-1)	0.00019	0.00002	0.000016	0.00002
K-S	0.0246	0.0121	0.0124	0.0149
Estimated median	45232	2581	163539	19380
Observed median	48705	2512	163061	19490
Gini ratio	0.636	0.736	0.528	0.444

Table 1. Parameter estimates and related statistics of the 1983 U.S. total wealth, total debt, human capital and income distributions.

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