

### Dipartimento di / Department of

Economia, Metodi Quantitativi e Strategie di Impresa

| Dottorato di Ricerca in / PhD program  | ECOSTAT   | Ciclo / CycleXXXV |
|--|-----------|-------------------|
| Curriculum in (se presente / if it is) | Economics |                   |

# Essays on household heterogeneity and financial frictions

| Cognome / Surname               | Ferlaino | Nome / Name | Francesco |
|---------------------------------|----------|-------------|-----------|
| Matricola / Registration number | r 863628 |             |           |

| Tutore / Tutor: | Stergios Athanasoglou |  |
|-----------------|-----------------------|--|
| Current de curr | Domenico Delli Gatti  |  |

Supervisor: Domenico Deili Galli

Coordinatore / Coordinator: Matteo Manera

ANNO ACCADEMICO / ACADEMIC YEAR 2022/2023

# Response to the review of the PhD thesis "Essays on household heterogeneity and financial frictions"

#### Francesco Ferlaino<sup>†</sup>

June 2024

## 1 Introduction

**IMPORTANT:** Given the system's restriction on uploading multiple files, the revised thesis has been appended to this very PDF file, directly after addressing the reviewers' comments.

Before discussing the key points and formulating a response to the evaluation of my doctoral dissertation, I wish to extend my deep appreciation to the two reviewers for their valuable feedback. The constructive critique and perceptive comments have significantly enhanced my comprehension of the merits and drawbacks of my study. I am confident that the recommendations put forth will play a crucial role, particularly in preparation for submission to academic journals.

In this reply, original comments by the reviewers are presented in italic, followed by my response. It is important to remark, in order to prevent any misunderstandings, that the figures and text cited in the italicized comments pertain to the previous version of the thesis. Figures and quotes from the new version of the thesis are often included in this response, to prevent unnecessary back and forth between the documents. If not directly quoted, I indicate the specific paragraph and page in the updated version of the thesis. To enhance readability, these parts in the new thesis file are colored in blue instead of black. The blue color is employed in this response to highlight a quotation from the thesis as well.

Unless stated otherwise, the citations referring to sections, pages, footnotes, and paragraphs in the thesis pertain to the updated version rather than the previous one, and they are highlighted in bold type. For figures reported in this response, I have provided the correspondent number of the figure in the updated version of the thesis as well.

To begin with, I will reply to the points raised by Professor Grazzini in Section 2. Subsequently, I will turn my attention to the comments made by Professor Massaro in Section 3.

<sup>&</sup>lt;sup>†</sup>DEMS, Università degli studi di Milano - Bicocca. Email: f.ferlaino@campus.unimib.it.

# 2 Response to comments on the chapter "Does the Financial Accelerator accelerate inequalities?"

### 2.1 General Assessment

This chapter studies the impact of the financial accelerator mechanism on household inequality, and in particular on the impact of a tightening monetary shock on inequality. It compares the results of a HANK model with and without financial frictions. The contribution of the paper is the addition of the financial frictions to a HANK model with heterogeneous households to show that financial frictions increase the impact on inequality of monetary policy. The topic is relevant because of social and economic consequences of inequality and to have a more complete picture on how monetary policy affects the economy. The paper in general is well written and sufficiently clear. There is the need to strengthen the motivation of the paper, to better clarify the mechanism leading to the theoretical results and to revise the empirical section. This is a major revision and I believe that 4 months should be enough to revise the thesis and to make it ready for the discussion.

**Response**: I would like to thank the reviewer for the positive evaluation.

#### 2.2 Main comments

#### 2.2.1 Motivation

The paper adds financial frictions to a HANK model with heterogeneous households (such as Luetticke, 2021). The model behaves very similarly with and without financial frictions, but financial frictions increase the distributional impact of monetary policy. Motivation/results of the paper should be strengthened. For example:

- Can you provide explicit policy or welfare implications? Is optimal monetary policy different in the two models?
- There has been a debate about the fact that monetary policy reacted too late to the recent inflation burst. Can your model rationalize this behavior of the FED?
- Can your model contribute to the debate on rising inequality in the US? If there is a positive causation between EFP and inequality, maybe rising inequality can be explained by an increase in financial frictions.

These are just suggestions, I do not ask you to do all these exercises, however you should try to provide a more compelling motivation to include financial frictions in the model, a part from just "it is interesting to see what happens". I agree that it is interesting, but to make the paper more relevant you need more to it. **Response**: I found all the three suggestion provided very interesting. As a matter of fact, I decided to follow the suggestion offered in the third bullet point above, also in light of the results from the suggested empirical analysis. I consistently modified the introductory Section 1.1, especially the second and third paragraph at page 8. I highlight, among the other, the paper by Caldara and Herbst (2019) that provides a valid argument on why financial frictions on non-financial firms are crucial when analyzing economic variables during the Great Moderation period, although the authors do not address the relationship between these frictions and inequality measures for households. In the second paragraph at page 8 I write:

... Caldara and Herbst (2019) employ a structural vector autoregressive model and discover that large effects of monetary policy shocks in the US during the Great Moderation period are explained by a strong systematic response of monetary policy to financial conditions. ...

As a preliminary check on whether we should expect a correlation between financial frictions and inequality before the empirical analysis of **Section 2.1**, I present Figure 1 in this response (**Figure 1.1** in the thesis), a descriptive statistic in which I show detrended series for a proxy of the magnitude of corporate financial frictions (I use the GZ spread developed by Gilchrist and Zakrajšek, 2012) and for a measure of consumption inequality (I opt for the 50/10 ratio of consumption percentiles).

In the first paragraph of page 9 I write:

To grasp intuition on a possible positive correlation between financial frictions and inequality measures in the data, let us consider Figure 1.1, which shows detrended series for the GZ spread and for a measure of consumption inequality, the ratio of the 50th percentile to that at the 10th percentile of the consumption distribution. The reference period is the so-called "Great Moderation", from the mid-1980s until 2007. These two measures appear to have a certain correlation, since consumption inequality responds with a lag to fluctuations in the spread until the beginning of the new millennium. After the burst of the Dot Com bubble, these co-movements appear to be even more contemporaneous.

The empirical analysis outcome points to a confirmation of the importance of the research question. First, an increase in financial frictions causes an increase in consumption inequality, regardless of what causes the increase in those frictions (**Figure 1.3**). Second, financial frictions seem to enhance the effects of monetary policy on consumption dispersion (**Figure 1.4**). Nonetheless, details on the empirical analysis suggested by the reviewer can be found in Section 2.2.3 of this response.



Figure 1: Corporate spread and consumption dispersion

The blue solid line shows the evolution of the GZ spread over time. The red dotted line displays the ratio of consumption at the 50th percentile to that at the 10th percentile of the consumption distribution. Both series have been detrended with a 8th-degree polynomial trend. The 50/10 ratio has also been logged, de-sesonalized with a quarterly dummy and smoothed with a centered three-quarter moving average.

Source: Gilchrist and Zakrajšek (2012) for the GZ spread. U.S. Bureau of Labor Statistics for consumption data.

#### 2.2.2 Mechanisms

The paper in general is well written. However, the mechanisms behind results should be explained better. In particular:

- How does leverage behave in the model? Is it pro-cyclical or counter-cyclical? In figure 1.C.1 you show that leverage reacts positively to a monetary shock. Since this variable is central to your story, you may want to show the IRF of leverage in the main text, comment on it and describe why it increases.
- How does EFP behave in the business cycle? The External Financial Premium is central to your story, so, as in the case of leverage, you should describe how it behaves and why. Surprisingly, I did not find an explicit comment on the behavior of the IRF of EFP. Since EFP increases with leverage and leverage increases after the shock, I guess that also EFP increases after the shock. You should add the IRF of EFP, maybe together with the IRF of leverage, to comment on what (and why!) happens to financial frictions after the monetary shock.
- The mechanism that causes an increase in inequality after the shock, and the mechanism behind the amplifying effect of financial frictions should be explained better. An increase of the interest rate should reduce wealth of the rich because of the inverse relation between interest rate and prices. In your model it seems that the opposite happens. What else is happening? They possibly increase savings due to



Figure 2: Impulse response to a monetary contraction for aggregate variables Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.

the higher interest rate. Is this compensating the first effect? Can you clarify what happens?

#### **Response**:

Bullet point 1 & 2 Firms' leverage and EFP both display counter-cyclical behavior, since their dynamics are strongly intertwined, and in line with the original mechanism developed by Bernanke et al. (1999). As the reviewer suggested, I integrated their IRFs in the main text, in Figure 2 in this response (Figure 1.6 in the thesis)

I tried to give a more compelling explanation on how this two variables fluctuate and why in the **first paragraph of page 35 and 36**, which I report below:

To illustrate how the financial accelerator works, the IRFs for the EFP, leverage, firm equity, and household liquidity are also displayed. An increase in the nominal interest rate depresses economic activity, leading to a lower demand for capital and, consequently, lower investment and capital price. On the other hand, a higher interest rate increases household liquidity, particularly liquidity directed to firms in the form of loans through financial intermediaries. As suggested by equation (1.11) and shown in the central panels of Figure 1.6, lower levels of capital and capital price and higher levels of firms' debt cause a decline in firms' equity and, therefore, a higher level of leverage.<sup>1</sup> Higher leverage implies higher firm financing costs, i.e., higher EFP, as pointed out by by eq. (1.10). Simultaneously, the entrepreneur's default threshold value,  $\bar{\omega}$ , also increases, which negatively affects the firm's equity level in the next period. With lower equity, firms need to resort to more external financing, but since the latter is more expensive as leverage and EFP increase, the level of capital that firms can afford is even lower, which means less investment and less goods production, generating the multiplier effect of the financial accelerator. The countercyclicality of leverage and EFP in the theoretical model is relevant for two reasons. First, it allows the replication of the financial accelerator mechanism developed by Bernanke et al. (1999). Second, it is consistent with the empirical evidence highlighted in Figure 1.6, in which a monetary contraction is followed by a co-movement of the corporate leverage and a proxy measure of financial frictions.

In addition, a comparison of the leverage and output behavior in the two scenarios (active or passive frictions) deserves a closer look. While output fluctuations are always enhanced by financial frictions for the entire horizon considered, this is not the case for leverage, where the leverage level with active frictions is relatively lower after three years. Although it may seem counter-intuitive, it is a common result in the theoretical literature,<sup>2</sup> and a possible explanation can be found in the power of the friction itself. In the "shut-off" version of the model, external funds are relatively cheaper because the EFP is fixed at its steady state level. Therefore, firms' deleveraging is slower in time, mainly because of the higher debt they contract with financial intermediaries, as shown in Figure 2. Nonetheless, active financial frictions can lead to a higher economic depression in terms of output and investment, even at relatively lower leverage levels in the economy.

**Bullet point 3** I tried to give a more detailed and comprehensive explanation of what happens in the household wealth distribution, starting from the first paragraph on page 41 until the end of the subsection, on page 44. In particular, to clarify the doubt about the increase of the interest rate that should reduce wealth of the rich because of the inverse relation between interest rate and prices, I write in the last paragraph on page 42:

To analyze what happens at the top of the distribution, it is important to remember that, according to model's assumptions, households can only

<sup>&</sup>lt;sup>1</sup>Recall that in this model leverage is defined as  $\frac{qK}{N}$ , or equivalently,  $\frac{D+N}{N}$ . <sup>2</sup>A similar dynamic occurs in the original Bernanke et al. (1999) model.

accumulate wealth in liquid assets. Government bonds and deposits have a fixed price (normalized to one), unlike capital; therefore, they are not affected by price fluctuations. This assumption neglects the fact that, in empirical data, a significant share of rich households' savings comprises illiquid assets, which usually bear a higher interest rate but are subject to price changes. The choice of a single liquid asset for household saving in the model has two main justifications. First, it does not add any further complications to the model structure, keeping it as simple as possible. Second, it provides continuity with the RANK model developed by Bernanke et al. (1999). It follows that IRFs for richer households' wealth could suffer from upward bias because they do not consider the negative effects of capital price fluctuations. However, this should not affect the validity of the results, since empirical evidence shows that rich households react to an increase in the interest rate by increasing the share of liquidity in their portfolio.<sup>3</sup>

#### 2.2.3 Empirics

The paper basically says that HANK models need to include financial friction to study inequality, and in particular to study the reaction of inequality after a monetary shock. You have to convince the reader that this point is empirically relevant. How do we distinguish between the model with and without financial frictions? Is the addition of financial frictions empirically relevant? These are the central questions you should address in the empirical section. In Eq. 1.36 you estimate the impact of a proxy of EFP on consumption dispersion. The proxy is endogenous to economic activity, so you basically are looking at a correlation. Since there may be other forces at work which influence both variables, this estimation does not say that an increase of EFP causes an increase of consumption dispersion. One possible way to show that your mechanism is empirically relevant is to show that an exogenous monetary shock has a significant positive impact on the EFP (as in your model) and on consumption dispersion. Then you should show that the EFP channel is actually increasing the impact of the monetary shock by designing a counterfactual. For example, if you have a SVAR (or better an instrumental SVAR to include externally identified monetary shocks) including both consumption dispersion, EFP and other variables, you can shut down the EFP channel and see if the other variables react differently. Something similar, in a completely unrelated empirical exercise was done in Figure 4 in Mumtaz and Theodoridis (2020). If the monetary shock increases consumption dispersion, you validate the HANK feature. If you show that EFP is affected by the shock and it increases the impact of the shock on consumption dispersion, you also validate the EFP feature. I believe that such result would greatly improve the relevance of

<sup>&</sup>lt;sup>3</sup>Luetticke (2021) shows with empirical estimates that wealthy households react to a contractionary monetary policy increasing their holdings of liquid wealth and portfolio liquidity.

#### the paper.

**Response**: Special thanks are due to the reviewer for the insightful recommendations put forth to improve the empirical analysis, specifically focusing on the methodology. **Section 1.2** of the thesis is entirely dedicated to the attainment of empirical results.<sup>4</sup> The purpose of this analysis is to provide empirical evidence supporting the significance of financial frictions in amplifying the impact of monetary policy on consumption inequality. This, in turn, seeks to establish a stronger justification for the creation of a theoretical model that can simulate these dynamics.

Following the recommendations of the reviewer, I use a proxy-SVAR. This model comprises of a policy rate, a proxy for the EFP (specifically the Excess Bond Premium, EBP, developed by Gilchrist and Zakrajšek, 2012, widely used in this literature), two economic variables representing the productive sector (industrial production and unemployment rate), and a measure for consumption dispersion. To ensure the reliability of the findings, the same SVAR model is executed twice, employing either the Gini index for consumption or the 50/10 ratio as inequality metrics. Furthermore, policy surprises identified as in Romer and Romer (2004) were selected as external instruments.

Regarding the instrument, I have also experimented with employing more contemporary identification approaches, such as those proposed by Gertler and Karadi (2015), Miranda-Agrippino and Ricco (2021), or Bauer and Swanson (2022) for external identification. However, these approaches have resulted in either weak instrument issues or inconsistent outcomes when compared to the existing economic literature. The primary issue with my empirical model likely stems from the relatively limited number of observations in the sample (less than 100), which restricts my ability to include additional variables without encountering concerns with the degrees of freedom in the regression analysis.

Currently, the approach proposed by Romer and Romer (2004) appears to be the only one that consistently produces reliable results for all variables. This is mostly due to the fact that the data collected for these surprises encompasses a wider time span, particularly during the Great Moderation era, in comparison to the previously mentioned approaches. However, I intend to enhance this specific aspect of the model by considering alternative identification approaches, adopting a Bayesian SVAR or exploring other methodologies in the near future.

In the thesis, **Figure 1.2** demonstrates that the implementation of a contractionary monetary policy leads to a statistically significant rise in the proxy for financial frictions and the two considered measures of consumption dispersion, in line with results by Coibion et al. (2017). In order to investigate the potential role of financial frictions in contributing to the observed rise in inequality, I first run a simple SVAR with Cholesky

 $<sup>^{4}</sup>$ Given that the whole section is a whole new addition to the original chapter, I have opted to avoid the color blue for the entire section in the thesis file.



Figure 3: Impulse response of consumption dispersion to a monetary shock Estimated responses to a standard deviation shock of monetary policy using an external instrument for shock identification. The counterfactual response is obtained after "shutting off" the financial friction channel.

identification and two variables: the EBP and a consumption dispersion measure. The outcomes presented in **Figure 1.3** indicate that an increase in financial frictions, regardless of the underlying cause, is accountable for a surge in consumption inequality.

To examine the potential amplifying effect of the EFP on the impact of the monetary shock on consumption dispersion, I adopt the methodology used in Lettau et al. (2002). This approach shares a similar underlying rationale with the paper recommended by the reviewer, Mumtaz and Theodoridis (2020).<sup>5</sup> By employing this methodology, it is possible to "shut-off" the influence of the variable of interest on other variables by assigning a value of zero to the specific elements of the structural model coefficient matrices in the SVAR. By neutralizing the impact of financial frictions, I construct a counterfactual scenario to analyze the IRFs of consumption inequality measures.

The results are shown in Figure 3 in this response (Figure 1.4 in the thesis) and commented at the end of Page 18:

In the unrestricted model, consumption inequality is consistently higher for most of the initial five years, indicating that the EBP tends to increase the dispersion of consumption. At certain intervals, the counterfactual scenario shows lower levels of consumption inequality than the baseline scenario's 68% confidence interval. Interestingly, this pattern seems to have a more pro-

<sup>&</sup>lt;sup>5</sup>The motivation behind my choice to consider the paper by Lettau et al. (2002) as the main source is that the model proposed by the authors does not incorporate second moments, similarly to my SVAR. Conversely, Mumtaz and Theodoridis (2020) restrict the effects of level shocks on these particular moments, as their research primarily revolves around macroeconomic volatility. Nonetheless, the underlying methodology is very similar.



Figure 4: Impulse responses to monetary policy shock Estimated responses to a standard deviation shock of monetary policy using an external instrument for shock identification. Bootstrapped median and confidence intervals are obtained after 2000 wild bootstrap.

nounced effect on the Gini index compared to the 50/10 consumption ratio, despite their relatively similar trends.

Similar comments hold for regression in eq. 1.37. At the beginning of the paper you show that leverage is increasing after a crisis, so the leverage is clearly endogenous to the business cycle. The estimation of eq. 1.37 shows a correlation. Moreover, the estimation in eq. 1.37 also raises some questions about the behavior of leverage. In the data leverage is high when interest rate is low (see figure 1 below). I am not claiming any causal relationship between the two, but this brings to the question: How does leverage behave in your model (as already pointed out in the "motivation" part of main comments)? You should analyze empirically the reaction of leverage to a monetary shock and determine if it is positive as shown in your IRF.

Section 1.2.3 of the dissertation addresses this comment by using the proxy-SVAR model established in earlier sections, focusing on just three variables: the policy rate, the financial frictions proxy, and the leverage for the non-financial sector. The findings, shown in Figure 4 in this response (Figure 1.5 in the thesis), validate the theoretical model mechanism by demonstrating a co-movement between the EBP and the non-financial leverage in the data subsequent to a monetary shock.

#### 2.2.4 Minor comments

**1** Page 8 you should provide the definition of leverage used in the paper. Now the definition of leverage can be found only in the title of Figure 1.1.

**Response**: I chose to delete Figure 1.1 in the introduction of the older version of the thesis, in favor of time series for corporate spread and consumption dispersion, which seem more relevant. However, I give a first definition of leverage in the first paragraph on page 10, where I introduce the empirical contribution:

... In addition, I estimate the relationship between financial frictions and corporate firms' leverage level, defined as the value of firms' capital over equity, which is crucial for the financial accelerator mechanism used in the theoretical model. ...

**2** on page 10, the preview of the mechanism is not clear.

**Response**: I tried to give a more compelling explanation in the last paragraph on page 10:

For the theoretical contribution, a HANK model capable of explaining the empirical findings is built. This model features asset market incompleteness, idiosyncratic income risk, sticky prices, and a *financial accelerator* on the production side, as in Bernanke et al. (1999). The "acceleration" effect arises due to friction in the way entrepreneurs obtain funds for the production of goods. Since an asymmetric information problem is introduced between lenders (financial intermediaries) and borrowers (entrepreneurs), lenders must pay auditing costs to check the actual production and to verify whether borrowers can repay their debt. This implies the existence of an "external finance premium", which is defined as the difference between the cost of funds raised externally (debt) and the opportunity cost of funds internal to the firm (net worth or equity).<sup>6</sup> This premium is linked to entrepreneurs' leverage: the more exposed the entrepreneurs, the higher the premium. Whereas lenders are riskaverse and borrowers are risk-neutral, audit costs are ultimately rebated to entrepreneurs themselves. Therefore, a contraction of economic activity that causes an increase in entrepreneurs' leverage will, in turn, result in higher auditing costs and a higher external finance premium. Entrepreneurs' net worth suffers a further depression due to these higher costs. Ceteris paribus, with lower equity to be used for production, entrepreneurs have to resort to more external funding, increasing their leverage and, consequently, incurring in a higher external finance premium, generating the financial acceleration in the economy. In short, higher leverage increases the cost of external funding, and vice versa, higher cost of external funding negatively affects entrepreneurs'

<sup>&</sup>lt;sup>6</sup>Throughout the paper, net worth and equity are intended as synonym.

net worth, increasing their leverage. Including this mechanism in a model with household heterogeneity helps to assess the impact of this acceleration on wealth and consumption distribution.

**3** At the beginning of section 1.2.2 you introduce the symbol  $\pi_t$ , without defining it. You should say explicitly that it is gross inflation (you introduce the Phillips curve only later in the text).

**Response**: Corrected. I introduce a definition for  $\pi_t$  just after the household budget constraint equation (1.5), on page 21, which is the first time  $\pi_t$  appears in the paper.

**4** The monetary shock should be normalized to produce a 0.25bp increase in the interest rate. This allows a clearer interpretation of the results.

**Response**: Corrected. Although, I am not sure but I assume there is a typo in the reviewer suggestion, probably meaning a 25 b.p. or 0.25% increase in the interest rate, in line with Bernanke et al. (1999). Therefore, I change the monetary shock to produce a quarterly increase in the int rate of 25 b.p.

**5** In the plots you should use different shapes for different lines, so that they can be read also in black and white.

**Response**: corrected.

**6** In figure 1.6 it seems that wealth of top 10% increases on impact after the shock. I mentioned this point also in the main comments, but this sounds weird to me. An increase in interest rates should reduce price of bonds, thereby reducing wealth.

**Response**: as I have already explained in Section 2.2.2 of this response (Bullet point 3), this is mostly due to absolute wealth liquidity and normalized price for bonds assumptions.

7 on page 33-34 you write "Since wages are equal for every one...". This phrase is misleading. You have that the hourly wage of workers is  $W \ x \ h$ , so it actually depends on productivity. You should specify what you mean with "wages" in your case. Usually, with wage you refer to hourly wage.

Response: Corrected. First paragraph of Section 1.6, on page 44:

... As already expressed in eq. (1.5), labor income for workers before taxes is defined as  $W_t h_{it} l_{it}$ . Since the wage level,  $W_t$ , is not idiosyncratic and is equal for everyone, if two workers with different productivity,  $h_{it}$ , were to provide the same quantity of labor,  $l_{it}$ , the high-productivity worker would obtain a higher salary. ...

# 3 Response to comments on the chapter "Effects of different financial frictions on households"

#### 3.1 Summary and general assessment

This review focuses on the second chapter of the thesis, titled "Effects of different financial frictions on households". The chapter studies how different financial frictions affect the wealth and consumption distribution of households following a contractionary monetary policy shock. The analysis is framed in a HANK model featuring households' heterogeneity due to idiosyncratic labor productivity. The first friction considered concerns the ability of households to obtain loans, while the second focuses on the ability of production firms to raise external funds. To this end, the model features two types of financial intermediaries: commercial banks, charging a premium on households' borrowing which depends on the aggregate level of households' debt, and investment banks, facing auditing costs and thus charging a premium on the cost of external funds for firms. The production sector is standard, with intermediate and final good producers acting in perfect competition, resellers facing quadratic costs of price adjustment, and capital producers facing quadratic adjustment costs to produce new capital. A central bank sets the nominal inter- est rate according to a Taylor rule with inertia, while the government sets the level of government spending, tax revenues and issuance of new bonds. The model is calibrated and the impact of a contractionary monetary policy shock is assessed by comparing impulse responses in a scenario with financial frictions on households and a scenario with financial frictions on firms.

The PhD candidate shows a good degree of knowledge of both modeling and numerical solution techniques. The modeling effort combines different off-the-shelve frameworks (Bernanke et al. 1999, Bayer et al. 2019, etc.), while the solution method follows Bayer et al. (2019) and Luetticke (2021). The research question is worthy of investigation and the topic is receiving in- creasing attention by scholars. Although reasonably polished, the current state of the chapter requires some revisions before it could be sent for publication in an international scientific journal with peer review. I therefore suggest a major revision and an extension of 4 to 6 months.

**Response**: I would like to thank the reviewer for the positive evaluation.

#### **3.2** Comments: Aggregate fluctuations

Figure 2.1 depicts impulse responses of aggregate variables to a monetary contraction. When describing the reactions of the composite consumption X and of the aggregate goods consumption C, it is stated that the reaction of the latter is similar in the case of financial frictions on firms and in the case of financial frictions on households. On the other



Figure 5: Impulse response to a monetary contraction for aggregate variables. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households.

hand, the reaction of the composite consumption differs in the two scenarios. This is not clear to me. In fact, the dynamic reaction of L is virtually identical in the two scenarios, as displayed in Figure 2.C.1 and stated on page 86. Given the definition  $C = X + L^{1+\gamma}/(1+\gamma)$ , it has to be the case that the differences observed in the dynamics of X in the two scenarios would imply very similar differences in the dynamics of C in the two scenarios, contrarily to what it is claimed in the main text on page 86. It seems to me that the reason why this is not visible in Figure 2.1 is simply due to the scaling of the two bottom panels: X ranges between -0.3 and 0.2, while C ranges between -4 and 0.5. I believe the candidate should further investigate the issue, e.g., by constructing numerical measures of the differences between scenarios.

**Response**: I completely agree with the reviewer, I acknowledge that my choice of words to describe the findings was inadequate. What I wanted to say is that the pattern of goods consumption may appear similar, but the responses for composite consumption exhibit distinct behaviors, although I did not address the fact that the magnitude on the Y-axis were different for the two variables, and therefore resulting in equivalent absolute differences between the scenarios due to comparable labor dynamics. In order to clarify this concept, I have restructured the presentation of these results. I first introduce IRFs for output and investment in **Figure 2.1** in the thesis, followed by a separate analysis of consumption and labor dynamics in **Figure 5** in this response (**Figure 2.2** in the thesis).

I introduce the comments suggested by the reviewer in the last paragraph of page 94, underlying the scaling issue of the Y-axis in Footnote 69 in the thesis (reported in

this response as Footnote 7):

Consumption and labor dynamics are displayed in Figure 2.2. Goods consumption, C, falls relatively more on-impact when considering active frictions on households. In the first nine quarters, the goods consumption response is lower but then overshoots and overtakes IRF values for the comparative scenario. Recall that goods consumption can be expressed as a function of composite consumption, X, and labor, L.

The right-hand side graph in Figure 2.2 shows that labor dynamics are fairly similar in the two scenarios. Therefore, the difference in responses occurring in C is almost entirely due to what occurs at the composite consumption level. Under active financial frictions on firms, X falls on-impact and then strongly overshoots, beginning its reversion to the steady state value almost immediately. Conversely, composite consumption under active financial frictions on households exhibits a relatively much greater fall on-impact. It follows that household borrowing frictions imply a relatively higher impact at the consumption level, suggesting an important role for household loan rate fluctuations. In this scenario, it takes the impulse response of X five quarters to overshoot, but then it keeps increasing for the remaining period considered in the figure. Approximately nine quarters after the shock, the value of composite consumption in this case exceeds that of frictions on the production sector. This is the same timing as that in the responses for goods consumption. This outcome is a consequence of the fact that, as mentioned above, labor dynamics are virtually similar in the two models. Given this result, and in light of the implications of equation (2.5), I focus on the dynamics of X rather than C to better understand the effects of the two financial frictions on household consumption.<sup>7</sup>

### 3.3 Comments: Wealth and consumption inequality

Figure 2.2 plots the responses of the Gini index of inequality for both wealth and consumption to a contractionary monetary policy shock. One interesting result is that inequality in wealth is higher in the case financial frictions on firms, while inequality in consumption is lower in the case financial frictions on firms. on page 88, it is stated that both wealth and consumption inequalities increase, but they do so at a relatively inverted pace. It is not clear to me whether this is a reference to the slopes of the reaction curves, also because the reaction of the Gini indexes for wealth is sort of hump-shaped (see also next

<sup>&</sup>lt;sup>7</sup>It must be noted that the visual difference in terms of "curve behavior" between responses for X and C is mostly due to the magnitude of the fluctuations. For instance, if we focus on the on-impact difference between the two models, we observe a similar differential in both composite and goods consumption, but the order of magnitude of the Y-axis in Figure 2.2 is different for these two variables.

comment), while the reaction of the Gini indexes seems to be monotonically decreasing over time. I think this point should be clarified.

**Response**: It was again a poor choice of words from my part. What I wanted to underline is exactly what the reviewer highlighted. After a monetary contraction, both Gini indices for wealth and consumption rise, in both scenarios. However, in case of financial frictions on firms, the wealth Gini index has a relatively higher response compared to the counterfactual scenario, while this response is relatively lower for the consumption Gini coefficient. I comment this behavior in the **first paragraph on page 97**:

A noteworthy observation emerges when examining which type of friction results in a more pronounced fluctuation in inequality for a given variable. Examination of the Gini index for wealth indicates that financial frictions affecting firms lead to a more significant response. Conversely, when analyzing the Gini index for consumption, it is evident that financial frictions related to household borrowing have a greater influence. Therefore, it can be concluded that wealth distribution is more sensitive to frictions in the production sector of the economy, whereas the dispersion of consumption is more impacted by frictions that hinder households' capacity to borrow liquidity.

Somewhat related to the previous point is the following. From Figure 2.2 it is clear that the difference in the response of the Gini indexes for wealth in the two scenarios (financial frictions on firms vs. on households) is increasing over time, while in the case of consumption the difference is hump-shaped, as also stated on page 87. It seems however that the response of the Gini index for wealth in the case of financial frictions on firms starts decreasing around period 14 or so. It would be useful to plot the impulse responses for periods beyond 20 to understand whether the hump-shape in the difference between the two responses is recovered. If that is the case, the distinction between the two differentials (wealth vs. consumption) would regard their persistence rather than their qualitative behavior (steadily increasing over time vs. hump-shaped) as stated on page 87.

**Response**: As suggested, I extended the IRFs beyond the 20 periods, considering a time span of 100 quarters. I report the results for both wealth and consumption Gini indices in Figure 6 in this response.

The differential for the Gini index of wealth appears to be hump-shaped actually, but it starts decreasing approximately after 60 quarters, so very forward in time. In the very long term, therefore, it is true that the difference between the two differential curves regard their persistence. Therefore, i think it is safe to assume that for a reasonable long period after the aggregate shock, the differential for the Gini index of wealth shows an upward trend. On the other hand, the differential curve for the consumption



Figure 6: Impulse responses to a monetary contraction for wealth and consumption inequality.

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households. The green line with asterisks is the percentage difference between the two curves using the lower curve as the base.

Gini index starts to rise again after approximately 35 periods. Note that this peculiar dynamic occurs not because of an unexpected rise of the Gini index in the future, but rather stems from a more rapid decline in the IRF for financial frictions on firms. It is crucial to acknowledge that while differential measures provide valuable insights, their interpretation becomes more challenging as the two analyzed curves approach the steady state level, leading to increased volatility in the results. In the thesis, I address this issue in **Footnote 71 on page 97**.

#### 3.4 Comments: Wealth and consumption dynamics

I have to admit that I had a bit of a hard time following the discussion in Sections 2.5.3 and 2.5.4. In particular, I was initially confused by the definition of Hand-to-Mouth (HtM) households in the model. Usual definitions of HtM households require them to have zero wealth by construction (e.g. TANK models), while in the proposed model there is a continuum of heterogeneous households endogenously choosing c and l (and thus a) without being exogenously constrained. I think that the discussion would definitely benefit by a clear definition of HtM households in the main text, e.g. at the beginning of Section 2.5.3.

**Response**: Corrected. I introduce my definition of HtM in the **first paragraph on page 98**:

Before moving forward with the analysis of the findings, it is important to address the calibration of HtM households in this model. In standard TANK models,<sup>8</sup> the proportion of HtM (or *rule-of-thumb*) households is externally determined, usually implying by construction that those households have zero wealth and exclusively spend their current income. Within HANK economies, households choose their optimal level of wealth and consumption endogenously in each period. This dynamics decision-making process allows for variations in the proportions of HtM households following aggregate shocks. In the HANK model proposed by Kaplan and Violante (2014), households are defined as HtM whenever they choose to either have zero liquid wealth or to lie at the credit limit. Due to technicalities of my model constructions, I have opted to employ a different definition of HtM. First, because I am already studying the fluctuation of the share of borrower households, I will not include agents who have reached their borrowing limit when calculating the HtM share. Second, given that the grid used to compute the wealth distribution is not evenly spaced and contains several grid points in close proximity to the zero-wealth threshold, households are classified as HtM if they possess zero or near-zero wealth, that is, a positive amount of wealth that does not surpass the minimum possible quarterly labor income realization.<sup>9</sup>

More generally, I think that in order to convey the main message, the candidate should better explain the economic intuition behind the obtained results.

My reading of the results derived in the paper is as follows. Monetary policy has both direct and indirect effects on households, with the former operating through changes in households' incentives to save and in households' net financial income, and the latter having an effect through the general equilibrium responses of prices and wages (hence of labour income and employment). After a monetary tightening, the considered financial frictions on households amplify the negative direct effect on borrowing households, while the considered financial frictions on firms amplify the negative indirect effect.

**Response**: I agree with the interpretation of the results given by the reviewer. I probably did not address enough the dichotomy direct-indirect effects, which was the main result of the consumption decomposition I carried out at the end of the previous version of the thesis. In the new version, I dedicate **Section 2.4.5** to the consumption decomposition of direct and indirect effects. The two figure presented in this section of the thesis are **Figure 2.6 and Figure 2.7**, that I report here in an unique figure as Figure 7.

In the second paragraph on page 105, I comment these findings highlighting the different dynamics of direct and indirect effects:

<sup>&</sup>lt;sup>8</sup>Such as Galí et al. (2007) or Bilbiie (2008)

<sup>&</sup>lt;sup>9</sup>The results remain almost unaffected when exclusively considering zero-wealth households as HtM.



Figure 7: Consumption decomposition for relevant prices

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The graph on the left-hand side represents the decomposition for the case of friction on firms. The graph on the center represents the decomposition considering the household borrowing spread in the liquidity return. In the graph on the right-hand side, I consider the same case with the borrowing penalty  $(\omega_t^H)$  as an individual variable.

Referring to the dichotomy proposed by Kaplan et al. (2018), it becomes possible to depict the results in terms of direct and indirect effects of monetary policy on household consumption. The presence of financial frictions within the production sector has a more significant influence on the indirect effects, particularly those associated with labor income, when compared to the counterfactual scenario. The wage component (yellow circled line) exhibits greater strength and persistence in the presence of active frictions and firms. This channel primarily contributes to the decline in composite consumption for this scenario, since profits and liquidity return contributions exert a positive influence for the period considered in Figure 2.6. On the other hand, financial frictions related to fluctuating household loan rates reassess the importance of direct effects in depressing consumption after a monetary contraction, primarily through changes in the borrowing penalty. It is important to underline, however, that significant indirect effects still exist in this context. The wage contribution remains a substantial factor in consumption reduction even with financial frictions on households.

In addition, I also included, in the **last paragraph of page 105**, an analysis for the different behavior of the liquidity return IRF in the left-hand side and right-hand side graphs of Figure 7 in this response:

In relation to direct effects, the behavior of the liquidity return contribution (net of the borrowing premium) in terms of response shape varies significantly. On impact, the contribution is marginally higher in the scenario in which there are frictions on firm borrowing. This can be observed by comparing the left-hand graph in Figure 2.6 with the right-hand one in Figure 2.7. The two responses reach their peak around the same time, with the former peaking in the third quarter and the latter in the fourth quarter. However, the rate of reversion differs significantly between the two. Reversion is much faster under firm financial frictions, whereas it is much slower under household frictions.<sup>10</sup> At first glance, this result may seem counter-intuitive. Financial frictions affecting household borrowing actually enhance the positive contribution of liquidity return in the long run, whereas the opposite happens when these frictions are shut off. Nevertheless, as explained in Section 2.4.3, this outcome is a logical consequence of the interplay between the demand and supply of borrowings in the production sector. First, most funds channeled to firms originate from the top 10% of households, who, as per the model's construction, are not impacted by the increase in the loan rate.<sup>11</sup> Second, under financial frictions on firms, entrepreneurs tend to resort to higher levels of debt initially, but subsequently aim to minimize their debt exposure due to higher costs associated with financial frictions. Therefore, in the last case, there is a faster decrease in firms' demand for borrowing. Conversely, under active frictions on households, entrepreneurs exhibit a relatively stronger inclination toward debt utilization, resulting in a slower reduction in their demand for funds. Therefore, this enduring dynamic also appears to have long-lasting effects on aggregate composite consumption, primarily through the contribution of liquidity returns on the latter.

Furthermore, I wish to highlight that I have consistently modified and rephrased Section 2.4.2, Section 2.4.3, and Section 2.4.4 in the revised edition of the thesis to offer a more precise interpretation of my results, particularly emphasizing the relevance of household behavior in proximity to the zero-wealth threshold. For instance, at the end of Section 2.4.4, on page 102, I write:

Differences in consumption responses offer insight into the dynamics near the zero-wealth boundary in Figure 2.4 and, therefore, in terms of consumption and wealth inequality. In presence of financial frictions on households,

<sup>&</sup>lt;sup>10</sup>Extending the duration of the IRFs reveals that consumption undershooting occurs approximately 24 quarters after the shock under financial frictions on firms. In the comparative scenario, even after 100 periods, the response value remains higher than the initial impact value.

<sup>&</sup>lt;sup>11</sup>Note that this model assumes net financial positions for household wealth. Therefore, households are restricted from simultaneously saving and borrowing funds.

a greater number of households opt to remain HtM, while fewer households choose to borrow with respect to the counterfactual scenario, due to the fluctuation of the borrowing penalty. This results in reduced wealth inequality among the population, as a larger proportion of households opt not to fall to the very bottom of the wealth distributions, unlike the situation with a fixed  $\omega^H$ . Conversely, an increase in HtM households leads to decreased consumption smoothing. In addition, individuals who choose to borrow end up consuming even less, as they must repay a higher interest rate, leading to greater consumption inequality compared to when there are frictions in the production sector. This clarifies both the lower Gini index for wealth and the higher Gini index for consumption in Figure 2.3 when households encounter frictions on borrowing.

Another source of confusion was the fact that it is stated, e.g. at page 89, that in the case of financial frictions on firms, borrowing is relatively cheaper. This is obviously true in comparison to the case with financial frictions on households and I think that this is what the candidate means, but it should be clarified in the text.

**Response**: This is precisely the intended message. In the revised edition of the thesis, I aimed to enhance the clarity of discussions regarding the comparisons among the scenarios across the entire analysis. For instance, I have rephrased the sentence mentioned by the reviewer on **page 100**:

... Financial frictions that exclusively impact productive firms lead to a scenario where borrowing for households becomes comparatively more affordable than when frictions directly affect households. This is due to the fixed loan premium,  $\omega^{H}$ , in the former case, while it increases in the latter case, leading to higher household loan costs. ...

The observed increase in the share of borrowers is the net outcome of a substitution effect (a monetary tightening makes borrowing more expensive and increases interest payments for households with net outstanding debt) and an income effect (induced by indirect general equilibrium responses). Do you have a sense of how robust are the results to different preferences' specifications?

**Response**: I must admit that I encountered some difficulty in understanding what the reviewer means with "different preferences' specifications". If the reviewer is referring to whether the results remain consistent when considering an alternative value for the relative risk aversion parameter, I have addressed this concern in **Appendix 2.F**. Specifically, I have included Gini coefficients and consumption decomposition IRFs in **Figure 2.F.7** and **Figure 2.F.8** for the case when the relative risk aversion parameter,  $\xi$ , is set to 2 instead of the baseline value of 4. In this scenario, despite the composite



Figure 8: Impulse responses to a monetary contraction for wealth and consumption inequality.

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households. The green line with asterisks is the percentage difference between the two curves using the lower curve as the base.

consumption exhibiting a positive value on impact in the case of financial frictions on firms, the main findings are robust.

If the reviewer suggests an alternative household preference model, such as separable preferences rather than GHH preferences, I performed supplementary robustness tests that were omitted from the revised thesis draft, but that I present in this response.

I modified the initial model by incorporating separable preferences and aligning it with the baseline calibration of specific wealth distribution moments. Also in this case, I introduced two different magnitudes for the monetary shock that resulted in a comparable response between the two scenarios in terms of output. The Gini indices of wealth and consumption are depicted in Figure 8. In this case, I am examining "goods consumption" instead of "composite consumption" as done within the model with GHH preferences.

There are clear differences from the baseline model. The Gini index for consumption consistently exhibits higher values for household frictions in the initial 5-year period, similar to the baseline model. However, the IRFs for the wealth Gini coefficient behave differently compared to the baseline model. In the latter, the IRF for financial frictions on firms consistently surpasses the scenario with household frictions. Conversely, in the case of separable preferences, the IRF for financial frictions on firms is relatively lower for approximately one year, after which it surpasses the values for the counterfactual scenario.

In order to determine whether this finding corresponds to radically different household



Figure 9: IRFs for the share of borrowing households, HtM households and wealth held by the top 10%

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households.

dynamics or whether the behavior of households around the zero-wealth threshold is similar to the baseline model, I illustrate in Figure 9 the wealth decomposition that mirrors the one performed in Figure 2.4 within the thesis.

Despite some differences that may account for the initial lower IRF in the Gini index of wealth under financial frictions on firms, it is evident that, also with separable preferences, financial frictions on firms lead to a higher proportion of households turning into borrowers and a lower proportion transitioning into HtM, as opposed to the situation with financial frictions on household borrowing. This particular trend is crucial, as it underscores the importance of the varying household borrowing penalty also for this model.

#### 3.5 Welfare analysis

Given that households are not exogenously constrained to be HtM, i.e. consume all their disposable income, and borrowing/saving behavior is optimally chosen, I was wondering which are the welfare consequences of the highlighted distributional effects. Would it be informative (and feasible) to conduct a welfare analysis?

**Response**: I express my gratitude to the reviewer for providing this suggestion, as it had not crossed my mind and could potentially serve as a significant and captivating expansion of this chapter. It is worth mentioning that the existing body of literature on the welfare effects and optimal monetary policy within a full-fledged HANK model is still in its early stages (e.g. McKay and Wolf, 2023, Acharya et al., 2023). Nevertheless, I have tried to conduct a welfare analysis, yielding certain outcomes that I intend to investigate more before incorporating them into the main text. However, I am eager to share these findings with the reviewer through this response.

In conducting my welfare analysis, I have opted to use the unconditional Consumption Equivalent (CE) as a metric (e.g. Lester et al., 2014, Born and Pfeifer, 2020), between the two scenarios considered so far: financial frictions on firms and financial frictions on households. The CE represents the percentage of goods consumption, C, that households are willing to forego (or obtain) in order to maintain the same level of welfare across both scenarios.

For instance, following the equations proposed by Lester et al. (2014) for household with GHH preferences in their Appendix B.4, let us consider the unconditional value function for households in the context of financial frictions on firms, and consider it the baseline scenario:

$$E(V^{F}) = E \sum_{j=0}^{\infty} \beta^{j} \left( \frac{1}{1+\xi} \left( c_{it+j} - h_{it+j} \frac{L_{t+j}^{1+\gamma}}{1+\gamma} \right)^{1+\xi} \right) .$$
(1)

Now let us consider the unconditional value function for households in the context of financial frictions on households, and consider it the counterfactual scenario. In order to distinguish between the two scenarios, I will employ a twiddle symbol on variables pertaining to frictions on households:

$$E(V^H) = E \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1+\xi} \left( \tilde{c}_{it+j}(1+\lambda) - \tilde{h}_{it+j} \frac{\tilde{L}_{t+j}^{1+\gamma}}{1+\gamma} \right)^{1+\xi} \right) .$$
(2)

The parameter  $\lambda$  is the unconditional CE, which is calibrated to ensure that the unconditional value functions for both scenarios are equal, that is,  $E(V^F) = E(V^H)$ .

When  $\lambda$  takes positive values, households that encounter constraints on their borrowing abilities would like to consume more to achieve indifference between the two scenarios. In contrast, when  $\lambda$  takes negative values, households that encounter constraints on their borrowing abilities could sacrifice a fraction  $\lambda$  of their consumption and still being indifferent. In other words, if  $\lambda > 0$ , households experience relatively higher welfare under active financial frictions on firms and inactive financial frictions on households, whereas the opposite holds true if  $\lambda < 0$ . As highlighted by Lester et al. (2014), there is no analytical solution for  $\lambda$  under GHH preferences, meaning that the solution must be approximated using numerical methods.

As usual in the welfare analysis literature for DSGE models, second-order perturbation solutions are required, since first-order approximations do not consider shock variances in welfare computation. This would result in unconditional means for the value function

#### Table 1: Welfare analysis

|                | $\lambda$ |           |
|----------------|-----------|-----------|
|                | $\xi = 4$ | $\xi = 2$ |
| Aggregate      | -1.03     | 29.09     |
| borrowers      | 3.85      | 8.66      |
| $\mathrm{HtM}$ | 4.71      | 5.21      |
| Top $10\%$     | -0.32     | -22.73    |

Values of  $\lambda$  are percentages.

being exactly the same for both scenarios. To address this issue, I adopt the second-order perturbation algorithm proposed by Bayer and Luetticke (2020). This algorithm enables the calculation of unconditional means for variables by considering second moment statistics, such as aggregate shock variances.

Following the second chapter of my thesis, I analyze the impacts of the two distinct types of friction by using different levels of aggregate shock that produce comparable outcomes in output. I compute CEs for the entire distribution (so, aggregate), for borrower households, for HtM households (as defined in Section 3.3 of this response), and for the top 10%. Furthermore, I compute CEs also for a model featuring risk aversion  $\xi = 2$ instead of the baseline value of 4. The findings, expressed in percentages, are displayed in Table 1.

The first finding is that aggregate  $\lambda$  is highly sensitive to household consumption preferences, involving a qualitative change in results. In the original model, at aggregate level, households in the counterfactual scenario could give up 1% of their consumption and still being indifferent between the two scenarios in terms of welfare. On the other hand, for a lower risk aversion value, households in the counterfactual scenario would like to consume approximately 30% more to be indifferent, meaning that they would be better off in the baseline scenario.

Interestingly, this qualitative change does not happen for the shares of population considered in the analysis. CEs for borrowers, HtM and the top 10% are qualitatively robust to a decrease in household risk aversion. Nonetheless, all the shares considered experiment an increase in the magnitude of CEs, since households with a lower risk aversion attach a higher utility to consumption in their value function.

Consistent with the main findings of the second chapter of my thesis, households around the zero-wealth threshold (borrowers and HtM) would be better off in the scenario of active financial frictions on firms. The household welfare function is increasing in consumption, and in my analysis I show that households under active financial frictions on firms have a better consumption smoothing ability. Household at the top 10%, on the contrary, are better off when financial frictions are active on household borrowing. This result, however, it is probably due to the fact that in this scenario the real interest rate is higher by construction. As already discussed, since wealth in my model is exclusively liquid, financial income for richer household is highly sensitive to changes in the monetary policy.

These, however, are just preliminary findings that I wanted to share with the reviewer, since he kindly suggested this potential extension of the study. Nevertheless, they seem to corroborate one of the main finding of the second chapter of my thesis, that is, the importance of household behavior at the lower end of the distributions, especially around the zero-wealth threshold.

#### 3.6 Empirical validation

The chapter is lacking an empirical validation of the results. I was therefore wondering whether it is possible to test some of the theoretical predictions of the model using e.g. household-level data from the Consumer Expenditure Survey to build consumption inequality measures, data on credit spread built by Gilchrist and Zakrajsek (2012) to proxy the EFP (and thus the level of financial frictions on firms), as well as data from identified monetary policy shocks (there is a variety of available series of monetary policy shocks identified using different methodologies, ranging from the pioneering approach of Romer and Romer (2004) to high-frequency approaches as in Gertler and Karadi (2015), Swanson and co-authors, Miranda-Agrippino and co-authors etc.). Obtaining data on wealth inequality is clearly more challenging, but the analysis could be carried for subgroups of respondents to the Consumer Expenditure Survey that can be classified as low net-worth households and high net-worth households following e.g. Coibion et al. (2017). I believe that the paper would greatly benefit from the addition of an empirical validation exercise, though at this stage I am not sure about its feasibility. I would nevertheless urge the candidate to think about such an exercise, at the very least as a possible direction of future research.

**Response**: I am particularly grateful for the detailed suggestions put forth by the reviewer in relation to the empirical analysis. As a matter of fact, also following the suggestion made by the other reviewer, I have conducted an empirical analysis in the first chapter of this updated thesis, and I refer to this part **on page 73**. I adopt a proxy-SVAR with a shut off mechanism á la Lettau et al. (2002) to assess the importance of firm financial frictions for the monetary policy transmission on household consumption inequality.

In line with the reviewer's suggestion, I use the Romer and Romer (2004) approach for external identification, along with consumption data from the CEX database. At this point in time, I have not incorporated wealth data into my analysis due to the challenges highlighted by the reviewer in obtaining and analyzing US household wealth data (noting that the Survey of Consumer Finances provides data on a triennial basis). Nevertheless, I intend to explore the methodology suggested by the reviewer and outlined in Coibion et al. (2017), to categorize households into low and high net-worth subgroups.

The main result of this analysis is that financial frictions seem to enhance the effects of monetary policy on household consumption dispersion. Nonetheless, if interested in more technical details, I would suggest looking Section 2.2.3 in this response, or **entire Section 1.2** in the thesis.

For empirical validation of financial frictions on households, I rely on existing findings from Lee et al. (2020), that I quote in the **last paragraph of page 72**:

Empirical evidence from recent studies demonstrates a positive correlation between these spreads and inequality indices, specifically consumption dispersion measures.<sup>12</sup> Lee et al. (2020) provide empirical support for the link between consumption dispersion and two measures of the spread in household borrowings: the spread between the two year personal loan rate and the three months T-Bill rate and the spread between the Commercial Bank interest on three months Credit Card plans and the three months T-Bill rate. Using Local Projection regression á la Jordà (2005), they show that an increase in the spread is associated with higher consumption inequality, regardless of the interest rate spread considered.<sup>13</sup>

#### 3.7 Minor comments

- 1 It would be useful to describe what  $c_{it}$ ,  $l_{it}$  and,  $h_{it}$  are immediately after Equation 2.1. Response: corrected.
- 2 Typo at page 68: Marginal Propensity to Earns. Response: corrected.

# **3** Unclear sentence at page 69: Total effective labor input, $\int l_{it}h_{it}di$ , is therefore to $L(W_t)$

<sup>&</sup>lt;sup>12</sup>The empirical literature often relies on consumption inequality as a preferred metric, thanks to the quarterly data on the US provided by the CEX. On the other hand, extrapolating wealth dynamics presents greater difficulty due to the triennial nature of the Survey of Consumer Finances.

<sup>&</sup>lt;sup>13</sup>In a more recent version of this paper (Faccini et al., 2024), the authors examine household data from Denmark and discover that higher spreads are connected to decreased consumption spending for indebted households, while the association is positive for wealthier households. They also construct an aggregate measure of the consumption-income elasticity that varies over time as a function of how households move across the wealth distribution and as a function of changes in the consumer credit spread. This index appears to exhibit volatility and countercyclicality due to changes in both net worth and the consumer credit spread.

**Response**: corrected.

**4** At page 70, after Equation (2.9), what is described as right-hand side should be lefthand side and viceversa.

**Response**: corrected.

5 Shouldn't  $V^w$  and  $V^r$  on the right-hand sides of Equations 2.3 and 2.4 be also indexed by i?

**Response**: I decided to omit the old equations (2.3) and (2.4) in the new updated version of the thesis, since they are not crucial for the understanding of the paper.

**6** In the right-hand side of Equation 2.42, shouldn't we have  $\int_i W_t h_{it} l_{it}$ , i.e. taxes collected *i* over the total labor income? If on the one hand it is true that  $l_{it} = L(w_t)$  for all workers, it is necessary to integrate to have  $\int_i h_{it} = 1$ .

**Response**: corrected.

7 Debt issuance by the government in Equation 2.43 does not react to tax revenues. Have you considered including them as an extension of the main analysis?

**Response**: After receiving this recommendation from the reviewer, I made the decision to incorporate a bond issuance rule that takes into account the response to the tax revenue level within the robustness section found in **Appendix 2.F**. At **page 122** I write:

Since I employ a HANK model, the Ricardian equivalence does not hold, and changes in fiscal policies could have significant effects. A modified version of equation (2.41) is taken into account, which also reacts to government tax revenues, T. Following the approach of Bayer et al. (2019), the alternative bond issuance rule is:

$$\frac{D_{t+1}^G}{\bar{D}^G} = \left(\frac{D_t^G \frac{R_t}{\pi_t}}{\bar{D}^G \frac{\bar{R}}{\bar{\pi}}}\right)^{\rho_{gov}} \left(\frac{T_t}{\bar{T}}\right)^{-\rho^T}$$

with  $\rho^T$  being the parameter determining the extent to which the rule is influenced by deviations in tax revenue from its steady state. When  $\rho^T = 0$ , the rule corresponds to equation (2.41). In this analysis, I assume a value of  $\rho^T = 1$ , indicating that the government responds actively to fluctuations in tax revenues. For example, if an adverse aggregate shock leads to a decrease in tax revenues, the government responds by increasing debt issuance to sustain higher public spending. Results are shown in Figure 2.F.5 and Figure 2.F.6. The primary outcome of this extension is that including this feature does not change the main results of the study qualitatively. The inequality acceleration is approximately the same in all the analysis conducted. Nevertheless, the increased government spending implemented through this alternative rule validates the effectiveness of expansionary fiscal policy in mitigating the impact of a contractionary monetary policy shock on output contraction. Furthermore, it helps to decrease disparities in wealth and consumption in absolute terms. Although differentials between the comparative scenarios remain similar to the baseline model, the absolute values of the Gini IRFs show a decrease both in for wealth and consumption.

### 8 Typo at page 87: there is a significant differences. Response: Corrected.

## References

- Acharya, S., Challe, E., and Dogra, K. (2023). Optimal monetary policy according to hank. American Economic Review, 113(7):1741–82.
- Bauer, M. D. and Swanson, E. T. (2022). A reassessment of monetary policy surprises and high-frequency identification. Working Paper 29939, National Bureau of Economic Research.
- Bayer, C. and Luetticke, R. (2020). Solving heterogeneous agent models in discrete time with many idiosyncratic states by perturbation methods. *Quantitative Economics*, 11:1253–1288.
- Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, 87(1):255–290.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.
- Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.
- Born, B. and Pfeifer, J. (2020). The New Keynesian Wage Phillips Curve: Calvo Vs. Rotemberg. *Macroeconomic Dynamics*, 24(5):1017–1041.
- Caldara, D. and Herbst, E. (2019). Monetary policy, real activity, and credit spreads: Evidence from bayesian proxy svars. American Economic Journal: Macroeconomics, 11(1):157–92.
- Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.
- Faccini, R., Lee, S., Luetticke, R., Ravn, M., and Renkin, T. (2024). Financial frictions: Macro vs micro volatility. Denmarks Nationalbank Working Papers 200.
- Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.
- Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.

- Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. American Economic Review, 102(4):1692–1720.
- Jordà (2005). Estimation and inference of impulse responses by local projections. *The American Economic Review*, 95(1):161–182.
- Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. American Economic Review, 108(3):697–743.
- Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239.
- Lee, S., Luetticke, R., and Ravn, M. (2020). Financial frictions: Macro vs micro volatility. CEPR Discussion Papers 15133.
- Lester, R., Pries, M., and Sims, E. (2014). Volatility and welfare. Journal of Economic Dynamics and Control, 38:17–36.
- Lettau, M., Ludvigson, S. C., and Steindel, C. (2002). Monetary policy transmission through the consumption-wealth channel. *Economic Policy Review*, 8(May):117–133.
- Luetticke, R. (2021). Transmission of monetary policy with heterogeneity in household portfolios. *American Economic Journal: Macroeconomics*, 13(2):1–25.
- McKay, A. and Wolf, C. K. (2023). Optimal policy rules in hank. Working Paper.
- Miranda-Agrippino, S. and Ricco, G. (2021). The transmission of monetary policy shocks. American Economic Journal: Macroeconomics, 13(3):74–107.
- Mumtaz, H. and Theodoridis, K. (2020). Dynamic effects of monetary policy shocks on macroeconomic volatility. *Journal of Monetary Economics*, 114(C):262–282.
- Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review*, 94(4):1055–1084.

# Essays on household heterogeneity and financial frictions

Francesco Ferlaino

Supervisor: Domenico Delli Gatti

## General abstract

This dissertation comprises two essays on the effects of monetary policy shocks in the presence of heterogeneous households and financial frictions. By employing a Heterogeneous Agents New Keynesian (HANK) model, I explore the role of financial frictions in shaping household wealth and consumption distributions after a conventional contractionary monetary shock, that is, an increase in the nominal interest rate by the central bank. Each essay forms a chapter of this thesis.

The initial chapter delves into the examination of the redistribution effects of monetary policy on households in the presence of financial frictions within the production sector of the economy. First, I conduct an empirical analysis using a proxy-SVAR model that incorporates a policy rate, a proxy measure for corporate financial frictions, economic variables, and a consumption dispersion measure. The findings validate that a contractionary monetary policy exerts a positive influence on both financial frictions and consumption inequality. Moreover, the results indicate that financial frictions amplify the impact of monetary policy on household inequality. Lastly, the study reveals that an increase in the nominal interest rate is associated with a rise in both frictions and leverage, validating the co-movement between these variables, which is crucial for the theoretical framework. Following this, I build a HANK model in which households can save only in one liquid asset, cannot borrow, and the production sector is subject to a financial accelerator that enhances the effects of aggregate shocks, in order to replicate some of the empirical results and conduct policy evaluation. The results show that (i) consumption inequality dynamics are qualitatively confirmed, with consumption inequality among households increasing after an interest rate rise and relatively higher when the financial accelerator is active; and (ii) that inequality measures for wealth are significantly higher when compared to the counterfactual scenario with no frictions. This finding suggests that financial frictions have considerable effects on household consumption and savings, since these agents rely differently on labor income for consumption smoothing and have different marginal propensities to consume along the wealth distribution.

In the second chapter, I build a model featuring household heterogeneity and two types of financial frictions: one on the production sector and one on household borrowing ability. The aim of this study is to examine how different financial frictions affect households' wealth and consumption after a contractionary monetary policy shock, whether these differences are significant, and why. The results show that (i) firms' financial accelerator affects household wealth inequality to a greater extent, while (ii) the friction on household loans induces a relatively higher dispersion of consumption among the population. The household borrowing penalty, which varies in the case of active financial frictions on households, plays a significant role in these dynamics. In the latter case, households are discouraged from moving to the bottom of the distribution, reducing loans and, therefore, consumption capacity. On the other hand, when only frictions in the production sector are active, more households are pushed to the bottom of the wealth distribution. This fluctuation relatively increases the Gini index of wealth, but simultaneously allows greater economy-wide consumption smoothing, resulting in less consumption inequality when compared to the previous case.

## Acknowledgments

First, I would like to thank my supervisor, Professor Domenico Delli Gatti, for his invaluable guidance, support, and encouragement. He has been very generous with his time and feedback, always available for discussions and consultations. I really appreciated his patience, trust and flexibility in allowing me to explore different research directions and develop my own ideas.

I would like to thank also some professors and researchers from Bicocca and Cattolica University: Alice Albonico, Jacopo Bonchi, Andrea Colciago, Marco Membretti and Emiliano Santoro for their time and their valuable and significant feedbacks, that surely helped me a lot in shaping my research. Nonetheless, I want to thank every faculty member that in this journey helped me, professionally and not.

I wish to thank Professor Davide Debortoli, from Universitat Pompeu Fabra, for allowing me to spend six wonderful months at the UPF, his availability, and his precious comments on my research. I also feel obliged to thank every member of the UPF who has dedicated some time to me, even just to answer a question or clarify a doubt of mine.

Obviously, I have to thank the people I shared this journey with: Pietro B., Pietro D.P., Matteo and Mene. Even though the pandemic did not allow us to fully experience the PhD life together, the mutual support that we gave each other was crucial in overcoming the challenges of this path.

I would also like to thank my office colleagues at the UPF, Diego and Barna. They have always been helpful with my research, but above all, they have helped me fit into a new environment, making me feel immediately part of the group.

A big thank you also to my lifetime friends, who probably had to endure long and boring conversations about my research topics.

This thesis is dedicated to my family, for their unconditional love and support, regardless of which path I would have chosen in my life. No man is an island, and without all this, it would have been impossible for me to achieve any goal.
# Contents

| 1.1  | Introd    | uction   |  |  |  |
|------|-----------|--|--|--|--|
| 1.2  | Empir     | ical analysis  |  |  |  |
|      | 1.2.1     | Methodology and data   |  |  |  |
|      | 1.2.2     | Empirical results  |  |  |  |
|      | 1.2.3     | Financial frictions and leverage   |  |  |  |
| 1.3  | The m     | odel   |  |  |  |
|      | 1.3.1     | Households   |  |  |  |
|      | 1.3.2     | Financial intermediaries   |  |  |  |
|      | 1.3.3     | Intermediate-goods producers   |  |  |  |
|      | 1.3.4     | Resellers  |  |  |  |
|      | 1.3.5     | Capital producers  |  |  |  |
|      | 1.3.6     | Final-goods producers  |  |  |  |
|      | 1.3.7     | Central bank   |  |  |  |
|      | 1.3.8     | Government   |  |  |  |
|      | 1.3.9     | Market clearing  |  |  |  |
|      | 1.3.10    | Numerical implementation $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ |  |  |  |
| 1.4  | Calibr    | ation  |  |  |  |
|      | 1.4.1     | Households   |  |  |  |
|      | 1.4.2     | Financial Intermediaries   |  |  |  |
|      | 1.4.3     | Production Sector  |  |  |  |
|      | 1.4.4     | Central Bank and Government  |  |  |  |
| 1.5  | 5 Results |  |  |  |  |
|      | 1.5.1     | Aggregate fluctuations   |  |  |  |
|      | 1.5.2     | Inequality among households: consumption   |  |  |  |
|      | 1.5.3     | Inequality among households: wealth $\ldots \ldots \ldots \ldots \ldots \ldots$                  |  |  |  |
| 1.6  | Inequa    | lity between households: skilled-unskilled workers and rentiers $\ .$                            |  |  |  |
| 1.7  | Conclu    | $iding \ remarks \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $   |  |  |  |
| Refe | rences    |  |  |  |  |
| App  | endix     |  |  |  |  |
| 1.A  | CEX o     | consumption data   |  |  |  |
| 1.B  | appen     | dix  |  |  |  |

|                       |      | 1.C.2  | Financial intermediaries' participation constraint and entrepreneur |     |  |
|-----------------------|------|--|---|-----|--|
|                       |      |  | j's optimization problem  | 54  |  |
|                       | 1.D  | Impuls   | se responses of MP contractionary shock                             | 57  |  |
|                       | 1.E  | Gini ir  | ndices for higher leverage at steady state                          | 57  |  |
| 1.F Robustness to ris |      |  | tness to risk aversion  | 57  |  |
|                       | 1.G  | Robus  | tness to investment cost  | 57  |  |
|                       | 1.H  | Robus  | tness to fiscal policy  | 59  |  |
|                       | 1.I  | Impuls   | se responses of a TFP shock   | 61  |  |
| <b>2</b>              | Effe | cts of   | different financial frictions on households                         | 70  |  |
|                       | 2.1  | Introd   | uction  | 70  |  |
|                       | 2.2  | The m  | odel  | 76  |  |
|                       |      | 2.2.1  | Households  | 76  |  |
|                       |      | 2.2.2  | Financial intermediaries  | 80  |  |
|                       |      |  | 2.2.2.1 Commercial Banks - Financial frictions on households .      | 80  |  |
|                       |      |  | 2.2.2.2 Investment Banks - Financial frictions on firms             | 81  |  |
|                       |      | 2.2.3  | Intermediate-goods producers  | 84  |  |
|                       |      | 2.2.4  | Resellers   | 85  |  |
|                       |      | 2.2.5  | Capital producers   | 86  |  |
|                       |      | 2.2.6  | Final-goods producers   | 87  |  |
|                       |      | 2.2.7  | Central bank  | 87  |  |
|                       |      | 2.2.8  | Government  | 87  |  |
|                       |      | 2.2.9  | Market clearing   | 88  |  |
|                       |      | 2.2.10   | Numerical implementation  | 89  |  |
|                       | 2.3  | $2.3  \text{Calibration}  \dots  \dots  \dots  \dots  \dots  \dots  \dots  \dots  \dots  $ |   |     |  |
|                       |      | 2.3.1  | Households  | 89  |  |
|                       |      | 2.3.2  | Financial intermediaries  | 92  |  |
|                       |      | 2.3.3  | Production Sector   | 92  |  |
|                       |      | 2.3.4  | Central Bank and Government   | 93  |  |
|                       | 2.4  | Result   | s   | 93  |  |
|                       |      | 2.4.1  | Aggregate fluctuations  | 93  |  |
|                       |      | 2.4.2  | Wealth and consumption inequality                                   | 96  |  |
|                       |      | 2.4.3  | Wealth dynamics   | 97  |  |
|                       |      | 2.4.4  | Consumption dynamics  | 100 |  |
|                       |      | 2.4.5  | Consumption decomposition   | 102 |  |
|                       | 2.5  | Conclu   | ıding remarks   | 106 |  |

| Refe           | rences   | 109 |  |
|----------------|--|-----|--|
| App            | endix  | 113 |  |
| 2.A            | Idiosyncratic productivity process and the joint distribution $\ldots$ .                                       | 113 |  |
| $2.\mathrm{B}$ | 2.B Investment banks optimal contract  |     |  |
|                | 2.B.1 Idiosyncratic shock on return on capital   | 114 |  |
|                | 2.B.2 Investment banks' participation constraint and entrepreneur $j$ 's                                       |     |  |
|                | optimization problem $\ldots$ | 114 |  |
| $2.\mathrm{C}$ | Impulse responses of MP contractionary shock $\ldots \ldots \ldots \ldots \ldots$                              | 117 |  |
| $2.\mathrm{D}$ | Impulse responses of MP contractionary shock - Same shock magnitude  | 117 |  |
| $2.\mathrm{E}$ | Consumption inequality analysis for goods consumption $C$  | 117 |  |
| $2.\mathrm{F}$ | Robustness checks  | 117 |  |

# Chapter 1 Does the financial accelerator accelerate inequalities?

#### Abstract

This study examines the redistribution effects of a conventional monetary policy shock among households in the presence of production-side financial frictions. A Heterogeneous Agents New Keynesian model featuring a financial accelerator is built after empirical evidence for consumption inequality. The results show that the presence of financial frictions significantly increases the magnitude of the Gini coefficient of wealth and other wealth inequality measures after contractionary monetary policy, compared to a scenario in which such frictions are inactive, proving that firms' financial characteristics affect household wealth inequality. Consumption dynamics are also affected: financial frictions have a significant impact on how households consume and save after a monetary contraction, because they rely differently on labor income to smooth consumption. The relative increase in consumption inequality confirms the empirical results.

# 1.1 Introduction

Macroeconomists, in particular those interested in theoretical models, did not raise much concern about the redistribution effects of monetary policy until recently. However, over the last decade, there has been an increasing interest in this topic for several reasons. The Great Recession has intensified the rise in wealth inequality that has been ongoing since the 1980s,<sup>1</sup> reaching a point where ignoring it could lead to missing important aspects of monetary transmission mechanisms. As a consequence, policymakers have expressed serious concerns about this issue (e.g. Bernanke, 2015).

<sup>&</sup>lt;sup>1</sup>See Piketty (2017) for a review of the history of inequality, especially in advanced economies.

The increasing power of computer processors and the development of new numerical techniques have made it possible to solve models featuring agent heterogeneity in a relatively short time. In the past, such models were highly time-consuming or simply impossible to solve. Fueled by these premises and building on frameworks developed by Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998), a new strand of Heterogeneous Agents New Keynesian (HANK) models took off in recent years, trying to assess the effects of aggregate shocks of different types on household wealth distribution and how the shape of this distribution could affect the propagation of such shocks.<sup>2</sup>

While the literature on the impact of monetary policy on inequality has blossomed in the last decade, very little has been said about the role of financial frictions in this regard, especially when these frictions affect the production side of the economy. Standard New Keynesian models aimed at studying monetary policy usually ignore the production sector's financial structure, in light of the Modigliani and Miller (1958) theorem of capital structure irrelevance. However, several recent findings indicate that firms' financial structure plays a significant role in the business cycle. For instance, Jordà et al. (2017) use a historical macro-financial database covering 17 advanced economies over the last 150 years to show that the leverage level of the economy has become an important factor in explaining business cycle moments, making the role of financial variables crucial in understanding aggregate economic dynamics. Adrian et al. (2019) study US data to find that negative GDP growth is positively correlated with a deterioration in financial conditions. Caldara and Herbst (2019) employ a structural vector autoregressive model and discover that large effects of monetary policy shocks in the US during the Great Moderation period are explained by a strong systematic response of monetary policy to financial conditions. Gilchrist and Zakrajšek (2012) focus their research on the relationship between corporate bond credit spreads and economic activity, building the "GZ credit spread", a reliable measure of the strength of financial frictions concerning the non-financial corporate sector in the US, and finding a correlation with substantial contractions in economic activity. In terms of theoretical contribution, the so-called "financial accelerator" was first introduced by Bernanke et al. (1996), and is based on a mechanism that amplifies initial shocks due to changes in financial conditions for non-financial companies.

However, the dynamics related to the effects of corporate financial frictions on monetary transmission to household wealth and consumption distributions have not been

 $<sup>^{2}</sup>$ For instance, Ahn et al. (2017) point out that the composition of micro-data incorporated in a Dynamic Stochastic General Equilibrium (DSGE) model could have significant effects on macroaggregate fluctuations, and vice versa.



Figure 1.1: Corporate spread and consumption dispersion

The blue solid line shows the evolution of the GZ spread over time. The red dotted line displays the ratio of consumption at the 50th percentile to that at the 10th percentile of the consumption distribution. Both series have been detrended with a 8th-degree polynomial trend. The 50/10 ratio has also been logged, de-sesonalized with a quarterly dummy and smoothed with a centered three-quarter moving average.

Source: Gilchrist and Zakrajšek (2012) for the GZ spread. U.S. Bureau of Labor Statistics for consumption data.

fully addressed, either theoretically or empirically. The rise of inequality, either at wealth or consumption level, has been one of the main topics of discussion in the last years in the US and in most advanced economies. To this extent, findings of a notable role of the financial accelerator in this process could be helpful understand such a complex phenomenon. The aim of this study is to fill this gap by providing empirical evidence and a theoretical framework to understand the underlying dynamics.

To grasp intuition on a possible positive correlation between financial frictions and inequality measures in the data, let us consider Figure 1.1, which shows detrended series for the GZ spread and for a measure of consumption inequality, the ratio of the 50th percentile to that at the 10th percentile of the consumption distribution.<sup>3</sup> The reference period is the so-called "Great Moderation", from the mid-1980s until 2007. These two measures appear to have a certain correlation, since consumption inequality responds with a lag to fluctuations in the spread until the beginning of the new millennium. After the burst of the Dot Com bubble, these co-movements appear to be even more contemporaneous.

From a theoretical point of view, the intuition comes from the influential paper by Kaplan et al. (2018). One of their main findings is that, when households are hetero-

 $<sup>^{3}</sup>$ See Appendix 1.A for details about the 50/10 consumption ratio.

geneous, most of the monetary policy transmission on households' consumption does not pass through direct effects (e.g., intertemporal substitution), but rather through indirect effects, such as labor dynamics, fiscal policy, and changes in asset prices.<sup>4</sup> More specifically, in their baseline model, labor income fluctuations are the most important component, accounting for more than half of the percentage change in aggregate consumption, leaving a marginal role for direct effects. Considering this result, I expect the presence of financial frictions in the production sector to be highly significant for wealth and consumption distribution shifts after a change in monetary policy, due to the existence of a share of households with zero or little wealth who rely mostly on their labor income for saving and consumption smoothing. Simultaneously, households with a high level of liquidity should also be affected, likely with the opposite effect.

My empirical contribution consists of a Structural Vector Auto-Regressive (SVAR) model with exogenous identification, including a monetary policy shock, financial frictions, and consumption inequality measures, along with other macroeconomic variables. The purpose of this exercise is twofold. First, it determines how these variables behave in the data after monetary policy innovation. Second, and most importantly, it assesses whether financial frictions have a significant influence on the pass-through of monetary policy to household consumption dispersion. In addition, I estimate the relationship between financial frictions and corporate firms' leverage level, defined as the value of firms' capital over equity, which is crucial for the financial accelerator mechanism used in the theoretical model. The results indicate a contraction of the economy, a strengthening of financial frictions, and an increase in consumption inequality. Financial frictions appear to be a statistically significant cause of an increase in consumption dispersion after the central bank increases the interest rate. Finally, monetary shocks generate a co-movement between financial frictions and leverage, consistent with the theoretical literature.

For the theoretical contribution, a HANK model capable of explaining the empirical findings is built. This model features asset market incompleteness, idiosyncratic income risk, sticky prices, and a *financial accelerator* on the production side, as in Bernanke et al. (1999). The "acceleration" effect arises due to friction in the way entrepreneurs obtain funds for the production of goods. Since an asymmetric information problem is introduced between lenders (financial intermediaries) and borrowers (entrepreneurs),

 $<sup>^{4}</sup>$ Table 1 in Kaplan et al. (2018) displays how in standard Representative Agent New Keynesian (RANK) models, direct effects account for almost 100% of the monetary transmission. This percentage could drop up to 50% in a Two Agents New Keynesian (TANK) model, indicating that heterogeneity among households actually matters. Nonetheless, in TANK models, direct effects are still the most important.

lenders must pay auditing costs to check the actual production and to verify whether borrowers can repay their debt. This implies the existence of an "external finance premium", which is defined as the difference between the cost of funds raised externally (debt) and the opportunity cost of funds internal to the firm (net worth or equity).<sup>5</sup> This premium is linked to entrepreneurs' leverage: the more exposed the entrepreneurs, the higher the premium. Whereas lenders are risk-averse and borrowers are risk-neutral, audit costs are ultimately rebated to entrepreneurs themselves. Therefore, a contraction of economic activity that causes an increase in entrepreneurs' leverage will, in turn, result in higher auditing costs and a higher external finance premium. Entrepreneurs' net worth suffers a further depression due to these higher costs. Ceteris paribus, with lower equity to be used for production, entrepreneurs have to resort to more external funding, increasing their leverage and, consequently, incurring in a higher external finance premium, generating the financial acceleration in the economy. In short, higher leverage increases the cost of external funding, and vice versa, higher cost of external funding negatively affects entrepreneurs' net worth, increasing their leverage. Including this mechanism in a model with household heterogeneity helps to assess the impact of this acceleration on wealth and consumption distribution.

The main finding is that the financial accelerator is also an accelerator of inequalities. The monetary contraction leads to a higher level of the Gini index for wealth and consumption when there are active financial frictions. This phenomenon occurs because households respond differently in terms of saving and consumption behaviors along their wealth distributions. Households experiencing the highest shifts are those closer to the borrowing constraint, which is in line with recent findings in the HANK literature, which aim to break the permanent income hypothesis. These agents are largely (if not fully) dependent on their current income for consumption, and they are unable to smooth it due to the lack of savings.<sup>6</sup> The further decline in production due to the financial accelerator has a significant impact on labor and wages and therefore has a greater impact on households relying more on labor income than on income from profits or savings. On the other hand, rich households benefit from the interest rate rise, accumulate more wealth and increase their consumption. Because the financial accelerator enhances these movements at the two tails of the distribution, the model generates even greater global inequality of wealth and consumption under active financial frictions.

My research lies in the rapidly growing literature on household heterogeneity within a New Keynesian framework. TANK models constitute a parsimonious yet powerful

<sup>&</sup>lt;sup>5</sup>Throughout the paper, net worth and equity are intended as synonym.

 $<sup>^{6}</sup>$ As I explain in Section 1.3, I assume that households cannot borrow resources to smooth consumption.

way to introduce household heterogeneity, with interesting results in monetary and fiscal policy evaluations (e.g., Galí et al., 2007; Bilbiie, 2008). Moreover, Debortoli and Galí (2017) showed how TANK models can reasonably approximate the predictions of a HANK model regarding the effects of an aggregate shock on aggregate variables. Nevertheless, they also point out that TANK models are not suitable for addressing other questions, such as the change in households' wealth distribution. Therefore, in such cases, we must resort to fully-fledged HANK models (e.g., Kaplan et al., 2018; Bayer et al., 2019; Auclert et al., 2021; Luetticke, 2021). Aside from the incredible contribution provided in creating algorithms that allow working with enormous amounts of grid points and, hence, with a great variety of households, a common goal is usually to match empirical micro data as accurately as possible. Differently, the aim of this study is to use a simpler model, taking advantage of such methodologies, to study the dynamics generated from the peculiar structure of the production sector in the economy.

My contribution is, obviously, also related to the financial frictions literature, in particular to the branch studying the financial accelerator generated by the existence of an "External Finance Premium" for firms (e.g., Carlstrom and Fuerst, 1997; Bernanke et al., 1999; Christiano et al., 2014; Carlstrom et al., 2016). In their seminal paper, Bernanke et al. (1999) articulate why the Modigliani and Miller (1958) assumption of financial structure irrelevance for real economic outcomes could be too limiting in certain cases, especially when frictions in financial markets are not small. The two main justifications the authors provide are that (i) even relatively small changes in entrepreneurial wealth could deliver important cyclical fluctuations (a line of thinking that goes back to Fisher, 1933) and (ii) empirical studies have appointed growing importance to credit market frictions, thereby increasing the need to fill the gap present in the theoretical literature at the time. My idea is to verify whether this acceleration mechanism takes place and has significant results also on inequality measures, a path that could not be explored by Bernanke et al. (1999) because they assumed a representative household in their model. Adding heterogeneity among households allows to understand not only how shocks affect the aggregate variables of the business cycle, but also the implications at idiosyncratic levels.

To the best of my knowledge, to date, only a few studies consider financial frictions in a HANK environment, such as Guerrieri and Lorenzoni (2017), Nakajima and Ríos-Rull (2019) and Fernández-Villaverde et al. (2023). Nonetheless, none of these studies seem to focus on the consequences of inequality arising from conventional monetary policy. The paper by Lee et al. (2020) is probably the closest to my research. However, the substantial difference with my study is in the type of friction in place and, hence, in the dynamics that are meant to be comprehended. They build on the works of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) to analyze the effects of frictions on banks' balance sheets, the presence of which directly affects households' chances to borrow resources from financial intermediaries (higher borrowing rates) and, consequently, to smooth consumption. On the other hand, my study focuses more on *direct* frictions on production firms that have *indirect* effects on households, mainly through changes in labor income. Since a consistent share of households relies entirely (or for a good part) only on labor income for their consumption, it becomes important to study how inequalities are shaped not only when the banking sector is not running smoothly, but also when firms' financing becomes more costly because of their financial structure.

In addition, I contribute to expanding the empirical literature concerning the effects of monetary shocks in the economy, accounting for financial frictions. Gertler and Karadi (2015) use shocks identified using high-frequency surprises around policy announcements as external instruments to obtain a consistent impulse response for corporate credit spreads. Caldara and Herbst (2019) employ a similar methodology to prove that a strong systematic response of monetary policy to financial conditions is crucial to account for the large effects of monetary policy during the Great Moderation. Although Coibion et al. (2017) show that contractionary monetary policy has an increasing effect on consumption dispersion measures, to the best of my knowledge, no study has investigated whether financial frictions on non-financial firms enhance monetary transmission to consumption inequality dynamics.<sup>7</sup>

The remainder of this chapter is organized as follows. Section 1.2 describe the empirical analysis conducted and its results. Section 1.3 outlines the model. Section 1.4 explains the calibration strategy. Section 1.5 displays quantitative results. Section 1.7 gives summary conclusions.

# **1.2** Empirical analysis

This section provides empirical evidence of the effects of a contractionary monetary policy shock on household consumption dispersion, taking into account corporate financial frictions. While I study two different types of inequalities (wealth and consumption) in the HANK model, conducting an empirical wealth analysis is less feasible. The Survey of Consumer Finances, the most reliable source of household wealth statistics in the United States, is a triennial survey. Therefore, I consider only consumption inequality

 $<sup>^{7}</sup>$ Lee et al. (2020) provide also empirical evidence for their model. However, as already explained, they focus on consumer credit spreads and not corporate spreads.

in this analysis, given that time series of household consumption in the US can be found with quarterly frequency.

# 1.2.1 Methodology and data

To this end, a Structural Vector Auto-Regression with external instrument identification (i.e., a proxy-SVAR) is employed. As pointed out by Gertler and Karadi (2015), adopting the classic Cholesky identification in a SVAR that includes both financial and real variables could generate results inconsistent with economic theory. The proxy-SVAR presented in this section contains both types of variables. I choose the 3-Month Treasury Bill rate (TB3MS) as the policy rate.<sup>8</sup> Exogenous monetary policy surprises are identified as in Romer and Romer (2004). The effects of increasing interest rate on financial frictions, output, occupation, and consumption dispersion are then evaluated. I use the Excess Bond Premium (EBP) built by Gilchrist and Zakrajšek (2012) as a proxy for the magnitude of financial frictions in the corporate non-financial sector. I then use the natural logarithm of industrial production (INDPRO) and the percentage level of unemployment (UNRATE) as measures of industrial output and employment, a choice in line with Caldara and Herbst (2019). To evaluate consumption dispersion, I use two measures commonly employed in the literature: the ratio between the 50th and 10th percentiles of consumption distribution and the Gini index. Data on consumption dispersion are constructed using the Consumer Expenditure Survey (CEX), a database built by the US Bureau of Labor Statistics;<sup>9</sup> more information about the CEX database are provided in Appendix 1.A. Romer and Romer (2004) innovations are collected by the series updated by Coibion et al. (2017). All other data, except the EBP, are obtained from the St. Louis FRED.

I choose to use quarterly data for the period 1984Q1–2007Q4. Two main reasons dictate this choice. First, in the theoretical model displayed in the next section, one period represents a quarter and the model is calibrated on the Great Moderation timespan, since the focus is on conventional monetary policy. Second, the CEX has been collected continuously since 1984 and on a quarterly basis. In addition, it would be difficult to carry out an analysis well beyond 2007, since the quarterly EBP series developed by Gilchrist and Zakrajšek (2012) covers the period up to 2010Q3.

<sup>&</sup>lt;sup>8</sup>Using other variables, such as the federal funds effective rate (FEDFUNDS) or the one-year government bond rate (GS1), does not significantly change the SVAR results.

<sup>&</sup>lt;sup>9</sup>https://www.bls.gov/cex/

### **1.2.2** Empirical results

Impulse response functions (IRFs) for the proxy-SVAR are shown in Figure 1.2.<sup>10</sup> I display results for two SVARs where the consumption dispersion measure is the only endogenous variable changing: 50/10 consumption ratio in the left column and the Gini index in the right column. As suggested by the Bayesian Information Criterion (BIC), the SVARs are estimated using two lags of each endogenous variable.<sup>11</sup> I show the mean values, 68%, and 90% confidence intervals after 2000 bootstraps.<sup>12</sup>

These results appear to be consistent with the existing literature. Values for Fstatistics in first-stage regressions suggest a good instrument validity, according to the threshold recommended by Stock et al. (2002) when only one instrument is employed. As in Gertler and Karadi (2015), a one standard deviation surprise monetary tightening induces a rise in the interest rate of approximately 0.25% and significantly increases the EBP, thus strengthening financial frictions. I also find a contraction in economic activity that is very similar to that shown in Caldara and Herbst (2019). Even though the two studies present substantial differences,<sup>13</sup> fluctuations in unemployment and industrial production are very similar both in magnitude and shape. Finally, an increase in the interest rate by the central bank enlarges consumption dispersion among households, a result in line with findings by Coibion et al. (2017). It should be noted that in the case of the 50/10 consumption ratio, the IRF undershoots after one year. However, it is never negatively significant at the 90% confidence interval. The Gini index for household consumption displays a similar behavior, although negative values are never statistically significant, not even for the 68% confidence interval.

These outcomes show how the theoretical model should behave, at least qualitatively, after the central bank increases the interest rate. However, they do not say much about how financial frictions affect consumption inequality. I employ two different methodologies to clarify this aspect. First, a two-variable SVAR with Cholesky identification is used, where the second variable, consumption dispersion, is assumed

<sup>&</sup>lt;sup>10</sup>To obtain IRFs, I employ the VAR toolbox for Matlab developed by Ambrogio Cesa-Bianchi (https://sites.google.com/site/ambropo/MatlabCodes)

<sup>&</sup>lt;sup>11</sup>Given the short sample employed in this SVAR (96 observations for each variable), the BIC seems to be the right criterion to consider because it places a higher weight on the sample size. On the other hand, the Akaike Information Criterion (AIC) suggests higher lag values, which would result in statistically insignificant IRFs. In fact, running the same SVAR with up to four lags does not substantially change the shape of the IRFs, but it generally increases the width of the confidence intervals.

<sup>&</sup>lt;sup>12</sup>Following Gertler and Karadi (2015) and Mertens and Ravn (2013), I use wild bootstrap, which generates valid confidence intervals under heteroscedasticity and strong instrument assumptions.

<sup>&</sup>lt;sup>13</sup>Caldara and Herbst (2019) adopt a Bayesian VAR, whereas I employ a frequentist VAR. In addition, the exogenous instrument and the policy rate are also different in their baseline model.



Figure 1.2: Impulse responses to monetary policy shock Estimated responses to a standard deviation shock of monetary policy using an external instrument for shock identification. Bootstrapped median and confidence intervals are obtained after 2000 wild bootstrap.



Figure 1.3: Impulse responses to a EBP shock Estimated responses to a standard deviation shock of EPB using Cholesky identification. Bootstrapped median and confidence intervals are obtained after 2000 residual bootstrap.

to have no contemporaneous effects on the first variable, EBP, using two lags of each endogenous variable.<sup>14</sup> This exercise does not consider the monetary policy contraction, but it aims to assess whether higher financial frictions have a significant effect on consumption inequality in the data, regardless of what causes an increase in the EBP. Again, in Figure 1.3, I consider two SVARs, one for the 50/10 consumption ratio and one for the Gini index, showing mean values and confidence intervals after 2000 bootstraps. The results show a positive relationship between an increase in financial frictions and a rise in consumption inequality, although it is less statistically significant when we consider the 50/10 consumption ratio. However, both consumption inequality measures have mostly positive mean values with a hump-shaped response. This latter feature is consistent with the descriptive statistics in Figure 1.1, where consumption dispersion seems to have a lagged response to corporate spread fluctuations.

Second, I resort again to the proxy-SVAR employed above, but I now "shut off" the effects of the EBP on other variables. To do so, I follow the methodology proposed by Lettau et al. (2002).<sup>15</sup> Let us consider the structural form of the VAR in Figure 1.2:

 $<sup>^{14}</sup>$ For these two SVARs, according to the inequality measure considered, the BIC suggests a different number of lags: In the case of the 50/10 ratio, the BIC still recommends the use of two lags, whereas in the case of the Gini index, it recommends six lags. However, adopting a higher lag value does not qualitatively change the results.

<sup>&</sup>lt;sup>15</sup>An approach following a similar logic, although different in the application, can be found in Mumtaz and Theodoridis (2020)



Figure 1.4: Impulse response of consumption dispersion to a monetary shock Estimated responses to a standard deviation shock of monetary policy using an external instrument for shock identification. The counterfactual response is obtained after "shutting off" the financial friction channel.

$$B_0 x_t = a + B_1 x_{t-1} + B_2 x_{t-2} + w_t, (1.1)$$

where  $x_t$  is the vector of endogenous variables, a is the deterministic trend containing a constant,  $w_t$  is the vector of mutually uncorrelated structural shocks, and  $B_i$  with i = 0, ..., 2 are matrices of structural model coefficients. To shut off the EBP effect on other variables, I set to zero the second column in  $B_1$  and  $B_2$  (since the EBP is ordered second in the vector of variables), except for the second element in the column. Thus, I cancel the effect that lagged values of EBP have on all endogenous variables except EBP itself. In Figure 1.4 I show consumption dispersion IRFs for (1.1) in the unrestricted (blue solid line) and counterfactual (green line with asterisks) models, that is, when the effects of EBP on other endogenous variables are present or shut off, respectively. I also show the confidence interval for the unrestricted scenario, computed after 2000 wild bootstraps. In the unrestricted model, consumption inequality is consistently higher for most of the initial five years, indicating that the EBP tends to increase the dispersion of consumption. At certain intervals, the counterfactual scenario shows lower levels of consumption inequality than the baseline scenario's 68% confidence interval. Interestingly, this pattern seems to have a more pronounced effect on the Gini index compared to the 50/10 consumption ratio, despite their relatively similar trends.

# **1.2.3** Financial frictions and leverage

Leverage plays a fundamental role in the financial accelerator dynamic, as discussed in the next section. According to the mechanism developed by Bernanke et al. (1999), financial frictions and leverage are strongly related, with one component positively affecting the other and vice versa. From an empirical point of view, the two variables should experience co-movement after a shock to the economy. To determine if this is also the case in the data, I employ a smaller proxy-SVAR, using the same monetary external instrument and featuring only three endogenous variables: the 3-Month Treasury Bill rate, the EBP, and the natural logarithm of non-financial corporate leverage. In the theoretical model, leverage is defined as firms' capital over equity. To be consistent with this definition, I compute the non-financial corporate leverage as the ratio of total assets of nonfinancial corporate business (TABSNNCB) to equity, which in turn is calculated as the difference between total assets and total liabilities (TLBSNNCB).<sup>16</sup> As before, I show the mean values, 68% and 90% confidence intervals after 2000 bootstraps, with two lags for each endogenous variable. The results are displayed in Figure 1.5. EBP and leverage have relatively similar responses in shape, but with some differences: the leverage response appears to be a little stronger at its peak and generally more persistent. However, both responses show a statistically significant hump-shaped increase, pointing to an empirical validity of the co-movement needed for the financial accelerator framework.

In the next section, I build a HANK model featuring financial frictions on productive firms to explain most of these empirical results and make estimates of changes in the wealth distribution.

# 1.3 The model

The theoretical model comprises households, financial intermediaries, a production sector, a central bank, and the government. Households consume, earn income (either from labor or profit, depending on the household type), and save in a liquid asset, which yields an interest rate. Financial intermediaries obtain deposits from households and lend them to the production sector, which, in turn, is responsible for the production of goods and capital. The central bank is in charge of monetary policy and sets the nominal interest rate, whereas the government acts as fiscal authority and chooses how to finance government spending. Time is discrete and infinite. The behavior of each

<sup>&</sup>lt;sup>16</sup>Data for non-financial corporate assets and liabilities are obtained from the St. Louis FRED.



Figure 1.5: Impulse responses to monetary policy shock Estimated responses to a standard deviation shock of monetary policy using an external instrument for shock identification. Bootstrapped median and confidence intervals are obtained after 2000 wild bootstrap.

agent is explained in detail below.<sup>17</sup>

# 1.3.1 Households

There is a continuum of ex ante identical households of measure one indexed by  $i \in [0, 1]$ . They are infinitely lived, have time-separable preferences with time-discount factor  $\beta$ and their utility function u is affected positively by consumption,  $c_{it}$ , and negatively by labor,  $l_{it}$ , with  $l_{it} \in [0, 1]$  being hours worked as a fraction of the time endowment, normalized to 1. The utility function u is strictly increasing and strictly concave in consumption and strictly decreasing and strictly convex in labor. Household i value function is the following:

$$V = E_0 \max_{\{c_{it}, l_{it}\}} \sum_{t=0}^{\infty} \beta^t u(c_{it}, l_{it}) , \qquad (1.2)$$

where I assume households have separable preferences with a Constant Relative Risk Aversion (CRRA) form:

$$u(c,l) = \frac{c^{1-\xi}}{1-\xi} - \psi \frac{l^{1+1/\nu}}{1+1/\nu} .$$
(1.3)

<sup>&</sup>lt;sup>17</sup>The model structure follows closely the 1-asset HANK version proposed in Luetticke (2021), with the exception of the introduction of financial frictions.

There are two types of household: workers and rentiers. Workers supply labor,  $l_{it}$ , in the production sector and have positive idiosyncratic labor productivity,  $h_{it} > 0$ . Because the global wage level,  $W_t$ , is the same for everyone, their income is given by  $W_t h_{it} l_{it}$ . Rentiers have zero labor productivity,  $h_{it} = 0$ , but collect a proportional share of total profits generated from the production sector,  $\Pi_t$ . Idiosyncratic labor productivity  $h_{it}$  follows an exogenous Markov chain according to the following firstorder autoregressive process and a fixed probability of transition between the worker and the rentier state:

$$h_{it} = \begin{cases} exp(\rho_h log(h_{it-1}) + \epsilon_{it}^h) & \text{with probability } 1 - \zeta \text{ if } h_{it-1} \neq 0 \\ h_t^H & \text{with probability } \iota \text{ if } h_{it-1} = 0 \\ 0 & \text{else} \end{cases}$$
(1.4)

where  $\epsilon_{it}^h \sim N(0, \sigma_h)$  and  $h_t^H$  is the highest possible productivity realization for workers. The parameter  $\zeta \in (0, 1)$  is the probability that a worker becomes a rentier, while  $\iota \in (0, 1)$  is the probability that a rentier becomes a worker. As stated above, workers who become rentiers leave the labor market  $(h_{it} = 0)$ , whereas rentiers that become workers are endowed with productivity  $h_t^H$ .<sup>18</sup> Workers and rentiers pay the same level of taxation,  $\tau$ , on their income.

The asset market is incomplete: there are no Arrow-Debreu state-contingent securities, households self-insure themselves only through savings in a non-state contingent risk-free liquid asset,  $a_{it}$ , and they cannot get indebted on that, that is, an *ad hoc* borrowing constraint exists ( $a_{it} \ge 0$ ). Thus, households cannot borrow from financial intermediaries to smooth their consumption. The household's budget constraint is:

$$c_{it} + a_{it+1} = \left(\frac{R_t}{\pi_t}\right) a_{it} + (1 - \tau) (W_t h_{it} l_{it} + \mathbf{I}_{h_{it}=0} \Pi_t) , \qquad (1.5)$$

where  $\mathbf{I}_{h_{it}=0}$  takes the value of 1 if the household is a rentier and 0 otherwise. On the right-hand side, we have households' expenditure, that is, consumption,  $c_{it}$  and 1-year-maturity savings,  $a_{it+1}$ . The left-hand side corresponds to households' total earnings: work/rent income net of taxes,  $(1 - \tau)(W_t h_{it} l_{it} + \mathbf{I}_{h_{it}=0} \Pi_t)$ , and the gross real interest rate on previous savings,  $(R_t/\pi_t)a_{it}$ , where  $\pi_t$  is the gross inflation rate.

Liquid assets held by households are a mix of deposits,  $D_t$ , and government bonds,  $B_t$ , so that we have the following relation:

$$A_t = D_t + B_t , \qquad (1.6)$$

<sup>&</sup>lt;sup>18</sup>Appendix 1.B contains details on the transition matrix for household productivity.

where  $A_t = \int a_{it} di$ . Deposits and bonds are perfect substitutes, which means that they carry the same real interest rate,  $\frac{R_t}{\pi_t}$ , and that households are indifferent to the composition of  $A_t$ .<sup>19</sup>

#### **1.3.2** Financial intermediaries

Financial intermediaries collect deposits from households and promise returns equal to the real risk-free interest rate,  $R/\pi$ , where  $\pi$  is the inflation level in the economy. For ease of display, I assume that the production sector is run by entrepreneurs, who are a mass-zero group of managers who are entitled to all the profits generated in the production sector and rebate them to rentier households. Financial intermediaries and entrepreneurs are responsible for the financial frictions considered in this model. Following Bernanke et al. (1999), I assume a continuum of entrepreneurs, indexed by j. Entrepreneur j acquires capital,  $K_j$ , from capital producers at the end of period t that is used at time t + 1. To buy capital for production, entrepreneurs rely on two type of financing: internal financing, that is, equity,  $N_j$ , and external financing,  $D_j$ .

Entrepreneur j balance sheet at period t + 1 is:

$$q_t K_{it+1} = N_{it+1} + D_{it+1} , \qquad (1.7)$$

where q is the price of capital at the time of the purchase.

One prerequisite for the functioning of this financial accelerator is that entrepreneurs are not indifferent to the composition of their balance sheet; that is, external financing is more expensive than internal financing. To introduce this feature, a Costly State Verification (CSV) problem à la Townsend (1979) exists, in which lenders (i.e., financial intermediaries) must pay a fixed auditing cost to observe the realized returns of borrowers (i.e., entrepreneurs). A relatively higher demand for debt increases auditing costs, resulting in a lower level of aggregate capital obtained for production.

Entrepreneurs repay investment banks with a portion of their realized returns on capital. In this framework, entrepreneurs are risk-neutral, while households are risk-averse. This implies a loan contract in which entrepreneurs absorb any aggregate risk on the realization of their profits. I also assume the existence of an idiosyncratic shock to entrepreneur j,  $\omega_j$ ,<sup>20</sup> on the gross return on aggregate capital,  $R^K$ . The idiosyncratic

<sup>&</sup>lt;sup>19</sup>I assume that each household has the same portfolio composition of liquid assets, which is equal to their aggregate level.

<sup>&</sup>lt;sup>20</sup>As noted by Christiano et al. (2014),  $\omega$  could be thought of as the idiosyncratic risk in actual business ventures: in the hands of some entrepreneurs, a given amount of raw capital is a great success, while in other cases may be not.

shock  $\omega$  has a log normal distribution of mean  $E(\omega) = 1$  that is i.i.d. across time and across entrepreneurs, with a continuous and once differentiable c.d.f.,  $F(\omega)$ .<sup>21</sup>

The optimal contract for financial intermediaries is:

$$\bar{\omega}_{jt+1}R_{t+1}^{K}q_{t}K_{jt+1} = Z_{jt+1}D_{jt+1}, \qquad (1.8)$$

where  $Z_j$  is the gross non-default loan rate and  $\bar{\omega}_j$  is the threshold value for entrepreneur j such that, for  $\omega_{jt+1} \geq \bar{\omega}_{jt+1}$ , entrepreneur j repays  $Z_{jt+1}D_{jt+1}$  to financial intermediaries and retains  $\omega_{jt+1}R_{t+1}^Kq_tK_{jt+1} - Z_{jt+1}D_{jt+1}$ . In the case of  $\omega_{jt+1} < \bar{\omega}_{jt+1}$ , instead, she cannot repay and defaults on her debt, obtaining nothing. Since entrepreneurs' future realizations of capital returns are only known by entrepreneurs ex-post, financial intermediaries must pay a fixed auditing cost,  $\mu$ , to recover what is left of entrepreneur j's activity after default, obtaining  $(1 - \mu)\omega_{jt+1}R_{t+1}^Kq_tK_{jt+1}$ .

Because of the optimal contract, financial intermediaries should receive an expected return equal to the opportunity cost of their funds. By assumption, they hold a perfectly safe portfolio (i.e., they are able to perfectly diversify the idiosyncratic risk involved in lending), and the opportunity cost for financial intermediaries is the real gross risk-free rate,  $R/\pi$ . It follows that the participation constraint for financial intermediaries that must be satisfied in each period t + 1 is:

$$[1 - F(\bar{\omega}_{jt+1})]Z_{jt+1}D_{jt+1} + (1 - \mu)\int_{0}^{\bar{\omega}_{jt+1}} \omega_j dF(\omega_j)R_{t+1}^K q_t K_{jt+1} \ge \frac{R_{t+1}}{\pi_{t+1}}D_{jt+1} , \quad (1.9)$$

where  $F(\bar{\omega}_j^F)$  is entrepreneur *j* default probability. Since financial markets are in perfect competition, (1.9) must hold with equality. The first term on the left-hand side of (1.9) represents the revenues received by financial intermediaries from the fraction of entrepreneurs that do not default, whereas the second term is what financial intermediaries can collect from defaulting entrepreneurs after paying monitoring costs.

Following the notation proposed in Christiano et al. (2014), I combine (1.7), (1.8), and (1.9) to write the following relationship:

$$EFP_{jt+1} = f(\bar{\omega}_{jt+1}, LEV_{jt+1})$$
, with  $f'(LEV_{jt+1}) > 0$ . (1.10)

where EFP is the "External Finance Premium" that Bernanke et al. (1999) define as the ratio between the return on capital and the real risk-free rate,  $R^{K}/(R/\pi)$ , and LEV = qK/N is entrepreneur's leverage. The EFP can be considered a measure of the

<sup>&</sup>lt;sup>21</sup>Section 1.C.1 provides analytical expressions for  $F(\omega)$  and other functions used in the following equations.

cost of external funds for the entrepreneur and, therefore, as a proxy for the strength of financial frictions. The  $(\bar{\omega}_{jt+1}, LEV_{jt+1})$  combinations that satisfy (1.10) define a menu of state (t+1)-contingent standard debt contracts offered to entrepreneur j, who chooses the contract that maximizes its objective.

In Appendix 1.C.2, I illustrate the entrepreneur j's optimization problem, which provides three important outcomes. First, the EFP increases monotonically with LEV. This means that entrepreneurs with a higher level of leverage pay a higher EFP. Second, the threshold value for entrepreneur j's default,  $\bar{\omega}_j$ , is endogenously defined by the EFP. Third, the fact that  $\bar{\omega}_j$  depends only on the aggregate variables  $(R, R^K \text{ and } \pi)$ implies that every entrepreneur will choose the same firm structure, that is,  $\bar{\omega}$  and LEV. Therefore, it is possible to drop the superscript j in the notation and consider a representative entrepreneur.

The other fundamental equation for the functioning of this financial accelerator is the law of motion for entrepreneurs' equity, which is expressed as follows:

$$N_{t+1} = \gamma \left[ q_{t-1} R_t^K K_t - \frac{R_t}{\pi_t} D_t - \mu G(\bar{\omega}_t) q_{t-1} R_t^K K_t \right] .$$
(1.11)

Equation (1.11) states that entrepreneurs' equity after the production process at time t is equal to the gross return on capital net of the loan repayment and auditing costs (which are borne by entrepreneurs because they are risk-neutral). Parameter  $\gamma$ represents the share of surviving entrepreneurs who bring their equity to the production process from one period to the next. Conversely, the share of entrepreneurs  $1 - \gamma$  dies and consumes equity at time t (we can think of this as entrepreneurial consumption). As explained by Carlstrom et al. (2016), this assumption avoids excessive entrepreneurs' self-financing in the long run.

Note that in (1.11) I did not included entrepreneurial labor, as usual in the literature (e.g., Bernanke et al., 1999, Christiano et al., 2014). The assumption of entrepreneurial labor was introduced mainly to justify the initial amount of equity for new entrepreneurs that take the place of the dead ones. However, to keep the model as simple as possible, I follow Carlstrom et al. (2016), assuming that new entrepreneurs' initial equity comes from a lump-sum transfer from existing entrepreneurs. Even so, since the funding can be arbitrarily small and since only aggregate equity matters, this transfer can be neglected in equation (1.11).<sup>22</sup>

Alternatively, (1.11) can be written in a more compact form as:

<sup>&</sup>lt;sup>22</sup>Bernanke et al. (1999) keep the share of income going to entrepreneurial labor at a very low level (on the order of 0.01), therefore neglecting this income sounds as a reasonable model simplification.

$$N_{t+1} = \gamma \left[ 1 - \Gamma(\bar{\omega}_t) \right] R_t^K q_{t-1} K_t , \qquad (1.12)$$

where  $[1 - \Gamma(\bar{\omega}_t)]$  is the share of capital returns to which non-defaulting entrepreneurs are entitled.<sup>23</sup> Equation (1.12), together with (1.10), explains the financial accelerator mechanism. Equation (1.10) states that an increase in entrepreneurs' leverage increases also the EFP. At the same time, (1.12) tells that an increase in the EFP increases  $\bar{\omega}$  as well, negatively affecting entrepreneurs' equity level for the next period and, therefore, impacting the aggregate leverage.

## **1.3.3** Intermediate-goods producers

Intermediate-goods producers adopt a standard Cobb-Douglas production function with constant returns to scale, employing aggregate capital, K, supplied by entrepreneurs and labor, L, from workers:

$$Y_t = z_t L_t^{\alpha} K_t^{1-\alpha} , \qquad (1.13)$$

where z represents the Total Factor Productivity (TFP).

TFP follows a first-order autoregressive process of type:

$$log(z_t) = \rho_z log(z_{t-1}) + \epsilon_t^z , \qquad (1.14)$$

with  $\epsilon_t^z$  following a normal distribution with mean 0 and variance  $\sigma^z$ .

Intermediate-good producers sell their production to resellers at a relative price  $MC_t$ . Therefore, their profit optimization is given by:

$$\Pi_t^{IG} = M C_t z_t L_t^{\alpha} K_t^{1-\alpha} - W_t L_t - r_t^K K_t .$$
(1.15)

Since they are in perfect competition, their profit optimization problem returns the wage paid per unit of labor and the rent paid per unit of capital:

$$W_t = \alpha M C_t z_t \left(\frac{K_t}{L_t}\right)^{(1-\alpha)} , \qquad (1.16)$$

$$r_t^K = (1 - \alpha) M C_t z_t \left(\frac{L_t}{K_t}\right)^{\alpha} .$$
(1.17)

 $^{23}$ See Appendix 1.C.2

#### 1.3.4 Resellers

Resellers are agents assigned to differentiate intermediate goods and set prices. Following Bayer et al. (2019), I assume that price adjustment costs follow a Rotemberg (1982) setup and that resellers are directly run by entrepreneurs, preserving their characteristics.<sup>24</sup> The demand for the differentiated good g is:

$$y_{gt} = \left(\frac{p_{gt}}{P_t}\right)^{-\eta} Y_t , \qquad (1.18)$$

where  $\eta > 1$  is the elasticity of substitution and  $p_g$  is the price at which good g is purchased.

Given (1.18) and the quadratic costs of price adjustment, resellers maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t Y_t \left\{ \left( \frac{p_{gt}}{P_t} - MC_t \right) \left( \frac{p_{gt}}{P_t} \right)^{-\eta} - \frac{\eta}{2\kappa} \left( \log \frac{p_{gt}}{p_{gt-1}} \right)^2 \right\} , \qquad (1.19)$$

with a time-constant discount factor.<sup>25</sup>

The New Keynesian Phillips Curve (NKPC) derived from the F.O.C. for price setting is as follows:

$$log(\pi_t) = \beta E_t \left[ log(\pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] + \kappa \left( MC_t - \frac{\eta - 1}{\eta} \right) , \qquad (1.20)$$

where  $\pi_t$  is defined as  $\frac{P_t}{P_{t-1}}$ .

# 1.3.5 Capital producers

After production at time t, entrepreneurs sell depreciated capital to capital producers at a price  $q_t$ . They refurbish depreciated capital at no cost,<sup>26</sup> and uses goods as investment inputs,  $I_t$ , to produce new capital,  $\Delta K_{t+1} = K_{t+1} - K_t$ , subject to quadratic adjustment

 $<sup>^{24}</sup>$ Bayer et al. (2019) make the further assumption that price setting is delegated to a mass-zero group of households (managers) that are risk neutral and compensated by a share in profits. Since in my model the whole production sector is run by entrepreneurs that, by assumption, are risk neutral and entitled to all the profits generated in this sector, I do no need to make this further assumption.

<sup>&</sup>lt;sup>25</sup>As explained by Bayer et al. (2019), only the steady state value of the discount factor matters in the resellers' problem, due to the fact that I calibrate to a zero inflation steady state, the same value for the discount factor of managers and households and approximate the aggregate dynamics linearly. This assumption simplifies the notation, since fluctuations in stochastic discount factors are virtually irrelevant.

<sup>&</sup>lt;sup>26</sup>The "no cost" assumption does not mean that  $\delta K$  is refurbished for free. Capital producers still need to buy the exact amount of I necessary to refurbish depreciated capital, but do not waste any further resources in this process. In fact, the law of motion for capital producers in the steady state (when  $\Delta K = 0$ ) is  $I = \delta K$ .

costs. Finally, they resell the newly produced capital to entrepreneurs before entering the next period (therefore still at price  $q_t$ ).

The law of motion for capital producers is:

$$I_t = \Delta K_{t+1} + \frac{\phi}{2} \left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \delta K_t . \qquad (1.21)$$

where  $\delta$  is the depreciation rate for capital.

Then, capital producers maximize their profit,  $q_t \Delta K_{t+1} - I_t$ , w.r.t. newly produced capital,  $\Delta K_{t+1}$ . This optimization problem delivers the optimal capital price:

$$q_t = 1 + \phi \frac{\Delta K_{t+1}}{K_t} \,. \tag{1.22}$$

This ensures that if the level of aggregate capital increases over time, so does its price.

It follows that entrepreneurs' return on capital does not depend only on goods production, but also on fluctuations of the capital price. Since entrepreneurs buy capital at the end of the period, they see that their capital at the beginning of the next period appreciated (depreciated) if q increases (decreases). The gross return on capital employed at time t can be written as:

$$R_t^K q_{t-1} K_t = r_t^K K_t + q_t K_t (1 - \delta) , \qquad (1.23)$$

where the first term on the right-hand side is the marginal productivity of capital derived in (2.27) and the second term represents the eventual capital gain (or loss) net of capital depreciation.

I can rearrange and finally derive the gross interest rate of capital as:

$$R_t^K = \frac{r_t^K + q_t(1-\delta)}{q_{t-1}} .$$
(1.24)

#### 1.3.6 Final-goods producers

Final-goods producers are perfectly competitive, buy differentiated goods from resellers at a given price, and produce a single homogeneous final good used for consumption, government spending, and investment. The optimization problem of final-goods producers is:

$$\max_{\{Y_t, y_{gt} \in [0,1]\}} P_t Y_t - \int_0^1 p_{gt} y_{gt} dg , \qquad (1.25)$$

subject to the following Constant Elasticity of Substitution (CES) function:

$$Y_t = \left(\int_0^1 (y_{gt})^{\left(\frac{\eta-1}{\eta}\right)} dg\right)^{\left(\frac{\eta}{\eta-1}\right)} . \tag{1.26}$$

From the zero-profit condition, the price index of the final good is:

$$P_t = \left(\int_0^1 (p_{gt})^{(1-\eta)}\right)^{\left(\frac{1}{1-\eta}\right)} .$$
 (1.27)

# 1.3.7 Central bank

The central bank is responsible for the monetary policy. It sets the gross nominal risk-free interest rate, R, reacting to the deviation from steady state inflation, and engages interest rate smoothing. The Taylor-type rule employed by the central bank is as follows:

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^{\rho_R} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\rho_R)\rho_\pi} \epsilon_t^R , \qquad (1.28)$$

where  $\epsilon_t^R$  is the monetary policy shock defined as  $log(\epsilon_t^R) \sim N(0, \sigma_R)$ . The parameter  $\rho_R \geq 0$  rules interest rate smoothing (if  $\rho_R = 0$ , the next-period interest rate depends only on inflation), whereas  $\rho_{\pi}$  captures the magnitude of the central bank's response to inflation fluctuations: the larger  $\rho_{\pi}$ , the stronger the central bank reaction (for the case limit  $\rho_{\pi} \to \infty$ , inflation is perfectly stabilized at its steady state level).

#### 1.3.8 Government

The government acts as fiscal authority. It determines the level of public expenditure,  $G_t$ , tax revenues,  $T_t$  and issuance of new bonds,  $B_{t+1}$ . Its budget constraint is given by:

$$B_{t+1} = \left(\frac{R_t}{\pi_t}\right) B_t + G_t - T_t , \qquad (1.29)$$

where  $T_t$  are taxes collected from both worker and rentier households:

$$T_t = \tau \left[ \int W_t h_{it} l_{it} d\Theta_t(a, h) + \mathbf{I}_{h_{it}=0} \Pi_t \right] , \qquad (1.30)$$

and  $\Theta_t(a, h)$  is the joint distribution of liquid assets and productivity across households on date t. Government bond issuance is regulated by the following rule:

$$\frac{B_{t+1}}{\bar{B}} = \left(\frac{B_t \frac{R_t}{\pi_t}}{\bar{B}\frac{\bar{R}}{\bar{\pi}}}\right)^{\rho_B} . \tag{1.31}$$

The parameter  $\rho_B$  captures how fast the government wants to balance its budget. When  $\rho_B \to 0$ , the government balances its budget by adjusting its spending. Instead, when  $\rho_B \to 1$ , the government is willing to roll over most of its outstanding debt.

# 1.3.9 Market clearing

The labor market clears when:

$$\int h \, l^*(a,h)\Theta_t(a,h)dadh = L_t \,, \qquad (1.32)$$

where  $l^*(a, h)$  is the optimal labor supply policy function of the household.

The liquid asset market clears when:

$$\int a^*(a,h)\Theta_t(a,h)dadh = A_t , \qquad (1.33)$$

where  $a^*(a, h)$  is the optimal saving policy function of the household.

The market for capital clears for (1.21) and (1.22).

Finally, good market clearing, which holds by Walras' law when other markets clear, is defined as:

$$Y_t = C_t + G_t + I_t + C_t^E + \mu G(\bar{\omega}_t) R_t^K q_{t-1} K_t , \qquad (1.34)$$

where on the left-hand side we have total output. On the right-hand side, apart from household consumption, public expenditure and investments, we also find entrepreneurial consumption,  $C^E$  (due to dying entrepreneurs), and auditing costs.<sup>27</sup>

# 1.3.10 Numerical implementation

To solve the model, I follow the solution proposed in Bayer et al. (2019) and Luetticke (2021). As the joint distribution,  $\Theta_t$ , is an infinite-dimensional object (and therefore not computable), it is discretized and represented by its histogram, a finite-dimensional object. I solve the household's policy function using the Endogenous Grid-point Method (EGM) developed by Carroll (2006), iterating over the first-order condition and ap-

<sup>&</sup>lt;sup>27</sup>Similarly to Kaplan et al. (2018), we can think of this last term as "financial services".

proximating the idiosyncratic productivity process using a discrete Markov chain with three states using the Tauchen (1986) method. The log grid for liquid assets comprises of 100 points. I solve for aggregate dynamics by first-order perturbation around the steady state, as in Reiter (2009). The joint distribution is represented by a bidimensional matrix (capital K does not display heterogeneity) with a total of 300 grid points, maintaining a sufficiently low computational time.

# 1.4 Calibration

The model is calibrated on the US economy, and because the focus is on conventional monetary policy, business cycle moments are targeted on the Great Moderation (i.e., 1983-2007). Periods in the model represent quarters; consequently, the following values for the calibrated parameters are intended quarterly unless otherwise specified. Table 1 provides a list of calibrated parameters for the model, whereas Table 2 shows the model's effectiveness in replicating wealth distribution and business cycle moments.

## 1.4.1 Households

For the households' utility function, I assume the coefficient of relative risk aversion  $\xi = 2$ , which is consistent with the findings of Attanasio and Weber (1995) and already used by Auclert et al. (2021). I set the Frisch elasticity of labor supply  $\nu = 1$ , in line with the results of Chetty et al. (2011). The parameter for the disutility of labor,  $\psi$ , is set to 5.5, to have an average value for hours worked equal to 1/2, as in Kaplan et al. (2018). The intertemporal discount factor,  $\beta$ , is equal to 0.987, so savings in deposits by households are sufficient to have a leverage for entrepreneurs of 2, the same value used by Bernanke et al. (1999) in their model, and a fair calibration given historical levels of corporate leverage. I decide on purpose to impose a non-borrowing condition for households, setting the borrowing limit for liquidity  $\underline{a} = 0$ , to highlight the transmission mechanism of monetary policy through financial frictions on the production sector rather than on the lending sector.<sup>28</sup>

The calibration of the productivity transition matrix, which determines how households move between the worker and rentier states, aims to provide a distribution of wealth consistent with empirical data. As in Luetticke (2021), I assume that the probability of becoming a rentier is the same for workers independent of their labor produc-

 $<sup>^{28}</sup>$ The lack of a negative ad hoc borrowing constraint denies a further instrument of parameterization, since in the literature this feature is often used to target the share of HtM or borrowing households. Nonetheless, the share of zero-wealth households generated by the model is still significant, approximately 16%.

| Parameter         | Value   | Description                      |
|-------------------|---------|----------------------------------|
| β                 | 0.987   | Discount factor                  |
| ξ                 | 2       | Relative risk aversion           |
| ν                 | 1       | Frisch elasticity of labor       |
| $\psi$            | 5.5     | Disutility of labor              |
| $\underline{a}$   | 0       | Borrowing constraint             |
| L                 | 0.0625  | Prob. of leaving entr. state     |
| $\zeta$           | 0.0005  | Prob. become rentier             |
| $ ho_h$           | 0.98    | Persistence of idio. prod. shock |
| $\sigma_h$        | 0.06    | SD if idio. prod. shock          |
| $\alpha$          | 0.7     | Labor share of production        |
| δ                 | 0.2     | Depreciation rate                |
| $\eta$            | 20      | Elasticity of substitution       |
| $\kappa$          | 0.09    | Price stickiness                 |
| $\phi$            | 5       | Adjustment cost of capital       |
| $\mu$             | 0.12    | Auditing costs                   |
| $\sigma_{\omega}$ | 0.27    | SD of the id. shock on entr.     |
| $\gamma$          | 0.985   | Entr. surviving rate             |
| $ ho_z$           | 0.95    | TFP shock persistence            |
| $\sigma_z$        | 0.00915 | TFP shock SD                     |
| R                 | 1.0063  | Nominal int. rate                |
| $ ho^R$           | 0.8     | Int. rate smoothing              |
| $ ho^{\pi}$       | 1.5     | Reaction to inflation            |
| $\sigma_R$        | 0.0025  | Monetary shock SD                |
| au                | 0.3     | tax rate                         |
| $ ho_B$           | 0.86    | Auto-correlation of debt         |

Table 1: Calibrated parameters

Table 2: Wealth distribution and business cycle moments

| Wealth | distribution | moments |
|--------|--------------|---------|
|        |              |         |

| Target          | Model | Target           |
|-----------------|-------|------------------|
| Gini wealth     | 0.78  | 0.78             |
| top 10% wealth  | 0.71  | 0.67             |
| zero-wealth HHs | 0.16  | $0.20 \sim 0.30$ |

Business cycle moments

| ·····                   |       |        |  |
|-------------------------|-------|--------|--|
| Target                  | Model | Target |  |
| SD of Y (%)             | 1.38  | 1.38   |  |
| $\sigma^{I}/\sigma^{Y}$ | 4.5   | 4.5    |  |
| SD of C $(\%)$          | 0.47  | 0.98   |  |
| Corr. of Y with Y       | 1     | 1      |  |
| Corr. of I with Y       | 0.99  | 0.92   |  |
| Corr. of C with Y       | 0.95  | 0.92   |  |

Real GDP, investment and consumption are in logs. All data for business cycle moment analysis are processed with a H-P filter with  $\lambda = 1600$ . The calibrated moments for wealth distribution is the Gini index for wealth. For business cycle moments, SD of Y and SD of I after a TFP shock.

tivity, and once they become workers again, they start with the highest productivity realization. The probability of leaving the rentier state is  $\iota = 0.0625$ , following the findings of Guvenen et al. (2014) on the probability of dropping out of the top 1% income group in the US. The probability of moving from the worker to the rentier state is  $\zeta = 0.0005$ , a value calibrated to obtain a Gini coefficient for wealth of 78%, in line with empirical data from the Survey of Consumer Finances (Luetticke, 2021), implying a share of rentier households of approximately 0,8%. Regarding idiosyncratic income risk for labor productivity, I set autocorrelation  $\rho_h = 0.98$  and standard deviation  $\sigma_h = 0.06$ , as estimated by Bayer et al. (2019).

## 1.4.2 Financial Intermediaries

The parameters concerning financial frictions on firms are in the ballpark of Bernanke et al. (1999) calibrations; therefore, the auditing cost is  $\mu = 0.12$  and the standard deviation of the idiosyncratic shock on the entrepreneur's returns is  $\sigma_{\omega} = 0.27$ , which are calibrated to have EFP = 1.005 (and, therefore, a credit spread of 2% p.a.) when the corporate leverage is 2. The share of surviving entrepreneurs,  $\gamma$ , is calibrated such that, at steady state, the equity level in (1.12) is equal to the equity implied by (1.10).

## 1.4.3 Production Sector

The labor share of production (accounting for profits) and capital depreciation rate follow standard values in the literature and are set respectively to  $\alpha = 0.7$  and  $\delta = 2\%$ . The mark-up is also standard, at 5%, which implies elasticity of substitution between goods varieties  $\eta = 20$ . The price stickiness parameter in the NKPC,  $\kappa = 0.09$ , is calibrated to generate a slope of the curve similar to the one that would arise in a model with sticky prices à la Calvo, with an average price duration of four quarters. The adjustment cost of the capital parameter is calibrated to  $\phi = 5$  to match investmentto-output volatility  $\sigma(I)/\sigma(Y) = 4.5$  after a TFP shock, in line with empirical data for the US elaborated by Bayer et al. (2019), in the model where the financial accelerator is in place. The persistence of the TFP shock is  $\rho_z = 0.95$ , while the standard deviation  $\sigma_z = 0.00915$  is calibrated to match the standard deviation of the US output (after HP filtering) in the targeted time period.

# 1.4.4 Central Bank and Government

Inflation at the steady state is set to 0% per annum, and the nominal (therefore real) interest rate on bonds is 2.5%, a value in line with the real average federal funds rate

for the Great Moderation period. I impose the same interest rate on all types of liquid savings (i.e., government bonds and deposits); otherwise, households would choose to invest only in one asset or the other. Regarding the Taylor rule adopted by the Central Bank, the parameter for interest rate smoothing is  $\rho_R = 0.8$ , according to the findings of Clarida et al. (2000), whereas the reaction to inflation fluctuations from the steady state is  $\rho_{\pi} = 1.5$ , which is a common value in the macroeconomic literature. For the magnitude of the monetary policy shock, I assume that the central bank raises the nominal interest rate by 25 b.p., a common value in the literature and consistent with the empirical results in Section 1.2.

The taxes set by the government are proportional to both labor income and profits, with a tax rate  $\tau = 0.3$  that targets the ratio of government spending to GDP to a standard value in the New Keynesian literature, G/Y = 20%. Since I am using a fiscal policy rule similar to the one adopted by Bayer et al. (2019), I also follow their estimation and set  $\rho_B = 0.86$ . This implies that the fiscal dynamic passes through government debt, with public spending adjusting to re-stabilize debt to its steady state level.

# 1.5 Results

To begin with, the fluctuations in aggregate variables are shown. This helps to assess the consistency of the results with respect to the findings of Bernanke et al. (1999). Subsequently, I examine the inequalities in the model, which are at the core of this research.

#### 1.5.1 Aggregate fluctuations

During the first period, the economy experiences an unexpected increase in the nominal interest rate (one-time innovation). Figure 1.6 compares the response of several aggregate variables to this shock when financial frictions are active (blue solid line) or not (red dashed line), i.e., when the EFP can fluctuate or is fixed to its steady state value.

The effect of the financial accelerator on aggregate variables has also been confirmed for heterogeneous households. Results are fairly similar to Figure 3 in Bernanke et al. (1999), with output and investment responses under financial frictions exhibiting higher magnitude on impact and higher persistence over time,<sup>29</sup> although IRFs in the HANK

 $<sup>^{29}\</sup>mathrm{Since}$  in Bernanke et al. (1999) there is a fall in the nominal interest rate, the two dynamics are mirrored.



Figure 1.6: Impulse response to a monetary contraction for aggregate variables Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.

model converge to the steady state (or even overshoot) more rapidly. Aggregate consumption fluctuations do not significantly differ between the two scenarios on impact, but this result is also consistent with the findings in Bernanke et al. (1999).<sup>30</sup>

To illustrate how the financial accelerator works, the IRFs for the EFP, leverage, firm equity, and household liquidity are also displayed.<sup>31</sup>. An increase in the nominal interest rate depresses economic activity, leading to a lower demand for capital and, consequently, lower investment and capital price. On the other hand, a higher interest rate increases household liquidity, particularly liquidity directed to firms in the form of loans through financial intermediaries. As suggested by equation (1.11) and shown in the central panels of Figure 1.6, lower levels of capital and capital price and higher levels of firms' debt cause a decline in firms' equity and, therefore, a higher level of leverage.<sup>32</sup> Higher leverage implies higher firm financing costs, i.e., higher EFP, as pointed out by by eq. (1.10). Simultaneously, the entrepreneur's default threshold value,  $\bar{\omega}$ , also increases, which negatively affects the firm's equity level in the next period. With lower equity, firms need to resort to more external financing, but since

<sup>&</sup>lt;sup>30</sup>The authors do not show impulse responses for consumption in their paper. Nonetheless, using replication codes as the one present in the Macroeconomic Model Data Base (https://www. macromodelbase.com) allows us to see this dynamic.

 $<sup>^{31}\</sup>mathrm{More}\ \mathrm{IRFs}$  for aggregate variables are shown in Appendix 1.D

<sup>&</sup>lt;sup>32</sup>Recall that in this model leverage is defined as  $\frac{q\bar{K}}{N}$ , or equivalently,  $\frac{D+N}{N}$ .

the latter is more expensive as leverage and EFP increase, the level of capital that firms can afford is even lower, which means less investment and less goods production, generating the multiplier effect of the financial accelerator. The countercyclicality of leverage and EFP in the theoretical model is relevant for two reasons. First, it allows the replication of the financial accelerator mechanism developed by Bernanke et al. (1999). Second, it is consistent with the empirical evidence highlighted in Figure 1.5, in which a monetary contraction is followed by a co-movement of the corporate leverage and a proxy measure of financial frictions.

In addition, a comparison of the leverage and output behavior in the two scenarios (active or passive frictions) deserves a closer look. While output fluctuations are always enhanced by financial frictions for the entire horizon considered, this is not the case for leverage, where the leverage level with active frictions is relatively lower after three years. Although it may seem counter-intuitive, it is a common result in the theoretical literature,<sup>33</sup> and a possible explanation can be found in the power of the friction itself. In the "shut-off" version of the model, external funds are relatively cheaper because the EFP is fixed at its steady state level. Therefore, firms' deleveraging is slower in time, mainly because of the higher debt they contract with financial intermediaries, as shown in Figure 1.6. Nonetheless, active financial frictions can lead to a higher economic depression in terms of output and investment, even at relatively lower leverage levels in the economy.

## 1.5.2 Inequality among households: consumption

To check whether the model is consistent with the empirical findings shown in Section 1.2, I first analyze how consumption dispersion evolves. Figure 1.7 displays IRFs for the Gini index of consumption and the ratio of consumption for the median percentile to consumption of the 10th percentile, measures already employed in the empirical analysis. The model replicates two main empirical results: (i) a contractionary monetary policy shock causes a rise in consumption dispersion both in terms of the Gini index and the 50/10 consumption ratio, and (ii) financial frictions increase the effect of monetary policy on consumption inequality.

The 50/10 consumption ratio rapidly converges to the steady state, but does not exhibit undershooting behavior, as observed in Figure 1.2. However, the Gini coefficient rapidly decreases during the first year, but then starts to flatten, resulting in a lower but long-term increase in general consumption inequality. Although this outcome seems to be at odds with findings in Section 1.2, where IRFs for the Gini index rapidly converge

 $<sup>^{33}</sup>$ A similar dynamic occurs in the original Bernanke et al. (1999) model.



Figure 1.7: Impulse responses for the 50/10 consumption ratio and Gini index Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from red line to the blue line.

to zero, it is a common result in the theoretical literature.<sup>34</sup>

The increasing effect of financial frictions on the magnitude and persistence of the IRFs in Figure 1.7 is evident. To better understand the effect of the financial accelerator on the Gini index, I plot a green line with circles that account for the percentage variation of the Gini index impulse response from the scenario with a muted financial accelerator to that where frictions are active. Acceleration of consumption inequality is actually hump-shaped and goes downward after one year. Therefore, the two curves representing consumption dispersion with and without financial frictions activity show some convergence in the medium term, even though the reversal to the steady state is much slower.

The Gini index is a powerful tool because it allows us to estimate the total inequality using a single number. However, it does not specify the distribution of the variable at stake (in this case consumption) among agents. Therefore, it is not possible to explain why inequality increases after the monetary contraction and why financial acceleration enhances this process. To clarify these points, in Figure 1.8, I decompose the aggregate impulse response for consumption to determine how it varies along the distribution. To

 $<sup>^{34}</sup>$ As explained by Luetticke (2021), the persistence of the increase in the Gini index is motivated by a prolonged time of higher wealth inequality, as shown in Section 1.5.3.



Figure 1.8: IRFs for consumption, aggregate and averages per wealth share The blue solid line represents the aggregate consumption. The dashed lines represent the average consumption of a specific share of households. Monetary shock  $\epsilon^R = 0.0025$ . On the left is the case with active financial frictions. On the right, financial frictions are shut off

study consumption behavior among poorer and richer agents, I track fluctuations in average consumption for the bottom 10% and 50%, and for the top 10% of households according to their wealth distribution.<sup>35</sup>

Figure 1.8 suggests two major considerations. First, monetary contraction results in lower average consumption for poorer households, confirming the literature findings. The average consumption in the bottom 10% of the distribution shows a significant contraction compared to aggregate consumption. As these households are perfectly constrained, the reduction in consumption is solely due to worsening labor conditions. Depression of consumption occurs up to the bottom half of the distribution. In this last case, however, consumption overshoots after around one year, since this share the population also includes households with a certain level of liquidity, who benefit from financial income. On the other hand, the average consumption at the top of the distribution increases steadily when the top 10% of the population is considered. Whereas in this model households can only save liquid assets, an increase in the interest rate would be very beneficial for rich households, who hold a significant amount of liquidity.<sup>36</sup> Even

 $<sup>^{35}{\</sup>rm Higher}$  household share values include consumption of lower shares. This means that the average consumption of the bottom 50% also includes consumption of the bottom 10%

<sup>&</sup>lt;sup>36</sup>A more comprehensive discussion about the implications of considering only liquidity for household savings can be found in Section 1.5.3.

though richer households have a lower marginal propensity to consume, their gains are significant enough to create a substantial increase in average consumption. However, the importance of the marginal propensity to consume can be appreciated by looking at fluctuations in consumption at the lowest decile: on impact, the percentage increase in average consumption at the top 10% is always less than a third of consumption at the bottom 10%, regardless of active or passive financial frictions.

Second, the financial accelerator does not drastically change the behavior of IRFs, but affects their magnitude. Although aggregate consumption is similar in both cases, fluctuations in average consumption per share of wealth increase significantly on impact in case of active financial acceleration. The higher decline in labour and wage levels due to active financial frictions is more significant for poor households, whereas wealthier households benefit from the relative increase in interest rate and profits.<sup>37</sup> This explains why the Gini index for consumption is higher when the financial accelerator is taken into account, even if consumption fluctuations appear similar at aggregate level.

A final remark on consumption inequality concerns the hump-shaped evolution of the percentage difference (green line with circles) between the two Gini coefficients in Figure 1.7. The average consumption per wealth share shown in Figure 1.8 helps us understand this behavior. On the left-hand panel, the average consumption fluctuation for the bottom 50% (green line with asterisks) overshoots earlier and stronger when compared with the muted financial accelerator case. In view of the fact that this share includes the bottom 10% (whose average consumption shows a relatively higher and more persistent depression), households around the middle percentile should be responsible for this overshoot. This dynamic is consistent with the fact that a part of households have a significant marginal propensity to consume, but are not wealthconstrained, and thus rely on both labor and financial income. Therefore, higher consumption in the middle of the distribution most likely contributes to pushing down the Gini index, causing a reversal of the trend for the green line with circles in the right panel of Figure 1.7.

## 1.5.3 Inequality among households: wealth

After observing that the results for consumption inequality are consistent with empirical evidence, I analyze how household wealth reacts to a monetary shock. Empirical evidence for these dynamics is more difficult to obtain because of the frequency of available data. Theoretical outcomes are therefore a good instrument for estimating wealth

 $<sup>^{37}\</sup>mathrm{As}$  I explain in Section 1.5.3, most rentiers belong to the top 10%.


Figure 1.9: Impulse responses of the Gini index for wealth Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation between the two IRFs.

inequality. I begin with the Gini index for the wealth distribution. Figure 1.9 displays how the Gini index changes from its steady state value, with active or passive financial accelerator.

First, let us focus on the Gini index dynamic for active financial frictions. The impulse response shows a significant and long-lasting increase. It reaches its apex one year after the shock and then reverts very slowly. After five years, the increase from the steady state value is still greater than the on-impact value. This long-lasting effect has already been seen in Figure 1.7 and is shared with household liquidity dynamics. In fact, in this model, the only type of wealth that households can accumulate is liquid, since by construction, they can only save in deposits and government bonds.<sup>38</sup> Contractionary monetary policy has a long-term impact not only on the total amount of wealth in the economy, but also on its distribution.

Also when considering wealth, the financial accelerator is an inequality accelerator. The red dashed line represents the IRFs for the Gini index with silenced frictions, which show significantly lower magnitude and persistence. In order to clarify the difference

<sup>&</sup>lt;sup>38</sup>Although it is not an exercise I undertake in this study, considering multiple assets should not significantly change the shape of the Gini index. For instance, Luetticke (2021) considers a contractionary MP shock in a model where households hold liquid and illiquid assets, and the evolution of the Gini index for wealth (see Figure 1 in its Appendix) is similar to that in Figure 1.9.

between the two scenarios, I plot again a green line with circles that account for the percentage variation of the Gini index impulse response from the scenario with a muted financial accelerator to that where frictions are active. On impact, financial friction implies a fluctuation in wealth inequality that is approximately 50% greater. Although both curves (solid and dashed) start reverting to the steady state value after about one year, their rate of reversion is different. The shape of the line with circles shows how this difference actually grows over time, reaching above 80% after four years. Therefore, the financial accelerator does not only increase inequality in wealth, but this increase is also constant, at least in the medium term. Interestingly, an increase in the magnitude of the monetary shock affects absolute values (the Gini index increases more with respect to its steady state values in both scenarios), but not relative values; that is, the shape and magnitude of the green line with circles are almost the same. Inequality acceleration has little to do with shock magnitude, but depends mostly on steady state dynamics such as leverage and initial wealth distribution. For instance, Figure 1.E.1 in Appendix 1.E. shows how the same aggregate shock applied to a similar model that features higher firm leverage (2.5, instead of 2) generates significantly higher Gini index differentials between the case of active and inactive financial frictions.

As already explained in the previous section, the Gini index cannot say how the our variable of interest is distributed among agents. For wealth analysis, fluctuations of two measures representing behavior at the two tails of the wealth distribution are computed: the share of perfectly constrained households (i.e., with zero wealth) and the share of wealth held by the richest 10% of the population. The results are shown in Figure 1.10.

Once again, let us first focus only on the scenario with active financial frictions (blue solid line). Similar to the Gini index, the two measures increase with a hump-shaped response. This suggests that (i) there is an increase in the number of poorer households because more households are pushed to the constraint, (ii) rich households are becoming richer, and (iii) the increase in the Gini index is caused by substantial movements on both tails of the wealth distribution. However, the dynamics triggering increasing responses in the two measures considered are completely different.

Constrained households have zero wealth, so an increase in the interest rate has essentially no effect on their financial income. On the other hand, indirect effects highlighted by Kaplan et al. (2018), in particular fluctuations related to labor income, are responsible for the rise in the share of constrained households. The economic depression brought about by contractionary monetary policy reduces the quantity of labor needed in the economy and the wage level (as can be seen in Appendix 1.D) and poor



Figure 1.10: Impulse responses for households' share measures

The graph on the left-hand side represents the fluctuations in the share of households with zero wealth. The one on the right-hand side represents the fluctuations in the share of wealth held by the richer 10%. Monetary shock  $\epsilon^R = 0.0025$ . The blue line refers to an economy with a financial accelerator. The red line refers to the case where financial frictions are shut off. The green dotted line (with values on the right side of the figure) represent the percentage variation from red line to the blue line.

households rely only or mostly on labor income for consumption and saving. Therefore, in addition to households already at the constraint, a share of households that was not perfectly constrained before the aggregate shock is pushed to the very bottom of the distribution.

To analyze what happens at the top of the distribution, it is important to remember that, according to model's assumptions, households can only accumulate wealth in liquid assets. Government bonds and deposits have a fixed price (normalized to one), unlike capital; therefore, they are not affected by price fluctuations. This assumption neglects the fact that, in empirical data, a significant share of rich households' savings comprises illiquid assets, which usually bear a higher interest rate but are subject to price changes. The choice of a single liquid asset for household saving in the model has two main justifications. First, it does not add any further complications to the model structure, keeping it as simple as possible. Second, it provides continuity with the RANK model developed by Bernanke et al. (1999). It follows that IRFs for richer households' wealth could suffer from upward bias because they do not consider the negative effects of capital price fluctuations. However, this should not affect the validity of the results, since empirical evidence shows that rich households react to an increase in the interest rate by increasing the share of liquidity in their portfolio.<sup>39</sup>

Households in the top 10% of the model are therefore highly affected by the direct effects of monetary policy, because they experience a significant increase in financial income and are less affected by labor dynamics. A further push toward wealth accumulation in the top decile comes from an increase in firms' profits. Although the share of rentiers (the only ones collecting profits) is quite small (approximately 0.8% of the total population), the vast majority of them belong to the top 10%.

Similarly to what happens for the Gini index, the financial accelerator increases the magnitude of impulse responses for these two measures of wealth fluctuation. The red dashed lines in Figure 1.10, in fact, always lie below the blue solid ones over the first four years, and the percentage differential on impact is very similar, between 45% and 50%. The striking difference in the dynamics at the two ends is the medium-term evolution of the differential (i.e., the green line with circles). At the bottom of the distribution, the effect of the financial accelerator continues to increase, whereas at the top 10%, the differential line starts decreasing immediately. Active financial frictions have a negative impact on constrained households as they lead to a further reduction in the quantity of labour and, in particular, a permanent reduction in wages. In subsection 1.D it can be seen that the financial accelerator further depresses the quantity of labor required for goods production, although it overshoots with respect to the counterfactual scenario after approximately three years. On the other hand, IRFs for the wage level are always lower for active financial frictions and at a very distant horizon. This latter dynamic is therefore probably the main reason for the constant increase in the differential line for constrained household wealth.

To understand why the differential of IRFs for wealth held by the richest 10% converges already in the short-medium term, we should look again at how aggregate household deposits evolve. As already explained above, firm financing is relatively cheaper when financial frictions are shut off. As a result, firms can take up relatively higher amounts of funds from households as the on-impact effects wane. It should also be noted that in this model, the household top decile holds 71% of the total wealth. The vast majority of firms'debt is likely to come from wealthy households' deposits. For this reason, we see a faster reversion to the steady state in the case of active financial frictions when considering fluctuations in the wealth held by the top decile. Fluctuations in the real interest rate obviously play an important role as well. The IRFs in Appendix 1.D show that in the case of active financial frictions, the real interest rate

 $<sup>^{39}</sup>$ Luetticke (2021) shows with empirical estimates that wealthy households react to a contractionary monetary policy increasing their holdings of liquid wealth and portfolio liquidity.

is indeed higher for the first three quarters, but then undershoots with respect to the scenario in which frictions are shut off. This helps to explain why the differential line for the top 10% wealth decreases even more rapidly after about one year.

## 1.6 Inequality between households: skilled-unskilled workers and rentiers

Households in this model are heterogeneous according to their wealth level and their taxed-income source. Consequently, an interesting analysis can be conducted on how inequality is shaped between household types, that is, workers (who collect income through labor) and rentiers (whose income is made of firms' profits). Workers can be further divided into two categories: those with low and high productivity. As already expressed in eq. (1.5), labor income for workers before taxes is defined as  $W_t h_{it} l_{it}$ . Since the wage level,  $W_t$ , is not idiosyncratic and is equal for everyone, if two workers with different productivity,  $h_{it}$ , were to provide the same quantity of labor,  $l_{it}$ , the high-productivity worker would obtain a higher salary. Therefore, with an abuse of notation, I sort households into three types: unskilled (low-productivity workers), skilled (high-productivity workers), and rentiers (profit collectors). I show how wealth inequality evolves between these households and how the financial accelerator affects these dynamics. To do so, I use the wealth Gini index for every household type. The results are presented in Figure 1.11. The financial accelerator not only increases the magnitude of the Gini index fluctuations but, in some cases, also changes the shape of the curves over time. For instance, while unskilled workers' Gini index variation seems to stabilize after one year when financial frictions are shut off, it continues to increase in the other scenario. Variations in the Gini index for rentiers seem to be exactly the same in the two cases, with the difference that they are enhanced in the presence of financial frictions.

Aside from financial accelerator effects, Figure 1.11 provides an interesting outcome: wealth inequality does not always increase. In fact, wealth inequality decreases among rentiers. The reason for this difference in the behavior of the Gini IRF is likely twofold. First, workers collect labor and financial income. Given that they are affected by both dynamics, it seems plausible that the shape of the evolution of their inequality mimics the shape of the global Gini index. On the other hand, rentiers always benefit from an increase in the interest rate, since both financial income and profits rise. Therefore, rentiers at the bottom of the wealth distribution are also better off.

Second, these trends could be caused not only by households moving along the



Figure 1.11: Gini index for wealth inequality according to households type

distribution but also by wealth movements between household types. To see this, Figure 1.12 shows how relative wealth changes after the contractionary monetary shock among workers and rentiers.<sup>40</sup> Fluctuations in the two scenarios have essentially the same shape, but differ in magnitude. The relative wealth of rentiers increases with a peak of over 3% in the case of financial frictions, whereas the relative wealth of workers decreases, albeit to a lesser extent. The workers who experienced the highest fluctuations are skilled workers who lose more relative wealth than unskilled workers. These results suggest that, aside from changes in household wealth distribution, the variations in the Gini index per household type could also be caused by wealth movements between groups, with the relative gains of rentiers more evenly distributed among them.

## 1.7 Concluding remarks

Empirical and theoretical evidence points to a relevant role of the financial accelerator in the monetary transmission, and this seems also to be true for consumption and

<sup>&</sup>lt;sup>40</sup>As "relative wealth" I mean the percentage of wealth in the hands of a certain household type over the whole wealth. Intuitively, if the relative wealth of a household category decreases, it does not necessarily mean that they have less wealth in absolute terms. In fact, as household savings increase after a rise in the nominal interest rate, the opposite is more likely. However, a decrease in relative wealth means a decrease in the weight of a household type's wealth compared with total wealth in the economy. This can be thought as a "relative drain" of wealth from certain household categories at the expense of others.



Figure 1.12: Relative wealth changes per households type

wealth inequality dynamics.

Adopting a proxy-SVAR with externally identified monetary shocks, I show that a contractionary monetary policy has an increasing effect on consumption dispersion, and financial frictions have a multiplier effect that is empirically significant.

Introducing financial frictions in the flavor of Bernanke et al. (1999) in a full-fledged HANK model, I am able to replicate relevant empirical results, showing that the financial accelerator not only causes a higher depression in aggregate variables such as output and investment after a monetary policy shock, but also an increase in inequality measures concerning wealth and consumption, coming to the conclusion that the financial accelerator is also an "inequality accelerator".

This acceleration is mostly due to movements at the two tails of the wealth distribution, with constrained households playing a crucial role. Since they cannot rely on savings or borrowing to smooth consumption, they rely solely on their income, which largely comes from labor. Frictions on the production side of the economy, such as those studied in this paper, depress labor income, pushing more people into the borrowing constraint and increasing wealth and consumption inequality. On the other hand, households in the top decile benefit from an increase in the interest rate, and their wealth and consumption increase. Nonetheless, to better understand the behavior of this latter share of households, an extension of the model that allows households to save also in illiquid assets is desirable. I will leave this as a possible venue for future research.

In addition, financial frictions not only enhance wealth changes among households but also between household types (workers and rentiers). In terms of relative wealth shares in the hands of a certain group, rentiers become relatively richer, and workers become relatively poorer after an increase in the interest rate. Wealth inequality in the economy is not only a matter of how households move along the wealth distribution but also of how wealth is redistributed between household types.

Although central bankers do not formally care about redistribution trends, their concern about this topic has increased over the last decade. From a technical perspective, the blooming literature on HANK models proves that wealth distribution has important effects on the transmission of monetary policies. Acknowledging that the financing structure of non-financial firms has important implications for the wealth and consumption redistribution of monetary policy shocks could be something to consider for future policy-making.

## References

- Adrian, T., Boyarchenko, N., and Giannone, D. (2019). Vulnerable growth. American Economic Review, 109(4):1263–89.
- Ahn, S., Kaplan, G., Moll, B., Winberry, T., and Wolf, C. (2017). When inequality matters for macro and macro matters for inequality. Working Paper 23494, National Bureau of Economic Research.
- Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. The Quarterly Journal of Economics, 109(3):659–684.
- Attanasio, O. P. and Weber, G. (1995). Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. Journal of Political Economy, 103(6):1121–1157.
- Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the sequencespace jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408.
- Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, 87(1):255–290.
- Bernanke, B., Gertler, M., and Gilchrist, S. (1996). The Financial Accelerator and the Flight to Quality. *The Review of Economics and Statistics*, 78(1):1–15.
- Bernanke, B. S. (2015). Monetary policy and inequality. *Brookings Institution blog, Washington DC.*
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.
- Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.
- Caldara, D. and Herbst, E. (2019). Monetary policy, real activity, and credit spreads: Evidence from bayesian proxy svars. American Economic Journal: Macroeconomics, 11(1):157–92.

- Carlstrom, C. and Fuerst, T. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review*, 87(5):893–910.
- Carlstrom, C. T., Fuerst, T. S., and Paustian, M. (2016). Optimal contracts, aggregate risk, and the financial accelerator. *American Economic Journal: Macroeconomics*, 8(1):119–47.
- Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.
- Chetty, N., Weber, A., Guren, A. M., and Day, M. (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. Technical report.
- Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk Shocks. American Economic Review, 104(1):27–65.
- Clarida, R., Galí, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, 115(1):147–180.
- Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.
- Debortoli, D. and Galí, J. (2017). Monetary policy with heterogeneous agents: Insights from TANK models. Economics Working Papers 1686, Department of Economics and Business, Universitat Pompeu Fabra.
- Fernández-Villaverde, J., Hurtado, S., and Nuño, G. (2023). Financial frictions and the wealth distribution. *Econometrica*, 91(3):869–901.
- Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–357.
- Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.
- Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. Journal of Monetary Economics, 58(1):17–34. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.

- Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.
- Gertler, M. and Kiyotaki, N. (2010). Chapter 11 financial intermediation and credit policy in business cycle analysis. volume 3 of *Handbook of Monetary Economics*, pages 547–599. Elsevier.
- Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. American Economic Review, 102(4):1692–1720.
- Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467.
- Guvenen, F., Kaplan, G., and Song, J. (2014). The glass ceiling and the paper floor: Gender differences among top earners, 1981-2012. Working Paper 20560, National Bureau of Economic Research.
- Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5):953–969.
- Jordà, O., Schularick, M., and Taylor, A. M. (2017). Macrofinancial history and the new business cycle facts. *NBER Macroeconomics Annual*, 31:213–263.
- Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. American Economic Review, 108(3):697–743.
- Krusell, P. and Smith, Jr., A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.
- Lee, S., Luetticke, R., and Ravn, M. (2020). Financial frictions: Macro vs micro volatility. CEPR Discussion Papers 15133.
- Lettau, M., Ludvigson, S. C., and Steindel, C. (2002). Monetary policy transmission through the consumption-wealth channel. *Economic Policy Review*, 8(May):117–133.
- Luetticke, R. (2021). Transmission of monetary policy with heterogeneity in household portfolios. *American Economic Journal: Macroeconomics*, 13(2):1–25.
- Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the united states. *American Economic Review*, 103(4):1212–47.
- Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48(3):261–297.

- Mumtaz, H. and Theodoridis, K. (2020). Dynamic effects of monetary policy shocks on macroeconomic volatility. *Journal of Monetary Economics*, 114:262–282.
- Nakajima, M. and Ríos-Rull, J.-V. (2019). Credit, Bankruptcy, and Aggregate Fluctuations. Working Papers 19-48, Federal Reserve Bank of Philadelphia.
- Piketty, T. (2017). Capital in the Twenty-First Century. Harvard University Press.
- Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. Journal of Economic Dynamics and Control, 33(3):649–665.
- Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review*, 94(4):1055–1084.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.
- Stock, J., Yogo, M., and Wright, J. (2002). A survey of weak instruments and weak identification in generalized method of moments. *Journal of Business and Economic Statistics*, 20:518 – 529.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.

## Appendix

## 1.A CEX consumption data

To build the consumption dispersion measures used in Figure 1.1 and Section 1.2, I use consumption expenditure data from the Consumer Expenditure Survey (CEX). The advantage of this dataset is that it contains consumption expenses on a quarterly basis continuously since 1984Q1. Because households in the dataset are representative of a portion of the US population (a weight is assigned to each of them), it is possible to build consumption distributions and consumption inequality measures. I use data from the FMLI interview file to compute households' total consumption: the total expenditure in a quarter (TOTEXPPQ) net of life and personal insurance (LIFINSPQ), cash contributions (CASHCOPQ), retirement, pensions and social security (RETPENPQ). As in Lee et al. (2020), I adopt a definition of consumption that includes durables, non-durables, and housing services, restricting the sample to households aged 20–60 years, working at least 10 hours a week, with a partner, and families with 10 members or less. As suggested by the US Bureau of Labor Statistics, the weight used to obtain a representation of the entire population in the US is the variable FINLWT21. The consumption ratio between the median and 10th percentiles is logged. Both consumption dispersion measures used, 50/10 ratio and Gini index, are smoothed with a centered three-quarter moving average and de-sesonalized with a quarterly dummy.

## 1.B Idiosyncratic productivity process and the joint distribution

Households can be workers, with productivity h > 0, or rentiers, with h = 0, which means that they do not earn labor income but only profit income. Furthermore, I assume that there are only two possible productivity realizations for workers: high productivity,  $h^H$ , and low productivity,  $h^L$ . This assumption, in addition to simplifying the computations, is useful for developing the analysis in Section 1.6 between skilled and unskilled workers. The Markov process generates the following transition matrix:

with probabilities, p, determined using the Tauchen method. In other studies using this household distribution framework, such as Luetticke (2021),rentiers who become workers are endowed with the median productivity level (h = 1). However, in this model, there are no states with median productivity levels.<sup>41</sup> Therefore, I assume that new workers are endowed with the highest productivity possible,  $h^H$ .

At the steady state, a joint distribution of households exists according to their wealth level, a, and their productivity, h. This joint distribution can be represented by the bi-dimensional matrix as follows:

#### wealth a

where  $H_{1,1}$  is the share of households with the lowest level of wealth and labor productivity (except for the last state  $h_m = 0$ , since in this model they are rentiers), and  $\int H da dh = 1$ . As the vector indicating possible household wealth levels is composed of 100 entries, this joint distribution matrix comprises 300 grid points ( $a_n = 100$  and  $h_m = 3$ ).

<sup>&</sup>lt;sup>41</sup>Following the calibration of the baseline model, I obtain that  $h^L = 0.786$  and  $h^H = 1.272$ 

## **1.C** Entrepreneurs optimal contract

#### 1.C.1 Idiosyncratic shock on return on capital

I assume that the Idiosyncratic shock  $\omega$  is distributed log-normally. i.e.  $\omega \in [0, +\infty)$ .<sup>42</sup> Using results from Appendix A.2 in Bernanke et al. (1999) I can write  $F(\omega)$ ,  $\Gamma(\omega)$  and  $G(\omega)$  in the analytical expressions that I use in my code to solve the model:

$$F(\omega) = \Phi\left[\left(\log(\bar{\omega}) + \frac{1}{2}\sigma_{\omega}^{2}\right)/\sigma_{\omega}\right] , \qquad (1.C.1)$$

$$\Gamma(\omega) = \Phi\left[\left(\log(\bar{\omega}) - \frac{1}{2}\sigma_{\omega}^{2}\right)/\sigma_{\omega}\right] + \bar{\omega}\left\{1 - \Phi\left[\left(\log(\bar{\omega}) + \frac{1}{2}\sigma_{\omega}^{2}\right)/\sigma_{\omega}\right]\right\}, \quad (1.C.2)$$

$$G(\omega) = \Phi\left[\left(\log(\bar{\omega}) + \frac{1}{2}\sigma_{\omega}^{2}\right)/\sigma_{\omega} - \sigma_{\omega}\right].$$
(1.C.3)

With  $\Phi(\cdot)$  being the normal cumulative distribution function and  $\sigma_{\omega}$  the standard deviation of the idiosyncratic shock on entrepreneurs' return on capital.

# 1.C.2 Financial intermediaries' participation constraint and entrepreneur j's optimization problem

After substituting (1.8) and (1.7) into (1.9), I obtain:

$$[1 - F(\bar{\omega}_{jt+1})]\bar{\omega}_{jt+1}R_{t+1}^{K}q_{t}K_{jt+1} + (1-\mu)\int_{0}^{\bar{\omega}_{jt+1}}\omega_{j}dF(\omega_{j})R_{t+1}^{K}q_{t}K_{jt+1} = \frac{R_{t+1}}{\pi_{t+1}}(q_{t}K_{jt+1} - N_{jt+1}) + (1.C.4)$$

Divide everything by  $R_{t+1}^R q_t K_{jt+1}$ :

$$\frac{\underline{R}_{t+1}^{K}}{\underline{R}_{t+1}} \left( \left[ 1 - F(\bar{\omega}_{jt+1}) \right] \bar{\omega}_{jt+1} + (1-\mu) \int_{0}^{\bar{\omega}_{jt+1}} \omega_{j} dF(\omega_{j}) \right) = \left( 1 - \frac{N_{jt+1}}{q_{t} K_{jt+1}} \right) . \quad (1.C.5)$$

Following the notation used in Bernanke et al. (1999) and Christiano et al. (2014):

 $<sup>^{42}</sup>$ Note that other kinds of distribution with values greater or equal to 0 could be used as well. Here I choose to adapt the same distribution as in Bernanke et al. (1999) to give a sense of continuity between the two studies.

$$\Gamma(\bar{\omega}_j) \equiv \int_0^{\bar{\omega}_j} \omega_j dF(\omega_j) + \bar{\omega}_j \int_{\bar{\omega}_j}^\infty dF(\omega_j) , \qquad \mu G(\bar{\omega}_j) \equiv \mu \int_0^{\bar{\omega}_j} \omega_j dF(\omega_j) , \quad (1.C.6)$$

where  $\Gamma(\bar{\omega}_j)$  is the expected gross share of profits going to the lender and  $\mu G(\bar{\omega}_j)$  is the expected monitoring cost paid by the lender.  $\Gamma(\bar{\omega}_j)$  can be rewritten as:

$$\Gamma(\bar{\omega}_j) = G(\bar{\omega}_j) + \bar{\omega}_j \left[1 - F(\bar{\omega}_j)\right] . \qquad (1.C.7)$$

I can now use (1.C.6) and (1.C.7) in (1.C.5) and rearrange to finally obtain:

$$\frac{R_{t+1}^K}{\left(\frac{R_{t+1}}{\pi_{t+1}}\right)} = \frac{1}{\Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1})} \left(1 - \frac{N_{jt+1}}{q_t K_{jt+1}}\right) , \qquad (1.C.8)$$

where  $\Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1})$  is the share of entrepreneur j's profits going to the lender (as loan repayment), net of auditing costs.

Equation (1.C.8) is the complete version of (1.10), which explain the function underlying  $f(\bar{\omega}_{jt+1}, LEV_{jt+1})$ . For a higher level of entrepreneur leverage, the EFP increases, raising the return on capital. However, it also increases the probability of an entrepreneur's default, thereby increasing the net share of profit demanded by financial intermediaries as loan repayment, resulting in higher financing costs for entrepreneurs. To see in detail how this mechanism works, I show the entrepreneur j's optimization problem below.

According to the optimal contract set by financial intermediaries, entrepreneur j's expected return can be expressed as:

$$E_t \left\{ \int_{\bar{\omega}_{jt+1}}^{\infty} \omega_j dF(\omega_j) R_{t+1}^K q_t K_{jt+1} - (1 - F(\bar{\omega}_j)) R_{t+1}^K q_t K_{jt+1} \right\} , \qquad (1.C.9)$$

with expectations taken with respect to the realization of  $R_{t+1}^K$ . The first term of (1.C.9) represents the entrepreneur's profit when she does not default on debt, while the second term is the amount of profits that she uses to repay the lender. Following the notation used above, and considering that the entrepreneur's return is subject to the participation constraint (1.9), I write entrepreneur j's optimal contracting problem as:

$$\max_{\{K_{jt+1},\bar{\omega}_{jt+1}\}} E_t \left\{ \left[ 1 - \Gamma(\bar{\omega}_{jt+1}) \right] R_{t+1}^K q_t K_{jt+1} \right\} ,$$

$$s.t. \quad \frac{R_{t+1}}{\pi_{t+1}} (q_t K_{jt+1} - N_{jt+1}) = \left[ \Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1}) \right] R_{t+1}^K q_t K_{jt+1} .$$

$$(1.C.10)$$

Deriving F.O.C. I obtain:

w.r.t. 
$$\omega_{jt+1}$$
:  $-\Gamma'(\bar{\omega}_{jt+1}) + \lambda_{jt+1} \left[\Gamma'(\bar{\omega}_{jt+1}) - \mu G'(\bar{\omega}_{jt+1})\right] = 0$ , (1.C.11)

w.r.t. 
$$K_{jt+1}$$
:  $E_t \left\{ \left[ 1 - \Gamma(\bar{\omega}_{jt+1}) \right] R_{t+1}^K - \lambda_{jt+1} \left[ \frac{R_{t+1}}{\pi_{t+1}} - \left( \Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1}) R_{t+1}^K \right) \right] \right\} = 0$ 
(1.C.12)

w.r.t. 
$$\lambda_{jt+1}$$
:  $E_t \left\{ \frac{R_{t+1}}{\pi_{t+1}} (q_t K_{jt+1} - N_{jt+1}) - [\Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1})] R_{t+1}^K q_t K_{jt+1} \right\} = 0$  (1.C.13)

where  $\lambda_j$  is the Lagrangian multiplier for entrepreneur *j*'s problem. By rearranging (1.C.11), it is possible to express  $\lambda_{jt+1}$  as a function of only  $\bar{\omega}_{jt+1}$ . Furthermore, rearranging (1.C.12):

$$E_t \left\{ \frac{R_{t+1}^K}{\frac{R_{t+1}}{\pi_{t+1}}} \right\} = \frac{\lambda_{jt+1}}{\left[1 - \Gamma(\bar{\omega}_{jt+1}) + \lambda_{jt+1} \left(\Gamma(\bar{\omega}_{jt+1}) - \mu G(\bar{\omega}_{jt+1})\right)\right]} .$$
(1.C.14)

It can be proven that there is a monotonically increasing relationship between the EFP and  $\bar{\omega}_j$ . According to (1.C.8), we can extend this relationship between the EFP and the leverage level of j, assessing that a higher entrepreneur's leverage implies a higher EFP.<sup>43</sup>

Furthermore, it is clear from (1.C.14) that  $\bar{\omega}_j$  is determined only by aggregate variables. Thus, any entrepreneur chooses the same threshold  $\bar{\omega}$  for the idiosyncratic shock on capital returns, below which they default, and the same leverage level.<sup>44</sup> This result

 $<sup>^{43}\</sup>mathrm{See}$  Appendix A.1 in Bernanke et al. (1999) for proofs.

<sup>&</sup>lt;sup>44</sup>According to (1.C.8), leverage is a function of the EFP (composed of only aggregate variables) and  $\bar{\omega}_j$ . If  $\bar{\omega}_j$  depends only on aggregate variables (since it is a function of the EFP, according to (1.C.14)), then the same can be said for the leverage.

allows to consider only the aggregate variables in the production sector part of the model, since every entrepreneur has the same firm structure.

## 1.D Impulse responses of MP contractionary shock

Figure 1.D.1 show several aggregate variables impulse responses for the monetary policy shock considered in the main text. This integrate IRFs present in Figure 1.6 in the main text.

## 1.E Gini indices for higher leverage at steady state

Figure 1.E.1 shows fluctuations of the Gini indices for wealth and consumption in a model with a higher initial level of firm leverage. I show results for the case where the latter is targeted to be equal to 2.5 (instead of 2, as in the baseline model). To reach this level of leverage while maintaining the general calibration, I slightly decrease the discount factor  $\beta$ , increase the labor disutility parameter  $\psi$  to 6.5, change the household probability to become a rentier,  $\zeta = 0.00056$ , and the parameter governing the adjustment cost of capital,  $\phi = 10$ .

### 1.F Robustness to risk aversion

For the baseline model, I used a parameter for households' risk aversion  $\xi = 2$ , which is already used in other HANK models in the literature. However, other models used different values; for instance, Bayer et al. (2019) and Luetticke (2021) assume  $\xi = 4$ . I recalibrated the model with this parameter to obtain relevant moments as in the baseline version. This implies a discount factor  $\beta = 0.986$ , labor disutility parameter  $\psi = 11$ , household probability of becoming a rentier  $\zeta = 0.00072$  and the parameter governing the adjustment cost of capital  $\phi = 7$ . Figure 1.F.1 and Figure 1.F.2 show fluctuations for aggregate variables and Gini indices, respectively.

## 1.G Robustness to investment cost

The baseline model features quadratic investment costs (the central term on the righthand side of Eq. (1.21)) where the parameter  $\phi$  is calibrated to match an investment volatility of 4.5. I display in Figure 1.G.1 and Figure 1.G.2 aggregate and Gini index fluctuations for the case limit where there is no investment cost, that is,  $\phi = 0$ . This means that the capital price q is fixed over time, and entrepreneurs do not make any profit from capital gains or creation of new capital  $\Delta K$ . This extreme calibration also



Figure 1.D.1: Aggregate fluctuations consequent to an increase of the nominal interest rate.

Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.





The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.

confirms the financial accelerator: the output, investment, consumption, and Gini indices are all greater when financial frictions are active. However, it is worth noting that some aggregate variables display completely different behaviors. For instance, the quantity of aggregate labor L increases after a MP contractionary shock. Interestingly, aggregate consumption fluctuations do not overshoot in the short-run with this parameterization.

### **1.H** Robustness to fiscal policy

Since I employ a HANK model, the Ricardian equivalence does not hold, and fiscal policies could have significant effects on monetary transmission. In the baseline model, I assume that the government adjusts its spending to bring debt to steady state values. In line with the empirical data, as in Bayer et al. (2019) and Luetticke (2021), I set the debt autocorrelation  $\rho_B = 0.86$ , meaning that the government is willing to roll over most of this debt, sustaining a higher level of public expenditure. I now consider the case in which the government wants to revert immediately to its steady state level of debt after a MP contractionary shock, setting  $\rho_B = 0$ . The results are shown in Figure 1.H.1 and Figure 1.H.2. The government achieves debt control by cutting



Figure 1.F.1: IRFs for aggregate variables,  $\xi = 4$ Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.





The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.

even more expenditure, inducing a higher economic depression in terms of output and consumption, and increasing inequalities even more when compared to the baseline specification of the model.

The government could also choose to maintain spending at its steady state level and adjust taxation through the tax parameter  $\tau$ . The results are shown in Figure 1.H.3 and Figure 1.H.4. While Output and investment do not display significant differences compared to the baseline calibration, consumption falls more, and the respective Gini index is higher. Taxes increase to balance the government budget constraint; however, taxation is proportional and not progressive. Therefore, poorer households (who rely more on labor income for consumption and have a higher marginal propensity to consume) are more affected by this tax rise. In this model, financial income is not taxed; therefore, wealthier households suffer less from an increase in tax rate.

## 1.I Impulse responses of a TFP shock

In this section, I show aggregate and inequality fluctuations when a positive TFP shock occurs, instead of a contractionary MP shock. The shock to  $z_t$  follows an AR(1) process with persistence  $\rho_z = 0.95$  and SD  $\sigma_z = 0.00915$ . The SD is calibrated such



Figure 1.G.1: IRFs for aggregate variables,  $\phi = 0$ Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.





The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.

that the autocorrelation of output is in line with values from US data. Figure 1.I.1 shows aggregate variable fluctuations, while Figure 1.I.2 shows fluctuations of the Gini indices for wealth and consumption. Interestingly, adopting a HANK model seems to solve the "financial accelerator dampening" of the TFP shock that occurs in Bernanke et al. (1999). In their results, the TFP shock confirms the financial accelerator only if persistence is set to  $\rho_z = 1$  such that the shock never reverts to zero over time. For a more standard value of the TFP shock persistence, such as  $\rho_z = 0.95$ , the model presented in Bernanke et al. (1999) shows a "financial deceleration". On the other hand, as shown in Figure 1.I.1, output, investment, and consumption also increase (albeit slightly) when persistence is less than 1.



Figure 1.H.1: IRFs for aggregate variables,  $\rho_B = 0$ Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.



Figure 1.H.2: IRFs for Gini indices,  $\rho_B = 0$ The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.



Figure 1.H.3: IRFs for aggregate variables,  $\tau$  adjustment Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.



Figure 1.H.4: IRFs for Gini indices,  $\tau$  adjustment

The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.



Figure 1.I.1: IRFs for aggregate variables to positive TFP shock TFP shock  $\sigma_z = 0.00915$  with  $\rho_z = 0.95$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off.



Figure 1.I.2: IRFs for Gini indices to positive TFP shock

The graph on the left-hand side represents fluctuations in the Gini index for wealth, while the one on the right-hand side for consumption. Monetary shock  $\epsilon^R = 0.0025$ . The blue solid line refers to an economy with a financial accelerator. The red dashed line refers to the case where financial frictions are shut off. The green line with circles (with values on the right side of the figure) represent the percentage variation from the solid line to the dashed line.

# Chapter 2 Effects of different financial frictions on households

#### Abstract

The objective of this study is to investigate the impact of different types of financial frictions on households' wealth and consumption following a contractionary monetary policy shock. The analysis focuses on two types of frictions: frictions on production firms and frictions on households' borrowing ability, both of which are incorporated into a HANK model. The findings indicate that the friction in the productive sector has a greater effect on wealth inequality, while the friction on household loans leads to a higher dispersion of consumption compared to the counterfactual scenario. These two dynamics are found to be interconnected. In the model with frictions on household borrowings, households are discouraged from moving towards the lower end of the wealth distribution, resulting in reduced borrowing and consequently lower consumption capacity. Conversely, when there is an active friction on the production side of the economy, more households are pushed towards the lower end of the distribution. This fluctuation increases the Gini index of wealth, but at the same time enables greater economy-wide consumption smoothing, thereby reducing consumption inequality compared to the previous case.

#### 2.1 Introduction

Although more than a decade has passed since the burst of the real estate bubble, the effects of the Great Financial Crisis remain tangible in economic research. Two branches of literature have drawn particular attention from academics interested in macro models to better understand the causes and effects of such events.

One concerns the implications of considering heterogeneous households as opposed to standard New Keynesian (NK) models, where a Representative Agent (RA) exists. This change in perspective is mostly motivated by the rising inequality (in terms of both wealth and income) experienced not only in the United States, but also in almost all advanced economies. Although this phenomenon has been ongoing for more than 40 years,<sup>45</sup> the 2008 financial crisis exacerbated this process. RA models offer a significant benefit as they can be potentially solved analytically. Consequently, they do not require extensive time when solved using computer software. This advantage was particularly crucial in the past due to limited computing power. However, it is important to note that the RA assumption is an extreme simplification. This simplification becomes even more stringent when a larger proportion of households fall under the "Hand-to-Mouth" (HtM) category. In this scenario, households predominantly consume their income without the ability to save, which is a characteristic often associated with increasing inequality. To address this issue, more complex models are necessary. A favorable tradeoff can be found in the form of Two-Agents New Keynesian (TANK) models. These models incorporate two distinct types of households: HtM and non-HtM. This approach maintains a relatively straightforward model while still yielding significant implications for overall economic dynamics and the transmission of monetary policy. Nonetheless, it is important to note that these models do not permit an analysis of changes in the distribution of household wealth. In contrast, full-fledged Heterogeneous Agents New Keynesian (HANK) models encompass multiple households that exhibit varying consumption and saving behaviors. These models allow for the matching of wealth distribution moments, which turn out to be relevant for explaining the consequences of aggregate shocks.

The other area of interest pertains to exploring amplification mechanisms within theoretical frameworks that could account for significant fluctuations in variables, even following a moderate aggregate shock. A prevalent assumption in conventional Real Business Cycle (RBC) and New Keynesian (NK) models is the notion of a frictionless economy, where financial intermediaries are almost nonexistent or simply transfer liquidity without impediments, ensuring that funds consistently reach individuals capable of optimizing their returns. Despite their relative simplicity, these models have proven to be effective in approximating historical business cycle statistics. However, they often fall short in explaining the magnitude and persistence of aggregate shock effects. The assumption of perfectly functioning financial markets, while convenient, is not realis-

 $<sup>^{45}\</sup>mathrm{Piketty}$  (2017) provides a thorough review of the recent inequality history, especially in advanced economies.

tic, particularly during times of financial crisis and credit rationing. The emergence of theories concerning imperfect financial markets can be traced back nearly a century ago (e.g., Fisher, 1933), in the aftermath of another disruptive financial crisis, the 1929 stock market crash. Although models capable of elucidating such dynamics have been formulated since the 1970s (e.g., Akerlof, 1970), they may not have garnered sufficient attention: until the advent of the Great Recession, financial crises were considered either relics of the past or primarily afflicting underdeveloped economies. However, since 2008, an increasing number of scholars have reevaluated the significance of financial frictions. They have sought to integrate existing mechanisms and introduce novel model features that could more effectively explain how a relatively minor disturbance can give rise to profound and enduring effects.

There is also a growing body of literature focusing on the impact of monetary policy shocks on household inequality. Coibion et al. (2017) is probably one of the most influential empirical contribution. Through the analysis of data from the Consumer Expenditure Survey (CEX) on consumption and income, the authors establish that contractionary monetary policies have significant effects, resulting in heightened levels of income, labor earnings, consumption, and total expenditure inequality. From a modeling perspective, the seminal paper by Kaplan et al. (2018) is probably one of the most important in demonstrating how monetary policy transmission mechanisms act very differently in a full-fledged HANK model compared to the relative RANK and TANK versions. However, to date, few studies have dealt with monetary shock effects on household behavior in an environment with financial frictions.

This study seeks to assess the implications of a conventional contractionary monetary policy shock (i.e., a rise in the nominal interest rate by the central bank) on the distribution of wealth and consumption patterns across households, depending on the type of financial friction considered in the model. I analyze two types of frictions: frictions on the ability of productive firms to raise external funds, and frictions on the ability of households to obtain loans. In both cases, the severity of these frictions is directly proportional to the spread between the relevant interest rate (gross return on capital in the case of frictions on firms, loan rate for frictions on households) and the risk-free rate. Consequently, when these interest rate differentials expand, a financial accelerator is triggered, intensifying the impact of the aggregate shock. Empirical evidence from recent studies demonstrates a positive correlation between these spreads and inequality indices, specifically consumption dispersion measures.<sup>46</sup> Lee et al. (2020)

<sup>&</sup>lt;sup>46</sup>The empirical literature often relies on consumption inequality as a preferred metric, thanks to the quarterly data on the US provided by the CEX. On the other hand, extrapolating wealth dynamics presents greater difficulty due to the triennial nature of the Survey of Consumer Finances.

provide empirical support for the link between consumption dispersion and two measures of the spread in household borrowings: the spread between the two year personal loan rate and the three months T-Bill rate and the spread between the Commercial Bank interest on three months Credit Card plans and the three months T-Bill rate. Using Local Projection regression á la Jordà (2005), they show that an increase in the spread is associated with higher consumption inequality, regardless of the interest rate spread considered.<sup>47</sup> In Section 1.2 of this dissertation, I employ a Structural Vector Autoregression model with external instrument identification to prove that the effects on monetary policy on consumption inequality are enhanced by financial frictions affecting the corporate sector, using the Excess Bond Premium computed by Gilchrist and Zakrajšek (2012) as a proxy of credit spread for non-financial firms.

Starting from these premises, I build a full-fledged HANK model featuring asset market incompleteness, idiosyncratic income risk, sticky prices, and two potential sources of financial frictions that come into play, depending on the case. It is important to highlight that my objective is not to provide precise quantitative outcomes, as the model's asset heterogeneity is monodimensional.<sup>48</sup> Instead, my approach involves conducting a qualitative analysis to investigate the movement of household distributions after a contractionary monetary shock in a particular economic state. Concerning frictions on productive firms, I resort to a financial accelerator similar to that proposed by Bernanke et al. (1999), which is one of the most seminal and recurring in the financial friction literature. In the case of friction on household borrowing ability, I take cues from the work of Cúrdia and Woodford (2016) and posit that the spread between deposit and loan interest rates is directly proportional to a non-decreasing convex function of the aggregate household debt in the economy: an increase in household debt leads to a corresponding expansion in this interest rate differential.<sup>49</sup> As the mechanisms and complexities of the two financial accelerators differ, the magnitude of the impulse responses could be different, but not necessarily because of economic dynamics. Hence, in

<sup>&</sup>lt;sup>47</sup>In a more recent version of this paper (Faccini et al., 2024), the authors examine household data from Denmark and discover that higher spreads are connected to decreased consumption spending for indebted households, while the association is positive for wealthier households. They also construct an aggregate measure of the consumption-income elasticity that varies over time as a function of how households move across the wealth distribution and as a function of changes in the consumer credit spread. This index appears to exhibit volatility and countercyclicality due to changes in both net worth and the consumer credit spread.

<sup>&</sup>lt;sup>48</sup>Accurate quantitative results are best suited for HANK models featuring two different assets (usually liquid and illiquid) as in Kaplan et al. (2018) or Luetticke (2021), since only with this kind of structure it is possible to target "wealthy hand-to-mouth" households.

 $<sup>^{49}</sup>$ Cúrdia and Woodford (2016) suggests that the spread could potentially be affected by households defaulting on unsecured debts. Nevertheless, to ensure simplicity, this particular aspect is omitted from the model.

order to facilitate a fair comparison, I opt not to apply the same monetary shock to both cases. Instead, I employ two distinct magnitudes that yield comparable fluctuations in output.

The main result is that the type of friction is important for changes in savings and consumption. The analysis presented in this research highlights that the difference in inequality fluctuations measures are predominantly driven by the decisions made by households in proximity to the zero-wealth threshold, particularly when they are faced with the choice of becoming borrowers or savers, as opposed to dynamics observed at the top end of the wealth distribution.<sup>50</sup> The contractionary monetary policy results in a reduction in labor income, which constitutes the primary source of earnings for poorer households. Individuals at the lower end of the distribution use their savings or opt to borrow funds to smooth consumption. In the presence of financial frictions within firms, the household borrowing premium remains constant, resulting in household borrowings being relatively more affordable than in the alternative scenario. Agents can move in a larger quantity to the bottom of the distribution, leading to a significant rise in wealth inequality. However, the impact on consumption is relatively smaller as agents can better smooth their consumption through borrowed liquidity. On the other hand, under financial frictions on households, the household borrowing premium increases after a monetary contraction. Consequently, fewer households are able to borrow, resulting in a deterioration of consumption smoothing. Moreover, borrower households experience even lower levels of consumption due to the higher interest rates on their debts. As a result, a greater proportion of households opt to remain HtM, preventing them from descending further down the distribution. This ultimately leads to a relatively lower Gini index for wealth but a higher one for consumption. Furthermore, the decomposition of aggregate consumption provides interesting insights on the dichotomy between direct and indirect effects introduced by Kaplan et al. (2018). The fluctuations in wages significantly contribute to consumption dynamics, which are further amplified by the presence of frictions within firms. Conversely, frictions within households play a pivotal role in accentuating the direct effects, primarily through fluctuations of the household borrowing premium.

This paper touches on different fields of macroeconomics. First, the model has roots in the literature concerning high heterogeneity among households, a path that started at the end of the 1980s (Imrohoroğlu, 1989; Huggett, 1993; Aiyagari, 1994). Krusell and Smith (1998) were the first to include aggregate uncertainty in such models, although their findings suggest that household heterogeneity has little impact on aggregate dy-

<sup>&</sup>lt;sup>50</sup>Both global wealth and consumption inequality are calculated as Gini indices.

namics, such as consumption, investment, and output. However, the Great Recession and increase in inequalities have raised interest in this topic, and an increasing number of scholars are striving to build algorithms capable of solving models with greater and more complex heterogeneity (e.g., Bayer and Luetticke, 2020; Auclert et al., 2021).

Second, this paper fits within the branch that examines the implications of household heterogeneity for monetary policy. Kaplan et al. (2018) is one of the most important contributions to the field, proving that household heterogeneity is fundamental in understanding monetary policy transmission. Nonetheless, concerns about monetary policy mechanisms with household heterogeneity were at the center of a blooming body of literature in recent years (e.g., Auclert, 2019; Luetticke, 2021). A thorough survey on this topic can be found in Colciago et al. (2019).

Third, my findings contribute to expanding the vast literature on financial frictions. Most frictions are built around the concept of asymmetric information between the lender-household and borrower-firm (e.g., Akerlof, 1970; Stiglitz and Weiss, 1981; Bernanke et al., 1999). Another common feature is the idea that a "moral hazard" exists that prevents the credit market from being frictionless (e.g., Holmstrom and Tirole, 1997; Gertler and Karadi, 2011; Farhi and Tirole, 2012). Papers on household borrowing frictions usually focus on unsecured loans and credit tightening (e.g., Iacoviello, 2005; Chatterjee et al., 2007; Cúrdia and Woodford, 2016). The survey by Brunnermeier et al. (2012) provides an excellent summary of the state-of-the-art in this branch of the economic literature.

By contrast, the theoretical literature on the effects of aggregate shocks on heterogeneous households in an environment featuring financial frictions is probably at the beginning of his existence. Guerrieri and Lorenzoni (2017) focuses on frictions on households and study the effects of a credit crunch on consumer spending by applying two types of shocks: a narrowing of the borrowing constraint and an increase of the borrowing spread for households. Nakajima and Ríos-Rull (2019) study the effects of earning shocks in a model with unsecured consumer credit and consumer bankruptcies. Fernández-Villaverde et al. (2023) analyze the nonlinear linkages between aggregate and financial variables in a framework featuring household heterogeneity and a leveraged "financial expert". In terms of monetary policy evaluation, Lee et al. (2020) investigate how an increase in the nominal risk-free rate (among other aggregate shocks) affects aggregate variables and household distributions of wealth and consumption when the household borrowing rate is affected by financial intermediaries' moral hazard. In the first chapter of this dissertation, I focus on the impact on heterogeneous households of a financial accelerator generated by a leveraged production sector.
Differently from what this study seeks to accomplish, none of the papers cited above inspect whether movements in inequality measures could depend on the types of frictions considered in the economy and, if that is the case, what is the reason behind the different responses.

The remainder of this chapter is organized as follows. Section 2.2 outlines the model. Section 2.3 explains the calibration strategy. Section 2.4 displays results. Section 2.5 gives summary conclusions.

# 2.2 The model

To obtain a better comparison between the two financial frictions, I do not compare two different models (one for each friction). Instead, I build a model incorporating both frictions so that the starting point for the analysis (i.e., the steady state) is the same. I then turn on one friction or the other and compare the impulse responses.<sup>51</sup> The model comprises households, financial intermediaries, a production sector, a central bank, and the government. Households consume, earn income (from either labor or profit, according to their household type), save, and borrow in a liquid asset. This asset yields an interest rate, that is augmented by a borrowing penalty in case of loans. There are two types of financial intermediaries: commercial banks, which intermediate household borrowings, and investment banks, which intermediate firm borrowings. The production sector produces goods and capital. The central bank is in charge of monetary policy and sets the nominal interest rate, whereas the government acts as fiscal authority and chooses how to finance government spending. The behavior of each agent is explained in detail below.<sup>52</sup>

#### 2.2.1 Households

There is a continuum of ex-ante identical households of measure one indexed by  $i \in [0, 1]$ . They are infinitely lived, have time-separable preferences with a time discount factor  $\beta$ .

Following Bayer et al. (2019), I assume households have Greenwood–Hercowitz–Huffman (GHH) preferences (Greenwood et al., 1988) and maximize the discounted sum of utility:

<sup>&</sup>lt;sup>51</sup>Because the two frictions are based on an interest rate spread fluctuating endogenously, the "active" friction, depending on the case, is the one for which the spread varies over time. To "shut off" a friction, I fix the relative spread.

 $<sup>^{52}</sup>$ The model structure is very similar to the 1-asset HANK version proposed in Bayer et al. (2019), with the exception of the introduction of financial frictions.

$$V = E_0 \max_{\{c_{it}, l_{it}\}} \sum_{t=0}^{\infty} \beta^t u \left( c_{it} - G(h_{it}, l_{it}) \right) .$$
(2.1)

where  $c_{it}$  is consumption for household *i* and  $G(h_{it}, l_{it})$  is a function of productivity,  $h_{it}$ , and labor supplied,  $l_{it}$ , representing household leisure.

Assuming GHH preferences instead of separable preferences has two major advantages and one flaw. First, from a computational perspective, it simplifies the numerical analysis. Second, as explained by Auclert et al. (2023), this prevents the model from generating an excessive Marginal Propensity to Earns (MPE), especially for households with high Marginal Propensity to consume (MPC), since GHH preferences dampen wealth effects on labor supply. However, using GHH preferences in models with household heterogeneity translates into higher fiscal and monetary multipliers. The latter should not be a problem in this model because both the scenarios compared in this analysis would be affected by this issue.<sup>53</sup>

The utility function features Constant Relative Risk Aversion (CRRA):

$$u(x_{it}) = \frac{x_{it}^{1-\xi}}{1-\xi} , \qquad (2.2)$$

where  $\xi \ge 0$  is the risk-aversion parameter, and  $x_{it} = (c_{it} - G(h_{it}, l_{it}))$  is household *i*'s composite demand for goods consumption and leisure. The function G measures the disutility from work.

Goods consumption bundles differentiated goods j according to a Dixit–Stiglitz aggregator:

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} .$$
(2.3)

Each of these differentiated goods is offered at price  $p_{jt}$ , so that for the aggregate price level,  $P_t = \left(\int p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$ , the demand for each of the varieties is given by:

$$c_{ijt} = \left(\frac{p_{jt}}{P_t}\right)^{-\eta} c_{it} .$$
(2.4)

The disutility of work,  $G(h_{it}, l_{it})$ , determines a household's labor supply given the

<sup>&</sup>lt;sup>53</sup>Auclert et al. (2023) call it the "New Keynesian models trilemma". Regarding HANK models, choosing separable preferences delivers consistent MPC and multipliers but not MPE. On the other hand, choosing GHH preferences delivers consistent MPC and MPE but not multipliers. One solution proposed by the authors is to consider a HANK model with separable preferences and sticky wages. As interesting as it may be as a model integration, I believe that for the time being, such a complication is not necessary.

aggregate wage rate,  $W_t$ , and a labor income tax,  $\tau$ , through the first-order condition:

$$\frac{\partial G(h_{it}, l_{it})}{\partial l_{it}} = (1 - \tau) W_t h_{it} . \qquad (2.5)$$

Assuming that G has a constant elasticity with respect to labor, I can write:

$$\frac{\partial G(h_{it}, l_{it})}{\partial l_{it}} = (1+\gamma) \frac{G(h_{it}, l_{it})}{l_{it}} , \qquad (2.6)$$

with  $\gamma > 0$  being the Frisch elasticity of labor supply. The expression of the composite good can be simplified, making use of (2.5) and (2.6):

$$x_{it} = c_{it} - G(h_{it}, l_{it}) = c_{it} - \frac{(1 - \tau)W_t h_{it} l_{it}}{1 + \gamma} .$$
(2.7)

Since the Frisch elasticity of labor supply is a constant parameter, the disutility of labor is always a constant fraction of labor income. Therefore, in both the household budget constraint and its utility function, only after-tax income enters, and neither hours worked nor productivity appears separately. This implies that, as suggested by Bayer et al. (2019), it can be assumed that  $G(h_{it}, l_{it}) = h_{it} \frac{l_{it}^{1+\gamma}}{1+\gamma}$  without further loss of generality, as long as we treat the empirical distribution of income as a calibration target. This functional form simplifies the household problem, as  $h_{it}$  drops out from the first-order condition, and all households supply the same number of hours  $l_{it} = L(W_t)$ . Total effective labor input,  $\int l_{it}h_{it}di$ , is therefore equal to  $L(W_t)$  since  $\int h_{it}di = 1$ . <sup>54</sup>

There are two types of household: workers and rentiers. Workers supply labor,  $L_t$ , in the production sector and have positive idiosyncratic labor productivity,  $h_{it} > 0$ . Their income is  $W_t h_{it} L_t$ . Rentiers have zero labor productivity,  $h_{it} = 0$ , but collect a proportional share of total profits generated from the production sector,  $\Pi_t$ . Idiosyncratic labor productivity  $h_{it}$  follows an exogenous Markov chain according to the following first-order autoregressive process and a fixed probability of transition between the worker and rentier state:

$$h_{it} = \begin{cases} exp(\rho_h log(h_{it-1}) + \epsilon_{it}^h) & \text{with probability } 1 - \zeta \text{ if } h_{it-1} \neq 0 \\ 1 & \text{with probability } \iota \text{ if } h_{it-1} = 0 \\ 0 & \text{else} \end{cases}$$
(2.8)

with  $\epsilon_{it}^h \sim N(0, \sigma_h)$ . The parameter  $\zeta \in (0, 1)$  is the probability that a worker becomes

<sup>&</sup>lt;sup>54</sup>More specifically, deriving the FOC with respect to labor of the households' optimization problem, making use of the new assumed  $G(h_{it}, l_{it})$ , and combining it with (2.5), we obtain  $l_{it} = [(1-\tau)W_t]^{\frac{1}{\gamma}} = L_t$ , since  $l_{it}$  depends only on aggregate variables.

a rentier and  $\iota \in (0,1)$  is the probability that a rentier becomes a worker. As stated above, workers that become rentiers leave the labor market  $(h_{it} = 0)$ , while rentiers that become workers are endowed with median productivity  $(h_{it} = 1)$ .<sup>55</sup> Workers and rentiers pay the same level of taxation, $\tau$ , on their income.

The asset market is incomplete: there are no Arrow-Debreu state-contingent securities; households self-insure themselves only through savings in a non-state contingent risk-free liquid asset,  $a_{it}$ , and they can borrow up to an exogenous borrowing limit. The household *i* budget constraint is:

$$c_{it} + a_{it+1} = \left(\frac{R_t^I}{\pi_t}\right) a_{it} + (1 - \tau) (W_t h_{it} L_t + \mathbf{I}_{h_{it}=0} \Pi_t) , \qquad a_{it} \ge \underline{a} , \qquad (2.9)$$

where  $\mathbf{I}_{h_{it}=0}$  takes value 1 if household *i* is a rentier, or 0 otherwise. On the left-hand side, we have households' expenditure, that is, consumption,  $c_{it}$  and 1-year-maturity savings,  $a_{it+1}$ . The right-hand side corresponds to households' total earnings, that is, the work/rent income net of taxes,  $(1-\tau)(W_th_{it}L_t + \mathbf{I}_{h_{it}=0}\Pi_t)$ , plus earnings (expenses) from savings (borrowings) in the liquid asset,  $\left(\frac{R_t^I}{\pi_t}\right)a_{it}$ .  $\pi_t$  is the gross inflation rate, while  $R_t^I$  is the gross nominal return on liquid assets. Borrowing households pay a "penalty",  $\omega_t^H$ , on the interest rate when they ask for a loan. Therefore,  $R_t^I$  has two definitions based on household *i*'s wealth:

$$R_t^I = \begin{cases} R_t & \text{if } a_{it} \ge 0\\ R_t(1+\omega_t^H) & \text{if } a_{it} < 0 \end{cases}$$
(2.10)

According to (2.7), total goods consumption can be expressed as  $c_{it} = x_{it} + \frac{(1-\tau)W_t h_{it} l_{it}}{1+\gamma}$ . By substituting this equation into (2.9), I can rewrite the household budget constraint in terms of composite consumption,  $x_{it}$ :

$$x_{it} + a_{it+1} = \left(\frac{R_t^I}{\pi_t}\right) a_{it} + (1 - \tau) \left(\frac{\gamma}{1 + \gamma} W_t h_{it} L_t + \mathbf{I}_{h_{it}=0} \Pi_t\right) , \qquad a_{it} \ge \underline{a} . \quad (2.11)$$

Equation (2.11) states that, in this model, what matters for households is the intertemporal allocation of composite consumption,  $x_{it}$ , rather than total goods consumption,  $c_{it}$ .

The model tracks only net household financial positions. This means that households cannot save and borrow simultaneously. Aggregate liquidity,  $A_t = \int a_{it} di$ , comprises household savings, and borrowings,  $B_t$ . In turn, households can save in three

<sup>&</sup>lt;sup>55</sup>Appendix 2.A contains details on the transition matrix for household productivity.

types of deposits that yield the same interest rate: deposits directed to commercial banks and used for household loans,  $D_t^H$ , deposits directed to investment banks and used for firm loans,  $D_t^F$ , and government bonds,  $D_t^G$ . Therefore, I can write the aggregate level of liquidity in the hands of households as:

$$A_t = D_t^H + D_t^F + D_t^G - B_t . (2.12)$$

Since these three saving instruments yield the same interest rate, households are completely indifferent to their portfolio composition.<sup>56</sup>

# 2.2.2 Financial intermediaries

Financial intermediaries collect deposits from households and promise returns equal to the risk-free interest rate. There are two types of intermediaries: commercial banks, which specialize in intermediations among households, and investment banks, which specialize in intermediation between households and the production sector.<sup>57</sup> These two types of financial intermediaries define the different types of financial frictions introduced in the model. First, I explain how commercial banks act before moving to investment banks.

# 2.2.2.1 Commercial Banks - Financial frictions on households

Commercial banks act similarly to the financial intermediaries in Cúrdia and Woodford (2016). I assume that banks can lend at most an amount that suffices to allow them to repay what they own to their depositors, considering the higher loan rate that households must pay when borrowing. This implies:

$$R_t(1+\omega_t^H)B_t = R_t D_t^H . (2.13)$$

Furthermore, when originating loans, commercial banks burn resources according to a non-decreasing, weakly convex function of the aggregate level of household debt,  $\Xi_t(B_t)$ . Therefore, end-of-the-period profits for commercial banks are:

$$\Pi_t^{com} = D_t^H - B_t - \Xi_t(B_t) .$$
 (2.14)

 $<sup>^{56}</sup>$ For sake of simplicity, I assume that the portfolio composition of any saver household is the same, and equal to the aggregate level of the three saving instruments.

<sup>&</sup>lt;sup>57</sup>This is obviously an abuse of terminology if we consider the real meaning of commercial and investment banks, but the idea is to give intuitive names to intermediaries that could clearly distinguish the functions of the two types of bank.

Using (2.13), (2.14) can be rewritten as:

$$\Pi_t^{com} = \omega_t^H B_t - \Xi_t(B_t) . \qquad (2.15)$$

Since commercial banks are in perfect competition, a bank chooses  $B_t$  that maximizes profits, leading to the F.O.C.:

$$\omega_t^H = \Xi_t'(B_t) , \qquad (2.16)$$

with the function  $\Xi_t(B_t) = \tilde{\Xi} B_t^{\eta^{FF}}$ , with  $\tilde{\Xi}$  and  $\eta^{FF}$  being calibrated parameters.

Result (2.16) directly links the penalty on household borrowings,  $\omega_t^H$ , to the aggregate level of household debt.<sup>58</sup> An increase in household indebtedness economy-wide results in a higher borrowing penalty, causing further depression in economic activities.

#### 2.2.2.2 Investment Banks - Financial frictions on firms

Investment banks collect deposits from households and promise returns equal to the real risk-free interest rate,  $R/\pi$ . For ease of display, I assume that the production sector is run by entrepreneurs, who are a mass-zero group of managers who are entitled to all the profits generated in the production sector and rebate them to rentier households. Investment banks and entrepreneurs are responsible for the other financial friction considered in this model. Following Bernanke et al. (1999), I assume a continuum of entrepreneurs, indexed by j. Entrepreneur j acquires capital,  $K_j$ , from capital producers at the end of period t which is used at time t + 1. To buy capital for production, entrepreneurs rely on two type of financing: internal financing (equity),  $N_j$ , and external financing (debt),  $D_i^F$ , borrowed from investment banks.

Entrepreneur j's balance sheet at period t + 1 is:

$$q_t K_{jt+1} = N_{jt+1} + D_{jt+1}^F , \qquad (2.17)$$

where q is the price of capital during the purchasing period.

One prerequisite for the financial accelerator to work is that entrepreneurs are not indifferent to the composition of their balance sheets; that is, external financing is more expensive than internal financing. To do so, I introduce a "Costly State Verification" (CSV) problem à la Townsend (1979) in which lenders (investment banks) must pay a

 $<sup>^{58}</sup>$ It is worth noting that empirical data usually show a higher degree of indebtedness in richer households, who can borrow more and at lower rates using as collateral their accumulated wealth (which, most of the time, is illiquid). However, we should keep in mind that this is a relatively simple model of net financial positions; therefore, it is impossible (and out of the scope of this study) to take track of such dynamics.

fixed auditing cost in order to observe the realized returns of borrowers (entrepreneurs). A relatively higher demand for debt increases auditing costs, resulting in a lower level of aggregate capital obtained for production.

Entrepreneurs repay investment banks with a portion of their realized returns on capital. In this framework, entrepreneurs are risk-neutral, whereas households are riskaverse. This implies a loan contract in which entrepreneurs absorb any aggregate risk on the realization of their profits. I also assume the existence of an idiosyncratic shock to entrepreneur j,  $\omega_j^F$ ,<sup>59</sup> on the gross return on aggregate capital,  $R^K$ . The idiosyncratic shock  $\omega^F$  has a log normal distribution of mean  $E(\omega^F) = 1$  that is i.i.d. across time and entrepreneurs, with a continuous and once differentiable c.d.f.,  $F(\omega^F)$ .<sup>60</sup>

The optimal contract for investment banks is:

$$\bar{\omega}_{jt+1}^F R_{t+1}^K q_t K_{jt+1} = Z_{jt+1} D_{jt+1}^F , \qquad (2.18)$$

where  $Z_j$  is the gross non-default loan rate and  $\bar{\omega}_j^F$  is the threshold value for entrepreneur j such that, for  $\omega_{jt+1}^F \geq \bar{\omega}_{jt+1}^F$ , entrepreneur j repays  $Z_{jt+1}D_{jt+1}$  to banks and retains  $\omega_{jt+1}^F R_{t+1}^K q_t K_{jt+1} - Z_{jt+1}D_{jt+1}$ . In the case of  $\omega_{jt+1}^F < \bar{\omega}_{jt+1}^F$ , instead, she cannot repay and defaults on her debt, obtaining nothing. Since entrepreneurs' future realizations of capital returns are only known by entrepreneurs ex-post, investment banks must pay a fixed auditing cost,  $\mu$ , to recover what is left of entrepreneur j's activity after default, obtaining  $(1 - \mu)\omega_{jt+1}^F R_{t+1}^K q_t K_{jt+1}$ .

Because of the optimal contract, investment banks should receive an expected return equal to the opportunity cost of their funds. By assumption, they hold a perfectly safe portfolio (i.e., they are able to perfectly diversify the idiosyncratic risk involved in lending), and the opportunity cost for investment banks is the real gross risk-free rate,  $R/\pi$ . It follows that the participation constraint for investment banks that must be satisfied in each period t + 1 is:

$$[1 - F(\bar{\omega}_{jt+1}^F)]Z_{jt+1}D_{jt+1} + (1 - \mu)\int_0^{\bar{\omega}_{jt+1}^F} \omega_j^F dF(\omega_j^F)R_{t+1}^K q_t K_{jt+1} \ge \frac{R_{t+1}}{\pi_{t+1}}D_{jt+1} , \quad (2.19)$$

where  $F(\bar{\omega}_j^F)$  is entrepreneur *j* default probability. Since financial markets are in perfect competition, (2.19) must hold with equality. The first term on the left-hand side

<sup>&</sup>lt;sup>59</sup>As noted by Christiano et al. (2014),  $\omega^F$  could be thought of as the idiosyncratic risk in actual business ventures: in the hands of some entrepreneurs, a given amount of raw capital is a great success, while in other cases may be not.

<sup>&</sup>lt;sup>60</sup>Appendix 2.B.1 provides analytical expressions for  $F(\omega^F)$  and other functions used in the following equations.

of (2.19) represents the revenues received by investment banks from the fraction of entrepreneurs that do not default, whereas the second term is what investment banks can collect from defaulting entrepreneurs after paying monitoring costs.

Following the notation proposed in Christiano et al. (2014), I combine (2.17), (2.18), and (2.19) to write the following relationship:

$$EFP_{jt+1} = f\left(\bar{\omega}_{jt+1}^F, LEV_{jt+1}\right)$$
, with  $f'(LEV_{jt+1}) > 0$ . (2.20)

where EFP is the "External Finance Premium" that Bernanke et al. (1999) define as the ratio between the return on capital and the real risk-free rate,  $R^{K}/(R/\pi)$ , and LEV = qK/N is entrepreneur j's leverage. The EFP can be considered a measure of the cost of external funds for the entrepreneur and, therefore, as a proxy for the strength of financial frictions. The  $(\bar{\omega}_{jt+1}^{F}, LEV_{jt+1})$  combinations that satisfy (1.10) define a menu of state (t+1)-contingent standard debt contracts offered to entrepreneur j, who chooses the contract that maximizes its objective.

In Appendix 2.B.2, I illustrate the entrepreneur j's optimization problem, which provides three important outcomes. First, the EFP increases monotonically with LEV. This means that entrepreneurs with a higher level of leverage pay a higher EFP. Second, the threshold value for entrepreneur j's default,  $\bar{\omega}_j^F$ , is endogenously defined by the EFP. Third, the fact that  $\bar{\omega}_j^F$  depends only on the aggregate variables  $(R, R^K \text{ and } \pi)$ implies that every entrepreneur will choose the same firm structure, that is,  $\bar{\omega}^F$  and LEV. Therefore, it is possible to drop superscript j in the notation and consider a representative entrepreneur.

The other fundamental equation for the functioning of this financial accelerator is the law of motion for entrepreneurs' equity, which is expressed as follows:

$$N_{t+1} = \gamma^F \left[ q_{t-1} R_t^K K_t - \frac{R_t}{\pi_t} D_t - \mu G(\bar{\omega}_t^F) q_{t-1} R_t^K K_t \right] .$$
 (2.21)

Equation (2.21) states that entrepreneurs' equity after the production process at time t is equal to the gross return on capital net of the loan repayment and auditing costs (which are borne by entrepreneurs because they are risk-neutral). Parameter  $\gamma^F$ represents the share of surviving entrepreneurs who bring their equity to the production process from one period to the next. Conversely, the share of entrepreneurs  $1 - \gamma^F$  dies and consumes equity at time t (we can think of this as entrepreneurial consumption). As explained by Carlstrom et al. (2016), this assumption avoids excessive entrepreneurs' self-financing in the long run.

Note that in (2.21) I did not included entrepreneurial labor, as usual in the literature

(e.g., Bernanke et al., 1999, Christiano et al., 2014). The assumption of entrepreneurial labor was introduced mainly to justify the initial amount of equity for new entrepreneurs that take the place of the dead ones. However, to keep the model as simple as possible, I follow Carlstrom et al. (2016), assuming that new entrepreneurs' initial equity comes from a lump-sum transfer from existing entrepreneurs. Even so, since the funding can be arbitrarily small and since only aggregate equity matters, this transfer can be neglected in equation (2.21).<sup>61</sup>

Alternatively, (2.21) can be written in a more compact form as:

$$N_{t+1} = \gamma^F \left[ 1 - \Gamma(\bar{\omega}_t^F) \right] R_t^K q_{t-1} K_t , \qquad (2.22)$$

where  $[1 - \Gamma(\bar{\omega}_t^F)]$  is the share of capital returns to which the non-defaulting entrepreneurs are entitled.<sup>62</sup> Equation (2.22), together with (2.20), explain this financial accelerator mechanism. Equation (2.20) states that an increase in entrepreneurs' leverage increases also the EFP. At the same time, (2.22) tells that an increase in the EFP increases  $\bar{\omega}^F$  as well, negatively affecting entrepreneurs' equity level for the next period and, therefore, impacting the aggregate leverage.

# 2.2.3 Intermediate-goods producers

Intermediate-goods producers adopt a standard Cobb-Douglas production function with constant returns to scale, employing aggregate capital, K, supplied by entrepreneurs and labor, L, from workers:

$$Y_t = z_t L_t^{\alpha} K_t^{1-\alpha} , \qquad (2.23)$$

where z is the Total Factor Productivity (TFP).

TFP follows a first-order autoregressive process of type:

$$log(z_t) = \rho_z log(z_{t-1}) + \epsilon_t^z , \qquad (2.24)$$

with  $\epsilon_t^z$  following a normal distribution with mean 0 and variance  $\sigma^z$ .

Intermediate goods producers sell their production to resellers at a relative price  $MC_t$ . Therefore, their profit optimization is given by:

$$\Pi_t^{IG} = M C_t z_t L_t^{\alpha} K_t^{1-\alpha} - w_t L_t - r_t^K K_t .$$
(2.25)

 $<sup>^{61}</sup>$ Bernanke et al. (1999) keep the share of income going to entrepreneurial labor at a very low level (on the order of 0.01), therefore neglecting this income sounds as a reasonable model simplification.

 $<sup>^{62}</sup>$ See Appendix 2.B.2

Since they are in perfect competition, their profit optimization problem returns the wage paid per unit of labor and the rent paid per unit of capital:

$$W_t = \alpha M C_t z_t \left(\frac{K_t}{L_t}\right)^{(1-\alpha)} , \qquad (2.26)$$

$$r_t^K = (1 - \alpha) M C_t z_t \left(\frac{L_t}{K_t}\right)^{\alpha} .$$
(2.27)

#### 2.2.4 Resellers

Resellers are agents assigned to differentiate intermediate goods and set prices. Price adjustment costs follow a Rotemberg (1982) setup, and resellers preserve entrepreneurial characteristics.<sup>63</sup> The demand for the differentiated good g is:

$$y_{gt} = \left(\frac{p_{gt}}{P_t}\right)^{-\eta} Y_t , \qquad (2.28)$$

where  $\eta > 1$  is the elasticity of substitution and  $p_g$  is the price at which good g is purchased.

Given (2.28) and the quadratic costs of price adjustment, the resellers maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t Y_t \left\{ \left( \frac{p_{gt}}{P_t} - MC_t \right) \left( \frac{p_{gt}}{P_t} \right)^{-\eta} - \frac{\eta}{2\kappa} \left( \log \frac{p_{gt}}{p_{gt-1}} \right)^2 \right\} , \qquad (2.29)$$

with a time-constant discount factor.<sup>64</sup>

The New Keynesian Phillips Curve (NKPC) derived from the F.O.C. for price setting is as follows:

$$log(\pi_t) = \beta E_t \left[ log(\pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] + \kappa \left( MC_t - \frac{\eta - 1}{\eta} \right) , \qquad (2.30)$$

where  $\pi_t$  is the gross inflation rate defined as  $\frac{P_t}{P_{t-1}}$ .

 $<sup>^{63}</sup>$ Bayer et al. (2019) make the further assumption that price setting is delegated to a mass-zero group of households (managers) that are risk neutral and compensated by a share in profits. Since in my model the whole production sector is run by entrepreneurs that, by assumption, are risk neutral and entitled to all the profits generated in this sector, I do no need to make this further assumption.

<sup>&</sup>lt;sup>64</sup>As explained by Bayer et al. (2019), only the steady state value of the discount factor matters in the resellers' problem, due to the fact that I calibrate to a zero inflation steady state, the same value for the discount factor of managers and households and approximate the aggregate dynamics linearly. This assumption simplifies the notation, since fluctuations in stochastic discount factors are virtually irrelevant.

#### 2.2.5 Capital producers

After production at time t, entrepreneurs sell depreciated capital to capital producers at a price  $q_t$ . They refurbish depreciated capital at no cost,<sup>65</sup> and uses goods as investment inputs,  $I_t$ , to produce new capital,  $\Delta K_{t+1} = K_{t+1} - K_t$ , subject to quadratic adjustment costs. Finally, they resell the newly produced capital to entrepreneurs before entering the next period (therefore still at price  $q_t$ ). The law of motion for capital producers is:

$$I_t = \Delta K_{t+1} + \frac{\phi}{2} \left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \delta K_t . \qquad (2.31)$$

where  $\delta$  is the depreciation rate for capital.

Then, they maximize their profits,  $q_t \Delta K_{t+1} - I_t$ , w.r.t. newly produced capital,  $\Delta K_{t+1}$ . This optimization problem delivers the optimal capital price:

$$q_t = 1 + \phi \frac{\Delta K_{t+1}}{K_t} \,. \tag{2.32}$$

Equation (2.32) ensures that if the level of aggregate capital increases over time, so does its price.

It follows that entrepreneurs' return on capital does not depend only on goods production, but also on fluctuations in capital price; since entrepreneurs buy capital at the end of the period, with the price of that period, they see their capital at the beginning of the next period appreciated (depreciated) if q increases (decreases). The gross return on capital employed at time t can be written as:

$$R_t^K q_{t-1} K_t = r_t^K K_t + q_t K_t (1 - \delta) , \qquad (2.33)$$

where the first term on the right-hand side is the marginal productivity of capital derived in (2.27), and the second term represents eventual capital gains (or losses) net of depreciation. I can rearrange and finally derive the gross interest rate of capital as:

$$R_t^K = \frac{r_t^K + q_t(1-\delta)}{q_{t-1}} .$$
(2.34)

<sup>&</sup>lt;sup>65</sup>The "no cost" assumption does not mean that  $\delta K$  is refurbished for free. Capital producers still need to buy the exact amount of I necessary to refurbish depreciated capital, but do not waste any further resources in this process. In fact, the law of motion for capital producers in the steady state (when  $\Delta K = 0$ ) is  $I = \delta K$ .

#### 2.2.6 Final-goods producers

Final-goods producers are perfectly competitive, buy differentiated goods from resellers at a given price, and produce a single homogeneous final good that is used for consumption, government spending, and investment. The optimization problem of final-goods producers is:

$$\max_{\{Y_t, y_{gt} \in [0,1]\}} P_t Y_t - \int_0^1 p_{gt} y_{gt} dg , \qquad (2.35)$$

subject to the following Constant Elasticity of Substitution (CES) function:

$$Y_t = \left(\int_0^1 (y_{gt})^{\left(\frac{\eta-1}{\eta}\right)} dg\right)^{\left(\frac{\eta}{\eta-1}\right)} . \tag{2.36}$$

From the zero-profit condition, the price index of the final good is:

$$P_t = \left(\int_0^1 (p_{gt})^{(1-\eta)}\right)^{\left(\frac{1}{1-\eta}\right)} .$$
 (2.37)

#### 2.2.7 Central bank

The central bank is responsible for the monetary policy. It sets the gross nominal risk-free interest rate,  $R_t$ , reacting to the deviation from steady state inflation, and engages interest rate smoothing. The Taylor-type rule employed by the central bank is as follows:

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^{\rho_R} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\rho_R)\rho_\pi} \epsilon_t^R , \qquad (2.38)$$

where  $\epsilon_t^R$  is the monetary policy shock defined as  $log(\epsilon_t^R) \sim N(0, \sigma_R)$ . The parameter  $\rho_R \geq 0$  rules the interest rate smoothing (if  $\rho_R = 0$ , the next-period interest rate depends only on inflation), whereas  $\rho_{\pi}$  captures the magnitude of the central bank's response to inflation fluctuations: the larger  $\rho_{\pi}$ , the stronger the central bank reaction (for the case limit  $\rho_{\pi} \to \infty$ , the inflation is perfectly stabilized at its steady state level).

# 2.2.8 Government

The government acts as fiscal authority. It determines the level of public expenditure,  $G_t$ , tax revenues,  $T_t$  and issuance of new bonds,  $D_{t+1}^G$ . Its budget constraint is given by:

$$D_{t+1}^{G} = \left(\frac{R_t}{\pi_t}\right) D_t^{G} + G_t - T_t , \qquad (2.39)$$

where  $T_t$  are the taxes collected from both workers and rentier households:

$$T_t = \tau \left[ \int W_t h_{it} L_t d\Theta_t(a, h) + \mathbf{I}_{h_{it}=0} \Pi_t \right] , \qquad (2.40)$$

and  $\Theta_t(a, h)$  the joint distribution of liquid assets and productivity across households on date t.

Bond issuance is regulated by the following rules:

$$\frac{D_{t+1}^G}{\bar{D}^G} = \left(\frac{D_t^G \frac{R_t}{\pi_t}}{\bar{D}^G \frac{\bar{R}}{\bar{\pi}}}\right)^{\rho_{gov}} .$$
(2.41)

Coefficient  $\rho_{gov}$  captures how fast the government wants to balance its budget. When  $\rho_{gov} \rightarrow 0$ , the government aims to balance its budget by adjusting spending. Instead, when  $\rho_{gov} \rightarrow 1$ , the government is willing to roll over most of the outstanding debt.

#### 2.2.9 Market clearing

The liquid asset market clears when:

$$\int a^*(a,h)\Theta_t(a,h)dadh = A_t , \qquad (2.42)$$

where  $a^*(a, h)$  is the optimal saving policy function of the household.

The market for capital clears for (2.31) and (2.32).

The labor market clears for (2.26).

Finally, good market clearing, which holds by Walras' law when other markets are clear, is defined as:

$$Y_t = C_t + G_t + I_t + C_t^E + \mu G(\bar{\omega}_t^F) R_t^K q_{t-1} K_t + \Upsilon_t , \qquad (2.43)$$

where on the left-hand side we have total output. On the right-hand side, apart from household consumption,<sup>66</sup> public expenditure and investments, we also find entrepreneurial consumption,  $C^E$  (due to dying entrepreneurs), auditing costs for investment banks, and resources used for household loans,  $\Upsilon_t = \Xi_t(B_t) + \omega_t^H B_t$ .<sup>67</sup>

<sup>&</sup>lt;sup>66</sup>Recall that household goods consumption consists of composite consumption and leisure. For aggregate quantities:  $C_t = X_t + \frac{L_t^{1+\gamma}}{1+\gamma}$ 

 $<sup>^{67}</sup>$ Similarly to Kaplan et al. (2018), the last two terms in (2.43) can be considered as expenses for "financial services".

#### 2.2.10 Numerical implementation

To solve the model, I follow the solution proposed in Bayer et al. (2019) and Luetticke (2021). As the joint distribution,  $\Theta_t$ , is an infinite-dimensional object (and therefore not computable), it is discretized and represented by its histogram, a finite-dimensional object. I solve the households' policy function using the Endogenous Grid-point Method (EGM) developed by Carroll (2006), iterating over the first-order condition and approximating the idiosyncratic productivity process using a discrete Markov chain with four states using the Tauchen (1986) method. The log grid for liquid assets comprises of 100 points. I solve for aggregate dynamics by first-order perturbation around the steady state, as in Reiter (2009). The joint distribution is represented by a bi-dimensional matrix (capital K does not display heterogeneity) with a total of 400 grid points, maintaining a sufficiently low computational time.

# 2.3 Calibration

The model is calibrated on the US economy, and because the focus is on conventional monetary policy, business cycle moments are targeted on the *Great Moderation* (i.e. 1985-2007). Periods in the model represent quarters; consequently, the following values for the calibrated parameters are intended quarterly, unless otherwise specified. Table 2.1 provides a list of calibrated parameters for the model, whereas Table 2.2 shows the model's effectiveness in replicating wealth distribution and business cycle moments.

#### 2.3.1 Households

For the households' utility function, I assume the coefficient of relative risk aversion  $\xi = 4$ , as in Bayer et al. (2019). I set the Frisch elasticity of labor supply  $\gamma = 1$ , in line with the results of Chetty et al. (2011). The intertemporal discount factor,  $\beta$ , is equal to 0.988, so that deposits in investment banks are sufficient to have a leverage for entrepreneurs of 2, the same value used by Bernanke et al. (1999) in their model. The borrowing limit,  $\underline{a}$ , is set such that 16% of households have a negative wealth position, a value in line with empirical data from the Survey of Consumer Finances (SCF) 1983–2007 (Luetticke, 2021).

The calibration of the productivity transition matrix, which determines how households move between the worker and rentier states, aims to provide a distribution of wealth consistent with empirical data. As in Luetticke (2021), I assume that the probability of becoming a rentier is the same for workers independent of their labor productivity, and that once they become workers again, they start with median productivity.

| Parameter         | Value           | Description                       |  |
|-------------------|-----------------|-----------------------------------|--|
| $\beta$           | 0.988           | Discount factor                   |  |
| ξ                 | 4               | Relative risk aversion            |  |
| $\gamma$          | 1               | Frisch elasticity of labor        |  |
| <u>a</u>          | -2.6            | Borrowing constraint              |  |
| l                 | 0.0625          | Prob. of leaving entr. state      |  |
| ζ                 | 0.00115         | Prob. become rentier              |  |
| $ ho_h$           | 0.98            | Persistence of idio. prod. shock  |  |
| $\sigma_h$        | 0.06            | SD if idio. prod. shock           |  |
| $\alpha$          | 0.7             | Labor share of production         |  |
| δ                 | 1.35%           | Depreciation rate                 |  |
| $\eta$            | 20              | Elasticity of substitution        |  |
| $\kappa$          | 0.09            | Price stickiness                  |  |
| $\phi$            | 7.5             | Adjustment cost of capital        |  |
| $\gamma^F$        | 0.986           | Entr. surviving rate              |  |
| $\rho_z$          | 0.95            | TFP shock persistence             |  |
| $\sigma_z$        | 0.87%           | TFP shock SD                      |  |
| R                 | 1.005           | Nominal int. rate                 |  |
| $ ho^R$           | 0.8             | Int. rate smoothing               |  |
| $ ho^{\pi}$       | 1.5             | Reaction to inflation             |  |
| $\sigma_R$        | 0.25% - $0.14%$ | Monetary shock SD                 |  |
| τ                 | 0.3             | tax rate                          |  |
| $ ho_{gov}$       | 0.86            | Auto-correlation of debt          |  |
| $\eta^{FF}$       | 51.62           | convex technology for HH<br>loans |  |
| Ĩ                 | $1.26e^{29}$    | comm. bank loans parameter        |  |
| $\mu$             | 0.12            | Auditing costs                    |  |
| $\sigma_{\omega}$ | 0.27            | SD of the id. shock on entr.      |  |

Table 2.1: Calibrated parameters

Table 2.2: Wealth distribution and business cycle moments

| Target             | Model | Target |
|--------------------|-------|--------|
| Gini wealth        | 0.78  | 0.78   |
| Share of borrowers | 0.16  | 0.16   |
| Top $10\%$ wealth  | 0.69  | 0.67   |

Business cycle moments

| Target                  | Model | Target |  |
|-------------------------|-------|--------|--|
| SD of Y (%)             | 1.38  | 1.38   |  |
| $\sigma^{I}/\sigma^{Y}$ | 3     | 3      |  |
| SD of C $(\%)$          | 0.64  | 0.98   |  |
| Corr. of Y with Y       | 1     | 1      |  |
| Corr. of I with Y       | 0.99  | 0.92   |  |
| Corr. of C with Y       | 0.99  | 0.92   |  |

Real GDP, investment and consumption are in logs. All data for business cycle moment analysis are processed with a H-P filter with  $\lambda = 1600$ . The calibrated moments for wealth distribution are the Gini index for wealth and the share of borrowing households. For business cycle moments, SD of Y and SD of I after a TFP shock.

The probability of leaving the rentier state is  $\iota = 0.0625$ , following the findings of Guvenen et al. (2014) on the probability of dropping out of the top 1% income group in the US. The probability of moving from the worker to the rentier state is set to  $\zeta = 0.00115$ , a value calibrated to obtain a Gini coefficient for wealth of 78% (in line with data from the SCF), which implies a share of rentier households equal to 1.8%. Regarding idiosyncratic income risk for labor productivity, I set autocorrelation  $\rho_h = 0.98$  and standard deviation  $\sigma_h = 0.06$ , as estimated by Bayer et al. (2019).

# 2.3.2 Financial intermediaries

I target the two spreads for the financial frictions to be equal to 2% p.a.. The reasons for this choice are twofold. First, it involves comparison purposes between the two scenarios. Second, they are the same values used in both Bernanke et al. (1999) and Cúrdia and Woodford (2016), allowing the model to be consistent with the existing literature.

Regarding commercial banks, I follow Cúrdia and Woodford (2016), assuming that a one-percent increase in the volume of credit increases the borrowing spread by one percentage point p.a.. Together with the targeted value  $\omega^H = 0.005$ , this implies  $\eta^{FF} = 51.62$  and  $\tilde{\Xi} = 1.26e^{29}$ .

The parameters concerning financial frictions on firms are in the ballpark of Bernanke et al. (1999) calibrations; therefore, the auditing cost is  $\mu = 0.12$  and the standard deviation of the idiosyncratic shock on the entrepreneur's returns is  $\sigma_{\omega} = 0.27$ , which are calibrated to have  $EFP_t = 1.005$  when the corporate leverage is 2. The share of surviving entrepreneurs,  $\gamma^F$ , is calibrated such that, at steady state, the equity level in (1.12) is equal to the equity implied by (2.20).

#### 2.3.3 Production Sector

The labor share of production (accounting for profits) and capital depreciation rate follow standard values in the literature and are set respectively to  $\alpha = 0.7$  and  $\delta =$ 1.35%. The mark-up is also standard, at 5%, which implies elasticity of substitution between goods varieties  $\eta = 20$ . The price stickiness parameter in the NKPC,  $\kappa = 0.09$ , is calibrated to generate a slope of the curve similar to the one that would arise in a model with sticky prices à la Calvo, with an average price duration of four quarters. The adjustment cost of capital parameter is calibrated to  $\phi = 7.5$  to obtain investmentto-output volatility of 3 after a TFP shock, a standard value for U.S. data, in a scenario where none of the frictions are active. The persistence of the TFP shock is  $\rho_z = 0.95$ , while the standard deviation is approximately  $\sigma_z = 0.09$ , standard measures in the literature.

#### 2.3.4 Central Bank and Government

Inflation at the steady state is set to 0, and the nominal (therefore real) interest rate on government bonds is 2%, a value in line with the real average federal funds rate for the Great Moderation period. I impose the same interest rate on all types of liquid savings (i.e., bonds and bank deposits); otherwise, households would choose to invest only in one asset or the other. Regarding the Taylor rule adopted by the Central Bank, the parameter for interest rate smoothing is  $\rho_R = 0.8$ , according to findings by Clarida et al. (2000), whereas the reaction to inflation fluctuations from the steady state is  $\rho_{\pi} = 1.5$ , which is a common value in the macroeconomic literature.

For comparison purposes, I apply two different magnitudes for the monetary policy shock in the two scenarios. The standard deviation of the monetary policy shock for the case with financial frictions on household borrowing ability is  $\sigma_R = 1\%$  p.a. (i.e., 0.25% quarterly). I then calibrate the shock for the other scenario to have a similar fluctuations in output between the two cases, delivering a parameter  $\sigma_R = 0.14\%$  quarterly. The persistence of the shock is zero, implying that it is a one-time innovation.

The taxes set by the government are proportional to labor income and profits, with a tax rate  $\tau = 0.3$  that targets the ratio of government spending to GDP to a standard value in the New Keynesian literature, approximately G/Y = 20%. Since I am using a fiscal policy rule similar to the one adopted by Bayer et al. (2019), I also follow their estimation and set  $\rho_B = 0.86$ . This implies that most of the fiscal dynamics goes through government debt, with public spending adjusting to re-stabilize debt to its steady state level.

# 2.4 Results

Before moving to inequality analyses, I examine aggregate fluctuations following the contractionary monetary policy shock. These results are not only useful for checking the consistency of my findings with the related literature, but also provide hints on differences at the idiosyncratic level between the two scenarios.

## 2.4.1 Aggregate fluctuations

As mentioned in Section 2.3.4, I assume two different magnitudes for monetary contraction. As a matter of fact, the two financial frictions are different in their mechanism



Figure 2.1: Impulse response to a monetary contraction for aggregate variables. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households.

complexity, with the one affecting firms being more complex and delivering a higher level of financial acceleration. Therefore, I believe that a fairer comparison would be between two shocks that have similar effects on the output, rather than between two identical shocks. However, as Appendix 2.D shows, considering the same magnitude for the monetary policy shock does not substantially change the main findings of this study.

Figure 2.1 shows the responses for output, Y, and investment,  $I.^{68}$  In the first year, the drop in output is almost identical in both scenarios, as intended. When considering active frictions on household borrowings, the recovery is more rapid and it slightly overshoots, whereas for frictions on firms the value remains below the steady state for the whole period considered in the figure. The investment level falls slightly more when considering financial frictions on firms, and its overshooting is considerably weaker and more short-lived than the alternative scenario considered in this analysis. Financial frictions on firms seem to generate a more pronounced impact on production-related variables even after a relatively weaker monetary contraction.

Consumption and labor dynamics are displayed in Figure 2.2. Goods consumption, C, falls relatively more on-impact when considering active frictions on households. In

<sup>&</sup>lt;sup>68</sup>More aggregate impulse responses can be found in Appendix 2.C.



Figure 2.2: Impulse response to a monetary contraction for aggregate variables. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households.

the first nine quarters, the goods consumption response is lower but then overshoots and overtakes IRF values for the comparative scenario. Recall that goods consumption can be expressed as a function of composite consumption, X, and labor, L.

The right-hand side graph in Figure 2.2 shows that labor dynamics are fairly similar in the two scenarios. Therefore, the difference in responses occurring in C is almost entirely due to what occurs at the composite consumption level. Under active financial frictions on firms, X falls on-impact and then strongly overshoots, beginning its reversion to the steady state value almost immediately. Conversely, composite consumption under active financial frictions on households exhibits a relatively much greater fall on impact. It follows that household borrowing frictions imply a relatively more powerful reaction at the consumption level. In this scenario, it takes the impulse response of X five quarters to overshoot, but then it keeps increasing for the remaining period considered in the figure. Approximately nine quarters after the shock, the value of composite consumption in this case exceeds that of frictions on the production sector. This is the same timing as that in the responses for goods consumption. This outcome is a consequence of the fact that, as mentioned above, labor dynamics are virtually similar in the two models. Given this result, and in light of the implications of equation (2.11), I focus on the dynamics of X rather than C to better understand the effects of



Figure 2.3: Impulse responses to a monetary contraction for wealth and consumption inequality.

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red dashed one when frictions are on households. The green line with asterisks is the percentage difference between the two curves using the lower curve as the base.

the two financial frictions on household consumption.<sup>69</sup> In Appendix 2.E, I analyze goods consumption as well, demonstrating that the main findings remain valid.

#### 2.4.2 Wealth and consumption inequality

Employing the Gini index of inequality for wealth and composite consumption, I now answer the initial question posed in this paper, that is, whether financial frictions affect household distribution of wealth and consumption differently after a contractionary monetary policy shock. The evolution of the indices for the two cases, the blue solid and red dashed lines, and the green line with asterisks representing the difference between the two curves in percentage terms, are shown in Figure 2.3.<sup>70</sup>

 $<sup>^{69}</sup>$ It must be noted that the visual difference in terms of "curve behavior" between responses for X and C is mostly due to the magnitude of the fluctuations. For instance, if we focus on the on-impact difference between the two models, we observe a similar differential in both composite and goods consumption, but the order of magnitude of the Y-axis in Figure 2.2 is different for these two variables.

<sup>&</sup>lt;sup>70</sup>As explained in the figure description, when comparing Gini indices between scenarios, the curve showing relatively lower values for the IRF is taken as the base for the differential calculation and, therefore, for the drawing of the green line with asterisks. For example, in the left-hand side graph in Figure 2.3 the green line with asterisks represents the percentage increase when switching from a scenario of active frictions on households (red dashed line) to active frictions on firms (blue solid line).

The Gini indices for both wealth and consumption increase following a contraction in monetary policy in both scenarios under consideration. The Gini index for wealth displays a hump-shaped trajectory, whereas the Gini index for consumption starts to revert instantaneously to its equilibrium value. However, this reversion process is longlasting for both indices. A noteworthy observation emerges when examining which type of friction results in a more pronounced fluctuation in inequality for a given variable. Examination of the Gini index for wealth indicates that financial frictions affecting firms lead to a more significant response. Conversely, when analyzing the Gini index for consumption, it is evident that financial frictions related to household borrowing have a greater influence. Therefore, it can be concluded that wealth distribution is more sensitive to frictions in the production sector of the economy, whereas the dispersion of consumption is more impacted by frictions that hinder households' capacity to borrow liquidity. In addition, the differential between the two scenarios displays a different pattern according to the frictions in place. When analyzing the wealth Gini index, the differential curve shows an increasing trend for the time span considered in the figure (five years), whereas the curve for the Gini index related to consumption displays a hump-shaped trajectory, gradually decreasing after approximately one year.<sup>71</sup>

Despite its utility, the Gini index falls short in revealing how the dispersion of wealth and consumption occurs across various individuals. Therefore, to understand the dynamics underlying the different responses in these indices, I first examine the distribution of households based on specific proportions of wealth held by individuals. Subsequently, I delve into an analysis of consumption patterns.

#### 2.4.3 Wealth dynamics

To investigate the dynamics of wealth inequality within households subsequent to the aggregate shock, I focus on three indicators that capture different aspects of household composition. These indicators encompass the share of households with borrowing obligations (i.e., those experiencing negative liquidity), the share of Hand-to-Mouth (HtM) households, and the percentage of wealth concentrated among the top 10% richest households in the distribution.

<sup>&</sup>lt;sup>71</sup>Note that the differential curve for the Gini index of wealth is expected to take on a hump-shaped form in the future, with reversion starting after 60 periods in the baseline model. On the other hand, the differential curve for the consumption Gini index, which initially exhibits a hump-shaped form in the short to medium term, begins to rise again after approximately 35 quarters. It is important to acknowledge, however, that this latter dynamic occurs not because of an unexpected rise of the Gini index in the future, but rather stems from a more rapid decline in the IRF for financial frictions on firms.

Before moving forward with the analysis of the findings, it is important to address the calibration of HtM households in this model. In standard TANK models,<sup>72</sup> the proportion of HtM (or *rule-of-thumb*) households is externally determined, usually implying by construction that those households have zero wealth and exclusively spend their current income. Within HANK economies, households choose their optimal level of wealth and consumption endogenously in each period. This dynamics decision-making process allows for variations in the proportions of HtM households following aggregate shocks. In the HANK model proposed by Kaplan and Violante (2014), households are defined as HtM whenever they choose to either have zero liquid wealth or to lie at the credit limit. Due to technicalities of my model constructions, I have opted to employ a different definition of HtM. First, because I am already studying the fluctuation of the share of borrower households, I will not include agents who have reached their borrowing limit when calculating the HtM share. Second, given that the grid used to compute the wealth distribution is not evenly spaced and contains several grid points in close proximity to the zero-wealth threshold, households are classified as HtM if they possess zero or near-zero wealth, that is, a positive amount of wealth that does not surpass the minimum possible quarterly labor income realization.<sup>73</sup>

The results of the IRFs for these three measures are displayed in Figure 2.4. Wealth held by the top 10% experiences a slightly higher increase on impact in the presence of active frictions on firms. However, the red dashed line, which represents fluctuations in case of active household borrowing frictions, surpasses the comparison scenario almost immediately and displays higher values for the remainder of the period depicted in the figure.

This dynamic is consistent with what happens both in terms of demand and supply of credit. Recall that in this model, all wealth held by households is liquid, and the richest top 10% holds almost 70% of all wealth in the economy. As a result, wealthier individuals benefit greatly from increases in the real interest rate, and they also serve as the main providers of credit. As shown in Figure 2.C.1 in Appendix 2.C, the real interest rate response is higher for financial frictions on households by construction. Therefore, richer households in this scenario are willing to take more credit because it yields relatively higher interests. Furthermore, firms in this model specification see their frictions shut off, resulting in a relative reduction in the cost of borrowing funds from households. It can be noted always in Figure 2.C.1 that the quantity of debt demanded by productive firms,  $D^F$ , shows relatively higher responses when financial

 $<sup>^{72}</sup>$ Such as Galí et al. (2007) or Bilbiie (2008)

<sup>&</sup>lt;sup>73</sup>The results remain almost unaffected when exclusively considering zero-wealth households as HtM.



Figure 2.4: IRFs for the share of borrowing households, HtM households and wealth held by the top 10%

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households.

frictions on households are present, expect on impact, where the IRF for the case of financial frictions on firms is slightly higher.

Wealth fluctuations at the top of the distribution, then, would suggest a higher wealth inequality in the case of financial frictions on households throughout the majority of the initial five-year period, but that is not the case according to the Gini coefficient displayed in Figure 2.3. Therefore, this highlights the significance of the main shifts happening at the lower end of the distribution, where low-wealth households and borrowers are situated.

According to Figure 2.4, households that are either HtM or resorting to debt increase in number after a monetary contraction. Interestingly, the proportion of borrowers shows a more pronounced increase in cases of frictions related to firm borrowing, whereas the percentage of HtM households exhibits a relatively higher increase in cases of frictions related to household borrowing.

Household behavior near the zero-wealth threshold and fluctuations in the household loan rate provide a plausible explanation for these dynamics. Following a contractionary monetary shock, households experience deteriorating labor conditions, particularly affecting poorer households the most, as they heavily rely on labor income for consumption and debt repayment. Consequently, an increasing number of households find themselves at the bottom of the wealth distribution, either depleting their savings or accumulating more debt to smooth consumption. Financial frictions that exclusively impact productive firms lead to a scenario where borrowing for households becomes comparatively more affordable than when frictions directly affect households. This is due to the fixed loan premium,  $\omega^H$ , in the former case, while it increases in the latter case, leading to higher household loan costs. Consequently, more households opt for borrowing when facing frictions on firms, while more remain near the zero-wealth threshold due to the deterrent effect of higher loan rates in presence of household borrowing frictions.

This interpretation aligns with the fluctuations observed in the Gini index for wealth in both scenarios, emphasizing the significance of the lower end of the distribution in generating disparities between the two cases. Moreover, it could also account for the dynamics observed at the consumption level. Households rely on their borrowing capacity to ensure a consistent level of consumption. If a larger share of agents are unable to borrow due to higher loan rates, this could result in a diminished ability to smooth consumption for a greater number of households (in comparison to a scenario with a fixed loan rate), consequently leading to a relatively higher dispersion in consumption. Nevertheless, to determine the plausibility of this intuition, I proceed with an analysis of consumption dynamics.

#### 2.4.4 Consumption dynamics

The decomposition of the impulse response of the aggregate consumption into average consumption responses for specific shares of the population provides valuable insights into the diverse consumption patterns observed after a contractionary monetary shock.<sup>74</sup> The consumption decomposition for the case of financial frictions of firms is depicted on the left-hand side of Figure 2.5, while the right-hand side illustrates the counterfactual scenario of frictions on household borrowing.

Households situated at the top of the wealth distribution tend to exhibit higher consumption levels when financial frictions on households are present, partially explaining why the Gini index for consumption increases more in this case. Note, however, that when we compare the on-impact response of the top 10% with that of the borrowers' share, the magnitude of the former is significantly lower in both frictions scenarios, although more persistent over time. This trend is in line with the fact that affluent

<sup>&</sup>lt;sup>74</sup>I choose to use average consumption fluctuations instead of absolute consumption fluctuations because I think they are better suited for comparisons between the two cases, but also for comparisons within the same case with respect to aggregate consumption fluctuations. Note that aggregate consumption is the case limit where the average consumption for the whole population is considered.



Figure 2.5: Average consumption fluctuation flor different shares of households. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise.

households are witnessing an uptick in their earnings, predominantly stemming from interest earned on savings, yet possess a lower MPC than their less wealthy counterparts. It is important to remember that under conditions of financial frictions on households, the real interest rate is higher by design. Therefore, it is expected to witness heightened consumption among the top 10% in this scenario, as their main source of income is financial.

Except differences in magnitudes mentioned above, the behavior of consumption at the top 10% is fairly similar. On the other hand, this does not seem the case when focusing on the lower half of the distribution. On impact, the decline in consumption for borrower households is roughly three times greater when financial frictions on households are present, compared to the situation for frictions on firms. This ratio diminishes as the percentage of households considered in the lower half of the wealth distribution increases, yet it remains significant. Upon analyzing the mean consumption of the whole bottom half of the population, it is evident that the immediate decline with household financial frictions is more than twice as high as in the alternative situation.<sup>75</sup>

Moreover, the contrast between the two model specifications reveals notable disparities in the persistence of the IRFs in the bottom half as well. Poorer households facing

 $<sup>^{75}</sup>$ Greater household share values encompass the consumption of lesser shares. This implies that the mean consumption of the lowest 50% also encompasses the consumption of the lowest 30%, which in turn encompasses the consumption of borrowers.

borrowing frictions not only witness a significant decline in average consumption at the outset, but also a notably sluggish convergence toward the steady state level. For instance, let us consider the dynamics for borrower households. In case of frictions only on the production sector, their average consumption overshoots after approximately one year. Conversely, if households confront constraints in their borrowing activities, their consumption never rebounds within the time-frame analyzed in Figure 2.5, that is, five years.<sup>76</sup>

The dynamics of consumption in proximity to the zero-wealth threshold can be explained by the behavior of the borrowing penalty,  $\omega^H$ . Under financial frictions on firms,  $\omega^H$  is fixed at its stead-state level. Consequently, a higher proportion of households choose to borrow money to ensure a more stable consumption pattern compared to the counterfactual situation where household borrowing is more expensive. Moreover, since they do not face frictions on borrowings, their consumption levels recover at a faster pace and are consistent with labor dynamics, given that their primary income source is labor earnings. Conversely, when there are financial frictions on households,  $\omega^H$  rises following a monetary tightening and gradually returns to its original level, as illustrated in Figure 2.C.1 in Appendix 2.C. Similarly, the IRF for consumption displays a slow recovery process.

Differences in consumption responses offer insight into the dynamics near the zerowealth boundary in Figure 2.4 and, therefore, in terms of consumption and wealth inequality. In presence of financial frictions on households, a greater number of households opt to remain HtM, while fewer households choose to borrow with respect to the counterfactual scenario, due to the fluctuation of the borrowing penalty. This results in reduced wealth inequality among the population, as a larger proportion of households opt not to fall to the very bottom of the wealth distributions, unlike the situation with a fixed  $\omega^H$ . Conversely, an increase in HtM households leads to decreased consumption smoothing. In addition, individuals who choose to borrow end up consuming even less, as they must repay a higher interest rate, leading to greater consumption inequality compared to when there are frictions in the production sector. This clarifies both the lower Gini index for wealth and the higher Gini index for consumption in Figure 2.3 when households encounter frictions on borrowing.

# 2.4.5 Consumption decomposition

In order to assess whether the rise in the household loan rate is the primary factor influencing the different household behavior near the zero-wealth threshold in the two

<sup>&</sup>lt;sup>76</sup>Extending the span for IRFs, the overshooting takes place roughly 55 quarters later.



Figure 2.6: Consumption decomposition for relevant prices

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The graph on the left-hand side represents the decomposition for the case of friction on firms, while the one on the right-hand side represents the case of friction on household borrowing.

comparative scenarios considered so far, or if there are other significant factors at play, I resort to the approach outlined by Luetticke (2021). In order to decompose the monetary transmission mechanism, I express the total composite consumption as a series of household policy functions that are determined by the equilibrium prices relevant to household consumption decisions, based on the budget constraint (2.9). The household policy functions are represented by the sequence  $\{\Omega_t\}_{t\geq 0}$ , where  $\Omega_t = \left\{\frac{R_t^I}{\pi_t}, W_t, \Pi_t\right\}$ . Therefore, the aggregate composite consumption can be written as:

$$X_t\left(\{\Omega_t\}_{t\geq 0}\right) = \int x_t\left(a,h;\{\Omega_t\}_{t\geq 0}\right)d\Theta_t,\tag{2.44}$$

where  $\Theta_t(da, dh; \{\Omega_t\}_{t\geq 0})$  is the joint distribution of liquid assets and idiosyncratic labor productivity. Totally differentiating (2.44), I decompose the total response to monetary shocks into parts explained by each single price.<sup>77</sup> Result are shown in Figure 2.6.

The profit contribution is virtually identical in both cases. The wage contribution exhibits a comparable pattern, albeit with greater strength in terms of magnitude when financial frictions are present within firms. This particular characteristic aligns with

<sup>&</sup>lt;sup>77</sup>A similar decomposition can be found also in Kaplan et al. (2018) and Auclert (2019).



Figure 2.7: Consumption decomposition for relevant prices

Note: monetary shock  $\epsilon^R = 0.0025$ . The graph on the left-hand side represents the decomposition considering the household borrowing spread in the liquidity return. In the graph on the right-hand side, I consider the same case with the borrowing penalty  $(\omega_t^H)$  as an individual variable.

the concept of these frictions impacting the primary source of income for a considerable portion of the population, particularly those who are at the bottom of the distribution. The most noteworthy distinction between the two scenarios, as expected, lies in the shape and magnitude of the contribution from liquidity return. In case of financial frictions on firms, it is positive and hump-shaped. The exclusive liquidity of wealth in this model could explain this pattern, as rich households (who possess more liquid assets) benefit greatly from an increase in liquid return and compensate for the impact on poorer households. Conversely, when active financial constraints are imposed on households, the liquidity return contribution is notably negative for more than one year, with a magnitude and persistence more in line with that of the aggregate composite consumption.

It is important to highlight that in Figure 2.6, when referring to "liquidity return", I also consider the household loan spread for borrowing households,  $(1 + \omega^H)$ , and remember that  $\omega^H$  is free to vary over time in the right-hand side of the figure, while it is fixed in the comparative scenario. Therefore, in order to evaluate the significance of this spread's impulse response, a further breakdown of the liquidity return is conducted, treating  $\omega^H$  as an individual price. The outcomes of this analysis are presented in Figure 2.7.

When analyzed independently, the borrowing penalty emerges as a key factor in reducing consumption for active frictions on households. Interestingly, its impact on the overall composite consumption decline is found to be even more significant than the effect of wages for a period lasting at least two years following the aggregate shock. Hence, it can be deduced that the moving household borrowing penalty plays a crucial role in shaping aggregate consumption, supporting the previous notion that most differences in wealth and consumption inequality between the two cases can be explained by household behavior surrounding the zero-wealth threshold.

Referring to the dichotomy proposed by Kaplan et al. (2018), it becomes possible to depict the results in terms of direct and indirect effects of monetary policy on household consumption. The presence of financial frictions within the production sector has a more significant influence on the indirect effects, particularly those associated with labor income, when compared to the counterfactual scenario. The wage component (yellow circled line) exhibits greater strength and persistence in the presence of active frictions and firms. This channel primarily contributes to the decline in composite consumption for this scenario, since profits and liquidity return contributions exert a positive influence for the period considered in Figure 2.6. On the other hand, financial frictions related to fluctuating household loan rates reassess the importance of direct effects in depressing consumption after a monetary contraction, primarily through changes in the borrowing penalty. It is important to underline, however, that significant indirect effects still exist in this context. The wage contribution remains a substantial factor in consumption reduction even with financial frictions on households.

In relation to direct effects, the behavior of the liquidity return contribution (net of the borrowing premium) in terms of response shape varies significantly. On impact, the contribution is marginally higher in the scenario in which there are frictions on firm borrowing. This can be observed by comparing the left-hand graph in Figure 2.6 with the right-hand one in Figure 2.7. The two responses reach their peak around the same time, with the former peaking in the third quarter and the latter in the fourth quarter. However, the rate of reversion differs significantly between the two. Reversion is much faster under firm financial frictions, whereas it is much slower under household frictions.<sup>78</sup> At first glance, this result may seem counter-intuitive. Financial frictions affecting household borrowing actually enhance the positive contribution of liquidity return in the long run, whereas the opposite happens when these frictions are shut off. Nevertheless, as explained in Section 2.4.3, this outcome is a logical consequence of the

<sup>&</sup>lt;sup>78</sup>Extending the duration of the IRFs reveals that consumption undershooting occurs approximately 24 quarters after the shock under financial frictions on firms. In the comparative scenario, even after 100 periods, the response value remains higher than the initial impact value.

interplay between the demand and supply of borrowings in the production sector. First, most funds channeled to firms originate from the top 10% of households, who, as per the model's construction, are not impacted by the increase in the loan rate.<sup>79</sup> Second, under financial frictions on firms, entrepreneurs tend to resort to higher levels of debt initially, but subsequently aim to minimize their debt exposure due to higher costs associated with financial frictions. Therefore, in the last case, there is a faster decrease in firms' demand for borrowing. Conversely, under active frictions on households, entrepreneurs exhibit a relatively stronger inclination toward debt utilization, resulting in a slower reduction in their demand for funds. Therefore, this enduring dynamic also appears to have long-lasting effects on aggregate composite consumption, primarily through the contribution of liquidity returns on the latter.

The robustness of the dynamics related to both Gini indices fluctuations and agents' behavior around the zero-wealth threshold appears to be unaffected by varying risk aversion levels among households. This is demonstrated in Figure 2.F.7 in Appendix 2.F, where a risk aversion value of  $\xi = 2$  is considered in households' preferences. Similarly, in this particular case, the Gini index of wealth is relatively higher for active frictions on firms compared with the counterfactual scenario, whereas the opposite holds true for the Gini index of composite consumption, with higher values for financial frictions on households. The breakdown of aggregate consumption depicted in Figure 2.F.8 confirms that indirect effects are magnified under financial frictions on firms, whereas financial frictions on  $\omega^{H}$ .

Additionally, in Appendix 2.F, I conduct robustness tests for various model specifications. I examine whether the outcomes remain consistent under extreme calibrations, such as a scenario with no quadratic costs for capital producers ( $\phi = 0$ ), a situation where the government fixes its bond issuance at the steady state level ( $\rho_{gov} = 0$ ), and a case where the government adjusts bond issuance also according to tax revenues. The key results appear to be robust across these alternative specifications.

# 2.5 Concluding remarks

Employing a full-fledged HANK model that encompasses two distinct financial frictions influencing different agents within the economy, I illustrate that these frictions yield varying effects on households in terms of wealth and consumption dynamics. When confronted with a contractionary monetary policy shock, both wealth and consumption inequality increase, regardless of whether the financial frictions affect firms or

<sup>&</sup>lt;sup>79</sup>Note that this model assumes net financial positions for household wealth. Therefore, households are restricted from simultaneously saving and borrowing funds.

households. Nevertheless, notable differences emerge between these two scenarios. In the presence of active financial frictions on firms, the Gini index for wealth exhibits a comparatively higher level. Conversely, when active friction is imposed on household borrowing, a greater dispersion in consumption arises.

This divergence in behavior can be attributed to the impact of these two frictions on the distribution of households around the zero-wealth threshold, which determines whether they fall into the category of borrowers or savers. Specifically, when considering only frictions related to the productive sector of the economy and keeping the consumer loan spread rate constant, a larger proportion of households opt for borrowing as it becomes relatively more affordable compared to an alternative scenario. This implies a rise in the Gini index of wealth inequality. Households are more inclined to transition towards the lower end of the wealth distribution, where they become borrowers. By doing so, they are able to better smooth their consumption due to the fixed loan spread. On the other hand, when frictions related to households' borrowing ability exist, households are discouraged from borrowing due to the rising borrowing penalty. As a result, a larger share of households opt to remain HtM, consuming only what they receive from their income. This leads to an increase in consumption dispersion but a relatively lower level of wealth inequality, as the percentage of borrowing households is comparatively reduced. The analysis of two distinct financial frictions in this research indicates differing consequences on the direct and indirect effects of monetary policy on household consumption. Specifically, frictions related to firms appear to have a greater influence on indirect effects, while frictions associated with households have a stronger impact on direct effects.

Despite using a relatively simple model, this study provides interesting guidance in terms of the redistribution effects of monetary policy according to a possible state of the economy. Even though wealth redistribution should not be a formal target for central banks, policymakers have undoubtedly been concerned with it over the last years, and recent findings have proven that this concern is also important from a macro-modeling point of view.

Nonetheless, there may be several dimensions across which this study could be extended to future research. For instance, this is a model concerning conventional monetary policy, but after the Great Financial Crisis, interest rates hit the zero lower bound for a prolonged period, forcing central banks to resort to unconventional policies. Another important topic in current theoretical macroeconomics is the heterogeneity of productive firms, which could be an extremely interesting extension of this model, that is, how productive sector heterogeneity is important when certain frictions are present. In addition, I assume that the friction generated by commercial banks is exclusively due to the resources they waste for their operations, with the intent of keeping the model as simple as possible. However, Cúrdia and Woodford (2016) also assume that a certain quantity of household debt is subject to default. Finally, the model proposed in this study assumes that households can only save through liquid assets. Empirical evidence indicates that affluent households typically hold the majority of their wealth in the form of illiquid assets. The introduction of illiquid assets as a method of accumulating wealth for individuals could have significant implications, particularly in relation to consumption, for the wealthiest segment of the population.

# References

- Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. The Quarterly Journal of Economics, 109(3):659–684.
- Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500.
- Auclert, A. (2019). Monetary policy and the redistribution channel. American Economic Review, 109(6):2333–67.
- Auclert, A., Bardóczy, B., and Rognlie, M. (2023). MPCs, MPEs, and Multipliers: A Trilemma for New Keynesian Models. *The Review of Economics and Statistics*, 105(3):700–712.
- Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the sequencespace jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408.
- Bayer, C. and Luetticke, R. (2020). Solving heterogeneous agent models in discrete time with many idiosyncratic states by perturbation methods. *Quantitative Economics*, 11:1253–1288.
- Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, 87(1):255–290.
- Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.
- Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.
- Brunnermeier, M. K., Eisenbach, T. M., and Sannikov, Y. (2012). Macroeconomics with financial frictions: A survey. Working Paper 18102, National Bureau of Economic Research.
- Carlstrom, C. T., Fuerst, T. S., and Paustian, M. (2016). Optimal contracts, aggregate risk, and the financial accelerator. *American Economic Journal: Macroeconomics*, 8(1):119–47.

- Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.
- Chatterjee, S., Corbae, D., Nakajima, M., and Rios-Rull, J.-V. (2007). A quantitative theory of unsecured consumer credit with risk of default. *Econometrica*, 75(6):1525–1589.
- Chetty, N., Weber, A., Guren, A. M., and Day, M. (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. Technical report.
- Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk Shocks. American Economic Review, 104(1):27–65.
- Clarida, R., Galí, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, 115(1):147–180.
- Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.
- Colciago, A., Samarina, A., and de Haan, J. (2019). Central bank policies and income and wealth inequality: A survey. *Journal of Economic Surveys*, 33(4):1199–1231.
- Cúrdia, V. and Woodford, M. (2016). Credit Frictions and Optimal Monetary Policy. Journal of Monetary Economics, 84(C):30–65.
- Faccini, R., Lee, S., Luetticke, R., Ravn, M., and Renkin, T. (2024). Financial frictions: Macro vs micro volatility. Denmarks Nationalbank Working Papers 200.
- Farhi, E. and Tirole, J. (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93.
- Ferlaino, F. (2024). Does the financial accelerator accelerate inequalities? *Working* paper.
- Fernández-Villaverde, J., Hurtado, S., and Nuño, G. (2023). Financial frictions and the wealth distribution. *Econometrica*, 91(3):869–901.
- Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–357.

- Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.
- Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. Journal of Monetary Economics, 58(1):17–34. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.
- Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. American Economic Review, 102(4):1692–1720.
- Greenwood, J., Hercowitz, Z., and Huffman, G. (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review*, 78(3):402–17.
- Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467.
- Guvenen, F., Kaplan, G., and Song, J. (2014). The glass ceiling and the paper floor: Gender differences among top earners, 1981-2012. Working Paper 20560, National Bureau of Economic Research.
- Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691.
- Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5):953–969.
- Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. American Economic Review, 95(3):739–764.
- Imrohoroğlu, A. (1989). Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy*, 97(6):1364–1383.
- Jordà (2005). Estimation and inference of impulse responses by local projections. *The American Economic Review*, 95(1):161–182.
- Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. American Economic Review, 108(3):697–743.
- Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239.
- Krusell, P. and Smith, Jr., A. (1998). Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy, 106(5):867–896.
- Lee, S., Luetticke, R., and Ravn, M. (2020). Financial frictions: Macro vs micro volatility. CEPR Discussion Papers 15133.
- Luetticke, R. (2021). Transmission of monetary policy with heterogeneity in household portfolios. *American Economic Journal: Macroeconomics*, 13(2):1–25.
- Nakajima, M. and Ríos-Rull, J.-V. (2019). Credit, Bankruptcy, and Aggregate Fluctuations. Working Papers 19-48, Federal Reserve Bank of Philadelphia.
- Piketty, T. (2017). Capital in the Twenty-First Century. Harvard University Press.
- Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. Journal of Economic Dynamics and Control, 33(3):649–665.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.
- Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. The American Economic Review, 71(3):393–410.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.

## Appendix

# 2.A Idiosyncratic productivity process and the joint distribution

Households can be workers, with productivity h > 0, or rentiers, , with h = 0, which means that they do not earn labor income but only profit income. Furthermore, I assume that there are three possible productivity realizations for workers: high productivity,  $h^H$ , median productivity,  $h^M$ , and low productivity,  $h^L$ . The Markov process generates the following transition matrix:

with probabilities, p, determined using the Tauchen method. I follow other studies using this household distribution framework, such as Bayer et al. (2019) and Luetticke (2021), and assume that rentiers who become workers are endowed with the median productivity level (h = 1).

At the steady state, a joint distribution of households exists according to their wealth level, a, and their productivity, h. This joint distribution can be represented by the bi-dimensional matrix as follows:

#### wealth a

where  $H_{1,1}$  is the share of households with the lowest level of wealth and labor productivity (except for the last state  $h_m = 0$ , since in this model they are rentiers), and  $\int H dadh = 1$ . As the vector indicating possible household wealth levels is composed of 100 entries, this joint distribution matrix comprises 400 grid points ( $a_n = 100$  and  $h_m = 4$ ).

#### 2.B Investment banks optimal contract

#### 2.B.1 Idiosyncratic shock on return on capital

Following Bernanke et al. (1999), I assume that the Idiosyncratic shock  $\omega^F$  is distributed log-normally. i.e.  $\omega^F \in [0, +\infty)$ .<sup>80</sup> Using results from Appendix A.2 in Bernanke et al. (1999) I can write  $F(\omega^F)$ ,  $\Gamma(\omega^F)$  and  $G(\omega^F)$  in the analytical expressions that I use in my code to solve the model:

$$F(\omega^F) = \Phi\left[\left(\log(\bar{\omega^F}) + \frac{1}{2}\sigma_{\omega^F}^2\right) / \sigma_{\omega^F}\right] , \qquad (2.B.1)$$

$$\Gamma(\omega^F) = \Phi\left[\left(\log(\bar{\omega}^F) - \frac{1}{2}\sigma_{\omega^F}^2\right) / \sigma_{\omega^F}\right] + \bar{\omega}^F \left\{1 - \Phi\left[\left(\log(\bar{\omega}^F) + \frac{1}{2}\sigma_{\omega^F}^2\right) / \sigma_{\omega^F}\right]\right\},\tag{2.B.2}$$

$$G(\omega^F) = \Phi\left[\left(\log(\bar{\omega}^F) + \frac{1}{2}\sigma_{\omega^F}^2\right) / \sigma_{\omega^F} - \sigma_{\omega^F}\right] , \qquad (2.B.3)$$

with  $\Phi(\cdot)$  being the normal cumulative distribution function and  $\sigma_{\omega^F}$  the standard deviation of the idiosyncratic shock on entrepreneurs' return on capital.

## 2.B.2 Investment banks' participation constraint and entrepreneur *j*'s optimization problem

After substituting (2.18) and (2.17) into (2.19), I obtain:

Divide everything by  $R_{t+1}^R q_t K_{jt+1}$ :

<sup>&</sup>lt;sup>80</sup>Note that other kinds of distribution with values greater or equal to 0 could be used as well. Here I choose to adapt the same distribution to give a sense of continuity between the two studies.

$$\frac{R_{t+1}^K}{R_{t+1}} \left( \left[ 1 - F(\bar{\omega}_{jt+1}^F) \right] \bar{\omega}_{jt+1}^F + (1-\mu) \int_0^{\bar{\omega}_{jt+1}^F} \omega_j^F dF(\omega_j^F) \right) = \left( 1 - \frac{N_{jt+1}}{q_t K_{jt+1}} \right) . \quad (2.B.5)$$

Following the notation used in Bernanke et al. (1999) and Christiano et al. (2014):

$$\Gamma(\bar{\omega}_{j}^{F}) \equiv \int_{0}^{\bar{\omega}_{j}^{F}} \omega_{j}^{F} dF(\omega_{j}^{F}) + \bar{\omega}_{j}^{F} \int_{\bar{\omega}_{j}^{F}}^{\infty} dF(\omega_{j}^{F}) , \qquad \mu G(\bar{\omega}_{j}^{F}) \equiv \mu \int_{0}^{\bar{\omega}_{j}^{F}} \omega_{j}^{F} dF(\omega_{j}^{F}) ,$$

$$(2.B.6)$$

where  $\Gamma(\bar{\omega}_j^F)$  is the expected gross share of profits going to the lender and  $\mu G(\bar{\omega}_j^F)$  is the expected monitoring cost paid by the lender.  $\Gamma(\bar{\omega}_j^F)$  can be rewritten as:

$$\Gamma(\bar{\omega}_j^F) = G(\bar{\omega}_j^F) + \bar{\omega}_j^F \left[1 - F(\bar{\omega}_j^F)\right] .$$
(2.B.7)

I can now use (2.B.6) and (2.B.7) in (2.B.5) and rearrange to finally obtain:

$$\frac{R_{t+1}^K}{\left(\frac{R_{t+1}}{\pi_{t+1}}\right)} = \frac{1}{\Gamma(\bar{\omega}_{jt+1}^F) - \mu G(\bar{\omega}_{jt+1}^F)} \left(1 - \frac{N_{jt+1}}{q_t K_{jt+1}}\right) , \qquad (2.B.8)$$

where  $\Gamma(\bar{\omega}_{jt+1}^F) - \mu G(\bar{\omega}_{jt+1}^F)$  is the share of entrepreneur j's profits going to the lender (as loan repayment), net of auditing costs.

Equation (2.B.8) is the complete version of (2.20), which explain the function underlying  $f(\bar{\omega}_{jt+1}^F, LEV_{jt+1})$ . For a higher level of entrepreneur leverage, the EFP increases, raising the return on capital. However, it also increases the probability of an entrepreneur's default, thereby increasing the net share of profit demanded by investment banks as loan repayment, resulting in higher financing costs for entrepreneurs. To see in detail how this mechanism works, I show the entrepreneur j's optimization problem below.

According to the optimal contract set by investment banks, entrepreneur j's expected return can be expressed as:

$$E_t \left\{ \int_{\bar{\omega}_{jt+1}}^{\infty} \omega_j^F dF(\omega_j^F) R_{t+1}^K q_t K_{jt+1} - (1 - F(\bar{\omega}_j^F)) R_{t+1}^K q_t K_{jt+1} \right\} , \qquad (2.B.9)$$

with expectations taken with respect to the realization of  $R_{t+1}^{K}$ . The first term of (2.B.9) represents the entrepreneur's profit when she does not default on debt, while

the second term is the amount of profits that she uses to repay the lender. Following the notation used above, and considering that the entrepreneur's return is subject to the participation constraint (2.19), I write entrepreneur j's optimal contracting problem as:

$$\max_{\{K_{jt+1},\bar{\omega}_{jt+1}^{F}\}} E_{t} \left\{ \left[ 1 - \Gamma(\bar{\omega}_{jt+1}^{F}) \right] R_{t+1}^{K} q_{t} K_{jt+1} \right\} ,$$

$$(2.B.10)$$

$$s.t. \quad \frac{R_{t+1}}{\pi_{t+1}} (q_{t} K_{jt+1} - N_{jt+1}) = \left[ \Gamma(\bar{\omega}_{jt+1}^{F}) - \mu G(\bar{\omega}_{jt+1}^{F}) \right] R_{t+1}^{K} q_{t} K_{jt+1} .$$

Deriving F.O.C. I obtain:

w.r.t. 
$$\omega_{jt+1}^F$$
:  $-\Gamma'(\bar{\omega}_{jt+1}^F) + \lambda_{jt+1} \left[ \Gamma'(\bar{\omega}_{jt+1}^F) - \mu G'(\bar{\omega}_{jt+1}^F) \right] = 0$ , (2.B.11)

w.r.t. 
$$K_{jt+1}$$
:  $E_t \left\{ \left[ 1 - \Gamma(\bar{\omega}_{jt+1}^F) \right] R_{t+1}^K - \lambda_{jt+1} \left[ \frac{R_{t+1}}{\pi_{t+1}} - \left( \Gamma(\bar{\omega}_{jt+1}^F) - \mu G(\bar{\omega}_{jt+1}^F) R_{t+1}^K \right) \right] \right\} = 0$ ,  
(2.B.12)

w.r.t. 
$$\lambda_{jt+1}$$
:  $E_t \left\{ \frac{R_{t+1}}{\pi_{t+1}} (q_t K_{jt+1} - N_{jt+1}) - \left[ \Gamma(\bar{\omega}_{jt+1}^F) - \mu G(\bar{\omega}_{jt+1}^F) \right] R_{t+1}^K q_t K_{jt+1} \right\} = 0$ ,  
(2.B.13)

where  $\lambda_j$  is the Lagrangian multiplier for entrepreneur *j*'s problem. By rearranging (2.B.11), it is possible to express  $\lambda_{jt+1}$  as a function of only  $\bar{\omega}_{jt+1}^F$ . Furthermore, rearranging (2.B.12):

$$E_t \left\{ \frac{R_{t+1}^K}{\frac{R_{t+1}}{\pi_{t+1}}} \right\} = \frac{\lambda_{jt+1}}{\left[ 1 - \Gamma(\bar{\omega}_{jt+1}^F) + \lambda_{jt+1} \left( \Gamma(\bar{\omega}_{jt+1}^F) - \mu G(\bar{\omega}_{jt+1}^F) \right) \right]} .$$
(2.B.14)

It can be proven that there is a monotonically increasing relationship between the EFP and  $\bar{\omega}_j^F$ . According to (2.B.8), we can extend this relationship between the EFP and the leverage level of j, assessing that a higher entrepreneur's leverage implies a higher EFP.<sup>81</sup>

<sup>&</sup>lt;sup>81</sup>See Appendix A.1 in Bernanke et al. (1999) for proofs.

Furthermore, it is clear from (2.B.14) that  $\bar{\omega}_j^F$  is determined only by aggregate variables. Thus, any entrepreneur chooses the same threshold  $\bar{\omega}^F$  for the idiosyncratic shock on capital returns, below which they default, and the same leverage level.<sup>82</sup> This result allows to consider only the aggregate variables in the production sector part of the model, since every entrepreneur has the same firm structure.

### 2.C Impulse responses of MP contractionary shock

Figure 2.C.1 show several aggregate variables impulse responses for the monetary policy shock considered in the baseline model. This integrate Impulse Response Functions (IRFs) present in Figure 2.1 and Figure 2.2 in the main text.

# 2.D Impulse responses of MP contractionary shock - Same shock magnitude

Below, I show the main aggregate and inequality fluctuations when the monetary shock magnitude used to produce IRFs is the same in both scenarios. The main findings relative to inequality fluctuations are qualitatively similar to that in the baseline model, where I consider two different shock magnitudes that have the same effect on output.

# 2.E Consumption inequality analysis for goods consumption C

In this section, I show the fluctuations in the Gini index and share averages for total goods consumption, C, for the baseline model. Also in this case, when financial frictions on households are active, the changes in the Gini index are stronger, as show in Figure 2.E.1. This can be explained by Figure 2.E.2: while fluctuations of aggregate C are similar in the two scenarios, average consumption for top e bottom shares of households are more scattered in the case of financial frictions on households.

#### 2.F Robustness checks

In this section, I show the Gini indices and consumption decomposition according to prices for different variants of the baseline model. Figure 2.F.1 and Figure 2.F.2 show

<sup>&</sup>lt;sup>82</sup>According to (2.B.8), leverage is a function of the EFP (composed of only aggregate variables) and  $\bar{\omega}_j^F$ . If  $\bar{\omega}_j^F$  depends only on aggregate variables (since it is a function of the EFP, according to (2.B.14)), then the same can be said for the leverage.



Figure 2.C.1: Impulse response to a monetary contraction for aggregate variables Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households.



Figure 2.D.1: Impulse response to a monetary contraction for aggregate variables Note: monetary shock is  $\epsilon^R = 0.0025$  in both the scenarios. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households.



Figure 2.D.2: Impulse response to a monetary contraction for aggregate variables Note: monetary shock is  $\epsilon^R = 0.0025$  in both the scenarios. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households.



Figure 2.D.3: Impulse responses to a monetary contraction for wealth and consumption inequality.

Note: monetary shock  $\epsilon^R = 0.0025$ . The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.D.4: Consumption decomposition for relevant prices



Figure 2.E.1: Impulse responses to a monetary contraction for consumption inequality. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.E.2: Average consumption fluctuation for different shares of households. Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise.

results when the parameter regulating the fiscal policy,  $\rho_{gov}$ , is equal to zero. Figure 2.F.3 and Figure 2.F.4 display results for the case limit of no quadratic costs for capital producer, that is,  $\phi = 0$ .

Since I employ a HANK model, the Ricardian equivalence does not hold, and changes in fiscal policies could have significant effects. A modified version of equation (2.41)is taken into account, which also reacts to government tax revenues, T. Following the approach of Bayer et al. (2019), the alternative bond issuance rule is:

$$\frac{D_{t+1}^G}{\bar{D}^G} = \left(\frac{D_t^G \frac{R_t}{\pi_t}}{\bar{D}^G \frac{\bar{R}}{\bar{\pi}}}\right)^{\rho_{gov}} \left(\frac{T_t}{\bar{T}}\right)^{-\rho^T} , \qquad (2.F.15)$$

with  $\rho^T$  being the parameter determining the extent to which the rule is influenced by deviations in tax revenue from its steady state. When  $\rho^T = 0$ , the rule corresponds to equation (2.41). In this analysis, I assume a value of  $\rho^T = 1$ , indicating that the government responds actively to fluctuations in tax revenues. For example, if an adverse aggregate shock leads to a decrease in tax revenues, the government responds by increasing debt issuance to sustain higher public spending. Results are shown in Figure 2.F.5 and Figure 2.F.6.

I also consider a model in which I change the parameter for households' risk aversion,  $\xi$ . In the baseline calibrations, I assume  $\xi = 4$  as in Bayer et al. (2019), but other models in the HANK literature(e.g., Auclert et al., 2021), assume a lower risk aversion for households. Therefore, in Figure 2.F.7 and Figure 2.F.8, I present the results when assuming a model with  $\xi = 2$ . In this case, however, to match wealth distribution moments, I need to change other parameters, such as  $\beta$ ,  $\zeta$  and  $\underline{a}$ .

The main findings of the baseline model, that is, relatively higher wealth inequality for financial frictions on firms, relatively higher consumption inequality for financial frictions on households, and the relevance of the borrowing penalty  $\omega^H$  for this dynamics, are robust to these changes in parameterization.



Figure 2.F.1: Impulse responses to a monetary contraction for wealth and consumption inequality,  $\rho_{gov} = 0$ .

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.F.2: Consumption decomposition for relevant prices,  $\rho_{gov} = 0$ 



Figure 2.F.3: Impulse responses to a monetary contraction for wealth and consumption inequality,  $\phi = 0$ .

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.F.4: Consumption decomposition for relevant prices,  $\phi = 0$ 



Figure 2.F.5: Impulse responses to a monetary contraction for wealth and consumption inequality, government reacts to tax revenues.

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.F.6: Consumption decomposition for relevant prices, government reacts to tax revenues.



Figure 2.F.7: Impulse responses to a monetary contraction for wealth and consumption inequality,  $\xi = 2$ .

Note: monetary shock  $\epsilon^R = 0.0025$  for active financial frictions on household borrowing,  $\epsilon^R = 0.0014$  otherwise. The blue line refers to an economy with financial frictions on firms, the red one when frictions are on households. The green dotted line is the percentage difference between the two curves using the lower curve as the base.



Figure 2.F.8: Consumption decomposition for relevant prices,  $\xi = 2$