Editors' Suggestion

Precise Predictions for the Associated Production of a W Boson with a Top-Antitop Quark Pair at the LHC

Luca Buonocore,[1] Simone Devoto,[2] Massimiliano Grazzini,[1] Stefan Kallweit,[3]
Javier Mazzitelli,[4] Luca Rottoli,[1] and Chiara Savoini[1]
[1]*Physik Institut, Universität Zürich, CH-8057 Zürich, Switzerland*
[2]*Dipartimento di Fisica, Università degli Studi di Milano and INFN, Sezione di Milano, I-20133 Milano, Italy*
[3]*Dipartimento di Fisica, Università degli Studi di Milano-Bicocca and INFN, Sezione di Milano-Bicocca, I-20126 Milano, Italy*
[4]*Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*

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The production of a top-antitop quark pair in association with a W boson ($t\bar{t}W$) is one of the heaviest signatures currently probed at the Large Hadron Collider. Since the first observation reported in 2015, the corresponding rates have been found to be consistently higher than the standard model predictions, which are based on next-to-leading order calculations in the QCD and electroweak interactions. We present the first next-to-next-to-leading order QCD computation of $t\bar{t}W$ production at hadron colliders. The calculation is exact, except for the finite part of the two-loop virtual corrections, which is estimated using two different approaches that lead to consistent results within their uncertainties. We combine the newly computed next-to-next-to-leading order QCD corrections with the complete next-to-leading order QCD plus electroweak results, thus obtaining the most advanced perturbative prediction available to date for the $t\bar{t}W$ inclusive cross section. The tension with the latest ATLAS and CMS results remains at the 1σ-2σ level.

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Introduction.—The final state of a $W^{\pm}$ boson produced in association with a top-antitop quark pair ($t\bar{t}W$) represents one of the most massive standard model (SM) signatures accessible at the Large Hadron Collider (LHC). Since the top quarks rapidly decay into a W boson and a b quark, the $t\bar{t}W$ process leads to two b jets and three decaying W bosons. This in turn gives rise to multilepton signatures that are relevant to a number of searches for physics beyond the standard model. In particular, $t\bar{t}W$ production is one of the few SM processes that provides an irreducible source of same-sign dilepton pairs. Additionally, the $t\bar{t}W$ signature is a relevant background for the measurement of Higgs boson production in association with a top-antitop quark pair ($t\bar{t}H$) and for four-top ($t\bar{t}t\bar{t}$) production.

Measurements of $t\bar{t}W$ production carried out by the ATLAS and CMS collaborations at center-of-mass energies of $\sqrt{s} = 8$ TeV [1,2] and $\sqrt{s} = 13$ TeV [3–5] lead to rates consistently higher than the SM predictions. A similar situation holds for $t\bar{t}W$ measurements in the context of $t\bar{t}H$ [6,7] and $t\bar{t}t\bar{t}$ [8,9] analyses. The most recent measurements [10,11], based on an integrated luminosity of about 140 fb^{-1}, confirm this picture, with a slight excess at the 1σ-2σ level.

In this context, it is clear that the availability of precise theoretical predictions for the $t\bar{t}W$ SM cross section is of the utmost importance. The next-to-leading order (NLO) QCD corrections to $t\bar{t}W$ production have been computed in Refs. [12–14], and electroweak (EW) corrections in Refs. [15,16]. Soft-gluon effects were included in Refs. [17–20]. NLO QCD effects to the complete off-shell $t\bar{t}W$ process have been considered in Refs. [21–23], while the complete off-shell NLO QCD + EW computation was reported in Ref. [24]. Very recently, even NLO QCD corrections to off-shell $t\bar{t}W$ production in association with a light jet were computed [25]. A detailed investigation of theoretical uncertainties for multilepton $t\bar{t}W$ signatures has been presented in Ref. [26] (see also Ref. [27]). Current experimental measurements are compared with NLO QCD + EW predictions supplemented with multijet merging [28,29], which are still affected by relatively large uncertainties. To improve upon the current situation, next-to-next-to-leading order (NNLO) QCD corrections are necessary.

In this Letter we present the first computation of $t\bar{t}W$ production at NNLO in QCD. While the required tree-level and one-loop scattering amplitudes can be evaluated with automated tools, the two-loop amplitude for $t\bar{t}W$ production is yet unknown. In this Letter, we estimate it by using two different approaches. The first parallels the approach successfully applied in Ref. [30] to $t\bar{t}H$ production, and is based on a *soft-W approximation*, which allows us to

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extract the $t\bar{t}W$ amplitude from the two-loop amplitudes for top-pair production [31] (see also Ref. [32]). The second is based on the NNLO calculation of Ref. [33], where an approximate form of the two-loop amplitude for the production of a heavy-quark pair and a W boson is obtained from the leading-color two-loop amplitudes for a W boson and four massless partons [34,35] through a *massification* procedure [36–38]. We demonstrate that the two approximations, despite their distinct conceptual foundations and the fact that they are used in a regime where their validity is not granted, yield consistent results within their respective uncertainties. Finally, we combine the computed NNLO QCD corrections with the complete NLO QCD + EW result, thus obtaining the most accurate theoretical prediction for this process available to date.

Calculation.—The QCD cross section for $t\bar{t}W$ production can be written as $\sigma = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{NLO}} + \Delta\sigma_{\mathrm{NNLO}} + \ldots$, where σ_{LO} is the leading-order (LO) cross section, $\Delta\sigma_{\mathrm{NLO}}$ the NLO QCD correction, $\Delta\sigma_{\mathrm{NNLO}}$ the NNLO QCD contribution, and so forth.

In addition to the inherent challenges involved in obtaining the relevant scattering amplitudes, the implementation of a complete NNLO calculation is a difficult task because of the presence of infrared (IR) divergences at intermediate stages of the calculation. In this work NNLO IR singularities are handled and canceled by using the q_T subtraction formalism [39], extended to heavy-quark production in Refs. [40–42]. According to the q_T subtraction formalism, the differential cross section $d\sigma$ can be evaluated as

$$d\sigma = \mathcal{H} \otimes d\sigma_{\mathrm{LO}} + [d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{CT}}]. \qquad (1)$$

The first term on the right-hand side of Eq. (1) corresponds to the $q_T = 0$ contribution. It is obtained through a convolution, with respect to the longitudinal-momentum fractions z_1 and z_2 of the colliding partons, of the perturbatively computable function $\mathcal{H}$ with the LO cross section $d\sigma_{\mathrm{LO}}$. The real contribution $d\sigma_{\mathrm{R}}$ is obtained by evaluating the cross section to produce the $t\bar{t}W$ system accompanied by additional QCD radiation that provides a recoil with finite transverse momentum q_T. When $d\sigma$ is evaluated at NNLO, $d\sigma_{\mathrm{R}}$ is obtained through an NLO calculation by using the dipole subtraction formalism [43–45]. The role of the counterterm $d\sigma_{\mathrm{CT}}$ is to cancel the singular behavior of $d\sigma_{\mathrm{R}}$ in the limit $q_T \to 0$, rendering the square bracket term in Eq. (1) finite. The explicit form of $d\sigma_{\mathrm{CT}}$ is completely known up to NNLO: it is obtained by perturbatively expanding the resummation formula of the logarithmically enhanced contributions to the q_T distribution of the $t\bar{t}W$ system [46–50].

Our computation is implemented within the MATRIX framework [51], suitably extended to $t\bar{t}W$ production, along the lines of what was done for heavy-quark production [41,42,52]. The method was recently applied also to the NNLO calculation of $t\bar{t}H$ [30] and $b\bar{b}W$ [33] production, for which the contributions from soft-parton emissions at low transverse momentum [53] had to be properly extended to more general kinematics [54]. The required tree-level and one-loop amplitudes are obtained with OpenLoops [55–57] and RECOLA [58–60]. In order to numerically evaluate the contribution in the square bracket of Eq. (1), a technical cutoff r_{cut} is introduced on the dimensionless variable q_T/Q, where Q is the invariant mass of the $t\bar{t}W$ system. The final result, which corresponds to the limit $r_{\mathrm{cut}} \to 0$, is extracted by computing the cross section at fixed values of r_{cut} and performing the $r_{\mathrm{cut}} \to 0$ extrapolation. More details on the procedure and its uncertainties can be found in Refs. [49,51].

The purely virtual contributions enter the first term on the right-hand side of Eq. (1), and more precisely the hard function H (related to $\mathcal{H}$ through $\mathcal{H} = H\delta(1 - z_1) \times \delta(1 - z_2) + \delta\mathcal{H}$) whose coefficients, in an expansion in powers of the QCD coupling $\alpha_{\mathrm{S}}(\mu_R)$, are defined as

$$H^{(n)} = \left. \frac{2\mathrm{Re}\left(\mathcal{M}_{\mathrm{fin}}^{(n)}(\mu_R, \mu_{\mathrm{IR}})\mathcal{M}^{(0)*}\right)}{|\mathcal{M}^{(0)}|^2} \right|_{\mu_R = Q}. \qquad (2)$$

Here, μ_R is the renormalization scale, and $\mathcal{M}_{\mathrm{fin}}^{(n)}$ are the perturbative coefficients of the finite part of the renormalized virtual amplitude for the process $u\bar{d}(d\bar{u}) \to t\bar{t}W^{+(-)}$, after the subtraction of IR singularities at the scale μ_{IR}, according to the conventions of Ref. [61]. In order to obtain an approximation of the NNLO coefficient $H^{(2)}$, we use two independent approaches, applied to both the numerator and the denominator of Eq. (2). The first relies on a soft-W approximation. In the high-energy limit, in which the colliding quark and antiquark of momenta p_1 and p_2 radiate a soft-W boson with momentum k and polarization $\varepsilon(k)$, the multiloop QCD amplitude in $d = 4 - 2\epsilon$ dimensions behaves as

$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim \frac{g}{\sqrt{2}} \left(\frac{p_2 \cdot \varepsilon^*(k)}{p_2 \cdot k} - \frac{p_1 \cdot \varepsilon^*(k)}{p_1 \cdot k} \right) \times \mathcal{M}_L(\{p_i\}; \mu_R, \epsilon), \qquad (3)$$

where g is the EW coupling and $\mathcal{M}_L(\{p_i\})$ the $q_L\bar{q}_R \to t\bar{t}$ virtual amplitude. In the second approach the two-loop coefficient $H^{(2)}$ is approximated in the ultrarelativistic limit $m_t \ll Q$ by using a massification procedure [36–38]. We start from the massless $W + 4$-parton amplitudes $\mathcal{M}^{m_t=0}$ evaluated in the leading-color approximation [35,62] to obtain

$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_{\mathrm{S}}(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right) \times \mathcal{M}^{m_t=0}(\{p_i\}, k; \mu_R, \epsilon), \qquad (4)$$

where Z are perturbative functions whose explicit expression up to NNLO can be found in Ref. [37]. This procedure [63] was successfully applied to evaluate NNLO corrections to $b\bar{b}W$ production in Ref. [33].

In order to use Eq. (3) to approximate the $t\bar{t}W$ amplitudes, we need to introduce a prescription that, from an event containing a $t\bar{t}$ pair and a W boson, defines a corresponding event in which the W boson is removed. This is accomplished by absorbing the W momentum into the top quarks, thus preserving the invariant mass of the event. On the other hand, for the application of Eq. (4) we map the momenta of the massive top quarks into massless momenta by preserving the four-momentum of the $t\bar{t}$ pair. In both cases we reweight the respective two-loop coefficients with the exact Born matrix elements. This approach effectively captures additional kinematic effects, which we expect to extend the region of validity of the approximations well beyond where it may be assumed in the first place.

For our numerical studies, we consider the on-shell production of a W boson in association with a $t\bar{t}$ pair in proton collisions, at a center-of-mass energy of $\sqrt{s} = 13$ TeV. We set the pole mass of the top quark to $m_t = 173.2$ GeV, while for the W mass we use $m_W = 80.385$ GeV. We work in the G_μ scheme for the EW parameters, with $G_\mu = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}$ and $m_Z = 91.1876$ GeV. We consider a diagonal Cabibbo-Kobayashi-Maskawa matrix. We use the `NNPDF31_nnlo_as_0118_luxqed` set for parton distribution functions (PDF) [64] and strong coupling, which is based on the LUXqed methodology [65] to determine the photon density. We adopt the LHAPDF interface [66] and use PineAPPL [67] grids through the new MATRIX+PineAPPL interface [68] to estimate PDF and α_S uncertainties. For our central predictions we set the renormalization (μ_R) and factorization (μ_F) scales to the value $\mu_0 = m_t + m_W/2 \equiv M/2$, and evaluate the scale uncertainties by performing a seven-point variation, varying them independently by a factor of 2 with the constraint $1/2 \le \mu_R/\mu_F \le 2$.

In order to test the quality of our approximations, we apply them to evaluate the contribution of the coefficient $H^{(1)}$ to the NLO correction, $\Delta\sigma_{\mathrm{NLO,H}}$. In Fig. 1 (upper panel) the two approximations are compared to the exact result, as functions of the cut on the transverse momenta of the top quarks, $p_{T,t/\bar{t}}$. We observe that both approximations get closer to the exact result if a harder cut is imposed, since the large-$p_{T,t/\bar{t}}$ region corresponds to a kinematical configuration where both of them are expected to reproduce the full amplitude. In particular, we observe that the soft approximation tends to undershoot the exact result, while the massification approach overshoots it. Remarkably, both approaches provide a good approximation also at the inclusive level.

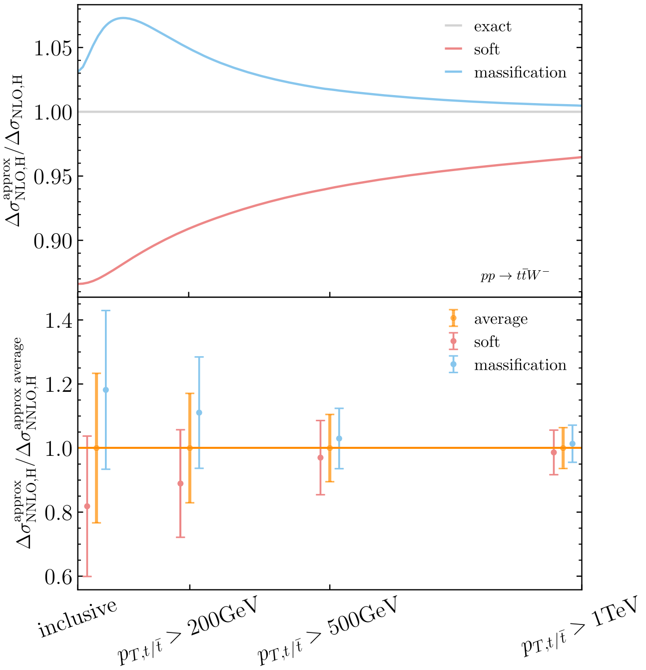

FIG. 1. Results for $\Delta\sigma_{\mathrm{NLO,H}}$ (upper panel) and $\Delta\sigma_{\mathrm{NNLO,H}}$ (lower panel), in the case of $t\bar{t}W^-$ production, obtained with the two approximations presented in this Letter, for different cuts on the transverse momenta of the top quarks. At NLO the approximations are normalized to the exact result, while at NNLO to their average. The uncertainties of each approximation at NNLO are presented, as well as their combination. Similar results are obtained for $t\bar{t}W^+$.

We now move on to the contribution of the coefficient $H^{(2)}$ to the NNLO correction, $\Delta\sigma_{\mathrm{NNLO,H}}$. In Fig. 1 (lower panel) the two approximations are compared, normalized to their average. The uncertainties of the soft and massification results are also depicted. These are evaluated starting from the assumption that the uncertainty of each approximation of $\Delta\sigma_{\mathrm{NNLO,H}}$ is not smaller than the relative difference between $\Delta\sigma^{\mathrm{approx}}_{\mathrm{NLO,H}}$ and the exact NLO result. We obtain a first estimate of the uncertainty on $\Delta\sigma_{\mathrm{NNLO,H}}$ by conservatively multiplying $\Delta\sigma^{\mathrm{approx}}_{\mathrm{NLO,H}}$ by a factor of 2. As an additional estimate, we consider variations of the subtraction scale μ_{IR}, at which our approximations are applied, by a factor of 2 around the central scale Q (adding the exact evolution from μ_{IR} to Q). For each of the two approximations, the uncertainty is defined as the maximum between these two estimates. From Fig. 1 we see that the two approximations are consistent within their respective uncertainties. We therefore conclude that our approach can provide a good estimate of the true NNLO hard-virtual contribution. Our best prediction for $\Delta\sigma_{\mathrm{NNLO,H}}$ is finally obtained by taking the average of the two approximations and linearly combining their uncertainties. We note that with such procedure the central values of the two approximations are enclosed within the uncertainty band of the average result. The final uncertainty on $\Delta\sigma_{\mathrm{NNLO,H}}$ turns out to be at the $\mathcal{O}(25\%)$ level. (We note that a similar control on the two-loop contribution is obtained in recent calculations

TABLE I. Inclusive cross sections for $t\bar{t}W^+$ and $t\bar{t}W^-$ production at different perturbative orders, together with their sum and ratio. The uncertainties are computed through scale variations and for our best prediction, $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$, are symmetrized as discussed in the text. Where NNLO QCD corrections are included, the error from the approximation of the two-loop amplitudes is also shown. The numerical uncertainties on our predictions are at the per mille level or below. The corresponding experimental results from the ATLAS [11] and CMS [10] collaborations are also quoted, with their statistical and systematic uncertainties.

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%\ +8.0\%}_{-5.8\%\ -7.5\%}$	$301^{+9.3\%\ +11.6\%}_{-9.0\%\ -10.3\%}$	$890^{+5.6\%\ +7.9\%}_{-5.6\%\ -7.9\%}$	$1.95^{+10.8\%\ +8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%\ +5.4\%}_{-5.4\%\ -5.4\%}$	$343^{+7.6\%\ +7.3\%}_{-7.6\%\ -7.3\%}$	$868^{+4.6\%\ +5.9\%}_{-4.6\%\ -5.9\%}$	$1.61^{+9.3\%\ +4.3\%}_{-9.3\%\ -3.1\%}$

for massless 2 → 3 processes employing the leading-color approximation, see, e.g., Ref. [69].) As we will observe in what follows, this leads to an uncertainty of the NNLO prediction which is significantly smaller than the residual perturbative uncertainties.

Results.—We now focus on our numerical predictions for the LHC. Our results for the total $t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections are presented in Table I. In the first three rows we consider pure QCD predictions, which are labeled $\mathrm{N}^n\mathrm{LO_{QCD}}$ with $n = 0$, 1, 2. The results in the fourth row, dubbed $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$, represent our best prediction. They include additively also EW corrections and all subleading (in α_S) terms up to NLO, originally computed in Ref. [16,70]. We recompute them here within the MATRIX framework, after validation against a recent implementation in WHIZARD [71]. Predictions for the sum and the ratio of the $t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections are also provided, and their scale uncertainties are evaluated by performing seven-point scale variations for each of them, keeping μ_R correlated, while the values of μ_F for the $t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections are allowed to differ by at most a factor of 2. (The uncertainty due to the approximation of the two-loop corrections is expected to largely cancel in the ratio.) Finally, the most recent results by the ATLAS [11] and CMS [10] collaborations are quoted.

We start by discussing the pattern of QCD corrections. The NLO cross section for both $t\bar{t}W^+$ and $t\bar{t}W^-$ production is about 50% larger than the corresponding LO result. The NNLO corrections are moderate, and increase the NLO result by about 15%, showing first signs of perturbative convergence. The ratio between the two cross sections shows a very stable perturbative behaviour. The size of the scale uncertainties is substantially reduced at NNLO, in line with the observed smaller corrections to the central prediction. The impact of the two-loop contribution is relatively large, about 6%–7% of the NNLO cross section. Nonetheless, we find that the ensuing uncertainty on our prediction is $\mathcal{O}(\pm 2\%)$, i.e., significantly smaller than the remaining perturbative uncertainties.

In addition to the value $\mu_0 = M/2$ used in Table I, we have also considered alternative choices for the central scale, specifically $\mu_0 = M/4$, $H_T/2$, and $H_T/4$, where H_T is the sum of the transverse masses of the top quarks and the W boson. Results for the different perturbative orders in the QCD expansion are presented in Fig. 2. At each order, the four predictions are fully consistent within their uncertainties, and in particular the $\mu_0 = M/2$ and $\mu_0 = H_T/4$ bands cover the central values of the other scale choices that have been considered. We note that symmetrizing the band of the $\mu_0 = M/2$ prediction at NNLO leads to an upper bound that is almost identical to that of the $\mu_0 = M/4$ and $\mu_0 = H_T/4$ scale variations. Therefore, to be conservative, the perturbative uncertainties affecting our final $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ results are estimated by symmetrizing the scale variation error. More precisely, we take the maximum among the upward and downward variations, assign it symmetrically, and leave the nominal prediction unchanged.

The EW corrections increase our $\mathrm{NNLO_{QCD}}$ cross sections by about 5%. While smaller than the NNLO QCD corrections, their inclusion is crucial for an accurate description of this process, as their magnitude is comparable to the $\mathrm{NNLO_{QCD}}$ scale uncertainties. The PDF (α_S) uncertainties, not shown in Table I, on the $t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections amount to $\pm 1.8\%$ ($\pm 1.8\%$) and $\pm 1.7\%$ ($\pm 1.9\%$), respectively. [We consider 68% confidence level PDF uncertainties, while the α_S uncertainties are computed as half the difference between the corresponding sets for $\alpha_S(m_Z) = 0.118 \pm 0.001$.] The PDF uncertainty on their ratio, derived by recalculating the ratio for each replica, is $\pm 1.7\%$. Its α_S uncertainty is negligible.

The current theory reference to which experimental data are compared is the FxFx prediction of Ref. [29], which reads $\sigma_{t\bar{t}W}^{\mathrm{FxFx}} = 722.4^{+9.7\%}_{-10.8\%}$ fb. Our $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ prediction for the $t\bar{t}W$ cross section in Table I is

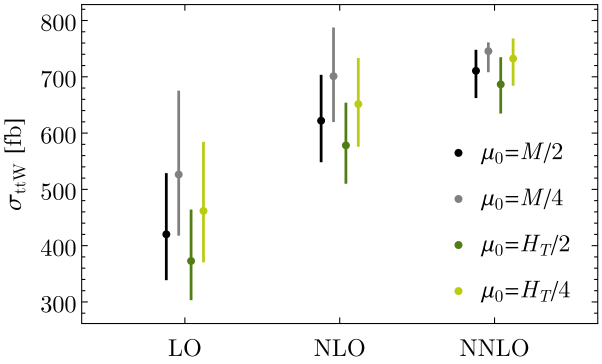

FIG. 2. Inclusive $t\bar{t}W$ cross sections at different orders in the QCD expansion, for different choices of the central renormalization and factorization scales.

fully consistent with this value, with considerably smaller uncertainties.

We now compare our theoretical predictions to the measurements performed by the ATLAS and CMS collaborations in Refs. [10,11], which represent the most precise experimental determination of the $t\bar{t}W^{\pm}$ cross sections to date. From Table I we observe that the individual measurements for the $t\bar{t}W^{+}$ and $t\bar{t}W^{-}$ cross sections are systematically above the theoretical predictions, but all within 2 standard deviations of our central results, except for the $t\bar{t}W^{-}$ measurement by the CMS Collaboration. The measurement of the ratio $\sigma_{t\bar{t}W^{+}}/\sigma_{t\bar{t}W^{-}}$ by the ATLAS Collaboration is in excellent agreement with our prediction, whereas the CMS result exhibits some tension.

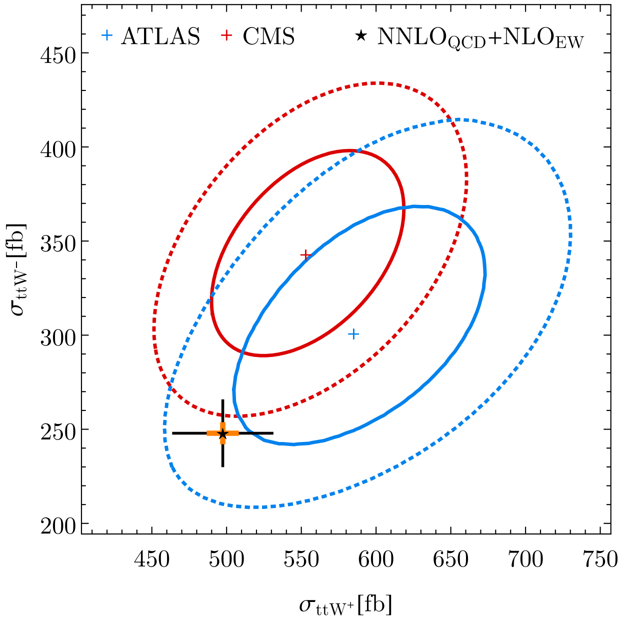

FIG. 3. Comparison of our $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ result to the measurement performed by the CMS (red) and ATLAS (blue) collaborations in Refs. [10,11], at 68% (solid) and 95% (dashed) confidence level. We indicate in black and orange the scale and the approximation uncertainties, respectively, of the $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ result.

Finally, we present in Fig. 3 our $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ results with their perturbative uncertainties in the $\sigma_{t\bar{t}W^{+}} - \sigma_{t\bar{t}W^{-}}$ plane, together with the 68% and 95% confidence level regions obtained by the two collaborations. The subdominant uncertainties due to the approximation of the two-loop corrections are also shown. When comparing to the data, we observe an overlap between the $\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ uncertainty bands and the 1σ and 2σ contours of the ATLAS and CMS measurements, respectively.

Summary.—In this Letter we have presented the first calculation of the second-order QCD corrections to the hadroproduction of a W boson in association with a top-antitop quark pair. Our results are exact, except for the finite part of the two-loop virtual corrections, which is computed by using two independent approximations. While these approximations are completely different in their conception, they lead to consistent results, thereby providing a strong check of our approach.

We have combined our results with the NLO EW corrections, obtaining the most precise theoretical determination of the inclusive $t\bar{t}W^{\pm}$ cross section available to date. Our results significantly reduce the size of the perturbative uncertainties, allowing for a more meaningful comparison to the results obtained by the ATLAS and CMS collaborations. The high level of precision attained by our theoretical predictions will enable even more rigorous tests of the SM, as more precise experimental measurements become available.

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