Eductive stability may not imply evolutionary stability in the presence of information costs

Ahmad Naimzada [a*], Marina Pireddu [b†]

[a] Dept. of Economics, Management and Statistics, University of Milano - Bicocca, U6 Building, Piazza dell'Ateneo Nuovo 1, 20126 Milano, Italy.

[b] Dept. of Mathematics and its Applications, University of Milano - Bicocca, U5 Building, Via Cozzi 55, 20125 Milano, Italy.

Abstract

Starting from a Muthian cobweb model, we extend the profit-based evolutionary setting in Hommes and Wagener (2010) populated by pessimistic, optimistic and unbiased fundamentalists, by assuming that agents face heterogeneous information costs, inversely proportional to the entity of their bias. Hommes and Wagener (2010) found that, when the unique steady state of their model is globally eductively stable in the sense of Guesnerie (2002), the equilibrium under evolutionary learning may be just locally, but not globally, stable. Thanks to the introduction of information costs, we prove that the equilibrium, when globally eductively stable, may be not even locally stable under evolutionary learning.

Keywords: Muthian cobweb model; heterogeneous agents; evolutionary learning; eductive stability; information costs; double stability threshold.

JEL classification: B52, C62, D84, E32

*Tel.: +39 0264485813. E-mail address: ahmad.naimzada@unimib.it

†Corresponding author. Tel.: +39 0264485767. E-mail address: marina.pireddu@unimib.it

1 Introduction

Hommes and Wagener (2010), which deals with the same profit-based share updating mechanism adopted in Brock and Hommes (1997) for the case without memory, considers a Muthian cobweb model framework in which producers can choose among three different forecasting rules: fundamentalists predict that prices will always be at their fundamental value, optimists predict that the price of the good will always be above the fundamental price, and pessimists always predict prices below the fundamental price. Despite the heterogeneity in the forecasting rules, in Hommes and Wagener (2010) all agents face a common zero information cost. Hommes and Wagener (2010) focuses on the case in which the Muthian model is globally eductively stable in the sense of Guesnerie (2002), that is, on the case in which the model is stable under naive expectations, as the slopes of demand and supply satisfy the familiar "cobweb theorem" by Ezekiel (1938). Hommes and Wagener (2010) shows that, under evolutionary learning, the steady state, which is always (locally or globally) stable, may coexist with a locally stable period-two cycle, along which prices fluctuate around the rational expectations price and most agents switch between optimistic and pessimistic strategies. This means that, although the model in Hommes and Wagener (2010) is globally eductively stable, the evolutionary system therein, contrarily to the setting in Brock and Hommes (1997), may not be globally evolutionary stable.

Extending the model in Hommes and Wagener (2010) by assuming that agents face heterogeneous information costs, inversely proportional to the degree of their bias,[1] we find that the equilibrium, when globally eductively stable, may be not even locally stable under evolutionary learning. Hence, the introduction of differentiated information costs, in addition to making the characterization of agents' heterogeneity more complete than in Hommes and Wagener (2010), allows us to obtain a stronger result, which gives a cleaner negative answer to the question *does eductive stability always imply*

[1] Namely, according to Hommes (2013), page 150, *A fundamentalists strategy, however, requires structural knowledge of the economy and information about "economic fundamentals", and therefore we assume positive information-gathering costs for fundamentalists.* Since biased agents do not perfectly know the economic fundamentals, we suppose that their information costs are lower than that of unbiased fundamentalists, but still non-negative. See also Anufriev et al. (2013) where, in a DSGE model with heterogeneous expectations, the equilibrium predictor for the inflation rate is available at cost $C \geq 0$, while biased agents do not face any information cost.

evolutive stability? addressed in that paper, and which was in turn inspired by the claim that *"reasonable" adaptive learning processes are asymptotically stable* in Guesnerie (2002).

Our setting is mainly analyzed by measuring the influence of agents' heterogeneity through the parameter describing the degree of optimism and pessimism. We find that the unique steady state, which coincides with the fundamental, may be stable either for all values of the bias or just for suitably small and for suitably large values of the bias, when the Muthian model is globally eductively stable.

In Section 2 we present the model, that we study in Section 3. The proofs of our results, as well as further comments and explanations, can be found in Naimzada and Pireddu (2019).

2 The model

We recall the discrete-time evolutionary cobweb setting in Hommes and Wagener (2010), to which we add information costs in the profits (see (2.5)).

The economy is populated by unbiased fundamentalists, that we will call just fundamentalists, and by two types of biased fundamentalists, i.e., optimists and pessimists. In order to obtain the same steady state as in Hommes and Wagener (2010), we assume that optimists and pessimists are symmetrically biased, with the former (latter) expecting that the price of the good they produce will always be above (below) the fundamental price. Moreover, for the sake of simplicity, along the paper we focus on the case in which, in addition to unbiased fundamentalists, just one group of optimists and one group of pessimists are present. In the Muthian farmer model, agents have to choose the quantity q of a certain good to produce in the next period and are expected profit maximizers. Assuming a quadratic cost function

$$\gamma(q) = \frac{q^2}{2s}, \tag{2.1}$$

with $s > 0$, the supply curve is given by

$$S(p^e) = sp^e, \tag{2.2}$$

where p^e is the expected price and s describes its slope. The demand function is supposed to be linearly decreasing in the market price, i.e.,

$$D(p) = A - dp,$$

with A and d positive parameters, representing respectively the market size and the slope of the demand function. We stress that the demand is positive for sufficiently large values of A.

At the fundamental price $p = p^*$ demand equals supply, i.e.,

$$p^* = \frac{A}{d+s}\,. \tag{2.3}$$

This is also the expression for the unique steady state in Hommes and Wagener (2010). Like in that paper, we will deal with the case in which the Muthian model is globally eductively stable in the sense of Guesnerie (2002), that is, on the case in which the model is stable under naive expectations, as the slopes of demand and supply satisfy the familiar "cobweb theorem" by Ezekiel (1938) and thus it holds that $s/d < 1$.

Agents have heterogeneous expectations about the price of the good they have to produce. In particular, assuming a symmetric disposition of the beliefs and characterizing the fundamentalists, pessimists and optimists by subscripts 0, 1, 2, respectively, in symbols we have that their expectations at time t are given by

$$p^e_{i,t} = p^* + b_i,\ i \in \{0,1,2\},\ \text{with}\ b_0 = 0,\ b_1 = -b,\ b_2 = b, \tag{2.4}$$

where $b > 0$ describes the bias of pessimists and optimists. In order to avoid a negative expectation for pessimists, we will restrict our attention to the bias values $b \in (0, p^*)$, with p^* as in (2.3).

Denoting by $\omega_{i,t}$ the share of agents choosing the forecasting rule $i \in \{0,1,2\}$ at time t, the total supply is given by $\sum_{i=0}^{2} \omega_{i,t} S(p^e_{i,t})$ and thus the market equilibrium condition reads as

$$A - dp_t = \sum_{i=0}^{2} \omega_{i,t} S(p^e_{i,t}).$$

As concerns the share updating mechanism, Hommes and Wagener (2010) deals with the discrete choice model in Brock and Hommes (1997) for the case without memory, in which only the most recently realized net profits $\pi_{j,t}$, $j \in \{0,1,2\}$, are taken into account. In symbols

$$\omega_{i,t} = \frac{\exp(\beta \pi_{i,t-1})}{\sum_{j=0}^{2} \exp(\beta \pi_{j,t-1})},\ i \in \{0,1,2\},$$

where $\beta > 0$ is the intensity of choice parameter.

When considering information costs, net profits $\pi_{j,t}$, $j \in \{0, 1, 2\}$, at time t are defined as

$$\pi_{j,t} = p_t S(p^e_{j,t}) - \gamma(S(p^e_{j,t})) - c_j, \tag{2.5}$$

with γ and S as in (2.1) and (2.2), respectively, and with the nonnegative parameter c_j representing the information cost deriving by the adoption of forecasting rule j. Since optimists and pessimists do not perfectly know the economic fundamentals and make symmetric errors in estimating them, we will assume that $c_1 = c_2 = c$, for a certain $c \geq 0$. On the other hand, unbiased fundamentalists exactly know the formulations of demand and supply functions and they are able to correctly compute the fundamental value. Due to their better information compared to optimists and pessimists, we will suppose that the information cost c_0 of fundamentalists satisfies $0 \leq c \leq c_0$. We stress that for $c = c_0 = 0$ we are led back to the framework in Hommes and Wagener (2010), while setting $c_0 = C$ and $c = 0$ we obtain the same information costs as in Anufriev et al. (2013), where $C \geq 0$ is the information cost associated with the equilibrium predictor for the inflation rate.

Introducing, like in Hommes and Wagener (2010), the variable $x_t = p_t - p^*$ and recalling (2.4), we can write our model dynamic equation in deviation from fundamental as

$$\begin{aligned} x_t &= \frac{sb}{d}(\omega_{1,t} - \omega_{2,t}) \\ &= \frac{sb}{d}\frac{\exp\left(-\frac{\beta s}{2}(x_{t-1}+b)^2\right)-\exp\left(-\frac{\beta s}{2}(x_{t-1}-b)^2\right)}{\exp\left(-\frac{\beta s}{2}(x_{t-1}+b)^2\right)+\exp\left(-\frac{\beta s}{2}(x_{t-1}-b)^2\right)+\exp\left(-\frac{\beta s}{2}x_{t-1}^2-\beta(c_0-c)\right)}. \end{aligned} \tag{2.6}$$

As we shall prove in Proposition 3.1, for the model formulation in terms of x_t, the unique steady state is given by $x^* = 0$. To such aim, it is expedient to rewrite (2.6) as

$$x_t = f(x_{t-1}), \tag{2.7}$$

where the one-dimensional map $f : (-p^*, +\infty) \to \mathbb{R}$ is defined by the right-hand side in (2.6). We stress that f is differentiable and that, recalling the expression of p^* in (2.3), its domain is enlarged by considering increasing values of A. Moreover, using the same argument employed in the proof of Theorem A in Hommes and Wagener (2010), it is possible to show that the map f is decreasing also for nonzero information costs. This excludes the possibility of complex dynamics in our framework, too, and indeed at most we observe a period-two cycle, either coexisting with the locally stable steady state, or being the unique attractor.

3 Analytical results and possible scenarios

We start our analysis by investigating which are the steady states for our model and by studying their local stability with respect to the intensity of choice parameter in the next result, where we summarize the possible resulting scenarios.

Along the section, we call a scenario destabilizing (stabilizing) with respect to a parameter when the steady state is stable (unstable) below a certain threshold of that parameter and unstable (stable) above it. We say that a scenario is mixed if the steady state is unstable inside an interval of intermediate parameter values and stable for lower and higher values of the parameter. We say that a scenario is unconditionally unstable (unconditionally stable) when the steady state is unstable (stable) for all the parameter values.

Proposition 3.1 *Equation* (2.7) *admits* $x = 0$ *as unique steady state. The equilibrium* $x = 0$ *is locally asymptotically stable for* (2.6) *if*

$$\beta < \frac{d\left(2 + \exp\left(\frac{\beta b^2 s}{2} - \beta(c_0 - c)\right)\right)}{2b^2 s^2}.$$

Hence, if $s/d < 1$*, according to the considered parameter configuration, on increasing* β *we may have an* (*i*) *unconditionally stable,* (*ii*) *mixed or* (*iii*) *destabilizing scenario.*

Since, as shown in Theorem A in Hommes and Wagener (2010) and as we recalled above, without information costs the steady state is always (locally or globally) asymptotically stable, cases (*ii*) and (*iii*) in Proposition 3.1 can not occur when setting $c = c_0 = 0$. Hence, if we neglected information costs we would not observe, in particular, the most classical scenario produced by the intensity of choice parameter, that is, the destabilizing scenario.

Despite such differences with the findings in Hommes and Wagener (2010) concerning the local stability of the steady state, similar conclusions to those drawn in Theorem A therein in regard to the existence of a locally stable period-two cycle for sufficiently large values of the intensity of choice parameter can be obtained in the presence of information costs, too. See Naimzada and Pireddu (2019) and Proposition 3.2 therein for further details.

Rewriting the stability condition in Proposition 3.1 in terms of the bias, we obtain the following result, while the effects of information costs on the equilibrium stability are studied in Corollary 3.2 in Naimzada and Pireddu (2019):

Corollary 3.1 *The equilibrium $x = 0$ is locally asymptotically stable for* (2.6) *if*

$$b^2 < \frac{d\left(2+\exp\left(\frac{\beta b^2 s}{2}-\beta(c_0-c)\right)\right)}{2\beta s^2}.$$

Hence, according to the considered parameter configuration, on increasing b we may have an unconditionally stable or mixed scenario.

Thus, Proposition 3.1 and Corollary 3.1 imply the existence of up to two possible stability thresholds for $x = 0$ with respect to β and b, respectively, and $x = 0$ may be locally stable just for sufficiently low and for sufficiently high values of the intensity of choice parameter and of the bias.[2] In particular, this means that the introduction of information costs may not only produce a destabilization of the system for intermediate values of the bias of optimistic and pessimistic agents, but that a strong enough beliefs' heterogeneity may be stabilizing in our setting.

In view of giving an economic interpretation of such counterintuitive result, let us start noticing that, in the absence of information costs, $x = 0$ is (locally or globally) stable for every value of b because in a neighborhood of the steady state it is relatively advantageous being fundamentalists. More precisely, if the bias is small, $x = 0$ is globally stable because profits of fundamentalists and biased agents do not differ very much and thus it does not happen that the profits of optimists or pessimists are much higher than those of fundamentalists when the initial condition is far from the steady state. On the other hand, the latter phenomenon does occur when b is large enough, as in this case the price forecast of optimists or pessimists is considerably more precise than that of fundamentalists when prices are far from the equilibrium. Then, in that region, a locally stable period-two cycle arises, along which agents switch between optimism and pessimism, even if $x = 0$ remains locally stable, because in a neighborhood of the steady state it is still relatively profitable being fundamentalists.

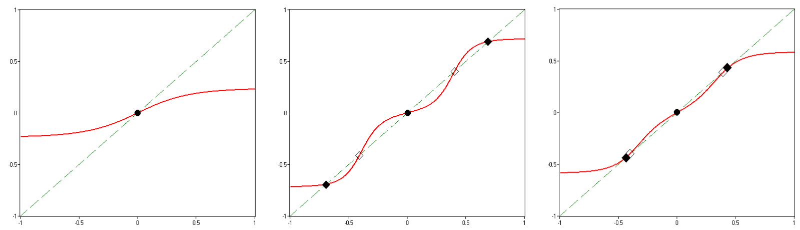
[2] We recall that also in the financial market setting considered in Chiarella et al. (2006) two stability thresholds for the unique steady state are detected. However, while in that context the stability region lies within the two thresholds, in the present framework the stability region lies outside them.

When introducing information costs, which are higher for fundamentalists than for biased agents, for intermediate values of the bias it may be relatively more profitable being optimists or pessimists than fundamentalists even for prices in a neighborhood of the steady state.[3] Hence, starting from lower values of b and increasing the bias, the basin of attraction of $x = 0$ shrinks, until a globally stable period-two cycle emerges, with agents switching between optimism and pessimism. However, when the bias is excessively large, for prices close to the equilibrium it becomes again relatively more profitable being fundamentalists, because the forecast error made by biased agents is too big, and thus $x = 0$ recovers its stability.

We conclude by illustrating in Figures 1 and 2 the possible scenarios found in Corollary 3.1 for increasing values of b, while fixing the other parameters as follows: $A = 8$, $s = 0.95$, $d = 1$, $\beta = 10$, $c = 0.1$. As concerns c_0, we focus in Figure 1 on the case $c_0 = 0.11$, in order to show that for almost coinciding values of the information costs for fundamentalists and for biased agents we obtain just the two frameworks which can occur in the absence of information costs. Namely, in Figure 1 (A) for $b = 0.4$ we find that the steady state $x = 0$ is globally stable and in Figure 1 (B) for $b = 0.8$ we observe, in addition to the locally stable steady state, denoted by a black dot, a stable and an unstable period-two cycles, denoted respectively by black and empty squares, which are born for $b \approx 0.690$ through a double fold bifurcation of the second iterate of f, that we illustrate in Figure 1 (C). Raising the value of the information cost for fundamentalists to $c_0 = 2$, in Figure 2 (A) for $b = 0.3$ we still find that the steady state $x = 0$ is globally stable, but in Figure 2 (B) for $b = 0.5$ we observe that the steady state is now unstable, and it is denoted by an empty dot, being surrounded by a globally stable period-two cycle, born for $b \approx 0.393$ through a pitchfork bifurcation of the second iterate of f, which corresponds to a flip bifurcation of f. In Figure 2 (C) for $b = 1$ the steady state $x = 0$ is again locally stable thanks to a further pitchfork bifurcation of the second iterate of f that has occurred for $b \approx 0.850$ at $x = 0$. The basin of attraction of $x = 0$ is separated by

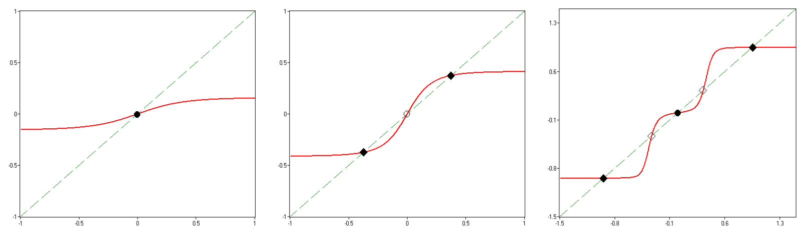
[3]In this respect, we stress that, for low values of b, when the difference in the information costs is high with respect to the bias, $x = 0$ may be globally asymptotically stable even if the net profits of fundamentalists are lower than the net profits of biased fundamentalists, also in a neighborhood of the steady state. This is indeed what happens for the parameter configuration considered in Figure 2. In fact, the introduction of information costs allows for a large variety of frameworks, whose correct interpretation requires to take into account the values both of the information costs and of the bias.

that of the locally stable period-two cycle by an unstable period-two cycle, born through the pitchfork bifurcation. We stress that in the frameworks considered in Figures 1 and 2 the period-two cycle persists for larger values of b and that the distance between the period-two points increases with b.



(A) (B) (C)

Figure 1: The graph of the second iterate of f for $c_0 = 0.11$, and $b = 0.4$ in (A), $b = 0.8$ in (B), and $b = 0.690$ in (C).



(A) (B) (C)

Figure 2: The graph of the second iterate of f for $c_0 = 0.2$, and $b = 0.3$ in (A), $b = 0.5$ in (B) and $b = 1$ in (C).

Declarations of interest: none.
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

Anufriev, M., Assenza, T., Hommes, C., Massaro, D., 2013. Interest rate rules and macroeconomic stability under heterogeneous expectations. Macroecon. Dyn. 17, 1574–1604.

Brock, W.A., Hommes, C.H., 1997. A rational route to randomness. Econometrica 65, 1059–1095.

Chiarella, C., He, X.-Z., Hommes, C., 2006. A dynamic analysis of moving average rules. J. Econ. Dyn. Control 30, 1729–1753.

Ezekiel, M., 1938. The cobweb theorem. Quart. J. Econ. 52, 255–280.

Guesnerie, R., 2002. Anchoring economic predictions in common knowledge. Econometrica 70, 439–480.

Hommes, C., 2013. Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems. Cambridge University Press, Cambridge.

Hommes, C., Wagener, F., 2010. Does eductive stability imply evolutionary stability? J. Econ. Behav. Organ. 75, 25–39.

Naimzada, A., Pireddu, M., 2019. Eductive stability may not imply evolutionary stability in the presence of information costs. Working paper version, available on Mendeley Data.