TIME INDEPENDENT CASE

STABILITY ESTIMATE : KN- unique solution

Hp. 1) $u, v \in H^2(D)$

$E_u \neq \emptyset$; meas $E_u = 0$

$\frac{1}{u_x} \in L^1(D)$

2) $E_u = \{ \xi_u^{(n)}, \{n\} \subseteq \mathbb{N} \}$; meas $\overline{E_u} = 0$

Def. 1) $I := \{ x \mid x \in \overline{D}, |x - \xi_u^{(n)}| \leq \varepsilon, \forall n \}, \varepsilon > 0$

Hp. 3) $v \in H^2(D)$, $E_v \neq \emptyset$

I	II
meas $E_v = 0$	meas $E_v > 0$
$\frac{1}{v_x} \in L^1(D)$	$\frac{1}{v_x} \in L^1(D \setminus E_v)$
$\left\| \frac{1}{v_x} \right\|_{0,1} \leq c_v$	$\left\| \frac{1}{v_x} \right\|_{L^1(D \setminus E_v)} \leq c_v$

4) $E_v \subseteq I$

5) $\exists (!)\ \hat{a}(u, f) \in \mathcal{C}^0(I) \cap A_{ad}$

Th.

I	II
$\| b - \hat{a} \|_{0,1} \leq$	$\| b - \hat{a} \|_{L^1(D \setminus E_v)} \leq$

$$\leq c_v (1 + 2 \| \hat{a} \|_{0,\infty}) \| v - u \|_{2,2}$$

Previous, related work :

P. Marcellini, 1982, <u>Ricerche di Matematica</u> $\underline{\underline{3}}$ #2, 223-43

$\hat{a} \in W^{1,2} \cap L^\infty$, $f = 1$, $u(x_0) = u(x_1) = 0$

STABILITY ESTIMATE
L^1 (KNI - unique solution)

Hp. 1) (reference potential)

$u \in \mathcal{X}$; $u \in KNI$ i.e, $E_u(\tau) \neq \emptyset$; $meas[E_u(\tau)] = 0$
$\tau \in \overline{T}$

$\left(\frac{1}{u_x}\right)(\tau) \in L^1(D)$

2) $E_u(\tau) = \{ \xi_u^{(n)}(\tau) ; \{n\} \subseteq \mathbb{N} \}$; $meas\, \overline{E_u(\tau)} = 0$

countably many points

countably many adherence points

Def 1) $I(\tau) := \{ x \mid x \in \overline{D}, |x - \xi_u^{(n)}(\tau)| \leq \varepsilon, \forall n \}$, $\varepsilon > 0$

Hp. 3) $v \in \mathcal{X}$; $E_v(\tau) \neq \emptyset$; $E_v(\tau) \subseteq I(\tau)$

I	II
$meas[E_v(\tau)] = 0$	$meas[E_v(\tau)] > 0$
$\left\| \frac{1}{v_x} \right\|_{L^1(D)}^{(\tau)} \leq C_v$	$\left\| \frac{1}{v_x} \right\|_{L^1(D \setminus E_v(\tau))}^{(\tau)} \leq C_v$

4) $\hat{a} \in \mathcal{A}_{ad} \cap \mathcal{C}^0(I(\tau))$

Th. $\| b - \hat{a} \|_{L^1(D)} \leq$ | $\| b - \hat{a} \|_{L^1(D \setminus E_v(\tau))} \leq$

$\leq C_v (1 + 2 \| \hat{a} \|_{0,\infty}) \| v - u \|_{\mathcal{X}(\tau)}$

I II

TECHNICAL LEMMATA: *the Gronwall – Bellmann Inequality Extended*

Lemma 1

$D := (x_0, x_1)$, $a \in A_{ad}$,

$q \in Q_{ad} := \{ q \mid q \in L^1(D),\ q \geq_{\text{a.e.}} 0\}$, $c_+ \geq 0$, const.

IF $a \leq_{\text{a.e.}} c_+ + \int_{x_0}^{x} aq\, ds$,

THEN $\|a\|_{0,\infty} \leq c_+ \exp \|q\|_{0,1}$

Lemma 2

Hp. $g \in B_{ad}$

$\frac{h_{xx}}{h_x} \in L^1(D)$; $\{(\frac{r}{h_x})^{[-1]}\} \subset L^\infty(D)$; $\exists \lim_{x \to x_0^+} (\frac{r}{h_x})^{[-1]}$

$g_x h_x + g h_{xx} - r =_{\text{d.w.}} 0$

Th. $\|g\|_{0,\infty} \leq \|(\frac{r}{h_x})_0^{[-1]}\|_{0,\infty} \exp \|\frac{h_{xx}}{h_x}\|_{0,1}$

TIME INDEPENDENT CASE : BASIC DEF AND HP

The Inverse Problem in $D := (x_0, x_1)$

given the *potential*

$$u \in U_{ad} := \{u \mid u \in H^1(D);\ u_x \in C^0(\bar{D} \setminus S_u)\}$$

the *source term*

$$f \in H^{-1}(D)$$

find the *conductivity*

$$a \in A_{ad}$$

s.t.

$$(a u_x)_x =_{\text{d.w.}} f$$

TIME INDEPENDENT CASE

STABILITY ESTIMATES: *Cauchy – unique solution*

Hp.1 $u, v \in H^2(D)$;

$\exists!\, \hat{a} \in \mathcal{C}^1(\bar{D} \setminus \{y_i\})$

(piecewise differentiable)

$|\frac{1}{v_x}| \leq c_V$, $\forall\, x \in \bar{D}$; $||v_{xx}||_{0,2} \leq c_2$

Def.1 $r := (a(v-u)_x)_x$; $R := \frac{r}{v_x}$

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Hp.2 $\exists \lim_{x \to x_0^+} \hat{a}_x$; $\exists \lim_{x \to x_1^-} \hat{a}_x$; $\{|\hat{a}_x|^{[-1]}\} \subset L^\infty(D)$

$g \in B_{ad}$; $\{R^{[-1]}\} \subset L^\infty(D)$; $\exists \lim_{x \to x_0^+} R^{[-1]}$

Th. $||\, b - \hat{a}\, ||_{0,\infty} \leq$

$\leq [1 + ||\hat{a}||_{0,\infty} + ||\, |\hat{a}_x|_0^{[-1]} ||_{0,\infty}]\, c_V\, ||v-u||_{2,2}$.

$.\exp[c_V c_2 \sqrt{\text{meas}[D]}\,]$

Previous, related work: F. Baumeister, K. Kunisch, 1991, Applicable Analysis

$\hat{a} \in W^{1,\infty}$, Cauchy uniqueness

UNIQUENESS *vs* TIME AVERAGING

Def. $\quad u := \dfrac{1}{\text{meas}[D]} \displaystyle\int_T u(t)\,dt$, etc.

Thm.

IF $\quad \{\, u,\ -u_t + f \,\} \longrightarrow \{\, u,\ f \,\}$

$\quad \exists\ \hat{a}\,(u, f) \in A_{ad}$; u satisfies *KN2.1*

THEN $\quad \exists!\ \hat{a} \cdot\!\ni\!\cdot\ (a u_x)_x =_{\text{d.w.}} f$

Rem. $\quad u$ satisfies either *KN1* or *KN2* or *KN2.2* $\not\Rightarrow$

$\not\Rightarrow\ \exists!\ \hat{a}$ from time averaged data.