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Fondo Crosta - Giovanni Franco

Subfondo

Raccolta

Fascicolo 1991-0912 - DCA - ICIAM

Sottofascicolo

Documento 1991-0912 - ICIAM - Vorstellung

(Allegato) 1991-0912 - ICIAM - Vorstel01
du mande 0 milia Vorstel02

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TIME INDEPENDENT CASE
STABILITY ESTIMATE : KN-unique solution

Hp. 1) $u, v \in H^2(D)$
 $E_u \neq \emptyset$; $\text{meas } E_u = 0$
 $\frac{1}{u_x} \in L^1(D)$

2) $E_u = \{ \xi_u^{(n)}, \{n\} \subseteq \mathbb{N} \}$; $\text{meas } \overline{E_u} = 0$

Def. 1) $I := \{ x \mid x \in \overline{D}, |x - \xi_u^{(n)}| \leq \varepsilon, \forall n \}$, $\varepsilon > 0$

Hp. 3) $v \in H^2(D)$, $E_v \neq \emptyset$

I
 $\text{meas } E_v = 0$
 $\frac{1}{v_x} \in L^1(D)$
 $\| \frac{1}{v_x} \|_{0,1} \leq C_v$

I
 $\text{meas } E_v > 0$
 $\frac{1}{v_x} \in L^1(D \setminus E_v)$
 $\| \frac{1}{v_x} \|_{L^1(D \setminus E_v)} \leq C_v$

4) $E_v \subseteq I$

5) $\exists (!) \hat{a}(u, f) \in \mathcal{C}^0(I) \cap \text{Ad}$

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Th.

I
 $\| b - \hat{a} \|_{0,1} \leq$

I
 $\| b - \hat{a} \|_{L^1(D \setminus E_v)} \leq$

$\leq C_v (1 + 2 \| \hat{a} \|_{0,\infty}) \| v - u \|_{2,2}$

Previous, related work :

P. Marcellini, 1982, Ricerche di Matematica \cong #2, 223-43

$\hat{a} \in W^{1,2} \cap L^\infty$, $f = 1$, $u(x_0) = u(x_1) = 0$

STABILITY ESTIMATE

L^1 (KNI - unique solution)

Hp. 1) (reference potential)

$$u \in \mathcal{X} ; u \in \text{KNI i.e., } E_u(\tau) \neq \emptyset ; \text{meas}[E_u(\tau)] = 0 \quad \tau \in \bar{T}$$

$$\left(\frac{1}{u_x}\right)(\tau) \in L^1(D)$$

$$2) E_u(\tau) = \{ \bar{x}_u^{(n)}(\tau) ; \{n\} \in \mathbb{N} \} ; \text{meas } \overline{E_u(\tau)} = 0$$

countably many points countably many adherence points

Def 1) $I(\tau) := \{x \mid x \in \bar{D}, |x - \bar{x}_u^{(n)}(\tau)| \leq \varepsilon, \forall n\}, \varepsilon > 0$

Hp. 3) $v \in \mathcal{X} ; E_v(\tau) \neq \emptyset ; E_v(\tau) \subseteq I(\tau)$

I

$$\text{meas}[E_v(\tau)] = 0$$

$$\left\| \frac{1}{v_x} \right\|_{L^1(D)}(\tau) \leq C_v$$

II

$$\text{meas}[E_v(\tau)] > 0$$

$$\left\| \frac{1}{v_x} \right\|_{L^1(D \setminus E_v(\tau))}(\tau) \leq C_v$$

4) $\hat{a} \in A_{ad} \cap \mathcal{C}^0(I(\tau))$

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Th. $\|b - \hat{a}\|_{L^1(D)} \leq$

$$\|b - \hat{a}\|_{L^1(D \setminus E_v(\tau))} \leq$$

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$$\leq C_v(1 + 2\|\hat{a}\|_{q,\infty}) \|v - u\|_{\mathcal{X}(\tau)}$$

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I

II

TECHNICAL LEMMATA: *the Gronwall – Bellmann Inequality Extended*

Lemma 1

$$D := (x_0, x_1), \quad a \in A_{ad},$$

$$q \in Q_{ad} := \{ q \mid q \in L^1(D), q \geq_{\text{a.e.}} 0 \}, \quad c_+ \geq 0, \text{ const.}$$

$$\text{IF} \quad a \leq_{\text{a.e.}} c_+ + \int_{x_0}^x a q \, ds,$$

$$\text{THEN} \quad \|a\|_{0, \infty} \leq c_+ \exp \|q\|_{0,1}$$

Lemma 2

Hp.

$$g \in B_{ad}$$

$$\frac{h_{xx}}{h_x} \in L^1(D); \quad \left\{ \left(\frac{r}{h_x} \right)^{[-1]} \right\} \subset L^\infty(D); \quad \exists \lim_{x \rightarrow x_0^+} \left(\frac{r}{h_x} \right)^{[-1]}$$

$$g_x h_x + g h_{xx} - r =_{\text{d.w.}} 0$$

Th.

$$\|g\|_{0, \infty} \leq \left\| \left(\frac{r}{h_x} \right)^{[-1]} \right\|_{0, \infty} \exp \left\| \frac{h_{xx}}{h_x} \right\|_{0,1}$$

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TIME INDEPENDENT CASE : BASIC DEF AND HP

The Inverse Problem in $D := (x_0, x_1)$

given the *potential* *Biblioteca*

$$u \in U_{ad} := \{u \mid u \in H^1(D); u_x \in C^0(\bar{D} \setminus S_u)\}$$

the *source term*

$$f \in H^{-1}(D)$$

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find the *conductivity*

$$a \in A_{ad}$$

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s.t.

$$(au_x)_x =_{\text{d.w.}} f$$

TIME INDEPENDENT CASE

STABILITY ESTIMATES: *Cauchy – unique solution*

Hp.1

$$u, v \in H^2(D);$$

$\exists! \hat{a} \in \mathcal{C}(\bar{D}, \{y_i\})$
(piecewise differentiable)

$$|\frac{1}{v_x}| \leq c_v, \forall x \in \bar{D}; \|v_{xx}\|_{0,2} \leq c_2$$

Def.1

$$r := (a(v - u)_x)_x; R := \frac{r}{v_x}$$

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Hp.2

$$\exists \lim_{x \rightarrow x_0^+} \hat{a}_x; \exists \lim_{x \rightarrow x_1^-} \hat{a}_x; \{|\hat{a}_x|^{[-1]}\} \subset L^\infty(D)$$

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$$g \in B_{ad}; \{R^{[-1]}\} \subset L^\infty(D); \exists \lim_{x \rightarrow x_0^+} R^{[-1]}$$

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Th.

$$\|b - \hat{a}\|_{0,\infty} \leq$$

$$\leq [1 + \|\hat{a}\|_{0,\infty} + \|\hat{a}_x|^{[-1]}\|_{0,\infty}] c_v \|v - u\|_{2,2} \cdot$$

$$\cdot \exp[c_v c_2 \sqrt{\text{meas}[D]}]$$

Previous, related work: F. Baummeister, K. Kunisch, 1991, Applicable Analysis
 $\hat{a} \in W^{1,\infty}$, Cauchy uniqueness

UNIQUENESS vs TIME AVERAGING

Def. $u := \frac{1}{\text{meas}[D]} \int_T u(t) dt$, etc. *Biblioteca*

Thm.

IF $\{u, -u_t + f\} \longrightarrow \{u, f\}$ *Zuadrelli*
 $\exists \hat{a}(u, f) \in A_{ad}$; u satisfies KN2.1

THEN $\exists! \hat{a} \cdot \exists \cdot (au_x)_x =_{\text{d.w.}} f$ *Crosta*

Rem. u satisfies either KN1 or KN2 or KN2.2 \Rightarrow
 $\Rightarrow \exists! \hat{a}$ from time averaged data.