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Introduction

This thesis contains two single-authored papers, each one being the outcome of an independently conducted project. Although the two papers have not been conceived as companions, both reflect my research interests in Industrial Economics, and more specifically in Regulation and Competition Policy. Theoretical models are therefore developed and applied to depict the behaviour of firms, consumers, and regulatory agencies, and then used to offer guidance with respect to normative issues. A common theme, connecting the two works and, more generally, characterizing my research, is the role of market power. In what follows, I loosely introduce the topic and focus of each paper without any pretense to be exhaustive or precise, instead I wish to provide the reader with some of the driving ideas and structures which can be found in my work. Currently a harshly debated topic, the role played by market power in many industries must be carefully understood before and whenever an agency wishes to intervene, in order to anticipate the reaction of the industry and fairly evaluate the consequences on consumers. I considered two among the most controversial ones: the renewable energy industry, and the digital industry. In each of these industries, market power cannot be considered a temporary or contingent feature, rather, it is a structural element which characterizes the competitive environment, and it cannot either be thought of as a negligible departure from the marginal cost pricing rule, the ideal efficient pricing rule. Take the case of the electricity market first. Due to a series of well-known factors: the regional nature of energy production, the scarce storage capacity, and chiefly the high installation costs that might make it a case of natural monopoly, competition has been a problem since the beginning (almost an original sin). Such a market is - and it has been for a long time – populated by many producers, most of them small suppliers, and by just a few big and important players. While the former most often act like price-takers, the latter strategically exploit their advantage position to manipulate the price and boost their profits. The most recent policies designed to cope with climate change and abate emissions involve the use of subsidies as monetary incentives to push firms to invest more in renewable capacity. However, in doing so, such policies create the opportunity for price-making firms to strategically plan the investments over time, adjusting timing and funds of the investments to cash-in more subsidies, hence partially nullifying the effects of the policies and making them more expensive. It follows that the regulator must carefully discern and disentangle the possible effects of a subsidy policy when facing such an environment, and for this reason I investigate the effects of a subsidy-and-cap strategy, which works in both directions: it provides a monetary incentive, and reduces the temptations of strategic delay with a price cap. To capture the issue, I follow a Real Options approach, which makes it possible to represent the long-term effects of policies under a tractable, yet rigorous framework.

Another market, way younger than the electricity one, is raising even more serious concerns for competition: the digital market. This latter one is also characterized by a few big players, however, there does not seem to be substantial barriers to entry. What seems to dampen competition are economies of scale and data. In my work, I present a model of a data intermediary who can extract data - a byproduct of some service she provides - from consumers, representing their preferences, and then allocate such data among product firms. The amount of data generated is connected to the quality of the service: the better the service, the more consumers will use it. The strategic allocation of information allows the intermediary to extract more extra-profits from the product firms through specifically designed tariffs. The source of extra-profits is the possibility for firms to use information to personalize prices offered to consumers, however, since data are strategically allocated by a gatekeeper, the known effects of competitive price discrimination are erased, and only a portion of consumers is offered personalized prices, the rest of them are offered

uniform prices. This sort of behaviour presents two conflicting effects that are puzzling to an Antitrust Authority: on one hand, more profits are generated, possibly decreasing the portion of surplus allocated to consumers, on the other hand uniform prices fall, increasing market participation. A fascinating feature of the current information technology is given by the scale economies they produce: in this model, the scale effect affects data production. As the number of firms intermediated (later called size of the intermediary) grows, more data are collected, but the number of firms, to which data are sold, does not affect the cost of running the service. Hence, more data allow for more extra-profits and more firms increase the marginal revenue of the intermediary, who will offer a better service, used by more consumers, generating more data. This cycle ultimately grants the intermediary the lion's share of the extra-profits. Furthermore, it gives the intermediary another weapon: by internalizing such a scale effect, she can make better offers than a potential identical entrant, and, if push comes to shove, engage in limit pricing (with respect to tariffs), preventing entry, and monopolizing the intermediation market. However, in this case, due to the internalized scale effect, the limit pricing strategy is profitable and might not result in low tariffs, potentially evading the scrutiny of the authorities.
Finally, *Chapter 3* contains some final thoughts.

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An enormous amount of gratitude is owed to my family and my mother: their constant support and love has been an essential ingredient in everything I have done, and hopefully continue to do.

Contents

Chapter 1

Renewable Capacity Expansion Policy, Subsidies and Market Power: a real options approach

Abstract. Industrialized countries are currently concerned with the adoption of "green" technologies. The EU is planning to abate CO2 emissions down to zero by 2050, devoting many resources to policies to incentivize investments in renewable plants, i.e. building new capacity. I use real options techniques to investigate the adoption and expansion paths of such capacity by a monopolistic plant, taking into account factors such as externalities, volatility and elasticity of demand, and land heterogeneity. By contrasting the investment strategies of a Monopolist and of a Ramsey Planner, I analyse the role of public subsidies (*price premia*) on both the optimal (and/or efficient) investment timing and the expected growth rates and long run distributions. I propose a subsidy-cap scheme composed by a price premium that increases over time and a price cap. The former increases the speed of the investment process, while the latter fights the incentive to delay that the Monopolist has, and helps controlling the expenditure.

JEL codes: C61,L51,L52,L94

1.1 Introduction

In light of the ambitious plans put forward by the USA and the EU to reduce emissions and pollution, investments in renewable energy sources and their production assume assume great importance. The goal of this paper is to argue for a support scheme for renewable energy which is financially feasible over time and can help both reach production targets and control the expenditure for subsidies. While it is true that many of the policies currently adopted invoke the use of subsidies to promote the adoption of green technologies, these policies often turn out to be quite expensive for the taxpayer. Furthermore, since market power has a prominent role in the energy industry, they are likely to give the price-making firms a chance to turn a substantial part of the subsidy into more profits, therefore reducing the effectiveness of the policy.
Most applied models, employing the Real Options framework, present cases of one-shot investments in new capacity, where subsidies alter the timing and the amount of the investment. This approach is fairly adequate and convincing when the idea is to promote the entry of new *fringe producers*, i.e. small producers who are likely to have a negligible market power, which could also help make the energy markets more competitive. However, there are also big producers - such as Enel in Italy, Endesa in Spain, etc...- who are interested in developing renewable electricity. It is then hard to imagine that such firms would simply invest all their capital in one lumpy investment, rather it appears more realistic that they would try to exploit their significant market power to programme and enact a long-term strategy. The question then becomes how to steer such expansion plans in order to reach regulatory goals and contain the cost of subsidies.

Regulatory Framework. The European Directive 2018/2001 (2018) enumerates various instruments to promote an increase in renewable energy production and diffusion among consumers. In particular, it sets a *target* for 32% of renewable energy production to be reached within 2030 (8), and encourages Member States to facilitate investments (12). It also points out the necessity

of *commitment* by the institution to reliable support scheme and *affordable* prices (29). Combining the above requirements, it can be deduced that reaching the production target is not the only thing that matters in this instance. In fact, while public support schemes are probably inevitable, they should be calibrated in such a way to control for their costs: it would be pointless to care about the final price to the consumer, without controlling the amount of resources (taxes) that go into the funding of the support plans. On the other side of the Atlantic Ocean, the American Renewable Energy Act (2021) has been approved. It promotes similar goals with respect to energy production and environmental protection, but goes even further. In fact, it sets an ambitious 70% renewable electricity production goal for 2030 (including hydroelectric).
In this context, it becomes crucial to understand what drives such investments and how capacity is expanded to reach the policy goals proposed by Governments.

Methodology. I apply Real Option theory to examine the case of generation capacity expansion by a monopolistic firm, undertaking *irreverible* investments. The market is characterized through an inverse demand perturbed by stochastic process. Of course the electricity markets are not only populated by monopolists, there are also smaller fringe firms, hence what will be referred to as a Monopolist in what follows should be construed as a monopolist on the residual demand. The justification for restricting the attention to one monopolist is twofold. Firstly, electricity markets exhibit a certain degree of market power, which is of primary interest in this paper. Secondly, the model presented in Grenadier (2002) easily allows to extend the analysis to a case of symmetric oligopolistic firms competing à la Cournot in a myopic fashion. In other words, the firms act strategically when choosing the optimal capacity, as in a Cournot oligopoly, but the decision to exercise of the options to expand over time is made without taking into account the timing of the others.
Seminal contributions, such as McDonald and Siegel (1986), examine the role of volatility in discussing irreversible investments, and find that it induces the firm to wait longer before investing, and in particular that the firm might de-

cide to wait until the value of the project is significantly above sunk costs.
On the front of capacity expansion over time, Dangl (1999) finds that uncertainty influences the decision in two ways: To greater uncertainty correspond both a delay in the investment timing and an increase in the chosen optimal capacity.
Furthermore, Pindyck (1988) argues that the possibility to gradually increase capacity (through marginal unitary investments, in his model) increases the value of the value of the firm, with respect to the case of one shot investment.

Application to Capacity Installation. Bøckman et al. (2008) applies Real Option valuation to the case of small hydro-power plant, identifying the determinants of size and timing, with particular regard to the case of Norway, when the project is subject to uncertainty in electricity prices. The issue of subsidies and their impact on capacity choice has been analysed, for instance, in Boomsma, Meade, and Fleten (2012) and Boomsma and Linnerud (2015) for the Nordic market. The authors argue that feed-in tariffs cause plants to be built before with respect to tradable green certificates, while the second seem to induce firms to install greater capacity. Policy uncertainty is also examined: if anticipated, policy uncertainty slows down investments, furthermore a calibrated mix of feed-in tariffs and certificates help reducing the risks connected to the Regulator's behaviour. Yang et al. (2008) argues that policy uncertainty (represented by carbon price uncertainty) also translates into a price premium in electricity investments (slowing them down).
In most of the works reported above, the firm is considered as a price taker in the context of a perfectly competitive market. Uncertainty sources, let them be from prices or regulatory interventions, are all exogenous. This set of hypothesis works well in the analysis of such fringe firms, as the one mentioned above, however it fails to confront the issue of market power in the electricity markets. The model proposed in Bigerna et al. (2019) introduces the monopolistic behaviour jointly with the issue of calibrating an optimal subsidy (in the form of a price premium). In fact, the authors propose a model in which the inverse demand is linear and the intercept is governed by a stochastic process,

causing a trade-off between timing and size of the investment: as the subsidy grows, the firm invests sooner, but adopt a lower capacity. Then Nagy, Hagspiel, and Kort (2021) introduces policy uncertainty in a similar context, but with different inverse demand, finding that policy withdrawal risk is detrimental to both welfare and the probability of reaching environmental targets.

Roadmap. In examining the role of market power, this paper is akin to those of Bigerna et al. (2019) and Nagy, Hagspiel, and Kort (2021), however there a fundamental difference in how the problem is put forward. I consider an expansion plan by a Monopolist, which has already some capital installed and is already inside the market: The firm[1] gradually expands her capacity in time, differently from the models mentioned above where the investment is always portrayed as a one-shot deal. Thus, in *Section 2* I move to analyse the differences between the investing behaviour of the Monopolist and the one of the Social Planner, keen on maximizing social surplus. *Section 3* develops a simple and tractable model of externalities, and finds in them the justification for "speeding up" the expansion process. Grounded in the analysis carried out in the previous section, *Section 4* presents a model of targeting policy, concluding that gradual expansion and market power render a fixed price premium (i.e. fixed proportion of current price) useless [2] to increase the speed of capital accumulation, arguing instead that the Regulator should commit to a subsidy scheme increasing over time. The introduction of such support schemes might solve a problem in terms of incentives, but creates another in terms of sustainability. In fact, how much and for how long can the taxpayer afford to keep these policies going? To set a limit to the expenditure, I introduce a price-cap[3], and show that a balance can be found between pushing

[1]I shall refer as "she" to the firm, or the Monopolist, and use "he" for the Social Planner and the Regulator.

[2]Technically, fixed subsidies decrease only the *time until first action*, i.e. the wait until the first, or only, investment is done.

[3]See also Willems and Zwart (2018) and Broer and Zwart (2013) for applications of Real Options technique in regulation.

the development and fiscal concerns. On the introduction of price-caps in real option models, the work of Dobbs (2004) provides a treatment considering a monopolist, while Roques and Savva (2009) consider an oligopolistic model extending the work of Grenadier (2002).
I approach the problem by standard arguments similar to Dixit (1993) and Dixit and Pindyck (1994) (Ch. 11) to examine the long-run trends in capacity adoption and generation of renewable energy. In the spirit and the methodology, this paper draws a lot of suggestions also from Guthrie (2010).
Proofs to propositions are rather repetitive application of standard Dynamic Programming techniques, not much more, hence they are relegated to *Appendix A*.

1.2 Basic model and Assumptions

This section is devoted to the laying out a simple model of monopolistic capacity expansion path. Such model will be extended in the following parts of the paper to take into account more realistic features in order to discuss policy issues.
The choice to model such as specific case, as it is the one of the monopolist, is grounded in the fact that the field of interest of this paper is the electricity market, which is indeed populated by a number of suppliers, however competition if far from perfection. The presence of *large producers*, common to many countries in Europe, combined to the scarce room left to import/export of electricity and the regional nature of some ancillary services, make for a reasonable argument about firms' market power. Although the assumption of monopolistic competition might be seen as extreme, quite similar conclusions could be argued in the context of a Cournot competition with *myopic firms*, as in Grenadier (2002).
As stated at the beginning, since the ultimate goal of the paper is the dynamics of investment, I will assume the monopolist to already have some capacity installed, and for the sake of simplicity depreciation is neglected. Consider then a monopolist with capital $k_0 > 0$ at the beginning of the infinite time

horizon $t = [0, +\infty)$, and a production function mapping capital $k_t \in R_+$ into capacity $q_t \in R_+$ given by

$$q_t = k_t^{\frac{1}{\theta}} \ , \tag{1.1}$$

where $\theta \in [1, \infty)$, a linear cost function

$$C(\Delta k_t) = c\Delta k_t, \ \ c \in R_+ \ , \tag{1.2}$$

while operative costs are normalized to zero. This assumption about costs has a twofold explanation. First, marginal cost of production of renewable plants are often very low, hence it is only convenient to neglect them, as they bear very little importance. However, analysis of investment models which take those into account can be found (with references) in e.g. Dixit (1993). Secondly, the presence of positive marginal cost of production could make it more convenient for the firm to temporarily stop production, which is extremely unlikely to happen in the case under examination, both because the increasing demand and regulatory interest for more renewable energy sources.
This specification guarantees decreasing return to scale, which will later turn out to be a condition for the solution of the optimization problem to converge. Notice indeed that:

$$\frac{\partial q_{it}}{\partial k_t} = \frac{1}{\theta} k_t^{-\frac{\theta-1}{\theta}}$$

and

$$\frac{\partial^2 q_{it}}{\partial k_t^2} = -\frac{\theta-1}{\theta^2} k_t^{-\frac{2\theta-1}{\theta}} < 0 \ .$$

However, in the present context of the "physical" expansion of a plant, it has also a more appealing and interesting interpretation. It can be understood, in fact, to be a parameter of *land heterogeneity*, which causes marginal return of capital to fall in time. To see this, imagine a firm[4] building her plant a piece at a time increasing overall capacity. Construction will start from more productive land lots, and only then the expansion process will move into less productive areas. This observation assumes a more particular meaning when studying the production of renewable energies. Indeed the rational expansion strategy will start from investing from lots where the exposition to the Sun is

[4] In the following also referred to as "she"

higher, in the case of solar energy or more windy, in the case of wind power.
On the demand side of the economy, I assume there are a number of identical consumers denoted by $i = 1, \ldots, n$, each of them a price-taker, with a continuous and (at least) twice differentiable utility function over R_+

$$u_i(q_{it}) = y_t \frac{q_{it}^{1-\eta}}{1-\eta} \ , \tag{1.3}$$

where the parameter η is such that $\eta \in (0,1)$. The stochastic process y_t follows instead a geometric Brownian motion (GBM) characterized as $\{y_t\}_{t\in[0,\infty)}$:

$$dy_t = \alpha y_t \, dt + \sigma y_t \, dz_t \ , \ y(0) = y_0$$

with dz_t being a standard Wiener process: $dz_t \sim N(0, dt)$.
It should be noticed that despite the stochastic disturbance, the utility function will still be isoelatic at any point in time, no matter the value takes by y_t.
I start by examining the equilibrium behaviour of the firm and comparing it to the investment plan that an ideal social planner would follow in a *first best* scenario. When evaluating the option to expand the two actors discount instantaneous inflows by the same rate, say $r > \alpha$. Both these cases are barrier control problems with an upper reflecting barrier: the goal is to find some threshold level for the process y_t, say $\hat{y}(k_t)$, such that whenever reached from below, the firm or the Social Planner will increase the invested capital k_t by the minimum amount necessary to restore the condition $y_{t+dt}(k_t) \leq \hat{y}_t(k_t)$.
The expansion of the plant is then a discontinuous process, which evolves by small strictly positive steps at random times over the horizon $[0, \infty)$.

1.2.1 Firm value

I begin by providing some microfoundations that will hold for the remainder of the paper. Consider, then, the individual choice of the consumer, deriving the individual demands first and then aggregating to find the market demand, which is assumed to be common knowledge.
At each instant t, each consumer maximizes his own payoff

$$\max_{q_{it}} v(q_{it}) = u_i(q_{it}) - p_t q_{it} \ ,$$

which delivers an individual (*Marshallian*) demand function:

$$q_{it}(p_t) = \left(\frac{p_t}{y_t}\right)^{-\frac{1}{\eta}} . \tag{1.4}$$

The aggregation process of individual demands into the (inverse) market demand, $q_t(p_t) = \sum_i q_{it}(p_t)$, simply yields

$$p = y_t n^{\eta} q_{it}^{-\eta} = y_t \, n^{\eta} k_t^{-\frac{\eta}{\theta}} , \tag{1.5}$$

which is the typical form of an isoelastic demand, with η being the inverse of the price-elasticity.

The monopolist approach the problem by choosing the capacity path which maximizes the value of the firm over time, taking into account the stochastic growth process of demand. Let $\pi(k_t)$ denote the deterministic instantaneous profit inflow at time t defined as

$$\pi(k_t) = n^{\eta} k_t^{\frac{1-\eta}{\theta}}$$

and let $V(k_t, y_t)$ denote the present value of inflow, which is given by

$$V(k_t, y_t) = y_t \pi(k_t) \, dt + e^{-r\,dt} E[V(k_t, y_t + dy_t)] . \tag{1.6}$$

Suppose now that at time t capacity is expanded by increasing the capital stock from k_t to k_t' and paying a unit cost c (as described in (1.2)), then the firm's value will be given by the equilibrium relationship

$$V(k_t, y_t) = V(k_t', y_t) - c(k_t' - k_t) .$$

The intuition is immediate: the firm will choose to invest further up to k_t' when the present value of the profits inflow given capital k_t equals the present value net of the cost of expansion.

To find a solution for the optimal expansion path, I adopt a standard Real Option analysis as outlined in Dixit and Pindyck (1994), by making use of Dynamic Programming. The set of necessary (and sufficient too, in this case) conditions is readily found as

$$\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2} + \alpha y \frac{\partial V}{\partial y} - rV + y\pi = 0 \tag{1.7}$$

$$\frac{\partial V}{\partial k} = c \tag{1.8}$$

$$\frac{\partial^2 V}{\partial k \partial y} = 0 \tag{1.9}$$

$$\lim_{y \to 0} V(y) = 0 \tag{1.10}$$

where (1.7) is the usual Bellman equation describing the optimal dynamics of investments, while (1.8) is the necessary condition to ensure that the marginal increase in the firm value offset by the increase in capital equals the marginal cost of expansion c. The last equation, (1.9) is a *smooth pasting* condition, a boundary condition which ensures the desirable feature of continuity of the value function along the optimal path[5]. Condition (1.10), also called *"no bubble"* condition, imposes that the Value Function should go to zero as the process y_t goes too, in order to avoid monetary speculations, which have no place in this model of irreversible investments. Loosely speaking, as the price falls, the value of the investment decreases because the cash flows generated by the investment decrease, however, when the price is very low, the probability of it increasing in the future is high, so one could speculate on that. This second component has sense if the investor can cash out of the investment, but in this case that is not possible, hence condition (1.10) captures the irreversibility feature of this model.

By standard arguments, I can now determine the optimal path as stated in the following proposition.

Proposition 1. *A monopolist having an initial capital* k_0*, facing market demand* (1.5)*, given* y_0*, with an expansion cost function as in* (1.2)*, will choose*

[5]The "smooth pasting" conditions belong to a family of requirements put in place so that the solution satisfies some degree of continuity. For this reason, they should be regarded as properties the author would like the solution to have, hence they can be justified out of economic intuition, not mathematical necessity. See Dixit (1993) for a discussion.

to invest each and every time the stochastic growth process hits the value

$$\hat{y}(k_t) = \frac{\beta}{\beta - 1} \frac{(r - \alpha)c}{n^\eta} \frac{\theta}{1 - \eta} k_t^{\frac{\theta+\eta-1}{\theta}} \tag{1.11}$$

just enough capital dk *to guarantee* $y_t = \hat{y}(k_t + dk)$*, where*

$$\beta = \frac{1}{2} - \frac{\alpha^2}{\sigma^2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{\alpha^2}{\sigma^2}\right)^2} > 1$$

is the positive root of the fundamental quadratic

$$\beta(\beta - 1)\sigma^2 + \beta\alpha - r = 0 \ .$$

Furthermore, for $y_t \leq \hat{y}(k_t)$*, the value of the firm is*

$$V(k_t, y_t) = y_t \frac{n^\eta k_t^{\frac{1-\eta}{\theta}}}{r - \alpha} + B(k_t) y_t^\beta \tag{1.12}$$

with $B(k_t)$ *being*

$$B(k_t) = \left(\frac{\beta - 1}{c}\right)^{\beta - 1} \left(\frac{n^\eta}{\beta(r - \alpha)} \cdot \frac{1 - \eta}{\theta}\right)^\beta \frac{\theta}{\beta(\theta + \eta - 1)} k_t^{\beta\frac{(\theta+\eta-1)}{\theta}} \ .$$

The factor $\beta/(\beta-1)$, also called the *real option multiplier*, is a common feature in this stream of literature. It can be shown that since

$$\frac{\partial \beta}{\partial \sigma} < 0$$

the multiplier $\beta/(\beta-1)$ increases in the volatility parameter σ. From this fact follows that the threshold level $\hat{y}_t(k_t)$ increases, implying that the firm would rather invest later then sooner.

The parameters used in the following graphs are as in the next table.

Table 1.1: Parameters' values

θ	η	σ	c	r	α	n
1.3	0.6	0.05	70	0.06	0.03	100

Figure 1.1: Value of $\hat{y}_t(k_t)$ at the varying of θ in different moments in time (k_t.)

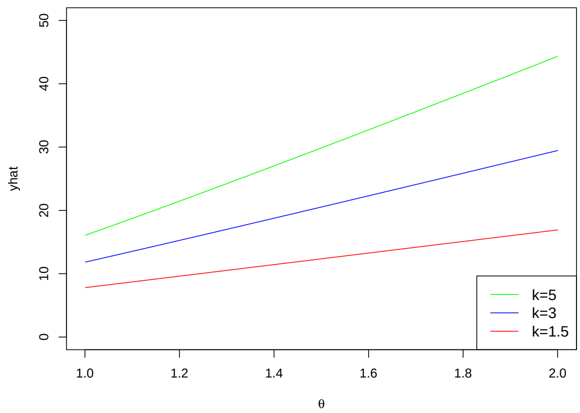
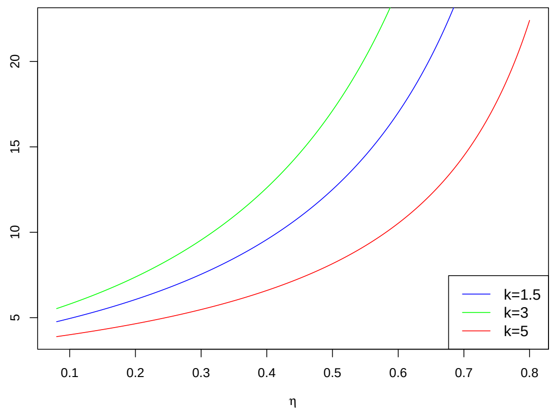


Figure 1.1 shows the impact of land heterogeneity on the investment threshold. Clearly, the parameter θ is a strong determinant in the decision of the firm about when to invest. As land heterogeneity grows, the firm would invest later since the marginal return on investments decreases with θ and in more substantial amounts when she does. Hence, the capital accumulation process will be more "fragmented": less frequent addition to capacity, but when a decision to expand is taken , the plant expands more, as θ grows. Over time, this effect becomes more important and can be seen in the fact that the curves shift upwards and become steeper. Intuitively this means that as the level of invested capital grows in time (dk_t is always strictly positive), investments, hence capacity capacity expansions, become rarer.

All together, land heterogeneity slows down the development of the plant, causes the expansion to happen with less regularity, and the bigger the plant is, the more important the size of these two effects becomes.

Figure 1.2: Value of $\hat{y}_t(k_t)$ at the varying of η in different moments in time (k_t).



The effect of the elasticity are instead less obvious. In a first moment, a high price-elasticity[6] of demand increases the value of waiting. As market demand becomes more inelatic, then the increase in market power favours a sooner development of the project. However, in cases where the demand is completely price-inelastic, the Monopolist delays the investments. As the demand for electricity is itself rather inelastic, the last part of the curves in Fig. 1.2 is likely to be the case.

As in Fig. 1.1, the expansion threshold increases over time, and in this case, an high value of η amplifies the upward twist in the last part of the curves. Overall, it can be seen that η likely decreases the investment threshold in the beginning, but later the effects can be reverted, and expansion delayed for high value of η.

1.2.2 Socially Optimal Investment

In order to asses the quality of the monopolist decision, it is necessary to provide a benchmark of comparison. Such benchmark can be found in assuming the firm is run by a Social Planner[7] whose intent is to maximize the social surplus at each point in time and programs the investment policy accordingly.

[6] I am referring to $\epsilon = 1/\eta$

[7] Also referred to as "SP" in what follows.

To this purpose, define

$$w(k_t) = \int_0^{q_t} n^\eta Q^{-\eta} dQ = \frac{n^\eta}{1-\eta} q_t^{1-\eta} = \frac{n^\eta}{1-\eta} k_t^{\frac{1-\eta}{\theta}}$$

to be the (deterministic part of the) instantaneous flow of social surplus at time t when the invested capital equals k_t. Analogously to the previous case, the social value of the firm is now given by

$$W(k_t, y_t) = y_t w(k_t)\, dt + e^{-r\, dt} E[W(k_t, y_t + dy_t)] \ . \tag{1.13}$$

and upon an expansion decision is made, the following equality must hold

$$W(k_t, y_t) = W(k_t', y_t) - c(k_t' - k_t)\, .$$

The socially optimal path can be found by a set of conditions similar to the ones I used for the monopolist:

$$\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 W}{\partial y^2} + \alpha y \frac{\partial W}{\partial y} - rW + yw = 0 \tag{1.14}$$

$$\frac{\partial W}{\partial k} = c \tag{1.15}$$

$$\frac{\partial^2 W}{\partial k \partial y} = 0 \tag{1.16}$$

$$\lim_{y \to 0} W(y) = 0 \tag{1.17}$$

with every equation keeping the same meaning and interpretation. By the same techniques, one gets the following proposition.

Proposition 2. *A Social Planner willing to maximize the social surplus as extracted from market demand* (1.5)*, with an expansion cost function as in* (1.2)*, will choose to invest each and every time the stochastic growth process hits the value*

$$\hat{y}^o(k_t) = \frac{\beta}{\beta - 1} \frac{(r-\alpha)c}{n^\eta} \theta k_t^{\frac{\theta+\eta-1}{\theta}} \tag{1.18}$$

just enough capital dk to guarantee $y_t = \hat{y}(k_t + dk)$. Furthermore, for $y_t \leq \hat{y}(k_t)$, the value of the firm is

$$W(k_t, y_t) = y_t \frac{n^\eta k_t^{\frac{1-\eta}{\theta}}}{(r-\alpha)(1-\eta)} + C(k_t) y_t^\beta \tag{1.19}$$

with $C(k_t)$ *being*

$$C(k_t) = \left(\frac{\beta-1}{c}\right)^{\beta-1}\left(\frac{n^\eta}{\beta(r-\alpha)\theta}\right)^{\beta}\frac{\theta}{\theta-\beta(\theta+\eta-1)}k_t^{\frac{\theta-\beta(\theta+\eta-1)}{\theta}} .$$

It can be easily observed that the threshold is lower for the SP than it is for the firm. This entails that the SP would invest sooner in the project at first, however the difference is not striking, it is only due to the inverse elasticity of demand. This has a somewhat dismal[8] interpretation. Given that the (absolute value of) price-elasticity of demand[9], call it $\epsilon > 0$ in the electricity field is rather low, see Lijesen (2007) , its inverse η is high, hence the wedge between the behaviour of the two actors is quite big. Though, the present model studies the decision-making process of a *residual monopolist* actually, which allows us to suppose a slightly more elastic residual demand.
Among the various interventions that policy makers put in place to increase the supply of renewable energy sources (e.g. price premia, feed-in tariffs, green certificates, etc...), various forms of subsidies seem to appear. What must be stressed is that these subsidies are often being paid to monopolists (or oligopolists), firms which posses market power and that will not hesitate to use it to make extra-profits. As such, these interventions do not look like strikingly innovative policy tools, on the contrary, they resemble an old idea, *á la* Loeb and Magat (1979).
Assume that the planner could, using a non-distortionary policy, attempt to reinstate the efficient outcome, by paying a subsidy to the monopolist in the form of a *price premium*, i.e. increasing the price the monopolist receives by a fixed share $s \in R_+$. Then, the following proposition characterizes such policy.

[8] As quite often occurs in Economics.

[9] To be precise consider $k_t = \frac{1}{y_t}p^{-\frac{1}{\eta}}$, then $\frac{dk_t}{dp_t}\frac{p_t}{k_t} = -\epsilon$, $\epsilon > 0$. The two parameters ϵ and η are linked as in

$$\epsilon = \frac{1}{\eta} .$$

Proposition 3. *The Social Planner could restore the efficient outcome by paying the monopolist a price premium s that equals*

$$s = \frac{\eta}{1-\eta} \ . \tag{1.20}$$

Such efficient subsidy works precisely in the way described by Loeb and Magat: it makes the interests of the Monopolist and the ones of the Planner converge by putting the whole social surplus in the hands of the first. The Monopolist is paid a subsidy that equals the consumer's surplus, hence her profits coincide with the total social surplus. Hence, profit maximization brings about the efficient outcome. Of course, this version of efficiency is all but desirable, so the operation is ideally concluded an ex-post lump-sum taxation on profits, or an ex-ante auction for the permit to enter the market. At any rate, this sort of approach, send us back to an old debate about regulated utilities and distributional issues, which are beyond the scope of this paper.
The analysis here conducted has a dynamic focus, more precisely the *rhythm* of capital accumulation, which transforms into installed capacity. In other terms, what I am interested in is the average growth rate of the capital invested which can be calculated accordingly to the methodology presented in Dixit and Pindyck (1994).

Proposition 4. *Provided that $\alpha > \frac{1}{2}\sigma^2$, the average growth rate of investment of both projects* (1.6) *and* (1.13) *between date t and $t + \Delta t$ equals*

$$E_t[\log k_{t+\Delta t} - \log k_t] = \frac{(\alpha - \frac{1}{2}\sigma^2)\theta}{\theta + \eta - 1}\Delta t \tag{1.21}$$

In terms of expectations, the logarithm of k_t grows linearly in time. The impact of the volatility, σ, is clear: in a more uncertain market, capital will be invested more slowly, hence capacity expanded later in time.
However, given the same initial capital k_0, the SP and the Monopolist invest with the same "speed", and independently on who is managing the project, at

time $t + \Delta t$ the expected invested capital is the same. Intuitively, this is due to the fact that the difference between the two optimal expansion programs is "how high" is the upper barrier, however once reached for the first time, the rhythm and quantity invested at each exercise time remains the same for both. The barrier is higher for the Monopolist at each time, but being the volatility of the process is always σ, the monopolist would have to invest more to be "reflected" downwards.

Heuristics. To help develop some intuition, consider the following heuristics[10]: assume both players start when $y_t = \bar{y}$ and that both find themselves in the inaction region, I can rewrite condition (1.11) and (1.18) as equilibrium level to hold over time

$$\bar{y}k_{t,m}^{-\frac{\theta+\eta-1}{\theta}} \leq \frac{\beta}{\beta-1}\frac{(r-\alpha)c}{n^{\eta}}\frac{\theta}{1-\eta} = L_m$$

and

$$\bar{y}k_{t,SP}^{-\frac{\theta+\eta-1}{\theta}} \leq \frac{\beta}{\beta-1}\frac{(r-\alpha)c}{n^{\eta}}\theta = L_{SP} = L_m(1-\eta) \ ,$$

being

$$L_m > L_{SP}$$

for the firm and the SP respectively. By noting that the exponent is the same and has negative sign, follows that

$$k_{t,m} < k_{t,SP} \ .$$

Whenever these two inequalities do not hold expansion occurs. Since the responsible for any deviation is the same process y_t, hence the distribution is the same, suppose that a date t', $y_{t'} > \bar{y}$, at t' the additional capital, say $dk_{t'}$ required to restore the equilibrium conditions, as spelled out above, is greater for the Monopolist. In fact, suppose that at time t both conditions hold with equality :

$$dk_{t',m} = \left(\frac{L_m}{y_{t'}}\right)^{-\frac{\theta}{\theta+\eta-1}} - k_{t,m} = \left(\frac{L_m}{y_{t'}}\right)^{-\frac{\theta}{\theta+\eta-1}} - \left(\frac{L_m}{\bar{y}}\right)^{-\frac{\theta}{\theta+\eta-1}}$$

[10]Warning: heavy abuse of notation.

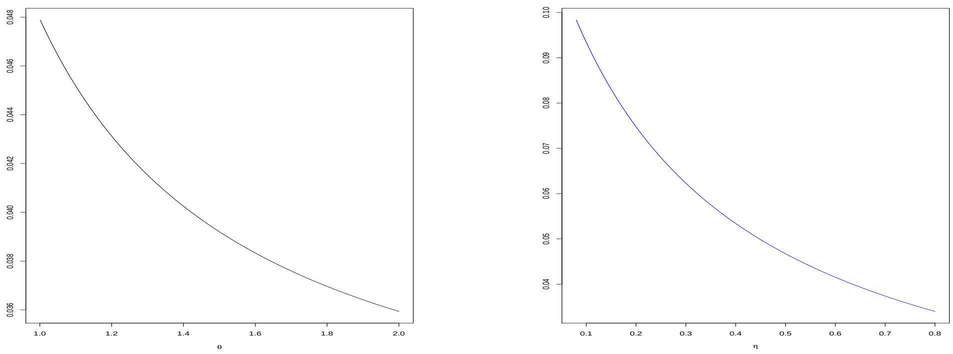
$$dk_{t',SP} = \left(\frac{L_{SP}}{y_{t'}}\right)^{-\frac{\theta}{\theta+\eta-1}} - k_{t,SP} = \left[\left(\frac{L_m}{y_{t'}}\right)^{-\frac{\theta}{\theta+\eta-1}} - \left(\frac{L_m}{\bar{y}}\right)^{-\frac{\theta}{\theta+\eta-1}}\right](1-\eta)^{-\frac{\theta}{\theta+\eta-1}}$$

hence

$$dk_{t',SP} < dk_{t',m} \ . \qquad \blacksquare$$

Figure 1.3: Average growth rate.

(a) Average growth rate as function of θ. (b) Average growth rates as function of η.



Both land heterogeneity and the inverse of price-elasticity have the same qualitative effect: as they increase the average growth rate of the investment decreases. As land becomes less productive, then investment is delayed to wait for the stochastic growth process to reach higher values, when the additional capital bears more profits. Whereas, a higher value for η implies more market power, which the Monopolist tries to exploit by waiting longer.

1.3 The Role of Externalities

The main reason adduced to the many policies in place, such as subsidies and incentive schemes[11] to investment is the bearing of a positive externality or the reduction of a negative one (such as pollution). In the specific case of renewable energy sources, Governments are paying large amounts of resources to induce a quicker and more stable provision of energy. There is very little doubt

[11]... which too often are just subsidies with a more appealing name.

about the negative externalities (e.g. air pollution) produced by production and consumption of fossil fuels and other traditional form of energy. In the case under examination, I represent the externality due to renewable sources as a positive production externality, which is due to the investments in the expansion of the plant.

I shall try to model this externality in a simple way: the installed capital (as it is) enters the utility function of the consumers. The findings, as limited as they come, show a possible partial convergence between the interest of the monopolistic firm and those of a SP.

Throughout this section, let's assume the consumers have a utility function as in

$$u_i(q_{it}) = y_t \frac{q_{it}^{1-\eta}}{1-\eta} k_t^e \ , \tag{1.22}$$

with e representing the elasticity of the utility with respect to the k_t:

$$\frac{\partial u_i}{\partial k_t} \frac{k_t}{u_i} = e \ .$$

In order to keep the demand curve decreasing in k_t and guarantee convergence, I assume $e \in [0, \eta/\theta]$.

Consumers maximize their payoff under an uniform marginal price, acting as price takers, with respect to the quantity of energy consumed q_{it}. They do not consider capital as a variable in as much as it is non-marketable to them.

Following the very same passages of the previous section, one can determine the market demand to be

$$p_t = y_t n^\eta q_t^{-\eta} k_t^e \tag{1.23}$$

which is constant over time. Hence the instantaneous profit flow for the firm is given by

$$\pi_t = n^\eta k_t^{\frac{1-\eta+\theta e}{\theta}} \ .$$

Adopting an utilitarian aggregator as a welfare measure, it delivers

$$U(k) = y_t \frac{n^\eta}{1-\eta} q_t^{1-\eta} k_t^e = y_t \frac{n^\eta}{1-\eta} k_t^{\frac{1-\eta+\theta e}{\theta}} \ . \tag{1.24}$$

The SP then is interested in the welfare maximization problem by considering the invested capital as the strategic variable, which means he solves

$$\max_k U(k_t) - p_t k_t$$

which yields

$$p_t = y_t \frac{n^\eta}{1-\eta} \frac{1-\eta+\theta e}{\theta} k_t^{\frac{1-\eta+\theta e-\theta}{\theta}} \quad . \tag{1.25}$$

Equation (1.25) represents the social marginal benefits given by the market exchange. It is evidently higher than the market demand in as in (1.23), because the SP manages to fully internalize the externality. The (deterministic part of the) instantaneous flow of social surplus is then calculated as

$$w(k_t) = \frac{n^\eta}{1-\eta} k_t^{\frac{1-\eta+\theta e}{\theta}} \quad .$$

By considerations analogous to the ones in the previous section, one can notice that:

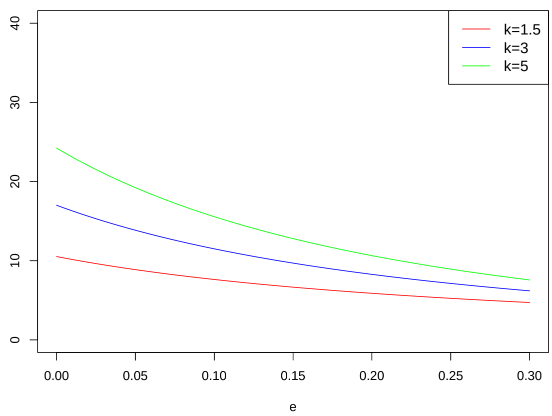
Proposition 5. *The Monopolist and the Social Planner would choose to expand the capacity of the project by investing new capital each and every time a threshold is hit by the process y_t, respectively given by*

$$\hat{y}^*(k_t) = \frac{\beta}{\beta-1} \frac{(r-\alpha)c}{n^\eta} \frac{\theta}{1-\eta+\theta e} k_t^{\frac{\theta+\eta-\theta e-1}{\theta}} \tag{1.26}$$

for the firm and by

$$\hat{y}^o(k_t) = \frac{\beta}{\beta-1} (1+\eta) \frac{(r-\alpha)c}{n^\eta} \frac{\theta}{1-\eta+\theta e} k_t^{\frac{\theta+\eta-\theta e-1}{\theta}} \quad . \tag{1.27}$$

Figure 1.4: Threshold value $\hat{y}$ with varying e.



From figure 1.4, one can notice that the externality parameter e decreases the investments threshold for the Monopolist, though the threshold keeps increasing in time as capacity is expanded. It can also be noticed that as the threshold increases over time, it tends to do it less and less rapidly with high values of e.

The modestly surprising result is that the efficient subsidy is indeed the very same as before

$$s = \frac{\eta}{1-\eta}$$

when I was not considering any externality in the model. Once again, the efficiency is reached through a Loeb and Magat (1979) mechanism. Such result can be rationalized by considering that since the externality I introduced in the model has only a "multiplicative" effect, the Monopolist, who programs her decision to expand using capital as a strategic variable, is able to take it into account. The only source of inefficiency remains market power: the monopolist will produce later to exploit the effect of growing demand as in *Proposition 1 & 2*.

The next thing worth investigating is the "speed" of the investment process. As noted in the previous section, and with the same tools the average growth rate of the invested capital depends, in this convenient specification, on

$$\frac{\partial \log \hat{y}}{\partial k_t} = \frac{\theta + \eta - \theta e - 1}{\theta} \ ,$$

which allows to deduce the next result.

Proposition 6. *If $\alpha > \frac{1}{2}\sigma^2$, should the project be run by the Monopolist or by the Social Planner, the average growth rate would be equal to*

$$E_t[\log k_{t+\Delta t} - \log k_t] = \frac{(\alpha - \frac{1}{2}\sigma^2)\theta}{\theta + \eta - \theta e - 1}\Delta t \ . \tag{1.28}$$

The reasons for this convergence are similar the ones leading to *Proposition 4*: if the Monopolist takes into account the externality when pricing, then again she will invest later, but higher amounts of capital with respect to the SP, who will invest in smaller amounts before her.

1.3.1 Extension

This set of results, of course, would not hold should the externality have an "additive" component along the presented multiplicative one. The introduction of such component undoubtedly complicates the functional form of the solution, but also allows for interesting suggestions about the dynamic nature of the efficient subsidy. In the previous models, partly due to the chosen specification, partly to the economics of monopolistic competition and complete information, the corrective subsidy was only function to the constant elasticity of the inverse market demand. Such feature made it simple to understand, and to use, but also a less than satisfying when it comes to fully capture the role of an externality. Now, I introduce a component which is not strategic to consumers, hence it will not be reflected in the demand curves, and hence in the market demand. This creates a more complex wedge between the behaviour of the monopolistic firm and the Social Planner: the problem for the firm, hence her optimizing behaviour, remains analogous to *Proposition 5*, while the SP takes into consideration a further element which increases the social marginal benefit.

Assume the individual utility for consumer i to be

$$u_i(q_i) = y_t \left(\frac{q_i^{1-\eta}}{1-\eta} k_t^e + \phi(k_t) \right) . \tag{1.29}$$

where $\phi(k) > 0$, a continuous strictly positive function of k_t , with $\phi'(k) >$, $\phi''(k) < 0$, represents the additive component of the externality. For the sake of simplicity, I assume that such component is the same for every consumer. It is immediate to verify that the market demand remains unaltered with respect to (1.23), while the SP problem gains a further element to take into account. The new social marginal benefit is then expressed as

$$p_t = y_t \left(\frac{n^\eta}{1-\eta} \frac{1-\eta+\theta e}{\theta} k_t^{\frac{1-\eta+\theta e-\theta}{\theta}} + n\phi'(k_t) \right) , \tag{1.30}$$

where the restrictions on the derivative of $\phi(k_t)$ ensure that it is decreasing. Analogous methods lead to the following result, which can be regarded as an extension of *Proposition 5.*

Proposition 7. *Given the social marginal benefit* (1.30)*, the Social Planner would increase the capital invested in the project when the process y_t reaches the threshold*

$$\hat{y}^o(k_t) = \frac{\beta - 1}{\beta}(r - \alpha)c\left(\frac{n^\eta}{1-\eta}\frac{1-\eta+\theta e}{\theta}k_t^{-\frac{\theta+\eta-\theta e-1}{\theta}} + n\phi'(k_t)\right)^{-1}, \quad (1.31)$$

and the efficient subsidy is $s(k_t)$

$$s(k_t) = \frac{\eta}{1-\eta} + n^{1-\eta}\phi(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}}. \quad (1.32)$$

It can be noticed that the efficient subsidy it is not necessarily constant any more. In fact, it might be either increasing, or constant or decreasing in time depending on the level of capital invested at the moment. Analytically,

$$\frac{\partial s(k_t)}{\partial k_t} > 0 \Leftrightarrow \phi'(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}} - \frac{1-\eta+\theta e}{\theta}\phi(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}-1} > 0$$

and symmetrically

$$\frac{\partial s(k_t)}{\partial k_t} < 0 \Leftrightarrow \phi'(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}} - \frac{1-\eta+\theta e}{\theta}\phi(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}-1} < 0 .$$

Furthermore, the efficient subsidy is not neutral with respect to market size, in fact

$$\frac{\partial s}{\partial n} > 0 .$$

This effect is due to the fact that, as population grows, the number of individual externalities not being taken into account by the utility-maximizing consumers, grows itself. Follows that to restore efficiency, a greater compensation must be offered to the monopolist. Intuitively, since the size of the market failure increases with the market size, it becomes more costly to mend it.
Moreover the presence of additive externalities actually provides a theoretical justification for the policy-maker to possibly alter the "speed" of the investment process, since market equilibrium fails to internalize part of an externality that would cause investments to increase, if fully captured. The following example provides an illustration for the efficient subsidy in a tractable framework.

Example: constant elasticity of ϕ. Suppose the following functional form for ϕ:

$$\phi(k_t) = k_t^{\xi}$$

and that $\xi < 1$. The efficient subsidy takes the form

$$s(k_t) = \frac{\eta}{1-\eta} + n^{1-\eta} k_t^{\xi - \frac{1-\eta+\theta e}{\theta}} .$$

Then, the subsidy will be increasing (decreasing) in k_t if and only if

$$\xi \gtrless \frac{1-\eta+\theta e}{\theta} .$$

Notice that this threshold has a particular relationship with land heterogeneity, θ; indeed, the higher is land heterogeneity, which means development is more costly , the the lower this threshold gets until it reaches the lower bound e. This implies that a plant built on harsher lands will be more likely *ceteris paribus* to need a subsidy increasing more rapidly over time, i.e. :

$$\xi > e .$$

The converse also holds true: as θ tends to 1, provided $\eta > e$ to ensure convergence, the subsidy will be decreasing if

$$\xi < 1 - \eta + e . \qquad \blacksquare$$

1.4 A Model of Targeting Policy

1.4.1 Model

Currently, the policy debate about increasing the share of renewable energy sources focuses around the implementation of supporting schemes that should induce the industry to expand its overall capacity within a predetermined period of time to reach a certain goal of abatement of emissions. In other words, the regulator is interested in finding a cost-efficient policy which can deliver that level of capacity such that the emissions are taken down to a certain level at a time in the future and the market demand is satisfied. Formally, set

$t = 0$ to represent the present, and assume that the current capacity is q_0, the regulator imposes a time target T^*, at which the Monopolist is expected to reach a target capacity q_{T^*}. The previous discussion reveals that part of the overall effect of externalities, given the assumption on the model, is already reflected into (inverse) elasticity of demand, that can actually be measured, therefore, I am going to assume now that the regulator faces a (inverse) demand function as in

$$p_t = y_t n^\gamma k^{-\frac{\gamma}{\theta}} \tag{1.33}$$

where γ is the observed elasticity parameter[12].

For simplicity, let $q_0 = 1$, and recalling that capacity maps into investments as $q^\theta = k$, then using the expected growth rate formula, I can write

$$E_0[\log q_T] = \frac{1}{\theta} E_t[\log k_T]$$

hence

$$E_0[\log q_T] = \frac{(\alpha - \frac{1}{2}\sigma^2)}{\theta + \gamma - 1} T \tag{1.34}$$

is the average growth rate of the capacity.

Once the regulator fixes the targets T^* and q_{T^*}, the goal is to find a proper support scheme to induce the right growth in the expansion of existing capacity, which is given by:

$$\frac{E_0[\log q_{T^*}]}{T^*} = g^* .$$

As it can be deduced from the previous sections and results, a fixed price premium as a share of current energy price is utterly unable to increase average growth rate of this process. It can only "bring down" the investment threshold, which means that in the short-run, the monopolist would invest a bit sooner, while in the long run the increase in capital would be unaltered. Follows that Regulator needs to commit to a time-varying subsidy scheme, increasing in the installed capacity/invested capital. In the next part of the analysis, I assume complete commitment from the Regulator and neglect the risk of withdrawal as analyzed in Boomsma and Linnerud (2015) and Nagy,

[12] I am using γ instead of η now to denote the observed by the Regulator to underline the fact that the policy maker might only be able to estimate through the available data.

Hagspiel, and Kort (2021).

The following proposition describes the subsidy scheme that induces the average growth rate g^*.

Proposition 8. *Suppose that the Regulator would induce the Monopolist to invest with an average growth rate g^*, this could be obtained by committing to paying a subsidy $s(k_t)$ that equals:*

$$s(k_t) = k_t^G - 1 \ , \tag{1.35}$$

where

$$G = \frac{1}{\theta}\left[(\theta + \gamma - 1) - \frac{(\alpha - \frac{1}{2}\sigma^2)}{g^*}\right] \tag{1.36}$$

and notice that the subsidy is increasing over time

$$\frac{\partial s}{\partial k_t} > 0 \qquad if \qquad G > 0$$

and vice versa. Furthermore, for $g^* > 0$, more land heterogeneity increases the required subsidy,

$$\frac{\partial G}{\partial \theta} \geq 0$$

as does the volatility of the process y_t,

$$\frac{\partial G}{\partial \sigma} \geq 0; .$$

Finally, the drift parameter α has a negative impact on $s(k_t)$,

$$\frac{\partial G}{\partial \alpha} < 0 \ .$$

From *Proposition 8*, it can be deduced that the targeting policy works by increasing the "speed" of investment for the Monopolist, and that this can be achieved if the Regulator commits to a subsidy scheme which is most likely increasing in time, depending on the currently installed capital. Of course commitment is crucial to this result. I assume no regulatory uncertainty, which is clearly a threat to reaching the target in time. Such issues have been analyzed for instance in Boomsma, Meade, and Fleten (2012). On the other hand the result of the proposition above indicates what matters in the long run to achieve the goals. Strikingly, assuming that the Monopolist gradually expands her capacity, instead of making a one-shot investment, installation costs do not affect the expected average growth rate of capital. Costs appear to be elements that regulate the frequency of the capital accumulation. In the long-run, a prominent role is played by the (inverse) elasticity of demand and by land heterogeneity. The role of the latter is explained by the fact that as marginal returns on capital decrease more quickly, the Regulator needs to compensate the firm to induce her to invest at the desired speed. Being the (inverse of) elasticity a proxy for market power, as γ increases the firm will have a greater incentive to reduce the output, hence slow down the investment process, to get higher rents.

1.4.2 Numerical Examples for 10-year goals

In what follows, I provide some numerical examples to better illustrate the shape of the subsidy and the role of the parameters that affect it. To do so, I present a middle term scenario of 10 years. One of the advantages of this model is that being the growth rate expressed in logarithmic terms, one can focus on the percentage expansion. So, the current capacity q_0 can be normalized as $q_0 = 1$, that brings to $k_0 = q_0^{\frac{1}{\theta}} = 1$. In this way $q_{T^*} = k_{T^*}^{\frac{1}{\theta}}$ represents the ratio between target capacity and current installed capacity.
I propose three level for each parameter to analyze the sensitivty of the target subsidy. The following table reports the values.

Table 1.2: Parameters' ranges

	θ	γ	σ
Low	1.2	0.5	0.03
Medium	1.3	0.6	0.05
High	1.4	0.7	0.07

The values for θ represents three scenarios where the quality of land is

causes diminishing return at various degrees. For example, consider the case of solar power generation. In this case θ measures the heterogeneity of exposition to light in the areas where the plant is developed, or the intensity with which wind blows, for and eolic plant.
As far as elasticity is concerned, electricity markets presents a low price elasticity of demand. However, this model does not allow for values of γ higher than 1 by hypothesis. Moreover, discussing the long-run expansion policy of a plant, one should take into account the possible entry of other small plants (a price-taking fringe, for instance), that in the long run might reduce the market power of the (dominant) firm. The values proposed here, nonetheless, are not inconsistent with certain ranges reported in Lijesen (2007), for the the long-run, although in this case the inverse demand represents the residual share of the market on which our firm exercises market power, hence the assumption of a higher elasticity is justified.
For the parameters characterizing the GBM, I have picked a drift $\alpha = 0.03$ and a range for volatility, σ, which is close to the calibration in Bigerna et al. (2019) for Italy. Though, differently from that paper, I do not discuss a precise target amount for the entire industry to be reached, rather the expansion of a single Monopolist, and as pointed out before, there is no need to name a specific number of GWs to be installed, rather the percentage increase contemplated in the expansion plan of the firm.
Over an horizon of 10 years, I propose an expansion of 50% of capacity, i.e. $q_{10} = 1.5$, and perform a sensitivity analysis.

At the varying of θ, the amount of capital necessary to reach the goal changes: specifically, a more heterogeneous land requires more capital over the years, since the rate of transformation of capital into capacity decreases in θ. From Fig.1.5, it can be observed that the subsidy increases in time reaching a value that ranges from about 4% to 12% of the invested capital. The role of θ is determinant in increasing the overall payment that must be made to the Monopolist, and the total amount of capital k to be invested in the project, which increases in a concave manner.

Figure 1.5: Subsidy $s(k_t)$ over time with different θ values

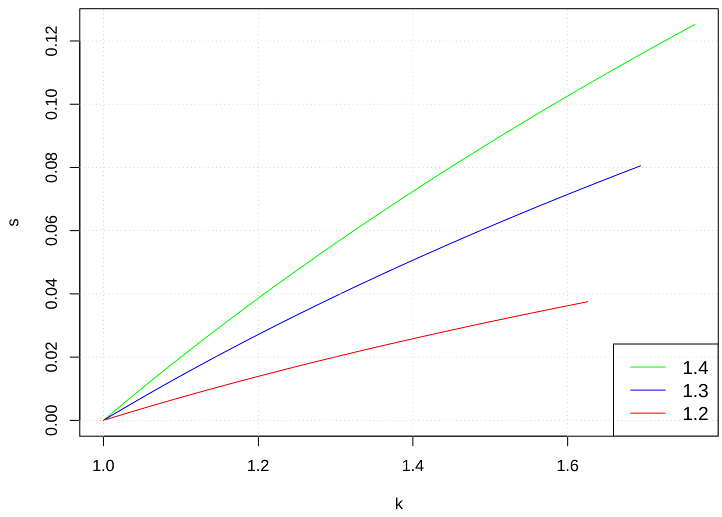
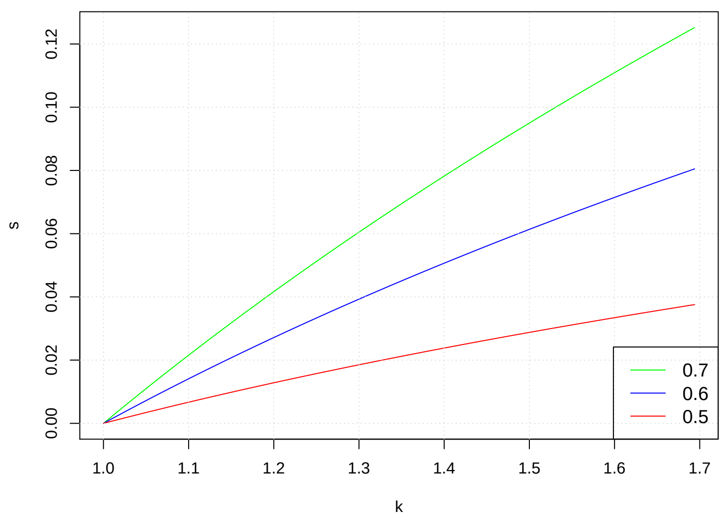
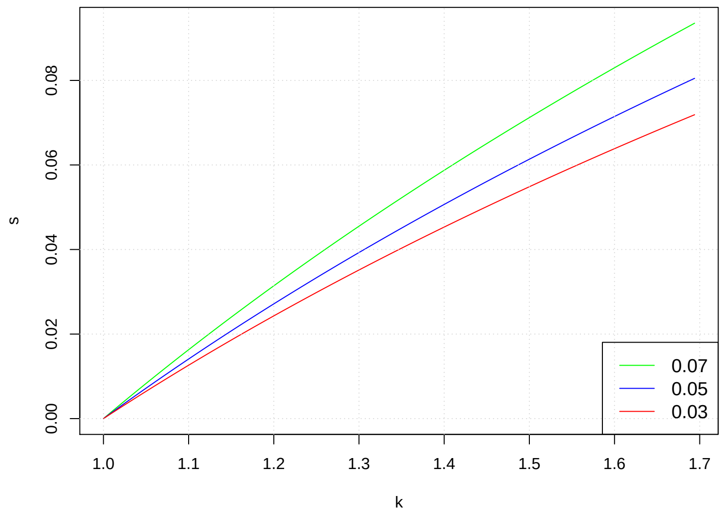


Figure 1.6: Subsidy $s(k_t)$ over time with different γ values



Considerations, analogous to the previous ones hold also for γ, see Fig 1.6. A gamma close to 1, as in this case, measures a substantial market power, which the monopolistic firm would like to exploit, increasing the $s(k_t)$ the Regulator needs to pay to force the required expected average growth rate. Although the overall effect is similar (as it can be appreciated from the analytic form for G), it appears that the subsidy is more sensitive to small increments of γ.

Figure 1.7: Subsidy $s(k_t)$ over time with different σ values



Volatility plays a significant role too, which can be appreciated from Fig. 1.7. At the end of the 10-year horizon, the value of the subsidy ranges from about 7% to over 9%. Intuitively, a more volatile demand would cause the monopolist to delay investments, as it is usual in the Real Option literature (see McDonald and Siegel (1986) or Dixit (1991)). Hence the Regulator must increase the magnitude of his intervention in the market.

At any rate, the subsidy scheme presents itself as a concave function increasing in the capital, i.e. over time. This implies that price premia should be lower at the beginning of the expansion plan and become significantly higher toward the horizon chosen by the Regulator.

1.4.3 Subsidy-and-Cap Strategy

As seen in the previous examples, the subsidy must be increasing in time in order to induce a stronger growth. On the other hand, this can easily turn out to be quite onerous for the taxpayers. It can be shown that a proper combination of increasing subsidy and a price cap can help regulate the growth process and simultaneously contain its cost. To show this result, I follow the methodology first developed by Grenadier (2002), and then adapted to include price-capping in oligopolistic markets by Roques and Savva (2009).

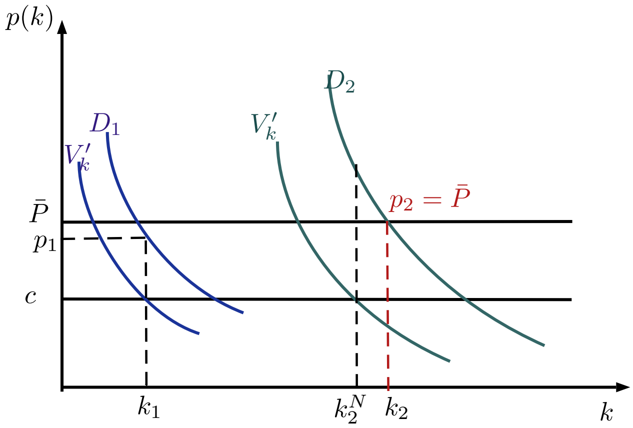


Figure 1.8: The picture shows how a binding price cap increases the installed capacity from k_2^N to k_2.

Since it will be convenient to work with prices, introduce the following notation: let P_t be the price gross of subsidy $s(k_t) = k_t^G - 1$

$$P_t = y_t n^\gamma k_t^{-\frac{\gamma}{\theta}+G} = y_t n^\gamma q_t^{-\gamma+G\theta}$$

and let instead the price-cap be denoted by $\bar{P}$. The instantaneous profit will be given by

$$\Pi_t = \begin{cases} P_t q_t & P < \bar{P} \\ \bar{P} q_t & P \geq \bar{P} \end{cases} \tag{1.37}$$

Proposition 9. *Fix a price-cap $\bar{P}$ and a subsidy of the form of $s(k_t)$, the Monopolist would invest every time the price P_t reaches the threshold*

$$\hat{P}_t = \left[\phi\left(\frac{\bar{P}}{r} - c\theta k_t^{\frac{\theta-1}{\theta}}\right)\bar{P}^{\beta_2-1}\right]^{\frac{1}{\beta_2}} , \tag{1.38}$$

where $\beta_2 < 0$,

$$\phi = \frac{\beta_1}{\frac{\beta_1-1}{\lambda(r-\alpha)} - \frac{\beta_1}{r}}$$

and

$$\lambda = 1 - \gamma + G\theta \ .$$

Expression (1.38) can be rewritten in the more familiar way

$$\hat{y}_t = n^{-\gamma} \left[\phi \left(\frac{\bar{P}}{r} - c\theta k_t^{\frac{\theta-1}{\theta}} \right) \bar{P}^{\beta_2 - 1} \right]^{\frac{1}{\beta_2}} k_t^{\frac{\gamma}{\theta} - G}$$

from which it can be noticed that the expected average growth rate, calculated as before, is not constant, rather it decreases over time, due to the presence of k_t in the square brackets in eq. (1.38). However, it is possible to identify some lower bound and study it analytically when $\theta = 1$. i.e. neglecting the role of land heterogeneity. Indeed, assume $\theta = 1$, then the threshold becomes

$$\hat{y}_t = n^{-\gamma} \left[\phi \left(\frac{\bar{P}}{r} - c \right) \bar{P}^{\beta_2 - 1} \right]^{\frac{1}{\beta_2}} k_t^{\gamma - G}$$

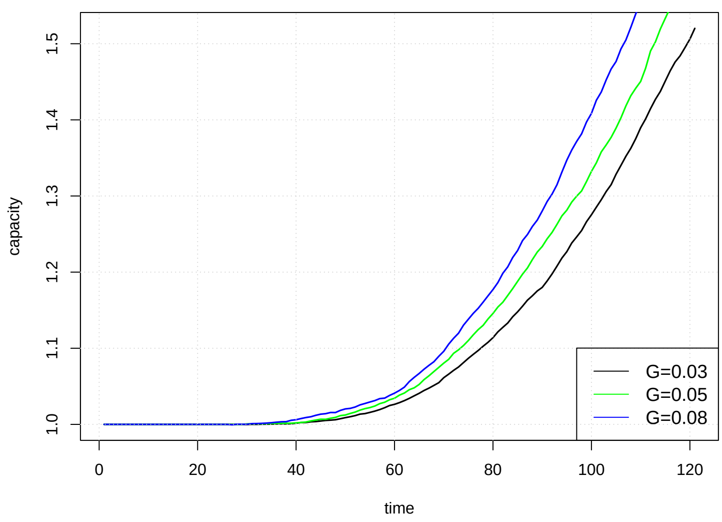
and fixed a level of growth rate g^* as before, the subsidy can be calibrated by setting G equal to

$$\underline{G} = \gamma - \frac{\alpha - \frac{1}{2}\sigma^2}{g^*}$$

which is the same as eq. (1.36), without land heterogeneity. In summation, the introduction of a price-cap helps control the expenditure on subsidies and still promotes a faster growth.

The following simulation shows the impact of price capping and a comparison with different levels of price premium (G).

Figure 1.9: Parameters values: $\gamma = 0.6$, $\theta = 1.4$, $y_0 = 0.8$, $\bar{P} = 0.9$, $c = 30$.



1.5 Conclusions

In this paper I have examined a dynamic model of investments to increase the capacity of production and focused in particular on the renewable electricity sector. The Dynamic Production approach provides a rigorous framework for the treatment of such expansion plans which are crucially important right now. The energy transition is first and foremost an industrial transition: a shift in the production technology. To the challenges already posed by the construction of new implants to, partially, substitute the old and many times inefficient ones, Governments have expressed the intention to speed this process up. The main reason given to justify heavy public intervention in energy (and not only) markets is the presence of an environmental externality, which renewable production should combat.

Absent public intervention, the behaviour of the monopolistic firm and the one of a Social Planner would differ substantially in terms of average growth rate, hence making impossible to reach the environmental targets (set by governments). To this end, a package of different policies is now being employed: among them, the payment of subsidies to firms which invest in these "green" technologies. As pointed out before, these subsidies end up in the pocket of firms with considerable market power, and in many an occasion these firms already have installed capacity, thus the issue becomes twofold. On one hand, the role of fixed subsidies, such as price premia, might help the entry of new firms, thereby reducing market concentration, though the possible trade-off, discussed in Bigerna et al. (2019), between capacity and timing might arise[13]. On the other hand, this kind of intervention proves useless to increase at the same time both speed and amounts of the investment. In fact, increasing subsidy schemes are called for to do so.

A first intuition is provided by investigating the First-Best solution, which delivers a result á la Loeb and Magat (1979): let, through some designed payments, the Monopolist appropriate the entire social welfare, and then find an efficient way to extract the rent. Of course, this is easiest said than done and

[13]Note that this trade-off depends on the linear demand assumption.

opens the issue of the reliability and the incentives given to the Regulator to behave. Furthermore, an increasing First-Best subsidy might prove rather heavy on the taxpayers, hence the need to find a balance.
The first approached I proposed aimed at investigating how could a simple analytical model capture the effect of an increasing scheme and what are the production and market variables that influence it and how. Elasticity of demand, land heterogeneity and volatility play an important role in determining the amount to be paid, and even the more so as time passes, posing once again the issue of fiscal sustainability. Turns out that the introduction of a price-cap can help control the expenditure, however strong land heterogeneity, i.e. decreasing marginal productivity of investment, causes the average growth rate to decrease over time. Such a complementarity between caps and subsidies has also been shown in Fabra (2018) in the context of an intuitive and straightforward model that deals with the fundamental of capacity mechanisms (also examined in Fabra, Fehr, and Frutos (2011)). Overall, given both the need to reach certain goals, and the need to cope with a substantial market power, the joint introduction of monetary incentives (such as subsidies) and tools of price regulation represents a useful strategy to promote long-term development.

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Appendix A: Proofs

Proof of Proposition 1. The proof, a pretty standard application of Dynamic Programming, follows the guess and verify approach. I guess that the solution to (1.7) takes the form

$$V(k_t, y_t) = B(k_t)y^{\beta_1} + b(k_t)y^{\beta_2} + y_t\frac{\pi(k_t)}{r-\alpha} ,$$

by substituting this functional form in (1.7) and simplifying, one gets the so called *fundamental quadratic*, which is the equation given by

$$\beta(\beta-1)\sigma^2 + \beta\alpha - r = 0 .$$

Now, this equation has real two roots: one is positive, say β_1, the other is negative, β_2.

By imposing the *"no bubbles"* condition, I can rule out the negative root, restricting the analysis the the positive one. The explanation is simple: consider the term $b(k_t)y^{\beta_2}$, since the exponent is negative, the value of such component grows and eventually booms when y_t approaches zero. Hence the value function would not go to zero when the demand reaches zero. However, $y_t = 0$ is an absorbing barrier for the process, and it implies that all future inflows $y_{t'}\pi(k_t) = 0$ for every $t' \geq t$, therefore the value of the firm equals zero itself. By these considerations, set $b(k_t) = 0$ and simply call $\beta_1 = \beta$. The value of β can be found by solving the fundamental quadratic:

$$\beta = \frac{1}{2} - \frac{\alpha^2}{\sigma^2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{\alpha^2}{\sigma^2}\right)^2} .$$

The terms inside the square root can be rearranged as

$$\frac{2r}{\sigma^2} + \frac{1}{4} - \frac{\alpha^2}{\sigma^2} + \frac{\alpha^4}{\sigma^4} =$$

$$\frac{1}{4} + \frac{2r-\alpha^2}{\sigma^2} + \frac{\alpha^4}{\sigma^4} > \frac{1}{2}$$

given that $r > \alpha$ and the square root in a monotone transformation. It follows that $\beta > 1$ and so that the *real option multiplier* is always larger than one, i.e. the expression

$$\frac{\beta}{\beta-1} > 1 .$$

Passing onto the value of $B(k_t)$ and $\hat{y}_t$, use the smooth pasting (1.9) and optimal capital (1.8) conditions to get

$$\beta B'(k)y^{\beta-1} + \frac{\pi'(k)}{r-\alpha} = 0$$
$$B'(k)y^{\beta} + y\frac{\pi'(k)}{r-\alpha} = c\ .$$

By substituting

$$\pi'(k_t) = n^{\eta}\frac{1-\eta}{\theta}k^{-\frac{\theta+\eta-1}{\theta}}$$

Those two equation can be combined to get eq. (1.11) and

$$B(k_t) = \left(\frac{\beta-1}{c}\right)^{\beta-1} \cdot \int_k^{\infty}\left(\frac{n^{\eta}}{\beta(r-\alpha)}\frac{1-\eta}{\theta}\right)^{\beta}\kappa^{-\frac{\theta+\eta-1}{\theta}}\ d\kappa$$

that upon integration delivers the result in *Proposition 1.* ■

Proof of Proposition 2. Being the problem analogous to the one presented in *Proposition 1*, the method is the same. Only this time the instantaneous inflow is

$$w(k_t) = \frac{n^{\eta}}{1-\eta}k^{\frac{1-\eta}{\theta}}$$

and

$$w'(k_t) = \frac{n^{\eta}}{\theta}k^{-\frac{\theta+\eta-1}{\theta}}\ . \qquad ■$$

Proof of Proposition 3. To restore efficiency, the interest of the Monopolist must be made converge with those of the SP. In other terms, the inflows that the firm gets must equal those that the SP at any moment in time. Therefore,

$$\pi_s(k_t) = n^{\eta}(1+s)k_t^{\frac{1-\eta}{\theta}} = \frac{n^{\eta}}{1-\eta}k^{\frac{1-\eta}{\theta}} = w(k_t)$$

which happens if and only if

$$s = \frac{1}{1-\eta} - 1 = \frac{\eta}{1-\eta}\ . \qquad ■$$

Proof of Proposition 4. The proof is rather scholastic and derivative. For the sake of clarity, I split it in two parts. First, an instrumental result is stated, next the proof of the proposition is completed through considerations analogous to Dixit and Pindyck (1994).

i)*Long-run distribution of a GBM with an upper reflecting barrier.* If M_t follows a GBM with drift μ and volatility σ, and an upper reflecting barrier $\bar{M}$, then the process $m_t = \log M_t$ follows an arithmetic (or absolute) Brownian motion with drift $\mu' = \mu - \frac{1}{2}\sigma^2$ and volatility σ. Following the result in Dixit (1993), p.60, m_t has density function

$$\psi(m_t) = \frac{2\mu'}{\sigma^2} \exp\left(2\frac{\mu'(m_t - m)}{\sigma^2}\right) .$$

Hence call $f(M_t)$ the density function of the original GBM, since

$$f(M_t) = \psi(\log M_t) \cdot \frac{d}{dM_t} \log M_t \ ,$$

it yields an exponential density function

$$f(M_t) = \frac{2\mu - \sigma^2}{\sigma \bar{M}} \left(\frac{M_t}{\bar{M}}\right)^{2\mu/\sigma^2 - 2} .$$

ii) Average growth rate. Consider equation (1.11) and rewrite it as

$$\bar{M} = \hat{y}(k_t) k_t^{-\frac{\theta+\eta-1}{\theta}} = \frac{\beta}{\beta - 1} \frac{(r-\alpha)c}{n^\eta} \frac{\theta}{1-\eta}$$

where the left-hand side represents the reflecting upper barrier in this case. Now take the logarithm of the stochastic process y_t to be $u_t = \log y_t$, and notice that this process is characterized as in *part i).*

Approximate the Brownian motion by discretization in the following way: divided time in arbitrarily short intervals, Δt, over which the process m_t can assume value on a grid made of small increments/decrements $\Delta m = \sigma\sqrt{\Delta t}$ one step at a time, with probability distribution given by:

$$p = \frac{1}{2}\left(1 + \mu' \frac{\sqrt{\Delta t}}{\sigma}\right) \quad ; \quad q = \frac{1}{2}\left(1 - \mu' \frac{\sqrt{\Delta t}}{\sigma}\right)$$

for an upward and a downward movement respectively. Intuitively, the monopolist would decide to invest additional capital at a given time t if the process m_t hits the barrier $\bar{m}$, which is possible only if at $t - \Delta t$ we have $m_{t-\Delta t} = \bar{m} - \Delta m$, and then an upward movement occurs. In such case the capital would be increased by Δk such that

$$\Delta m - \frac{\theta + \eta - 1}{\theta} \Delta \log k = 0 \ .$$

In the long-run the average growth rate can be written as the product of the a) probability that m_t is close to $\bar{m}$, i.e. just a step below it, b) the probability of an upward movement that brings $m_t > \bar{m}$, and c) the percent change in $\log k_t$ necessary to bring the process m_t below the barrier again, all divided by Δt. Hence, call g the average growth rate, it equals

$$g = \frac{1}{\Delta t} \left(\frac{2\mu}{\sigma^2} - 1 \right) \Delta m \cdot \frac{1}{2} \left[1 + \left(\alpha - \frac{1}{2}\sigma^2 \right) \frac{\sqrt{\Delta t}}{\sigma} \right] \cdot \frac{\theta \Delta m}{\theta + \eta - 1}$$

and by simplifying and letting $\Delta t \to 0$, one finally obtains

$$g = \frac{(\alpha - \frac{1}{2}\sigma^2)\theta}{\theta + \eta - 1} \ . \qquad \blacksquare$$

Proof of Proposition 5. Completely analogous to proposition *Propositions 1 & 2.* The only variation is that this time

$$\pi_t' = n^\eta \frac{1 - \eta + \theta e}{\theta} k_t^{\frac{1-\eta+\theta e-\theta}{\theta}}$$

and

$$w(k_t) = \frac{n^\eta}{1-\eta} \frac{1 - \eta + \theta e}{\theta} k_t^{\frac{1-\eta+\theta e-\theta}{\theta}} \ . \qquad \blacksquare$$

Proof of Proposition 6. Follows the exact same procedure of *Proposition 4*, with the difference of $\log k$ being

$$\Delta \log k = \Delta m \frac{\theta}{\theta + \eta - \theta e - 1} \ . \qquad \blacksquare$$

Proof of Proposition 7. First of all, notice that $\phi(k_t)$ does not affect the consumer's decisions, since it is not a function of q_{it}. Hence the results of *Proposition 5 & 6* still hold as far as the Monopolist is concerned. The function ϕ instead matters for the SP, who has to plan a subsidy scheme such that the payoff of the Monopolist equals the total social surplus. The threshold value is obtained through the very same passages already seen in *Proposition 1 & 2*, while the optimal subsidy is obtained by finding that $s(k_t)$ such that

$$\pi_t = (1 + s(k_t))n^\eta k_t^{\frac{1-\eta+\theta e}{\theta}} = \frac{n^\eta}{1-\eta} k_t^{\frac{1-\eta+\theta e}{\theta}} + n\phi(k_t) = w_t \ .$$

The efficient subsidy policy is then obtained as

$$s(k_t) = \frac{\eta}{1-\eta} + n^{1-\eta}\phi(k_t)k_t^{-\frac{1-\eta+\theta e}{\theta}} \ .$$

Proof of Proposition 8. Suppose the subsidy takes the form

$$1 + s(k_t) = k_t^G \ ,$$

then the deterministic part of the instantaneous profit flow reads

$$\pi_t = n^\gamma k^{\frac{1-\gamma}{\theta}+G} \ ,$$

it follows that

$$\pi_t' = n^\gamma \left(\frac{1-\gamma}{\theta} + G \right) k_t^{-\frac{\theta+\gamma-1-\theta G}{\theta}} \ .$$

By solving the usual Dynamic Programming problem, and by *Proposition 4*, once normalized $k_0 = 1$, one gets

$$E_0[\log k_T] = \frac{(\alpha - \frac{1}{2}\sigma^2)\theta}{\theta + \gamma - 1 - \theta G} T$$

that can be rewritten in terms of desired capacity installed at time T^* as

$$E_0[\log q_{T^*}] = \frac{(\alpha - \frac{1}{2}\sigma^2)\theta}{\theta + \gamma - 1 - \theta G} T^* \ .$$

Hence, find G so to satisfy

$$\frac{(\alpha - \frac{1}{2}\sigma^2)}{\theta + \gamma - 1 - \theta G} = g^*$$

which yields

$$G = \frac{1}{\theta}\left[(\theta+\gamma-1) - \frac{(\alpha-\frac{1}{2}\sigma^2)}{g^*}\right] .$$

The comparative statics of the two is readily understood from the sign of G, in fact

$$G \geq 0 \qquad \text{whenever} \qquad g^* \geq \frac{\alpha-\frac{1}{2}\sigma^2}{\theta+\gamma-1}$$

and

$$G < 0 \qquad \text{whenever} \qquad g^* < \frac{\alpha-\frac{1}{2}\sigma^2}{\theta+\gamma-1} .$$

Otherwise, one can take the derivative

$$\frac{\partial G}{\partial \theta} = -\frac{1}{\theta^2}\left[(\gamma-1) - \frac{\alpha-\frac{1}{2}\sigma^2}{g^*}\right]$$

and notice that

$$\frac{\partial G}{\partial \theta} \geq 0 \qquad \text{whenever} \qquad g^* \geq -\frac{\alpha-\frac{1}{2}\sigma^2}{1-\gamma} .$$

The impact of the volatility parameter can be seen simply by computing

$$\frac{\partial G}{\partial \sigma} = \frac{1}{\theta g^*}\sigma > 0 ,$$

while the role of α can be found, analogously, by examining the partial derivative

$$\frac{\partial G}{\partial \alpha} = -\frac{1}{\theta g^*} < 0 .$$

Proof of Proposition 9. The proof follows the methodology introduced by Grenadier (2002) and then expanded by Roques and Savva (2009). So consider the value function for the Monopolist expressed as a function of price and quantity $M(P_t, q_t)$ with $P_t = P_t(q_t)$ and $q_t = q(k_t)$, and define the marginal value with respect to k_t as (I am dropping the subscripts for economy of notation)

$$\frac{\partial M}{\partial k} = \frac{\partial M}{\partial q}\frac{\partial q}{\partial k} = m\frac{\partial q}{\partial k}$$

hence the Bellman Equation is

$$\frac{\partial q}{\partial k}\left(\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 m}{\partial P^2} + \alpha y \frac{\partial m}{\partial P} - rm + (1-\gamma+G\theta)P\right) = 0 \qquad P < \bar{P} \tag{A.1}$$

and

$$\frac{\partial q}{\partial k}\left(\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 m}{\partial P^2} + \alpha y \frac{\partial m}{\partial P} - rm + \bar{P}\right) = 0 \qquad P \geq \bar{P} \tag{A.2}$$

which is null if and only if the terms in brackets is 0, since $\frac{\partial q}{\partial k} > 0$. Hence, I will focus on it. Then guess the solution takes the form

$$m = M_i P^{\beta_1} + M_j P^{\beta_2} + A_i P + B_i$$

which once substituted in (A.1) and (A.2), respectively delivers the system

$$m = M_1 P^{\beta_1} + \frac{1 - \gamma + G\theta}{r - \alpha} P \qquad P < \bar{P}$$

$$m = M_2 P^{\beta_1} + M_3 P^{\beta_2} + \frac{\bar{P}}{r} \qquad P \geq \bar{P}$$

where the β-s are the two solution to the fundamental quadratic:

$$\beta_{1,2} = \frac{1}{2} - \frac{\alpha^2}{\sigma^2} \pm \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{\alpha^2}{\sigma^2}\right)^2},$$

with $\beta_2 < 0$. Then, introduce the value-matching and smooth pasting conditions

$$m(\bar{P}) = c\left(\frac{\partial q}{\partial k}\right)^{-1} \tag{A.3}$$

$$\frac{\partial}{\partial P} m(\bar{P}) \frac{\partial q}{\partial k} = 0 \tag{A.4}$$

furthermore, we are going to require continuity at the cap by imposing

$$m(\bar{P}^{(-)}) \frac{\partial q}{\partial k} = m(\bar{P}^{(+)}) \frac{\partial q}{\partial k} \tag{A.5}$$

$$\frac{\partial}{\partial P} m(\bar{P}^{(-)}) \frac{\partial q}{\partial k} = \frac{\partial}{\partial P} m(\bar{P}^{(+)}) \frac{\partial q}{\partial k} \tag{A.6}$$

once again notice that $\frac{\partial q}{\partial k}$ does not play any role except for eq.(A.3), so I can follow the methodology as outlined in Roques and Savva (2009). Substituting the functional form for the solution into (A.3)-(A.6), I get a system of four equations

$$M_2 \hat{P}^{\beta_1} + M_3 \hat{P}^{\beta_2} + \frac{\bar{P}}{r} = c\theta k^{\frac{\theta - 1}{\theta}} \tag{A.7}$$

$$M_2\beta_1\hat{P}^{\beta_1-1} + M_3\beta_2\hat{P}^{\beta_2-1} = c\theta k^{\frac{\theta-1}{\theta}} \tag{A.8}$$

$$M_1\bar{P}^{\beta_1} + \frac{1-\gamma+G\theta}{r-\alpha}\bar{P} = M_2\bar{P}^{\beta_1} + M_3\bar{P}^{\beta_2} + \frac{\bar{P}}{r} \tag{A.9}$$

$$M_1\beta_1\bar{P}^{\beta_1-1} + \frac{1-\gamma+G\theta}{r-\alpha} = M_2\beta_1\bar{P}^{\beta_1-1} + M_3\beta_2\bar{P}^{\beta_2-1}\ . \tag{A.10}$$

Using (A.8) in (A.7) and (A.10) in (A.9) one can eliminate both M_1 and M_2, resulting in

$$H_3 = \hat{P}^{-\beta_2}\left(c\theta k^{\frac{\theta-1}{\theta}} - \frac{\bar{P}}{r}\right)\frac{\beta_1}{\beta_1-\beta_2} \tag{A.7a}$$

and

$$H_3 = \bar{P}^{1-\beta_2}\frac{\frac{\beta_1-1}{\lambda(r-\alpha)} - \frac{\beta_1}{r}}{\beta_1-\beta_2} \tag{A.8a}$$

finally, combining (A.7a) and (A.8a) delivers (1.38). ■

Chapter 2

Data Intermediation and Price Targeting: anticompetitive features

Abstract. Many applications and social media collect data about their users, and then they sell such data to product firms to improve pricing or advertising. In my model, the data intermediary allocates the information among firms, selling differentiated products, in order to maximize the tariffs she can extract from them. Firms can then target consumers with personalized prices. This results in the coexistence of uniform and personalized prices, which undermines the positive effects of competitive price discrimination. When intermediating a number of markets, the intermediary is able to internalize a data production externality: due to the negligible marginal cost of selling data to several firms, as their number (size of intermediation) grows, more data are generated and the intermediary's profits increase. I also argue that such externality provides the opportunity to prevent the entry of an equally efficient rival: if the incumbent practises limit pricing, she still makes positive profits, which increase in her size.

JEL codes: D80, L11, L12, L13, L86.

2.1 Introduction

Many on-line applications, most prominently social networks, such as Instagram and Facebook, or websites, offer services to users, often for free, and collect data related to them. Then said apps sell (or "share") such information to producers of various goods (or services). Often, in particular on social networks, users are shown ads with offers that are likely to fit their preferences on the basis of the collected data. I explore a model in which an intermediary can strategically allocate bits of information among firms allowing them to offer personalize prices. The intermediary's behaviour leads to two effects in this model. First, by pursuing profit-maximization, the intermediary alters competition in product markets: information allocation is designed in order to extract as much profits as possible from firms. Second, the intermediary can design tariffs to prevent the entry of a similar rival, thereby monopolizing the intermediation market.

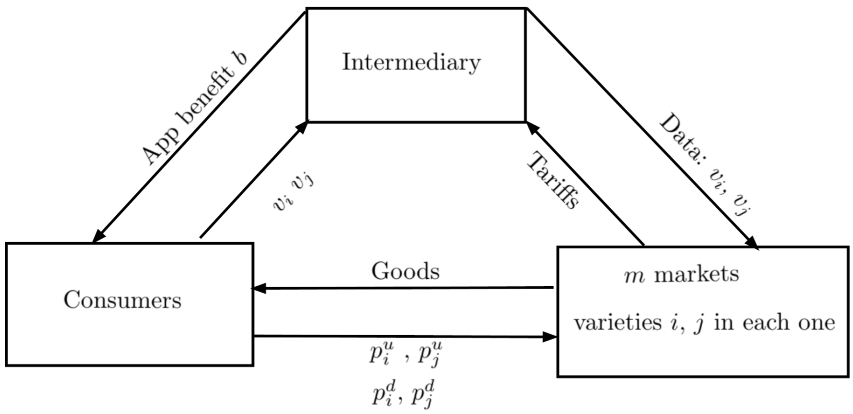
Hence, the main goal of this work is to bridge the collection of data, and their use in personalized pricing, with the role of intermediation. In particular, I study the case in which data are available to firms (competing in prices with differentiated products) through a deal with an intermediary, who collects data by offering a service (or App) to consumers.

Data have two main features of interest in this case. Firstly, they are information, as such they might be used or not: being informed might improve the pay-offs of firms, but, at the same time, carelessly distributed information might lead to lower profits (intensifying competition). Given the possibility, an agent might decide to use only part of the the total amount of data at his disposal to personalize prices: it will in fact be shown how uniform and personalized prices might coexist in equilibrium. Secondly, nothing comes from nothing: data are a byproduct of some economic activity, and possibly an economic activity in a different market. In this instance, the *other* economic activity will be represented by an intermediary, supplying consumers with some *free service*, which might very well be understood as an on-line one (such as a social network).

Hence the main focus of this paper: connecting the economics of intermediation with the economics of pricing and price discrimination in competitive contexts (for product markets). It follows that the scope of the paper will be twofold: one point of the analysis will be the optimal and/or equilibrium prices in the product markets, depending on the possibility of credibly committing to use only a certain subset of the information agents have. The other will be how the intermediary interacts with said agents (product firms) and affects the prices. The results will later be used to discuss competition between intermediaries, with a particular emphasis on the anticompetitive nature of intermediation. Relying on the fact that current technologies in this field allow an intermediary to serve a large number of clients (product firms, in this paper) without incurring in substantial marginal costs, the idea of a *zero cost of replication*, meaning that the *size per se* - i.e. the number of firms intermediated - of the intermediary is ideally not costly, rather what is costly is the quality of the service (App) and its maintenance (i.e. designing, programming, and updating the app), is useful in understanding how a network externality raises. The increasing size of an intermediary increases its volume of business, therefore increasing the quality of the service provided.
Here starts the cycle: the higher the quality of the App, more people will probably use it, hence more data, and more data means more profits, *and* the opportunity of more profits can draw in more firms. As the *size* grows, so does the marginal return on the quality of the app, hence the equilibrium level of quality increases. Repeating this sort of reasoning, under mild assumptions, it can be seen that, absent large differences between two intermediaries, most likely only one will be left, working like a magnet for both sides of the market. With a large size, comes a great market power[1], i.e. the opportunity to extract the extra-profits (the ones generated through the use of data). Thus the issue of *profit concentration* makes its appearance: keeping all the above discussion in mind, the ability to create and extract more profits combined with a large market power (boosted by network externalities) provides the monopolistic intermediary with ability to appropriate the lion's share of the extra-profits. In

[1] ...and fewer responsibilities! No Spider-man here, sorry.

other words: more profits are generated in this economy, but they end up in one guy's pockets. On top of that, the position that the intermediary achieves allows her to practice limit pricing, in order to keep an entrant out of the market, without pricing at marginal cost, due to the network externality. Should the Intermediary's size be large enough, the limit prices might be high enough to elude the scrutiny of an Antitrust Authority. It comes as no surprise that the combination of huge amounts of data, economies of scale, network externalities characterizing digital markets has brought the Authorities to reconsider Competition Policy standards, see Crémer, Montjoye, and Schweitzer (2019). Here below, I propose some references for this work.



Different Approaches to Competitive Price Discrimination. Most of the analysis concerning competitive price discrimination revolves around the Hotelling model[2], which has become somehow the workhorse of this literature: Thisse and Vives (1988) stands out as its seminal paper. As the authors point out, when firms are unable to commit to uniform prices, and they engage in price discrimination, this overall might benefit consumers and increase efficiency. The ideal[3] rationale would be that if the valuation of each consumer is known to every firm, then the consumer becomes a little market by itself in which firms compete á la Bertrand. It follows, in such an extreme case, that prices will generally go down and the measure of consumer surplus increases, see Corts (1998). Then, the profits would be determined by the

[2]The seminal contribution dates back to Hotelling (1929).

[3]Ideal.

relative preferences for one variety over the others, and should these preferences be relatively close, the equilibrium price would generally be close to the marginal cost of production. Later on, Armstrong and Vickers (2001) proposed a different framework to investigate the effects of competitive price discrimination in which firms compete directly in the utility they offer to consumers. In this work, also, is pointed out how allowing firms to price discriminate might improve the overall efficiency. Such framework has also been proposed in De Corniere and Taylor (2020) to investigate the role of data in shaping competition.

A rather different approach to model this topic has been provided by Stole (1995) and in Rochet and Stole (2002), who employ optimal control techniques. This kind of approach provides a useful representation for an extension of both the traditional discrete choice model -such as Hotelling and the circular Salop model (see Salop (1979))- but also another stream of literature -see Mussa and Rosen (1978) and Maskin and Riley (1984).

Both the approaches above, though delivering interesting results, might also prove quite hard to handle should one be intended to study the "fine details" of pricing. The first (the linear Hotelling one) tends to be quite restrictive [4] in terms of the preferences that consumers are allowed to have, and may require very tedious calculations. The latter instead, though more general, presents a number of technical difficulties (conditions required by the optimal control theory). Another stream of literature -the one followed here-, on the other hand, uses extended and adapted versions of the model proposed in Perloff and Salop (1985) which is discrete choice model where valuations are the values that some random variable takes; recent examples are Anderson, Baik, and Larson (2019) in the context of advertising and Rhodes and Zhou (2021), which proposes a model where consumers might want to disclose their private information and allows for dependent distributions of valuations.

Modelling Data and Intermediation. Given the increasing importance of data and the multiplicity of their uses, currently economic literature has been

[4] In Appendix B I articulate the reasons why I chose a different model.

devoting much attention to the issue. Models for instance have been developed to investigate the role of information technology in pricing, see Calvano et al. (2020), Bergemann and Bonatti (2011), Bergemann and Bonatti (2019) and Bergemann, Bonatti, and Gan (2020) for an introduction to the issue. At the same time, others put more emphasis on the role of a "biased" intermediary with her own agenda, see De Corniere and Taylor (2014), also De Corniere and Taylor (2019). The main ideas in this stream of literature is to connect data acquisition by an intermediary of some kind, and their role in shaping competition in product markets.
My approach in this respect is to view data as a tool that allows to *endogeneize* price discrimination (personalized prices), but also as a good that the intermediary allocates strategically to boost her profits.

Concerns for Competition. As the importance of data and the *Tech industry* grows, it is becoming patent that there is a general lack of competition. Not that it usually ranks among the top priorities of firms to fiercely compete, however in this case there is reason to believe that this is due to more than the general dislike for harsh price wars et similia. One important feature, present almost everywhere in this sector, are the so called network effects: basically a positive externality on others' utility or profits given by the number of agents consuming or joining the same good or firm. These effects might create the conditions for a winner-takes-all sort of game, so that in the long run only one big player is left in the market, either because competitors find it convenient to exit, or due to aggressive acquisition policies, see e.g. Motta and Peitz (2021). As mentioned in Kamepalli, Rajan, and Zingales (2020), there is also another aspect in these killer acquisitions, namely: why should a small start up want to fight against a big tech giant? It might just be better to accept a generous offer, been bought, and put your effort in a new project. The traditional metaphor of the big fish preying on the smaller ones might not prove up to the task of representing what is going on. In fact, the opposite story might hold true: investors pour money in promising projects that they already expect to be acquired by some big player if successful, either because they might pose a threat, or to acquire their proprietary innovative technologies, see e.g. Motta and Shelegia (2021).

The Road Ahead. *Section 2* of this paper deals with pricing and competitive price discrimination, applied by just one or the whole industry. The goal is to find how information can be used to maximize firm's or industry's profits (provided some commitment device), in contrast to the equilibrium use of information. At first I will assume that one firm or the whole industry know the valuation of every consumer, later I will restrict this kind of knowledge to only a part of the market, say δ, exogenously given. In *Section 3* a monopolistic Intermediary will be introduced, hence δ will be made endogenous: data are a by-product of the service that she offers and they increase as the quality of the app increases. Then, the analyses moves towards the role of the *size*, i.e. the number of markets intermediated, and how this affects the surplus generated and its *distribution*. *Section 4* presents some kind of "thought experiment" aimed at showing how the economic activity of the Intermediary, as previously defined, *"naturally"* leads to the monopolization of the intermediation market. To this end, the threat of a fictional identical entrant will be considered. I will show how such an entry can still be prevented even if entry and switching (for the firms from on intermediary to the other) costs are negligible and how the practice of limit pricing still allows profits to be made. *Appendix A* contains the technical proofs, while *Appendix B* develops an alternative pricing model à la Hotelling.

Notation. In this work, to keep the notation as simple as possible, I will slightly abuse the standard symbols used to denote derivatives: I will use the symbol "∂" to refer to the simple derivative, partial or not, with respect one variable, and the symbol "d" to identify the total derivative, which takes into account the direct and indirect effects of a the change in one variable on the outcome.

2.2 Optimal Price Targeting in a Competitive Industry

As mentioned above, in order to carry out the analysis with differentiated goods, I adapt the model first proposed by Perloff and Salop (1985). Assume there two firms in the market, denoted as i and j, selling differentiated products to the same set of consumers $h \in H$, where H is a continuum of measure 1. Firms can sell up to one unit of one good to each consumer. Each consumer h is characterized by a vector $v^h = (v_i^h, v_j^h)$ of valuations, which are private information. Valuations for each good are distributed according to a continuously differentiable distribution $G_k(v_k) = G(v)$, for $k = i, j$: the two measures are identical and independently[5] distributed over the support $[\underline{v}, \bar{v}]$, and the probability density function will be denoted by $g(v)$ and it is defined to be positive over the support, and nil everywhere else. I will also assume *full market coverage*: every consumer always has the chance to buy from one and only one firm. In this setting, the marginal cost of production can be set zero without loss of generality. Furthermore, I am not going to consider entry or exit in the product market(s): the number of competitors will remain fixed throughout the analysis. It follows that each market can be represented by a duopoly, which simplifies analysis and notation without altering the conclusions. To this purpose, define the variable $s = v_i - v_j$, and call its cumulative distribution and density function, respectively, $F(s)$ and $f(s)$:

$$f(s) = \int_{\underline{v}}^{\bar{v}} g(s+v)g(v)dv; ,$$

they inherit differentiability from $G(v)$ and they have support $[\underline{v} - \bar{v}, \bar{v} - \underline{v}]$, and in particular, the distribution is symmetric: $f(s) = f(-s) \quad \forall s \in [\underline{v} - \bar{v}, \bar{v} - \underline{v}]$ and $F(s) = 1 - F(-s) \ \ \forall s \in [\underline{v} - \bar{v}, \bar{v} - \underline{v}]$. The variable s represents the difference in the valuations of the two goods, and it is useful to think of

[5]Should the two measures be dependent, the results would still hold. However, they would require to be framed as in Caplin and Nalebuff (1991), and Rhodes and Zhou (2021). Such a generalization would lead me to use a much more sophisticated toolbox which is beyond the more prosaic goals of this paper.

it as a measure of *relative preference*, but also of relative advantage of one firm over the other. In fact, if the two firms knew the private information of every consumer in the market and compete in personalized price schedules, $p_i^d = s$, for $s \geq 0$, and $p_j^d = -s$, otherwise. It is also a representation of the mark-up that product variety allows firms to charge the consumers with. Using this newly defined variable will help study the model without computing a specific convolution of two random variables. Lastly, a word about the assumption of symmetric product firms: while one could assume some kind of asymmetry between the two competitors, the goal of this paper is to show how the role of intermediation might or might not impress ex-post asymmetries on the market. Thus, assuming ex-ante asymmetric firms would make the analysis more complex and outside the scope of the present work.
In order to carry out a complete analysis of the optimal pricing behaviour, I will first analyse the uninformed uniform pricing case (Uninformed Uniform Prices, UUP), then move to the case in which one firm can propose personalized prices (Unilateral Price Targeting, UPT), and finally discuss the implications of both firms targeting consumers with personalized prices (Symmetric Price Targeting, SPT). As in the title of the section, the idea is to identify optimal pricing strategies, not necessarily equilibrium ones: in fact, the equilibrium analysis will be the object of the remainder of the paper. Optimality will be investigated in two different situations, both from the perspective of one firm and with respect to the whole industry.

2.2.1 Uninformed Uniform Prices.

Assume neither firm has access to information about consumers, hence they are bound to offer uniform prices, respectively denoted by p_i and p_j. A consumer h will buy from firm i if and only if $v_i^h - p_i \geq v_j^h - p_j$, i.e. when $s = v_i - v_j \geq p_i - p_j$, otherwise she will buy from j: the demand functions are then given respectively by:

$$Q_i(p) = [1 - F(p_i - p_j)]$$

$$Q_j(p) = F(p_i - p_j) \ .$$

Both existence and uniqueness of the equilibrium have to be verified. The most common way to check for uniqueness is to see if the best response map is a contraction and thereby satisfies the following condition for $k = i, j$

$$\frac{\partial^2 \pi_k}{\partial p_k^2} + \frac{\partial^2 \pi_k}{\partial p_i \partial p_j} < 0 \ , \tag{2.1}$$

(see e.g. Vives (1999)). However, as we will see, while that condition is easily satisfied, I need to make an assumption to ensure a that the changes in uniform prices caused by different degrees of price targeting are well behaved. Let $\epsilon_i \geq 0$ and $\epsilon_j \geq 0$ be two non-negative parameters.
To ensure the existence of an equilibrium in uniform pricing competition, I will make the following assumption:

Assumption 1. The density functions $g(v)$ are log-concave.

This implies that also the cumulative distribution $G(v)$ is log-concave. Logarithmic concavity is also preserved by the sum of two independent variables, so $f(s)$ and its c.d.f. $F(s)$ are log-concave too, see Bagnoli and Bergstrom (2005) for a detailed treatment. If a distribution is log-concave, then it is also *regular*, in terms of increasing hazard rate[6].
Note that $\frac{\partial^2 \pi_k}{\partial p_k^2} < 0$ (due to Assumption 1, the revenue function is concave), while $\frac{\partial^2 \pi_k}{\partial p_i \partial p_j} > 0$ since price are strategic complements in this model. It is then useful to state a lemma that assures the existence and uniqueness of equilibria in uniform prices in a more general case and other features, that will come in handy. Define $Q_i(p, \epsilon) = [F(\epsilon_i - p_j) - F(p_i - p_j)]$ and $Q_i(p, \epsilon) = [F(p_i - p_j) - F(p_i - \epsilon_j)]$, for $\epsilon = (\epsilon_i, \epsilon_j)$.

Lemma 1. *Let $\epsilon_i \geq 0$ and $\epsilon_j \geq 0$ be two non-negative parameters. Then:*

(i) An equilibrium in uniform prices always exists and is unique.

(ii) The derivatives of the equilibrium prices with respect to the parameters are always positive: $\frac{\partial p_k}{\partial \epsilon_l} \geq 0$. Furthermore, $\frac{\partial p_k}{\partial \epsilon_k} \geq \frac{\partial p_l}{\partial \epsilon_k}$.

[6] See Myerson (1981).

(iii) The derivatives of the equilibrium prices with respect to any of the two parameters are lower than unity: $\frac{\partial p_k}{\partial \epsilon_k} < 1$, $\frac{\partial p_l}{\partial \epsilon_k} < 1$.

The equilibrium result in uniform prices is reported in the following lemma.

Lemma 2. *When competing in uniform prices, the equilibrium price is given by* $p_i^* = p_j^* = p^*$*, where*

$$p^* = \frac{1}{2}\frac{1}{f(0)} \tag{2.2}$$

and each firm's profit are given by $\pi_i^u = \pi_j^u = \pi^u$*, with*

$$\pi^u = \frac{1}{4}\frac{1}{f(0)} \ , \tag{2.3}$$

and the equilibrium is unique.

Since the two firms are ex-ante symmetrical, the equilibrium prices coincide and every consumer buys at the same price. Note also that every consumer buys her preferred variety of product: if $v_i^h \geq v_j^h$, then the consumer purchases the i-variety of the good, and vice-versa. This particular situation in which none of the competitors is informed or able to apply any kind of price discrimination will be regarded as the benchmark case, as some state of "natural ignorance" that will be "tampered with" later on. It will serve as a comparison to evaluate whether or not certain strategies of price discrimination may be profitable, or desirable. Being the firms i and j symmetric, the computation of welfare at the symmetric equilibrium is quite easy. Consumers have mass 1, half of them buy from i, half from j, given that $f(v)$ is symmetric. It follows that if the equilibrium is symmetric, then welfare depends only on the equilibrium price p

$$W^s(p) = \int_p^{\bar{v}} v g(v) dv \ . \tag{2.4}$$

It follows that when the symmetric equilibrium price decreases, welfare always increases

$$\frac{\partial W^s}{\partial p} = -p\, g(p) \leq 0 \ .$$

Let Π denote industry profits, then consumer surplus can be stated as

$$CS = W^s - \Pi = W^s - \frac{1}{2}\frac{1}{f(0)}$$

and it is easy to see that in any symmetric equilibrium, an increase in profits causes the consumer surplus to decrease. This is a consequence of the total welfare being a constant: since there is no "mismatch" between consumer and firm, the size of the cake is at its maximum, the only thing which can change is who gets to eat more of it.

2.2.2 Unilateral Price Targeting

Now I introduce information for the first time, in an asymmetric manner: one firm knows the private information of everyone in the market, the other knows nothing. This case too is somehow extreme and perhaps not realistic (though not necessarily unrealistic), however, it will be useful in understanding the limits of price targeting in a competitive context, when one firm is given all the informational advantage.

To begin with, consider the case in which there firm i is completely informed about the valuations of every consumer in the market, while firm j does not, and denote with $S \equiv [\underline{v} - \bar{v},\ \bar{v} - \underline{v}]$ the whole set of consumers arranged according to their s. A negative s means they prefer, ceteris paribus, the good offer by Firm j, and a positive s the opposite. Firm i can therefore propose a personalized price to consumers, p_i^d, while the rival can only propose one price market wise, p_j^u. Call $E_i \equiv [e_i, \bar{v} - \underline{v}]$ the set of consumers that are offered discriminatory prices. Assume that a firm offers personalized prices to a subset of consumers of the form $E_i \equiv [e_i, \bar{v} - \underline{v}] \subseteq S$. Consider the following scenario: the price-targeting firm will try to offer a price that makes each consumer indifferent between buying her product or the rival's variety. Namely, fix a value for p_j^u, then the price offered to a consumer by i is $p_i^{dh} = v_i^h - v_j^h + p_j^u = s^h + p_j^u$, on the condition that $v_i^h - v_j^h + p_j^u \geq 0$, ruling out negative prices[7]. In

[7]This assumption is not problematic in this context, because it is a static one. On the other hand, should we be discussing a (not further specified) dynamic interaction, negative prices might be a useful anticompetitive instrument, for instance to prevent a rival from entering the market.

that case, the profit of firm i would be given by

$$\pi_i^d(p_j^u) = \int_{-p_j^u}^{\bar{v}-\underline{v}} (s + p_j^u) f(s) ds = \int_{-p_j^u}^{\bar{v}-\underline{v}} s f(s) ds + p_j^u [1 - F(-p_j^u)] \ , \qquad (2.5)$$

and, as one can see, the effect of an increase in p_j^u is always positive on firm's i profits from price targeting. This leads to a consideration: given the strategic complementarity of prices, too harsh a competition, also through the price targeting, pushes down the rival's uniform price, causing a backlash on firm i's profits.

Consider now a different case: firm i has the possibility to propose either a personalized price to consumers or a uniform price. Define then $\epsilon_i \in [0, \bar{v} - \underline{v} - p^*]$ and call it *degree of price discrimination.* The value of ϵ_i is the price equivalent to the *lowest discriminatory price* offered by firm i. Given *Lemma 1*, there is a one-to-one map between ϵ_i and e_i, which allows me to define $E_i \equiv \{ \ s \in S \mid s \geq \epsilon_i - p_j^u(\epsilon_i) \ \}$ [8]. Suppose the game unfolds as follows:

0 Firm i acquires the information, firm j learns i has access to information,.

1 Both firms compete in uniform prices, p_i^u and p_j^u.

2 Firm i proposes a personalized price p_i^d to consumers such that $v_i - v_j + p_j^u \geq \epsilon_i$.

The timing of the game is essential for this discussion, and common in the related literature, see Chen et al. (2022) . The rival must know that the other firm is informed and also to which extent she is informed (completely, in this case) in order to anticipate the price schedules offered by i. This timing allows j to calibrate her optimal response, hence the existence of both personalized and uniform prices on the market: vice versa, after observing the personalized prices offered by i, j could lower her uniform price of one ι to poach all the chunk of price-targeted consumers. One interpretation of these strategy is to view consumers who get targeted as the "good ones" (from the point of

[8] This exercise could be carried out using e_i as the independent variable. However, since the whole point of this section is to investigate prices, I find it more instructive to use ϵ_i, which is a price itself.

view of the firm), and consider the following: to target everyone increases the competition, hence drives the rival's price down too. Since the personalized prices are increasing in the rival's uniform one, such an aggressive behviour might be self-defeating in the end. Perhaps, some properly calibrated mix of personalized prices to some of the "best customers" and a uniform price to the rest of them might avoid depressing the rival's price and lead to higher profits. Firm's i profits are then $\Pi_i(\epsilon_i, p^u) = \pi_i^d(\epsilon_i, p_j^u) + \pi_i^u(\epsilon_i, p^u)$, with

$$\pi_i^d(\epsilon_i, p_j^u) = \int_{\epsilon_i - p_j^u}^{\bar{v}-\underline{v}} (s + p_j^u) f(s) ds = \int_{\epsilon_i - p_j^u}^{\bar{v}-\underline{v}} s f(s) ds + p_j^u [1 - F(\epsilon_i - p_j^u)] \ , \quad (2.6)$$

$$\pi_i^u(\epsilon_i, p^u) = p_i^u [F(\epsilon_i - p_j^u) - F(p_i^u - p_j^u)] \ , \quad (2.7)$$

while firm j's profits are still

$$\pi_j^u(p) = p_j^u F(p_i^u - p_j^u) \ . \quad (2.8)$$

Keeping the rival's price fixed, it can easily be observed that while the profit from price discrimination decreases when ϵ_i increases, the profit from uniform pricing increases.
The next proposition shows a first divergence between the optimal pricing strategy and the equilibrium one.

Proposition 1. *Suppose firm i knows the private valuations of every consumer, then:*

(i) The optimal strategy for firm i is to offer a personalized price if $p_i^d \geq \epsilon_i^*$, and a uniform $p_i^u(\epsilon_i^*)$ otherwise, while firm j offers $p_j^u(\epsilon_i^*)$ and ϵ_i^* is unique.

(ii) At the optimum, uniform prices increase in ϵ_i, i.e. $\frac{\partial p_i^u}{\partial \epsilon_i}(\epsilon_i^*) \geq 0$ and $\frac{\partial p_j^u}{\partial \epsilon_i}(\epsilon_i^*) \geq 0$.

(iii) The equilibrium strategy is given by $\epsilon_i = 0$.

Intuitively, though her profits would benefit from less price targeting, firm i is not able to credibly commit to such an optimal degree of price discrimination, i.e. to limit the set of consumers being offered personalized prices. The rival, firm j, would also benefit from a less aggressive pricing strategies:

$$\frac{d\pi_j^u(\epsilon_i)}{d\epsilon_i} = \frac{\partial \pi_j^u}{\partial p_i}\frac{\partial p_i}{\partial \epsilon_i} > 0$$

thereby increasing also industry profits. In other words: both firms would prefer the outcome give by ϵ_i^*,with respect to the actual equilibrium of the game. At any rate, absent a solid commitment device, this cannot be implemented.

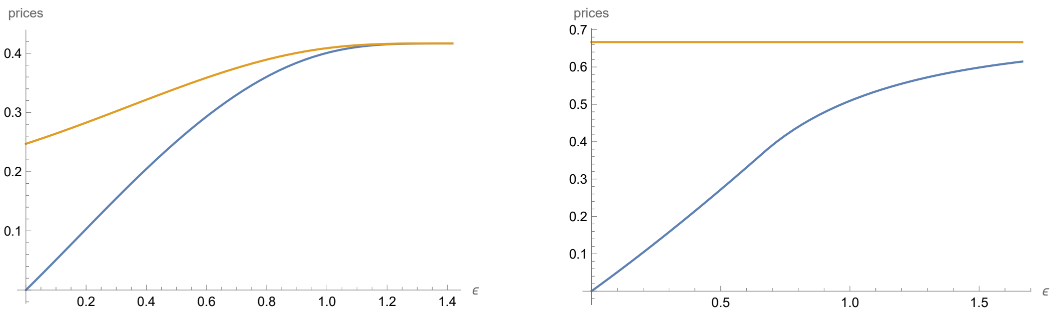


Figure 2.1: On the left, results for $G(v) \sim Beta(2,2)$; on the right, results for $G(v) \sim Exp(1.5)$. Yellow is for p_j^u, blue is for p_i^u.

Observe also that, thanks to Lemma 1, uniforms prices reach their maximum at p^*, when there is no price targeting, and since $\frac{\partial p_i}{\partial \epsilon_i} > \frac{\partial p_j}{\partial \epsilon_i}$, we also have that $p_i^u \leq p_j^u$. At the equilibrium, $\epsilon_i = 0$, the uniform price of firm j reaches its minimum (while i only offers personalized prices), and then increases as ϵ_i does.

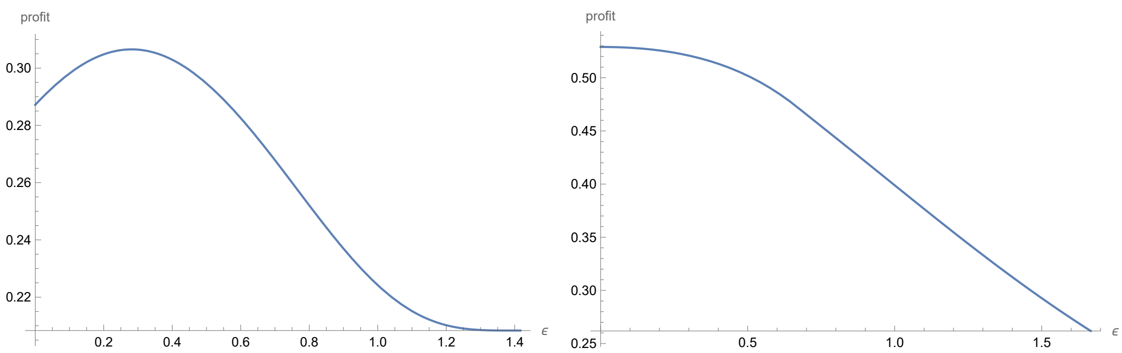


Figure 2.2: On the left, results for $G(v) \sim Beta(2,2)$; on the right, results for $G(v) \sim Exp(1.5)$.

Remark: Welfare in the Asymmetric case. Welfare in this case is considerably more complicated to calculate (see Appendix A). This complication is given by the fact that there is a *mismatch* between the preference for a product and the product that is bought by the consumer. Prices alter the allocation of the two goods with respect to the previous case. In fact, since the uniform price offered by Firm i is lower than the one offered by Firm j, then there will be consumers with a higher valuation for good j buying good i. Though this increases the mass of consumer that make a purchase, some of them will buy from their least favourite variety. More specifically, there will be a fraction of consumer for whom $v_j^h \geq v_i^h$, but $v_i^h - p_i^u \geq v_j^h - p_j^u$, hence prices distort the matching between consumers and firms. Such an effect makes the effects of UPT ambiguous on both Welfare and Consumer Surplus, that will ultimately depend on the distribution $G(v)$.

2.2.3 Symmetric Price Targeting

Another situation has to be investigated: namely, the one when both firms know everything. Again, though not being particularly realistic, this case is often analysed in many papers as the competitive price discrimination benchmark. It is often (and rightly so) stressed how competitive price discrimination might benefit consumers by fostering competition. *Ideally*, when the tastes of every consumer are known to the firms, the allocation should be efficient and prices driven down by price competition. The idea being that each consumer becomes like a small market in which firms compete á la Bertrand, and if products are homogeneous enough, the prices fall close enough to marginal cost. This of course hardly happens in real markets. As in the previous case, I will allow for the two firms to propose a mixed price schedule of both personalized prices and uniform prices; keeping the same notation as before, $p_i^{dh} \geq \epsilon_i$ and $p_j^{dh} \geq \epsilon_j$ will denote the personalized prices of the two firms, offered to their "good customers", and let $\epsilon = (\epsilon_i, \epsilon_j)$. As above, the sets of consumers who receive personalized offers can be written as $E_i \equiv \{ \ s \in S \mid s \geq \epsilon_i - p_j^u(\epsilon) \ \}$ and $E_j \equiv \{ \ s \in S \mid s \leq p_i^u(\epsilon) - \epsilon_j \ \}$ .The purpose of this paragraph is to investigate profitable deviation from the ideal results that such price wars might deliver, i.e. competitive price discrimination, observing instead how a certain degree of price discrimination might be chosen and result in an anticompetitive conduct. In other words, I am interested in how firms might like to act in order to boost both their profits at detriment of consumers. Of course, another scenario could be taken into account: the one in which the two firms collude. Such scenario would require the ability of the two firms to "punish" the other in a next period to enforce the collusive agreement, which would make the analysis a dynamic one: form the time being, I restrict the attention to a one-period interaction. With this in mind, the industry maximizing degree of price discrimination is to be understood as the optimal mix of uniform and personalized prices that, absent the guarantee of a collusive agreement, firms would like to play in a semi-competitive context in which both cannot commit not to make offers to every consumer in the market. This said, firm's i profits are given by $\Pi_i(\epsilon_i, p^u) = \pi_i^d(\epsilon_i, p_j^u) + \pi_i^u(\epsilon_i, p^u)$, with

$$\pi_i^d(\epsilon_i, p_j^u) = \int_{\epsilon_i - p_j^u}^{\bar{v}-\underline{v}} (s + p_j^u) f(s) ds = \int_{\epsilon_i - p_j^u}^{\bar{v}-\underline{v}} s f(s) ds + p_j^u [1 - F(\epsilon_i - p_j^u)] \ , \quad (2.9)$$

$$\pi_i^u(\epsilon_i, p^u) = p_i^u [F(\epsilon_i - p_j^u) - F(p_i^u - p_j^u)] \ , \quad (2.10)$$

analogously, firm j's profits are given by $\Pi_j(\epsilon_j, p^u) = \pi_i^d(\epsilon_j, p_i^u) + \pi_j^u(\epsilon_j, p^u)$, where

$$\pi_j^d(\epsilon, p_i^u) = \int_{\underline{v}-\bar{v}}^{p_i^u - \epsilon_j} (-s + p_i^u) f(s) ds = \int_{\underline{v}-\bar{v}}^{p_i^u - \epsilon_j} -s f(s) ds + p_i^u F(p_i^u - \epsilon_j) \ , \quad (2.11)$$

$$\pi_j^u(p) = p_j^u [F(p_i^u - p_j^u) - F(p_i^u - \epsilon_j)] \ . \quad (2.12)$$

If with only one firm using targeted prices it could have been obvious that profits would increase, at least for such a firm, in this case the interactions take on a further degree of complexity. The decrease in the rival's price offset by price discrimination goes both ways and lowers one firm's personalized prices as well. Hence, it does not appear obvious whether, when the whole industry adopts personalized prices, such a pricing structure might or might not increase profits. In what follows, the analysis is greatly simplified by the ex-ante symmetry assumed at the beginning, so it should not be surprising that the optimal as well the equilibrium strategies are symmetric. Consider the derivatives of the industry profits with respect to the two parameters:

$$\frac{d\Pi}{d\epsilon_i} = \frac{d\Pi_i}{d\epsilon_i} + \frac{d\Pi_j}{d\epsilon_i} = 0$$

$$\frac{d\Pi}{d\epsilon_j} = \frac{d\Pi_i}{d\epsilon_j} + \frac{d\Pi_j}{d\epsilon_j} = 0 \; .$$

For any of these two derivatives to be nil, the value of the parameter ϵ_k has to be higher than in the UPT case: since the competitor profits increases in ϵ_l $(l \neq k)$ and it is always positive, i.e. as the number of people who get targeted decreases, the other derivative has to be negative, implying less of the "best ones" of the respective high-valuations customers. The results are summarized in what follows.

Proposition 2. *Suppose Firms i and j know the private valuations of every consumer, then:*

(i) The industry maximizing strategy for is to offer a personalized price if $p_i^d \geq \hat{\epsilon}_i$ $p_j^d \geq \hat{\epsilon}_j$, *with* $\hat{\epsilon}_i = \hat{\epsilon}_j = \hat{\epsilon}$ *and uniform prices* $p_i^u(\hat{\epsilon}) = p_j^u(\hat{\epsilon})$ *and* $\hat{\epsilon}$ *is unique.*

(ii) At the optimum, uniform prices increase in ϵ_i, *i.e.* $\frac{\partial p_i^u}{\partial \epsilon_k}(\hat{\epsilon}) \geq 0$ *and* $\frac{\partial p_j^u}{\partial \epsilon_k}(\hat{\epsilon}) \geq 0$, $k = i, j$.

(iii) The equilibrium strategy is given by $\hat{\epsilon_i} = \hat{\epsilon_j} = 0$.

(iv) SPT increases Welfare.

Let's consider the optimum strategies. Interestingly, there are both differences and analogies with the previous case. Start from the differences: the symmetry grants that each customer buys from his favourite producer, which was not the case in UPT: at the optimum, prices do not alter the matching between consumers and firms any more. On the other hand, the more polarized customers for each firm are offered personalized prices, while the "average guys" are offered the same uniform prices. The pro-competitive effects of

competitive price discrimination are weakened when industry profits are maximized: firms (not surprisingly) would rather soften competition, and caress the idea of a strategy that includes both uniform and personalized prices. For both firms, there is a trade off between rent extraction from the *more polarized* consumers and the profits gained from the *less polarized* ones since a more aggressive targeting lowers the uniform prices. It follows that firms would like to coordinate around a price system that allows for surplus extractions from the customers with very heterogeneous valuations for the two goods, and a uniform price for the rest of them, avoiding harsh competition. More consumers participate in the purchase and profits increase.
Once again, and in complete analogy with the bulk of existing literature, firms are not able to commit to an industry-optimal strategy, hence the equilibrium result in this case does not include any uniform price and drives personalized prices down respectively to $p_i^{dh} = s^h$, when $s^h \geq 0$, and $p_j^{dh} = -s^h$, when $s^h < 0$. The only margin the firms make in equilibrium is brought to them by product differentiation.

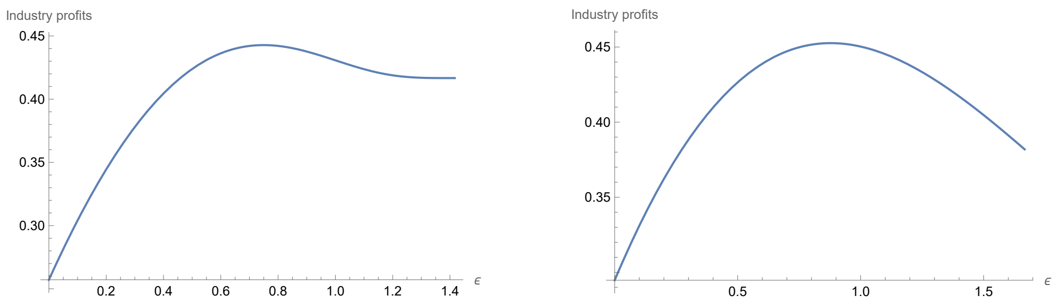


Figure 2.3: On the left, results for $G(v) \sim Beta(2,2)$; on the right, results for $G(v) \sim Exp(1.5)$.

2.2.4 The case of Partial Knowledge

All the above discussion referred to extreme circumstances when producers know all or nothing about consumers, in what follows partial knowledge will be introduced, i.e. I allow for the valuations of a subset of consumers to be known, while the valuations of the rest of them remains private information. Call the share of those consumers whose valuations are known, whether to one

or both firms, $\delta \in [0,1]$. Depending on who knows what, the prices will be determined and profits made, however it should be pointed out that in this case there is a portion of consumers, δ, that might be targeted by one or both firms with personalized prices, like in UPT and SPT cases, while there is a share $1-\delta$ which will be offered uniform prices as in the UUP case. While the determination of the equilibrium use of information essentially remains the same as in the previous paragraphs, the optimal degree of price discrimination (loosely denoted by) ϵ changes since the benefit from uniform prices and the losses from less price targeting are differently weighted.
Starting from the case of UPT, given a value of δ, profit are given by $\Pi_i(\epsilon_i, p^u, \delta) = \pi_i^d(\epsilon_i, p_j^u, \delta) + \pi_i^u(\epsilon_i, p^u, \delta)$, with

$$\pi_i^d(\epsilon_i, p_j^u, \delta) = \delta \left[\int_{\epsilon_i - p_j^u}^{\bar{v} - \underline{v}} sf(s)ds + p_j^u[1 - F(\epsilon_i - p_j^u)] \right] ,$$

$$\pi_i^u(\epsilon_i, p^u, \delta) = \delta \left[p_i^u[F(\epsilon_i - p_j^u) - F(p_i^u - p_j^u)] \right] + (1-\delta) p_i^u[1 - F(p_i^u - p_j^u)] ,$$

while firm j's profits remain the same. The following proposition describes how partial information impacts the optimal degree of price discrimination.

Proposition 3. *If only firm i knows the valuations of the share δ of consumers, then (i) there exists only one profit-maximizing $\epsilon_i^*(\delta)$, and (ii) within the δ-consumers, $\epsilon_i^*(\delta) > \epsilon_i^*(1)$. Also notice that*

$$\left. \frac{d\Pi_i}{d\delta} \right|_{\epsilon_i^*(\delta)} = \left. \frac{\partial \Pi_i}{\partial \delta} \right|_{\epsilon_i^*(\delta)} > 0 .$$

Indeed observe that, by the envelope theorem, and that by differentiating the first order condition for profit maximization w.r.t. δ around the optimum one gets

$$\left. \frac{d}{d\delta} \left(\frac{d\Pi_i}{d\epsilon_i} \right) \right|_{\epsilon_i^*(\delta)} = \frac{d^2 \Pi_i}{d\epsilon_i^2} \frac{\partial \epsilon_i}{\partial \delta} + \left. \frac{\partial \Pi_i}{\partial \epsilon_i \partial \delta} \right|_{\epsilon_i^*(\delta)} = 0$$

with

$$\left. \frac{\partial \Pi_i}{\partial \epsilon_i \partial \delta} \right|_{\epsilon_i^*(\delta)} < 0$$

hence

$$\frac{\partial \epsilon_i^*}{d\delta} = -\frac{\frac{\partial \Pi_i}{\partial \epsilon_i \partial \delta}}{\frac{d^2 \Pi_i}{d\epsilon_i^2}}\Bigg|_{\epsilon_i^*(\delta)} < 0 \tag{2.13}$$

Within the set of the consumers that could be targeted, less get targeted than in the case of full knowledge, i.e. $\delta = 1$. This is simply rationalized by the higher weight associated to profits derived from uniform prices when $\delta < 1$: by investing in more price discrimination uniform prices are pushed down both within the δ-consumers and outside said set, but in this case the benefit from price discrimination are circumscribed to a smaller set of consumers, while relative dimension of those who buy at uniform prices is bigger.
Turning to the case of SPT, now also the profits of firm j depend on δ, in a completely analogous fashion as i's ones:

$$\pi_j^d(\epsilon, p_i^u, \delta) = \delta\left[\int_{\underline{v}-\bar{v}}^{p_i^u-\epsilon_j} -sf(s)ds + p_i^u F(p_i^u - \epsilon_j)\right] ,$$

$$\pi_j^u(\epsilon, p, \delta) = \delta\Bigg[p_j^u[F(p_i^u - p_j^u) - F(p_i^u - \epsilon_j)]\Bigg] + (1-\delta)[p_j^u F(p_i^u - p_j^u)] .$$

As reported in the proposition below, the results of Proposition 2 and 3 extend to this case, confirming that industry profits are higher when both firms engage in personalized pricing, provided it is properly calibrated.

Proposition 4. *If both firms know the valuations of the share δ of consumers, then (i) there exist a unique couple of industry profit-maximizing $\hat{\epsilon}_i(\delta) = \hat{\epsilon}_j(\delta) = \hat{\epsilon}(\delta)$, with (ii) $\hat{\epsilon}(\delta) > \hat{\epsilon}(1)$. Also in this case, at the optimum,*

$$\frac{\partial \hat{\epsilon}}{d\delta} = -\frac{\frac{\partial \Pi}{\partial \epsilon_k \partial \delta}}{\frac{d^2 \Pi}{d\epsilon_k^2}}\Bigg|_{\hat{\epsilon}(\delta)} < 0 . \tag{2.14}$$

(iii) Welfare increases in δ.

The two results above indicate that the less informed are the firms, the less price discrimination the would do in a competitive environment. The rationale for this is found again in the increased relative importance of profits made from uniform prices with respect to those from personalized pricing: uniform prices are pushed down as the set of consumers offered personalized prices increases, and in this case there exist a set of consumer to whom only uniform prices can be offered, therefore too an aggressive price targeting drives down profits on a wider market segment (with respect to the case of full information).

2.3 The Economics of the Intermediary: Monopoly

Once analysed the possible equilibria and optimal pricing strategies for the firms, now I introduce an Intermediary. The intermediary offers a service (an on-line service, for example) to consumers for free. I will assume that the higher the quality of service, the higher the number of users. While benefiting from such service, consumers reveal their private information to the intermediary, who can then sell it to firms. In other words, now information becomes endogenous in this model and its allocation a strategy played by the intermediary. Assume that the intermediary, denoted also with I, can provide a service, referred to as App (A), of benefit $b \in R^+$ to consumers at a positive marginal cost $c > 0$, $c(b) = c \cdot b$. Each consumer h incurs in a attention cost of using the App $\underline{b}^h$ distributed according to a measure $Prob(\underline{b} \leq b) = H(b)$, where $H(b)$ is independently distributed from $G(v)$, thus also from $F(s)$, and $h(b)$ is the respective p.d.f..

Assumption 2. $H(b)$ is non-negative and strictly concave, i.e. $h'(u) > 0$ and $h''(b) < 0$.

The attention cost can be interpreted as the effort of using a certain device: in this respect consumers are heterogeneous. In fact, given a level of quality of the service offered by I, not every consumers will derive the same utility from using it, actually in order to use it they have to pay attention to it. It follows that as the quality of the app increases, the number of consumers using it increases, meaning also that more consumers will reveal their valuations. In other words, the share of δ-consumers will increase in b, $\delta(b) = H(b)$:

$$\frac{\partial \delta}{\partial b} = h(b) \geq 0 \ .$$

Thus, the amount of consumers that might be offered personalized prices is made endogenous, and shall be pinned down by the profit-maximizing equilibrium strategy of the intermediary, and assume for the moment that the Intermediary can credibly commit to sell only the data she wants to whom she wants. In this model, the Intermediary has also another advantage: she comes to know information first and can possibly decide how to allocate it among the product firms i and j. The analysis conducted in the previous sections should suggest that the intermediary might not want to sell out all the information he has gathered, rather there exist optimal informational structures that can make the industry more profitable. What remains to be seen is whether or not such structures can be sustained by equilibrium behaviours. To begin with, assume that the intermediary is a monopolist, facing no competition or the threat of entry by other competitors. In such scenario, if firms make extra-profits by using the data purchased from I, I will assume that the bargaining power over such profits is all in the hands of the intermediary. Denote the extra-profits made by firms using data to price-discriminate as $\Delta\Pi_i(\epsilon) = \Pi_i(\epsilon) - \pi_i^u(\emptyset, \emptyset)$ and $\Delta\Pi_j(\epsilon) = \Pi_j(\epsilon) - \pi_j^u(\emptyset, \emptyset)$, where $\epsilon = (\epsilon_i, \epsilon_j)$ stands for the allocation decided by the intermediary: they do not depend directly on δ, rather they depend on the degree of price discrimination they are allowed to do according to the data they have bought. The tariffs that the intermediary applies are given by $T_i = \Pi_i(\epsilon) - \pi_i^u(\emptyset, \epsilon_j)$ and $T_j = \Pi_j(\epsilon) - \pi_j^u(\epsilon_i, \emptyset)$, since the intermediary can always threat to sell to the rival. The tariffs the Intermediary proposes as a monopolist must take into account that while increasing the profits of one firm (by providing a credible commitment to less price discrimination), she might also increase the rival firm's profits. It follows that there might be *overprovision* of data with respect to cases studied in *Section 2*.
The profit function of the intermediary thus reads as

$$R(\epsilon(\delta(b))) = T(\epsilon(\delta(b))) - c \cdot b$$

observe that

$$\hat{\epsilon}^M(\delta) = \arg\max_{\epsilon}[\Pi_i(\epsilon) - \pi_i^u(\emptyset, \hat{\epsilon}_j)] + [\Pi_j(\epsilon) - \pi_j^u(\hat{\epsilon}_i, \emptyset)] - cb \tag{2.15}$$

and it is ultimately a function of b. Observe that as the first element of the equation reaches a maximum, the second is always decreasing in ϵ, hence in b. Although the volume of businesses (the size) and the cost of maintenance of the App affect the amount of data δ produced, the optimal allocation of information is not affected by the production cost of the intermediary.
A further characterization of the activity of the Intermediary needs to be specified.

Assumption 3 (Timing). Tariffs are paid to the Intermediary only after profits are realized in the goods market. Such contracts are enforceable in a court of law.

In other words, I am assuming that the Intermediary can draft data provision contracts with firms that imply delayed payment. Notice that this feature is also consistent with the economic environment of the model since I have not ever assumed any external funds i.e. the presence of banks, or lending institution, nor savings on any side of the interactions. Adding such feature to the present framework would demand a description of the access modality to a financial sector, which is beyond the scope this analysis. The utility of *Assumption 3* lays in the fact that allows to remove the commitment issues that the Intermediary might face, and her credibility. In fact, were the payment be requested upfront, there would be no reason for the product firms to trust the Intermediary, who could always go back on her word and reveal more information to the rival. To sum up the import of this assumption, it could be said that guarantees the credibility of the Intermediary, removing what could otherwise be a commitment issue, and also prevents the opportunism of firms by introducing judicial consequences in case of missed payments.
The game unfolds as follows:

1. The intermediary decides a value for b.

2. The intermediary decides how and how much information ϵ to allocate among product firms. The intermediary stores said amount of informa-

tion. *Information storage is costly and publicly observable.*

3. If the firms accept the deal they agree to pay a tariff T_k.

4. Competition ensues and profits are realized.

5. Tariffs T_k are paid to the Intermediary.

The following proposition describes the impact of intermediation in altering the market structure.

Proposition 5.

(i) If the intermediary finds it profitable to sell only to Firm i, then the optimal amount of information allocated is $\epsilon_i^{*M}(\delta) = \epsilon_i^*(\delta)$.

(ii) If the intermediary can sell to both firms, the optimal allocation of information is $\hat{\epsilon}^M(\delta) \leq \hat{\epsilon}(\delta)$.

(iii) The equilibrium quality b^* *and* $\hat{b}$ *are given as*

$$b^* = h^{-1}\left(\frac{c}{\frac{\partial T_i}{\partial \delta}}\right)\Bigg|_{b^*} \tag{2.16}$$

$$\hat{b} = h^{-1}\left(\frac{c}{\frac{\partial T_i}{\partial \delta} + \frac{\partial T_j}{\partial \delta}}\right)\Bigg|_{\hat{b}} . \tag{2.17}$$

To put it simply: under *Assumption 3*, intermediation makes it possible to sustain at equilibrium a different use of information. First notice that optimal and equilibrium degrees of price discrimination do not coincide when the Intermediary is a monopolist. The monopolistic position of the Intermediary makes it such that the information provided is *more* than what the product firms would like to use if they were dealing with the data directly.
Secondly, the intermediary can impress on the market, through the product firms, the price structures that maximize her own profit, and, since in this case she has got all the bargaining power, he can appropriate all the extra-profits resulting from the use of data. Under this dynamic, another issue emerges: profit concentration. Anyway, as far as the above setting is concerned, this is

an implication of the assumptions rather than a result; however, all the extra-profits generated are taken away by the Intermediary, which suggest that while increasing the profitability of the industry, intermediation might not lead to a boost in the balance sheet of the producers depending on the power the actors have when splitting the extra-gains. Later on, I will present how competition might rebalance the profit-sharing, at any rate some intuition can be gained right now. The digital industry, where such data intermediaries belong, is arguably dominated by incredibly large firms, that can hardly be pictured as fierce rivals. It follows that the, admittedly extreme in this model, assumption concerning the distribution of bargaining power might very well be not so extreme in reality.

2.3.1 Multiple Markets Intermediation

Many papers investigate the impact of the number of competing product firms in the same market, in this model however, given the assumption that the firms are symmetric and there is full market coverage the number of competitors would not deliver new results: as competition increases, when competing in uniform prices, the equilibrium price would decrease, hence more consumers purchase the differentiated good resulting in equal market shares for the product firms. What instead can offer more insight on the intermediary's behaviour is the number of different markets in which the intermediary operates, which will be denoted by $m \geq 1$ and referred to as the intermediary's size . To keep things simple then, assume these m markets are identical, i.e. can be described by the same distributions $G(v)$ and $F(s)$, and independent from one another. It can already been anticipated that m will not affect the equilibrium and optimal degree of price discrimination directly, but only indirectly. Keeping the marginal cost of quality constant, it is easy to guess that an increase in the volume of business (increased m) translates into an higher investment in the quality of the App, b, which in turn maps into a higher share δ of known consumers. In this section, I will simply refer to the industry profits with $\Pi(b)$ and to the optimal level of quality when operating in m different markets by

$\hat{b}^m$. The intermediary problem therefore reads

$$\max_{b^m} R(b^m) = mT(b^m) - cb^m \tag{2.18}$$

the following proposition collects the results describing the impact that the intermediary size has on the markets.

Proposition 6. *Suppose a monopolistic Intermediary operates in m independent identical markets. Then:*

(i) The quality of the service increases with the size $\hat{b}^{m+1} \geq \hat{b}^m$.

(ii) As the size increases, the intermediary's profits increase too.

(iii) As the size grows, the degree of price discrimination tends to the one of full knowledge.

(iv) With SPT, Welfare increases as the number of intermediated markets grows $\frac{\partial W^s}{\partial m} > 0$

These results are quite intuitive. As the volume of the business for the intermediary increases, then the investment in the App quality increases, since its marginal productivity keeps growing and marginal costs are constant. Logically, more quality increases the amount of consumers that will find it convenient to use the App, for no particular reason rather that the opposite would sound quite absurd. This mechanism widens the share of δ-consumers, who are those whose valuation is known, and finally, exploiting the results of Propositions 4 and 5, as more consumers' valuations become known, i.e. δ nears 1, the degree of price discrimination will grow.

What should be discussed in this respect is the technological foundation for these results: namely, the *zero cost of replication.* By zero cost of replication I mean that the job that the intermediary actually does is investing and taking care of the App, which collects data about consumers, he does not incur in any cost when selling (or these cost are negligible). There are two effects then: fix a certain level of quality, then the size of the intermediation does not affect the costs directly. Secondly, this generates a positive externatlity on profits:

if more firms join the same intermediary, then the investment in quality b increases, which means more data are collected.
The role of the size of intermediation m is then clear: the bigger it is, the closer to case of complete information about consumers, and the ability to shape competition in the way the intermediary sees fit grows. One could also argue in this setting that the balance of bargaining power shifts in the intermediary's favour as m grows, thereby making the initial (arbitrary) imbalance in bargaining power look much less extreme: it would rather seem the result endogenously determined.

2.4 Competition between Intermediaries

Let's introduce a rival in the mix. The, perhaps, most striking features of the digital industry appears to be a general lack of competition. One possible reason might be represented by the set of anticompetitive commercial practices: tying, bundling, forms of price discrimination, exclusive dealings and alike. Another reason might be the existence of network effects on the demand side, possibly due to a reduction in search costs: one might want to join a social media like Facebook because everyone also is on Facebook, while joining another media, perhaps identical if not better in some instances, might prove idle since the value of the joining depends on many others using the same service. The existing literature, though, often tends to model such effects in a "implicit" way, i.e. it employs specifications of the utility function containing some externality term that increases in the number of participants, rather than showing how the technology of production can allow for an economy of scale that abates the costs. In simpler words, consider the *nightclub example*: people, ceteris paribus, find it convenient to go to crowded clubs because they guess they have better chances to find good company (according to their tastes). Given that if there are many other people there, the probability of meeting someone you like is higher. The (unreasonable) alternative is to wander from one club to the other, which is not only annoying, but may very well manifest as a lot of many spent in gas. The trending attitude [9] in the literature is more in line with modelling the idea that people enjoy certain things because other people do as well.

However, my arguments differ from these views, in hope to complement them. Firstly, the network effect I will describe is on the suppliers' side: producers prefer to join the same intermediary because by increasing his size, the intermediary can increase the profits from price discrimination. This provides a justification for network effect grounded in technology[10] and pricing rather than a generic desire for good company. Secondly, I will attempt to show how, due to such an effect, anticompetitive practices, limit pricing in this case, might not be easily viable and not look at all like limit pricing. Connecting the dots, the monopolistic position of the intermediary might be reached without any particular structure of consumers' preferences, it is instead founded in the technology of data production, and the size of the intermediary allows for an aggressive attitude in pricing, without having to significantly lower them.

In order to argue for the first, I need to establish how firms behave one with respect to another.

Assumption 4. Firms are not able to strategically coordinate and play in teams.

The assumption above establishes that firms cannot coordinate in accepting or rejecting offers made by competing intermediaries. Consequently, for a firm (or an industry, which is represented by two firms in this model) to stick with an intermediary, it suffices to satisfy her individual rationality constraint, i.e. the intermediary cannot offer a deal to a firm counting on the behaviour of another.

To argue for the second point, I have to make another assumption, namely a tie-breaking to determine when the rival actually enters and compete.

[9] See the work of Armstrong (2006), or Rochet and Tirole (2006) for great seminal examples.

[10] Ultimately delivering a result similar to the "only one man left standing" of Arthur (1989), though in a much simplified context.

Assumption 5 (Tie-breaking rule). The rival intermediary, J, has entry cost in the amount of ι, small at pleasure. Firms have switching costs when moving from one intermediary to the other in the amount of κ, small at pleasure.

The tie-breaking rule above implies that the rival intermediary decides to enter if and only if it makes strictly positive profits, and that firms prefer, *ceteris paribus*, not to switch[11] unless there is a strictly positive gain from doing it. The reason for this assumption is that any consistent positive amount of entry cost would make it harder for the rival J to enter the market, which would favour the incumbent I. The same holds for switching costs. I am removing those advantages, or reducing them to a minimum. Furthermore, the two intermediaries I and J compete in prices offering the same service (to begin with) á la Bertrand, which means that competition is as intense as it gets. In other words, the environment in which I discuss the model is the harshest possible environment for the incumbent: entry is extremely cheap and the rival is at least as good as he is.

Lemma 3. *Suppose there is an incumbent intermediary I , serving m industries, and an entrant rival J, with a technology that allows it to serve up to $n \leq m$, as efficiently as I. Then:*

(i) The rival cannot poach any firm from I.

(ii) If the incumbent I engages in limit pricing, he makes non-negative profits.

(iii) The limit tariffs increase as the difference between m and n becomes wider.

Proof of Lemma 3. (i) I will prove this for the SPT case, assuming that the cost of generating extra-profits are split evenly among the firms. The

[11] See Farrell and Klemperer (2007) for a detailed treatment. Although, in this case, switching costs are way less powerful and not strategic variables.

amount of extra profits generated by I is equal to $\Delta\Pi^m(b^m) = m[\Pi(b^m) - \pi^u(\emptyset)] - C(b^m)$, and since profits are increasing in the size , it follows that $\Delta\Pi^m(b^m) \geq \Delta\Pi^n(b^n)$. The best offer the rival J can make is to leave to the producers all of the extra profits, i.e. $T_J^n = \frac{1}{2}\frac{C(b^n)}{n}$ and each firms makes profit as $\Pi = \frac{1}{2}[\Pi(b^n) - \frac{C(b^n)}{n}]$. The incumbent, being at least as big as the rival, can make the same offer, and still make positive profits (nil if $n = m$): $mR_I(b^m) - nR_J(b^n) \geq 0$. Since the profits of the entrant would be nil, by assumption, the rival does not enter.
(ii) At the equilibrium, the tariffs proposed to firms by the incumbent I are given as

$$T_I^m = \frac{1}{2m}[mR_I(b^m) - nR_J(b^n)] \ .$$

This way the rival profits are nil, hence there is no entry.
(iii) The claim follows from the positive impact of size on industry profits: as profits increase, T_I^m-s grow. ■

The lemma above should not come as a surprise: it follows from the zero cost of replication technology already described. In particular, *claim (ii)* is a direct implication since the the bigger intermediary generates a higher externality than the smaller one, he can lower prices and still make profits. In the same fashion as the intermediary grows bigger, the impact of competition on the amount of extra-profits he can extract from firms becomes weaker weaker.

2.4.1 From Zero... to Monopolist

When discussing the size of the intermediary, we are discussing of how a network is formed, which is most naturally represented by a series of offers, done one after the other, to firms: the network increases step by step. Consider the following set-up:

Set-up. There are m industries and two identical intermediaries I and J: firms, or industries, decide which intermediary to join one at a time, sequentially. I will use industries rather than single firms to simplify the analysis.

However, there is a rationale: if there is a point in foreclosing, then only one firm gets intermediated, if there is not, then both firms get intermediated. Since they choose sequentially, then the scale effect drives them to choose the same intermediary. The game unfolds this way:

0 Both I and J are ready to enter the market, paying ι.

1 Firms in the first industry, say 1, choose and intermediary I.

2a Then the second has to decide which intermediary to join.

2b Industry 1 has to decide whether to switch or not.

- Repeat until industry m.

3a Contracts are signed, tariffs are agreed upon and data collected.

3b Profits are finally realized.

4 Tariffs are paid to the Intermediary.

Proposition 7. *(i) Under the above set-up, at the equilibrium there is only one intermediary in the market and practices limit pricing. (ii) For a given* b*, the intermediary allocates information according to* $\hat{\epsilon}(\delta)$*. i.e. the industry profit-maximizing allocation.*

Proof of Proposition 7. The tariff offered to the firms in the industry 1 is $T_I^1 = T_J^1 = \frac{1}{2}C(b^1) > 0$ so that if accepted, the intermediary enters the market. The first industry is indifferent to joining one or the other: say it joins I. When addressing the second industry, the tariffs differ (numbers at the top are notation for the industry, they don't identify powers):

$$T_J^2 = \frac{1}{2}C(b^1)$$

$$T_I^2 = \frac{1}{2}[R_I^2(b^2) - R_J(b^1)] \; ;$$

the firms in industry 2 also join I. Notice that industry 1 has no incentive to switch, since it would incur in a cost κ, and would not gain anything. Iterating this argument until industry m one gets that the equilibrium tariff is

$$T_I^m = \frac{1}{2}[R_I^m(b^m) - R_J^1(b^1)] \ ,$$

which corresponds to limit pricing. Since firms can only decide to switch one at a time, again they cannot lower the tariffs they pay, and they would incur in a switching cost κ, so none of them has the incentive to join J.
(ii) The intermediary's problem is to find

$$\hat{b}^m = \arg\max_b [R_I^m(b^m) - R_J(b^1)] = \arg\max_b R_I^m(b^m)$$

which pins down $\hat{b}^m$. To any $\hat{b}^m$, by *Assumption 2*, corresponds only one $\delta(\hat{b}^m)$, and since the allocation of data does not depend on b directly, given a certain value of δ, then profit maximization for the intermediary implies industry profit maximization

$$\hat{\epsilon}(\delta^m) = \arg\max_{\epsilon(\delta)} R_I^m(\epsilon(\delta)) = \arg\max_{\epsilon(\delta)} m\Pi(\epsilon(\delta)) \ ,$$

which corresponds to the allocations of *Propositions 1-4*. ■

There are two features I want to stress about this situation. To begin with, competition just look likes it is expected to look at early stages: two identical firms, competing in prices and selling an homogeneous good: prices are low. As more and more firms join the same intermediary, competition does not soften, simply it does not look like harsh competition since tariffs keep getting higher and higher. Prices are significantly different from zero, and they grow at each round, yet they are competitive as before.
Secondly, there is a first come best served standard: firms which get in first pay lower tariffs than those who get in later. The margins that the intermediary makes increase at each round, and in the size m.
The fact that prices can be high and still be limit prices can easily trick the observer into believing that the conduct of the intermediary is fair, while actually he is engaging in anticompetitive behaviour, which simply does not look

like it. The simple economics of the intermediation, in this case, can reveal that competition can be hindered in markets for intermediation without being very costly if the potential pool of firms is large, and very aggressive commercial practices do not necessarily need to look aggressive. Furthermore, as the size grows, it becomes easier to keep an aggressive attitude at a lower cost. In this model, at any rate, this does not affect the efficiency in the market for intermediation, it only affects the way in which profits are shared between the intermediary and the intermediated firms.

2.5 Conclusions

In this work, I have attempted to show a mechanism through which data intermediation and their strategic allocation can impact on prices, and dampen competition in product markets, and at the same time, due to an economy of scale, provide grounds for anticompetitive behaviour in the intermediation market itself, leading to its monopolization. To conclude this paper, a few remarks are in order. First, the effects on consumers are ambiguous. There is no clear way, unless one can justify the adoption of a specific distribution, to see whether or not consumers are harmed by the segmentation impressed on product markets. While some consumers will clearly see their share of the surplus decrease due to price targeting, more consumers will participate in the purchase, increasing welfare, and benefiting from the exchange that now happens due to lower prices offered to non-discriminated consumers. This phenomenon, though not unknown, might be puzzling to Antitrust Authorities, trying to understand the impact data intermediation, precisely because the effect on consumers are hard to disentangle from the overall effect on welfare. A second point is that intermediation brings about *market segmentation*. In this model, the more *polarized* consumers are targeted with personalized prices, while the less polarized are offered uniform prices, and this segmentation depends on the availability of information. It follows that privacy regulations impact and possibly halt the actions of the intermediary, thus affecting welfare. There is then a strong relationship between the legal protection of consumers' data and price schedules in product markets and competition in the digital sector, which should be carefully examined when legislating in one direction or the other. Restricting access to data and their exchange to shield consumers from fraud or protect their sensitive information, depending on the circumstances, can tamper with a socially desirable increased market participation, revealing another trade-off to debate and deal with. *Privacy, prices, and market structure* are interdependent and all together affect social surplus.

A third point is provided by *scale* and *network effects*, which in this model are on the product firms' side, but in reality exist also on the consumers' side. The technologies on which digital firms rely generate those kind of self-boosting phenomena which bring more firms and consumers to be associated with few digital firms to generate more surplus, since there the amount of surplus is generally positively related to the masses of agents on both sides of a market interacting through the same intermediary. Such technology then is likely to simultaneously increase efficiency and smother competition, and to show how this might happen one does not need to resort to sophisticated anticompetitive strategies, since the most elementary ones can work. However, even in the simple settings of this model, this discussion should not lead to the conclusion that, since - in the short-run - welfare increases, there are no further issues worth investigating. One issue worth commenting upon, for instance, is the matter of *profit concentration*. In the last section, I have produced a simple scheme of sequential contracting, that, even under competitive pressure and little incumbent's advantage, makes it possible for the digital firm to appropriate most of the profits generated. Hence, there is a distributional issue to be faced. The traditional idea that the regulator should only care about the size of the "cake" generated by market exchanges, and only later about how to possibly redistribute it more fairly, has been proven hard to realize. Most of such digital firms come in considerable sizes themselves and operate globally, which gives them the ability to shape their legal and fiscal status to shrink the amount of taxes they pay, making it difficult to enact redistributive policies. This should suggest a more integrated approach where wealth generation and distributional concerns can be considered concurrently.

In summation, the picture in front of us is complex and puzzling in many directions and it is difficult to propose easy solutions due to the old and new trade-offs: privacy is to be weighted against market participation, efficiency against market concentration, and the necessity to integrate the standards for the analysis of wealth generation and distribution.

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2.6 Appendix A: Proofs

Proof of Lemma 1. (i) Existence follows from *Assumption 1*: log-concavity implies regularity, so it also follows that for any pair $\epsilon_i \geq 0, \epsilon_j \geq 0$, the following functions are increasing respectively in p_i^u and p_j^u

$$MR_i^u(\epsilon_i) = p_i^u - \frac{[F(\epsilon_i - p_j^u) - F(p_i^u - p_j^u)]}{f(p_i^u - p_j^u)} \tag{2.19}$$

$$MR_j^u(\epsilon_j) = p_j^u - \frac{[F(p_i^u - p_j^u) - F(p_i^u - \epsilon_j)]}{f(p_i^u - p_j^u)} \;, \tag{2.20}$$

hence the existence of the equilibrium is assured.

To prove uniqueness, consider the case of firm i's profits (j's case is completely analogous). Then, compute the following derivatives (the superscript u will be dropped for economy of notation):

$$\frac{\partial \pi_i}{\partial p_i} = -p_i f(p_i - p_j) + [F(\epsilon_i - p_j) - F(p_i - p_j)] \tag{2.21}$$

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -p_i f'(p_i - p_j) - 2f(p_i - p_j) < 0 \tag{2.22}$$

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = p_i f'(p_i - p_j) + f(p_i - p_j) - f(\epsilon_i - p_j) > 0 \tag{2.23}$$

and

$$\frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i} = f(\epsilon_i - p_j) > 0 \; . \tag{2.24}$$

To guarantee the uniqueness of such result, it suffices to show that the contraction condition is satisfied:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = -f(p_i - p_j) - f(\epsilon_i - p_j) < 0 \; ,$$

observe also that

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} + \frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i} = -f(p_i - p_j) < 0 \; .$$

(ii) As above I analyse the case for i, j being completely analogous. Differentiate the first order conditions with respect to ϵ_i and get

$$\frac{\partial^2 \pi_i}{\partial p_i^2}\frac{\partial p_i}{\partial \epsilon_i} + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\frac{\partial p_j}{\partial \epsilon_i} + \frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i} = 0 \tag{2.25}$$

and

$$\frac{\partial^2 \pi_j}{\partial p_j^2}\frac{\partial p_j}{\partial \epsilon_i} + \frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\frac{\partial p_i}{\partial \epsilon_i} + \frac{\partial^2 \pi_j}{\partial p_j \partial \epsilon_i} = 0 \; , \tag{2.26}$$

notice that $\frac{\partial^2 \pi_j}{\partial p_j \partial \epsilon_i} = 0$, then solve the system to obtain the two derivatives:

$$\frac{\partial p_i}{\partial \epsilon_i} = \frac{-\frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i}}{\frac{\partial^2 \pi_i}{\partial p_i^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\frac{\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi_j}{\partial p_j^2}}} \geq 0 \tag{2.27}$$

and

$$\frac{\partial p_j}{\partial \epsilon_i} = \frac{\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\frac{\frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i}}{\frac{\partial^2 \pi_i}{\partial p_i^2}}}{\frac{\partial^2 \pi_j}{\partial p_j^2} - \frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\frac{\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi_i}{\partial p_i^2}}} \geq 0 \; . \tag{2.28}$$

By inspection of equation (2.26), one can see that, for it to be nil, it must be that $\frac{\partial p_i}{\partial \epsilon_i} > \frac{\partial p_j}{\partial \epsilon_i}$ since $\frac{\partial^2 \pi_j}{\partial p_j^2} + \frac{\partial^2 \pi_j}{\partial p_i \partial p_j} < 0$.
Rewrite the expression for $\frac{\partial p_i^u}{\partial \epsilon_i} \leq 1$ using absolute values

$$\left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right| - \left|\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\frac{\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi_j}{\partial p_j^2}}\right| - \frac{\partial^2 \pi_i}{\partial p_i \partial \epsilon_i} > 0$$

since

$$\left|\frac{\frac{\partial^2\pi_j}{\partial p_i\partial p_j}}{\frac{\partial^2\pi_j}{\partial p_j^2}}\right| < 1$$

then it suffices to show that

$$\left|\frac{\partial^2\pi_i}{\partial p_i^2}\right| - \frac{\partial^2\pi_i}{\partial p_i\partial p_j} - \frac{\partial^2\pi_i}{\partial p_i\partial\epsilon_i} > 0 \ ,$$

which is equal $f(p_i - p_j) > 0$. By inspection of the FOCs, one can see that $\frac{\partial p_i}{\partial\epsilon_i} > \frac{\partial p_j}{\partial\epsilon_i}$ and hence $\frac{\partial p_j}{\partial\epsilon_i} < 1$ too. ■

Proof of Lemma 2. Consider the profits of the two firms: $\pi_i^u = p_i[1-F(p_i - pj)]$ and $\pi_j^u = p_j F(p_i - pj)$, taking the first order conditions with respect p_i and p_j (respectively) delivers

$$p_i = \frac{1 - F(p_i - pj)}{f(p_i - p_j)}$$

$$p_j = \frac{F(p_i - pj)}{f(p_i - p_j)} \ .$$

Exploiting the ex-ante symmetry between the firms, the symmetric equilibrium in prices is

$$p^* = \frac{1}{2}\frac{1}{h(0)} \ .$$

Uniqueness follows as a special case of the precedent lemma. ■

Proof of Proposition 1. (i) Consider the second stage of the game and take the first order conditions form profit maximization of (2.7) and (2.8), and then rearrange to get the equilibrium prices (I have omitted ϵ_i from the expressions for the sake of brevity)

$$p_i^u(\epsilon_i) = \frac{F(\epsilon_i - p_j^u) - F(p_i^u - p_j^u)}{f(p_i^u - p_j^u)} \quad (2.29)$$

$$p_j^u(\epsilon_i) = \frac{F(p_i^u - p_j^u)}{f(p_i^u - p_j^u)} \ . \quad (2.30)$$

The uniqueness of ϵ^* follows from the log-concavity of $F(s)$ which makes the function $\Pi(\epsilon_i)$ quasi-concave. Although there might be more points satisfying the FOC $\frac{d\Pi_i}{d\epsilon_i} = 0$, there is only one such that $\frac{d^2\Pi_i}{d\epsilon_i^2} < 0$. (ii) It follows from

Lemma 1.
(iii)Assume firms are playing their best responses in the second stage of the game and take the derivative with respect to $\epsilon = 0$ of the total profits of i, Π_i (again, I will use p_j^u instead of $p_j^u(\epsilon_i)$ for brevity)

$$\begin{aligned}\frac{d\Pi_i}{d\epsilon_i} &= -(\epsilon_i - p_j^u) f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + \frac{\partial p_j^u}{\partial \epsilon_i}[1 - F(\epsilon_i - p_j^u)] \\ &- p_j^u f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + p_i^u \left[f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + f(p_i^u - p_j^u)\frac{\partial p_j^u}{\partial \epsilon_i}\right]\end{aligned}$$

by setting it equal to zero one gets

$$\epsilon^* = p_i^u + \frac{\frac{\partial p_j^u}{\partial \epsilon_i}}{1 - \frac{\partial p_j^u}{\partial \epsilon_i}}\left[\frac{1 - F(\epsilon_i - p_j^u)}{f(\epsilon_i - p_j^u)} + p_i^u \frac{f(p_i^u - p_j^u)}{f(\epsilon_i - p_j^u)}\right]\Bigg|_{\epsilon_i^*}. \tag{2.31}$$

(iii) Assume firm i plays ϵ_i^*, and firm j responds with $\pi_j^u(\epsilon_i^*)$, then she has the incentive to unilaterally deviate decreasing the value of ϵ_i. In fact, take the first derivative of Π_i at ϵ_i^* keeping $p_j^u = p_j^u(\epsilon_i^*)$ fixed and obtain

$$\frac{d\Pi_i}{d\epsilon_i} = \frac{\partial \pi_i^d}{\partial \epsilon_i} + \frac{\partial \pi_i^u}{\partial p_i^u}\frac{\partial p_i^u}{\partial \epsilon_i} + \frac{\partial \pi_i^u}{\partial \epsilon_i}\Bigg|_{\epsilon_i^*} = -(\epsilon_i^* - p_i^u) f(\epsilon_i^* - p_j) < 0$$

where

$$\frac{\partial \pi_i^u}{\partial p_i^u}\Bigg|_{\epsilon_i^*} = 0$$

by the envelope theorem. This means that by lowering the value of ϵ_i played the profits increase. It follows that firm i cannot credibly commit not to use all the information she has. Hence at the equilibrium $\epsilon_i = 0$. ■

Remark: Welfare in the Asymmetric case. The calculations for welfare and consumer surplus in this case do not deliver a clear answer with respect to the price targeting affecting positively or else the consumers. Divide consumers into three different areas. Area 1: in this segment consumers are such that $v_i \geq v_j$ and they buy from firm i, i.e. their favourite variety, hence welfare is given by

$$W_1 = \frac{1}{2}\int_{p_i^u}^{\bar{v}} v\, g(v)\, dv. \tag{2.32}$$

Since $p_i^u \leq p^*$, welfare in this area increases because more consumers participate in the exchange.
Area 2: due to $p_i^u \leq p_j^u$, some consumers that prefer variety j over i, $v_j \geq v_i$, will buy from firm i because $v_i - p_i^u \geq v_j - p_j^u$.

$$W_2 = \int_{p_i^u}^{p_j^u} \int_{p_i^u}^{v_j} v_i g(v_i) dv_i g(v_j) dv_j + \int_{p_i^u}^{\bar{v}} \int_{p_i^u}^{v_j - p_j^u + p_i^u} v_i g(v_i) dv_i g(v_j) dv_j \ . \tag{2.33}$$

In this area there are two opposite effects: prices alter the matching between consumers preferences and the variety they purchase, decreasing efficiency; however, both uniform prices are lower than p^*, so more consumer will buy, and this increases welfare.
Area 3: the remaining consumers are such that $v_j \geq v_i$ and they buy from j, furthermore $v_j \geq v_i - p_i^u + p_j^u$, so

$$W_3 = \int_{\underline{v}}^{p_i^u} \int_{p_j^u}^{\bar{v}} v_j g(v_j) dv_j g(v_i) dv_i + \int_{p_i^u}^{\bar{v} + p_i^u - p_j^u} \int_{v_i - p_i^u + p_j^u}^{\bar{v}} v_j g(v_j) dv_j g(v_i) dv_i \ . \tag{2.34}$$

In this last area the matching between consumers and firms is correct, however the two conflicting effects of the asymmetry may affect the welfare both ways. As above, since the uniform price is lower, $p_j^u \leq p^*$, market participation increases, but the mass of consumers might be reduced due to the effect of the misplacement represented by Area 2. Ultimately, Welfare will depend on the particular distributions of $G(v)$. ■

Proof of Proposition 2. (i) Start by imposing the following first order condition

$$\frac{d\Pi}{d\epsilon_i} = \frac{d\Pi_i}{d\epsilon_i} + \frac{d\Pi_j}{d\epsilon_i} = 0 \ ,$$

then expand it as follows

$$
\begin{aligned}
\frac{d\Pi}{d\epsilon_i} =& -(\epsilon_i - p_j^u)f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + \frac{\partial p_j^u}{\partial \epsilon_i}[1 - F(\epsilon_i - p_j^u)] \\
& - p_j^u f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + p_i^u\left[f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + f(p_i^u - p_j^u)\frac{\partial p_j^u}{\partial \epsilon_i}\right] \\
& - (p_i^u - \epsilon_j)f(p_i^u - \epsilon_j)\frac{\partial p_i^u}{\partial \epsilon_i} + \frac{\partial p_i^u}{\partial \epsilon_i}F(p_i^u - \epsilon_j) + p_i f(p_i^u - \epsilon_j)\frac{\partial p_i^u}{\partial \epsilon_i} \\
& + p_j^u \frac{\partial p_i^u}{\partial \epsilon_i}[f(p_i^u - p_j^u) - f(p_i^u - \epsilon_j)] \, .
\end{aligned}
\tag{2.35}
$$

Consider the symmetry of the two firm's strategies: $1-F(\epsilon_i-p_j^u) = F(p_j^u-\epsilon_j)]$, $f(\epsilon_i-p_j^u) = f(p_i^u-\epsilon_j)$ and $p_j^u = p_i^u \equiv p$ (at the maximum), hence the expression can be reduced to

$$
\begin{aligned}
\hat{\epsilon} =& p + \frac{\frac{\partial p_i^u}{\partial \epsilon_i} + \frac{\partial p_j^u}{\partial \epsilon_i}}{1 + \frac{\partial p_i^u}{\partial \epsilon_i} - \frac{\partial p_j^u}{\partial \epsilon_i}} \frac{1 - F(\epsilon_i - p)}{f(\epsilon_i - p)} \\
& + \frac{\frac{\partial p_i^u}{\partial \epsilon_i} + \frac{\partial p_j^u}{\partial \epsilon_i}}{1 + \frac{\partial p_i^u}{\partial \epsilon_i} - \frac{\partial p_j^u}{\partial \epsilon_i}} p \frac{f(0)}{f(\epsilon_i - p)}
\end{aligned}
$$

(ii) These results follow by arguments completely identical to the ones in Proposition 1. (iii) In the same fashion of Proposition 1, keeping the rival's price fixed one gets

$$
\left.\frac{d\Pi_i}{d\epsilon_i}\right|_{\hat{\epsilon}} = -(\hat{\epsilon}_i - p_i^u)f(\hat{\epsilon}_i - p_j^u) < 0
$$

and

$$
\left.\frac{d\Pi_j}{d\epsilon_j}\right|_{\hat{\epsilon}} = -(\hat{\epsilon}_j - p_j^u)f(p_i^u - \hat{\epsilon}_j) < 0
$$

those equations imply that both firms find it profitable to deviate, reducing $\hat{\epsilon}$. (iv) The bit about welfare is immediate. Check the derivative

$$
\frac{\partial W^s}{\partial p} = -p\, g(p) < 0
$$

hence, since $p \leq p^*$, Welfare increases with SPT. ■

Proof of Proposition 3. (i) Take the derivative w.r.t ϵ_i^*

$$\begin{aligned}\frac{d\Pi_i}{d\epsilon_i} =&\delta\Bigg[- (\epsilon_i - p_j^u) f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + \frac{\partial p_j^u}{\partial \epsilon_i}[1 - F(\epsilon_i - p_j^u)] \\ &- p_j^u f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + p_i^u\left[f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + f(p_i^u - p_j^u)\frac{\partial p_j^u}{\partial \epsilon_i}\right]\Bigg] \\ &+ (1-\delta)\left[p_i^u f(p_i - p_j)\frac{\partial p_j^u}{\partial \epsilon_i^*}\right] = 0 \ ,\end{aligned} \tag{2.36}$$

call

$$\epsilon^*(1,\delta) = p_i^u + \frac{\frac{\partial p_j^u}{\partial \epsilon_i}}{1 - \frac{\partial p_j^u}{\partial \epsilon_i}}\left[\frac{1 - F(\epsilon_i - p_j^u)}{f(\epsilon_i - p_j^u)} + p_i^u\frac{f(p_i^u - p_j^u)}{f(\epsilon_i - p_j^u)}\right] \ ,$$

then rewrite the FOC to deliver the equation

$$\epsilon_i^*(\delta) = \epsilon_i^*(1,\delta) + \frac{1-\delta}{\delta}p_i^u\frac{f(p_i^u - p_j^u)}{f(\epsilon_i - p_j^u)}\left(\frac{\frac{\partial p_j^u}{\partial \epsilon_i}}{1 - \frac{\partial p_j^u}{\partial \epsilon_i}}\right)\Bigg|_{\epsilon_i^*(\delta)} . \tag{2.37}$$

(ii) By inspection of the resulting equation. ■

Proof of Proposition 4. Analogously to the previous proposition, consider the derivative

$$\begin{aligned}\frac{d\Pi}{d\epsilon_i} =&\delta\Bigg[- (\epsilon_i - p_j^u) f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + \frac{\partial p_j^u}{\partial \epsilon_i}[1 - F(\epsilon_i - p_j^u)] \\ &- p_j^u f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + p_i^u\left[f(\epsilon_i - p_j^u)\left(1 - \frac{\partial p_j^u}{\partial \epsilon_i}\right) + f(p_i^u - p_j^u)\frac{\partial p_j^u}{\partial \epsilon_i}\right] \\ &- (p_i^u - \epsilon_j) f(p_i^u - p_j^u)\frac{\partial p_i^u}{\partial \epsilon_i} + \frac{\partial p_i^u}{\partial \epsilon_i}F(p_i^u - \epsilon_j) + p_i^u f(p_i^u - p_j^u)\frac{\partial p_i^u}{\partial \epsilon_i} \\ &+ p_j^u\frac{\partial p_i^u}{\partial \epsilon_i}[f(p_i^u - p_j^u) - f(p_i^u - \epsilon_j)]\Bigg] \\ &+ (1-\delta)p_j^u\frac{\partial p_i^u}{\partial \epsilon_i}[f(p_i^u - p_j^u) - f(p_i^u - \epsilon_j)] = 0 \ ,\end{aligned} \tag{2.38}$$

then use symmetry to simplify and rearrange to obtain the result

$$\hat{\epsilon}(\delta) = \hat{\epsilon}(1,\delta) + \frac{1-\delta}{\delta}p_i^u\left[\frac{f(0)}{f(\epsilon_i - p_j)} - 1\right]\left(\frac{\frac{\partial p_j^u}{\partial \epsilon_i}}{1 + \frac{\partial p_i^u}{\partial \epsilon_i} - \frac{\partial p_j^u}{\partial \epsilon_i}}\right)\Bigg|_{\hat{\epsilon}(\delta)} \tag{2.39}$$

(ii) By inspection.

(iii) Since at $\hat{\epsilon}(\delta)$ decreases in δ, being the equilibrium symmetric, uniform prices decrease too, and this implies that W^s increases in δ. ■

Proof of Proposition 5. The proof for claims (i) and (ii) simply follows from the maximization of the tariff, i.e. in case (i) for a given level of b

$$\arg\max_{\epsilon_i} \Pi_i(\epsilon) - \pi_i^u(\emptyset, \epsilon_j^*)$$

by taking the FOC

$$\frac{\partial \Pi_i(\epsilon)}{\partial \epsilon_i} - \frac{\partial \pi_i^u(\emptyset, \epsilon^*)}{\partial \epsilon_i} = 0$$

and observing that

$$\frac{\partial \pi_i^u(\emptyset, \epsilon^*)}{\partial \epsilon_i}$$

notice that the maximum is reached at $\epsilon_i^{*M} = \epsilon_i^*$. Claim (ii) follows from analogous arguments:

$$\hat{\epsilon}^M \arg\max_{\epsilon} \Pi_i(\epsilon) + \Pi_j(\epsilon) - \pi_i^u(\emptyset, \epsilon_j) - \pi_j^u(\epsilon_i, \emptyset) ,$$

hence the FOC w.r.t. ϵ_i and ϵ_j respectively read

$$\frac{\partial \Pi(\epsilon)}{\partial \epsilon_i} - \frac{\partial \pi_j^u(\epsilon_i, \emptyset)}{\partial \epsilon_i} = 0$$

$$\frac{\partial \Pi(\epsilon)}{\partial \epsilon_j} - \frac{\partial \pi_i^u(\emptyset, \epsilon_j)}{\partial \epsilon_i} = 0 .$$

To be satisfied, those conditions require $\frac{\partial \Pi(\epsilon)}{\partial \epsilon_k} > 0$ for $k = i, j$, hence $\hat{\epsilon}^M \leq \hat{\epsilon}$.

(iii) The expression for the optimal level of quality b^* can be derived as follows from the first order condition for the profit maximization of the intermediary:

$$\left.\frac{dR}{db}\right|_{b^*} = \left.\frac{dT_i}{d\delta}\frac{d\delta}{db}\right|_{b^*} - c$$

hence

$$h(b^*) = \left.\frac{c}{\frac{\partial T_i}{\partial \delta}}\right|_{b^*}$$

and finally

$$b^* = h^{-1}\left(\frac{c}{\frac{\partial T_i}{\partial \delta}}\right)\Bigg|_{b^*}.$$

Analogous passages deliver the expression for $\hat{b}$:

$$\frac{dR}{db}\bigg|_{\hat{b}} = \frac{d(T_i + T_j)}{d\delta}\frac{d\delta}{db}\bigg|_{\hat{b}} - c$$

and then

$$\hat{b} = h^{-1}\left(\frac{c}{\frac{\partial T_i}{\partial \delta} + \frac{\partial T_j}{\partial \delta}}\right)\Bigg|_{\hat{b}}. \qquad \blacksquare$$

Proof of Proposition 6. (i) Notice first that $T(b)$ are concave in b. To prove the first claim, simply consider the two first order conditions for the sizes m and $m+1$:

$$\frac{dR}{db^m} = m\frac{dT}{db^m} - c = 0$$

$$\frac{dR}{db^{m+1}} = (m+1)\frac{dT}{db^{m+1}} - c = 0$$

rearranging, they deliver respectively

$$\frac{dR}{db^m} = \frac{c}{m}$$

$$\frac{dR}{db^{m+1}} = \frac{c}{m+1}$$

and since profits are concave in b, it follows that $\hat{b}^{m+1} \geq \hat{b}^m$.

(ii) The increase is twofold. First, when the size increases from m to $m+1$, keeping b fixed, the profits would go from $R^m(b^m) = mT^m(b^m) - cb^m$ to $R^{m+1}(b^m) = (m+1)T^m(b^m) - cb^m$. However this would not be the optimal level for the App quality, which would be instead b^{m+1} yielding the corresponding profit $R^{m+1}(b^{m+1}) = (m+1)T^{m+1}(b^{m+1}) - cb^{m+1}$. It follows that intermediary's profits increase in m.

(iii) Follows logically from the previous claims. As m grows, b grows, and so does δ. As delta approximates 1, the $\hat{\epsilon}^M(\delta^m)$ gets closer to $\hat{\epsilon}^M(1)$.

(iv) By inspection of the previous *Propositions*. $\blacksquare$

2.7 Appendix B: A Hotelling Counterpoint

This section is dedicated to a simple model of price competition with heterogeneous consumers à la Hotelling. The purpose of this Appendix is, of course, to show a different model specification, and the differences that such model delivers at the equilibrium/optimum, compared to the one previously presented, but also to comment on some of the limits of the Hotelling model.

Hotelling models are presently one of the workhorses in investigating the many issues in competition in digital markets: the main reason being they can provide a manageable representation of product differentiation and heterogeneous consumers' preferences. This said, on a technical side, the framework often becomes quite hard to handle as more sophisticated elements are introduced, and the appealing simplicity of linearity, suddenly is replaced by equations of higher order, for which solutions are not always clean. On the other hand, a more theoretical point should be made. In Hotelling models the representation of heterogeneity can be quite restrictive to the point of being unreasonable when it comes to discuss tastes. The consumer's attitudes toward two products is pinned down uniquely by one parameter, namely the location l, and the valuations for the two products are strongly negatively correlated. This scheme is perhaps a most adequate and convenient depiction of a situation in which two duopolist are selling the same quality of the same good, but there are physical transport or search costs in which the consumer incurs. Imagine a simple market in which two firms sell apples, the very same ones, but to buy your apples you need to go to the shop: the physical distance determines the difference in valuations, yet you enjoyment for the apple is the same. When it comes instead to the representation of preferences for differentiated products, the interpretation is way less appealing: suppose there are Fuji apple and Gala apples. In this setting, you simply cannot like apples very much and prefer one variety more than the other, as well as (conversely) you cannot just hate all apples. If you are extremely fond of Fuji-s, in this model, you must absolutely hate Gala-s, and vice versa. If you feel mildly positive towards one variety, by force, you feel as mildly negative about the other, and so on. Such characterization rules therefore out many different possibilities and a variety of combination of tastes, at the cost of sounding rather unreasonable put to the test of economic interpretation.

Suppose there is a continuum of consumers buying one unit of a (durable) good, uniformly distributed over the $[0,1]$ interval. There are two firms, producing only two variety of that good, called i and j , respectively located at $l_i = 0$ and $l_j = 1$. The valuation of the consumer located in $x \in [0,1]$ for each good is given by

$$u(x,l) = v - |l - x| - p(x)$$

where $v > 2$ [12], $l = \{0,1\}$, and $p^d(x)$ denotes that the price a consumer pays might depend on his location.

Absent any further characterization, the equilibrium prices are uniform $p_i^u = p_j^u = 1$, the demand regions are given by $q_i = q_j = \frac{1}{2}$, hence both firms make profits $\pi_i^u = \pi_j^u = \frac{1}{2}$. Consumer surplus can also be computed as

$$CS = \int_0^{\frac{1}{2}} [v - x - 1]dx + \int_{\frac{1}{2}}^1 [v - (1-x) - 1]dx = v - \frac{5}{4}\ ,$$

hence the total welfare is $W = CS + \pi_i + \pi_j = v - \frac{1}{4}$. Now, I move to analyse the optimal information structures in this model, which, as we will see, provide different implications.

2.7.1 Price Targeting

Consider the case in which only firm i has access to information, and receives e_i, while firm j can only observe the chosen reach ϵ_i^*. The timing of the game is the following:

0 Firm i knows the location of every consumer, and firm j knows that, but she does not know the location of any.

1 Firm i chooses the consumers $[0, e_i^*]$ to target with a personalized price $p_0(x, e_i^*)$. Consumers offered a personalized price can either accept it, or buy from the competitor.

[12] This assumption guarantees that every consumer would be better off buying even in case of a fully informed monopolist practising first degree price discrimination.

2 Firm 1 observes e_i^*, then both firms compete with uniform prices over $[e_i^*, 1]$.

Furthermore, a small technical condition should be imposed: the the maximum reach should be smaller than the indifferent consumer threshold $e_i \leq \hat{x}(e_i)$, which is verified if $e_i \in [0, 3/4]$

Lemma B1. *For $e_i \in [0, 3/4]$, when firm i receives information e_i*

i) price discrimination is profitable;

ii) there exists a $e_i^* = 3/7$;

$$p_i^d(x, e_i) = 2 - \frac{2}{3}e_i - 2x \qquad x \in [0, e_i] \tag{2.40}$$

$$p_i^u(e_i) = 1 - \frac{4}{3}e_i \quad x \in [e_i, \hat{x}(e_i)] \tag{2.41}$$

$$p_j^u(e_i) = 1 - \frac{2}{3}e_i \quad x \in [\hat{x}(e_i), 1] \tag{2.42}$$

Analogously to the model in the main body of the paper, there exist an optimal degree of price discrimination which is, in this case, not only less than 1, but also strictly less than 1/2. The price schedule proposed is decreasing in the distance from the location of the firm, and drives uniform prices down, intensifying competition as we move closer to the rival's location.

Lemma B2.

1. *If both firms use personalized prices, industry profits decrease: for any couple $e = (e_i, e_j) > 0$*

$$\Pi_i(e) + \Pi_j(e) < 1 \ .$$

Lastly, by putting together the results above, one can conclude the following:

Proposition B1 *(i) A monopolistic intermediary would rather sell data to both competitors. (ii) An intermediary practising limit pricing under SPT, Assumptions 3 & 4, would apply UPT.*

The conclusion is that the Hotelling model, in this case, not only imposes strong and unnecessary restrictions on the preferences consumers are allowed to have, but, and most importantly, significantly alters the findings obtained in the main text. Under the assumption of Hotelling preferences, the Intermediary would completely foreclose one firm from accessing data and proposing personalized prices, which would translate into picking one firm and making it grow. Though this might also happen in the more general setting used in the main text, in this case is the only outcome, given a specific set of preferences. It follows that the adoption of such preferences should be justified by real-world phenomena. Furthermore, if one would like to adventure into considering multiple markets intermediation, as I do, those preferences become a straitjacket, because they would require a number of markets to fit such representation, not only one.

2.7.2 Proofs.

Proof of Lemma B1. For ease of notation, I will refer to the data as simply e_i in the proof. Start from stage 2 and work backwards: given e_i, the contended market is $[e_i, 1]$. The location of the indifferent consumer can be found as

$$\hat{x} = \frac{1}{2} + \frac{p_j - p_i}{2} \; .$$

Hence

$$\pi_i^u = p_i \left(\frac{1}{2} + \frac{p_j - p_i}{2} - e_i \right)$$

$$\pi_j^u = p_j \left(\frac{1}{2} - \frac{p_j - p_i}{2} \right)$$

that delivers the system of FOCs

$$\frac{1}{2} + \frac{p_j}{2} - p_i - e_i = 0$$

$$\frac{1}{2} - p_j + \frac{p_i}{2} = 0$$

from which the equilibrium uniform prices can be derived as

$$\begin{cases} p_i^u(e_i) &= 1 - \frac{4}{3}e_i \\ p_j^u(e_i) &= 1 - \frac{2}{3}e_i \end{cases}$$

the indifferent consumer is determined to be $\hat{x} = 1/2 + e_i/3$.
Go back to stage 1: Firm i will target consumers with prices $p_i^d(x, \epsilon_i)$ in a way to make them indifferent between buying from her or from the rival:

$$v - x - p_i^d(x, \epsilon_i) = v - (1 - x) - p_j$$

by substituting the equilibrium value for p_j^u, one gets

$$p_i^d(x, e_i) = 2 - \frac{2}{3}e_i - 2x \ .$$

Compute now the total profits for firm i as

$$\Pi_i(e_i) = \int_0^{e_i} p_i^d(x, \tilde{e})d\tilde{e} + p_i^u(e_i)(\hat{x}(e_i) - e_i)$$

$$= 2e_i - \frac{5}{3}e_i^2 + \frac{1}{18}(16e_i^2 - 24e_i + 9)$$
$$= \frac{1}{2} + \frac{2}{3}e_i - \frac{7}{9}e_i^2 \ .$$

i) Check that $\Pi_i(e_i) \geq \Pi_i(0)$ if $\frac{2}{3}e_i - \frac{7}{9}e_i^2 \geq 0$, i.e. for $e_i \in [0, 6/7]$ but the analysis is restricted to $e_i \in [0, 3/4]$. Hence, price discrimination is always more profitable than uniform pricing.
ii) Furthermore

$$\frac{\partial \Pi_i}{\partial e_i} = \frac{2}{3} - \frac{14}{9}e_i \geq 0$$

iff $e_i \in [0, 3/7]$, hence $e_i^* = \frac{3}{7}$. ■

Proof of Lemma B2. Remember $e = (e_i, e_j) > 0$. Starting from the second stage, the indifferent consumer is found as before over the interval $[e_i, 1 - e_j]$. This delivers the two demand regions contended with uniform prices

$$q_i^u = \frac{1}{2} + \frac{p_j - p_i}{2} - e_i$$

$$q_j^j = \frac{1}{2} - \frac{p_j - p_i}{2} - e_j \ .$$

Taking the FOCs for profit maximization yields the system

$$\frac{1}{2} + \frac{p_j}{2} - p_i - e_i = 0$$

$$\frac{1}{2} - p_j + \frac{p_i}{2} - e_j = 0$$

solving for the equilibrium prices one gets

$$\begin{cases} p_i^u(e) &= 1 - \frac{4}{3}e_i - \frac{2}{3}e_j \\ p_j^u(e) &= 1 - \frac{2}{3}e_i - \frac{4}{3}e_j \end{cases}$$

In the first stage, each firm sets the price schedule to make the targeted consumers indifferent between buying from her or from the rival, obtaining (symmetrically)

$$\begin{cases} p_i^d(x,e) &= 2 - \frac{2}{3}e_i - \frac{4}{3}e_j - 2x \\ p_j^d(x,e) &= -\frac{2}{3}e_j - \frac{4}{3}e_i + 2x \end{cases}$$

furthermore, the indifferent consumer can be calculated as $\hat{x}(e) = \frac{1}{2} + \frac{e_i - e_j}{3}$. The equilibrium profits are then

$$\begin{cases} \Pi_i(x,e) &= \frac{1}{2} + \frac{2}{3}e_i - \frac{7}{9}e_i^2 - \frac{4}{9}e_ie_j - \frac{2}{3}e_j + \frac{2}{9}e_j^2 \\ \Pi_j(x,e) &= \frac{1}{2} + \frac{2}{3}e_j - \frac{7}{9}e_j^2 - \frac{4}{9}e_ie_j - \frac{2}{3}e_i + \frac{2}{9}e_i^2 \ . \end{cases}$$

It can be noticed that under price discrimination the sum of profits is less than the profits under uniform prices:

$$\Pi_i(x,e) + \Pi_j(x,e) = 1 - \frac{5}{9}e_i^2 - \frac{5}{9}e_j^2 < 1 \ .$$

■

Proof of Proposition B1. (i) Using the previous *Lemmas*, one can see that under UPT, the revenue would amount to $T = \frac{19}{49}$ for UPT and to $T = \frac{1}{2}$ for SPT. Hence, SPT is more profitable than UPT for a monopolistic intermediary. (ii) By invoking *Proposition 5 (iii)* and *Lemma B2*, the revenue-maximizing allocation is $\hat{e} = (0,0)$, which would imply $T = 0$. ■

Chapter 3

Final Remarks

With the papers in this thesis, I have tried to contribute to the fields of Regulation and Antitrust theory and policy. Both these subfields of Industrial Economics essentially deal with the many issues related to fair competition and consumer welfare, though often from different perspectives: while Regulation Theory mainly focuses on protecting consumers from natural monopolies and market design, Antitrust authorities have a more general scope, levelling the playing field and making sure competition remains alive and fair.

In the first chapter, I have shown a model of dynamic regulatory intervention aimed at subsidizing the development and expansion of a good with positive externalities (renewable energy plants). In that instance, market power appears as a challenge for the Regulator, who must anticipate the ability of a price-setting firm to manipulate the timing and size of the expansion projects to embark the largest possible amount of funds in the long run, exacerbating the cost of the public intervention and wasting the taxpayers money. A cunning Social Planner would recognize the possibility of such self-serving behaviour (on the producer's side), and therefore plan a more sophisticated, Second Best, support scheme that at the same time provides the right incentives and removes the temptation of exploiting public finances.

The second chapter instead is more positive in its analysis (rather than normative). The role of data production, intermediation and allocation is shown to be the source of market power in possibly many markets and the intermedi-

ation market itself. Furthermore, the anticompetitive features present in each one seem to reinforce each other and grow together. This result would be a no-brainer for a policy maker if it were not for the fact that, as the departure from competitive behaviour grows, efficiency follows, brought about by a widened market participation. Thus, the puzzle that these new forms of interactions in digital markets pose to contemporary society, an Antitrust authorities in particular: it is indeed the case that large sizes, driven by economies of scale and network effects, are often tied together with efficiency gains, and you cannot have one without the other. This phenomenon reveals then a rather perplexing (if not disturbing) side when private information being divulged is the source of such welfare gains. Going out on a limb, Economics, left alone, can only take note of that, since there are few weapons in the standard economic arsenal to reach general policy prescription in this context. Hence the necessity of a larger debate which takes into account social preferences to accompany and guide the normative economic analysis of these emerging issues.
To conclude these final reflections, I would only add that the problem of market power, which is the *fil rouge* of this thesis, tends to present itself in different forms and in different environments: it can appear as a "bad behaviour" (collusion, unfair commercial practices, etc...), but also as the expected and "natural" result of the adoption of a technology, such as in the two cases I have treated. It also stands out that in both the cases I have discussed, public intervention must always be questioned and carefully evaluated, given the possibility that what might seem a good idea to begin with can always have its drawbacks, and perhaps worsen the overall situation by altering the equilibrium of the variety of factors that concur to determine economic realities.