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Firm size and the Macroeconomy

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Firm size and the Macroeconomy

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Introduction

This dissertation collects two essays on firm size dynamics and aggregate shocks. By employing a model with heterogeneous firms I investigate the role of firm death on long-run employment growth and aggregate dynamics, in particular with respect to technology and entry cost shocks. The thesis is divided into two chapters.

In the first chapter I identify the effects of entry cost shocks through a Bayesian VAR model using data on US firms from the Business Dynamics Statistics (BDS) database and the Federal Reserve (FRED). From an unexpected increase in entry costs three facts emerge:

1. new firms are on average larger;
2. the employment distribution across firms becomes more concentrated toward large firms;
3. unemployment decreases.

To address these facts I develop a heterogeneous-firm model with search frictions in the labor market and endogenous firm dynamics - namely, costly entry of new businesses and endogenous exit of firms - calibrated on data from BDS.
The model explains that the resulting fall in competition due to the decrease in the entry of new firms increases incumbents' profitability, promoting job creation and leading to a tighter labor market. This causes an increase in labor costs (i.e. wage and marginal cost of hiring increase) inducing new firms to be larger to break even their costs, whereas small unproductive incumbents are forced to exit. Hence the share of workers employed in large

firms increases.

In the second chapter I provide empirical evidence on two facts:

1. in the long run, the difference in the average net job creation rates between small and large firms is mainly due to the higher exit rate of small firms;

2. in response to a negative technology shock, small firms destroy more jobs by exiting than large firms do.

The heterogeneous-firm model with search frictions and endogenous firm dynamics replicates these empirical facts.
Moreover, contrary to frameworks with exogenous exit, it can account for the volatility of exit and the differential of job destruction due to exit between small and large firms conditional to technology shocks.
Finally, results show that endogenous exit amplifies substantially the response of unemployment to technology shocks.

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Chapter 1

Firm size and entry cost shocks

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Abstract This study explores how an increase in entry costs affects the size of new entrants and the concentration of employment according to firm size, along with its effects on macro-variables such as unemployment and the exit rate. To this aim we use a BVAR model to estimate the response of such variables to an entry cost shock, then we develop a heterogeneous-firm model with search frictions and endogenous entry/exit dynamics calibrated on data from Business Dynamics Statistics (BDS) database to address our empirical results.
We find that positive entry cost shocks increase the average size of entrants and move employment shares toward the largest firms. These results reveal the role of entry costs' fluctuations in explaining the dynamics at business cycle horizons of both firm and employment share distributions according

to size.

1.1 Introduction

Entry of new firms is receiving increasing attention from the macroeconomic literature, particularly regarding its importance for growth, aggregate job creation and economic recovery following downturns[1].

Recently, entry cost shocks - i.e. unexpected changes in the cost to create new businesses - have gained interest from both a trend and business cycle prospective. Most notably Gutierrez, Jones and Philippon (2021) show that entry cost shocks might explain the downward trend of the entry rate observed in the last decade[2] and argue that "entry cost shocks can account for much of the increase in aggregate concentration and [...] have large effects on aggregate investments, the natural interest rate, and the stance of monetary policy". Similarly, abstracting from trend dynamics and focusing on temporary fluctuations, Sédlaček (2020) shows that if an entry cost shock is sufficiently large - as it has been during the Great Recession - its effects on the economy, specifically on unemployment, are persistent[3].

These shocks can be clearly distinguished from other well-known perturbations such as technology shocks; for instance the entry rate and the firms' discounted sum of future profits react oppositely in

[1]For instance Sédlaček and Sterk (2017) observe that the initial conditions of firm birth have persistent effects in future dynamics of aggregate employment since young firms disproportionally contribute to both job creation and destruction as shown by Haltiwanger et al. (2013).

[2]Similarly, Pugsley and Sahin (2019) document a declining startup rate in recent years and prove that it affects directly the decline in trend growth rate of employment and the asymmetric response of employment to output's fluctuations.

[3]Given that new firms contribute significantly to aggregate job creation (see Haltiwanger et al. (2013)), entry cost shocks have large effects on aggregate employment.

response to the former, in the same direction following the latter[4]. In particular entry cost shocks, by decreasing the entry of new firms, decrease market contendibility among incumbent firms and have persistent effects on the number of firms in the following years as shown by Sédlaček (2020). This work studies how the firm size and employment share distributions react to an exogenous, temporary increase in entry costs; specifically it focuses on the response of the average size of new firms and employment concentration (defined as the ratio between employment of large firms and employment of all the other firms) to an unexpected increase in entry costs from a *business-cycle* perspective rather than by looking at trend dynamics. This means that the focus is on *temporary* - rather than *permanent* - changes in the cost of entry faced by new firms. As an example, temporary variations in borrowing costs or advertising expenses can be considered factors explaining fluctuations in entry costs at business cycle frequencies.

To show that entry costs do vary over the business cycle let us consider figure 1.1; there we show the trend (left) and business cycle (right) dynamics of the entry rate (blue curve) and number of restrictions (red curve) due to regulatory stringency at the industry-level[5]. As in Gutierrez and Philippon (2019) the number of restrictions serves as a proxy of regulation, which in turn is presumably the most rigid component of entry costs[6] (i.e. we expect it to vary the least over the business cycle compared to any other component of entry costs).

To point out the economic significance of fluctuations in entry restrictions at business cycle frequency, notice that the latter entail important economic effects in the short run. Fluctuations in

[4]See again Gutierrez, Jones and Philippon (2021).

[5]Entry rate observations come from the Business Dynamics Statistics (BDS) database whereas observations of the number of restrictions are taken from the RegData database. The series in the right box are smoothed through a one-sided HP filter with a smoothing parameter equal to 100.

[6]We need to emphasize that regulation constitute only one dimension of entry costs, although an important one.

entry restrictions might act either on the access to credit of startups through variations in borrowing costs, or on the time needed to complete bureaucratic procedures, or on the amount of fees to be paid for starting a new business to mention a few. In turn these effects operate on a macroeconomic level by affecting the entry rate, the number of firms in the following periods and the average size of firms, with consequences on aggregate unemployment as shown in our BVAR evidence in the following section.

From a trend perspective, by looking at the plot on the left of figure 1.1 Gutierrez and Philippon (2019) suggest that regulation might hurt entry of new firms since the number of restrictions and the entry rate display opposite trends (although they argue that this could be due to other common factors). Instead from a business-cycle viewpoint, the plot on the right contributes to the motivation of our study on temporary entry cost shocks by showing that the HP-filtered component of regulation - which is the most rigid dimension of entry costs - indeed varies over the business cycle. To stress further the relevance of entry cost shocks we provide empirical evidence on fluctuations of industry concentration at business cycle frequency, which might be affected, among other factors, also by changes in entry rates due to variations in entry costs. Let us consider figure 1.2; there we show the cyclical component of the Herfindahl-Hirschman Index (HHI) obtained from a HP filter[7] (red series). This index is equal to the sum of the squared market shares of each firm in the sample and it is used as indicator of the degree of industry concentration. Compared to the unemployment rate in first difference (blue series) - that is used here as a business cycle indicator - the cyclical component of HHI displays fluctuations at business cycle frequency[8]. This fact - coupled with the previous evidence on entry restrictions' cyclical component - suggests that fluctuations in entry

[7]We rely on a dataset of firms' sales from 1984 to 2018 based on Compustat observations.

[8]Moreover the correlation coefficient between the cyclical component of HHI and the first-differenced unemployment rate is equal to 0.2908 with p-value 0.0901, indicating significant countercyclicality of the former.

costs at business cycle frequency are worth studying.

Our analysis proceeds as follows. First we document the BVAR evidence on the effects of entry

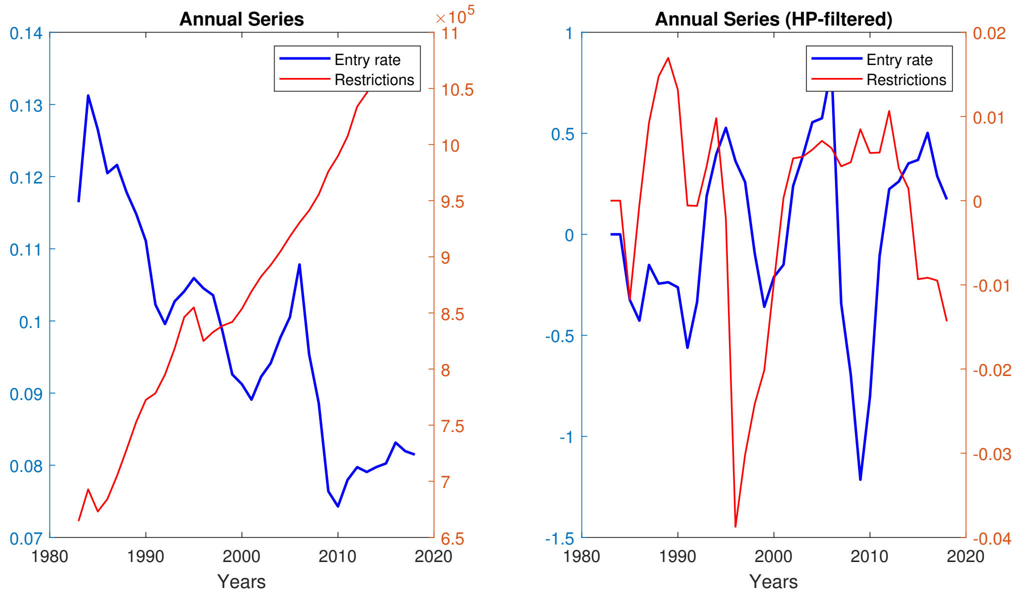

Figure 1.1: Annual series of entry rate (blue) and number of restrictions (red): trend (left box) and business cycle (right) dynamics.

cost shocks on our variables of interest; then we develop a heterogeneous-firm model with search frictions on the labor market (inducing involuntary unemployment) and endogenous firm dynamics (namely costly entry and endogenous exit of firms) calibrated on data from the Business Dynamics Statistics (BDS) database to address our empirical results.

The importance of entrants' average size has been emphasized by Haltiwanger et al. (2013) who highlight the disproportionate contribution of new firms to aggregate job creation. Moreover the relevance of employment concentration toward large firms has been pointed out by Colciago, Lin-

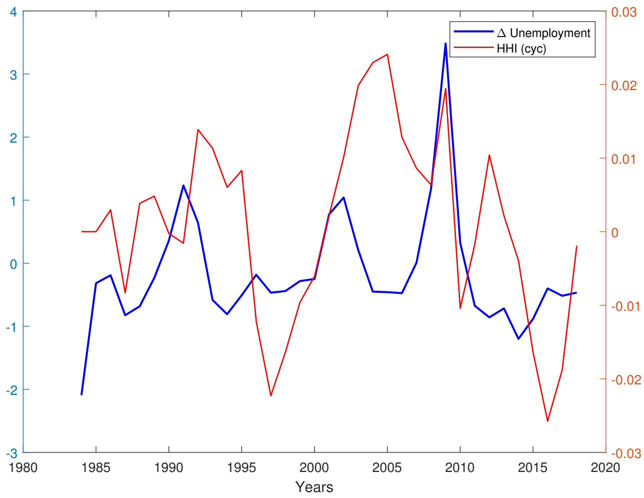

Figure 1.2: Annual series of first-differenced unemployment (blue) and the cyclical component of HHI (red).

denthal and Trigari (2019), who show that large firms contribute the most to the overall variability of aggregate job flow rates because they hold the largest employment share[9]. Hence the behavior of the employment share of large firms relatively to the shares of all other firms - i.e. our employment concentration indicator - is important to explain fluctuations in aggregate employment at business cycle horizons.

The BVAR evidence suggests that the increase in the cost of entry and the following shortage of new firms increases the average size of entrants and induces employment to be concentrated toward large firms, together with a decrease in unemployment. The model explains that incumbent firms

[9]Specifically, by using data from the Business Dynamics Statistics (BDS) database they show that mature/large firms hold the 42% of aggregate employment.

benefit from lower market contendibility due to the lack of new entrants, facing an higher stock market value. Therefore by increasing their size they induce labor market tightness (i.e. the ratio between vacancies issued and unemployed workers) to increase along with wages. The increase in labor costs implies that new firms have to be larger to break even on their costs, whereas small, low productivity incumbent firms exit from the market. Consequently the fraction of workers employed by large firms and the average size of new firms increase, consistently with the empirical evidence.
A first contribution comes from the empirical part of this study: we assess the effects of an unexpected, temporary increase in entry costs on aggregate unemployment, the size distribution of firms and the employment distribution according to firm size. Specifically, we document the short-run dynamics following an entry cost shock of the employment share of large firms with respect to the shares of all other firms (we define this indicator as employment concentration), the average size of new firms and the unemployment rate.
A second contribution comes from the theoretical part: we show that a heterogeneous-firm model featuring costly entry of new firms, endogenous exit of incumbents and search frictions in the labor market can explain our empirical results, suggesting that entry cost shocks affect the degree of firm heterogeneity which, in turn, affects aggregate job flow dynamics and unemployment.

Literature. There has been an increasing interest in the role of new firms in the economy since Haltiwanger et al. (2013), who find that firm birth contributes substantially to aggregate employment growth dynamics. More recently Pugsley and Şahin (2019) argue that the increasing startup deficit and the consequent shift of employment toward mature firms implies a lower aggregate elasticity of employment to business cycle conditions; Pugsley, Sédlaček and Sterk (2020) find that economic growth is largely driven by few startups with high-growth potential and that differences between these firms and all others are driven mainly by *ex-ante* heterogeneity; Sédlaček and Sterk (2017) point at the strong persistent effect of firm birth's initial conditions on future dynamics of aggregate employment.

Relative to the empirical responses of macro-variables to an exogenous variation in the number of new firms, Gourio, Messer and Siemer (2016) estimate the response of GDP, aggregate productivity and exit to an exogenous increase in entry; they find that despite the increase in exit, the total number of firms increases as well, leading to a rise in GDP. Similarly, Gutierrez, Jones and Philippon (2021) develop and estimate a model where the markup decreases with the number of firms, finding that an exogenous temporary increase in entry costs leads to a fall in the number of new entrants and in output. Differently from them, we focus on the short-run dynamics of unemployment and the firm size distribution following an exogenous, temporary increase in the cost of firm entry.

The literature is equally rich of models studying the propagation channel of entry, for instance Clementi and Palazzo (2013), Clementi et al.(2015) and Lee and Mukuyama (2018), whose models show that entry induces more persistence in the response of aggregate variables to a technology shock; our model featuring heterogeneous firms and endogenous entry and exit is similar to theirs, with the difference that ours is characterized also by equilibrium unemployment and, instead of technology shocks, it studies entry cost shocks. Siemer (2014) shows that a missing generation of entrants due to a large financial shock results in a long-lasting recession; our model does not feature the financial side of the economy, however it takes into account the average market value of firms and additionally replicates the response of unemployment to a shortage in new firms. Colciago, Fasani and Rossi (2022) find that by adding investment in new firms in the household's budget constraint the response of unemployment to a technology shock is amplified. With these studies we share the modeling framework of entry, exit and search frictions; differently from them, instead of technology shocks we focus on the temporary variations in entry costs at business cycle frequency.

Finally, notable works on the response of labor market variables to exogenous variations in the number of new firms are the already mentioned Sédlaček (2020), showing that the large increase in entry costs occurred during the Great Recession had persistent effects on unemployment and the distribution of firms due to the missing generation effect, and Cacciatore and Fiori (2016) studying the consequences of a permanent decrease in entry barriers. Differently from these works we study fluctuations in entry costs at business cycle frequency and how they affect the size distribution of firms and aggregate unemployment.

Our paper is structured as follows. The first section reports the empirical facts; the second describes the model; the third shows the details of the calibration; the fourth reports the results; the fifth concludes.

1.2 Empirical analysis

In this section we provide empirical evidence on the macroeconomic effects of unexpected, temporary changes in entry costs in a Bayesian VAR framework.

1.2.1 Data

To estimate the impulse responses to an entry cost shock we collect data from two different sources:

1. firm-level annual observations of exit and entry rates, average size of entrants and employment shares by firm size come from the 2018 release of Business Dynamics Statistics (BDS) database, collecting information on job-flows by location, state, industry, size and age between March 1983 and March 2018. In this context, firm size is defined as the number of workers employed in a firm;

2. quarterly observations of NASDAQ composite index, consumption and unemployment between 1982Q3 and 2018Q1 from the Federal Reserve Economic Data (FRED).

One complication arises from the different frequency of observations in the two datasets: annual for BDS, quarterly for FRED. In order to address this problem we estimate a mixed-frequency Bayesian VAR and identify entry cost shocks through sign restrictions[10].

Some words need to be spent on how entrants and their job flows are treated in our analysis. BDS classifies entrants according to their *final* size -i.e. the number of workers employed in a firm at the date of the survey - , since their size at $t-1$ is simply zero. Instead Moscarini and Postel-Vinay (2012) assign them to the "small-firms" class since entrants are mostly small; contrary to them, in this paper entrants are distinguished from any other size class and put into a "size-0" group in order to separate job flows of incumbents from that of new firms. Moreover we classify firms by their *average size*[11] - to mitigate issues arising from *regression to the mean*[12] - and the classes' cutoffs are chosen following Fort et al.(2013)'s classification. According to the *average size* definition, the size of firm i is defined as the average between its size at $t-1$ (i.e. the size registered in March $t-1$) and its current size at t.

Variables from BDS are built as follows:

- entry and exit rates are respectively equal to the number of firm births and deaths over the total number of firms in the current year;
- average size of entrants is equal to

$$AvgSize_t^{EN} = \sum_j med_j^{size} \frac{N_{jt}^{EN}}{N_t^{EN}}$$

[10]We use the MATLAB toolbox developed by Canova and Ferroni and set a Conjugate prior.

[11]That is $size_t = \frac{size_t + size_{t-1}}{2}$

[12]As explained by Haltiwanger et al.(2013) and quoting Friedman (1992), it is "the most common fallacy in the statistical analysis of economic data." The main problem arising from this fallacy is that the classification of firms by a specific size at time t or $t-1$ leads to an inverse relationship between size and growth: if for instance a firm experienced a transitory positive shock yesterday it is less likely it will grow again today.

where med_j^{size} is the median size of class j, N_{jt}^{EN} represents the number of new entrants belonging to size class j (according to BDS classification) and N_t^{EN} is the total number of new firms[13];

- employment concentration is the ratio

$$\frac{\text{employment large firms}}{\text{employment all ther firms}}$$

where large firms are classified as businesses with 500 employees or more. This variable is an indicator of variations in the distribution of employment across firms.

FRED's variables are either constructed or transformed as follows:

- NASDAQ composite index is taken in logs and it is used to represent the average market value of firms;

- real consumption is the log of the sum of personal consumption expenditures of non-durable goods and services;

- unemployment coincides with the unemployment rate.

1.2.2 Entry cost shock: identification strategy

The identification of entry cost shocks is based on sign restrictions to the responses of a subset of our dataset; as shown in table 2.3 we assume that following an increase in the entry cost the NASDAQ composite index and aggregate profits increase, whereas the entry rate decrease. Why the latter responds negatively to the shock is straightforward: since entry of new firms is more costly fewer

[13]Such variable is an approximation of the average size; we consider the medians of all the available 10 size classes from BDS to minimize the discrepancy from the true measure.

of them start a business. Consequently, the response of entry implies a lower competitive pressure that does not erodes the market share of incumbents. Thus the latter face higher profits, hence higher stock market values. [14].

Variable	**Sign**	**Quarters**
NASDAQ composite	+	1-8
Entry rate	-	1-4
Consumption	unrestricted	unrestricted
Employment concentration	unrestricted	unrestricted
Unemployment	unrestricted	unrestricted
Average size entrants	unrestricted	unrestricted
Exit rate	unrestricted	unrestricted

Table 1.1: Sign restriction.

1.2.3 BVAR: results

The estimated responses are reported in figure 2.1. Unemployment and firm exit decrease following the shock. The response of the average size of entrants is not significant on impact, however few

[14]We impose that the restrictions last for 8 quarters for NASDAQ and 4 quarters for entry. The length of the restrictions is set to minimize the width of the credible bands of unemployment and exit. However we verify that restricting the sign of NASDAQ and entry rate only for the first quarter does not alter the significance of average size of entrants and employment concentration's responses.

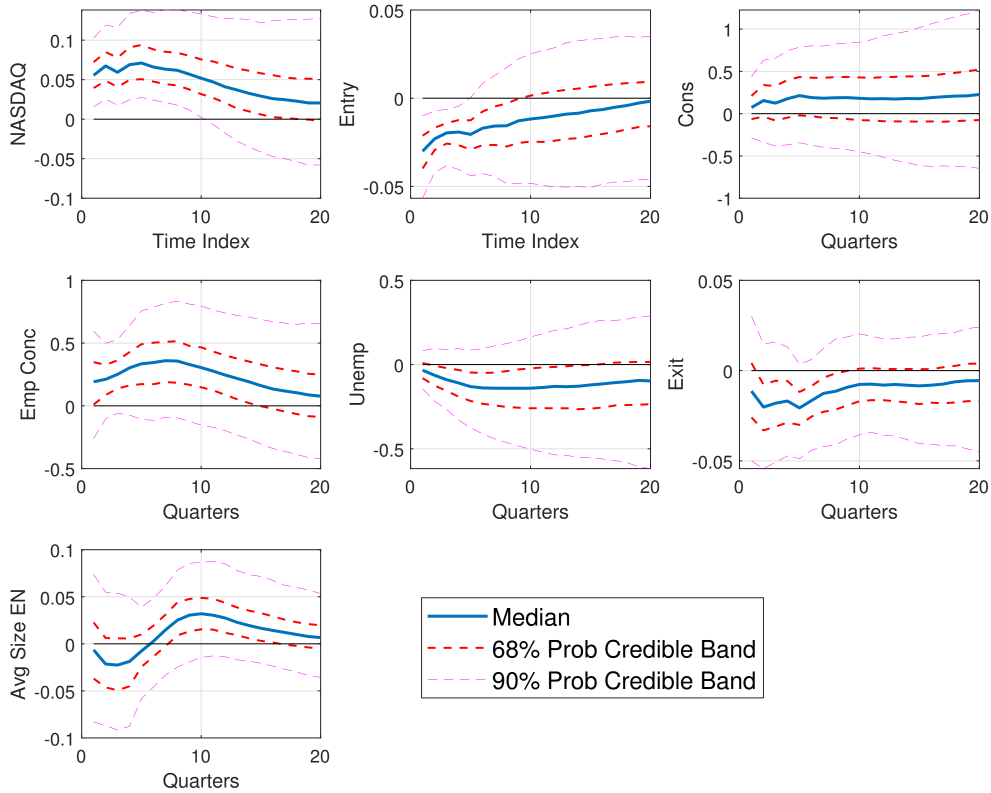

Figure 1.3: Estimated impulse responses to a 1 std deviation shock to entry cost.

periods later it increases. Employment concentration increases, suggesting that, following the entry cost's increase, larger business absorb more employment relatively to all other firms and increase their employment share. Finally, the response of consumption is not significant.

In order to explain these results we build a heterogeneous-firm model of the labor market with endogenous entry and exit as shown in the following section.

1.3 The Model

We consider a model with heterogeneous firms and a representative households in discrete time and infinite horizon. The labor market is subject to search frictions as in Mortensen and Pissarides (1994); entry and exit are endogenous as in Hopenhayn (1992) and Clementi and Palazzo (2016); finally, the representative household owns all firms in equilibrium and pays the entry cost.

1.3.1 Labor Market

Firms post vacancies by paying a linear cost κ per vacancy. The workforce is characterized by a constant mass I comprising employed and unemployed workers belonging to the representative household. Firms and unemployed workers meet according to a constant returns to scale matching function determining the mass of new hires - i.e. matches - through the rule

$$M_t = M(U_{t-1}, V_t) = \mu U_{t-1}^{1-\gamma} V_t^{\gamma} \tag{1.1}$$

where U_{t-1} and V_t are the unemployed and vacancies masses respectively. By defining $\theta_t = \frac{V_t}{U_{t-1}}$ as labor market tightness, the matching function can be rewritten as

$$M_t = \mu \theta_t^{1-\gamma} V_t$$

The probability that a vacancy is filled is $q_t = \frac{M_t}{V_t} q(\theta) = \mu \theta_t^{1-\gamma}$ and it is taken as given by all firms; the probability to find a job for an unemployed is $\phi_t = \frac{M_t}{U_{t-1}} = \mu \theta_t^{\gamma}$. As soon as unemployed are matched they become productive.

1.3.2 Firms

There is a continuous mass of perfectly competitive producers that are heterogeneous with respect to their size. Their productivity is made up by two components, one subject to idiosyncratic shocks, the other drawn upon entry from a time-invariant distribution $G(Z)$.
Firm j produces output y_{jt} according to the function $y_{jt} = A_t Z_j z_{jt} f(n_{jt})$ where n_{jt} is the amount of labor. Following Elsby and Michaels (2013) we choose $f(n) = n^\alpha$ so that the marginal product of labor declines with employment.
Following entry cost shocks, but before idiosyncratic shocks, firms draw a fixed cost of production c_o from a time-invariant, Log-normal distribution with parameters μ_o and σ_o: if their expected value is negative they exit; on the contrary, they continue to operate (or start to produce if they are new firms)[15].
The value of a firm net of the fixed cost of production - before any idiosyncratic shock occurs - is

$$\tilde{F}_{jt} = \left(E^z(F_{jt}|z_{jt-1}) - \int_0^{E^z(F_{jt}|z_{jt-1})} xg(x)dx \right) G[E^z(F_{jt}|z_{jt-1})] \tag{1.2}$$

where $g()$ and $G()$ are respectively the density and cumulative distribution functions of the Log-normal distribution, $E^z(F_{jt}|z_{jt-1})$ is defined as the expected discounted sum of future profits conditional on past idiosyncratic shock z_{jt-1} and F_{jt} is the value of current profits plus the discounted sum of future profits after the idiosyncratic shock z_{jt} has taken place.
Conditional on survival, firms choose the optimal employment level maximizing their value F_{jt}; separations from workers happens at zero cost, whereas vacancy-posting is subjected to the cost κ

[15] We follow Clementi and Palazzo (2016) in the design of the exit policy.

per vacancy. After the matching is complete, production and wage setting take place simultaneously.

Timing is as follows:

- the entry cost shock takes place;
- potential entrants pay the entry cost and will enter effectively next period;
- all firms - both new entrants and incumbents - draw the fixed cost of production c_o from the distribution $G()$ and continue to operate whether their expected value is positive. If not, they exit;
- idiosyncratic productivity shocks hit incumbent firms; entrants draw the permanent productivity component and, as the rest of incumbent firms, are hit by the idiosyncratic shock;
- given their initial size - which is zero for entrants -, firms set their employment adjustment policies and, in case, post vacancies;
- after the matching process takes place, firms bargain the wage and produce.

Firm j's problem is of the form

$$F_{jt} = \max_{n,v} \left\{ A_t Z_j z_{jt} n^{\alpha} - w_{jt} n - \kappa v + E_t \Lambda_{t,t+1} \tilde{F}_{jt+1} \right\}$$

subject to

$$n_{jt} = \begin{cases} n_{jt-1} + q_t v_{jt} \text{ if } n_{jt} > n_{jt-1} \\ n_{jt-1} - f_{jt} \text{ if } n_{jt} \leq n_{jt-1} \end{cases}$$

where $\Lambda_{t,t+1} = \beta E_t \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor and f_{jt} represents number of workers fired.

If the firm decides to hire new workers the firm-level employment dynamics reads as $n_t = n_{t-1} + q_t v_t$, hence the objective function can be maximized only in terms of n (to ease the notation, from now on we ignore the permanent idiosyncratic productivity component Z_j):

$$F_{jt} = \max_{n} \left\{ A_t z_{jt} n^{\alpha} - w_{jt} n - \frac{\kappa}{q_t}(n - n_{jt-1})\mathbf{1}^{+} + E_t \Lambda_{t,t+1} \tilde{F}_{jt+1} \right\} \tag{1.3}$$

where 1^{+} is an indicator function equal to one if $n > n_{t-1}$ and zero otherwise.
The first order condition of this problem delivers the Job Creation Condition (JCC)

$$\alpha A_t z_{jt} n_{jt}^{\alpha-1} - \frac{\partial w_{jt}}{\partial n_{jt}} n_{jt} - w_{jt} + E_t \Lambda_{t,t+1} D_{jt} = \frac{\kappa}{q_t}\mathbf{1}^{+} = J_{jt} \tag{1.4}$$

where $D_{jt} = \frac{\partial E_t \tilde{F}_{jt+1}}{\partial n_{jt}}$ denotes the marginal value of current employment choice on future profits. As explained by Elsby and Michaels (2013), because of diminishing marginal product of labor the firm can affect its wage. Given the existence of rents due to labor market frictions, firms and workers bargain over such rents to determine the optimal wage. Constant marginal product of labor implies that these rents are the same irrespective of firm size. On the other hand, diminishing marginal product of labor implies that these rents depend on firm-level employment; specifically such rents decrease as n increases. This means that a firm can reduce the cost of labor - namely the wage - by increasing its labor force.
Finally, let us consider D_{jt} again. It can be shown[16] that this can be written as

$$\begin{aligned} D_{jt} = E_t \Bigg[P^{na}_{jt+1} \Bigg(& \alpha A_{t+1} z_{jt+1} n_{jt}^{\alpha-1} - \frac{\partial w_{jt+1}}{\partial n_{jt}} n_{jt} - w_{jt+1} + \\ & E_{t+1} \Lambda_{t+1,t+2} J_{jt+1} \Bigg) + P^{h}_{jt+1} \frac{\kappa}{q_{t+1}} \Bigg] \end{aligned} \tag{1.5}$$

[16]For more details see the appendix in Elsby and Michaels (2013)'s paper.

where P^{na}_{jt+1} and P^{h}_{jt+1} denotes the probabilities of non-adjustment and hiring of firm j depending on its future employment choice and shock realizations. The intuition is as follows. For given values of z_{jt+1} firm j will either freeze employment, separate from some of its workers, hire new workers or exit in $t+1$ with positive probability. In case it decides to keep its employment level unchanged, the marginal effect of the current employment choice will continue to affect firm j's value until the firm will find optimal to change it. At the same time if the firm finds profitable to increase its labor force next period, current employment will decrease the cost of vacancy-posting at $t+1$.

1.3.3 Endogenous entry

Each period entry is determined by the free-entry condition

$$E_t \tilde{F}^e_{jt+1} = c_{et} \tag{1.6}$$

stating that the expected value of entrants $E_t \tilde{F}^e_{jt+1}$ - whose expectation is computed over $G(Z_j)$ - must be equal to an entry cost c_{et} of the form

$$c_{et} = \psi_t (N^e_t)^{\xi} \tag{1.7}$$

where $\psi_t = \psi + \zeta_t$ and $\zeta_t = \rho_A \zeta_{t-1} + \epsilon^A_t$ with $\epsilon_t \sim N(0, \sigma_A)$.

Notice that potential entrants are *ex-ante* the same, since only upon paying the entry cost will draw their permanent productivity component Z_j from the time-invariant distribution $G()$ in the following period; only then they become productive.

Following Gutierrez Gallardo, Jones and Philippon (2019) c_{et} is an increasing function of the mass of entrants N^e_t so that entry responds inelastically to changes in the aggregate conditions. Upon paying the entry cost, new firms become productive next period.

Given entry, the mass of firms follows the law of motion

$$N_t = (1-\delta)N_{t-1}^e + N_{t-1}^{surv} = \sum_j N_{jt} \tag{1.8}$$

where N_{t-1}^{surv} denotes the mass of surviving firms from endogenous and exogenous exit.

1.3.4 Household

There is a representative risk-averse household maximizing its life-time utility

$$\mathbf{U}_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{1.9}$$

subject to the budget constraint

$$\begin{aligned} C_t + B_t + x_{t+1} F_t N_t + x_{t+1}^e F_t^e N_t^e = \\ \sum_j w_{jt} L_{jt} + b(I - L_t) + (\Pi_t + F_t) N_{t-1}^{surv} x_t + \\ (\Pi_t^e + F_t) N_{t-1}^e x_t^e + B_{t-1} R_t - T_t \end{aligned} \tag{1.10}$$

the aggregate employment law of motion

$$L_t = (1 - s_t^{tot}) L_{t-1} + \phi_t (I - L_{t-1}) \tag{1.11}$$

and aggregate labor supply

$$\sum_j L_{jt} = L_t \tag{1.12}$$

where

- B_t is a state-contingent asset exchanged by members of the household
- x_{t+1} and x^e_{t+1} are shares in a mutual fund of incumbents and new entrants respectively
- F_t and F^e_t are the average value of incumbents and entrants respectively
- b is unemployment benefit and T_t is government transfers to finance b
- Π_t and Π^e_t are dividends from incumbents and entrants, that is revenues minus labor, vacancies and fixed costs
- L_{jt} is the amount of labor provided to firm j and $L_t = I - U_t$ is aggregate employment
- s^{tot}_t is the total separation rate comprising both firings and exit.

The household maximizes its lifetime utility subject to the previous constraints through $\{C_t, L_{jt}, B_t, x_{t+1}, x^e_{t+1}\}$ These are chosen by solving the first-order conditions of the problem

$$F_t N_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \big(\Pi_{t+1} + F_{t+1}\big) N^{surv}_t \tag{1.13}$$

$$F^e_t N^e_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \big(\Pi^e_{t+1} + F_{t+1}\big) N^e_{t-1} \tag{1.14}$$

$$\lambda_t = u(C_t) \tag{1.15}$$

$$\frac{1}{R_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{1.16}$$

$$H_{jt} = (w_{jt} - b)\lambda_t + \beta E_t H_{jt+1}\big(1 - s^{tot}_{t+1} - \phi_{t+1}\big) \tag{1.17}$$

Equation 2.17 describes how the household's shadow value H_t of an additional worker hired in firm j is related to the increase in utility given by being employed $w_{jt}\lambda_t$, the decrease in utility due to the loss of unemployment benefit $b\lambda_t$ and the continuation utility value. For our purpose it might be useful to express it as[17]

$$H_{jt} = W_{jt} - \Gamma_t \tag{1.18}$$

where W_{jt} and Γ_t denotes the value of employment at firm j and unemployment respectively. These are equal to

$$\begin{aligned} W_{jt} = \lambda_t w_{jt} + \beta E_t \lambda_{t+1} \Bigg\{ \delta \Gamma_{t+1} + \Bigg[P^f_{jt+1} \Big(\tilde{s}_{jt+1} \Gamma_{t+1} + (1 - \tilde{s}_{jt+1}) W^f_{jt+1} \Big) + \\ P^{na}_{jt+1} W^{na}_{jt+1} + P^h_{jt+1} W^h_{jt+1} \Bigg] \Bigg\} \end{aligned} \tag{1.19}$$

$$\Gamma_{jt} = b\lambda_t + \beta E_t \lambda_{t+1} \Bigg[(1 - \phi_{t+1}) \Gamma_{t+1} + \phi_{t+1} \sum_j P^h_{jt+1} W_{jt+1} \Bigg] \tag{1.20}$$

where P^f_{jt+1}, P^{na}_{jt}, P^h_{jt} are the probabilities of firing, non-adjustment and hiring of firm j. Equation 2.19 describes the value of employment to a worker; it takes into account the probability of losing the job due to exogenous firm exit and the probability that, conditional on survival, the firm will decide to separate. $\tilde{s}_{jt}$ represents the probability that employer j will fire that particular worker conditional on the probability that it is optimal to decrease firm j's workforce. Finally, the worker distinguishes between W^f_{jt}, W^{na}_{jt} and W^h_{jt} - that are the wages chosen when the firm separate from workers, does not adjust or hires new workers - because they will differ according to the employment level chosen by the firm, since the employer and its employees will bargain over wages every time an employment adjustment takes place.

The value of unemployment is simpler; equation 2.20 states that an unemployed worker must take into account expected value of being hired by any firm in the economy plus the expected value of

[17]Here we follow Elsby and Michaels (2013).

remain unemployed.

1.3.5 Wage bargaining

Firm j and its workers maximize the joint surplus

$$\max_{w_{jt}} J_{jt}^{1-\eta} H_{jt}^{\eta} \tag{1.21}$$

whose first-order condition delivers

$$\eta J_{jt} = (1-\eta)\frac{H_{jt}}{\lambda_t} \tag{1.22}$$

Substituting 2.18 into 2.22 yields

$$\left(W_{jt} - \Gamma_t\right) = \frac{\eta}{(1-\eta)} J_{jt}\lambda_t$$

Notice that if firm j is hiring then from the JCC

$$\left(W_{jt+1}^{h} - \Gamma_{t+1}\right) = \frac{\eta}{(1-\eta)} \frac{\kappa}{q_{t+1}} \lambda_{t+1} \tag{1.23}$$

whereas if it is firing

$$\left(W_{jt+1}^{f} - \Gamma_{t+1}\right) = 0 \tag{1.24}$$

and finally in case of non-adjustments

$$\left(W_{jt+1}^{na} - \Gamma_{t+1}\right) = \frac{\eta}{1-\eta} J_{jt+1}\lambda_{t+1} \tag{1.25}$$

Given 1.23 and 1.24, let us consider W_{jt} and Γ_t separately.

By performing some substitutions, the value of unemployment 2.20 becomes

$$\Gamma_t = \lambda_t b + \beta E_t \lambda_{t+1}\Gamma_{t+1} + \beta\frac{\eta}{1-\eta} E_t \lambda_{t+1}\phi_{t+1}\frac{\kappa}{q_{t+1}} \tag{1.26}$$

whereas by substituting 1.23, 1.24 1.25, and 1.26 into 2.19, the value of employment becomes

$$W_t = \lambda_t w_{jt} + \beta E_t \lambda_{t+1}\Gamma_{t+1} + \beta \frac{\eta}{1-\eta} E_t \lambda_{t+1}\left[\frac{\kappa}{q_{t+1}} P^h_{jt+1} + J_{jt+1} P^{na}_{jt+1}\right]. \tag{1.27}$$

Since $H_{jt} = W_{jt} - \Gamma_{jt}$, by subtracting 1.27 from 1.26 we obtain

$$H_{jt} = (w_{jt} - b)\lambda_t + \beta \frac{\eta}{1-\eta} E_t \lambda_{t+1}\left[\frac{\kappa}{q_{t+1}} P^h_{jt+1} + J_{jt+1} P^{na}_{jt+1}\right] - \beta \frac{\eta}{1-\eta} E_t \lambda_{t+1}\phi_{t+1}\frac{\kappa}{q_{t+1}}. \tag{1.28}$$

By combining the first-order condition from wage bargaining $\frac{\eta}{(1-\eta)} J_{jt} = \frac{H_{jt}}{\lambda_t}$ with the JCC in 2.4 and the previous equation we obtain

$$\frac{\eta}{1-\eta}\left[\alpha A_t z_{jt} n_{jt}^{\alpha-1} - \frac{\partial w_{jt}}{\partial n_{jt}} n_{jt} - w_{jt} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(P^{na}_{jt+1} J_{jt+1} + P^h_{jt+1}\frac{\kappa}{q_{t+1}}\right)\right] =$$
$$w_{jt} - b + \beta \frac{\eta}{1-\eta} E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\kappa}{q_{t+1}} P^h_{jt+1} + J_{jt+1} P^{na}_{jt+1}\right] - \beta \frac{\eta}{1-\eta} E_t \frac{\lambda_{t+1}}{\lambda_t}\phi_{t+1}\frac{\kappa}{q_{t+1}}.$$

This finally gives the differential equation

$$w_{jt} = \eta\left[\alpha A_t z_{jt} n_{jt}^{\alpha-1} - \frac{\partial w_{jt}}{\partial n_{jt}} n_{jt} + \beta\kappa E_t \frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + (1-\eta)b$$

by solving which we find the wage bargaining equation

$$w_{jt} = \eta\left[\frac{\alpha A_t z_{jt} n_{jt}^{\alpha-1}}{1-\eta(1-\alpha)} + \beta\kappa E_t \frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + (1-\eta)b. \tag{1.29}$$

1.3.6 Aggregation, market clearing and equilibrium

In equilibrium the representative household owns the whole portfolio of firms, hence $x_t = x_t^e = 1$. Moreover

- labor demand equals labor supply

$$\sum_j L_{jt} = \sum_j n_{jt} N_{jt} \tag{1.30}$$

- good market clearing implies

$$C_t + N_t^e c_{et} = Y_t - \kappa V_t - \sum_j N_{jt} \int_0^{E_{t-1}F_{jt}} xg(x)dx \tag{1.31}$$

  where $Y_t = \sum_j y_{jt}$, $V_t = \sum_j v_{jt}$ and the last term represents the sum of all fixed costs drawn by firms that choose to continue to operate

- firms set their employment policy function by solving JCC (2.4)

- the wage is set in order to split the joint surplus of a match by satisfying equation 2.23

- labor market tightness $\theta_t = \frac{V_t}{U_{t-1}}$ is determined in equilibrium and taken as given by firms and the representative household

- the dynamics of aggregate unemployment follows the rule

$$U_t = (1 - \phi_t)U_{t-1} + s_t^{tot} L_{t-1} \tag{1.32}$$

- the aggregate separation rate is determined by firm-level exit and firing

$$s_t^{tot} = \frac{\left[\sum_j n_{jt} N_{jt-1}^{exit} + \sum_j |n_{jt} - n_{jt-1}| N_{jt}^{firing}\right]}{L_{t-1}} \tag{1.33}$$

  where N_{jt-1}^{exit} and N_{jt}^{firing} denote the mass of firms that exit endogenously and that choose to separate from their workers respectively.

1.3.7 Numerical solution

The steady-state is solved numerically by local approximation of the value function F_{jt}. The algorithm proceeds as follows:

- by using the discretized grid of employment choices and idiosyncratic productivity[18] we find

[18]We use the Rouwenhorst's method to discretize AR(1) processs.

a first solution of the problem by iterating on the Bellman equation;

- if such solution does not hit the boundaries of the employment grid, continue; otherwise enlarge the grid and come back to the previous point;

- for each idiosyncratic productivity's value, regress the objective function of the problem only on the employment grid nodes immediately before and after the solution found in the previous step; the regressors are employment, employment squared and a constant. Specifically

  - given the objective function $g_z(n)$, consider only $g_z(n_{k-1})$ and $g_z(n_{k+1})$ such that n_k is the discrete maximizer of g_z on the grid and $n_{k-1} \leq n_k \leq n_{k+1}$;

  - compute the OLS coefficients of

$$g_z(n) = \beta_0 + \beta_1 n + \beta_2 n^2$$

    by using the three nodes n_{k-1},n_k and n_{k+1} from the previous point;

  - take the first derivative of the interpolated g_z with respect to n and find the value of n that maximizes g_z by solving the first-order condition

$$\tilde{n} = -\frac{\beta_1}{2\beta_2}$$

  - calculate the objective function implied by $\tilde{n}$:

$$\tilde{g_z} = \beta_0 + \beta_1 \tilde{n} + \beta_2 \tilde{n}^2$$

- approximate F_j with $\tilde{g_z}$ computed for the firm with initial size equal to n_j;

- iterate until convergence of the value function.

As for solving the dynamic stochastic equilibrium, we follow the method of Reiter (2009)[19]: stochastic aggregate dynamics are compute by linearization for each grid point around the steady state. In this way the Bellman equation is treated not as a function but as a system of difference equations. The model can be summarized as

$$E_t\mathbf{F}_t\Big(\mathbf{X}_t, \mathbf{X}_{t+1}, \log A_t, \log A_{t+1}\Big) = 0 \tag{1.34}$$

where $\mathbf{X}_t$ comprises all model variables, both firm-level and aggregate. In steady state equation 2.28 reads

$$E_t\mathbf{F}\Big(\mathbf{X}, \mathbf{X}, 0, 0\Big) = 0 \tag{1.35}$$

By computing the Jacobian of 2.28 evaluated at the steady state equilibrium, for sufficiently small aggregate shocks the model can be approximated linearly as

$$E_t\mathcal{A}\big(\mathbf{X}_{t+1} - \mathbf{X}\big) + \mathcal{B}\big(\mathbf{X}_t - \mathbf{X}\big) + \mathcal{C}\log A_{t+1} + \mathcal{D}\log A_t \tag{1.36}$$

where

- $\mathcal{A} = \frac{\partial \mathbf{F}_t}{\partial \mathbf{X}_{t+1}}$
- $\mathcal{B} = \frac{\partial \mathbf{F}_t}{\partial \mathbf{X}_t}$
- $\mathcal{C} = \frac{\partial \mathbf{F}_t}{\partial \log A_{t+1}}$
- $\mathcal{D} = \frac{\partial \mathbf{F}_t}{\partial \log A_t}$

This form allows the model to be solved through the QZ decomposition method outlined by Klein (2000). In this way we avoid any bounded rationality assumption as in the Krusell and Smith (1998)'s method.

[19]We use the MATLAB code developed by Costain and Nakov (2011).

1.4 Calibration

The model follows a quarterly calibration. We set $\beta = 0.99$ and $\alpha = 0.65$ as standard in the literature. As for labor market parameters, we normalize steady-state θ to one and set the matching elasticity $\gamma = 0.72$ as in Shimer (2005). Moreover we let the worker's bargaining power η to be equal to γ in order to satisfy the Hosios condition. The matching efficiency is set in order to provide a job-finding probability equal to 0.7 as in Blanchard and Galí (2010). Unemployment benefit b is chosen in order to provide a replacement ratio equal to 40% of the average wage as in Shimer (2005), whereas the marginal cost of vacancies κ is set so that the total cost of vacancies is 1% of GDP as in Blanchard and Galí (2010).

Turning to the exit policy parameters, μ_{of} and σ_o are chosen in order to match jointly a quarterly exit rate equal to 2.5% and the exit rate differential between small (1-19) and large (500+) firms[20]. The elasticity of entrants ξ in the entry cost is equal to 1.5 as estimated by Gutierrez Gallardo, Jones and Philippon (2019), whereas the parameter ψ is set so that the entry cost is 5% of GDP. Firm-level permanent productivity components Z_j and the corresponding frequencies $F(Z_j)$ are parameterized so as to match the distributions of employment share and firm share distributions according to size in the U.S. [21]

The standard deviation of idiosyncratic productivity shocks σ_z is set to target an overall separation rate of 9% which is the median value of the rage 8% - 10% reported by Hall (1995); the persistence of idiosyncratic shock ρ_z is chosen to match the average NJC rate[22]. Finally, the standard deviation σ_A of the entry cost shock is set to match the response of stock market values on impact whereas its persistence ρ_A is equal to 0.72 as in Gutierrez, Jones and Philippon (2021).

[20] Source: Business Dynamics Statistics (BDS) database.

[21] Source: BDS dataset, observations between 1983-2018.

[22] Ibidem.

The calibration is summarized in table 2.5.

1.5 Results

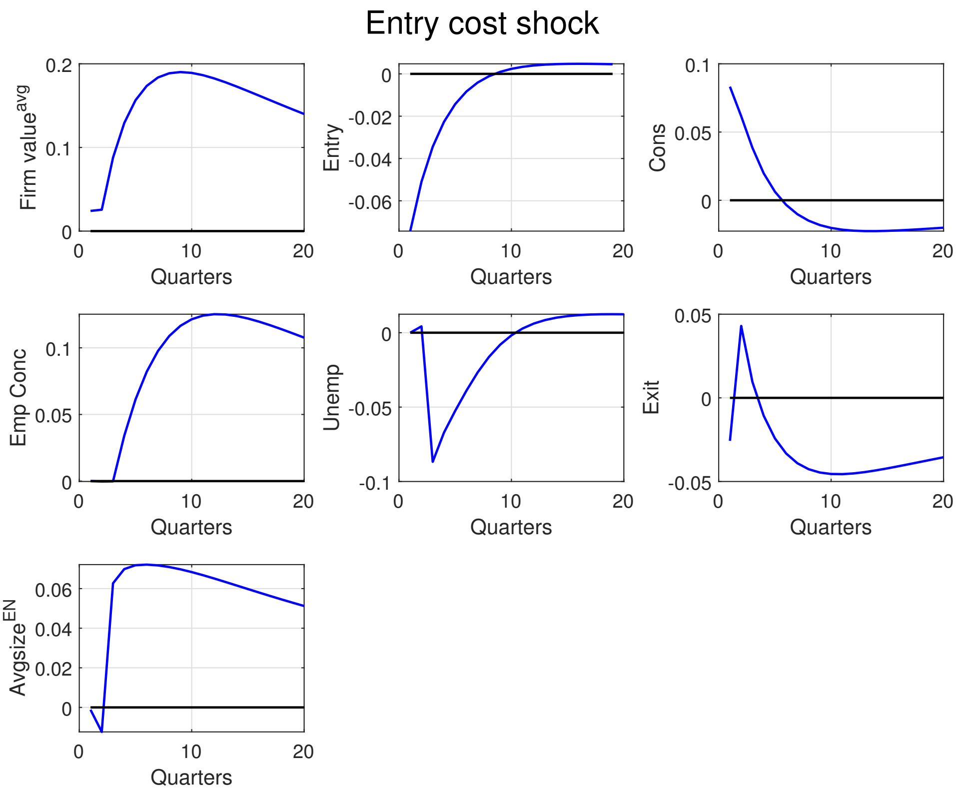

Figure 1.4: Model's responses to a 1 std deviation increase in entry cost.

Figure 2.2 reports the theoretical responses to the shock. Average size of entrants is the weighted average of firm-level employment chosen by each new firms, whose weights are the shares of each entrant with respect to the total mass of new firms. Employment concentration is measured as the ratio between the mass of employment from large firms over the mass of employment from all other firms, similarly to the indicator used in the BVAR.

Due to the increase of the entry cost, the representative household disinvests from new entrants and consumes more. The rise in aggregate demand induces incumbent firms to increase their size by issuing more vacancies; the increase of aggregate vacancies drives unemployment down and labor market tightness θ_t up. The dynamics of the average size of entrants depends on the response of θ_t: when the labor market is tighter, marginal costs of hiring $\frac{\kappa}{q_t}$ and wages increase, hence new firms have to be larger to break even on their costs.

The increase in firms' expected value makes less likely for firms to exit. However, despite the higher survival probability, the overall number of firms falls because of the fall in entrants. Moreover notice that, because of the tighter labor market and the subsequent increase in labor costs, small, low productivity incumbent firms exit from the market more than the others do as shown by the response of employment concentration. Coherently with our BVAR results the latter increases, indicating that following the shock large firms increase their employment share relative to firms in other classes. This effect emerges from both the general increase in firm's size due to higher profitability - shown by the increase in the firm market value - and the exit of small unproductive firms, causing large businesses to gain employment shares. Hence, following the exogenous increase in entry costs, employment is more concentrated toward larger firms.

1.6 Conclusion

We develop a heterogeneous-firm model with search frictions and endogenous entry/exit dynamics to explain the empirical response of the average size of entrants and employment concentration to an entry cost shock.

We find that positive entry cost shocks increase the average size of entrants and move employment shares toward the largest firms. These results reveal the role of entry costs' fluctuations in explaining the dynamics at business cycle horizons of both firm and employment share distributions according to size.

Parameter/SS value	Definition	Value	Source/Target
β	Discount factor	0.99	Annual interest rate = 4%
α	Returns to scale parameter	0.65	Labor share
θ	Labor market tightness	1	Normalization, Shimer (2005)
γ	Matching elasticity	0.72	Shimer (2005)'s estimate
μ	Matching efficiency	0.7	$\phi = 0.7$, BG (2010)
η	Workers' bargaining power	$=\gamma$	Hosios condition
b	Unemployment benefit	1.22	$\frac{b}{w_{avg}} = 40\%$ Shimer (2005)
κ	Vacancy marginal cost	0.7	$\frac{\kappa V}{GDP} = 1\%$ BG (2010)
ξ	Elasticity of entrants	1.5	GJP (2019)
ψ	Entry cost parameter	1.2	$\frac{c_e}{GDP} = 5\%$
μ_o	Log-norm. par.	-8.2866	exit = 2.5% (BDS)
σ_o	Log-norm. par.	4.56;	exit differential = 2.05%
Z_j	Permanent productivity component	[1.2; 2.5; 4.95; 10.9]	Employment share (BDS)
$F(Z_j)$	Probability $Z = Z_j$	[0.9573; 0.0393; 0.0026; 0.0009]	Firm share (BDS)
σ_z	Std deviation id shocks	0.075	$s^{tot} = 9\%$, Hall (1995)
ρ_z	Persistence id shocks	0.87	avg $NJCR$ (BDS)
σ_A	Std dev entry cost shock	0.03	Stock mkt value response
ρ_A	Persistence entry cost shock	0.72	GJP (2021)

Table 1.2: Quarterly calibration.

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1.A Appendix: BVAR robustness checks

This section reports the estimated impulse responses for different specifications of the VAR.

1.A.1 Time intervals

The sample is split into two subsamples: 1982Q2-2008Q1 and 1992Q2-2018Q1. Results are shown in figures 1.5 and respectively.

1.A.2 Sign restrictions

In this subsection sign restrictions are imposed only on impact (i.e. they last just one period). Responses are shown in figure 1.7.

1.A.3 Different priors

In this subsection we set different priors for the coefficient matrix in the reduced VAR: Minnesota and Jeffrey priors instead of the Conjugate prior. Responses are shown in figures 1.8 and 1.9

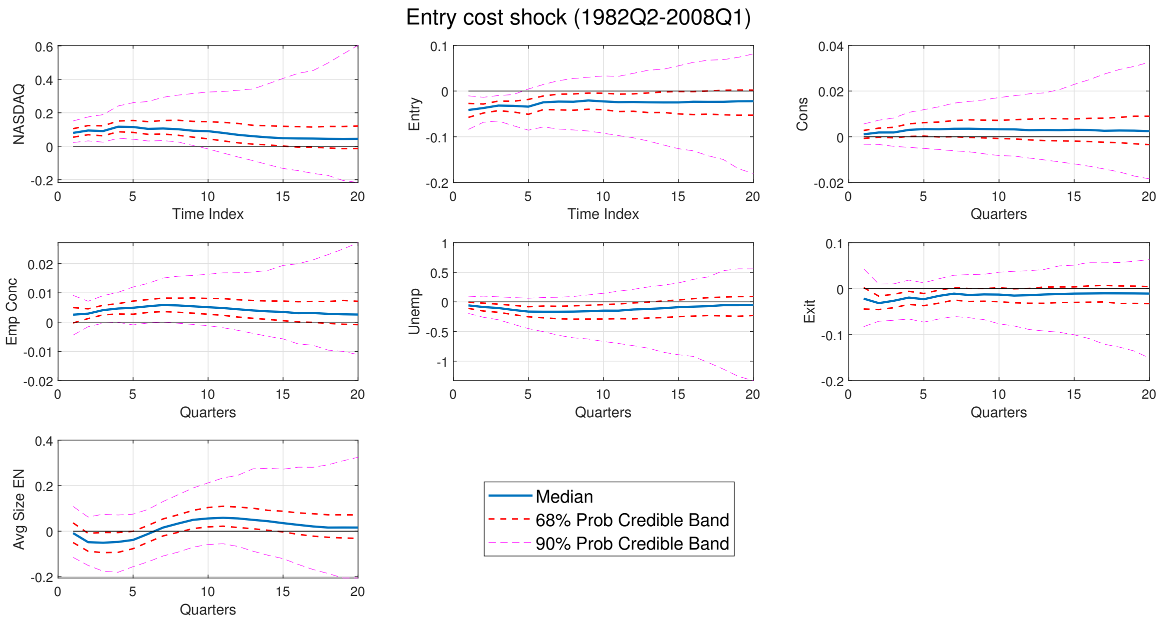

Figure 1.5: 1-std deviation entry cost shock (1982Q2-2008Q1).

respectively.

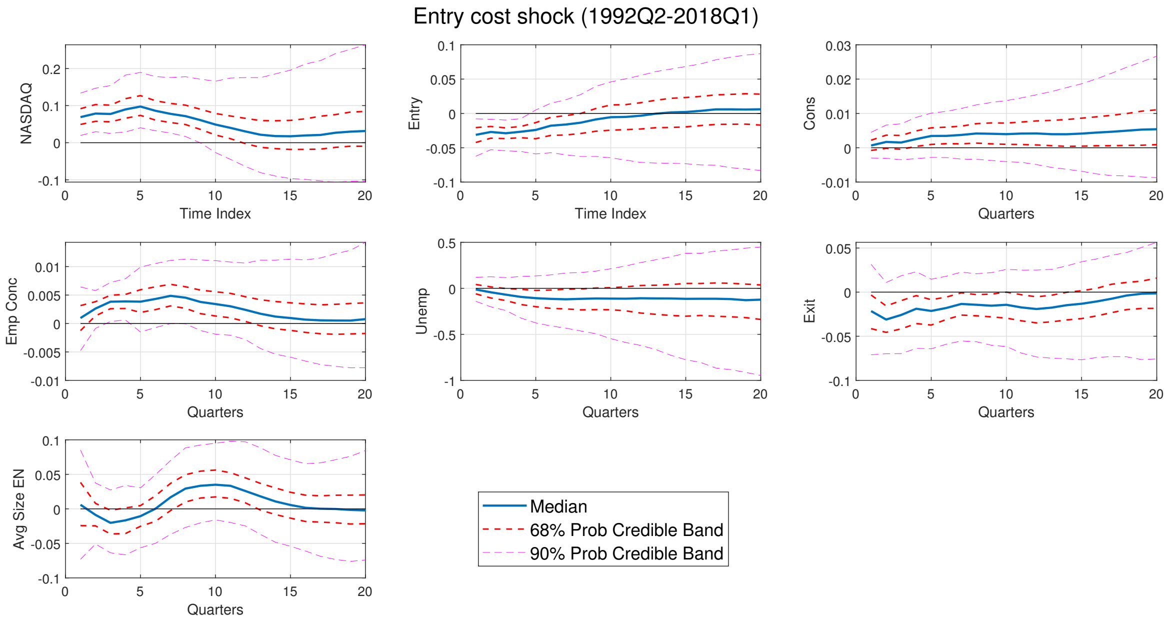

Figure 1.6: Estimated responses to a 1-std deviation entry cost shock (1992Q2-2018Q1).

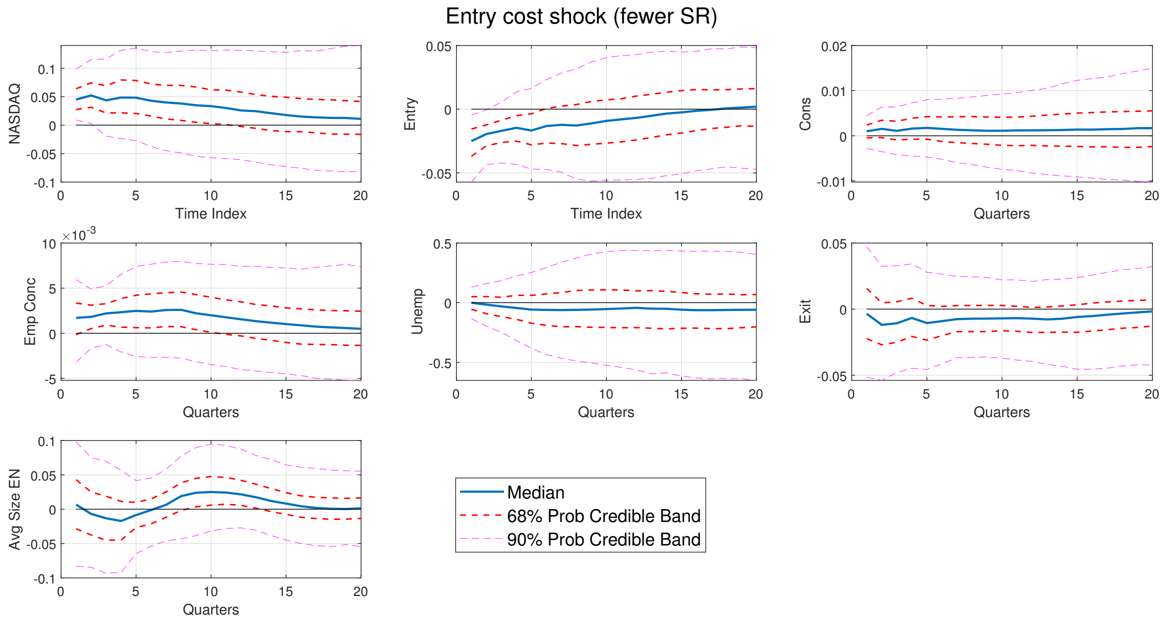

Figure 1.7: Estimated responses to a 1-std deviation entry cost shock (fewer SR).

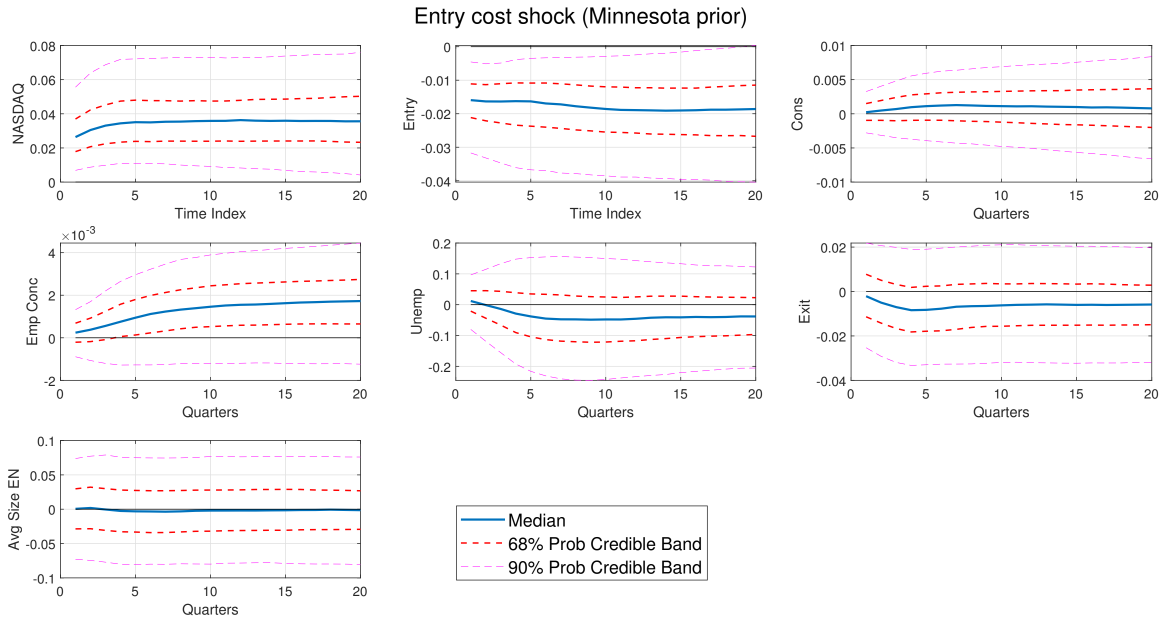

Figure 1.8: Estimated responses to a 1-std deviation entry cost shock (Minnesota prior).

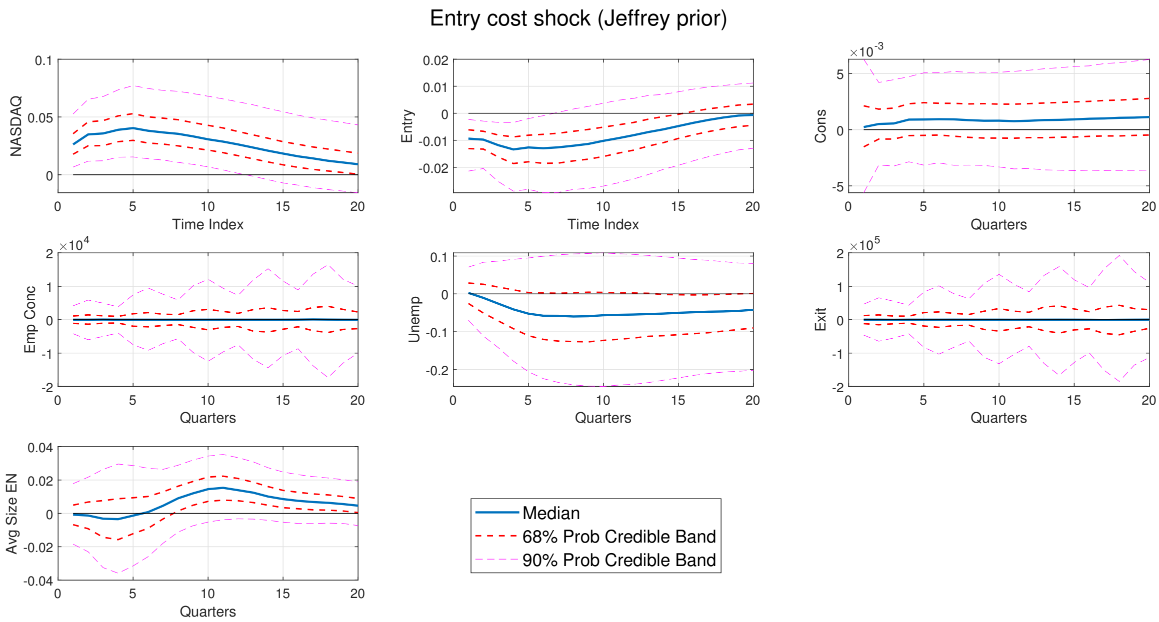

Figure 1.9: Estimated responses to a 1-std deviation entry cost shock (Jeffrey prior).

Chapter 2

Firm size, exit and aggregate fluctuations

Andrea Colciago and Marco Membretti

Abstract Data from Business Dynamics Statistics (BDS) show that job destruction due to exit drives the long-run difference in employment growth rates between small and large firms. Moreover, following a negative technology shock small firms destroy more jobs by exiting than large firms. Hence job destruction due to exit is a key dimension to explain both facts. To this aim we develop a heterogeneous-firm model with search frictions and endogenous entry/exit replicating the long-run differential of job destruction due to exit between small and large firms and its empirical response to technology shocks. Contrary to frameworks with *exogenous* exit, our model can account for the volatility of exit and the differential of job destruction due to exit between small and large firms conditional to the technology shock. Additionally, we point out that endogenous firms exit leads

to a substantial amplification of unemployment in response to technology shocks.

2.1 Introduction

Since the benchmark heterogeneous-firm models of Hopenhayn (1992) and Hopenhayn and Rogerson (1993), there has been an increasing interest in modeling firm entry and exit of firms particularly in relation to aggregate dynamics[1]. Empirically, the relevance of the entry process for aggregate employment and output's dynamics has been shown by Pugsley and Sahin (2019) who report that the recently declining startup rate has amplified the employment's response to output contractions and dampened its growth during expansions. On the same line Haltiwanger et al. (2013) and Sedláček and Sterk (2017) emphasize the importance of startups for aggregate job creation.
Although earlier studies have found exit less relevant than entry for business cycle fluctuations in aggregate variables such as output or unemployment, its role has been recently revised. For instance, contrary to the literature finding that *establishment* death in the manufacturing sector is less cyclical than birth[2], Tian (2018) concentrates on *firm* death and observes that its apparent acyclical nature is due to both business cycle proxies and timing[3]. Additionally, Casares, Khan and Portineau (2020) observe that *net entry* - i.e. the difference between births and deaths - has become more cyclical and that a model with endogenous exit provides a better fit to the data than one with acyclical exit. From this literature emerges that both entry and exit matter for aggregate

[1] Clementi and Palazzo (2013), Clementi et al. (2015) and Lee and Mukuyama (2018) to name a few.

[2] See Lee and Mukuyama (2015) for instance.

[3] By considering all sectors she finds that exit positively lags business cycles measured as variables in *levels* - such as real GDP or unemployment rate - whereas when the cyclical indicator is a *growth* rate the countercyclicality of firm exit emerges.

dynamics.

This work studies the business cycle dynamics of job destruction due to exit according to size motivated by two empirical facts:

1. firm exit is necessary to match the difference in the long-run net job creation (NJC) rates between small and large firms. Specifically, small firms are characterized by *negative* NJC rates mainly due to a larger job destruction (JD) from exit[4];

2. small firms destroy less jobs through exit than large firms following a positive technology shock.

This evidence, reporting a different cyclicality in job flows according to firm size, is linked to the work of Moscarini and Postel-Vinay (2012) showing that the net job creation rate of large firms is more strongly and significantly correlated with unemployment than the net job creation rate of small firms. We follow a similar path in comparing job flows of large versus small firms, but differently from them we focus on the exit process depending on size and ask whether, in addition to the different long-run behavior of exit as reported by fact 1, job destruction flows due to exit of the two types of firms display different cyclicalities conditional to technology shocks.

Fact 2 provides the answer: following a positive technology shock small firms destroy less jobs through exit than large. Moreover we assess that the correlation, conditional to technology shocks, of the differential $JD^{ex}_{small} - JD^{ex}_{large}$ with labor productivity is negative and significant.

To address these results we generalize the benchmark search model of Mortensen and Pissarides (1994) to include heterogeneous multi-worker firms and endogenous entry/exit dynamics. Contrary to models with exogenous exit we can reproduce the empirical evidence; besides in our framework

[4] As noted by Haltiwanger et al.(2013) small firms are more likely to exit than larger firms *even controlling for age.*

unemployment reacts more sizeably to technology shocks relative to the model with *exogenous* exit and *endogenous* entry, suggesting that not only entry but also exit is a viable source of amplification.

Our contribution is twofold: first we provide empirical evidence documenting the different cyclicality and volatility of job destruction due to exit between small and large firms conditional to technology shocks; secondly we show that a model with equilibrium unemployment where firms differ in their size, enter the market according to a free-entry condition and exit endogenously depending on both idiosyncratic and aggregate conditions can replicate our empirical results. By construction models with exogenous exit fail to replicate both the response of exit and job destruction due to exit and their conditional second moments, whereas our model is in fact successful. Finally, we show that the endogenous exit framework contributes to the amplification of unemployment's response relative to models with exogenous exit and endogenous entry, therefore linking our work to the literature studying amplification channels in models with search and matching frictions.

Literature. This work is related to the literature on firm dynamics started by Hopenhayn (1992) and Hopenayn and Rogerson (1993), with later developments such as Clementi and Palazzo (2016), Clementi et al. (2015) and Lee and Mukuyama (2018). These works study aggregate dynamics of the benchmark model with endogenous entry and exit and size heterogeneity: incumbent firms are hit by idiosyncratic shocks determining whether to exit or not, in case of survival they choose employment adjustments in order to maximize their value and a free-entry condition governs the mass of new entrants in equilibrium. They find that entry and exit amplify the response of the aggregates to technology shocks, survival rates increase with size and both entrants and exiters are smaller in expansions. Our model follows their design of firm-level exit policy functions: firms draw a fixed cost of production from a time-invariant distribution and exit such cost exceeds their expected value. In addition, we further generalize their model by considering frictions in the labor

market and showing both the higher cyclicality and volatility of job destruction due to exit of small firms relative to large business, conditional to technology shocks, and the amplification effect of endogenous exit on the response of unemployment.
An increasing literature generalizes search models by including firm dynamics. Coles and Moghaddasi (2011) focus mainly on endogenous entry, leaving exit exogenous, and find that entry amplifies aggregate productivity and separation shocks. Garibaldi (2006) builds a model of multi-worker firms, search frictions and entry/exit showing that such framework generates amplification of unemployment's fluctuations. Our model can be considered a further development of his since we allow for marginal decreasing returns to scale, endogenous job destruction and a stationary size distribution in order to replicate firm-level NJC rates and job destruction due to exit differentials by size. Kaas and Kircher (2015) add heterogeneity with respect to job-filling rates to generate sluggish dynamics of macroeconomic variables as in the data and obtain a tractable model though long-term wage contracts; our model is instead closer to the benchmark models of Mortensen and Pissarides (1994) and Hopenhayn (1992); despite our setup is simpler than Kaas and Kircher's (2015) we are able to replicate the NJC rate distribution by size, the JD due to exit differential between small and large firms and its response to technology shocks. More recently Berstein et al. (2021) have studied the interplay between net entry and aggregate job flows in a model with search frictions and endogenous entry/exit; with them we share the amplification effect due to endogenous exit, but additionally we consider firm heterogeneity with respect to size to take into account the differential of job destruction due to exit between small and large firms. Colciago, Fasani and Rossi (2022) develop and estimate a model with search frictions and endogenous firm entry; they find that the success of the model in replicating the dynamics of macro-variables is due to a form of endogenous wage moderation spreading from the extensive margin of investment. Our model is a simpler version of theirs in that it features perfect rather than monopolistic competition and abstracts from capital and investment; moreover, contrary to their framework, we allow firms to

be different with respect to size. Our work is closely related to Elsby and Michaels (2013) who develop a heterogenous-firm model with search frictions and a constant mass of firms. Our model relies on their derivation of firm-level wage schedule, but additionally it allows for endogenous entry and exit. Recently, firm age has become a prevailing topic in labor macroeconomics modeling, for instance Acemoglu and Hawkins (2014) develop a model featuring non-linear hiring costs, firm dynamics and consider heterogeneity also in terms of age, and Sedláček (2020) uses a model with firm age, permanent heterogeneity in terms of size and non-linear hiring costs to study the effect of lost generations of firms on aggregate job flow dynamics.
In recent years there has been an increasing interest in modeling job-to-job transitions: Schaal (2017) uses a model of direct search with heterogeneous firms, endogenous entry/exit and workers voluntarily quitting their job for more valuable positions to study the effects of time-varying idiosyncratic volatility; Bilal et al. (2021) develop a similar model but with random search to study job reallocation and poaching; Elsby and Gottfries (2022) find that the on-the-job-search framework enriched with endogenous entry, exogenous exit and firm heterogeneity create imperfect labor market competition since hiring rates are increasing in MPL and decreasing in quit rates, giving rise to gradual mean reversion in marginal product among hiring firms. These models are richer than ours in many aspects; however, since our focus is specifically on the extensive margin of job destruction in relation to firm size, relying on a more parsimonious model does not imply a significant loss of explanatory power. At the same time our framework allows to match the differential in job destruction due to exit between small and large firms to replicate the empirical response of macrovariables and to assess the amplification effect due to endogenous firm dynamics.

Our paper is structured as follows. The first section reports the empirical facts; the second describes the model; the third shows the details of the calibration; the fourth reports the results; the fifth concludes.

2.2 Empirical analysis

In this section we address two facts:

1. job destruction due to firm exit determines the empirical difference in the long-run net job creation (NJC) rate between small and large firms. Specifically, by dropping job destruction due to firms' death all NJC rates by size become positive and quantitatively similar;

2. the response of job destruction(JD) due to exit of small firms to a technology shock is more countercyclical than that of large firms.

The annual series of average size, NJC by size, JD of small/large exiters and exit rate are computed from the Business Dynamics Statistics (BDS) database, based on the Longitudinal Business Database (LBD) and containing information on both establishment- and firm- level job flows by location, state, industry, size and age. This paper utilizes the 2018 release including firm-level observations from March 1983 to March 2018.
BDS classifies firms according to size following two alternatives:

1. *average size*, i.e. the average of firm's size between $t-1$ and t, that mitigates issues arising from *regression to the mean*[5];

2. *initial size*, i.e. the size of the firm at $t-1$; this strategy avoids the *reclassification bias*[6].

[5]As explained by Haltiwanger et al.(2013) and quoting Friedman (1992), it is "the most common fallacy in the statistical analysis of economic data." The main problem arising from this fallacy is that the classification of firms by a specific size at time t or $t-1$ leads to an inverse relationship between size and growth: if for instance a firm experienced a transitory positive shock yesterday it is less likely it will grow again today.

[6]Namely the fact that job flows might be incorrectly attributed to the right firms: if firms changes from one size class to another between two periods, attributing job flows to the previous or the following class may lead to very different results.

Entrants are classified in BDS according to their *final* size instead, since their size at $t-1$ is simply zero. Moscarini and Postel-Vinay (2012) assign them to the “small-firms” class since entrants are mostly small; contrary to them, in this paper entrants are distinguished from any other size class and put into a “size-0” group in order to separate job flows of incumbents from that of new firms. Moreover firms are classified following the *average size*'s strategy and the classes' cutoffs are chosen following Fort et al.(2013)'s classification.
Finally, job flow (JF) are defined as follows:

$$JF_{It} = \sum_{i \in I} JF_{it}$$

where I is an index for the size class and i identifies each firms' subgroup belonging to that class. The job flow rate jf is simply

$$jf_{It} = \frac{JF_{It}}{\sum_{i \in I} 1/2(L_{it} + L_{it-1})}$$

where L_{it} is firm i's size at time t.

2.2.1 Long-run NJC

Size Class	**NJC rates** All sample	**JD**^{ex}	**NJC rates** Survivors only
1-19	-0.0694	0.0816	0.0121
20-99	-0.0076	0.0308	0.0232
100-499	0.0102	0.0163	0.0265
500 or more	0.0068	0.0018	0.0086

Table 2.1: Annual long-run NJC rates from BDS: all sample vs survivors only.

Size Class	**NJC rates** All sample	**JD^{ex}**	**NJC rates** Survivors only
YOUNG (0-5 years)			
1-19	-0.0863	0.1348	0.0485
20-99	0.0121	0.0602	0.0723
100-499	0.0429	0.0460	0.0889
500 or more	0.0605	0.0061	0.0666
OLD 6+ years			
1-19	-0.0631	0.0574	-0.0057
20-99	-0.0144	0.0234	0.0090
100-499	0.0042	0.0126	0.0168
500 or more	0.0049	0.0012	0.0061

Table 2.2: Annual long-run NJC rates from BDS: all sample vs survivors only (conditional on age).

Net job creation (NJC) rates are heterogeneous across size; by looking at their annual long-run averages in the second column of table 2.1 it is evident that small firms destroy more jobs on net with respect to larger firms. The third column indicates that such heterogeneity is due to the different amount of jobs destroyed by firm death: small firms are more subject to death than larger firms, hence a sizeable fraction of their job destruction comes from exit. By dropping jobs lost due to firm deaths (last column) all NJC rates turn positive and quantitatively similar, suggesting that job destruction due to exit is a source of heterogeneity in employment growth rates according to size.

This evidence is robust to conditioning on age; table 2.2 shows that any difference in employment growth rates of firms with different size *conditional on age* is dampened focusing on businesses that survive[7]. Moreover notice that, although job destruction due to exit of young firms is on average larger than those of old firms, the fact that it decreases with size *independently on age* still holds[8]. These facts suggest that Gibrat's law is violated if the NJC rate distribution by size including both leaving and continuing businesses is considered: there seems to exist a direct relationship between size and growth such that large firms create jobs and small firms destroy them (on net). However by conditioning the analysis on firm's survival, Gibrat's law is re-established: any difference in growth rates is strongly dampened and there is no clear relationship between firm size and growth.

2.2.2 BVAR

This part presents empirical evidence on the response of job destruction due to exit of small firms relative to large firms to a technology shock, identified through a BVAR with sign restrictions. To this aim we take the difference of job destruction by exit between small and large firms, which we label as ΔJD^{ex}_{S-L}.

The series used in the estimation belong to two different datasets:

1. firm exit, entry and job destruction due to exit for each size class are retrieved from quarterly observations in the BED database 1993Q3 and 2018Q1[9];

2. unemployment rate, log of real GDP, wage and labor productivity at a quarterly frequency

[7]Small firms' rates are still negative only for old firms but closer to zero in the same way as all other firms.

[8]This is coherent with the finding that exit decreases with size even conditioning on age, as shown by Haltiwanger et al. (2013)

[9]Entry and exit rates in BED are observed at the establishment level. Following Rossi (2019) we use these data as proxies for firm-level entry and exit rates.

in the interval 1993Q3-2018Q1 are provided by Federal Reserve Economic Data (FRED).

We estimate a Bayesian VAR where technology shocks are identified through sign restrictions[10]. Specifically the responses labor productivity, GDP, wage and entry rate are restricted to be positive as shown in table 2.3; only for labor productivity the restriction is valid for the first 20 quarters as in Colciago, Fasani and Rossi (2022). No restrictions are imposed on the exit rate, ΔJD^{ex}_{S-L} and unemployment.

Figure 2.1 reports the estimated responses to a 1% increase in labor productivity. Unemployment's

Variable	**Sign**	**Quarters**
Labor productivity	+	1-20
$\mathrm{JD}^{ex}_{small-large}$	unrestricted	unrestricted
Entry rate	+	1
Exit rate	unrestricted	unrestricted
Wage	+	1
GDP	+	1
Unemployment	unrestricted	unrestricted

Table 2.3: Sign restrictions.

response is in line with the effects of an exogenous increase in technology: it slowly falls in the first seven quarters, then it reverts back to its pre-shock level. Both firm exit and ΔJD^{ex}_{S-L} decrease, then they overshoot their pre-shock level. ΔJD^{ex}_{S-L} reveals that small firms destroy less jobs by exiting than large firms when the economy is hit by a positive technology shock.

[10]We use the MATLAB toolbox developed by Cesa-Bianchi avalable at https://sites.google.com/site/ambropo/MatlabCodes.

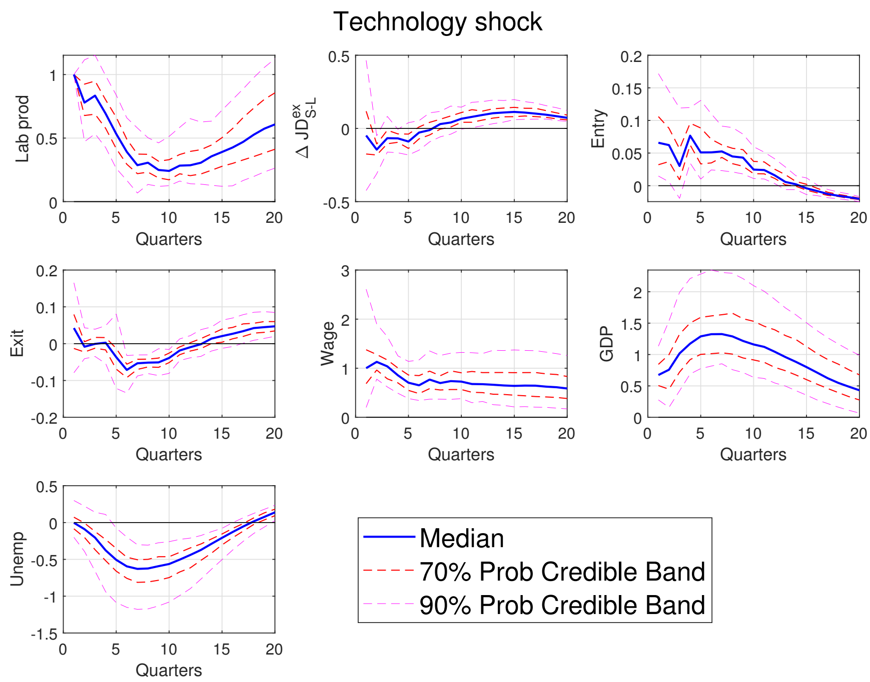

Figure 2.1: Estimated responses to a 1% increase in labor productivity.

To further appreciate this result table 2.4 reports the 90% probability credible interval estimates of conditional moments to a technology shock from the BVAR[11]. Conditional to the technology shock both exit and ΔJD^{ex}_{S-L} are almost as volatile as labor productivity, since their relative standard deviation is close to 1. Moreover they are both negatively correlated with the latter, and in particular ΔJD^{ex}_{S-L} is more strongly correlated than the exit rate.

[11]We follow Gali (1999) to convert our reduced form VAR representation into a structural MA representation, needed to compute the empirical conditional moments.

	Conditional moments	
	Exit	ΔJD^{ex}_{S-L}
Relative std deviation $std(variable)/std(prod)$	[0.8518 1.0324]	[1.4155 1.6001]
Correlation with lab prod	[-0.4048 -0.1502]	[-0.6783 -0.5915]

Table 2.4: Conditional moments (in parentheses the 90% probability credible interval estimates) from the BVAR (conditional to the technology shock).

2.3 The Model

We consider a model with heterogeneous firms and a representative households in discrete time and infinite horizon. The labor market is subject to search frictions as in Mortensen and Pissarides (1994); entry and exit are endogenous as in and Clementi and Palazzo (2016) and the entry cost function is similar to Gutierrez, Jones and Philippon (2021); finally, the representative household owns all firms in equilibrium and pays the entry cost.

2.3.1 Labor Market

Firms post vacancies by paying a linear cost κ per vacancy. The workforce is characterized by a constant mass I comprising employed and unemployed workers belonging to the representative household. Firms and unemployed workers meet according to a constant returns to scale matching function determining the mass of new hires - i.e. matches - through the rule

$$M_t = M(U_{t-1}, V_t) = \mu U_{t-1}^{1-\gamma} V_t^{\gamma} \tag{2.1}$$

where U_{t-1} and V_t are the unemployed and vacancies masses respectively. By defining $\theta_t = \frac{V_t}{U_{t-1}}$ as labor market tightness, the matching function can be rewritten as

$$M_t = \mu \theta_t^{1-\gamma} V_t$$

The probability that a vacancy is filled is $q_t = \frac{M_t}{V_t} q(\theta) = \mu \theta_t^{1-\gamma}$ and it is taken as given by all firms; the probability to find a job for an unemployed is $\phi_t = \frac{M_t}{U_{t-1}} = \mu \theta_t^{\gamma}$. As soon as unemployed are matched they become productive.

2.3.2 Firms

There is a continuous mass of perfectly competitive producers that are heterogeneous with respect to their size. Their productivity is made up by two components, one subject to idiosyncratic shocks, the other drawn upon entry from a time-invariant distribution $G(Z)$.

Firm j produces output y_{jt} according to the function $y_{jt} = A_t Z_j z_{jt} f(n_{jt})$ where n_{jt} is the amount of labor. Following Elsby and Michaels (2013) we choose $f(n) = n^{\alpha}$ so that the marginal product of labor declines with firm-level employment.

Following entry cost shocks, but before idiosyncratic shocks, firms draw a fixed cost of production c_o from a time-invariant, Log-normal distribution with parameters μ_o and σ_o: if their expected value is negative they exit; on the contrary, they continue to operate (or start to produce if they are new firms)[12].

[12]We follow Clementi and Palazzo (2016) in the design of the exit policy.

The value of a firm net of the fixed cost of production - before any idiosyncratic shock occurs - is

$$\tilde{F}_{jt} = \left(E^z(F_{jt}|z_{jt-1}) - \int_0^{E^z(F_{jt}|z_{jt-1})} xg(x)dx \right) G[E^z(F_{jt}|z_{jt-1})] \tag{2.2}$$

where $g()$ and $G()$ are respectively the density and cumulative distribution functions of the Log-normal distribution, $E^z(F_{jt}|z_{jt-1})$ is defined as the expected discounted sum of future profits conditional on past idiosyncratic shock z_{jt-1} and F_{jt} is the value of current profits plus the discounted sum of future profits after the idiosyncratic shock z_{jt} has taken place.

Conditional on survival, firms choose the optimal employment level maximizing their value F_{jt}; separations from workers happens at zero cost, whereas vacancy-posting is subjected to the cost κ per vacancy. After the matching is complete, production and wage setting take place simultaneously.

Timing is as follows:

- aggregate productivity shocks take place;
- potential entrants pay the entry cost and will enter effectively next period;
- all firms - both new entrants and incumbents - draw the fixed cost of production c_o from the distribution $G()$ and continue to operate whether their expected value is positive. If not, they exit;
- idiosyncratic productivity shocks hit incumbent firms; entrants draw the permanent productivity component and, as the rest of incumbent firms, are hit by the idiosyncratic shock;
- given their initial size - which is zero for entrants -, firms set their employment adjustment policies and, in case, post vacancies;
- after the matching process takes place, firms bargain the wage and produce.

Firm j's problem is of the form

$$F_{jt} = \max_{n,v} \left\{ A_t Z_j z_{jt} n^{\alpha} - w_{jt} n - \kappa v_{jt} + E_t \Lambda_{t,t+1} \tilde{F}_{jt+1} \right\}$$

subject to

$$n_{jt} = \begin{cases} n_{jt-1} + q_t v_{jt} \text{ if } n_{jt} > n_{jt-1} \\ n_{jt-1} - f_{jt} \text{ if } n_{jt} \leq n_{jt-1} \end{cases}$$

where $\Lambda_{t,t+1} = \beta E_t \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor and f_{jt} represents number of workers fired.

If the firm decides to hire new workers the firm-level employment dynamics reads as $n_t = n_{t-1} + q_t v_t$, hence the objective function can be maximized only in terms of n (to ease the notation, from now on we ignore the permanent idiosyncratic productivity component Z_j):

$$F_{jt} = \max_{n} \left\{ A_t z_{jt} n^{\alpha} - w_{jt} n - \frac{\kappa}{q_t}(n - n_{jt-1})\mathbf{1}^{+} + E_t \Lambda_{t,t+1} \tilde{F}_{jt+1} \right\} \tag{2.3}$$

where 1^{+} is an indicator function equal to one if $n > n_{t-1}$ and zero otherwise.

The first order condition of this problem delivers the Job Creation Condition (JCC)

$$\alpha A_t z_{jt} n_{jt}^{\alpha-1} - \frac{\partial w_{jt}}{\partial n_{jt}} n_{jt} - w_{jt} + E_t \Lambda_{t,t+1} D_{jt} = \frac{\kappa}{q_t}\mathbf{1}^{+} = J_{jt} \tag{2.4}$$

where $D_{jt} = \frac{\partial E_t \tilde{F}_{jt+1}}{\partial n_{jt}}$ denotes the marginal value of current employment choice on future profits. As explained by Elsby and Michaels (2013), because of diminishing marginal product of labor the firm can affect its wage. Given the existence of rents due to labor market frictions, firms and workers bargain over such rents to determine the optimal wage. Constant marginal product of labor implies that these rents are the same irrespective of firm size. On the other hand, diminishing marginal product of labor implies that these rents depend on firm-level employment; specifically such rents decrease as n increases. This means that a firm can reduce the cost of labor - namely the wage - by increasing its labor force.

Finally, let us consider D_{jt} again. It can be shown[13] that this can be written as

$$D_{jt} = E_t \Bigg[P^{na}_{jt+1} \Bigg(\alpha A_{t+1} z_{jt+1} n^{\alpha-1}_{jt} - \frac{\partial w_{jt+1}}{\partial n_{jt}} n_{jt} - w_{jt+1} + E_{t+1} \Lambda_{t+1,t+2} J_{jt+1} \Bigg) + P^{h}_{jt+1} \frac{\kappa}{q_{t+1}} \Bigg] \tag{2.5}$$

where P^{na}_{jt+1} and P^{h}_{jt+1} denotes the probabilities of non-adjustment and hiring of firm j depending on its future employment choice and shock realizations. The intuition is as follows. For given values of z_{jt+1} firm j will either freeze employment, separate from some of its workers, hire new workers or exit in $t+1$ with positive probability. In case it decides to keep its employment level unchanged, the marginal effect of the current employment choice will continue to affect firm j's value until the firm will find optimal to change it. At the same time if the firm finds profitable to increase its labor force next period, current employment will decrease the cost of vacancy-posting at $t+1$.

2.3.3 Endogenous entry

Each period entry is determined by the free-entry condition

$$E_t \tilde{F}^{e}_{jt+1} = c_{et} \tag{2.6}$$

stating that the expected value of entrants $E_t \tilde{F}^{e}_{jt+1}$ - whose expectation is computed over $G(Z_j)$ - must be equal to an entry cost c_{et} of the form

$$c_{et} = \psi (N^{e}_{t})^{\xi} \tag{2.7}$$

[13]For more details see the appendix in Elsby and Michaels (2013)'s paper.

Notice that potential entrants are *ex-ante* the same, since only upon paying the entry cost will draw their permanent productivity component Z_j from the time-invariant distribution $G()$ in the following period; only then they become productive.

Given entry, the mass of firms follows the law of motion

$$N_t = (1-\delta)N^e_{t-1} + N^{surv}_{t-1} = \sum_j N_{jt} \tag{2.8}$$

where N^{surv}_{t-1} denotes the mass of surviving firms from endogenous and exogenous exit.

2.3.4 Household

There is a representative risk-averse household maximizing its life-time utility

$$\mathbf{U}_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{2.9}$$

subject to the budget constraint

$$\begin{aligned} C_t + B_t + x_{t+1}F_tN_t + x^e_{t+1}F^e_tN^e_t = & \\ & \sum_j w_{jt}L_{jt} + b(I-L_t) + (\Pi_t + F_t)N^{surv}_{t-1}x_t + \\ & (\Pi^e_t + F_t)N^e_{t-1}x^e_t + B_{t-1}R_t - T_t \end{aligned} \tag{2.10}$$

the aggregate employment law of motion

$$L_t = (1-s^{tot}_t)L_{t-1} + \phi_t(I - L_{t-1}) \tag{2.11}$$

and aggregate labor supply

$$\sum_j L_{jt} = L_t \tag{2.12}$$

where

- B_t is a state-contingent asset exchanged by members of the household
- x_{t+1} and x^e_{t+1} are shares in a mutual fund of incumbents and new entrants respectively
- F_t and F^e_t are the average value of incumbents and entrants respectively
- b is unemployment benefit and T_t is government transfers to finance b
- Π_t and Π^e_t are dividends from incumbents and entrants, that is revenues minus labor, vacancies and fixed costs
- L_{jt} is the amount of labor provided to firm j and $L_t = I - U_t$ is aggregate employment
- s^{tot}_t is the total separation rate comprising both firings and exit.

The household maximizes its lifetime utility subject to the previous constraints through $\{C_t, L_{jt}, B_t, x_{t+1}, x^e_{t+1}\}^{\infty}_{t=0}$. These are chosen by solving the first-order conditions of the problem

$$F_t N_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\big(\Pi_{t+1} + F_{t+1}\big) N^{surv}_t \tag{2.13}$$

$$F^e_t N^e_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\big(\Pi^e_{t+1} + F_{t+1}\big) N^e_{t-1} \tag{2.14}$$

$$\lambda_t = u(C_t) \tag{2.15}$$

$$\frac{1}{R_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{2.16}$$

$$H_{jt} = (w_{jt} - b)\lambda_t + \beta E_t H_{jt+1}\big(1 - s^{tot}_{t+1} - \phi_{t+1}\big) \tag{2.17}$$

Equation 2.17 describes how the household's shadow value H_t of an additional worker hired in firm j is related to the increase in utility given by being employed $w_{jt}\lambda_t$, the decrease in utility due to the loss of unemployment benefit $b\lambda_t$ and the continuation utility value. For our purpose it might be useful to express it as[14]

$$H_{jt} = W_{jt} - \Gamma_t \tag{2.18}$$

where W_{jt} and Γ_t denotes the value of employment at firm j and unemployment respectively. These are equal to

$$\begin{aligned} W_{jt} = \lambda_t w_{jt} + \beta E_t \lambda_{t+1} \Bigg\{ \delta \Gamma_{t+1} + (1-\delta) \Bigg[P^f_{jt+1} \Big(\tilde{s}_{jt+1} \Gamma_{t+1} + (1-\tilde{s}_{jt+1}) W^f_{jt+1} \Big) + \\ P^{na}_{jt+1} W^{na}_{jt+1} + P^h_{jt+1} W^h_{jt+1} \Bigg] \Bigg\} \end{aligned} \tag{2.19}$$

$$\Gamma_{jt} = b\lambda_t + \beta E_t \lambda_{t+1} \Bigg[(1-\phi_{t+1}) \Gamma_{t+1} + \phi_{t+1} \sum_j P^h_{jt+1} W_{jt+1} \Bigg] \tag{2.20}$$

where P^f_{jt+1}, P^{na}_{jt}, P^h_{jt} are the probabilities of firing, non-adjustment and hiring of firm j. Equation 2.19 describes the value of employment to a worker; it takes into account the probability of losing the job due to exogenous firm exit and the probability that, conditional on survival, the firm will decide to separate. $\tilde{s}_{jt}$ represents the probability that employer j will fire that particular worker conditional on the probability that it is optimal to decrease firm j's workforce. Finally, the worker distinguishes between W^f_{jt}, W^{na}_{jt} and W^h_{jt} - that are the wages chosen when the firm separate from workers, does not adjust or hires new workers - because they will differ according to the employment level chosen by the firm, since the employer and its employees will bargain over wages every time an employment adjustment takes place.

The value of unemployment is simpler; equation 2.20 states that an unemployed worker must take into account expected value of being hired by any firm in the economy plus the expected value of

[14]Here we follow Elsby and Michaels (2013).

remain unemployed.

2.3.5 Wage bargaining

Firm j and its workers maximize the joint surplus

$$\max_{w_{jt}} J_{jt}^{1-\eta} H_{jt}^{\eta} \tag{2.21}$$

whose first-order condition delivers

$$\eta J_{jt} = (1-\eta)\frac{H_{jt}}{\lambda_t} \tag{2.22}$$

by solving which we find the wage bargaining equation

$$w_{jt} = \eta\left[\frac{\alpha A_t z_{jt} n_{jt}^{\alpha-1}}{1-\eta(1-\alpha)} + \beta\kappa E_t\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + (1-\eta)b \tag{2.23}$$

2.3.6 Aggregation, market clearing and equilibrium

In equilibrium the representative household owns the whole portfolio of firms, hence $x_t = x_t^e = 1$. Moreover

- labor demand equals labor supply

$$\sum_j L_{jt} = \sum_j n_{jt} N_{jt} \tag{2.24}$$

- good market clearing implies

$$C_t + N_t^e c_{et} = Y_t - \kappa V_t - \sum_j N_{jt}\int_0^{E_{t-1}F_{jt}} xg(x)dx \tag{2.25}$$

  where $Y_t = \sum_j y_{jt}$, $V_t = \sum_j v_{jt}$ and the last term represents the sum of all fixed costs drawn by firms that choose to continue to operate

- firms set their employment policy function by solving JCC (2.4)
- the wage is set in order to split the joint surplus of a match by satisfying equation 2.23
- labor market tightness $\theta_t = \frac{V_t}{U_{t-1}}$ is determined in equilibrium and taken as given by firms and the representative household
- the dynamics of aggregate unemployment follows the rule

$$U_t = (1-\phi_t)U_{t-1} + s_t^{tot} L_{t-1} \tag{2.26}$$

- the aggregate separation rate is determined by firm-level exit and firing

$$s_t^{tot} = \frac{\left[\sum_j n_{jt} N_{jt-1}^{exit} + \sum_j |n_{jt} - n_{jt-1}| N_{jt}^{firing}\right]}{L_{t-1}} \tag{2.27}$$

  where N_{jt-1}^{exit} and N_{jt}^{firing} denote the mass of firms that exit endogenously and that choose to separate from their workers respectively.

2.3.7 Numerical solution

The steady-state is solved numerically by local approximation of the value function F_{jt}. The algorithm proceeds as follows:

- by using the discretized grid of employment choices and idiosyncratic productivity[15] we find a first solution of the problem by iterating on the Bellman equation;
- if such solution does not hit the boundaries of the employment grid, continue; otherwise enlarge the grid and come back to the previous point;

[15]We use the Rouwenhorst's method to discretize AR(1) processs.

- for each idiosyncratic productivity's value, regress the objective function of the problem only on the employment grid nodes immediately before and after the solution found in the previous step; the regressors are employment, employment squared and a constant. Specifically
  - given the objective function $g_z(n)$, consider only $g_z(n_{k-1})$ and $g_z(n_{k+1})$ such that n_k is the discrete maximizer of g_z on the grid and $n_{k-1} \leq n_k \leq n_{k+1}$;
  - compute the OLS coefficients of

    $$g_z(n) = \beta_0 + \beta_1 n + \beta_2 n^2$$

    by using the three nodes n_{k-1},n_k and n_{k+1} from the previous point;
  - take the first derivative of the interpolated g_z with respect to n and find the value of n that maximizes g_z by solving the first-order condition

    $$\tilde{n} = -\frac{\beta_1}{2\beta_2}$$

  - calculate the objective function implied by $\tilde{n}$:

    $$\tilde{g_z} = \beta_0 + \beta_1 \tilde{n} + \beta_2 \tilde{n}^2$$

- approximate F_j with $\tilde{g_z}$ computed for the firm with initial size equal to n_j;
- iterate until convergence of the value function.

As for solving the dynamic stochastic equilibrium, we follow the method of Reiter (2009)[16]: stochastic aggregate dynamics are compute by linearization for each grid point around the steady state. In

[16] We use the MATLAB code developed by Costain and Nakov (2011).

this way the Bellman equation is treated not as a function but as a system of difference equations. The model can be summarized as

$$E_t\mathbf{F}_t\Bigg(\mathbf{X}_t, \mathbf{X}_{t+1}, \zeta_t, \zeta_{t+1}\Bigg) = 0 \tag{2.28}$$

where $\mathbf{X}_t$ comprises all model variables, both firm-level and aggregate. In steady state equation 2.28 reads

$$E_t\mathbf{F}\Bigg(\mathbf{X}, \mathbf{X}, 0, 0\Bigg) = 0 \tag{2.29}$$

By computing the Jacobian of 2.28 evaluated at the steady state equilibrium, for sufficiently small aggregate shocks the model can be approximated linearly as

$$E_t\mathcal{A}\big(\mathbf{X}_{t+1} - \mathbf{X}\big) + \mathcal{B}\big(\mathbf{X}_t - \mathbf{X}\big) + \mathcal{C}\zeta_{t+1} + \mathcal{D}\zeta_t \tag{2.30}$$

where

- $\mathcal{A} = \frac{\partial \mathbf{F}_t}{\partial \mathbf{X}_{t+1}}$
- $\mathcal{B} = \frac{\partial \mathbf{F}_t}{\partial \mathbf{X}_t}$
- $\mathcal{C} = \frac{\partial \mathbf{F}_t}{\partial \zeta_{t+1}}$
- $\mathcal{D} = \frac{\partial \mathbf{F}_t}{\partial \zeta_t}$

This form allows the model to be solved through the QZ decomposition method outlined by Klein (2000). In this way we avoid any bounded rationality assumption as in the Krusell and Smith (1998)'s method.

2.4 Calibration

The model follows a quarterly calibration. We set $\beta = 0.99$ and $\alpha = 0.65$ as standard in the literature. As for labor market parameters, we normalize steady-state θ to one and set the matching elasticity $\gamma = 0.72$ as in Shimer (2005). Moreover we let the worker's bargaining power η to be equal to γ in order to satisfy the Hosios condition. The matching efficiency is set in order to provide a job-finding probability equal to 0.7 as in Blanchard and Galí (2010). Unemployment benefit b is chosen in order to provide a replacement ratio equal to 40% of the average wage as in Shimer (2005), whereas the marginal cost of vacancies κ is set so that the total cost of vacancies is 1% of GDP as in Blanchard and Galí (2010).

Turning to the exit policy parameters, μ_{of} and σ_o are chosen in order to match jointly a quarterly exit rate equal to 2.5% and the exit rate differential between small (1-19) and large (500+) firms[17]. The elasticity of entrants ξ in the entry cost is equal to 1.5 as estimated by Gutierrez Gallardo, Jones and Philippon (2019), whereas the parameter ψ is set so that the entry cost is 5% of GDP. Firm-level permanent productivity components Z_j and the corresponding frequencies $F(Z_j)$ are parameterized so as to match the distributions of employment share and firm share distributions according to size in the U.S. [18]

The standard deviation of idiosyncratic productivity shocks σ_z is set to target an overall separation rate of 9% which is the median value of the rage 8% - 10% reported by Hall (1995); the persistence of idiosyncratic shock ρ_z is chosen to match the average NJC rate[19]. Finally, the standard deviation σ_A of the aggregate productivity shock is chosen to match to the response on impact of labor productivity in the BVAR whereas the persistence ρ_A is equal to 0.9 as in Rossi (2019).

[17] Source: Business Dynamics Statistics (BDS) database.

[18] Source: BDS dataset, observations between 1978-2018.

[19] Ibidem.

The calibration is summarized in table 2.5.

Parameter/SS value	Definition	Value	Source/Target
β	Discount factor	0.99	Annual interest rate = 4%
α	Returns to scale parameter	0.65	Labor share
θ	Labor market tightness	1	Normalization, Shimer (2005)
γ	Matching elasticity	0.72	Shimer (2005)'s estimate
μ	Matching efficiency	0.7	$\phi = 0.7$, BG (2010)
η	Workers' bargaining power	$=\gamma$	Hosios condition
b	Unemployment benefit	1.22	$\frac{b}{w_{avg}} = 40\%$ Shimer (2005)
κ	Vacancy marginal cost	0.7	$\frac{\kappa V}{GDP} = 1\%$ BG (2010)
ξ	Elasticity of entrants	1.5	GJP (2019)
ψ	Entry cost parameter	1.2	$\frac{c_e}{GDP} = 5\%$
μ_o	Log-norm. par.	-8.2866	exit = 2.5% (BDS)
σ_o	Log-norm. par.	4.56;	exit differential = 2.05%
Z_j	Permanent productivity component	[1.2; 2.5; 4.95; 10.9]	Employment share (BDS)
$F(Z_j)$	Probability $Z = Z_j$	[0.9573; 0.0393; 0.0026; 0.0009]	Firm share (BDS)
σ_z	Std deviation id shocks	0.075	$s^{tot} = 9\%$, Hall (1995)
ρ_z	Persistence id shocks	0.87	avg $NJCR$ (BDS)
σ_A	Std dev tech shock	0.26/100	BVAR Lab prod
ρ_A	Persistence tech shock	0.9	Rossi (2019)

Table 2.5: Quarterly calibration.

2.5 Results

2.5.1 Baseline model: exit and NJC rates

Table 2.6 reports the steady state distributions of firm size (second column), employment share (third column) and NJC rates by size (fourth column) of the model, compared with the corresponding quarterly long-run distributions computed through observations of firm-level job flows and firm dynamics from the Business Dynamics Statistics (BDS) database between 1983 and 2018[20].

The theoretical NJC rates with respect to size are close to the data: small firms are characterized by

Size Class	**Firm Share**	**Employment Share**	**NJC rates**
Model			
0	0.0276	0.0071	2
1-19	0.7906	0.2226	-0.0141
20-99	0.1502	0.1438	-0.0011
100-499	0.0196	0.0965	0.0048
500 or more	0.0120	0.5301	0.0065
BDS data (quarterly)			
0	0.0267	0.0260	2
1-19	0.8517	0.1803	-0.0174
20-99	0.1006	0.1706	-0.0017
100-499	0.0168	0.1378	0.0027
500 or more	0.0042	0.4854	0.0018

Table 2.6: SS distributions: model vs data.

[20] In order to approximate the quarterly series starting from the annual, we impose that in each quarter the job flow is always the same, so that $JF^{quarterly} = JF^{annual}/4$.

negative employment growth rates, whereas all other firms have rates close to zero. This happens because small firms exit endogenously more often than the others, hence their job destruction due to exit is larger. By construction a model with exogenous exit will provide counterfactual positive NJC rates for each size class.

2.5.2 Baseline model: response to technology shocks

The theoretical responses to a 1% increase in aggregate productivity reported in figure 2.2 are in line with the BVAR IRFs: wage and GDP are procyclical, whereas unemployment slowly decreases and the entry rate rises. Exit and job destruction due to exit are both falls on impact: following a positive technology shock fewer firms leave the market and the probability that small unproductive firms remain operative increases, reducing the amount of jobs destroyed by their exit.
Notice that the response of ΔJD^{ex}_{S-L} overshoots its pre-shock level some periods after the shock similarly to the estimated response. Such behavior depends on the increase in labor market tightness (see figure 2.3) which is procyclical as standard in search models: as θ_t increases, labor costs - wage and hiring cost - increase as well; initially the rise in labor costs is offset by the increase in aggregate productivity, however when the latter starts to revert to its steady state value the cost of a tight labor market reduces the benefit from hiring new workers. This is particularly true for those small unproductive firms who entered/survived thanks to favourable macroeconomic conditions due to the exogenous increase in aggregate productivity and now that the economy reverts back to the pre-shock state are forced to exit.

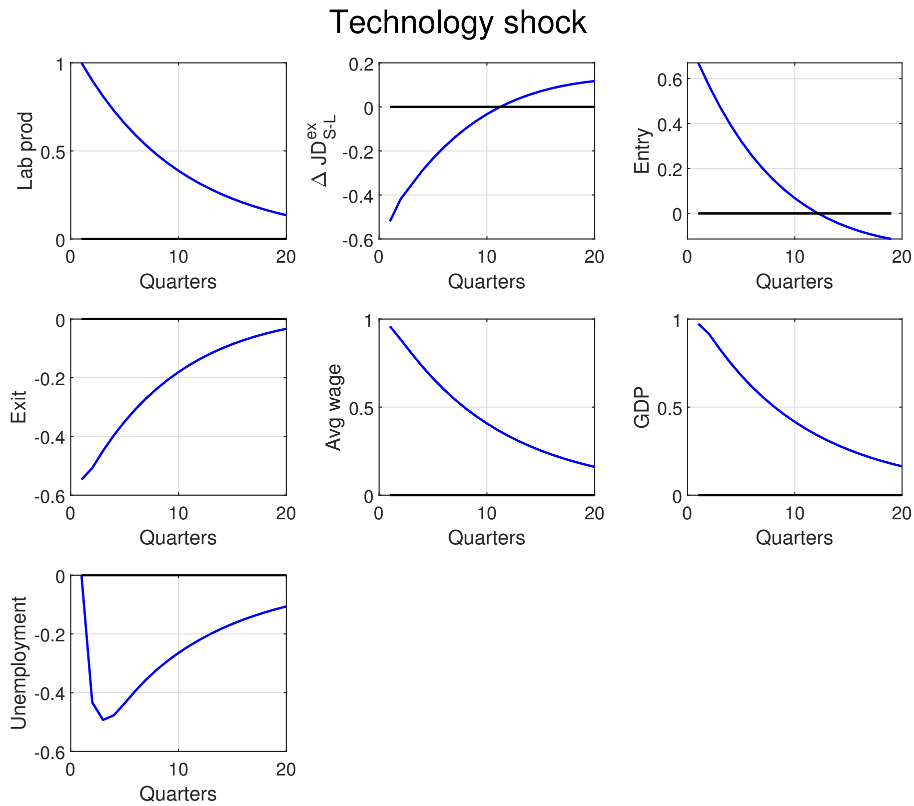

Figure 2.2: Theoretical responses to a 1% increase in aggregate productivity.

2.5.3 The role of endogenous exit

In this section we investigate the role of endogenous exit for the amplification of technology shocks. To this end we build a version of the model with constant exit rates, no differences in long-run NJC rates according to size and costly entry. In this way we can focus on the effects of exit only, leaving

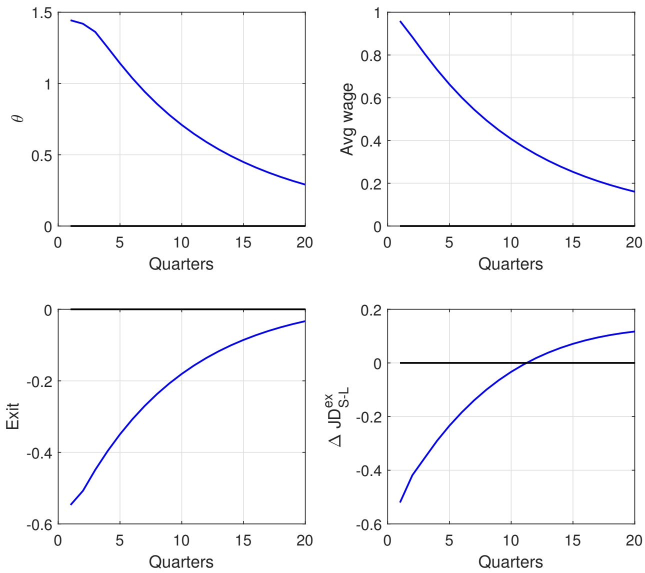

Figure 2.3

entry's contribution to unemployment's response outside. Additionally, we consider a standard model of the labor market without any firm dynamics to use it as benchmark for the other two versions.

Table 2.7 reports in the first column the 90% probability credible interval estimates of conditional moments to a technology shock from our BVAR, while the subsequent columns report moments from the simulations of the three models.

Clear differences emerge between our model and the other two by looking at the conditional moments of exit and ΔJS^{ex}_{S-L}. Our BVAR estimates indicate that their conditional standard deviation is comparable with that of productivity since their ratio is close to one; only the model with endogenous exit is able to address this fact, whereas the other two models cannot by construction.

Moreover notice that these two variables are significantly countercyclical conditional to technology shocks, since their correlation with productivity is negative even at the upper bound of the 90% credibility band.

Figure 2.4 compares the response of unemployment under the three alternatives and suggests that

	Exit			
	Data	**Rep firm**	**Exogenous exit**	**Endogenous exit**
Relative std deviation $std(variable)/std(prod)$	[0.8518 1.0324]	0	0	0.9531
Correlation with lab prod	[-0.4048 -0.1502]	0	0	-0.8504
	ΔJD^{ex}_{S-L}			
	Data	**Rep firm**	**Exogenous exit**	**Endogenous exit**
Relative std deviation $std(variable)/std(prod)$	[1.4155 1.6001]	0	0	0.9264
Correlation with lab prod	[-0.6783 -0.5915]	0	0	-0.7415

Table 2.7: Conditional moments. The first column shows 90% probability credible interval estimates; second, third and fourth columns reports moments from simulations of each model version.

the model with endogenous exit amplifies the response of unemployment to technology shocks. Indeed the other two models lack of an amplification channel, namely the number of jobs saved from destruction due to exit thanks to the exogenous increase in aggregate productivity; since these jobs are filled by unemployed workers, total unemployment will decrease even more than in the models where exit is acyclical.

To understand the role of endogenous exit and heterogeneity in exit rates let us examine figure 2.5, showing the responses of unemployment and the difference of job destruction due to exit between small and large firms from our model (first row) and data (second row). On impact, the probability of drawing a fixed cost of production larger than the firm's value decreases because the latter, being an increasing linear function of aggregate productivity, increases. The right-hand side panel of figure 2.4 shows that such effect is different depending on firm size. Following the shock small firms destroy less jobs by exiting than large firms do and the subsequent reduction in overall job destruction makes unemployment falling even more. Without endogenous exit, all firms leave the market at the same rate independently on their size, and no change in job destruction due to exit takes place following aggregate shocks.

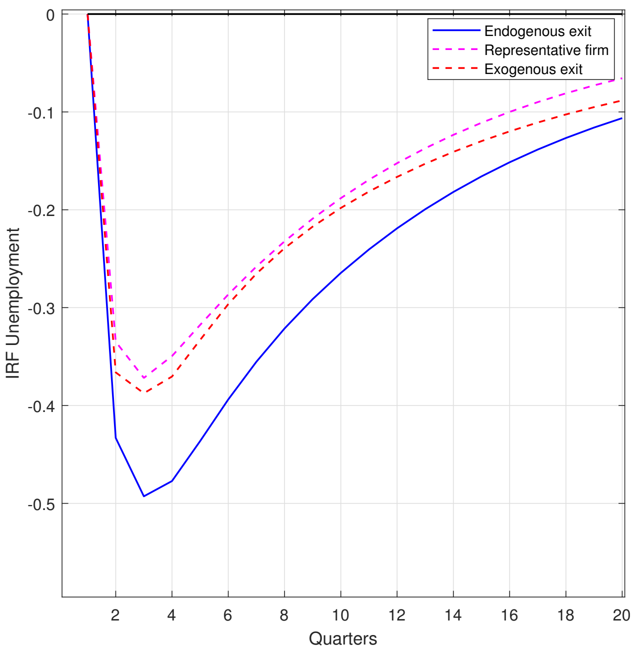

Figure 2.4: Comparison endogenous vs exogenous exit vs representative-firm model.

2.6 Conclusion

We develop a heterogeneous-firm model with search frictions and endogenous entry/exit dynamics reproducing the fact that, in the long run, small firms are characterized by negative employment growth rate because they are more exposed to exit than all other firms. Moreover the model replicates the empirical response of job destruction from exit of small firms relative to large firms to a technology shock, along with the their conditional standard deviation and correlation with productivity, contrary to models with exogenous exit.

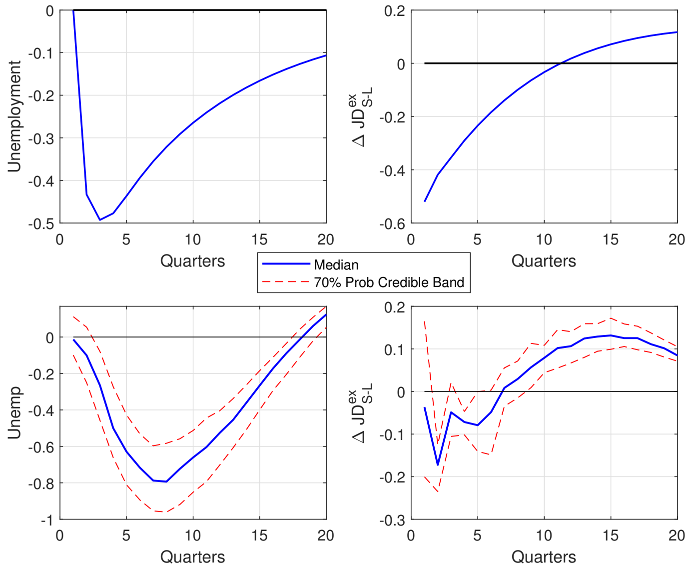

Figure 2.5: IRF of unemployment (left) and ΔJD^{ex}_{S-L} (right) in the model with endogenous exit (first row: model; second row: data).

Finally we show that not only entry but also exit is a viable source of amplification of technology shocks and that in our model job destruction due to exit is responsible for the larger unemployment's response.

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2.A Appendix: Conditional Moments

To compute conditional moments to a technology shock we follow Gali (1999) and convert the reduced form VAR representation

$$B(L)\boldsymbol{x_t} = \nu_t \tag{2.31}$$

into

$$\boldsymbol{x_t} = C(L)\epsilon_t \tag{2.32}$$

where $B(L)$ and $C(L)$ are lag polynomials (the latter of infinite order), $\boldsymbol{x_t}$ is the vector of the detrended variables, ν_t collects the innovations and vector ϵ_t collects structural shocks. Moreover $B(0) = I$, $E_t\nu_t\nu_t' = \Sigma$ and $E_t\nu_t\boldsymbol{x_{t-j}} = 0$ for $j = 1, 2, 3....$

The conditional variance of variable x_j to shock i is written as

$$\text{var}(x_j|i) = \sum_{t=0}^{\infty}(C_t^{ij})^2 \tag{2.33}$$

where C_t^{ij} denotes the coefficient of polynomial $C(L)$ with respect to structural shock i at time t for variable x_j.

The conditional correlation coefficient of variable x_j and x_k to shock i is expressed as

$$\rho(x_j, x_k) = \frac{\sum_{t=0}^{\infty} C_t^{ij} C_t^{ik}}{\sqrt{\text{var}(x_j|i)\text{var}(x_k|i)}} \tag{2.34}$$

As in Gali (1999) we assume that each innovation is an independent linear combination of each structural shock through the relation

$$\nu_t = S\epsilon_t \tag{2.35}$$

Given these elements, the polynomial $C(L)$ can be retrieved by inverting $B(L)$ and multiplying it by matrix S as follows:

$$C(L) = B(L)^{-1}S \tag{2.36}$$

where $B(L)^{-1}$ denotes the inverse of $B(L)$. To estimate S, Gali (1999) observes that $C(1)$ is the Cholesky factor of $B(1)^{-1}\Sigma B(1)^{-1'}$, hence S is equal to $B(1)$ chol $B(1)^{-1}\Sigma B(1)^{-1'}$. In our framework we use the candidate S satisfying the sign restrictions for each draw in the estimation. We then use all draws to create interval estimates of the conditional moments.

2.B Appendix: BED and BDS

Following Rossi (2019) we use entry and exit rates from BED at the establishment level as proxies for the entry and exit rates at the firm level. Figure 2.6 reports their annual series from the two different datasets, showing analogous dynamics for both.

2.C Appendix: BVAR robustness checks

This section reports the estimated impulse responses for different specifications of the BVAR.

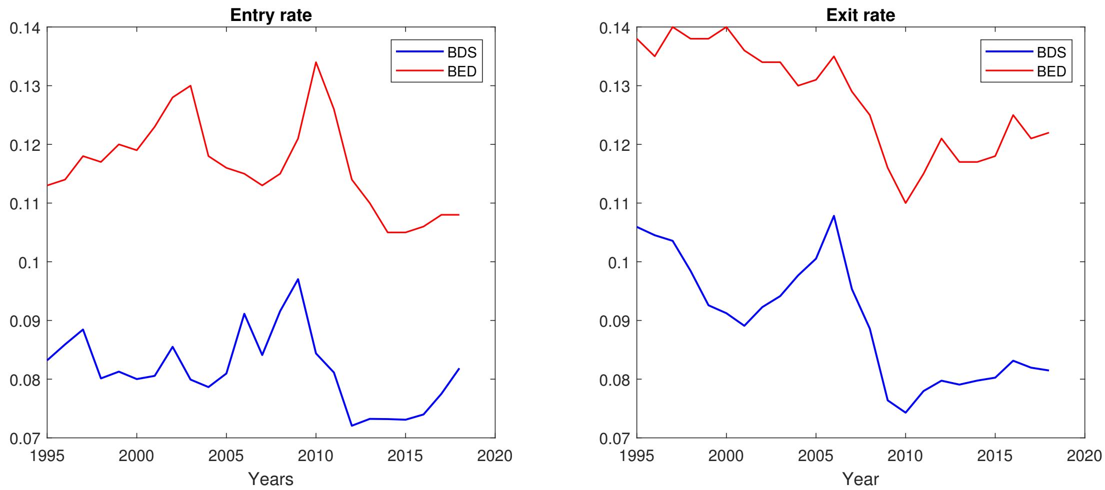

Figure 2.6: Entry (left) and exit (right) rates series from BDS (blue) and BED (red) datasets between 1995 and 2018.

2.C.1 Time interval (1993Q3-2008Q1)

The sample is restricted to the time interval 1993Q3-2008Q1. Responses are shown in figure 2.7.

2.C.2 HP filter

In this subsection are smoothed through a one-sided HP filter. Results are shown in figure 2.8.

2.C.3 Sign restriction

In this subsection sign restrictions are imposed only on impact to labor productivity, wage and

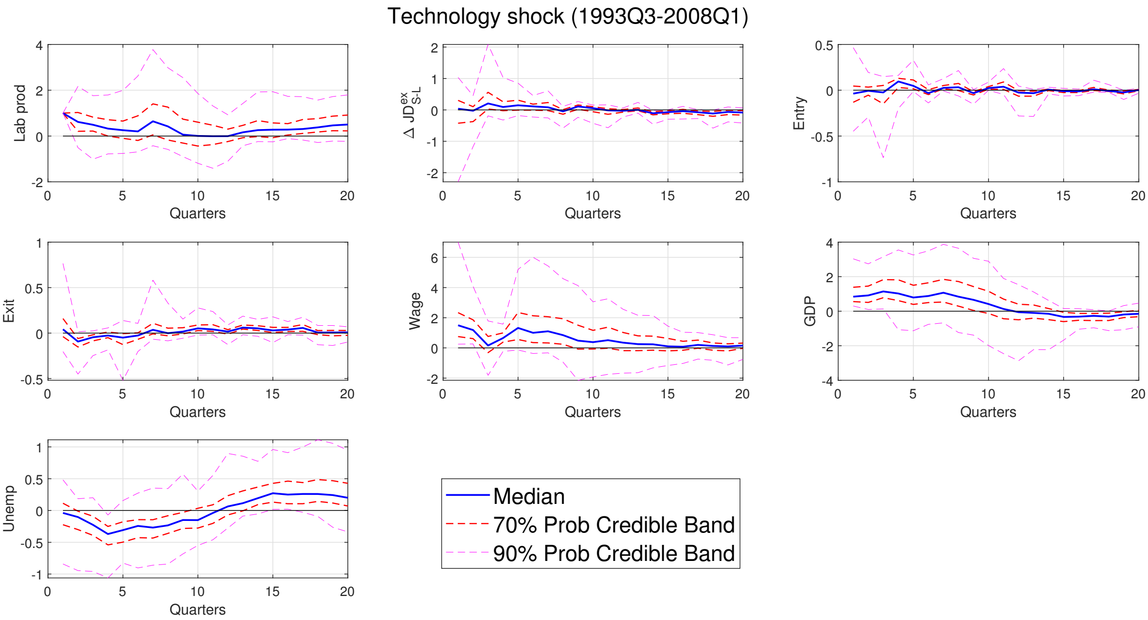

Figure 2.7: Estimated responses to a 1% technology shock (1993Q3-2008Q1).

GDP. Results are shown in figure 2.9.

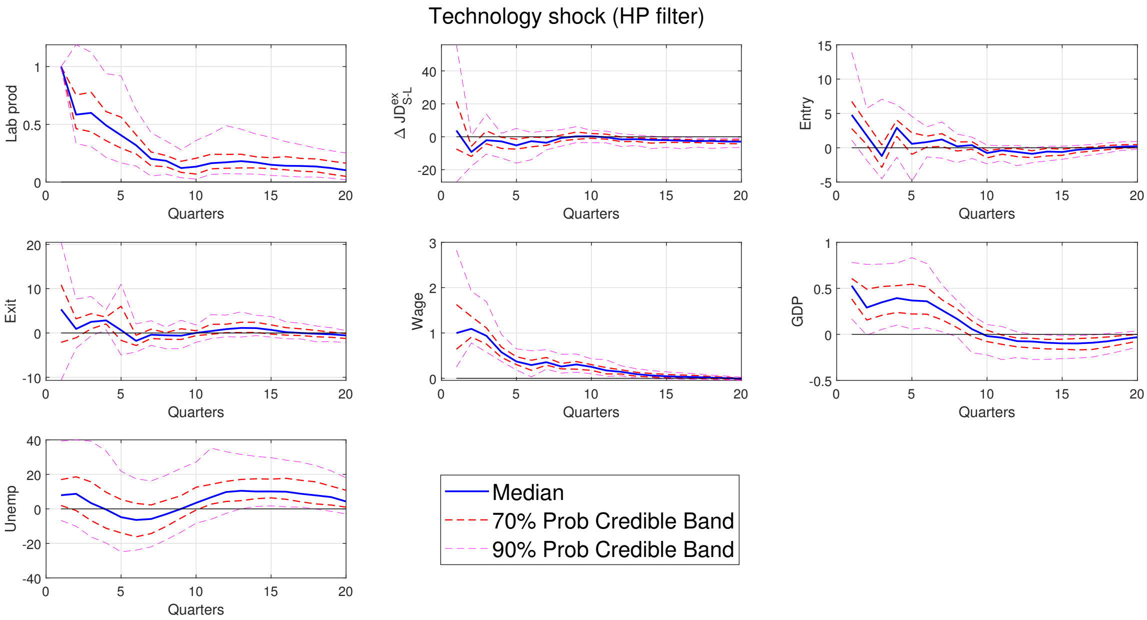

Figure 2.8: Estimated responses to a 1% technology shock (one-sided HP filter).

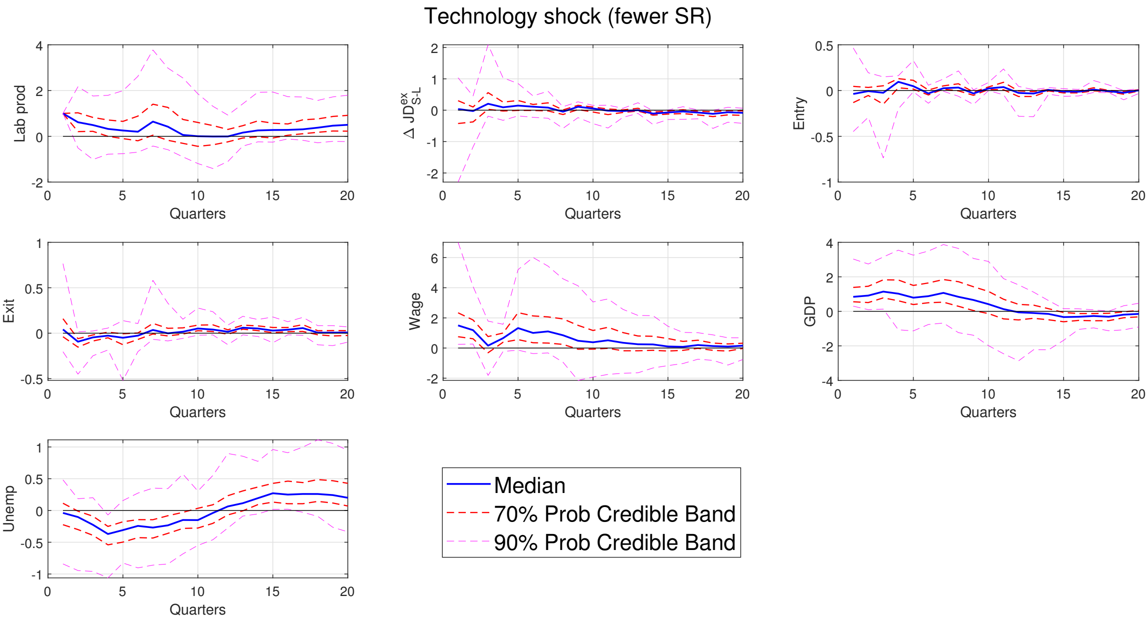

Figure 2.9: Estimated responses to a 1% technology shock.