# The Green Sequencing and Routing Problem * 

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#### Abstract

The paper deals with a sequencing and routing problem originated by a real-world application context. The problem consists in defining the best sequence of locations to visit within a warehouse for the storage and/or retrieval of a given set of items during a specified time horizon, by considering some specific requirements and operating policies which are typical of the kind of warehouse under study. A fleet composed of both electric (i.e., equipped with a lithium-ion battery) and conventional (i.e., with internal combustion engine) forklifts is considered. We model the problem in terms of constrained multicommodity flows on a space-time network, and we extend a matheuristic approach proposed for the case of only conventional vehicles. Preliminary computational results are also presented.


Keywords: Green Logistics • Warehouse management • Matheuristic.

## 1 Introduction

Warehouses are an essential component of any supply chain. Warehousing concerns receiving, storing, order picking, and shipping of goods. The large majority of the warehouses (especially in Western Europe, according to [8]) are operated pursuing the picker-to-parts principle, i.e., workers walk or drive through the warehouse to perform either picking or put-away operations. The former concern the movement of items from the storage locations towards the output point of the warehouse to respond to a customer order, the latter instead concern the movement of items from the input points of the warehouse towards the storage area to store the items in the assigned storage locations. Picking and put-away are recognized as the most labor and time consuming internal logistics processes, and their careful and efficient planning plays a major role in improving productivity and decreasing the operational costs of a warehouse.

The problem addressing this issue is known in the literature as Sequencing and Routing Problem (SRP). Precisely, the SRP has the scope of defining the

[^0]most efficient sequence of operations to move items within the warehouse to perform order picking and put-away operations, by typically minimizing the total material handling cost or travel efforts (measured either in time or distance traveled by the workers), and respecting some additional and peculiar requirements related to the application context [8].

Warehouses are also major contributors of greenhouse gas emissions in supply chains, especially raised by the use of diesel forklifts [3]. Consequently, besides traditional operational and economic objectives, increasing attention is now given by companies to usually overlooked aspects, such as sustainability and environmental-friendly issues in warehouse management. The Green SRP is thus emerging as a new topic of research. It is a variant of the classic SRP where some electric vehicles perform operations within the warehouse. Although the use of electric forklifts has been recognized as a way to both reduce long term management costs [5] and improve healthiness for workers (e.g., reduced noise, better local air quality), it contributes to increasing the problem complexity since peculiar activities, such as the scheduling of recharging periods, as well as the limited autonomy of the vehicles, need to be considered when planning ordinary picking and put-away operations.

A very few contributions discussing SRPs with electric forklifts are available in the literature, highlighting the novelty of the topic. In [7], picking and putaway operations need to be planned by using a fleet of electric forklifts, whose battery may be replaced once the state of charge is too low. A similar problem is discussed in [2], where besides battery replacement also the recharging process of the batteries is considered. Both problems are formulated as job-shop problems.

Recently, [6] addressed a SRP related to a large production site of an Italian company. The SRP is characterized by some specific requirements, originated by the layout design of the warehouse, and also by the particular kind of products stocked, i.e., tissue products for sanitary and domestic use. Conventional vehicles, i.e., with an internal combustion engine, are considered to perform picking and put-away operations. In this paper, we investigate the green extension of the above mentioned problem, where some of the vehicles are electric and equipped with a lithium-ion battery. This technology is considered as the most promising for the near future by the majority of literary sources, for its high efficiency and long lifespan [1]. Indeed, it is also the technology adopted in the studied warehouse.

The paper is organized as follows. The Green SRP is presented in Section 2. The main features of the mathematical model proposed for its formulation and an overview of the matheuristic approach used to solve it are described in Sections 3 and 4, respectively. Section 5 presents some preliminary numerical results on the Green SRP. Finally, Section 6 concludes the paper and identifies some future research directions.

## 2 The Green SRP

The addressed problem is defined in a warehouse characterized by two disjoint areas. The first area is a transit zone connecting the input points of the warehouse, where items wait to be stored, to the storage area. The second area instead is the storage area, where storage locations are situated together with a collection area where, according to the pick-and-sort policy followed, retrieved items are gathered to establish order integrity before loading trucks. Items are homogeneously stocked with respect to their type of product within the storage locations, which have different capacities, depending on their location within the warehouse. The input points and the collection area are capacitated as well.

During a specified time horizon, i.e., an eight hours work-shift, a number of items of different product types require the transportation from the input points to their preassigned storage locations and, at the same time, a certain number of items need to be picked from their storage locations and transported to the collection area. We define these flows of items as incoming and outgoing, respectively. Incoming items are available at a known availability date, while outgoing items are required to reach the collection area before a known due date. The amount of items to move and their product types are also known in advance.

The movements of items are performed by capacitated vehicles belonging to two different types of fleets, defined in the following as F1 and F2. The routing of the two fleets of vehicles is restricted to only one of the above described disjoint areas of the warehouse. In particular, F1 can only travel in the transit zone, thus moving incoming items from the input points towards collectors, i.e., capacitated zones located at the entrance of the storage area, whereas F2 can only circulate within the storage area, thus moving both incoming items from the collectors towards the assigned storage locations, and outgoing items from the storage locations towards the collection area. Incoming items thus need to follow a two-echelon movement towards their storage locations, using vehicles of fleet F1 and F2 sequentially. In addition, the routing of the vehicles has to be planned by considering:
i) anticipation of outgoing movements with respect to the planned due dates; this is particularly relevant when a shift with a low demand is followed by a shift with a high demand, thus items planned to leave the site in the second shift may be moved towards the collection area already during the first one;
ii) a strict management policy for both picking and put-away operations prescribing that, separately per product type, storage locations have to be emptied/filled up one at a time following a given order of precedence, implying that a new storage location may be utilized for picking/storing only if the previous one in the considered order is already completely empty/full;
iii) safety requirements for workers.

We refer to [6] for a more detailed and comprehensive description of the features above. Here we consider the case where a subset of the vehicles of type F2 are electric and equipped with a lithium-ion battery. The battery is discharged when vehicles move or lift items from the ground, and its state of
charge needs to be maintained within a given range to ensure a long lifetime to the battery. As opposed to traditional lead-acid batteries needing full recharging operations, a lithium-ion battery may benefit from partial recharging, which may occur during even short break times between operations at the available charging station. Thus, besides planning the routing of the vehicles in order to move inbound items (from the input points towards the preassigned storage locations) and outbound items (from storage locations towards the collection area), battery charging operations need to be scheduled as well. As in [6], the primary aim is to minimize the travel time of all the vehicles within the warehouse.

## 3 Mathematical formulation

The problem is formulated in terms of constrained multicommodity flows on a space-time network, and a Mixed Integer Linear Programming (MILP) model based on this formulation is proposed.

Let $\mathcal{K}$ be the set of the product types, or commodities, requiring movement in a given time horizon. It is composed of the subset of the incoming commodities $\mathcal{K}_{\text {in }}$ and the subset of the outgoing commodities $\mathcal{K}_{\text {out }}$. Let $\mathcal{V}^{1}$ and $\mathcal{V}^{2}$ be the sets of vehicles of type F1 and F2, respectively, in charge of moving commodities inside the warehouse. Moreover, let $\mathcal{V}^{E} \subseteq \mathcal{V}^{2}$ denote the subset of the electric vehicles of type F2.

Let $\mathcal{G}^{P}=\left(\mathcal{N}^{P}, \mathcal{A}^{P}\right)$ be the directed graph representing the physical network on which vehicles operate. The set of nodes $\mathcal{N}^{P}$ includes:

- the set $\mathcal{S}_{i n}^{k}$ of the storage locations preassigned to the product types in $\mathcal{K}_{i n}$, and the set $\mathcal{S}_{\text {out }}^{k}$ of the storage locations occupied by items of product types in $\mathcal{K}_{\text {out }}$ at the beginning of the time horizon;
- the parking areas for vehicles of type F1 and F2, denoted by $\omega^{1}$ and $\omega^{2}$, respectively;
- the set $\mathcal{R}$ of the input points (within the transit zone);
- the set $\mathcal{B}$ of the collectors;
- the output point (or collection area) $\pi$;
- the available charging station $c$.

The set of $\operatorname{arcs} \mathcal{A}^{P}$ represent direct connections between pairs of distinct locations of the warehouse. The dynamics of the problem are modelled through a space-time network $\mathcal{G}=(\mathcal{N}, \mathcal{A})$. The time horizon is discretized into $T$ time periods of equal length through $T+1$ time instants. The set $\mathcal{N}^{P}$ is then replicated $T+1$ times, resulting in set $\mathcal{N}$. A node in $\mathcal{N}$ is defined by a couple $(i, t)$, with $i \in \mathcal{N}^{P}$ and $t \in\{0, \ldots, T\}$, and represents one of the locations of the warehouse at one of the considered $T+1$ time instants. The set of $\operatorname{arcs} \mathcal{A}$ is composed of two subsets: the subset of holding $\operatorname{arcs} \mathcal{A}^{H}$, including arcs of type $((i, t),(i, t+1))$, for any $i \in \mathcal{N}^{P}$ and $t \in\{0, \ldots, T-1\}$, used to model idle time of items or vehicles in a given node for one time period, and the subset of moving $\operatorname{arcs} \mathcal{A}^{M}$, including arcs of type $\left((i, t),\left(j, t^{\prime}\right)\right)$ with $(i, j) \in \mathcal{A}^{P}, t \in\left\{0, \ldots, T-\tau_{i, j}\right\}$ and $t^{\prime}=t+\tau_{i, j}$, where $\tau_{i, j}$ denotes the travel time from $i$ to $j$ in the directed graph
$\mathcal{G}^{P}$. The travel time $\tau_{i, j}$ is determined by considering the allowed speed of the vehicles and by assuming that vehicles always follow a shortest path from $i$ to $j$ along the network. The subset of arcs $\mathcal{A}^{M}$ is thus used to model movements of items or vehicles between two different locations in different time periods.

Several parameters are introduced to describe the features of vehicles and incoming and outgoing commodities. We refer to [6] for a complete description. We introduce here only those related to the energy consumption model for the battery.

Let $e^{i j}$ be the battery energy consumed by an electric vehicle in $\mathcal{V}^{E}$ to move empty along $(i, j) \in \mathcal{A}^{P}$, while $e^{i j k}$ be the additional battery energy consumed by the vehicle, per unit of load, to move along $(i, j) \in \mathcal{A}^{P}$ if it is loaded with items of product type $k \in \mathcal{K}$. Moreover, let $e^{k}$ be the energy consumed by a vehicle to lift one unit of product type $k \in \mathcal{K}$. This operation is necessary only at certain nodes of $\mathcal{N}^{P}$, i.e., nodes in $S_{\text {in }}^{k} \cup \mathcal{S}_{\text {out }}^{k} \cup \mathcal{B}$. Furthermore, let $e^{r}$ denote the increase of the battery energy, for one period of time, if the vehicle recharges at the charging station. These parameters have been calculated according to the comprehensive energy consumption model described in [4], which takes into account speed, acceleration, deceleration, load cargo and gradients. Finally, let $\left[B^{-}, B^{+}\right.$] define the range in which the charge of the battery should always be maintained, while $\psi_{0}^{v} \in\left[B^{-}, B^{+}\right]$be the charge that vehicle $v \in \mathcal{V}^{E}$ has at the beginning of the time horizon. Regarding the battery, $\Theta \in\left[B^{-}, B^{+}\right]$will denote the minimum charge required for each electric vehicle at the end of the time horizon. Parameter $\Theta$ has been introduced to ensure enough charge at the beginning of the next time horizon, to perform basic operations such as traveling to the charging station.

Now, let us introduce the main families of variables used to formulate the addressed Green SRP. The following four families of variables model the routing of vehicles and commodities along the network. In the variable definition, $\mathcal{A}_{F 1}$, $\mathcal{A}_{F 2}, \mathcal{A}_{\text {in }}$ and $\mathcal{A}_{\text {out }}$ denote the subsets of arcs of the network where vehicles of type F1, vehicles of type F2, incoming commodities and outcoming commodities are permitted to move, respectively:
$-x_{(i, t)\left(j, t^{\prime}\right)}^{v} \in\{0,1\}$, for any $v \in \mathcal{V}^{1}$ and $\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 1}$, indicates whether vehicle $v$ passes on the arc $\left((i, t),\left(j, t^{\prime}\right)\right)$ or not;
$-x_{(i, t)\left(j, t^{\prime}\right)}^{v} \in\{0,1\}$, for any $v \in \mathcal{V}^{2}$ and $\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 2}$, indicates whether vehicle $v$ passes on the arc $\left((i, t),\left(j, t^{\prime}\right)\right)$ or not;
$-y_{(i, t)\left(j, t^{\prime}\right)}^{k} \in \mathbb{Z}_{+}$, for any $k \in \mathcal{K}_{\text {in }}$ and $\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{\text {in }}$, indicates the number of items of product type $k$ passing on the arc $\left((i, t),\left(j, t^{\prime}\right)\right)$;
$-y_{(i, t)\left(j, t^{\prime}\right)}^{k} \in \mathbb{Z}_{+}$, for any $k \in \mathcal{K}_{\text {out }}$ and $\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{\text {out }}$, indicates the number of items of product type $k$ passing on the $\operatorname{arc}\left((i, t),\left(j, t^{\prime}\right)\right)$.

Moreover, we define:
$-\psi_{t}^{v} \in \mathbb{R}_{+}$, for any $v \in \mathcal{V}^{E}$ and $t \in\{1, \ldots, T\}$, which indicates the state of charge of the battery of the electric vehicle $v$ at time $t$.

The objective function of the MILP model is defined as follows:

$$
\begin{align*}
& \min \sum_{v \in \mathcal{V}^{1}} \sum_{\substack{\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 1}: \\
i \neq \omega^{1}, j \neq \omega^{1}}} \tau_{i, j} x_{(i, t)\left(j, t^{\prime}\right)}^{v}+\sum_{\substack{v \in \mathcal{V}^{2}}} \sum_{\substack{\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 2}: \\
i \neq \omega^{2}, j \neq \omega^{2}}} \tau_{i, j} x_{(i, t)\left(j, t^{\prime}\right)}^{v} \\
& \quad+\psi \sum_{k \in \mathcal{K}_{i n}} \sum_{\substack{\left((i, t),\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{i n}: \\
i, j \in \mathcal{R}}} y_{(i, t)\left(j, t^{\prime}\right)}^{k}+\xi \sum_{k \in \mathcal{K}_{o u t}} P^{k} . \tag{1}
\end{align*}
$$

It is composed of four parts. The first two summations define the primary optimization goal, i.e., minimizing the travel time of all the vehicles within the warehouse. Notice that arcs entering or leaving the parking areas are not considered for both vehicle types to encourage vehicles to come back to their parking areas when idle, so limiting congestion situations along the network. The third and fourth summations define soft objectives. In particular, the third summation relates to the time of permanence of the items on the input points, so as to favor the movements of items towards other spots of the warehouse. The fourth relates to the anticipation movements to perform. The latter summations are weighted through parameters $\psi$ and $\xi$, respectively, to state their mutual priorities. Being $\mathcal{N}^{-}\left(\pi, t^{\prime}\right)$ the set of nodes linked to $\pi \in \mathcal{N}^{P}$ via an entering arc, the terms $P^{k}$ are defined as follows:

$$
\begin{equation*}
P^{k}=\max \left\{0, \sum_{t=0}^{\tilde{T}} d_{o u t}^{k}(\pi, t)-\left[u_{\pi}^{k}+\sum_{t=0}^{T} \sum_{(j, t) \in \mathcal{N}^{-}\left(\pi, t^{\prime}\right)} y_{(j, t)\left(\pi, t^{\prime}\right)}^{k}\right]\right\} \tag{2}
\end{equation*}
$$

for any $k \in \mathcal{K}_{\text {out }}$. The rationale of this penalty is to compare the amount of items of type $k$ at the beginning of the time horizon, i.e., $u_{\pi}^{k}$, plus the items of type $k$ transported to the collection area $\pi$ during the considered time horizon, given by the last two addendum of (2), with the overall demand of $k$ from the time instant $t=0$ to an extended time instant $\tilde{T}>T$, given by the first addendum of (2). Input parameter $\tilde{T}$ relates to the future time periods addressed for the anticipation moves, while $d_{o u t}^{k}(\pi, t)$ denotes the number of items of type $k$ which are requested in the collection area at the latest time $t$. The penalty is equal to 0 if, during the considered time horizon, an amount of items of type $k$ enough to satisfy both the demand of $k$ in the time horizon and also in the extended one, is moved to the collection area. Otherwise, the penalty to be paid is set proportionally to the amount of future demand that cannot be moved in advance.

Several constraints need to be defined to formulate the MILP model, such as flow conservation constraints for incoming and outgoing product types as well as for vehicles, to ensure their correct moving and routing within the warehouse, linking capacity constraints for vehicles and incoming and outgoing flows, demand constraints to ensure the respect of due dates for outgoing product types, location capacity constraints, constraints ensuring the correct application of the management policy in the warehouse, and finally constraints ensuring the security requirements for workers. The latter, in particular, impose that at most
one vehicle can be present in any arc of the space-time network, except for the holding arcs representing dwell time at the parking areas.

For the sake of brevity, we do not report the above mentioned constraints, which can be found in [6]. On the other hand, we present below those constraints which regulate the energy behavior of the electric vehicles, since they are peculiar to the Green SRP. In such constraints, $M$ is an input parameter defined as $M=B^{+}-B^{-}$. Moreover, $L_{j}$ is a parameter which assumes value 1 if $j \in S_{\text {in }}^{k} \cup \mathcal{S}_{\text {out }}^{k} \cup \mathcal{B}$, and 0 otherwise. The constraints are defined as follows:

$$
\begin{align*}
& \psi_{t^{\prime}}^{v} \leq \psi_{t}^{v}-e^{i j} x_{(i, t)\left(j, t^{\prime}\right)}^{v}-\sum_{k \in \mathcal{K}} e^{i j k} y_{(i, t)\left(j, t^{\prime}\right)}^{k}-L_{j} \sum_{k \in \mathcal{K}} e^{k} y_{(i, t)\left(j, t^{\prime}\right)}^{k}  \tag{3}\\
& \quad+M\left[1-x_{(i, t)\left(j, t^{\prime}\right)}^{v}\right]^{\forall} \quad \forall v \in \mathcal{V}^{E}, \forall\left((i, t)\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 2}: i \neq j, \\
& \psi_{t^{\prime}}^{v} \geq  \tag{4}\\
& \psi_{t}^{v}-e^{i j} x_{(i, t)\left(j, t^{\prime}\right)}^{v}-\sum_{k \in \mathcal{K}} e^{i j k} y_{(i, t)\left(j, t^{\prime}\right)}^{k}-L_{j} \sum_{k \in \mathcal{K}} e^{k} y_{(i, t)\left(j, t^{\prime}\right)}^{k} \\
& -M\left[1-x_{\left.(i, t)\left(j, t^{\prime}\right)\right]}^{v} \quad \forall v \in \mathcal{V}^{E}, \forall\left((i, t)\left(j, t^{\prime}\right)\right) \in \mathcal{A}_{F 2}: i \neq j,\right.  \tag{5}\\
& \\
& \psi_{t+1}^{v} \leq \psi_{t}^{v}+e^{r} x_{(c, t)(c, t+1)}^{v} \quad+M\left[1-x_{(c, t)(c, t+1)}^{v}\right] \quad \forall v \in \mathcal{V}^{E}, \forall((c, t)(c, t+1)) \in \mathcal{A}_{F 2},
\end{align*}
$$

$$
\begin{align*}
\psi_{t+1}^{v} \geq & \psi_{t}^{v}+e^{r} x_{(c, t)(c, t+1)}^{v}  \tag{6}\\
& -M\left[1-x_{(c, t)(c, t+1)}^{v}\right] \quad \forall v \in \mathcal{V}^{E}, \forall((c, t)(c, t+1)) \in \mathcal{A}_{F 2},
\end{align*}
$$

$$
\begin{equation*}
\psi_{t+1}^{v} \leq \psi_{t}^{v}+M\left[1-x_{(i, t)(i, t+1)}^{v}\right] \tag{7}
\end{equation*}
$$

$$
\forall v \in \mathcal{V}^{E}, \forall((i, t)(i, t+1)) \in \mathcal{A}_{F 2}: i \neq c,
$$

$$
\begin{equation*}
\psi_{t+1}^{v} \geq \psi_{t}^{v}-M\left[1-x_{(i, t)(i, t+1)}^{v}\right] \tag{8}
\end{equation*}
$$

$$
\forall v \in \mathcal{V}^{E}, \forall((i, t)(i, t+1)) \in \mathcal{A}_{F 2}: i \neq c,
$$

$$
\begin{equation*}
B^{-} \leq \psi_{t}^{v} \leq B^{+} \quad \forall v \in \mathcal{V}^{E}, \forall t \geq 0 \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
\psi_{T}^{v} \geq \Theta \quad \forall v \in \mathcal{V}^{E} \tag{10}
\end{equation*}
$$

Constraints (3)-(8) model the state of charge of the battery, which decreases if the vehicle travels along a moving arc, increases if the vehicle idles on a holding arc corresponding to the charging station, or remains constant if the vehicle idles on any other location of the warehouse. Specifically, the discharge of the battery is modelled by constraints (3)-(4). By recalling that a moving arc can be used
by at most one vehicle, when a vehicle $v$ travels along a moving arc $\left((i, t)\left(j, t^{\prime}\right)\right)$ then constraints (3)-(4) imply

$$
\psi_{t^{\prime}}^{v}=\psi_{t}^{v}-e^{i j}-\sum_{k \in \mathcal{K}} e^{i j k} y_{(i, t)\left(j, t^{\prime}\right)}^{k}-L_{j} \sum_{k \in \mathcal{K}} e^{k} y_{(i, t)\left(j, t^{\prime}\right)}^{k}
$$

thus defining the state of charge of the battery of the vehicle at the time instant $t^{\prime}$ as the state of charge of the battery of the vehicle at the time instant $t$ minus the energy necessary for the vehicle to move empty on the arc (the second term in the equation), the additional energy used if the vehicle is loaded (the third term in the equation) and the energy used to lift items at location $j$, if necessary (the last term in the equation). If the arc is not travelled by the vehicle, then constraints (3)-(4) are satisfied since weaker than constraints (9). The latter define the lower and upper thresholds for the ideal operating conditions of the battery. When the vehicle is at the charging station $c$, i.e., it is on a holding arc of form $((c, t)(c, t+1))$, constraints (5)-(6) define the state of charge of the battery at time instant $t+1$ as the state of charge of the battery at the time instant $t$ plus the energy recharged during one time period at the charging station. Constraints (7)-(8), instead, indicate that the state of charge of the battery remains unchanged if the vehicle is idling on a location of the warehouse other than $c$. Finally, constraints (10) impose that the state of charge of the electric vehicles at the end of the time horizon is greater than or equal to the minimum threshold $\Theta$. This is to ensure that, at the beginning of the next shift, their state of charge is enough to perform some basic operations rather than being completely discharged.

## 4 Matheuristic resolution approach

In [6], a matheuristic approach based on a decomposition strategy has been proposed for the conventional SRP since real size instances, such as those provided to us by our industrial partner, could not be directly addressed through the state-of-the-art commercial solver CPLEX. The approach has shown a very good performance, as detailed in [6].

Specifically, the original planning horizon is divided into $\Lambda$ subperiods of equal length. Each subperiod gives rise to a subproblem, whose features are those of the original problem restricted to the considered subperiod. The $\Lambda$ subproblems are then sequentially solved by using CPLEX in such a way that the final state of the system obtained solving subproblem $\lambda-1$ becomes the initial state of the system when solving subproblem $\lambda$, for any $\lambda=2, \ldots, \Lambda$. In particular, the state of the system in each subproblem takes into account the position of vehicles and items within the warehouse. Once the $\Lambda$ subproblems have been solved, in order to construct a solution for the original problem, and thus the complete schedule for the entire time horizon, it is sufficient to concatenate the $\Lambda$ solutions in an increasing order with respect to the subperiod addressed, i.e., from subperiod 1 to subperiod $\Lambda$. The matheuristic approach is summarized in Algorithm 1. We refer to [6] for a more detailed description.

```
Algorithm 1 The matheuristic approach
    Divide the time horizon into \(\Lambda\) subperiods
    for \(\lambda=1, \ldots, \Lambda\) do
        Solve the \(\lambda\)-th subproblem
    end for
    Concatenate the subproblem solutions from 1 to \(\Lambda\)
```

The matheuristic approach has been extended to deal with the green aspects previously introduced. In particular, the initial state of the system in each subproblem takes into account, in addition to the position of vehicles and items within the warehouse at the end of the previous subperiod, also the state of charge of each battery. Specifically, for any $\lambda=2, \ldots, \Lambda$, the state of charge of a vehicle, say $v$, at the initial time instant of subproblem $\lambda$, say $0_{\lambda}$, is defined as $\psi_{0_{\lambda}}^{v}=\psi_{T_{\lambda-1}}^{v}$, where $\psi_{T_{\lambda-1}}^{v}$ is the state of charge of the battery of vehicle $v$ at the final time instant of subproblem $\lambda-1$, here denoted by $T_{\lambda-1}$.

## 5 Numerical experiments

We present some preliminary results on the Green SRP by solving the set of five artificial instances in the dataset used in [6], suitably generalized to the green context. Generally, such instances turned out to be too difficult to address directly with CPLEX, thus the matheuristic previously introduced has been used to solve the Green SRP. Experiments have been performed by varying the number of the electric vehicles, by analysing both the efficiency of the matheuristic approach in terms of percentage gap and solution time, and also investigating the quality of the returned solutions in terms of some crucial performance indicators suggested by our industrial partner.

The matheuristic has been implemented using the language OPL and solved via CPLEX 12.6 (IBM ILOG, 2016) with a time limit of 3 hours. The experiments have been run on an Intel Xeon 5120 with 2.20 GHz and 32 GB of RAM.

### 5.1 The reference case study

The instances refer to the production site of a company, leader in the tissue sector, which works daily on three shifts of 8 hours and produces more than 300 different types of products. Items are arranged in unit-loads and wrapped in so-called columns of pallets. There are 3 input points with capacity 10,14 and 8 columns, respectively, while the storage area has 858 storage locations, with different capacities ranging from 8 to 17 columns. The number of collectors is 6 , with capacities ranging from 2 to 8 columns. Finally, the collection area has a capacity of 700 columns. The fleet of the company is composed of 5 LGV shuttles, corresponding to vehicles of type F1, and 7 forklifts, corresponding to vehicles of type F2, some of which are electric. Both types of vehicles, hereafter LGV and FKL for short, may transport 2 columns at most at the same time. On
average, during each 8 hours shift, 320 columns of 9 product types need to be moved from the input points towards their storage locations, while 1110 columns of 28 product types need to be moved from their storage locations towards the collection area. Regarding parameters $B^{-}, B^{+}$and $\Theta$, related to the state of charge of the battery, they have been set to the $30 \%$, the $80 \%$ and the $35 \%$ of the total capacity of the battery, respectively. Parameters $B^{-}$and $B^{+}$have been set up in accordance with the features described in the user manual of the specific electric FKL used by the industrial partner.

The Green SRP instances have been generated starting from the five artificial SRP instances in the dataset used in [6]. In turn, such five SRP instances have been generated starting from a real dataset provided by the company, which comprises a pool of selected 8 hours shifts, by shortening the duration of a shift from 8 to 4 hours, and reducing the number of product types and columns to move accordingly. The main features of the Green SRP instances are reported in Table 1. Specifically, the number of available storage locations is reported (column SL) together with the number of the product types in $K_{\text {in }}$ and in $K_{\text {out }}$ (columns $K_{\text {in }}$ and $K_{\text {out }}$, respectively), and the corresponding number of items to move (columns $C_{\text {in }}$ and $C_{\text {out }}$, respectively).

Table 1. The Green SRP instances.

|  | Main features |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Instance | SL | $K_{\text {in }}$ | $K_{\text {out }}$ | $C_{\text {in }}$ | $C_{\text {out }}$ |
| 1 | 10 | 3 | 6 | 142 | 288 |
| 2 | 4 | 2 | 4 | 108 | 234 |
| 3 | 9 | 3 | 5 | 78 | 188 |
| 4 | 4 | 2 | 3 | 132 | 226 |
| 5 | 8 | 3 | 5 | 134 | 364 |
| Average | 6.6 | 2.2 | 4.2 | 90.8 | 117.2 |

### 5.2 Computational results

In order to solve the Green SRP by means of the matheuristic approach, we split the time horizon into four subshifts, thus obtaining subshifts of about 60 minutes. As reported in [6], longer subshifts may lead to hardly solvable subproblems, while shorter subshifts seem to negatively affect the quality of the solutions obtained. The resolution of each subproblem has been performed via CPLEX by stopping the execution as soon as an optimality gap less than $1 \%$ or, alternatively, a time limit of 15 minutes were reached. Most subproblems, however, were solved to optimality. Finally, parameters $\psi$ and $\xi$ in (1) have been set equal to 10 , since this combination proved to be very effective in [6].

Each of the five Green SRP instances has been solved three times, by varying the number of the electric vehicles. Specifically, 1,2 and 3 electric FKL have been considered (recall that the total number of FKL is 7). In the following, we refer to the composition of the fleet of FKL as a pair of numbers in brackets of type $\left(\left|\mathcal{V}^{D}\right|-\left|\mathcal{V}^{E}\right|\right)$, where the first position indicates the number of traditional FKL used, while the second position gives the number of the electric FLK. After some preliminary tests, we considered three different settings for the initial state of the charge of the electric vehicles, depending on their number. Specifically, if one electric FKL is used, its initial state of charge is set to the half of the range $\left[B^{-}, B^{+}\right]$; if two electric FKL are used, the range $\left[B^{-}, B^{+}\right.$] is split into two parts of equal length, and the initial state of charge of one vehicle is set to the half of the first range, while the initial state of charge of the second vehicle is set to the half of the second range; finally, if three electric FKL are used, then the range $\left[B^{-}, B^{+}\right]$is split into three parts of equal length, and the initial state of charge of the vehicles is set to the half of the first, of the second and of the third range, respectively.

For each instance, Table 2 reports the time, in seconds, required by the matheuristic to find a solution to the Green SRP for each of the three fleet compositions mentioned before (calculated as the sum of the times needed to solve the subproblems). It also reports the percentage optimality gap, calculated with respect to the optimal value found by CPLEX by solving the Green SRP with only one electric FKL. This is the only variant of Green SRP that CPLEX was able to solve to optimality, and the corresponding optimal values thus represent lower bounds for all the addressed variants. The times required to compute such lower bounds are reported in column LB. Moreover, to perform a comparison with the traditional SRP, the times required by the matheuristic to solve SRP are reported in column $(7-0)$, to emphasize that SRP is the special case of Green SRP with 0 electric vehicles.

Table 2 shows that the Green SRP is more difficult to address than the traditional SRP. In the case of no electric vehicles, the average time required by the matheuristic is in fact about 9 seconds, whereas when electric vehicles are present the time increases a lot, especially in the case of 3 electric FKL.

Table 2. Performance of the matheuristic for Green SRP.

|  | $\left(\left\|\mathcal{V}^{D}\right\|-\left\|\mathcal{V}^{E}\right\|\right)$ |  |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Instance | (7-0) | $(6-1)$ |  | $(5-2)$ |  | $(4-3)$ |  | LB |
|  | Time | Time | Gap | Time | Gap | Time | Gap | Time |
|  | 10.91 | 757.73 | $10.65 \%$ | 87.10 | $9.70 \%$ | 1014.75 | $8.69 \%$ | 4496.23 |
| 2 | 5.37 | 66.99 | $6.71 \%$ | 365.52 | $6.38 \%$ | 1063.37 | $7.69 \%$ | 316.13 |
| 3 | 10.45 | 14.51 | $0.32 \%$ | 106.54 | $3.09 \%$ | 124.41 | $4.87 \%$ | 1792.26 |
| 4 | 7.54 | 463.36 | $2.21 \%$ | 111.95 | $20.64 \%$ | 717.77 | $21.32 \%$ | 699.05 |
| 5 | 13.35 | 1816.47 | $13.52 \%$ | 2097.74 | $6.60 \%$ | 2530.83 | $15.00 \%$ | 4642.82 |
| Avg. | 9.53 | 623.81 | $6.68 \%$ | 553.77 | $9.28 \%$ | 1090.22 | $11.51 \%$ | 2389.30 |

Table 3. Features of solutions in terms of crucial performance indicators.

|  | $\mid\left(\mathcal{V}^{D}\left\|-\left\|\mathcal{V}^{E}\right\|\right)\right.$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $(7-0)$ | $(6-1)$ | $(5-2)$ | $(4-3)$ |
| LGV Avg. Travel Time (min.) | 75.84 | 76.56 | 76.16 | 76.16 |
| FKL Avg. Travel Time (min.) | 122 | 122.51 | 122.80 | 123.03 |
| Conventional FKL Avg. Travel Time (min.) | 122 | 122.20 | 125.84 | 117.10 |
| Electric FKL Avg. Travel Time (min.) | - | 124.4 | 115.2 | 130.93 |
| Electric FKL Avg. Charging Time (min.) | - | 10.4 | 4.6 | 6.5 |
| Input point Avg. Idle Time per item (min.) | 0.127 | 0.129 | 0.123 | 0.131 |
| \% of saturation of collection area after 3h | $99.43 \%$ | $99.43 \%$ | $98.29 \%$ | $99.43 \%$ |
| \% of saturation of collection area after 4h | $99.43 \%$ | $99.08 \%$ | $98.23 \%$ | $98.12 \%$ |

Notice that, as reported in [6], CPLEX was able to optimally solve the five SRP instances in 217 seconds on average, whereas the optimal solution of the Green SRP with just one electric vehicle required about 2389 seconds on average (see column LB). Nevertheless, the matheuristic is still efficient, being able to find good solutions for the Green SRP with an average optimality gap of about $7 \%$ in the case of 1 electric FKL, $9 \%$ in case of 2 , and $11 \%$ in case of 3 .

To better analyse the results in Table 2 as well as the quality of the computed solutions, Table 3 reports some aggregated features of the solutions in terms of crucial performance indicators suggested by our industrial partner.

Specifically, the primary goal is analysed in terms of the average time, in minutes, travelled by a LGV and by a FKL over the 5 instances. Disaggregated results are also reported separately for conventional and electric vehicles (averages are calculated over the corresponding number of conventional and electric vehicles used). Moreover, we report the average charging time of the electric vehicles, always in minutes. The secondary goals, i.e., emptying the input points and anticipation moves, are evaluated by considering the average time, in minutes, an incoming item idles on an input point before been moved to an available collector, and the percentage of saturation of the collection area both 60 minutes before the end of the planning horizon (\% of saturation of collection area after 3 h ) and also at the end of the planning horizon (\% of saturation of collection area after 4h).

The average time travelled by a FKL (conventional and electric) is almost the same for both the traditional and the Green SRP. However, the number of electric vehicles used strongly influences the usage of the fleet of FKL. This is especially remarkable in the case of 2 electric FKL, where just a few operations are committed to the electric vehicles, whereas more operations are instead performed by conventional vehicles. On the other hand, in the case of 3 electric vehicles, i.e., when almost half of the fleet of FKL is electric, then electric vehicles travel more on average. Interestingly, the average charging time of the battery is greater in the case of a single electric vehicle, probably because, in
the absence of conflicts with other electric vehicles towards the unique charging station, it tends to recharge more frequently. The LGV travel time, instead, slightly increases with respect to SRP. This may be explained as an additional way to prevent the discharge of the batteries of the electric FKL, as the majority of the routes from the input points towards the assigned storage locations is built in such a way to favor short trips on-board of electric FKL and longer trips on-board of LGV.

Regarding the secondary goals, being electric FKL not always available, this slightly slows down the rate of the anticipation moves to be performed with respect to SRP. Except for the case in which 2 electric FKL are used, after 3 hours of operations the collection area seems not to be affected by the composition of the fleet of FKL (see the row \% of saturation of collection area after 3 h and compare with SRP). However, the $\%$ of saturation of the collection area after 4 hours highlights a lower readiness of the fleets including electric vehicles to promptly respond with replenishment operations when some new space is made available in the collection area. Similarly, the average idle time of incoming items on the input points generally increases with respect to the traditional SRP. Notice that the different impact of the number of electric vehicles on the primary and secondary goals may explain the reduction of time and/or gap sometimes observed in Table 2 when the number of the electric vehicles increases (like instance 1 in the case of one and two electric vehicles).

## 6 Conclusions

The Green SRP has been proposed and studied, where some of the vehicles of the fleet performing operations within the warehouse are electric. A pool of instances has been solved with a time decomposition matheuristic, which extends the one originally built for the traditional SRP. The experimental results, although preliminary, highlight the greater computational complexity of the Green SRP compared to SRP, and the good performance of the resolution approach in terms of efficiency and quality of the returned solutions. Future research will investigate additional scenarios in terms of number of electric vehicles used, also proposing alternative Green SRP resolution approaches.

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