Effective transverse momentum in multiple jet production at hadron colliders

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We consider the class of inclusive hadron collider processes in which several energetic jets are produced, possibly accompanied by colorless particles [such as Higgs boson(s), vector boson(s) with their leptonic decays, and so forth]. We propose a new variable that smoothly captures the $N+1$ to N-jet transition. This variable, that we dub k_T^{ness}, represents an effective transverse momentum controlling the singularities of the $N+1$-jet cross section when the additional jet is unresolved. The k_T^{ness} variable offers novel opportunities to perform higher-order calculations in quantum chromodynamics by using nonlocal subtraction schemes. We study the singular behavior of the $N+1$-jet cross section as $k_T^{\text{ness}} \to 0$ and, as a phenomenological application, we use the ensuing results to evaluate next-to-leading-order corrections to $H+\text{jet}$ and $Z+2$-jet production at the LHC. We show that k_T^{ness} performs extremely well as a resolution variable and appears to be very stable with respect to hadronization and multiple-parton interactions.

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I. INTRODUCTION

Most of the new-physics searches and precision studies of the Standard Model at the Large Hadron Collider (LHC) are carried out by identifying events with a definite number of energetic leptons, photons, and jets. Jets are collimated bunches of hadrons that represent the fingerprints of the high-energy partons (quarks and gluons) produced in the hard-scattering interaction. A precise description of jet processes requires observables capable of efficiently capturing the dynamics of the energy flow in hadronic final states. A classic example is provided by dimensionless "shape variables" in e^+e^- collisions [1] and their generalization to proton-proton collisions [2,3]. These variables are sensitive to different aspects of the theoretical description of the underlying hard-scattering process and are designed to measure the deviation from the leading-order (LO) energy flow, which characterizes the bulk of the events.

For processes with N jets at the Born level, observables describing the $N+1 \to N$-jet transition are particularly useful to veto additional jets, for instance, to discriminate signal over backgrounds. When no jets are produced in the final state and only a colorless system is tagged, a prominent example of a dimensionful variable which inclusively describes the initial-state radiation is given by the transverse momentum of the colorless system (q_T). Another commonly used variable is (N-)jettiness [4] τ_N, which is defined on events containing at least N hard jets. Requiring $\tau_N \ll 1$ constrains the radiation outside the signal (and beam) jets, effectively providing an inclusive way to veto additional jets. These resolution variables have been used to formulate nonlocal subtraction methods for QCD calculations at next-to-next-to-leading order (NNLO) and beyond [5,6]. Both q_T and τ_N are also used as resolution variables in the matching of NNLO computations to parton shower simulations [7–11].

In the context of nonlocal subtraction schemes, the efficiency of the calculation is subject to the size of the (missing) power corrections, which in general depends on the choice of the resolution variable. For multijet processes, jettiness is the only viable variable proposed to date, and it has been successfully used to compute NNLO corrections to several color-singlet processes [6,12–15] and to the production of a vector or Higgs boson in association with a jet [16–18]. Nonetheless, it is well known that τ_N is affected by large power corrections. Indeed, even in the case of the production of a colorless final state, the power suppressed contributions are linear and logarithmically enhanced already at next-to-leading order (NLO), see e.g., Refs. [19–23]. On the other hand, nonlocal subtraction methods based on q_T are subject to milder power corrections, which can even be quadratic [24–26] in the absence of cuts [27–30].

An important probe of QCD dynamics in the infrared region is obtained when fixed-order perturbative predictions are supplemented with the all-order resummation of soft and collinear emissions and eventually compared to experimental data. This comparison largely depends on low-energy physics phenomena, such as hadronization or multiple-parton interactions (MPIs), which must be properly included to realistically simulate collider events. In particular, the constraining power of the observable is substantially diluted when these effects, which go beyond a purely perturbative description, become dominant. While hadronization corrections and MPI effects are mild in the case of q_T, they are particularly significant in the case of observables like N-jettiness [31].

In this work we introduce a new global dimensionful observable, which we call k_T^{ness}, to describe the $N+1 \to N$-jet transition. This variable, which takes its name from the k_T-jet clustering algorithm [32,33], represents an effective transverse momentum describing the limit in which the additional jet is unresolved. When the unresolved radiation is close to the colliding beams, the variable coincides with the transverse momentum of the final-state system. When the unresolved radiation is emitted close to one of the final-state jets, the variable describes the relative transverse momentum with respect to the jet direction. As we will show, k_T^{ness} offers a number of attractive features. It is affected by relatively mild power corrections, thereby allowing us to efficiently compute NLO corrections to processes with a vector or Higgs boson plus one or more jets. We also show that the new variable is very stable with respect to hadronization and MPIs. Due to these appealing properties, k_T^{ness} might prove useful as a resolution variable in higher-order computations and in their matching to parton shower simulations, as well as in studies of nonperturbative physics associated with the underlying event in proton-proton collisions.

The paper is organized as follows. In Sec. II we define k_T^{ness} and we discuss the formulation of a subtraction scheme based on this observable. We present numerical results for Higgs + jet production and for $Z+2$-jet production at NLO in Sec. III. We also study the impact of MPI effects and hadronization on the k_T^{ness} observable in $Z+1$-jet production. Finally, we summarize our findings and discuss future prospects in Sec. IV. Further details on the perturbative ingredients needed at NLO are given in the Appendix.

II. DEFINITION AND NLO IMPLEMENTATION

We consider the inclusive hard-scattering process

$$h_1(P_1) + h_2(P_2) \to j(p_1) + j(p_2) + \cdots + j(p_N) + F(p_F) + X, \tag{1}$$

where the collision of the two hadrons h_1 and h_2 with momenta P_1 and P_2 produces N final-state hard jets with momenta $p_1, p_2 \ldots p_N$, possibly accompanied by a generic colorless system F with total momentum p_F. The QCD radiative corrections to the process in Eq. (1) receive contributions from final states including up to $N+k$ partons, where k is the order of the computation. The dimensionful quantity $(N\text{-})k_T^{\mathrm{ness}}$ for an event featuring $N+m$ partons (with $1 \le m \le k$) is defined by using the *exclusive* k_T clustering algorithm [32,33]. We first introduce the distances

$$d_{ij} = \min(p_{Ti}, p_{Tj})\Delta R_{ij}/D, \qquad d_{iB} = p_{Ti}, \tag{2}$$

where D is a parameter of order unity, $i, j = 1, 2 \ldots, N+m$ and $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$ is the standard separation in rapidity (y) and azimuth (ϕ) between the (pseudo) particles i and j. The quantity d_{iB} is the “particle-beam” distance, given by the transverse momentum p_{Ti}.[1] The k_T^{ness} variable is defined via a recursive procedure through which close-by particles are combined with each other or with the beam until $N+1$ jets remain. The procedure goes as follows:

(1) Compute the minimum of the d_{ij} and the d_{iB}. If there are at least $N+2$ final-state pseudoparticles, perform step 2. If there are only $N+1$ pseudoparticles, perform step 3.
(2) If the minimum is one of the d_{iB}, then recombine i with the beam and remove it from the list of pseudoparticles. The recombination is done starting from a recoil momentum initialized to $p_{\mathrm{rec}} = 0$ at the beginning of the procedure and collecting the recoiled momenta with $p_{\mathrm{rec}} \to p_{\mathrm{rec}} + p_i$. If the minimum is a d_{ij} then replace the pseudoparticles i and j with a new pseudoparticle with momentum $p_i + p_j$. Go back to step 1 with a configuration which has one pseudoparticle less.
(3) When $N+1$ pseudoparticles are left, if the minimum is one of the d_{iB} add the recoil to p_i through $p_i \to p_{\mathrm{rec}} + p_i$ and set $k_T^{\mathrm{ness}} = p_{T,i}$. If instead the minimum is a d_{ij} then set $k_T^{\mathrm{ness}} = \min(d_{ij})$.

To the best of our knowledge, k_T^{ness} has not been considered before in the literature. The variable depends on the parameter D entering the distance in Eq. (2). We note that other prescriptions to treat the recoil (for instance, by neglecting it in step 3 are in principle possible, and the differences start to appear from NNLO (i.e., $k=2$).

We have computed the singular behavior of the cross section for the production of a colorless system accompanied by an arbitrary number of jets at NLO as $k_T^{\mathrm{ness}} \to 0$. The computation starts by organizing the terms relevant in each singular region and removing the double counting, similar to what is done in Refs. [35,36]. The terms containing initial-state collinear singularities produce the so-called “beam” functions, while those containing final-state collinear singularities give rise to the “jet” functions. The remaining contributions, describing soft radiation at large angles, produce the so-called “soft” function. The

[1]To be precise, Refs. [32,33] use pseudorapidities in the definition of d_{ij}. Here we use rapidities as in the implementation of the k_T algorithm in the FASTJET code [34].

soft and collinear singularities, regulated by working in $d = 4 - 2\epsilon$ dimensions, cancel out with those of the virtual contribution, to obtain a finite cross section. The results of our computation are used to construct a subtraction formula for the partonic cross section $d\hat{\sigma}_{\rm NLO}^{\rm F+N\,jets+X}$ as follows:

$$d\hat{\sigma}_{\rm NLO}^{\rm F+N\,jets+X} = \mathcal{H}_{\rm NLO}^{\rm F+N\,jets} \otimes d\hat{\sigma}_{\rm LO}^{\rm F+N\,jets} + [d\hat{\sigma}_{\rm LO}^{\rm F+(N+1)jets} - d\hat{\sigma}_{\rm NLO}^{\rm CT,F+N\,jets}]. \quad (3)$$

The real contribution $d\hat{\sigma}_{\rm LO}^{\rm F+(N+1)jets}$ is obtained by integrating the tree-level matrix elements with one additional parton and is divergent in the limit $k_T^{\rm ness} \to 0$. The "counterterm" $d\hat{\sigma}_{\rm NLO}^{\rm CT,F+Njets}$ is constructed by combining the singular contributions discussed above and matches the real contribution in the $k_T^{\rm ness} \to 0$ limit. Its explicit expression in the partonic channel ab reads

$$d\hat{\sigma}_{{\rm NLO}\,ab}^{\rm CT,F+Njets} = \frac{\alpha_{\rm S}}{\pi}\frac{dk_T^{\rm ness}}{k_T^{\rm ness}}\left\{\left[\ln\frac{Q^2}{(k_T^{\rm ness})^2}\sum_\alpha C_\alpha - \sum_\alpha \gamma_\alpha - \sum_i C_i \ln(D^2) - \sum_{\alpha\neq\beta}\langle \mathbf{T}_\alpha\cdot\mathbf{T}_\beta\rangle \ln\left(\frac{2p_\alpha\cdot p_\beta}{Q^2}\right)\right]\right.$$
$$\left. \times\, \delta_{ac}\delta_{bd}\delta(1-z_1)\delta(1-z_2) + 2\delta(1-z_2)\delta_{bd}P_{ca}^{(1)}(z_1) + 2\delta(1-z_1)\delta_{ac}P_{db}^{(1)}(z_2)\right\} \otimes d\hat{\sigma}_{{\rm LO}\,cd}^{\rm F+Njets}, \quad (4)$$

where $\gamma_q = 3C_F/2$, $\gamma_g = (11C_A - 2n_F)/6$, $C_F = 4/3$, and $C_A = 3$ are the QCD color factors with n_F the number of active flavors and D is the parameter entering the definition of $k_T^{\rm ness}$ [see Eq. (2)]. The index i labels the final-state partons with color charges $\mathbf{T}_i$ ($\mathbf{T}_i^2 = C_i$) and momenta p_i ($\sum_i p_i = q$, $Q^2 = q^2$), while α and β label initial- and final-state partons. The symbol $\langle \mathbf{T}_\alpha\cdot\mathbf{T}_\beta\rangle = \langle \mathcal{M}_{cd\to{\rm F+N\,jets}}|\mathbf{T}_\alpha\cdot\mathbf{T}_\beta|\mathcal{M}_{cd\to{\rm F+N\,jets}}\rangle/|\mathcal{M}_{cd\to{\rm F+N\,jets}}|^2$ is the normalized color-correlated tree-level matrix element for the partonic process contributing to $d\hat{\sigma}_{{\rm LO}\,cd}^{\rm F+N\,jets}$, and a sum over all the possible final-state parton flavors is understood. The functions $P_{ab}^{(1)}(z)$ are the LO Altarelli-Parisi kernels (in $\alpha_{\rm S}/\pi$ normalization) [37–39], and the symbol $\otimes$ denotes the convolutions with respect to the longitudinal-momentum fractions z_1 and z_2 of the colliding partons. The square bracket in Eq. (3) is evaluated by requiring $k_T^{\rm ness}/M > r_{\rm cut}$, where $M \sim Q$ is a hard scale which can be chosen depending on the specific process under consideration. The first term on the right-hand side of Eq. (3) is obtained by convoluting the LO cross section $d\hat{\sigma}_{\rm LO}^{\rm F+N\,jets}$ with the perturbative function $\mathcal{H}_{\rm NLO}^{\rm F+N\,jets}$. The latter contains the virtual correction after subtraction of the infrared singularities, additional finite contributions of collinear origin (beam and jet functions) and of soft origin (soft function). More details on the evaluation of $\mathcal{H}_{\rm NLO}^{\rm F+N\,jets}$ can be found in the Appendix. The physical cross section is formally obtained by taking the limit $r_{\rm cut} \to 0$ in Eq. (3).

We have implemented Eq. (3) to evaluate H + jet (in the limit of an infinite top-quark mass) and Z + 2-jet production at the LHC. The real contribution is evaluated with MCFM [40], while the subtraction counterterm $d\hat{\sigma}_{\rm NLO}^{\rm CT,F+Njets}$ and the $\mathcal{H}_{\rm NLO}^{\rm F+N\,jets} \otimes d\hat{\sigma}_{\rm LO}^{\rm F+N\,jets}$ contribution are computed with a dedicated implementation. In particular, for $H+$-jet production the required tree-level and one-loop amplitudes are still obtained from MCFM, while for $Z + 2$ jet all the (color-correlated) amplitudes are evaluated with OpenLoops [41–43].

III. RESULTS

We consider proton-proton collisions at the LHC at a center-of-mass energy of 13 TeV. We use the NNPDF31_nlo_as_0118 parton distribution functions [44] with $\alpha_{\rm S}(m_Z) = 0.118$ through the LHAPDF interface [45]. As for the electroweak couplings we use the G_μ scheme with $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, $m_W = 80.385$ GeV, $m_Z = 91.1876$ GeV, and $\Gamma_Z = 2.4952$ GeV. We define jets via the anti-k_T algorithm [46] with $R = 0.4$.

We start the presentation of our results with H + jet production. We compute the corresponding cross section through Eq. (3) by setting the parameter $D = 1$ and requiring the leading jet to have $p_T^j > 30$ GeV. The factorization and renormalization scales μ_F and μ_R are set to the Higgs boson mass $m_H = 125$ GeV. In order to compare our results to those that can be obtained with jettiness subtraction, we have implemented the corresponding calculation in a modified version of the MCFM code [40], which we have benchmarked against the numerical results of Ref. [23]. The 1-jettiness variable is defined as

$$\mathcal{T}_1 = \sum_i \min_l \left\{\frac{2q_l\cdot p_i}{Q_l}\right\}, \quad (5)$$

where q_l ($l = 1, 2, 3$) are the momenta of the initial-state partons and of the hardest jet present in the event and the sum over i runs over the final-state parton momenta p_i. Following Ref. [23] we compute $\mathcal{T}_1$ in the hadronic center-of-mass frame and we set the normalization factors $Q_l = 2E_l$. To compare the results obtained with a 1-jettiness cut to those obtained using 1-$k_T^{\rm ness}$ we define

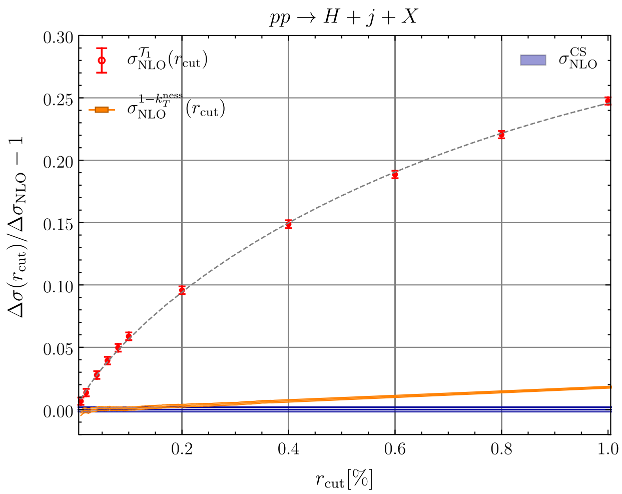

FIG. 1. The NLO correction $\Delta\sigma$ for the $H + $jet cross section computed with 1-jettiness (red points) and 1-k_T^{ness} (orange). The r_{cut} dependence is compared to the (r_{cut} independent) result obtained with dipole subtraction using MCFM (blue).

the minimum r_{cut} on the dimensionless variable $r = \mathcal{T}_1/\sqrt{m_H^2 + (p_T^j)^2}$ $[r = k_T^{\mathrm{ness}}/\sqrt{m_H^2 + (p_T^j)^2}]$.

In Fig. 1 we study the behavior of the NLO correction $\Delta\sigma$ as a function of r_{cut} for the jettiness and k_T^{ness} calculations, normalized to the result obtained with Catani-Seymour (CS) dipole subtraction [47,48] by using MCFM (which is independent of r_{cut}). Both jettiness and k_T^{ness} results nicely converge to the expected result but the r_{cut} dependence is very different for the two calculations. The r_{cut} dependence in the case of jettiness is rather strong: At $r_{\mathrm{cut}} = 1\%$ the difference with the exact result is about 25% of the computed correction. The observed r_{cut} dependence is consistent with a logarithmically enhanced linear behavior. We note that performing the computation in other frames may improve the convergence [23], but the functional behavior remains the same. By contrast, in the case of k_T^{ness} the dependence is rather mild. At $r_{\mathrm{cut}} = 1\%$ the difference with the exact result is only about 3% of the computed correction. The r_{cut} dependence is consistent with a purely linear behavior (i.e., without logarithmic enhancements).

We now move to consider $Z + 2$-jet production. We compute the cross section to obtain a dilepton pair in the invariant-mass range $66 \leq m_{\ell\ell} \leq 116$ GeV together with (at least) two jets with $p_T > 30$ GeV and pseudorapidity $|\eta| < 4.5$. The leptons have $p_T \geq 20$ GeV and pseudorapidity $|\eta_\ell| \leq 2.5$. The minimum separation between the leptons is $\Delta R_{\ell\ell} > 0.2$ while leptons and jets have $\Delta R_{\ell j} > 0.5$. The factorization and renormalization scales are set to the Z boson mass m_Z. Our calculation is carried out by using the transverse mass of the dilepton system as a hard scale M to define r_{cut} and the parameter D is set to

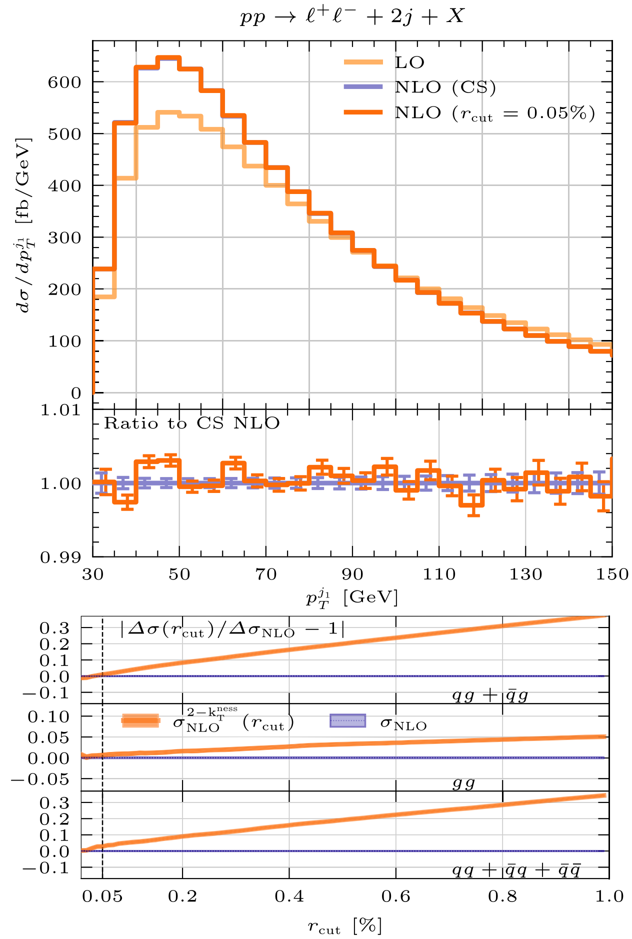

FIG. 2. $Z + 2$-jet production at NLO: k_T^{ness}-subtraction against CS. The p_T distribution of the leading jet (upper and center) at LO (yellow) and NLO (orange, k_T^{ness}; blue, CS). NLO corrections $\Delta\sigma$ as a function of r_{cut} in the three partonic channels (lower).

$D = 0.1$ in this case. We have checked that similar results can be obtained by choosing different values of D. We compare our results with those obtained using the implementation of $Z + 2$-jet production of [49], which is based on CS subtraction. In Fig. 2 (upper panel) we show the p_T distribution of the hardest jet at LO and NLO, computed with k_T^{ness} subtraction (using $r_{\mathrm{cut}} = 0.05\%$) and with CS. The central panel shows the relative difference between the two calculations. We observe an excellent agreement between the two results at the few permille level. The three lower panels display the NLO correction $\Delta\sigma$ as a function of r_{cut} in the (anti)quark-gluon, gluon-gluon, and (anti)quark-(anti)quark partonic channels compared to the corresponding result obtained with CS. The results nicely converge to the CS values in all the channels, and also in this case the r_{cut} dependence is linear.

Finally, in view of potential applications of k_T^{ness} as a probe of jet production in hadron collisions, we study the stability of our new variable under hadronization and MPIs. We have generated a sample of LO events for $Z + $jet with the POWHEG Monte Carlo event generator [50–52] and showered them with PYTHIA8 [53] using the A14 tune [54]. We use the same setup as for $H + \text{jet}$, now setting $\mu_R = \mu_F = m_Z$ and adding an additional requirement on the leading jet rapidity $|y^{j_1}| < 2.5$. We define the (dimensionless) 1-jettiness event shape τ_1 as in Ref. [4]; the jet axis coincides with the direction of the leading jet reconstructed using the FASTJET code [34] (which we also use in each step of the k_T clustering algorithm used to compute k_T^{ness}). This simply corresponds to choosing Q_l as the partonic center-of-mass energy Q in Eq. (5) and to defining $\tau_1 = \mathcal{T}_1/Q$. Our results are shown in Fig. 3. The left panel shows the 1-jettiness distribution, while the right panel depicts the 1-k_T^{ness} result. The result obtained at parton level (red) is compared with the result including hadronization corrections (blue) and further adding MPIs (green). The bands are obtained by varying μ_F and μ_R by a factor of 2 around their central value with the constraint $1/2 < \mu_F/\mu_R < 2$. The 1-jettiness distribution has a Sudakov peak at $\tau_1 \sim 0.02$. The hadronization corrections are relatively large in the region of the peak and remain on the order of 10% as τ_1 increases. The inclusion of MPIs drastically changes the shape of the distribution, the peak moving to $\tau_1 \sim 0.15$. The 1-k_T^{ness} distribution displays a Sudakov peak at 1-$k_T^{\text{ness}} \sim 15$ GeV, similar to what we would expect from a transverse-momentum distribution of a colorless system produced by gluon fusion. The position of the peak and the shape of the distribution remain rather stable when hadronization is included, while the inclusion of MPIs makes the distribution somewhat harder. Comparing the left and right panels of Fig. 3 we clearly see that the k_T^{ness} distribution is significantly more stable against the inclusion of hadronization and MPIs, and it is therefore a good candidate for QCD studies in multijet production at hadron colliders. This could have maybe been expected, since variables based on transverse momenta are known [3] to be mildly sensitive to hadronization and underlying events.

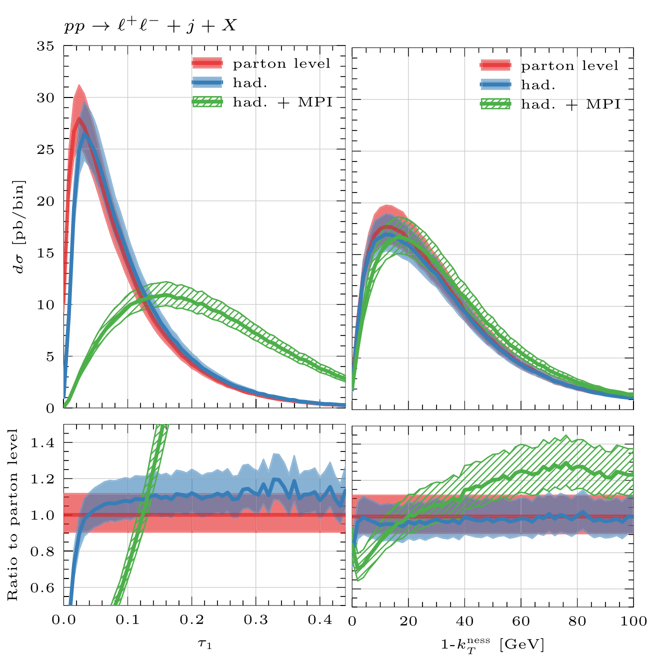

FIG. 3. $Z + \text{jet}$ at LO + parton shower: τ_1 (left) and 1-k_T^{ness} (right) spectra at the parton level (red) and including hadronization (blue) or hadronization and MPIs (green).

IV. SUMMARY AND OUTLOOK

In this work we have introduced the new variable k_T^{ness} to describe multiple jet production in hadronic collisions. The variable represents an effective transverse momentum controlling the singularities of the $N + 1$-jet cross section when the additional jet is unresolved. The k_T^{ness} variable can be used to perform higher-order QCD calculations by using a nonlocal subtraction scheme, analogously to what is done for q_T and jettiness. We have computed the singular limit of the $N + 1$-jet cross section as $k_T^{\text{ness}} \to 0$ and we have used the results to evaluate NLO corrections to $H + \text{jet}$ and $Z + 2$-jet production at the LHC, finding complete agreement with results obtained with standard NLO tools. Compared to jettiness, power corrections are under much better control and are linear, without logarithmic enhancements. This scaling behavior, which is due to the fact that k_T^{ness} is an effective transverse momentum, is expected to be a general property holding for an arbitrary number of jets, since no additional perturbative ingredients appear beyond the two jet case.[2] The extension of our calculations to NNLO will require a significant amount of conceptual and technical work, but our results suggest that, once the missing perturbative ingredients are available, an NNLO scheme based on k_T^{ness} could be constructed and implemented in a relatively simple way, as done for q_T in the case of colorless final states and heavy quarks [27,57,58]. Additionally, the perturbative coefficients entering the NLO and NNLO calculations are necessary ingredients to study the all-order structure of k_T^{ness} at next-to-leading logarithmic accuracy and beyond. We have also shown that k_T^{ness} appears to be very stable with respect to hadronization and multiple-parton interactions, thereby offering new opportunities for QCD studies in multijet production at hadron colliders. Being an effective transverse momentum, we also expect that k_T^{ness} could be used as a resolution variable when matching NNLO computations to k_T-ordered parton showers for processes with one or more jets at the

[2]We note that such linear scaling behavior has a *dynamical* origin and, therefore, cannot be captured through recoil effects as in Refs. [55,56].

Born level. We look forward to further studies in these directions.

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APPENDIX: JET AND SOFT FUNCTIONS

In the following we discuss the evaluation of the perturbative coefficient $\mathcal{H}^{\mathrm{F+N\,jets}}_{\mathrm{NLO}}$ controlling the first term in the subtraction formula (3). To set up our notation we consider the N-jet partonic process,

$$c(p_c) + d(p_d) \to j(p_1) + j(p_2) + \cdots + j(p_N) + F(p_F), \tag{A1}$$

which contributes to Eq. (1) at the Born level. The perturbative coefficient $\mathcal{H}^{\mathrm{F+N\,jets}}_{cd;ab}$ to be convoluted with the partonic cross section $d\hat{\sigma}^{\mathrm{F+N\,jets}}_{\mathrm{LO}\,cd}$ can be written as

$$\mathcal{H}^{\mathrm{F+N\,jets}}_{cd;ab} = (\mathbf{HS})_{cd} C_{ca} C_{db} \prod_{i=1,\ldots,N} J_i, \tag{A2}$$

where C_{ca} and C_{db} are the customary collinear functions appearing in the q_T subtraction formalism, J_i are the jet functions describing collinear radiation to each of the final-state partons, and an appropriate sum over final-state parton flavors is understood. The explicit expressions of the jet functions read

$$J_i^f = \begin{cases} 1 + \frac{\alpha_S(\mu_R)}{\pi}\left\{C_A\left[\frac{131}{72} - \frac{\pi^2}{4} - \frac{11}{6}\ln(2D) - \ln(D)\ln\left(\frac{Q^2}{4E_i^2}\right) - \ln^2(D)\right] + T_R n_f\left[-\frac{17}{36} + \frac{2}{3}\ln(2D)\right]\right\} + \mathcal{O}(\alpha_S^2) & \text{if } f = g \\ 1 + \frac{\alpha_S(\mu_R)}{\pi} C_F\left[\frac{7}{4} - \frac{\pi^2}{4} - \frac{3}{2}\ln(2D) - \ln(D)\ln\left(\frac{Q^2}{4E_i^2}\right) - \ln^2(D)\right] + \mathcal{O}(\alpha_S^2) & \text{if } f = q, \bar{q} \end{cases}$$

where E_i is the jet energy in the partonic center-of-mass frame. The contribution $(\mathbf{HS})_{cd}$ is given by

$$(\mathbf{HS})_{cd} = \frac{\langle \mathcal{M}_{cd} | \mathbf{S} | \mathcal{M}_{cd} \rangle}{|\mathcal{M}^{(0)}_{cd}|^2}, \tag{A3}$$

where $|\mathcal{M}_{cd}\rangle$ is the UV renormalized virtual amplitude after the subtraction of infrared singularities,[3] which admits a perturbative expansion in $\alpha_S(\mu_R)$. The soft-parton factor $\mathbf{S}$ is an operator in color space and can be expanded as

$$\mathbf{S} = 1 + \frac{\alpha_S(\mu_R)}{\pi}\mathbf{S}^{(1)} + \mathcal{O}(\alpha_S^2). \tag{A4}$$

The computation of the soft factor $\mathbf{S}^{(1)}$ can be carried out as follows. We consider the emission of a soft gluon with momentum k from the Born level process in Eq. (1). We work in the partonic center-of-mass frame and we parametrize the momentum k as

$$k = k_t(\cosh(\eta), \sin(\phi), \cos(\phi), \sinh(\eta)). \tag{A5}$$

We start from the squared eikonal current $\mathbf{J}^2 = \mathbf{J}^2(k)$ and we reorganize it to explicitly subtract initial- and final-state collinear singularities. We obtain

$$\begin{aligned} \mathbf{J}^2_{\mathrm{sub}} = {} & \left(-\mathbf{T}_c \cdot \mathbf{T}_d \omega_{cd} - \sum_i (\mathbf{T}_c \cdot \mathbf{T}_i \omega_{ci} + (c \leftrightarrow d)) - \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \omega_{ij}\right)\Theta(r_{\mathrm{cut}} - k^{\mathrm{ness}}_{T,S}/Q) \\ & - (\mathbf{T}_c^2 \omega^c_d + (c \leftrightarrow d))\Theta(r_{\mathrm{cut}} - k_t/Q) - \sum_i \mathbf{T}_i^2 \omega_{C_i S}\Theta(r_{\mathrm{cut}} - k^{\mathrm{ness}}_{T,C_i S}/Q), \end{aligned} \tag{A6}$$

where the sum runs over the labels of the final-state jets (i.e., $i, j = 1, \ldots, N$). The eikonal kernels $\omega_{\alpha\beta}$, ω^α_β are defined as

$$\omega_{\alpha\beta} \equiv \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k)(p_\beta \cdot k)} \qquad \omega^\alpha_\beta \equiv \frac{p_\alpha \cdot p_\beta}{(p_\alpha \cdot k)((p_\alpha + p_\beta) \cdot k)} \tag{A7}$$

[3] Our definition of the finite part of the one-loop amplitude corresponds to the conventions of Binoth Les Houches Accord [59].

and ω_{C_iS} as

$$\omega_{C_iS} \equiv \frac{p_i \cdot (p_c + p_d)}{(p_i \cdot k)((p_c + p_d) \cdot k)}, \tag{A8}$$

where α and β denote initial- and/or final-state partons. In the expression in Eq. (A6) we have included phase space constraints that limit the integration to the region $k_T^{\rm ness}/Q < r_{\rm cut}$. In particular, $k_{T,S}^{\rm ness}$ is the soft limit of the resolution variable $k_T^{\rm ness}$,

$$k_{T,S}^{\rm ness} = \min(1, \{\Delta R_{ik}\}/D) k_t, \tag{A9}$$

while $k_{T,C_iS}^{\rm ness}$ is the soft limit of the $k_T^{\rm ness}$ approximation used in the final-state collinear limit and it can be expressed, in the partonic center-of-mass frame, as

$$(k_{T,C_iS}^{\rm ness})^2 = k_t^2 \frac{\cosh(\eta) 2(\cosh(\eta - y_i) - \cos(\phi - \phi_i))}{\cosh(y_i) D^2}. \tag{A10}$$

The integration of the initial-state collinear contributions produce the customary collinear coefficient functions C_{ca}, while the integration of the final-state collinear contribution produces the jet functions J_i. The leftover soft contributions can be obtained by integrating the subtracted soft current $\mathbf{J}^2_{\rm sub}$ over the radiation phase space. More precisely, the soft integrals in Eq. (A6) produce $1/\epsilon$ poles and logarithmic terms in $r_{\rm cut}$. In order to analytically extract them we define a new subtracted current as follows:

$$\mathbf{J}^2_{\rm sub} = \mathbf{J}^2_{\rm sing} + (\mathbf{J}^2_{\rm sub} - \mathbf{J}^2_{\rm sing}) \equiv \mathbf{J}^2_{\rm sing} + \mathbf{J}^2_{\rm fin}, \tag{A11}$$

where $\mathbf{J}^2_{\rm sing}$ is still singular in the soft-wide-angle limit, while

$$[\mathbf{J}^2_{\rm fin}] \equiv 8\pi^2 \mu^{2\epsilon} \frac{1}{(2\pi)^{D-1}} \int d^D k \delta_+(k^2) \mathbf{J}^2_{\rm fin} \tag{A12}$$

is finite in $D = 4$ dimensions and can be computed numerically. The soft-singular term $\mathbf{J}^2_{\rm sing}$ can be defined as

$$\mathbf{J}^2_{\rm sing} = \sum_i \mathbf{T}_c \cdot \mathbf{T}_i ((\omega^c_d - \omega^c_i)\Theta(r_{\rm cut} - k_t/Q) + (\omega_{C_iS} - \omega^i_c)\Theta(D r_{\rm cut} - k_{i\perp}/Q)) + (c \leftrightarrow d)$$
$$+ \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j (\omega_{C_iS} - \omega^i_j)\Theta(D r_{\rm cut} - k_{i\perp}/Q), \tag{A13}$$

where the sum runs over the labels of the final-state partons and $k_{i\perp}$ is the transverse momentum of k with respect to the i-jet direction (in the partonic center-of-mass frame),

$$k_{i\perp}^2 = k_t^2 \frac{2(\cosh(\eta - y_i) - \cos(\phi - \phi_i))(\cosh(\eta + y_i) + \cos(\phi - \phi_i))}{\cosh(2y_i) + 1}. \tag{A14}$$

The integration of $\mathbf{J}^2_{\rm sing}$ produces poles in $1/\epsilon$ and logarithmic terms in $r_{\rm cut}$. The $1/\epsilon$ poles, together with the $1/\epsilon$ and $1/\epsilon^2$ poles coming from the initial- and final-state collinear integrals, cancel the corresponding poles in the virtual contribution. The logarithmic contributions in $r_{\rm cut}$ produce the counterterm in Eq. (4). The finite remainder from the integration of $\mathbf{J}^2_{\rm sing}$ is

$$[\mathbf{J}^2_{\rm sing}]_{\rm fin} = -\frac{1}{2}\left\{ \sum_i \mathbf{T}_c \cdot \mathbf{T}_i \left[\mathrm{Li}_2\left(-\frac{(p_d \cdot p_i)}{(p_c \cdot p_d)} \right) + \mathrm{Li}_2\left(-\frac{(p_d \cdot p_i)(p_c \cdot p_d)}{((p_c + p_d) \cdot p_i)^2} \right) + 2\ln(D) \ln\left(\frac{(p_c \cdot p_i)(p_c \cdot p_d)}{(p_i \cdot (p_c + p_d))^2} \right) \right] + (c \leftrightarrow d) \right.$$
$$\left. + \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left[\mathrm{Li}_2\left(-\frac{((p_c + p_d) \cdot p_j)^2}{4(p_c \cdot p_d)(p_i \cdot p_j)} (1 - \cos^2\theta_{ij}) \right) + 2\ln(D) \ln\left(\frac{(p_i \cdot p_j)(p_c \cdot p_d)}{p_i \cdot (p_c + p_d) p_j \cdot (p_c + p_d)} \right) \right] \right\}, \tag{A15}$$

where

$$\cos\theta_{ij} = 1 - \frac{2(p_c \cdot p_d)(p_i \cdot p_j))}{p_i \cdot (p_c + p_d) p_j \cdot (p_c + p_d)}. \tag{A16}$$

Finally, the soft factor $\mathbf{S}^{(1)}$ can be evaluated as

$$\mathbf{S}^{(1)} = [\mathbf{J}^2_{\rm sing}]_{\rm fin} + [\mathbf{J}^2_{\rm fin}]. \tag{A17}$$

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