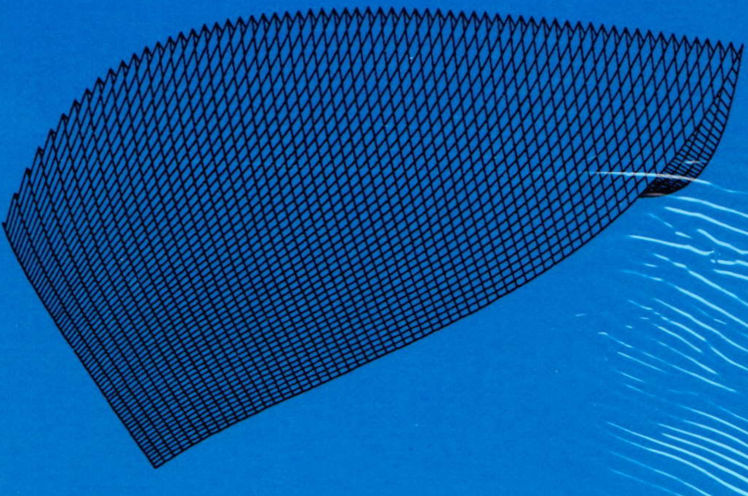
IDENTIFICATION AND CONTROL IN SYSTEMS GOVERNED BY PARTIAL DIFFERENTIAL EQUATIONS



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Proceedings of the conference on Control and Identification of Partial Differential Equations, part of the Summer Research Conferences in the Mathematical Sciences. Mount Holyoke College, South Hadley, Massachusetts, July 11–16, 1992.

This conference was sponsored by the American Mathematical Society, the Society for Industrial and Applied Mathematics, and the Institute of Mathematical Statistics and was supported in part by a grant from the National Science Foundation.

PREFACE

The 1992 AMS-IMS-SIAM Joint Summer Research Conference on Control and Identification of Partial Differential Equations was held at Mount Holyoke College, July 11-16. It was an active and stimulating conference, with about 40 participants from the United States as well as Canada, France, and Austria. The objectives of this conference were to provide up-to-date information on recent developments and results in control, identification, and mathematical modeling of partial differential equations and to stimulate further research in the area of control and identification for distributed parameter systems. This volume, which provides a record of some of the presentations and discussions that took place during this meeting, reflects current trends in control and system identification for partial differential equations. In the following we summarize the contributions contained in this volume. The summaries are grouped according to research topics.

Mathematical Modeling

Mathematical models for piezoceramic actuators in smart material structures are presented by H. T. Banks and R. C. Smith. The models are developed for actuation via bending moments and in-plane forces induced by imbedded piezoceramic patches. Use of the control concepts is illustrated by feedback synthesis for an example from structural acoustics. H. T. Tran, J. F. Scroggs, and K. J. Bachmann consider mathematical models for flow dynamics in a vertical reactor for high pressure vapor transport of materials for compound semiconductors. Numerical simulations are performed to address design issues for the Scholz geometry of the reactor. In addition, various control problems related to the optimal reactor design are outlined.

Parameter Estimation

G. Crosta considers the problem of estimating spatially varying conductivity in the one dimensional heat equation. Injectivity and stability of the inverse of parameter-to-solution mappings are studied under various Cauchy data on the parameter. A statistical analysis of error distribution in the least squares parameter estimation problem is studied by B. G. Fitzpatrick and G. Yin. A bootstrap method for inference, including confidence intervals and tests of normality, is developed. M. Kroller and K. Kunisch develop a software based on MATLAB for the estimation of parameters in two point boundary value problems. This can be used to investigate how different discretizations and regularization terms affect the behavior of solutions to an ill-posed inverse problem. The problem of estimating flexural rigidity in a von Kármán plate equation using dynamical point-observations of deformations is discussed by L. White. The problem is formulated as a norm-constrained minimization problem in Hilbert spaces. A numerical approximation based on Galerkin approximations is developed, analyzed, and illustrated with numerical test examples.

Optimal Control

A. Bensoussan and P. Bernhard consider robust stabilization of infinite dimensional systems. The problem is formulated as a standard problem of H_∞-optimal control. The optimal feedback synthesis based on Riccati equations is derived. A shape optimization problem that arises in design of a forebody simulator for a free-jet engine test facility is studied by J. Borggard, J. Burns, E. Cliff, and M. Gunzberger. Sensitivity equations for the sate equation with respect to the design parameters are derived and used for accurate gradient calculations. A linear quadratic regulator problem for systems governed by second order elliptic equations with Neumann boundary control is studied by L. Ji and G. Chen. The cost functional is defined by the square of norm of tracking error at the sensored points on the boundary. Regularity results for the optimal state and numerical results based on the boundary element method are presented. J. A. Reneke considers a control algorithm based on the reproducing kernel Hilbert space method for hereditary systems where the system is described by the input-output covariance function. Numerical examples are given to illustrate the approach.

Feedback Stabilization

The problem of global exponential stabilization of a von Kármán plate by boundary velocity feedback is discussed by M. E. Bradley and I. Lasiecka. The energy multiplier method and microlocal analysis are used to obtain global exponential stability of the controlled dynamics. C. I. Byrnes, D. S. Gilliam, and V. I. Shubov consider a boundary control for Burgers' equation with flux control at one end. It is shown that the uncontrolled dynamics are not asymptotically stable. Global existence, stability, and compactness of solutions to controlled dynamics with velocity feedback are established. Well-posedness of feedback control systems described by the triple (A, B, C) on Hilbert spaces is discussed by K. A. Morris. It is shown that well-posed systems remain stable under bounded feedback control. H. Ozbay and J. Turi consider a class of control systems described by singular integro-differential equations. Existence of a finite dimensional stablizing compensator (output feedback control) is shown employing modern frequency domain techniques. Feedback control of a one-link flexible robot arm is discussed by T. J. Tarn, A. K. Bejczy, and C. Guo. The problem is formulated as a nonlinear distributed parameter control problem. A sampled output feedback control with periodic gain is proposed and analyzed for non-colocated sensor/actuator system dynamics.

The success of the conference was greatly enhanced by active participation of attendees, and the organizers/editors are most grateful to them for their contributions. Special thanks and appreciation are due Carole Kohanski, the conference coordinator for the American Mathematical Society, and the AMS staff for their help in all organization, preparation, and administrative matters. The conference was sponsored by the National Science Foundation through a grant to the AMS.

CONTENTS

Chapter 6
Some Stability Estimates for the Identification of Conductivity in the One - Dimensional Heat Equation*

Giovanni Crosta†

Abstract

The stability of position - dependent conductivity in one spatial dimension is considered. This inverse problem is assumed throughout to have at least one measurable, bounded and strictly positive solution. Since conductivity satisfies an ordinary differential equation (ODE), uniqueness conditions may result from information of local or non - local type. Local information corresponds to a Cauchy datum, which can be supplied either at a regular or at a critical point; at the latter temperature is stationary (singular Cauchy problem). Non - local information is supplied as the domain average of thermal flux at a given instant of time. The main purpose of the paper is to provide a unified view over the stability estimates pertaining to the unique solution. Some additional restrictions (regularization) are imposed on the temperature data. If uniqueness is due to a regular Cauchy problem, L^{∞}– estimates are obtained. Singular problems, on the other hand, yield L^r -estimates, $1 \leq r < \infty$. Non local conditions are treated similarly. The unifying device is the defect equation, an ODE for conductivity differences in a space of distributions. Estimates are arrived at by suitably integrating said ODE. Some examples and counterexamples are provided.

Introduction

One of the best known inverse problems is the identification of position dependent conductivity $a(.)$, the leading coefficient appearing in the one - dimensional heat equation

$$(0.1) \qquad (au_x)_x = u_t + f \quad \text{in } Q := (x_0, x_1) \times (t_0, t_1) \ ,$$

from knowledge of the (thermal) *potential* u and of the source term f in the whole interval at one or more instants of time.

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