



# Social sustainability in Italy: a model-based hierarchical composite indicator

Giorgia Zaccaria<sup>1</sup> · Cinzia Di Nuzzo<sup>2</sup>

Received: 22 October 2024 / Accepted: 23 June 2025 / Published online: 2 September 2025  
© The Author(s) 2025

## Abstract

This study aims to develop a hierarchical composite indicator using a model-based approach to measure social sustainability in Italy. The proposed indicator is built by integrating manifest variables, including education, quality of life, social relationships, and participation in cultural activities, while examining specific and broader latent dimensions they reflect or form. The hierarchical structure of the social sustainability composite indicator is derived using the Hierarchical Disjoint Principal Component Analysis. Ultimately, this methodology identifies intermediate latent concepts between the manifest variables and the general concept, offering meaningful insights for comprehensively understanding the phenomenon under study, and also allows for testing the nature of the relationships between two sequential levels of the hierarchy. Comparisons across the Italian regions, years (pre- and post-COVID), and genders are provided, enhancing the understanding of social dynamics affecting sustainability in a context marked by significant regional diversity.

**Keywords** Hierarchical models · Parsimony · Reflective model · Formative model · Multidimensional concepts · Italian disparities

## 1 Introduction

Sustainability is a multidimensional concept that traditionally comprises three dimensions: environmental, economic and social (Purvis et al., 2019). This is a key part of the Agenda 2030 through the Sustainable Development Goals (United Nations General Assembly, 2019), which are 17 thematic areas toward which countries should address their efforts.

---

✉ Giorgia Zaccaria  
giorgia.zaccaria@unimib.it  
Cinzia Di Nuzzo  
cinzia.dinuzzo@unict.it

<sup>1</sup> Department of Statistics and Quantitative Methods, University of Milano-Bicocca, Via Bicocca degli Arcimboldi 8, 20100 Milan, Italy

<sup>2</sup> Department of Economics and Business, University of Catania, Corso Italia, 55, 95129 Catania, Italy

When focusing specifically on social sustainability, this represents a complex and dynamic concept that can be conceptualized as the synthesis of a set of dimensions contributing to its definition (McGuinn et al., 2020). It pertains to quality of life, both objective and perceived (Schiefer & Van der Noll, 2017), well-being (Berger-Schmitt & Noll, 2000), social capital (Dempsey et al., 2010), social inclusion, cultural participation, civic engagement, and education (Shirazi & Keivani, 2019). Cultural aspects are part of social sustainability, as culture influences how individuals perceive their roles in society and how they collaborate in addressing collective challenges. Social structures contribute, in turn, to the preservation and evolution of cultural identity, making these two dimensions strongly interconnected. For these reasons, in recent years, cultural sustainability has increasingly been considered as a separate dimension (Hawkes, 2001). Nonetheless, since its recognition is still under debate and not fully established, this paper adopts the traditional approach to defining sustainability, which considers only social sustainability as a proper dimension alongside the environmental and economic ones.

Due to its multidimensional nature, we study social sustainability by defining a set of Composite Indicators (CIs) that quantify its latent dimensions. In general, CIs are useful tools for conveying and synthesizing information on phenomena that are not directly observable. They are defined as latent variables resulting from the aggregation of manifest variables into a single synthetic measure (Nardo et al., 2005; OECD-JRC, 2008). Two types of CIs can be distinguished based on different levels of abstraction in the dimensions they measure. Specifically, the General Composite Indicator (GCI) quantifies the phenomenon under study, while the Specific Composite Indicators (SCIs) represent its latent dimensions. However, due to their complexity, multidimensional phenomena often exhibit nested latent dimensions from the most specific (first-order SCIs) to broader ones (higher-order SCIs). This hierarchical structure reflects the intrinsic nature of such phenomena and highlights the need for a hierarchical approach to build a comprehensive GCI and explore its dimensions.

In defining the hierarchy that leads to GCI, the nature (“direction”) of the relationships among variables in two sequential hierarchical levels—which formally constitutes the measurement model (Blalock, 1964; Bollen & Bauldry, 2011)—must be analyzed. Two kinds of measurement models exist (Bollen, 2001): reflective, where a set of correlated observed variables reflects an unobserved one (top-down approach), and formative, where the latent variable is formed by manifest variables that are not interchangeable (bottom-up approach). It should be noted that the measurement model is usually chosen a priori by the researcher rather than tested via a statistical procedure. However, a theoretical definition of the relationships among variables is not always available or experimentally confirmed, and a test to determine the proper direction is required. Following the same rationale, a second distinction concerns the research strategy. If a theory on a multidimensional phenomenon is available, all the relationships among manifest variables and latent constructs are known and set a priori in a confirmatory (theory-driven) approach. On the other hand, an exploratory analysis can be conducted when all the relationships among manifest variables and latent constructs are not known a priori and must be explored (data-driven approach). These two approaches can be combined when some relationships are known according to a specific theory, while others remain unknown and need to be assessed through an exploratory analysis.

The specialized literature on the construction of composite—and more generally, synthetic—indicators divides methods as either non-aggregative or aggregative. The former

includes Partially Ordered SET (POSET, Neggers & Kim, 1998; Schroder, 2002), applicable to both ordinal (Fattore, 2016; Fattore & Alaimo, 2023) and mixed systems (Brugemann & Patil, 2011; Alaimo et al., 2021; Alaimo, 2022), and avoids the need to determine arbitrary aggregation or weighting schemes. In contrast, the aggregative methods are based on the definition of weights. The latter are usually assigned subjectively, based on expert evaluation, which often draws criticism for the lack of quantitative assessment. To overcome this limitation, a set of CIs can be constructed using a statistical procedure, i.e., an approach based on a model and hence referred to as “model-based”, with a mathematical formalization that limits potentially arbitrary choices. We focus here on the literature concerning aggregative and hierarchical models, where the latter properly take the nature of multidimensional and complex phenomena into account. Existing methodologies for studying hierarchical relationships among variables via an exploratory approach based on covariances or correlations include Higher-Order Factor Models (HOFMs, Cattell, 1978; Rindskopf & Rose, 1988; Le Dien & Pagès, 2003), Second-Order Disjoint Factor Analysis (2O-DFA, Cavicchia & Vichi, 2022), and Ultrametric Composite Indicator Model (UCIM, Cavicchia et al., 2024). HOFMs are sequential procedures that repeat the application of Factor Analysis (Spearman, 1904) with a reduced number of factors each time. Although they build a hierarchy of latent factors, the main drawback of HOFMs is that they do not optimize an overall objective function, which may lead to inaccurate detection of hierarchical relationships among variables. On the other hand, 2O-DFA and UCIM address this limitation through a simultaneous approach. The former obtain first-order SCIs via a Disjoint Factor Analysis (Vichi, 2017) and aggregates them into the GCI, while the latter detects a hierarchical structure through the estimation of an ultrametric correlation matrix derived from the observed one. Although these are simultaneous models that search for an optimal hierarchical solution, 2O-DFA is restricted to two hierarchical levels, which prevents detecting nested and broader latent dimensions between the first-order SCIs and GCI. In contrast, UCIM allows for the construction of a hierarchy composed of several levels, but it involves the quantification of latent dimensions only a posteriori, rather than within the estimation process itself.

In this paper, we delve into social sustainability in Italy by considering a simultaneous and exploratory methodology that constructs a GCI, detects and quantifies its nested latent dimensions, and tests the nature of the relationships (reflective or formative), while avoiding any subjective choices. The key feature of the model, which is novel in the literature on CI construction, is dual. Firstly, it lies in its hierarchical nature, which allows for exploring intermediate levels of SCIs and their aggregations (in order) into broader dimensions of social sustainability. Secondly, it differentiates between the reflective and formative approach without imposing a priori constraints on the relationships or their nature, instead testing them via a statistical tool. For the reflective part of the hierarchy, a Principal Component Analysis (PCA, Pearson, 1901; Hotelling, 1933) approach is used by following Mazziotta and Pareto (2019), who demonstrated this is further suitable for reflective than formative approaches. The methodology is parsimonious, as it first identifies a partition of the variable space into disjoint groups, each associated with a latent concept quantified via a first-order SCI, and then builds a hierarchy of higher-order SCIs over them under the nestedness assumption. It is worth noting that although the disjoint structure may be restrictive in some cases, it improves latent concept interpretation. Moreover, the methodology can reconstruct potential interactions among dimensions in subsequent hierarchical levels,

avoiding the need for overlapping variable groups, i.e., cross-loadings between observed and latent variables. Since the number of first-order SCIs must be fixed in advance, we extend herein the generalized cross-validation criterion introduced by Josse and Husson (2012) for PCA to the case in which the principal components synthesize information from smaller groups of variables in a disjoint structure. Furthermore, the unidimensionality of the first-order SCIs and the internal consistency of the variable groups are analyzed in constructing the GCI for social sustainability. The definition of a set of CIs for measuring this phenomenon is particularly relevant in Italy, given the country's complexity and diversity, which are reflected in strong regional differences at both the GCI and SCI levels. The detection and quantification of specific and broader dimensions allows for comparisons across regions, providing a useful tool for policymakers to formulate more effective and targeted public policies.

The rest of the paper is organized as follows. Section 2 introduces the notation and nomenclature used in Sect. 3, where we describe the model-based approach for building a set of CIs with a hierarchical configuration. In Sect. 4, we present the data and the analysis of social sustainability across the Italian regions in 2019 (pre-COVID) and 2022 (post-COVID). A final discussion completes the paper in Sect. 5.

## 2 Notation

The notation and nomenclature used in the following sections are listed below.

|                  |                                                                                                                                                                                                                     |
|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $n$              | Number of rows (units) of a data matrix.                                                                                                                                                                            |
| $J$              | Number of columns (observed, manifest variables) of a data matrix.                                                                                                                                                  |
| $Q$              | Number of variable groups and corresponding first-order SCIs of the first dimensionality reduction of the data.                                                                                                     |
| $M$              | Bottom-up hierarchical level defining the model type: if $M = 1$ , the model is reflective; if $M > 1$ , the model is mixed reflective/formative.                                                                   |
| $X$              | $(n \times J)$ data matrix.                                                                                                                                                                                         |
| $V_q$            | $(J \times q)$ binary matrix defining the partition (membership) of the $J$ observed variables into $q$ groups, with $q = M, \dots, Q$ .                                                                            |
| $Y_q$            | $(n \times q)$ component score matrix, where each disjoint principal component (SCI) is associated with a group of the variable partition into $q$ groups.                                                          |
| $B_q$            | Diagonal matrix of order $J$ , whose diagonal elements represent the loading of each variable on one of the disjoint principal components associated with the groups in the partition of variables into $q$ groups. |
| $g$              | $(n \times 1)$ vector, whose elements represent the GCI scores for the $n$ units of a data matrix.                                                                                                                  |
| $E_q$            | $(n \times J)$ error matrix associated with the variable partition into $q$ groups.                                                                                                                                 |
| $R_{Y_q}$        | Correlation matrix of the $q$ disjoint principal components in $Y_q$ .                                                                                                                                              |
| $\text{diag}(a)$ | Diagonal matrix whose diagonal entries correspond to the elements of the vector $a$ .                                                                                                                               |
| $\text{diag}(A)$ | Diagonal matrix whose diagonal entries correspond to the diagonal elements of the matrix $A$ .                                                                                                                      |
| $1_q, 1_J$       | $(q \times 1)$ and $(J \times 1)$ unitary vectors, respectively.                                                                                                                                                    |
| $I_q$            | Identity matrix of order $q$ .                                                                                                                                                                                      |

### 3 Method

The Hierarchical Disjoint Principal Component Analysis (HierDPCA) is a model that enables the building of hierarchies of latent concepts from observed (manifest) variables. It starts with a dimensionality reduction of  $J$  variables into  $Q$  disjoint groups, each associated with a principal component of maximum variance. After that, HierDPCA tests the correlation among the  $Q$  disjoint principal components allowing a switch from a reflective to a formative approach. Indeed, if the correlation turns out to be statistically significant, HierDPCA selects two groups from the variable partition to merge and computes the corresponding principal component of maximum variance for this broader group, thus reducing their number by one. The choice of which pair of variable groups to merge is based on the maximization of the variance explained by the corresponding  $Q - 1$  disjoint principal components. The model continues aggregating variable groups until no correlation among the disjoint principal components is statistically significant. When this occurs, HierDPCA completes the hierarchy via a regression model on the higher-order principal component, using principal components of lower-order as predictors.

Formally, by considering an  $(n \times J)$  centered data matrix  $X$ , HierDPCA is defined by the system of simultaneous equations

$$\left\{ \begin{array}{l} X = gV_1' B_1 + E_1 \quad \text{if } M = 1 \\ g = Y_M \beta + \epsilon \quad \text{if } M > 1 \\ X = Y_M V_M' B_M + E_M \\ \dots \quad \dots \quad \dots \\ X = Y_{Q-1} V_{Q-1}' B_{Q-1} + E_{Q-1} \\ X = Y_Q V_Q' B_Q + E_Q \end{array} \right. \quad (1)$$

where, for  $q = M, \dots, Q$ , with  $Q \leq J$ , the following constraints hold

$$V_q = [v_{jp(q)} \in \{0, 1\} : j = 1, \dots, J, p = 1, \dots, q] \quad (\text{binary}); \quad (2)$$

$$V_q 1_q = 1_J \quad (\text{row stochastic}); \quad (3)$$

$$\begin{aligned} V_{q-1} &= [V_q \setminus \{v_{q-1(q)}, v_{q(q)}\}, v_{q-1(q-1)}] \quad (\text{nested partitions}); \\ v_{q-1(q-1)} &= v_{q-1(q)} + v_{q(q)} \end{aligned} \quad (4)$$

$$B_q = \text{diag}([b_{1(q)}, \dots, b_{J(q)}]) \quad (\text{diagonal}); \quad (5)$$

$$V_q' B_q B_q V_q = I_q \quad (\text{semi-orthogonal}). \quad (6)$$

In detail, at the  $q$ -th level of the hierarchy, starting from  $q = Q$ ,  $V_q$  represents the  $(J \times q)$  variable-group membership matrix,  $B_q$  is the  $(J \times J)$  diagonal matrix with diagonal elements identifying to the weights (loadings) of the observed variables on the corresponding principal components (one loading per variable),  $Y_q$  is the  $(n \times q)$  component score matrix, and  $E_q$  is the  $(n \times J)$  error matrix. The parameterization of the loading matrix via  $B_q V_q$ , subject to constraints (2), (3), and (5), ensures a sparse structure that facilitates the inter-

pretation of the principal components, as each variable belongs to only one group of the partition into  $q$  groups and therefore contributes to defining a single principal component. Additionally, the disjoint configuration implies relaxing the assumption of uncorrelated principal components, supporting the construction of a hierarchy over them. Specifically, the hierarchical structure in (1) is guaranteed by constraint (4), which allows for computing the variable partition at the higher-order level (i.e.  $V_{q-1}$ ) by merging two columns from the partition of the lower-order level (i.e.  $V_q$ ). As a result,  $V_{q-1}$  and  $V_q$  share  $q - 2$  identical columns. The merged columns are identified as the last two of  $V_q$  in constraint (4), although this is merely a convention for mathematical formalization.

The first two equations in (1) distinguish between a fully reflective and a mixed reflective/formative model. The value of  $M$ , which identifies this distinction, is set as the first bottom-up level at which no statistically significant correlation occurs among the disjoint principal components. This is determined by testing whether the correlation matrix of  $Y_q$  ( $q = Q, \dots, 2$ ) equals the identity matrix, with  $R_{Y_q} = I_q$  as the null hypothesis. If the latter is not rejected, the  $q$  disjoint principal components in  $Y_q$  are uncorrelated and the hierarchy is completed using a formative approach. Conversely, if the null hypothesis is rejected, the disjoint principal components in  $Y_q$  are significantly correlated and a reflective approach is used to synthesize two of them into a higher-order component. The test used is the one proposed by Chen et al. (2010), which is non-parametric since no distributional assumptions are made about the data. Its test statistic is asymptotically normally distributed under the null hypothesis, with a p-value threshold of 0.05 in our experiments. If  $M = 1$ , the hierarchy built by HierDPCA is fully reflective, as the correlation among the disjoint principal components in  $Y_q$  is statistically significant for all  $q = Q, \dots, 2$ . The last aggregation yields a principal component  $g$ , that quantifies a general concept derived from all observed variables. Although this is not directly connected to the previous hierarchical level—as it could be by substituting  $X$  with  $Y_2$  in the first equation of (1)—it allows for the proper construction of a *composite* indicator at the last bottom-up level of the hierarchy, enabling the comparison of two GCIs with different underlying hierarchical structures. On the other hand, when  $M > 1$ , the model remains reflective up to  $M$  and then becomes formative; the last aggregation is obtained by regressing the uncorrelated  $Y_M$  on  $g$ . It is worth noting that, if  $Q < J$ , the first bottom-up aggregation of the  $J$  observed variables into  $Q$  disjoint groups always follows a reflective approach, by identifying the first-order SCIs represented by  $Y_Q$ . If the additional  $Y_q$  for  $q = Q - 1, \dots, M$  are computed, the higher-order SCIs are identified. Finally,  $g$  represents the GCI.

Before illustrating the estimation process of the model, we provide an example of the output obtained by HierDPCA. Let us consider  $J = 11$  observed variables partitioned into  $Q = 4$  groups, as reported in the following membership matrix, transposed for space reasons.

$$V'_4 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

We generate a data matrix in two different scenarios: a) the variables range from strongly to moderately correlated both within and between groups; b) all variables are strongly correlated within groups, while the variables in the first three groups are correlated among

themselves but show near-zero correlation with the last two. If we apply HierDPCA in scenario a), it constructs a reflective hierarchy over the four disjoint principal components in  $Y_4$ , each associated with a group of  $V_4$ , since all variables are correlated with each other (see Fig. 1a). The columns of  $Y_4$  correspond to the four first-order SCIs, and binary aggregations lead to the second- and third-order SCIs,  $Y_3$  and  $Y_2$  respectively, culminating in the GCI (g). The membership matrices corresponding to the disjoint principal components in  $Y_3$  and  $Y_2$  are

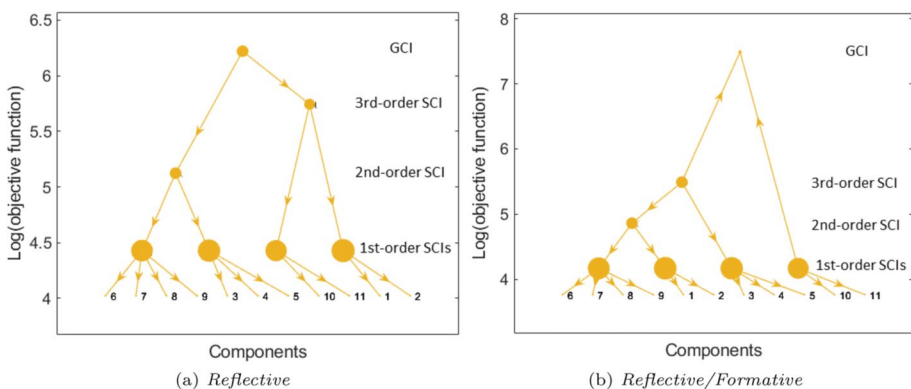
$$V'_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}, \quad V'_2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

while  $V_1 = 1_J$ . It is worth noting that since the variable groups are paired,  $q - 2$  columns of  $Y_q$  and  $Y_{q-1}$ , for  $q = 3, 4$ , are identical, as are those of the corresponding membership matrices. In scenario b), three levels of the hierarchy are obtained via a reflective approach, i.e.,  $q = 2, 3, 4$ . However,  $M = 2$ , since when  $q = 2$ , the correlation between the disjoint principal component aggregating variables 10 and 11 and the other component, which pertains to all other variables, is not statistically different from zero. Indeed, variables 10 and 11 are generated as uncorrelated with the others, and therefore, for the last level, the model switches to a formative approach, as shown in Fig. 1b (see the arrows changing direction, from top-down for the reflective approach to bottom-up for the formative one).

HierDPCA is estimated within the least-squares framework by minimizing the following loss function

$$F(\theta) = \sum_{q=M}^Q \|X - Y_q V'_q B_q\|^2 + \|X - g V'_1 B_1\|^2 I(M = 1) + \|g - Y_M \beta\|^2 I(M > 1) \quad (7)$$

subject to constraints (2)–(6), where  $I(\cdot)$  denotes the indicator function and  $\theta$  is the parameter set. If the model is fully reflective, the second term in (7) is added to the first and  $\theta = \{B_1, \dots, B_Q, V_2, \dots, V_Q, g, Y_2, \dots, Y_Q\}$ ; if it is mixed reflective and formative, the third term is added and  $\theta = \{\beta, B_M, \dots, B_Q, V_M, \dots, V_Q, g, Y_M, \dots, Y_Q\}$ . The minimization of the loss function in (7) represents a mixed continuous (for the estimation of the



**Fig. 1** Example of reflective and reflective/formative hierarchies built by HierDPCA

vectors  $\mathbf{g}$  and  $\beta$ , and the matrices  $B_q$  and  $Y_q$ ) and discrete (for the estimation of the matrices  $V_q$ ) problem.

The estimates of  $Y_q$  correspond to the disjoint principal components  $\hat{Y}_q = X\hat{B}_q\hat{V}_q$ , where the  $p$ -th column of  $\hat{V}_q$  is obtained as the principal component of maximum variance for the variables in the corresponding  $p$ -th group of  $\hat{V}_q$ ,  $p = 1, \dots, q$ . The diagonal elements of  $\hat{B}_q$  represent the weights of the linear combination. Therefore, if the  $j$ -th variable belongs to the  $p$ -th group of  $\hat{V}_q$ , its loading is  $\hat{b}_{j(q)}$ , that is the unique loading for the  $j$ -th variable at level  $q$ . This is obtained as the element of the eigenvector corresponding to the largest eigenvalue of the covariance matrix for the subset of variables belonging to the  $p$ -th group of  $\hat{V}_q$ .  $\hat{\mathbf{g}}$  is the principal component of maximum variance computed from all manifest variables. If  $M > 1$ ,  $\hat{\beta}$  is the coefficient of the linear model that regresses  $\hat{Y}_M$  on  $\hat{\mathbf{g}}$ . It should be noted that - when  $M = 1$  -  $\hat{V}_1$  is trivial, as aggregating the columns of  $\hat{V}_2$ , regardless of its configuration, results in a unitary vector.

A coordinate descent algorithm is used to implement HierDPCA. It initializes the parameters by starting from a random partition of the  $J$  manifest variables into  $Q$  groups, and then computes the loadings and the corresponding disjoint principal components. Next, the aforementioned test for correlation is performed, and the subsequent level of the hierarchy is obtained through a reflective or formative approach, depending on the test outcome, up to the completion of the hierarchy. This results in the achievement of  $\hat{\theta}^{(0)}$ . The procedure is iterated until  $F(\hat{\theta}^{(t)}) - F(\hat{\theta}^{(t-1)}) < \epsilon$ , where  $\epsilon$  is a small positive arbitrary constant (set to  $10^{-6}$  in our experiments). Given its iterative nature, the solution can depend on the random initial membership matrix. Therefore, a multistart approach is implemented to increase the chances of obtaining a global optimum, rather than a local one. In our experiments, we set 100 as the number of random starts (i.e., initial partitions), being conservative to ensure a stable solution (see Cavicchia et al., 2023, for further details).

The HierDPCA algorithm requires to fix  $Q$  in advance. This can be chosen according to the methods usually used in PCA, such as the Kaiser rule (see Jolliffe, 2002, among others). However, these methods are usually heuristic or subjective. In this paper, we use a model selection criterion that penalizes the residual sum of squares by the degrees of freedom of the residuals, balancing model fit and complexity. Specifically, we extend the Generalized Cross-Validation (GCV) criterion introduced by Josse and Husson (2012) in the PCA framework to the Disjoint Principal Component Analysis (DPCA, Ferrara et al., 2016), which is employed in (1) to obtain the first (bottom-up) partition  $V_Q$  of the variable space into  $Q$  disjoint groups, along with the corresponding matrices  $B_Q$  and  $Y_Q$ . The number of parameters in DPCA is  $nQ + 2J - Q$ , which is calculated by considering  $nQ$  parameters for estimating the  $Q$  disjoint principal components in  $Y_Q$ ,  $J$  for the diagonal elements of  $B_Q$ , and  $J - Q$  for the parameters in  $V_Q$ , where only one non-zero element per row must be considered, subject to the constraint of non-empty groups. GCV for DPCA is therefore

$$\text{GCV}_{\text{DPCA}}(Q) = \frac{nJ \times \left( \sum_{i=1}^n \sum_{j=1}^J (x_{ij} - \hat{x}_{ij}^{(Q)})^2 \right)}{(nJ - nQ - 2J + Q)^2}, \quad (8)$$

where  $\hat{x}_{ij}^{(Q)}$  refers to the  $i$ -th row and  $j$ -th column of the reconstructed matrix  $\hat{X}^{(Q)} = \hat{Y}_Q \hat{V}_Q' \hat{B}_Q$ . This model selection criterion emulates well-known information criteria such as the Akaike information criterion (Akaike, 1974) and the Bayesian information criterion (Schwarz, 1978), which are typically used in the maximum likelihood framework.

For constructing a proper set of CIs, it is essential that the resulting estimates are both internally consistent and unidimensional. Internal consistency refers to the ability of a group of manifest variables to coherently measure the same underlying concept. This is typically reflected by positive correlations among manifest variables within the same group, suggesting that they are associated with a reliable concept. If this does not occur, it may be necessary to change the polarity of some variables to avoid non-compensability in the CI, which should be performed as a pre-processing step (Cavicchia & Vichi, 2021). To assess the internal consistency for the variable groups identified by HierDPCA, we use Cronbach's  $\alpha$  (Cronbach, 1951), which can be computed at each level of the hierarchy. George and Mallery (2003) defined the reliability thresholds based on the Cronbach's  $\alpha$  as follows:  $0.70 \leq \alpha \leq 0.79$  as acceptable,  $0.80 \leq \alpha \leq 0.89$  as good, and  $\alpha \geq 0.90$  as excellent.

Unidimensionality, on the other hand, concerns whether a SCI measures a single underlying concept. In HierDPCA, this property is required only at the first bottom-up level of the hierarchy, i.e., for the SCIs corresponding to the  $Q$  disjoint principal components. Indeed, as we move up the hierarchy, where broader concepts are formed by combining multiple dimensions, unidimensionality is no longer required. To evaluate unidimensionality for each variable group, we assess the magnitude of the second eigenvalue of their covariance (or correlation) matrix. If this second eigenvalue is smaller than the average eigenvalue (or 1, in case of correlations), the corresponding SCI can be considered unidimensional. GCV usually selects a number of variable groups  $Q$  that corresponds to unidimensional SCIs. Indeed, by minimizing the residual sum of squares while balancing model complexity, it seeks the minimum number of disjoint principal components—those with maximum variance within each group—that best reconstruct the variability of the data.

These two properties are essential for obtaining a correct GCI based on nested dimensions. Therefore, it is crucial to verify their validity for the set of CIs identified by HierDPCA, as their fulfillment is desirable for drawing reliable conclusions about the phenomenon under study. If the first-order SCIs do not satisfy unidimensionality, the researcher may consider increasing the number of variable groups by one. Additionally, it is recommended to conduct an in-depth analysis of the manifest variables included in the study of the target phenomenon to better understand their relationships and how they relate to the phenomenon itself.

Finally, HierDPCA is not scale invariant due to its relationship with principal component analysis and regression. Therefore, we recommend standardizing the data if the observed variables have different measurement units or scales, as we will do in the analyses presented in the following section.

## 4 The hierarchical composite indicator of social sustainability

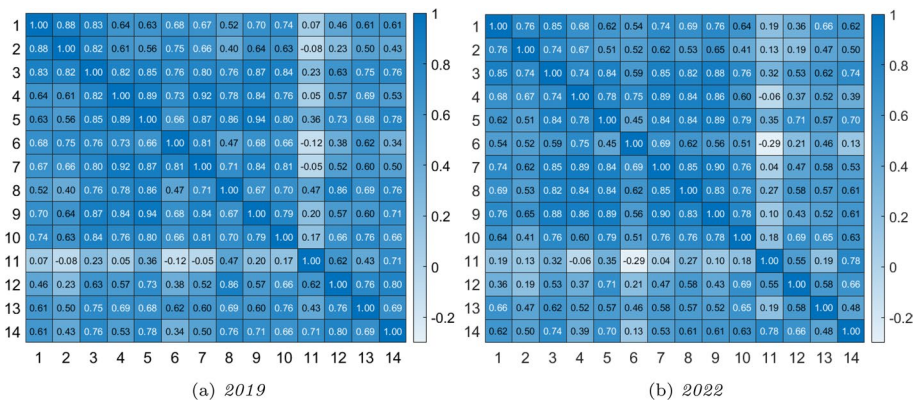
### 4.1 Data

The data used in the following analyses are provided by the Italian National Institute of Statistics, specifically from the BES (Equitable and Sustainable Well-being) system. The selection of manifest variables for constructing a composite indicator of social sustainability in Italy was guided by the aim of capturing various aspects of this multidimensional phenomenon, including education, cultural and social participation, subjective well-being, safety, and environmental perceptions, as discussed in Sect. 1. Both the conceptual relevance of the variables and their ability to adequately represent the analytical framework were considered. We selected 14 manifest variables, all measured as percentages, but standardized to z-score because of their different variability. These variables represent a balanced coverage of the main domains related to social sustainability. The data are collected for the 20 Italian regions (NUTS2) and pertain to the years 2019 (pre-COVID) and 2022 (post-COVID). Additionally, we consider them differentiated by gender for each year.

The full list of variables is reported in Table 1, and their correlation matrices for the two years considered are illustrated in Fig. 2. In detail, the first two variables—“Individuals with at least a diploma (aged 25–64)” and “Graduates or tertiary education holders (aged 25–34)”, which are strongly correlated—are essential for measuring the level of education and training of the Italian population, which is a key factor for social sustainability. Indeed, educational attainment is directly linked to the individuals’ ability to participate in economic, social, and cultural life, as well as their capacity to adapt to social and technological changes. The presence of NEET (variable 3) is instead a critical measure of social vulnerabilities, especially among younger generations since it is closely related to the future well-being of a society and its capacity to ensure the active participation of young people in social and economic life. Due to its meaning, we reverse this variable such that to obtain the right polarity of all manifest variables (“positive meaning” in this case with respect to social sustainability). The cultural engagement of the population, which is a central aspect of (social) sustainability, is reflected by the variables “Cultural participation outside the home” and “Reading books and newspapers” (variables 4 and 5, respectively). Indeed, participation in cultural activi-

**Table 1** Variable description and polarity

| ID | Variable name                                                 | Polarity |
|----|---------------------------------------------------------------|----------|
| 1  | Individuals with at least a diploma (aged 25–64)              | +        |
| 2  | Graduates or tertiary education holders (aged 25–34)          | +        |
| 3  | Young people not in employment, education, or training (NEET) | –        |
| 4  | Cultural participation outside the home                       | +        |
| 5  | Reading books and newspapers                                  | +        |
| 6  | Cultural and creative employment                              | +        |
| 7  | Regular internet users                                        | +        |
| 8  | Social participation                                          | +        |
| 9  | Civic and political participation                             | +        |
| 10 | Satisfaction with family relationships                        | +        |
| 11 | Perception of safety when walking alone at night              | +        |
| 12 | Life satisfaction                                             | +        |
| 13 | Satisfaction with leisure time                                | +        |
| 14 | Satisfaction with the environmental situation                 | +        |



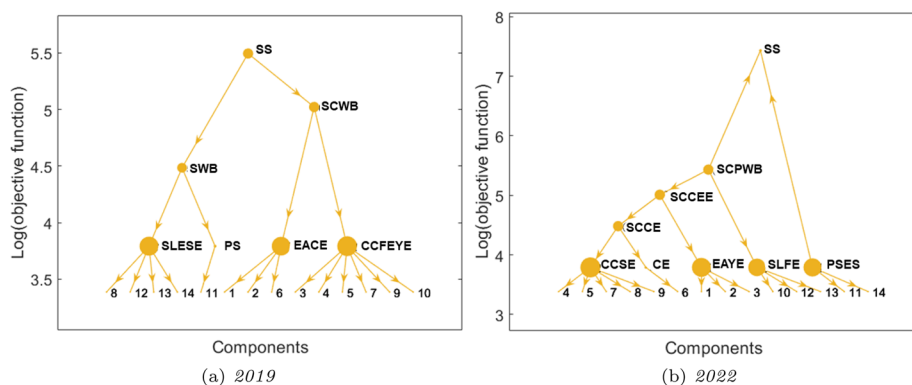
**Fig. 2** Correlation matrices of the data by year

ties and reading are factors that not only enhance quality of life but also contribute to social cohesion and the spread of cultural capital. Moreover, cultural and creative work along with regular internet use (variables 6 and 7) are crucial for innovation and sustainable development, since cultural employment promotes social and cultural identity, while internet access facilitates both economic and cultural participation. To measure active participation which contributes to social cohesion and democratic stability—fundamental aspects of a sustainable society—we consider the variables “Social participation” (variable 8) and “Civic and political participation” (variable 9). The remaining five variables measure satisfaction and perceptions for several aspects of life, from personal to interpersonal aspects, like “Satisfaction with leisure time” and “Satisfaction for family relationships” (variables 13 and 10, respectively), across safety and environmental thinking (variables 11 and 14), up to “Life satisfaction” (variable 12). Figure 2a shows that in 2019, variable 11, i.e., “Perception of safety when walking alone at night”, has negative or near-zero correlations with all variables except variables 8, 12, 13, and 14, with which it exhibits a high or moderately high correlation. In 2022, its correlation with variable 14 increases to 0.78, where variable 14 pertains to “Satisfaction with the environment situation”, while being very low with the other variables. Meanwhile, the correlation between variable 14 and the others decreases (Fig. 2b). This reflects into a shift from a reflective to a formative approach in the hierarchy construction across the two years, as we will see in the following section.

## 4.2 Results

We firstly illustrate the results of the HierDPCA application on the aforementioned data set across the two years, 2019 (pre-COVID) and 2022 (post-COVID). In both cases, to perform an exploratory analysis, we select  $Q$  according to the GCV criterion in Eq. (8). For 2019,  $Q = 4$ , while for 2022,  $Q = 5$ . The corresponding hierarchies are depicted in Fig. 3.

In 2019, the model identifies a fully reflective GCI of *Social Sustainability* (SS), which arises from the aggregation of the four variable groups at the first bottom-up level of the hierarchy (Fig. 3a). The four groups are composed as follows: the two variables measuring the level of education (variables 1 and 2) and “Cultural and creative employment” are merged representing the group of *Educational Achievement and Creative Employment*



**Fig. 3** Path diagram of the hierarchy estimated by HierDPCA to obtain the social sustainability composite indicator across the Italian regions by year

(EACE), with a higher loading of the first two, as reported in Table 3; a second group associated with the construct *Cultural, Civic, Family Engagement and Youth Employment* (CCFEYE) is defined by “Young people not in employment, education, or training (NEET)”, the variables representing cultural engagement (variables 4 and 5), “Regular internet users”, “Civic and political participation”, and “Satisfaction for family relationships” (whose loading is the lowest in the group). The combination of these variables reflects how engagement in society contributes to a broader concept of integration into the social fabric. The third group lumps the variables related with satisfaction for several aspects of life (variables from 12 to 14) together with “Social participation” and identifies the concept of *Satisfaction with Life, Environment and Social Engagement* (SLESE). Finally, the singleton “Perception of safety when walking alone at night” (*Perceived Security*, PS) does not seem to be strongly related to the other variables, preventing it from merging with them at the first bottom-up level of the hierarchy, and it will represent a distinguishing factor between males and females, as we will see later on. All the groups are unidimensional and internally consistent, with  $\alpha_{EACE} = 0.91$ ,  $\alpha_{CCFEYE} = 0.97$ ,  $\alpha_{SLESE} = 0.93$ , while this is not measurable for singletons. The second-order and the third-order SCIs are characterized by the aggregation of SLESE and PS into *Subjective Well-Being* (SWB), and of EACE and CCFEYE into *Socio-Cultural Well-Being* (SCWB), respectively. SWB captures the subjective experience of individuals’ quality of life and perceptions of personal safety characterizing social sustainability, with higher loadings for variables 8, 12, and 14 (Table 3). On the other hand, SCWB integrates various forms of education and active participation in society representing both social and cultural aspects of sustainability. As shown by  $B_1^{2019}$  in Table 3, in the GCI of SS, the last three variables exhibit the lowest loadings, following PS, educational aspects and creative employment, which constitute the first-order SCI EACE. The loading value for variable 11, equal to 0.10, reflects its weak correlation with the other variables, apart from those it is merged with to obtain the second-order SCI SWB (see Fig. 2a). However, SWB and SCWB turn out to be interrelated, as well as the majority of their variables, as highlighted by the reflective approach chosen by HierDPCA. Additionally, all the groups in the hierarchy are internally consistent.

As is well known, the COVID-19 pandemic strongly affected the social and cultural lives of individuals, leading to significant restrictions on social gatherings, cultural events, and

other aspects of everyday life, such as education and work, throughout the country. These changes in the social and cultural fabric are also reflected in the dimensions that contribute to the definition of social sustainability. As previously mentioned, the first modification occurs in the number of variable groups on which the hierarchy is built by HierDPCA to obtain the GCI of SS, which is now equal to 5. Secondly, the model identifies a reflective hierarchical structure up to  $M = 2$ , after which it turns into a formative approach, as shown in Fig. 3b (with arrows pointing from the bottom up).

The five groups identified at the first bottom-up level in 2022 share some similarities with those from 2019. Specifically, the group previously associated with the first-order SCI CCFEYE recurs in 2022, except for variables 3 and 10, while including “Social participation”; therefore, it can be interpreted as *Cultural, Civic, and Social Engagement* (CCSE). The first two variables pertaining to education, which were combined together with variable 6 in 2019, are now joined with “Young people not in employment, education, or training (NEET)”, representing the construct of *Educational Achievement and Youth Employment* (EAYE), while variable 6 (*Creative Employment*, CE) is a singleton in 2022. It is important to recall that the COVID-19 pandemic significantly impacted education by imposing remote learning, which could result in disparities across different layers of the Italian population. Additionally, the economic downturn caused by the pandemic increased the number of young people classified as NEET, as job opportunities diminished and educational institutions struggled to engage students effectively. Crucial differences occur in the other groups, which can be traced back to the COVID-19 pandemic itself. In detail, “Satisfaction with family relationships”, “Life satisfaction”, and “Satisfaction with leisure time” are merged into a group representing *Satisfaction with Life and Family Engagement* (SLFE), where variable 10 has the highest loading (as shown in Table 3). Indeed, the COVID-19 crisis necessitated prolonged periods of confinement at home, which brought families closer together but also intensified the challenges of familial interactions, while diminishing overall life satisfaction and fulfillment in leisure time. Finally, the last first-order SCI, referred to as *Perceived Security and Environmental Satisfaction* (PSES), reflects into the manifest variables “Perception of safety when walking alone at night” and “Satisfaction with the environmental situation”. This indicates that perceptions of safety are not solely tied to personal experiences; they also encompass concerns about the broader environmental context. These concerns have intensified post-pandemic with the sudden resurgence of economic activities, leading to increased industrial output, vehicle pollution, and other environmental hazards. The importance of this aspect becomes evident as it represents one of the two SCIs that form the GCI of SS. It is worth noting that, as in 2019, all the first-order SCIs are unidimensional and internally consistent, with  $\alpha_{CCSE} = 0.97$ ,  $\alpha_{EAYE} = 0.92$ ,  $\alpha_{SLFE} = 0.84$ , and  $\alpha_{PSES} = 0.87$ , while the Cronbach’s  $\alpha$  for CE is not available as it is a singleton.

The aggregations in pairs of four out of the five groups obtained for the year 2022 concern first CCSE and CE, resulting in the second-order SCI of *Social, Cultural and Creative Engagement* (SCCE), in which the latter has the lowest loading (Table 3). Secondly, this group is combined with EAYE to form the third-order SCI of *Social, Cultural, Creative and Educational Engagement*, SCCEE, and lastly, SCCEE is merged with SLFE, representing the fourth-order SCI of *Socio-Cultural and Personal Well-Being* (SCPWB). This broader SCI turns out not to be significantly correlated with PSES, leading to a shift in the approach to building the GCI of SS from reflective to formative, as indicated by the weakening correlations of variables 11 and 14 with the others (Fig. 2b). According to the second equation

reported in (1), the regression parameters for SCPWB and PSES are 0.98 and 0.24, respectively. This suggests that the former is strongly indicative of social sustainability, but needs to be integrated with the latter to accurately reflect the complexities of this multidimensional phenomenon in 2022. For completeness, a description of the SCIs for both years is provided in Table 2.

Notably, each column of Table 3, except for those corresponding to  $B_4^{2019}$  and  $B_5^{2022}$ , provides the loadings for the variables that are merged into broader groups. For variables remaining in the same group, a dash is reported, as their corresponding loading remain the same as in the previous levels. Table 3 also displays the variance explained by the disjoint principal components at each hierarchical level. As expected, this percentage decreases with the number of disjoint principal components, i.e., moving up the hierarchy. Indeed, with fewer disjoint principal components, less variability of the original data can be reconstructed. Nonetheless, across the hierarchical levels, the percentage of variance explained remains high or moderately high, reaching approximately 86% in both years, with 4 and 5 disjoint principal components, respectively.

Based on the described structures, we can analyze the differences across the Italian regions. The rankings reported in Table 4 for the two years are based on the min-max normalization of the SS scores, which are illustrated in Fig. 4. From the latter, it is evident that Trentino-Alto Adige outdistances all other regions by remaining stable in the first position in both years. Meanwhile, three other “groups” of regions can be identified: central and northern regions with GCI values between 0.5 and 0.75; Abruzzo, Molise, Sardegna, and Basilicata, with GCI scores between 0.15 and 0.45; and Puglia, Calabria, Sicilia, and Cam-

**Table 2** Description of the first-order and higher-order SCIs for the two years

| CI type       | Acronym | Description                                               |
|---------------|---------|-----------------------------------------------------------|
| 2019          |         |                                                           |
| 1st-order SCI | EACE    | Educational Achievement and Creative Employment           |
| 1st-order SCI | CCFEYE  | Cultural, Civic, Family Engagement and Youth Employment   |
| 1st-order SCI | SLESE   | Satisfaction with Life, Environment and Social Engagement |
| 1st-order SCI | PS      | Perceived Security                                        |
| 2nd-order SCI | SWB     | Subjective Well-Being                                     |
| 3rd-order SCI | SCWB    | Socio-Cultural Well-Being                                 |
| 2022          |         |                                                           |
| 1st-order SCI | EAYE    | Educational Achievement and Youth Employment              |
| 1st-order SCI | CE      | Creative Employment                                       |
| 1st-order SCI | CCSE    | Cultural, Civic, and Social Engagement                    |
| 1st-order SCI | SLFE    | Satisfaction with Life and Family Engagement              |
| 1st-order SCI | PSES    | Perceived Security and Environmental Satisfaction         |
| 2nd-order SCI | SCCE    | Social, Cultural and Creative Engagement                  |
| 3rd-order SCI | SCCEE   | Social, Cultural, Creative and Educational Engagement     |
| 4th-order SCI | SCPWB   | Socio-Cultural and Personal Well-Being                    |

**Table 3** Variable weights for the reflective part of the hierarchy: a dash indicates that the variable does not contribute to defining the CI at that level of the hierarchy. The variance explained by all disjoint principal components at each level of the hierarchy is reported in the last row

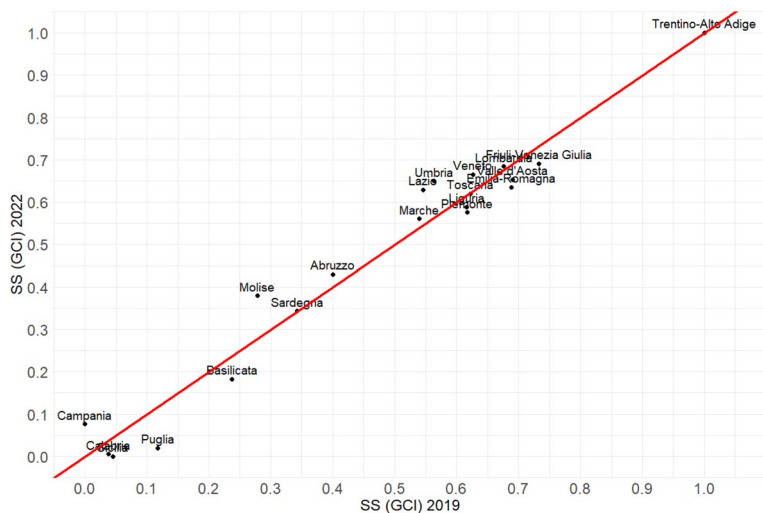
| Var. ID  | $B_1^{2019}$ | $B_2^{2019}$ | $B_3^{2019}$ | $B_4^{2019}$ | $B_2^{2022}$ | $B_3^{2022}$ | $B_4^{2022}$ | $B_5^{2022}$ |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1        | 0.26         | 0.32         | –            | 0.58         | 0.29         | 0.32         | –            | 0.59         |
| 2        | 0.24         | 0.31         | –            | 0.60         | 0.25         | 0.29         | –            | 0.56         |
| 3        | 0.31         | 0.36         | –            | 0.41         | 0.32         | 0.36         | –            | 0.58         |
| 4        | 0.29         | 0.34         | –            | 0.41         | 0.31         | 0.35         | 0.43         | 0.44         |
| 5        | 0.31         | 0.34         | –            | 0.42         | 0.31         | 0.33         | 0.40         | 0.44         |
| 6        | 0.25         | 0.32         | –            | 0.55         | 0.24         | 0.27         | 0.33         | 1.00         |
| 7        | 0.29         | 0.35         | –            | 0.41         | 0.32         | 0.36         | 0.43         | 0.45         |
| 8        | 0.28         | –            | 0.46         | 0.51         | 0.31         | 0.34         | 0.42         | 0.44         |
| 9        | 0.29         | 0.34         | –            | 0.41         | 0.32         | 0.36         | 0.42         | 0.45         |
| 10       | 0.29         | 0.33         | –            | 0.39         | 0.29         | –            | –            | 0.59         |
| 11       | 0.10         | –            | 0.38         | 1.00         | –            | –            | –            | 0.71         |
| 12       | 0.25         | –            | 0.49         | 0.52         | 0.21         | –            | –            | 0.57         |
| 13       | 0.27         | –            | 0.43         | 0.48         | 0.25         | –            | –            | 0.56         |
| 14       | 0.26         | –            | 0.48         | 0.50         | –            | –            | –            | 0.71         |
| Exp. Var | 67.22%       | 77.48%       | 82.23%       | 86.10%       | 71.91%       | 78.09%       | 82.36%       | 86.05%       |

For simplicity,  $B_q^{\text{year}}$  denotes the loadings on the main diagonal of  $B_q$

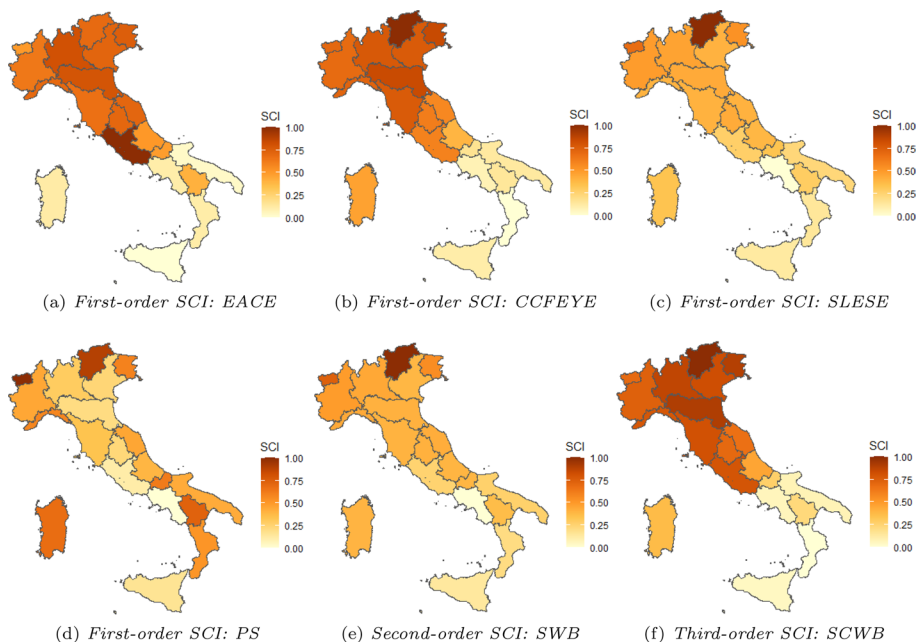
**Table 4** Rankings based on the social sustainability composite indicator (normalized scores). For each region in the 2019 ranking (baseline), the third column indicates the advancement (+), decrease (–), or same position (=) in the 2022 ranking

| Position | 2019                  | 2022                  | Changing |
|----------|-----------------------|-----------------------|----------|
| 1        | Trentino–Alto Adige   | Trentino–Alto Adige   | =        |
| 2        | Friuli–Venezia Giulia | Friuli–Venezia Giulia | =        |
| 3        | Valle d’Aosta         | Lombardia             | –2       |
| 4        | Emilia–Romagna        | Veneto                | –3       |
| 5        | Lombardia             | Valle d’Aosta         | +2       |
| 6        | Veneto                | Umbria                | +2       |
| 7        | Toscana               | Emilia–Romagna        | –2       |
| 8        | Piemonte              | Lazio                 | –3       |
| 9        | Liguria               | Toscana               | –1       |
| 10       | Umbria                | Liguria               | +4       |
| 11       | Lazio                 | Piemonte              | +3       |
| 12       | Marche                | Marche                | =        |
| 13       | Abruzzo               | Abruzzo               | =        |
| 14       | Sardegna              | Molise                | –1       |
| 15       | Molise                | Sardegna              | +1       |
| 16       | Basilicata            | Basilicata            | =        |
| 17       | Puglia                | Campania              | –1       |
| 18       | Sicilia               | Puglia                | –2       |
| 19       | Calabria              | Calabria              | =        |
| 20       | Campania              | Sicilia               | +3       |

pania, which significantly differ from the other regions, exhibiting very low GCI values. Figures 5 and 6 display the normalized scores of the first-order SCIs and the higher-order SCIs (at the second-to-last hierarchical level for 2022), which can help properly understand which dimensions contribute to the final rankings, along with the variable weights in Table

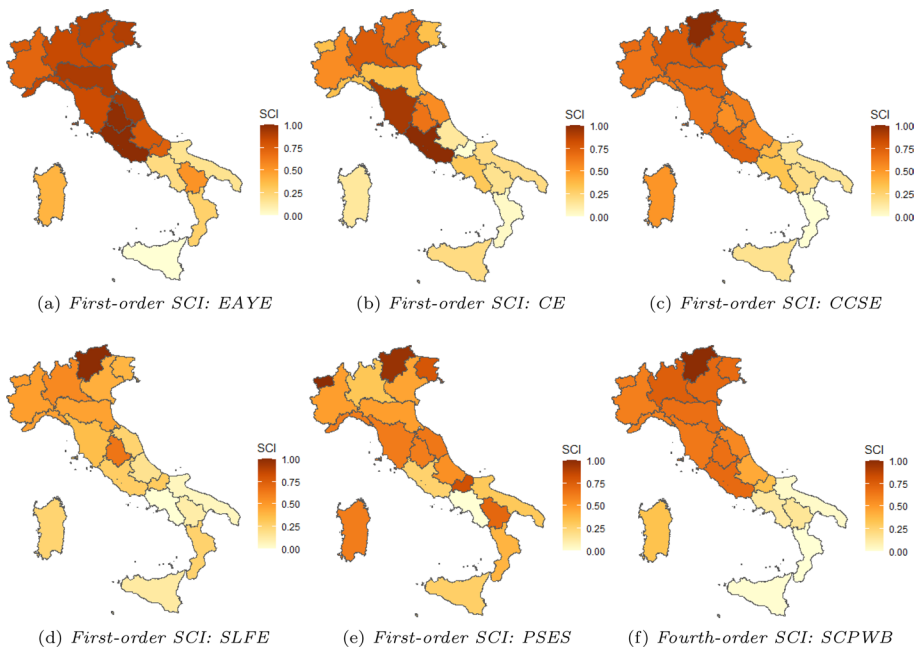


**Fig. 4** Representation of the Italian regions normalized scores for 2019 and 2022 (red line: bisector)



**Fig. 5** Normalized scores of the four first-order SCIs and two higher-order SCIs for the 20 Italian regions, year 2019

3. Specifically, the dominance of Trentino–Alto Adige in the two rankings is the result of its significantly higher scores across all SCIs, by about 0.25 to 0.50 points from the central and northern regions, except for EACE (Fig. 5a) and PS (Fig. 5d) in 2019, and EAYE (Fig. 6a) and CE (Fig. 6b) in 2022. However, the variables defining EACE and PS in 2019 load



**Fig. 6** Normalized scores of the five first-order SCIs and the fourth-order SCI for the 20 Italian regions, year 2022

less than the majority of the others on the GCI and on the corresponding higher-order SCIs, as well as those associated with EAYE and CE in 2022 on the fourth-order SCI SCPWB (Fig. 6f). Contrarily, for instance, Friuli–Venezia Giulia exhibits more stable performance across the six SCIs in 2019, though in 2022, it has a relatively lower score for CE (Fig. 6b). Although it holds the second position in the ranking, the final score of Friuli–Venezia Giulia is far from that of Trentino–Alto Adige and decreases in 2022. As shown in Fig. 4, its position is below the bisector line, with GCI equal to 0.73 in 2019 and 0.69 in 2022. It is worth noting that the representation of the SS scores for the two years allows to appreciate the change in the regions' positions both in terms of rankings and magnitude, where the latter, being relative (obtained by min-max normalization), quantifies the actual differences.

Within the group of the central and northern regions, a notable improvement in the ranking, corresponding to an approximate increase of 0.10 in the SS score, is observed for Umbria and Lazio, which gain four and three positions, respectively, in 2022. Specifically, in 2019, Umbria has moderately high values only for the first-order SCIs EACE (Fig. 5a) and CCFEYE (Fig. 5b), resulting in a high value for the third-order SCI SCWB (Fig. 5f); in 2022, it achieves a significantly higher score for EAYE (Fig. 6a) and relevant values for the other first-order SCIs. As for Lazio, it has the highest score for the first-order SCI EACE (Fig. 5a) in 2019, and in 2022, it reaches the highest scores for both EAYE (Fig. 6a) and CE (Fig. 6b), which overlap with EACE except for variable 3. Therefore, educational achievement, and youth and creative employment in Lazio increases compared to 2019. In contrast, Emilia-Romagna exhibits an opposite trend, losing three positions and approximately 0.05 points in GCI in 2022. While its scores in 2019 are similar to those of other northeastern regions, with high values for the first-order SCIs EACE (Fig. 5a) and CCFEYE (Fig. 5b),

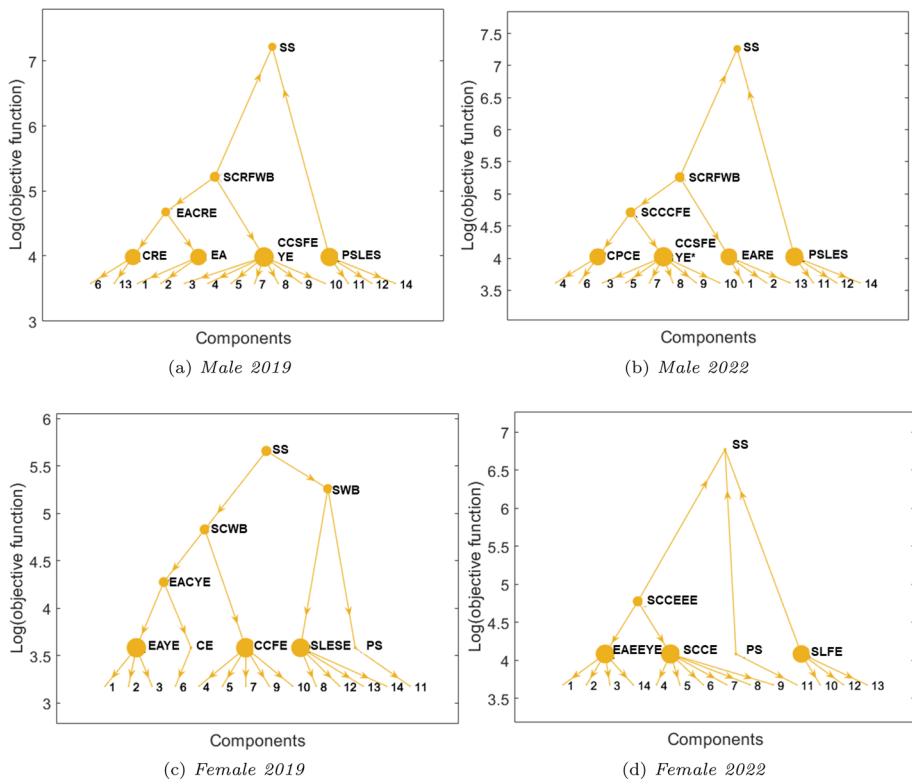
i.e., for education, civic and cultural engagement, as well as for the corresponding third-order SCI (Fig. 5f) that merges the previous two, it shows a very low value for the first-order SCI CE in 2022 (Fig. 6b), which is comparable to those of the southern and insular regions.

The lower end of the rankings is occupied by the southern and insular regions, with slight changes between the two years. For instance, Campania, which ranks last in SS in 2019, moves up to fourth from last in 2022, with a corresponding increase in the score, while Puglia loses one position and approximately 0.10 points in the score, and Sicilia drops to the last position. Meanwhile, Abruzzo and Basilicata remain stable in their positions in the ranking, even if in 2022, the former increases its SS score and the latter decreases it. Figures 5 and 6 show that the southern and insular regions generally display the lowest scores, usually significantly lower than those of the northern regions. Notable exceptions include Sardegna, Molise and Basilicata, where, for instance, the perception of safety in 2019 (Fig. 5d) is very high than the regions in the same geographical area. Similarly, in 2022, when variable 11 is merged with variable 14 (Fig. 6e), these regions still stand out. Additionally, Abruzzo has relatively stable scores across SCIs in 2019, aligning more closely with the northern regions, although its scores shift more in 2022. Nevertheless, within the corresponding geographical area, these four regions hold the highest positions.

#### 4.2.1 Analysis by gender

The multidimensional phenomenon of social sustainability in Italy can also be analyzed across genders. Following an exploratory approach, we choose  $Q$  via the GCV criterion, as done for the complete data set. It is worth noting that the data for males and females exhibit different patterns of relationships among variables (see also the correlation matrices reported in the Supplementary Material), requiring a specific choice of the number of first-order SCIs within these data sets to accurately represent the phenomenon under study. According to the GCV criterion, in 2019,  $Q = 4$  for males and  $Q = 5$  for females; in 2022,  $Q = 4$  for both males and females. The results of the HierDPCA application by gender and year are presented in Fig. 7, along with Table 5, which provides the definition of the SCIs. First, we can observe that, in addition to a moderate difference in the first-order SCIs, the hierarchy in 2019 is mixed reflective/formative with  $M = 2$  for males (Fig. 7a), and fully reflective for females (Fig. 7c)—similarly to the complete case. In 2022, the hierarchical structure is mixed reflective/formative for both genders, as well as in the complete case, with  $M = 2$  for males (Fig. 7b) and  $M = 3$  for females (Fig. 7d).

The first-order SCIs obtained for males and females are sometimes similar, or even identical, to those identified in the complete data set. For instance, in 2019, the first-order SCI “Cultural, Civic, Social, Family Engagement and Youth Employment” (CCSFYEY) for males is the same as the first-order SCI CCFEYE, except for variable 8, i.e., “Social participation”. For females, four first-order SCIs match those identified in the complete data set: two from 2019 (SLESE and PS) and two from 2022 (EAYE and CE). In 2022, two first-order SCIs for females are equivalent to SCIs identified in the complete data set for the same year, i.e., the first-order SCI SLFE and the second-order SCI SCCE. Furthermore, both males and females retain one first-order SCI from the 2019 gender-specific analysis—PSLES (i.e., “Perceived Security, Life and Environmental Satisfaction”) and PS, respectively. Moreover, for males, CCSFYEY only loses variable 4 in 2022, and the first-order SCI “Educational Achievement and Recreational Engagement” (EARE), composed of variables



**Fig. 7** Path diagram of the hierarchy estimated by HierDPCA to obtain the social sustainability composite indicator across the Italian regions by gender and year

1, 2, and 13, is similar to the second-order SCI “Educational Achievement, Creative and Recreational Engagement” (EACRE) identified in 2019, which includes variables 1, 2, 13, and 6. For females, the first-order SCI “Cultural, Civic, and Family Engagement” (CCFE) in 2019 overlaps with “Social, Cultural, and Creative Engagement” (SCCE) for variables 4, 5, 7, and 9.

In addition to the aforementioned similarities, we explore the gender-specific features of social sustainability to highlight the discrepancies in the underlying structures. In 2019, a significant difference between genders is observed in “Cultural and creative employment” (variable 6). In fact, for males, this variable is merged with “Satisfaction with leisure time” (variable 13) into the first-order SCI “Creative and Recreational Engagement” (CRE). For females, however, variable 6 is a singleton in the first bottom-up level of the hierarchy, before being merged with the first three variables pertaining to education and youth employment (forming the first-order SCI EAYE), which results in the second-order SCI “Educational Achievement, Creative, and Youth Employment” (EACYE). This outcome suggests that for males, cultural and creative activities may be more closely associated with leisure time and recreation, whereas for females, these activities appear to be an important aspect by itself. In both cases, they are linked with education and youth employment at the second bottom-up hierarchical level, indicating that the higher the latter, the greater the involve-

**Table 5** Description of the first-order and higher-order SCIs for the two years by gender

| Gender | CI type       | Acronym  | Description                                                            |
|--------|---------------|----------|------------------------------------------------------------------------|
| 2019   |               |          |                                                                        |
| Male   | 1st-order SCI | EA       | Educational Achievement                                                |
|        | 1st-order SCI | CRE      | Creative and Recreational Engagement                                   |
|        | 1st-order SCI | CCSFYEY  | Cultural, Civic, Social, Family Engagement and Youth Employment        |
|        | 1st-order SCI | PSLES    | Perceived Security, Life and Environmental Satisfaction                |
|        | 2nd-order SCI | EACRE    | Educational Achievement, Creative and Recreational Engagement          |
| Female | 3rd-order SCI | SCRFWB   | Socio-Cultural, Recreational, and Familiar Well-Being                  |
|        | 1st-order SCI | EAYE     | Educational Achievement and Youth Employment                           |
|        | 1st-order SCI | CE       | Creative Employment                                                    |
|        | 1st-order SCI | CCFE     | Cultural, Civic, and Family Engagement                                 |
|        | 1st-order SCI | SLESE    | Satisfaction with Life, Environment and Social Engagement              |
|        | 1st-order SCI | PS       | Perceived Security                                                     |
|        | 2nd-order SCI | EACYE    | Educational Achievement, Creative and Youth Employment                 |
|        | 3rd-order SCI | SCWB     | Socio-Cultural Well-Being                                              |
|        | 4th-order SCI | SWB      | Subjective Well-Being                                                  |
| 2022   |               |          |                                                                        |
| Male   | 1st-order SCI | EARE     | Educational Achievement and Recreational Engagement                    |
|        | 1st-order SCI | CPCE     | Cultural Participation and Creative Employment                         |
|        | 1st-order SCI | CCSFYEY* | Cultural, Civic, Social, Family Engagement and Youth Employment        |
|        | 1st-order SCI | PSLES    | Perceived Security, Life and Environmental Satisfaction                |
|        | 2nd-order SCI | SCCCFE   | Social, Cultural, Creative, Civic, Family Engagement                   |
| Female | 3rd-order SCI | SCRFWB   | Socio-Cultural, Recreational, and Familiar Well-Being                  |
|        | 1st-order SCI | EAEYEY   | Educational Achievement, Environmental Engagement and Youth Employment |
|        | 1st-order SCI | SCCE     | Social, Cultural, and Creative Engagement                              |
|        | 1st-order SCI | SLFE     | Satisfaction with Life and Family Engagement                           |
|        | 1st-order SCI | PS       | Perceived Security                                                     |
|        | 2nd-order SCI | SCCEEE   | Social, Cultural, Creative, Educational, and Environmental Engagement  |

\* The name is identical to the first-order SCI CCSFEYE for males in 2019. However, the latter also includes variable 4, “Cultural participation outside the home”, which represents the cultural aspects along with variables 5 and 7

ment in or attention to creative and cultural activities. However, its loading is the second highest in magnitude for defining EACRE, and the lowest in EACYE (see Table 6). In 2022, variable 6 is lumped with “Cultural participation outside the home” (variable 4) for males, while for females it is encompassed in the broader group represented by the first-order SCI “Social, Cultural, and Creative Engagement” (SCCE). Another important difference between genders is in the “Perception of safety when walking alone at night” (variable 11). This is a crucial aspect for females, who may be exposed to higher risks than males, while for the latter, it is related to “Life satisfaction” (variable 12) and “Satisfaction with the environmental situation” (variable 14) in both years—they form the first-order SCI PSLES.

At the second-to-last level, the hierarchical structures for males identify the same disjoint principal components across years: the third-order SCI SCRFBW (i.e., “Socio-Cultural,

**Table 6** Variable weights for the reflective part of the hierarchy: a dash indicates that the variable does not contribute to defining the CI at that level of the hierarchy. The variance explained by all disjoint principal components at each level of the hierarchy is reported in the last row

| <i>Male</i>   |              |              |              |              |              |              |              |
|---------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Var. ID       | $B_2^{2019}$ | $B_3^{2019}$ | $B_4^{2019}$ | $B_5^{2022}$ | $B_3^{2022}$ | $B_4^{2022}$ |              |
| 1             | 0.28         | 0.53         | 0.71         | 0.30         | –            | 0.59         |              |
| 2             | 0.23         | 0.48         | 0.71         | 0.24         | –            | 0.58         |              |
| 3             | 0.34         | –            | 0.39         | 0.34         | 0.37         | 0.42         |              |
| 4             | 0.32         | –            | 0.38         | 0.32         | 0.36         | 0.71         |              |
| 5             | 0.33         | –            | 0.39         | 0.31         | 0.36         | 0.40         |              |
| 6             | 0.28         | 0.52         | 0.71         | 0.25         | 0.27         | 0.71         |              |
| 7             | 0.32         | –            | 0.38         | 0.34         | 0.38         | 0.41         |              |
| 8             | 0.30         | –            | 0.36         | 0.33         | 0.36         | 0.41         |              |
| 9             | 0.33         | –            | 0.39         | 0.34         | 0.37         | 0.42         |              |
| 10            | 0.31         | –            | 0.36         | 0.30         | 0.34         | 0.39         |              |
| 11            | –            | –            | 0.58         | –            | –            | 0.59         |              |
| 12            | –            | –            | 0.56         | –            | –            | 0.54         |              |
| 13            | 0.26         | 0.47         | 0.71         | 0.24         | –            | 0.56         |              |
| 14            | –            | –            | 0.59         | –            | –            | 0.60         |              |
| Exp. var      | 72.46%       | 78.40%       | 83.22%       | 70.97%       | 78.00%       | 82.22%       |              |
| <i>Female</i> |              |              |              |              |              |              |              |
| Var. ID       | $B_1^{2019}$ | $B_2^{2019}$ | $B_3^{2019}$ | $B_4^{2019}$ | $B_5^{2019}$ | $B_3^{2022}$ | $B_4^{2022}$ |
| 1             | 0.28         | –            | 0.33         | 0.52         | 0.59         | 0.33         | 0.51         |
| 2             | 0.26         | –            | 0.32         | 0.53         | 0.58         | 0.30         | 0.49         |
| 3             | 0.30         | –            | 0.35         | 0.50         | 0.57         | 0.34         | 0.53         |
| 4             | 0.30         | –            | 0.34         | –            | 0.45         | 0.33         | 0.43         |
| 5             | 0.31         | –            | 0.34         | –            | 0.46         | 0.33         | 0.41         |
| 6             | 0.22         | –            | 0.29         | 0.44         | 1.00         | 0.24         | 0.31         |
| 7             | 0.30         | –            | 0.35         | –            | 0.45         | 0.35         | 0.44         |
| 8             | 0.26         | 0.47         | –            | –            | 0.50         | 0.31         | 0.41         |
| 9             | 0.29         | –            | 0.34         | –            | 0.45         | 0.35         | 0.43         |
| 10            | 0.29         | –            | 0.34         | –            | 0.43         | –            | 0.60         |
| 11            | 0.05         | 0.33         | –            | –            | 1.00         | –            | 1.00         |
| 12            | 0.24         | 0.49         | –            | –            | 0.52         | –            | 0.56         |
| 13            | 0.27         | 0.44         | –            | –            | 0.48         | –            | 0.57         |
| 14            | 0.27         | 0.49         | –            | –            | 0.50         | 0.27         | 0.47         |
| Exp. var      | 66.31%       | 76.05%       | 80.92%       | 85.61%       | 88.72%       | 76.09%       | 81.57%       |

For simplicity,  $B_q^{\text{year}}$  denotes the loadings on the main diagonal of  $B_q$

Recreational, and Familiar Well-Being”) and the first-order SCI PSLES. These components turn out to be uncorrelated in both years, completing the hierarchy that defines the GCI of SS in a formative approach, with a regression coefficient of 0.91 in 2019 and 0.92 in 2022 for SCRFWB, and of 0.41 in 2019 and 0.39 in 2022 for PSLES. Conversely, for females, the hierarchy is fully reflective in 2019, where the two higher-order SCIs reflected by the GCI are the same as those identified in the complete data set (i.e., SCWB and SWB), with higher loadings for the variables pertaining to cultural, civic, and family engagement. In 2022, three aspects form the GCI of SS switching to a formative approach: SCCEEE (“Social, Cultural, Creative, Educational, and Environmental Engagement”), SLFE, and PS. The lat-

ter has a very low regression coefficient (0.02), while those for SCCEEE and SLFE are equal to 0.91 and 0.42, respectively. For all four hierarchies illustrated in this section, the variable groups are internally consistent, and the first-order SCIs are unidimensional. Additionally, the variance explained by the disjoint principal components exceeds 70% at each hierarchical level, except for the last one in the 2019 hierarchy for females (see Table 6).

As shown in Fig. 7, the COVID-19 pandemic affected social sustainability by altering the magnitude of its pivotal aspects. This, in turn, influenced the order of aggregation among the first-order SCIs and the direction of relationships across hierarchical levels, with different impacts for males and females. These changes are also reflected in the rankings reported in Table 7, and further in Fig. 8, which illustrates the comparison of the GCI normalized scores between 2019 and 2022 per gender. For males, regions such as Emilia–Romagna and Liguria experience noticeable drops in their rankings (five and six positions, respectively), with a corresponding decrease in the GCI of SS of approximately 0.10 points. Meanwhile, regions like Umbria and Lazio show increases of six and three positions, respectively, although their growth in GCI values is less pronounced. The group of regions composed of Abruzzo, Sardegna, Molise, and Basilicata exhibits smaller ranking shifts, corresponding to differences in the scores of about 0.10 points between the two years (e.g., for Sardegna and Molise). Puglia, Sicilia, Calabria, and Campania remain at the lower end of the hierarchy, as in the complete case, though Campania's performance improved in 2022. For females, Lombardia rises to second place in 2022, surpassing Friuli–Venezia Giulia, despite similar scores in 2022 and a more pronounced difference in 2019. Lazio and Umbria each gain three positions in 2022, with a notable increase in the SS GCI scores. On the other hand, for instance, Liguria and Piemonte experience declines in the ranking, despite an increase in their GCI scores in 2022—for this reason, it is fundamental to interpret Table 7 together with Fig. 8. In general, GCI scores for females tend to be higher than those for males, except in some central and northern regions such as Piemonte and Marche and in southern and insular regions in 2019. This highlights the differences in the gender-specific impacts of the pandemic on social sustainability across the Italian regions. The geographical representation of SCI scores reported in the Supplementary Material provides additional insights into the importance of the underlying SS dimensions by gender. Finally, similar to Fig. 4 for the complete data set, we can identify four “groups” of regions also in Fig. 8. Specifically, Trentino–Alto Adige consistently holds the first position, significantly outdistancing the central and northern regions, which have GCI scores between 0.5 and 0.75. The third group includes Abruzzo, Molise, Basilicata, and Sardegna, with scores ranging from 0.15 to 0.45, while the fourth and final group consists of Puglia, Calabria, Sicilia, and Campania, which exhibit very low scores.

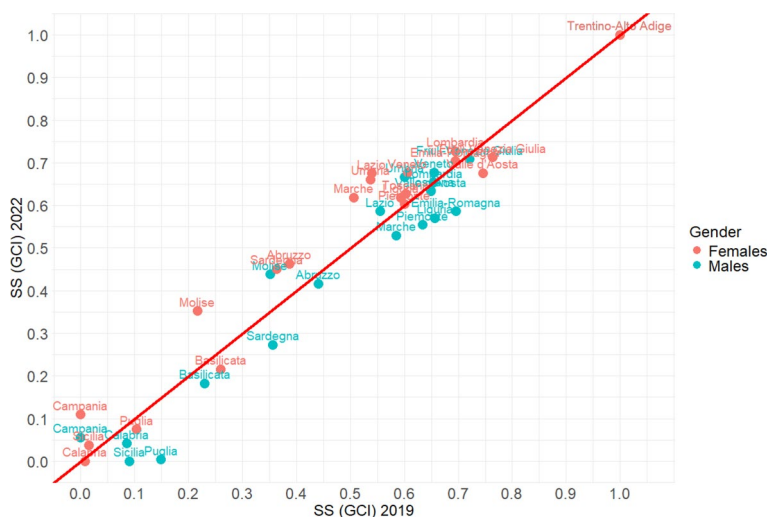
## 5 Conclusions

This study presented a novel hierarchical composite indicator for evaluating social sustainability in Italy, using a statistical approach grounded in the Hierarchical Disjoint Principal Component Analysis (HierDPCA). This methodology allows for inspecting hierarchical structures within the data and provides a tool for testing the direction of the relationships between sequential levels of the hierarchy, thereby avoiding any subjective choices. Additionally, a generalized cross-validation criterion for selecting the number of specific latent

**Table 7** Rankings based on the social sustainability composite indicator (normalized scores)

| Position      | 2019                  | 2022                  | Changing |
|---------------|-----------------------|-----------------------|----------|
| <i>Male</i>   |                       |                       |          |
| 1             | Trentino–Alto Adige   | Trentino–Alto Adige   | =        |
| 2             | Friuli–Venezia Giulia | Friuli–Venezia Giulia | =        |
| 3             | Emilia–Romagna        | Veneto                | –5       |
| 4             | Liguria               | Umbria                | –6       |
| 5             | Veneto                | Lombardia             | +2       |
| 6             | Lombardia             | Toscana               | +1       |
| 7             | Valle d’Aosta         | Valle d’Aosta         | =        |
| 8             | Toscana               | Emilia–Romagna        | +2       |
| 9             | Piemonte              | Lazio                 | –2       |
| 10            | Umbria                | Liguria               | +6       |
| 11            | Marche                | Piemonte              | –1       |
| 12            | Lazio                 | Marche                | +3       |
| 13            | Abruzzo               | Molise                | –1       |
| 14            | Sardegna              | Abruzzo               | –1       |
| 15            | Molise                | Sardegna              | +2       |
| 16            | Basilicata            | Basilicata            | =        |
| 17            | Puglia                | Campania              | –2       |
| 18            | Sicilia               | Calabria              | –2       |
| 19            | Calabria              | Puglia                | +1       |
| 20            | Campania              | Sicilia               | +3       |
| Position      | 2019                  | 2022                  | Changing |
| <i>Female</i> |                       |                       |          |
| 1             | Trentino–Alto Adige   | Trentino–Alto Adige   | =        |
| 2             | Friuli–Venezia Giulia | Lombardia             | –1       |
| 3             | Valle d’Aosta         | Friuli–Venezia Giulia | –3       |
| 4             | Emilia–Romagna        | Emilia–Romagna        | =        |
| 5             | Lombardia             | Veneto                | +3       |
| 6             | Veneto                | Valle d’Aosta         | +1       |
| 7             | Toscana               | Lazio                 | –2       |
| 8             | Piemonte              | Umbria                | –4       |
| 9             | Liguria               | Toscana               | –2       |
| 10            | Lazio                 | Marche                | +3       |
| 11            | Umbria                | Liguria               | +3       |
| 12            | Marche                | Piemonte              | +2       |
| 13            | Abruzzo               | Abruzzo               | =        |
| 14            | Sardegna              | Sardegna              | =        |
| 15            | Basilicata            | Molise                | –1       |
| 16            | Molise                | Basilicata            | +1       |
| 17            | Puglia                | Campania              | –1       |
| 18            | Sicilia               | Puglia                | –1       |
| 19            | Calabria              | Sicilia               | –1       |
| 20            | Campania              | Calabria              | +3       |

For each region in the 2019 ranking (baseline), the third column indicates the advancement (+), decrease (–), or same position (=) in the 2022 ranking



**Fig. 8** Representation of the Italian regions normalized scores for 2019 and 2022 per gender (red line: bisector)

dimensions was provided, extending the one proposed by Josse and Husson (2012) to Dis-joint Principal Component Analysis.

The application of HierDPCA to study social sustainability highlighted significant regional disparities across Italy, confirming that this phenomenon is shaped by a complex interplay of factors, including education, cultural participation, well-being, and perceptions of safety. The comparative analysis between 2019 (pre-COVID) and 2022 (post-COVID) demonstrated that the pandemic impacted various dimensions of social and cultural life, notably altering patterns of engagement and satisfaction. Regarding the ranking of the Italian regions, Trentino-Alto Adige consistently ranked at the top, benefiting from strong performances across most dimensions, while southern and insular regions like Campania, Calabria, Puglia, and Sicilia steadily lagged behind. The post-pandemic context revealed a shift in the composite indicator's structure, emphasizing the growing importance of socio-cultural engagement and personal well-being in shaping social sustainability. Gender-specific analyses further illustrated distinct patterns, with males and females experiencing different impacts on their social and cultural engagement, educational achievements, and perceptions of safety, particularly due to the pandemic. This gendered perspective underscores the need for tailored policy interventions to address specific challenges faced by different demographic groups. Overall, the composite indicator developed in this study might represent a valuable tool for policymakers, enabling a comprehensive understanding of the various facets of social sustainability in Italy. Indeed, identifying specific latent dimensions and broader ones, along with their quantification, provides insights into the key areas that should be strengthened to promote an inclusive society in line with the goals of sustainable development.

Finally, in this paper, we consider only continuous manifest variables. However, extending HierDPCA to include qualitative variables could be of interest for future studies and may further enhance our understanding of the phenomenon of social sustainability.

**Supplementary Information** The online version contains supplementary material available at <https://doi.org>

/10.1007/s10479-025-06731-4.

**Acknowledgements** The work of Cinzia Di Nuzzo is funded by the European Union—NextGenerationEU, in the framework of the GRINS—Growing Resilient, INclusive and Sustainable project (GRINS PE00000018—CUP E63C22002120006). The views and opinions expressed are solely those of the authors and do not necessarily reflect those of the European Union, nor can the European Union be held responsible for them.

**Data availability** The data that support the findings of the study reported in Sect. 4 are available from the corresponding author upon reasonable request.

## Declarations

**Conflict of interest** All Authors have no conflict of interest.

**Open Access** This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License, which permits any non-commercial use, sharing, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if you modified the licensed material. You do not have permission under this licence to share adapted material derived from this article or parts of it. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by-nc-nd/4.0/>.

## References

- Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6), 716–723.
- Alaimo, L. (2022). *Complexity of social phenomena: Measurements, analysis, representations and synthesis*. Sapienza University Press.
- Alaimo, L., Arcagni, A., Fattore, M., & Maggino, F. (2021). Synthesis of multi-indicator systems over time: A poset-based approach. *Social Indicators Research*, 157(1), 77–99.
- Berger-Schmitt, R., & Noll, H. (2000). *Conceptual framework and structure of a european system of social indicators*. Zentrum für Umfrage, Methoden und Analysen (ZUMA): Technical report.
- Blalock, H.M. (1964). *Causal inferences in nonexperimental research*. The University of North Carolina Press.
- Bollen, K. A. (2001). Indicator: Methodology. In N. J. Smelser & P. B. Baltes (Eds.), *International Encyclopedia of the Social and Behavioral Sciences* (pp. 7282–7287). Oxford: Elsevier Science.
- Bollen, K. A., & Bauldry, S. (2011). Three cs in measurement models: causal indicators, composite indicators, and covariates. *Psychological methods*, 16(3), 265–284.
- Bruggemann, R., & Patil, G. (2011). *Ranking and prioritization for multi-indicator systems: Introduction to partial order applications*. Springer Science & Business.
- Cattell, R. B. (1978). *The scientific use of factor analysis in behavioral and life sciences*. Plenum.
- Cavicchia, C., Sarnacchiaro, P., Vichi, M., & Zaccaria, G. (2024). A model-based ultrametric composite indicator for studying waste management in Italian municipalities. *Computational Statistics*, 39, 21–50.
- Cavicchia, C., & Vichi, M. (2021). Statistical model-based composite indicators for tracking coherent policy conclusions. *Social Indicators Research*, 156, 449–479.
- Cavicchia, C., & Vichi, M. (2022). Second-order disjoint factor analysis. *Psychometrika*, 87, 289–309.
- Cavicchia, C., Vichi, M., & Zaccaria, G. (2023). Hierarchical Disjoint Principal Component Analysis. *AStA Advances in Statistical Analysis*, 107(3), 537–574.
- Chen, S. X., Zhang, L., & Zhong, P. (2010). Tests for high-dimensional covariance matrices. *Journal of the American Statistical Association*, 105(490), 810–819.
- Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika*, 16(3), 297–334.

- Dempsey, N., Bramley, G., Power, S., & Brown, C. (2010). The social dimension of sustainable development: Defining urban social sustainability. *Sustainable Development*, 19, 289–300.
- Fattore, M. (2016). Partially ordered sets and the measurement of multidimensional ordinal deprivation. *Social Indicators Research*, 128(2), 835–858.
- Fattore, M., & Alaimo, L. (2023). A partial order toolbox for building synthetic indicators of sustainability with ordinal data. *Socio-Economic Planning Sciences*, 88, 101623.
- Ferrara, C., Martella, F., & Vichi, M. (2016). Dimensions of well-being and their statistical measurements, In *Topics in Theoretical and Applied Statistics*, eds. Allea, G. and A. Giommi, 85–99. Springer International Publishing.
- George, D., & Mallery, P. (2003). *SPSS for Windows step by step: A simple guide and reference. 11.0 update (4th ed.)*. Boston, MA: Boston, MA: Allyn & Bacon.
- Hawkes, J. (2001). *The fourth pillar of sustainability: culture's essential role in public planning*. Common Ground Publishing Pty Ltd.
- Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. *Journal of Educational Psychology*, 24(6): 417–441, 498–520.
- Jolliffe, I. T. (2002). *Principal Component Analysis* (2nd ed.). New York: Springer.
- Josse, J., & Husson, F. (2012). Selecting the number of components in principal component analysis using cross-validation approximations. *Computational Statistics & Data Analysis*, 56(6), 1869–1879.
- Le Dien, S., & Pagès, J. (2003). Analyse factorielle multiple hiérarchique [Hierarchical multiple factor analysis]. *Revue de statistique appliquée*, 51(2), 47–73.
- Mazziotta, M., & Pareto, A. (2019). Use and misuse of PCA for measuring well-being. *Social Indicators Research*, 142(2), 451–476.
- McGuinn, J., Fries-Tersch, E., Jones, M., Crepaldi, C., Masso, M., Kadarik, I., Samek Lodovici, M., Drufuca, S., Gancheva, M., & Geny, B. (2020). Social sustainability. concepts and benchmarks. Technical report, European Union, Policy Department for Economic, Scientific and Quality of Life Policies Directorate-General for Internal Policies.
- Nardo, M., Saisana, M., Saltelli, A., & Tarantola, S. (2005). Tools for composite indicators building. Technical Report EUR 21682, Join Research Centre, Ispra, Italy.
- Neggers, J., & Kim, H. (1998). *Basic posets*. World Scientific Publishing.
- OECD-JRC (2008). Handbook on constructing composite indicators: Methodology and user guide. Technical report, OECD Publishing.
- Pearson, K. (1901). On lines and planes of closest fit to systems of points in space. *Philosophical Magazine and Journal of Science*, 2(11), 559–572.
- Purvis, B., Mao, Y., & Robinson, D. (2019). Three pillars of sustainability: In search of conceptual origins. *Sustainability*, 14(3), 681–695.
- Rindskopf, D., & Rose, T. (1988). Some theory and applications of confirmatory second-order factor analysis. *Multivariate Behavioral Research*, 23(1), 51–67.
- Schiefer, D., & Van der Noll, J. (2017). The essentials of social cohesion: A literature review. *Social Indicators Research*, 132, 579–603.
- Schroder, B. (2002). *Ordered sets*. An introduction: Birkhauser.
- Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, 6(2), 461–464.
- Shirazi, M. R., & Keivani, R. (2019). The triad of social sustainability: Defining and measuring social sustainability of urban neighbourhoods. *Urban Research & Practice*, 12(4), 448–471.
- Spearman, C. E. (1904). General intelligence, objectively determined and measured. *The American Journal of Psychology*, 15(2), 201–293.
- United Nations General Assembly. (2019). *Transforming our world: The 2030 agenda for sustainable development*. United Nations: Technical report.
- Vichi, M. (2017). Disjoint factor analysis with cross-loadings. *Advances in Data Analysis and Classification*, 11(3), 563–591.