Letter

# Bare-bones de Sitter vacua

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We compute the supersymmetry-breaking three-form fluxes generated by the addition of anti-D3 branes at the tip of a Klebanov-Strassler throat. These fluxes give rise to nontrivial terms in the superpotential when the throat is embedded in a flux compactification. We describe these terms both from a ten-dimensional and from a four-dimensional perspective and show that, upon including Kähler-moduli stabilization, the resulting potential admits de Sitter minima. Our proposed de Sitter construction does not require additional supersymmetry-breaking (0,3) fluxes, and hence is more minimalist than the KKLT proposal.

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## I. INTRODUCTION

The accelerated expansion of our Universe points toward the existence of a positive vacuum energy density, whose value is about 120 orders of magnitudes smaller than the value expected from field-theory estimates. On the other hand, there are by now several arguments [1] that stable de Sitter vacua cannot be constructed in controlled low-energy effective theories that are consistent with quantum gravity. This leaves only two open possibilities: either the accelerated expansion of our Universe comes from a time-dependent vacuum energy density, or there is a problem with the no-de-Sitter conjecture, which can be disproved by an explicit construction.

Unfortunately, constructing metastable de Sitter vacua is notoriously difficult in string theory. Despite its intricate ingredients, shortcomings and potential instabilities, the almost twenty-year-old construction of Kachru, Kallosh, Linde and Trivedi (KKLT) [2] still stands out as one of the very few generic proposals that has not been fully proven to be unstable. It is a three-step construction that combines fluxes, nonperturbative phenomena and anti-D3 branes in a warped Calabi-Yau compactification with a deformed conifold-type throat. In order to obtain a positive and small cosmological constant, the fluxes required in the first step need to break supersymmetry generating a very small superpotential $W_{0,\mathrm{KKLT}}$. This has been criticized on two counts: theory and practice. On the formal side, these supersymmetry-breaking runaway solutions are not protected against corrections, and it was argued in [3] that they are not a good ground onto which one can add the nonperturbative ingredients necessary in the second step to prevent the runaway and stabilize the volume moduli. On the practical side, it is very hard to obtain explicit solutions with a sufficiently small superpotential, although there has been recent progress in engineering this type of flux vacua [4–6].

The purpose of this paper is to take a step toward bridging the conflict between the no-de-Sitter swampland arguments [1] and what can be constructed explicitly and controllably in string theory. We propose a new method to construct de Sitter vacua, which has one less ingredient than the KKLT construction, and hence is potentially plagued by less problems. More precisely, we show that one can construct de Sitter vacua with a small cosmological constant without the need of a flux superpotential $W_{0,\mathrm{KKLT}}$. Note that this requires restricting to manifolds that can support supersymmetric flux vacua. Manifolds admitting supersymmetric flux vacua were conjectured to require a "geometric modularity" property [7]. While many Calabi-Yau manifolds seem to have the desired modularity property and admit supersymmetric vacua [8], a few of them were shown not to allow for such solutions (see [9] and references therein).

Our key observation is that the anti-D3-branes necessary to uplift the cosmological constant source fluxes that generate precisely a small superpotential. Therefore, in our "bare bones" de Sitter construction, only supersymmetric fluxes are needed in the first step, thus avoiding the problems mentioned above.

## II. FLUXES GENERATED BY $\overline{\text{D3}}$ BRANES

A strongly warped region in a Calabi-Yau compactification can be engineered as a Klebanov-Strassler (KS) throat [10]. This is a cone over an $S^2 \times S^3$ base (see Fig. 1). The two-sphere of the base shrinks at the tip of the cone while the three-sphere has always finite size, parametrized by a modulus $Z$. The base can be also thought as a $U(1)$ fibration over $S^2 \times S^2$. The symmetries of the geometry consist of two $SU(2)$ factors acting on the base two-spheres and a $\mathbb{Z}_2$ swapping them.

The most general deformation of the conifold metric with fluxes preserving the $\mathrm{SU}(2)^2 \times \mathbb{Z}_2$ symmetry can be written in terms of eight functions of a radial coordinate $\{\Phi_i(r)\}$ [11]; this space of type-IIB supergravity solutions includes the Klebanov-Strassler [10], Maldacena-Nuñez [12], and baryonic branch solutions [13]. In this paper, we are interested in the deformation of the KS solution caused by the addition of $\bar{N}$ anti-D3 branes at the tip of the throat. In particular, we calculate how the anti-D3 branes affect the complexified three-form flux $G_3$, whose (p,q) components can be put in correspondence with various quantities in the effective four-dimensional low-energy theory describing the system.

Assuming that the backreaction of the anti-D3 branes on the geometry is small and can be studied in perturbation theory, the deformed geometry is given by

$$\Phi_i = \Phi_i^{\mathrm{KS}} + \lambda \phi_i + O(\lambda^2), \tag{1}$$

where the analytical dependence of the fluctuations $\phi_i$ has been computed in [14–16] and the small expansion parameter is:

$$\lambda = \frac{\bar{N}}{g_s M^2}, \tag{2}$$

where $M$ is the integral of the Ramond-Ramond $F_3$-flux on the $S^3$. Usually, the number of anti-D3 branes is taken to be $\bar{N} = 1$, since configurations with multiple anti-D3 branes have a tachyon [17], but we will keep track of it for completeness.

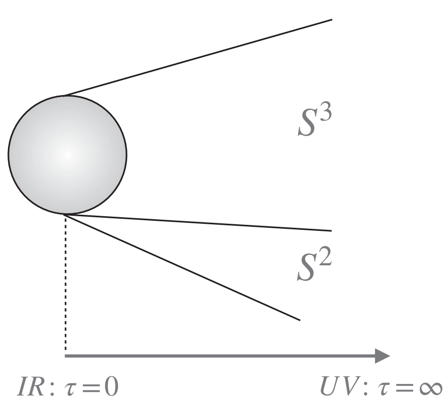


FIG. 1. An artist's impression of the KS geometry.

In the KS solution, the complexified three-form flux, $G_3$, is (2,1) with respect to the choice of complex structure picked by supersymmetry [18]. When the anti-D3 branes are added at the tip of throat, the three-form flux also gets corrections, $G_3 = G_3^{\mathrm{KS}} + G_3^{\overline{\mathrm{D3}}}$ and, at the same time, the complex structure is rotated. This implies that in general $G_3$ is not of (2,1)-type anymore [and neither is imaginary-self-dual (ISD)] but it develops all other components: for example, the (0,3) component is

$$G_{(0,3)}^{\overline{\mathrm{D3}}} = \frac{\sqrt{6}}{g_s h^{3/4}(\tau)} \frac{\partial_\tau \varphi(\tau)}{\sinh^2(\tau)} \bar{\Omega} + O(\lambda^2),$$
$$\varphi = g_s \sinh(\tau)\phi_7 + \cosh\left(\frac{\tau}{2}\right)^2 \phi_5 - \sinh\left(\frac{\tau}{2}\right)^2 \phi_6, \tag{3}$$

where $\Omega$ is the (3,0) form defined by the KS complex structure, $h(\tau)$ is proportional to the warp factor and $\phi_{5,6,7}$ are functions of the radius, whose UV and IR expansions are in [14], and whose full analytic expression can be found in [15].[1]

Even though the solution is nonsupersymmetric, one can analyse it using off-shell supersymmetry methods. Indeed, the nonsupersymmetric back-reacted anti-D3-brane solution computed in [14–16] uses the Papadopulos-Tseytlin ansatz which is based on the existence of an SU(3) structure (or analogously a globally defined spinor). This is an algebraic property required to use off-shell supersymmetric methods, that the cone over $S^2 \times S^3$ satisfies. On-shell supersymmetry then requires this spinor to be covariantly constant (with respect to some connection, which in the supersymmetric Klebanov-Strassler solution is just the Levi-Civita one, i.e., the manifold is Calabi-Yau). In the nonsupersymmetric solution, there is no such simple condition, and one has instead to solve the second order equations. Nevertheless, the fact that there is an underlying off-shell supersymmetry due to the globally defined spinor, allows to attack this problem using off-shell $\mathcal{N} = 1$ supersymmetry methods, in particular using the flux-induced superpotential. For the complex structure (or analogously, the globally defined spinor) of the Klebanov-Strassler zeroth order solution, such superpotential is nothing but the Gukov-Vafa-Witten (GVW) superpotential [20], given in Planck units by:

$$W_{\overline{\mathrm{D3}}} = \int G_3^{\overline{\mathrm{D3}}} \wedge \Omega_{\mathrm{KS}}, \tag{4}$$

where $\Omega_{\mathrm{KS}}$ is the holomorphic three-form of the unwarped internal manifold of the KS solution.

[1]Detailed information on the calculations of this section can be found in Secs. 2 and 3 of [19], noting that $S = \frac{1}{(2\pi)^2\alpha'} Z$.

Given that the anti-D3-brane generates a (0,3) component of flux, it thus gives rise to a superpotential. Using (3), the integral can be performed explicitly, giving an on-shell value[2]

$$W_{\overline{D3}} = -0.87 i\lambda M Z_{KS} + O(\lambda^2), \tag{5}$$

where $Z_{KS}$ is the value of the conifold modulus for the KS solution. The anti-D3-brane not only generates this flux component, but also imaginary-antiself-dual (IASD) pieces. These generate F-terms for the axio-dilaton and conifold moduli, given by

$$D_\tau W = -\frac{i g_s}{2}\int G_3^{*\overline{D3}} \wedge \Omega_{KS}, \qquad D_Z W = \int G_3^{\overline{D3}} \wedge \chi_{KS}, \tag{6}$$

where $\chi_{KS}$ is a (2,1)-form (whose first-order expression in $\lambda$ can be found in [21]). Such integrals can be numerically evaluated using the explicit form of $G_-^{\overline{D3}}$ given in [22]:

$$D_Z W = -1.5 i\lambda M + O(\lambda^2),$$
$$D_\tau W = 0.6\lambda g_s M Z_{KS} + O(\lambda^2). \tag{7}$$

This on-shell superpotential and F-terms, computed using the ten-dimensional solution, will be used in Sec. IV to compute an effective potential for the Kähler modulus in a KKLT-like construction.

## III. 4D SUPERGRAVITY DESCRIPTION

Before adding the $\overline{D3}$ branes at the tip of the throat, the superpotential and Kähler potential describing the conifold-modulus dynamics in a warped compactification have been computed in [23,24][3]:

$$W = \frac{M}{2\pi i}\left(Z \log\frac{\Lambda_{UV}^3}{Z} + Z + w_Z\right) + i\frac{K}{g_s}Z,$$
$$\mathcal{K} = -3\log\left(\rho + \bar\rho - \frac{\xi}{3}|Z|^{2/3}\right) + \log(2\gamma^4), \tag{8}$$

where $\gamma^2 = 16\sqrt{2}\pi^7\|\Omega\|^2$,[4] $\xi = 9c' g_s M^2$ and $c' \approx 1.18$ is a numerical factor coming from the warping [25]. Notice that the Kähler potential for the $Z$ modulus is known in a small-field expansion, and only the $Z^{2/3}$ term was worked-out explicitly. To avoid cumbersome expressions in what follows, we use the log form of the Kähler potential above (8), but it is understood that in the final results only the leading term in $Z^{2/3}$ is kept.

The supersymmetric Minkowksi vacuum is given by:

$$\partial_Z W|_{Z_{KS}} = 0 \quad\Rightarrow\quad Z_{KS} = \Lambda_{UV}^3 e^{-\frac{2\pi K}{g_s M}}. \tag{9}$$

Since the KS scalar potential and superpotential have to be zero on-shell in a supersymmetric Minkowski vacuum, this fixes the constant $w_Z$ in (8):

$$W_{\text{on-shell}} = 0 \quad\Rightarrow\quad w_Z = -\Lambda_{UV}^3 e^{-\frac{2\pi K}{g_s M}}. \tag{10}$$

We can promote this to an off-shell superpotential for the axion-dilaton as well, given by

$$W_{KS} = \frac{M}{2\pi i}\left[Z\left(\log\frac{\Lambda_{UV}^3}{Z} + 1\right) - \Lambda_{UV}^3 e^{\frac{2\pi i\tau K}{M}}\right] + K\tau Z. \tag{11}$$

This satisfies the supersymmetry condition in the axion-dilaton direction $D_\tau W|_{Z_{KS}} = \partial_\tau W|_{Z_{KS}} = 0$.

We now add anti-D3 branes, whose backreaction can be captured in the language of the four-dimensional effective theory by:

(i) an uplift term in the scalar potential, breaking supersymmetry and shifting the conifold modulus vev from $Z_{KS}$ to $Z_0$ (to be computed below).

(ii) A (0,3) flux giving rise to an additional superpotential $W_{\overline{D3}}$, whose dependence on the conifold and dilaton-axion moduli will be determined by requiring consistency with the ten-dimensional computation (5) and (7).

To compute the former, it is useful to describe the antibrane uplift potential in a manifestly supersymmetric way (more precisely in a nonlinearly supersymmetric way) introducing a nilpotent chiral multiplet $X$ [26,27], with the following Kähler potential and superpotential [24]:

$$\mathcal{K} = -3\log\left(\rho + \bar\rho - \frac{|X|^2}{3} - \frac{\xi}{3}|Z|^{\frac{2}{3}}\right) - \log\left(\frac{\mathrm{Im}\tau}{\gamma^4}\right),$$
$$W = W_{KS} + \frac{1}{M}\sqrt{\frac{c''\bar N}{\pi}}Z^{2/3}\tau X + Ae^{-a\rho} + W_{\overline{D3}}, \tag{12}$$

where $c'' \approx 1.75$ is a numerical factor related to the anti D3 brane energy [28] and we have also included the non-perturbative contribution to the superpotential coming from gaugino condensation or D3-brane instantons. The usual $\mathcal{N} = 1$ four-dimensional scalar potential

$$V = e^{\mathcal{K}}\{G^{a\bar b} D_a W D_{\bar b}\bar W - 3|W|^2\} \tag{13}$$

[2]Here by on-shell we mean that this is the value on the nose, where the complex structure moduli are fixed to those of the zeroth order Klebanov-Strassler solution.

[3]Here we assume that all the other complex structure moduli have been stabilized at a higher scale in a supersymmetric way and only consider the conifold modulus.

[4]We normalize the holomorphic three-form of the warped geometry such that the integral of the unwarped Klebanov-Strassler three-form $\Omega_{KS}$ over the $S^3$ at the bottom of the throat is equal to $Z$. This gives $\|\Omega\|^2 = 3/\pi^4$.

can then be written in the convenient form

$$V = \frac{\gamma^4 g_s}{r^2}\left\{\frac{9}{\xi}|Z|^{4/3}|\partial_Z W|^2 + |\partial_X W|^2 + \frac{4}{g_s^2 r}|D_\tau W|^2 + \frac{\rho+\bar\rho}{3}\left|\partial_\rho W_{\rm eff} - \frac{3}{\rho+\bar\rho}W_{\rm eff}\right|^2 - \frac{3}{\rho+\bar\rho}|W_{\rm eff}|^2\right\}, \quad (14)$$

where

$$W_{\rm eff} = \frac{M}{2\pi i}\left(Z - \Lambda_{\rm UV}^3 e^{-\frac{2\pi K}{g_s M}}\right) + Ae^{-a\rho} + W_{\overline{\rm D3}}, \quad (15)$$

and $r \equiv \rho + \bar\rho - \frac{\xi}{3}|Z|^{2/3}$ and where we used the on-shell axion-dilaton value $\mathrm{Im}\tau = g_s^{-1}$.

In deriving (14) we used the following relations

$$G^{z\bar j}\partial_{\bar j}\mathcal{K} = G^{i\bar z}\partial_i\mathcal{K} = \mathcal{O}\left(\frac{Z^{4/3}\bar Z^{1/3}}{\rho+\bar\rho}\right) \quad (16)$$

and omitted subleading terms. From now on we will also not take into account the $|D_\tau W|^2$ term as it scales like $\mathcal{O}(|Z|^2/r^3)$ and hence is subleading as well.

In the absence of the nonperturbative term (setting $A = 0$), the second line in (14) is zero and the scalar potential becomes

$$V = \frac{\gamma^4|Z|^{4/3}}{c'(\rho+\bar\rho)^2}\left\{\left|\frac{1}{2\pi i}\log\frac{\Lambda_{\rm UV}^3}{Z} + \frac{iK}{g_s M}\right|^2 + \frac{c'c''}{\pi}\lambda\right\}, \quad (17)$$

where we used that in the strongly warped regime $r \approx \rho + \bar\rho$. Equation (17) is the KS + uplift scalar potential, which as a function of the conifold modulus Z has a minimum at

$$Z_0 = \Lambda_{\rm UV}^3 e^{-\frac{2\pi K}{g_s M}} e^{-\frac{3}{4}\left(1-\sqrt{1-\frac{64\pi c' c'' \bar N}{9 g_s M^2}}\right)} \simeq \left(1 - \frac{8\pi c' c''}{3}\lambda\right) Z_{\rm KS} \equiv Z_{\rm KS} + \delta Z, \quad (18)$$

where we expanded to the first order in the $\overline{\rm D3}$ uplift parameter $\lambda$, defined in (2), in order to compare to the 10d computation.

The on-shell value of the superpotential is then computed to first order

$$W(Z_0) \simeq W_{\rm KS}(Z_{\rm KS}) + \partial_Z W_{\rm KS}(Z_{\rm KS})\delta Z + W_{\bar{\rm D}3} = W_{\overline{\rm D3}} = -0.87 i\lambda M Z_{\rm KS}, \quad (19)$$

where in the first line we used (9) and (10) and in the second line we inserted the 10d input (5).

The F-term of the conifold modulus can similarly be evaluated

$$D_Z W(Z_0) = \partial_Z W(Z_0) + \mathcal{K}_Z W(Z_0) \simeq \partial_Z^2 W_{\rm KS}(Z_{\rm KS})\delta Z + \mathcal{K}_Z W_{\overline{\rm D3}} \simeq \frac{4c'c''}{3i}\lambda M \simeq -2.75 i\lambda M, \quad (20)$$

where in deriving the result, we anticipated, using the explicit form of the Kähler potential in (12), that the term $\mathcal{K}_Z W_{\overline{\rm D3}} \sim O(Z_0^{2/3})$ and is therefore subleading. This F-term has the same parametric dependence as its 10d counterpart (7), with a different numerical coefficient.

Finally, the F-term of the axion-dilaton is

$$D_\tau W(Z_0) \simeq \partial_Z\partial_\tau W(Z_{\rm KS})\delta Z + \partial_\tau W_{\overline{\rm D3}} + \mathcal{K}_\tau W_{\overline{\rm D3}} = -\frac{8\pi c' c'' K}{3}\lambda Z_{\rm KS} + \frac{0.435}{M}Z_{\rm KS} + \partial_\tau W_{\overline{\rm D3}}. \quad (21)$$

In order to obtain the correct parametric dependence of the ten-dimensional result (7) we impose

$$\partial_\tau W_{\overline{\rm D3}} = \frac{8\pi c' c'' K}{3}\lambda Z_{\rm KS} \simeq -K(Z_0 - Z_{\rm KS}), \quad (22)$$

and thus

$$D_\tau W(Z_0) = 0.435 g_s M\lambda Z_{\rm KS}. \quad (23)$$

The off-shell value of $W_{\overline{\rm D3}}$ should therefore be considered as an expansion

$$W_{\overline{\rm D3}}(\tau) = W_{\overline{\rm D3}}(\tau_0) + \partial_\tau W_{\overline{\rm D3}}(\tau_0)(\tau - \tau_0) + \cdots, \quad (24)$$

where we have determined the first two coefficients $W_{\overline{\rm D3}}(\tau_0)$ and $\partial_\tau W_{\overline{\rm D3}}(\tau_0)$ by consistency with the 10d results.

Let us stress that we do not expect exact numerical agreement between the ten and four-dimensional results, but we do get the same parametric dependence. One of the reasons that the numerical factors might not exactly match is that the four-dimensional theory misses the effects of massive but light modes of the compactification [23].

Before closing this section, note that the complete scalar potential (14) has an approximately decoupled structure

$$V = V_{\rm KS+uplift} + V_{\rm KKLT}, \quad (25)$$

where

$$V_{\rm KKLT} = \frac{\gamma^4 g_s}{r^2}\left\{\frac{\rho+\bar\rho}{3}\left|\partial_\rho W_{\rm eff} - \frac{3W_{\rm eff}}{\rho+\bar\rho}\right|^2 - \frac{3|W_{\rm eff}|^2}{\rho+\bar\rho}\right\} \quad (26)$$

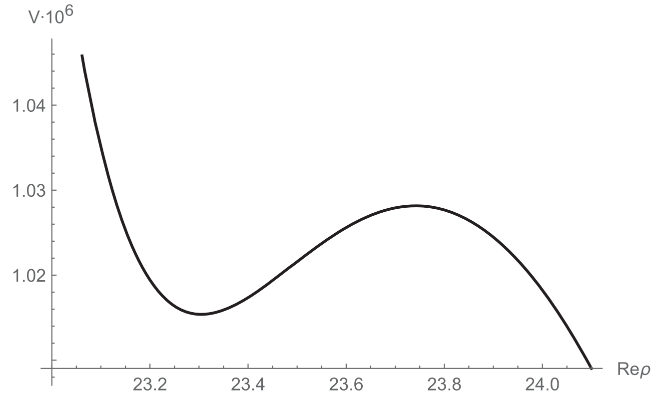


FIG. 2. The potential for the choice of parameters $a = \frac{\pi}{3}$, $g_s = \frac{1}{2}$, $A = 3 \times 10^3$, $K = 134$, $M = 200$. This gives $Z_0 \approx 10^{-4}$.

and $V_{\mathrm{KS+uplift}}$ is given in (17). Furthermore, the KKLT small superpotential constant $W_0$ is given in our construction by the on-shell value of the $\rho$-independent term in (15)

$$W_{0,\mathrm{KKLT}} = \frac{M}{2\pi i}(Z_0 - \Lambda_{\mathrm{UV}}^3 e^{-\frac{2\pi K}{g_s M}}) + W_{\overline{\mathrm{D3}}} \simeq -i\left(0.87 - \frac{4}{3}c' c''\right)\lambda M Z_{\mathrm{KS}}. \tag{27}$$

## IV. BARE-BONES DE SITTER

In this section, we show that for certain choices of the parameters, the potential:

$$V = e^{\mathcal{K}}(G^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2) \tag{28}$$

leads to de Sitter vacua. The potential is computed using the 10d input $D_i W$, $W = W_{\overline{\mathrm{D3}}} + Ae^{-a\rho}$ and Kähler potential as in (12).

In Fig. 2 we plot the potential for a particular choice of parameters. This choice is not unique: we have performed a partial scan of the parameters and we are able to find several other de Sitter vacua.

For our de Sitter minimum the hierarchy between the bottom of the KS throat and the UV scale is of order $\frac{2\pi K}{g_s M} \approx 8$. For other de Sitter constructions without a large warping, see [29].

In the future, it would be important (but rather nontrivial) to check if the existence of this minimum survives higher order corrections in $\lambda$ and $Z$, as well as quantum corrections.

## V. CONCLUSIONS

A nonvanishing on-shell Gukov-Vafa-Witten superpotential is crucial in a KKLT-like construction of de Sitter vacua. In this paper, we have shown that a small GVW superpotential, dubbed $W_{\overline{\mathrm{D3}}}$ above, is generated by $\overline{\mathrm{D3}}$ branes at the tip of a KS throat. This superpotential, together with the anti D3-brane-generated F-terms provide all that is needed to obtain a compactification with a positive cosmological constant.

As we explained above, our proposal for constructing de Sitter solutions is more bare-bones and hence more robust than the KKLT one, because it has one less ingredient. Of course, as in all phenomenological constructions, adding more ingredients gives one more freedom to tune the resulting physical parameters. Hence, one can argue that our proposal, though more robust, is less accommodating that the KKLT construction for obtaining a parametrically small cosmological constant. However, the aim of our paper is not phenomenological, bur rather to understand which ingredients are absolutely necessary to construct de Sitter, and which are optional, with an ultimate purpose of achieving a robust construction that may provide a way to escape the no-go arguments of [1]. We believe our result represents a step in that direction.

Another interesting result of the calculation presented in this paper is the parametric agreement between the first-principle, ten-dimensional computation of the effective potential (in Sec. II) and the four-dimensional-supergravity computation (in Sec. III). To our knowledge, this is the first confirmation of the validity of the off-shell four-dimensional warped effective action [25] and the analysis of [28].

Last, but not least, our proposal does not avoid some of the known constraints on KKLT-like models. It would be interesting to explore whether the problems underlined in [30] also apply to our model. Furthermore, the minimum we found requires the contribution to the D3 tadpole of the fluxes in the KS throat to be of order $KM \approx 2 \times 10^4$. In [31] it was conjectured that such throats cannot be embedded in a flux compactification with stabilized moduli. It would be interesting to use our procedure to search for vacua where this tadpole contribution is smaller.

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