



NLO QCD corrections to off-shell top–antitop production with semi-leptonic decays at lepton colliders

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Abstract The study of top-quark properties will be a central aspect of the physics programme of any future lepton collider. In this article, we investigate the production of top-quark pairs in the semi-leptonic decay channel in e^+e^- collisions, whose experimental signature is one charged lepton, jets, and missing energy. We present for the first time fiducial cross sections and differential distributions at next-to-leading-order accuracy in QCD for the full off-shell process. We find that the QCD corrections for the considered process are strongly dependent on the beam energies and range from few per cent up to more than 100% (near threshold and above 1 TeV). We focus, in particular, on two scenarios: one close to threshold (365 GeV), dominated by top-pair production, and one at the TeV scale (1.5 TeV), for which irreducible-background contributions become relevant. An assessment of polarised-beam effects is also provided.

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1 Introduction

What large-scale collider experiment will come after the end of the Large Hadron Collider (LHC) is currently an open question. At present, several options are being considered which include lepton colliders such as the International Linear Collider (ILC) [1–3], the FCC-ee [4], or the Compact Linear Collider (CLIC) [5]. In all cases, the study of top-quark properties will play a central role in the physics programme of those facilities.

The main advantage of lepton colliders over hadron ones is the possibility to tune very precisely the centre-of-mass (CM) energy of the experiment. Thus, one can perform a scan of energies covering the threshold region for the production of a pair of top quarks. This provides a very clean access to key properties of the top quark such as its mass and width [6].

On the experimental side, significant prospective work has been done [7–9] to estimate the potential gain in performing such measurements and to assess their experimental limitations. On the theory side, great efforts have been put in providing precise predictions using non-relativistic QCD and resummation techniques at threshold [10–13]. Differential predictions including also the transition to the continuum described by fixed-order QCD have been obtained in Ref. [14].

Above threshold, several predictions at fixed order in QCD have been provided for the on-shell production of a top–antitop pair, i.e. $e^+e^- \rightarrow t\bar{t}$, reaching next-to-next-to-next-to-leading-order (N³LO) accuracy for the inclusive cross section [15, 16] and next-to-next-to-leading-order (NNLO) accuracy at the differential level [17–20]. For the off-shell top–antitop pair production with leptonic decays, i.e. $e^+e^- \rightarrow j_b j_{\bar{b}} \ell^- \ell'^+ \nu_{\bar{\ell}} \nu_{\ell'}$, which is well-defined both below and above threshold, several next-to-leading-order (NLO) QCD predictions have been provided [21–23]. Regarding

electroweak (EW) corrections, NLO accuracy for the inclusive cross section has been achieved long ago [24–26] and later supplemented with $\mathcal{O}(\alpha^2)$ ISR effects [27]. Recently, the QED ISR effects at NLL in collinear factorisation have been matched to NLO EW corrections for on-shell production [28].

It is worth emphasising that for off-shell predictions the fully leptonic final state has been usually considered in the literature, with the exception of some sensitivity and background studies relying on LO off-shell simulations in the lepton-plus-jets channel [29–32].

In particular, the NLO QCD corrections for the semi-leptonic final state, i.e. $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$ are still unknown.¹ The lepton+jets channel has the advantage to possess a larger cross section owing to the larger W-decay branching ratio. It also allows to fully reconstruct the momenta of the top quarks.

In the present work, we fill this gap by computing for the first time NLO QCD corrections for the process $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$. In particular, we discuss phenomenological results in the case where all final-state particles are well separated, which corresponds to a so-called *resolved* topology as opposed to the case where light jets are allowed to be clustered in a large-radius b jet (*boosted* topology). We provide cross sections and differential distributions for different CM energies.

A further advancement of this calculation concerns the implementation of the FKS subtraction scheme [34] in the Monte Carlo integration code MOCANLO. Among others, the present calculation served to validate the implementation of the FKS subtraction terms for processes with only final-state soft and collinear singularities.

This article is organised as follows. In Sect. 2, the process under investigation is presented (Sect. 2.1), the input parameters and event selections are listed (Sect. 2.2), and several remarks are provided regarding our implementation (Sect. 2.3). Section 3 discusses numerical results for the fiducial cross section and differential distributions. Finally, in Sect. 4 the main results obtained are summarised.

2 Calculation details

2.1 Definition of the process

In the present work, we consider the production of a top-antitop pair in e^+e^- collisions in the semi-leptonic decay channel,

$$e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu, \quad (1)$$

¹ For the LHC, the semi-leptonic final state has been computed by some of us few years ago [33].

at NLO QCD accuracy. All final-state particles (quarks and leptons) are considered massless, and no quark mixing is taken into account (unit CKM matrix). With such a choice, the NLO corrections of order $\mathcal{O}(\alpha_s \alpha^6)$ are genuine QCD corrections to the leading-order (LO) EW [$\mathcal{O}(\alpha^6)$] cross section, as the EW corrections to the LO interference [$\mathcal{O}(\alpha_s \alpha^5)$] vanish thanks to colour algebra. The real corrections are made of all possible gluon emissions from any of the coloured particles. The virtual corrections consist in the interference of Born amplitudes with one-loop ones, which are obtained by the insertion of a gluon in the tree-level matrix elements.

As illustrated in Fig. 1 for the leading order, all possible non-resonant and off-shell contributions are accounted for. In the top row, on the left-hand side, the typical production of a pair of top quarks and their semi-leptonic decay is depicted. The middle diagram shows a Higgs-strahlung type contribution where the Higgs boson decays into a pair of W bosons and the Z boson into a bottom-antibottom pair. The diagram on the right-hand side shows a contribution to the same final state that does not involve any resonant top or antitop quark or Higgs boson. In the second row, a tri-boson (left) and a single-top (right) contribution are shown. We do not consider initial-state-radiation (ISR) and beam-strahlung effects of QED type as we restrict ourselves to QCD corrections.

2.2 Input parameters and kinematic selections

The on-shell weak-boson masses and decay widths are fixed as [35]

$$\begin{aligned} M_W^{\text{OS}} &= 80.379 \text{ GeV}, & \Gamma_W^{\text{OS}} &= 2.085 \text{ GeV}, \\ M_Z^{\text{OS}} &= 91.1876 \text{ GeV}, & \Gamma_Z^{\text{OS}} &= 2.4952 \text{ GeV}, \end{aligned} \quad (2)$$

and then converted into the pole values [36], leading to the numerical values

$$\begin{aligned} M_W &= 2.08429889367 \text{ GeV}, & \Gamma_W &= 80.3519715945 \text{ GeV}, \\ M_Z &= 91.1534806192 \text{ GeV}, & \Gamma_Z &= 2.49426637877 \text{ GeV} \end{aligned} \quad (3)$$

used in the calculation. The Higgs-boson and top-quark pole masses are chosen as [35]

$$\begin{aligned} M_H &= 125 \text{ GeV}, & \Gamma_H &= 4.07 \times 10^{-3} \text{ GeV}, \\ m_t &= 173 \text{ GeV}, & \Gamma_t &= 1.3448 \text{ GeV}. \end{aligned} \quad (4)$$

While the Higgs-boson width is taken from Ref. [37], the numerical value of the top-quark width is obtained by applying relative QCD corrections from Ref. [38] to the LO top-quark width computed following Ref. [39]. The resulting value of the top width is used at both LO and NLO accuracy

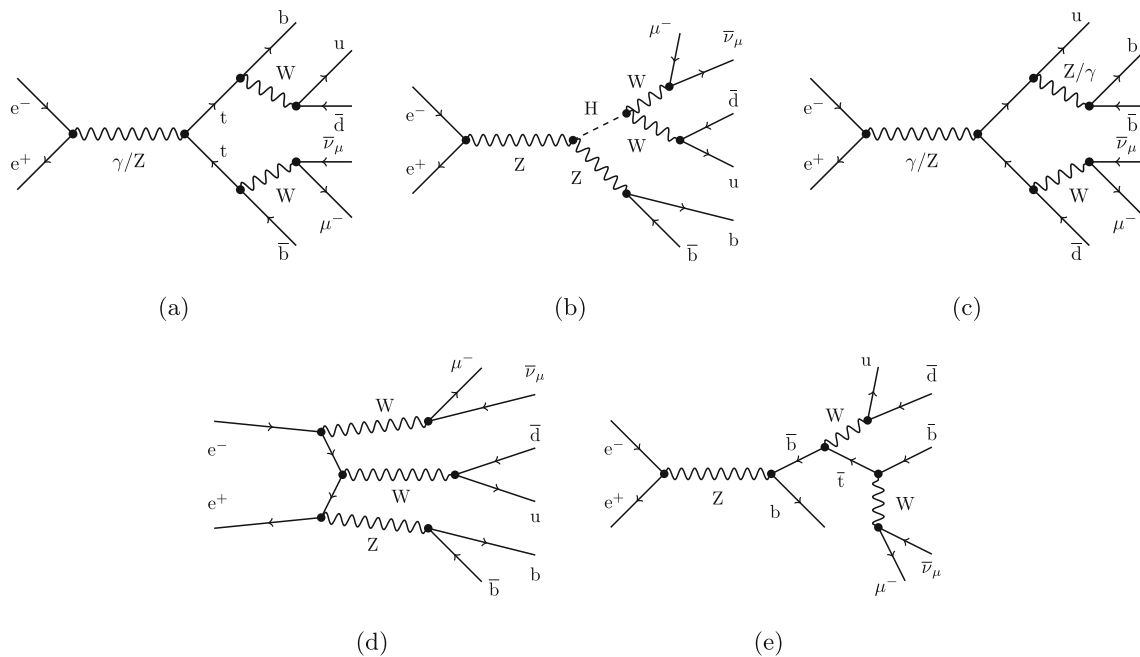


Fig. 1 Sample tree-level Feynman diagrams for the process $e^+e^- \rightarrow jjb j \mu^+ \nu_\mu$

and kept fixed in the scale variation. All unstable particles are treated within the complex-mass scheme [40–43].

The EW coupling constant α is computed within the G_μ scheme [44] with the Fermi constant set to

$$G_\mu = 1.16638 \cdot 10^{-5} \text{ GeV}^{-2}. \tag{5}$$

We neglect the bottom mass throughout the calculation and determine the running of the strong coupling constant α_s at two loops using the RECOLA program [45], assuming $\alpha_s(M_Z) = 0.118$ with 5 active flavours. Finally, the renormalisation scale is set to $\mu_R = m_t$, and the scale uncertainty is obtained by varying μ_R by a factor 2 up and down.

In the following, we consider e^+e^- collisions at several CM energies. In addition to a scan of the integrated cross sections between 300 GeV and 2 TeV, shown in Sect. 3.1, we focus on two particular CM energies. Specifically, in Sect. 3.2 we provide differential results for 365 GeV, i.e. the highest collision energy envisioned for the FCC-ee [4]. A similar energy is planned for the first operating scenario of CLIC [8,46], targeting the production of $t\bar{t}$ pairs above threshold. In Sect. 3.3, we show differential results for 1.5 TeV, i.e. the second operating stage of CLIC [9].

In our setup, the jet clustering is carried out with a resolution radius $R = 0.4$ as suggested in Ref. [8], using an own implementation of the anti- k_T algorithm. This algorithm employs the distance criteria of Ref. [47] with an exponent -1 for final-state partons (quarks and gluons). Partons close to the beams are discarded for angles below 0.7721° , which corresponds to a rapidity of 5. We would like to point out that a common choice in the literature for lepton-collider stud-

ies is the usage of k_T -based algorithms with different radii [7,8,48]. While the details of the clustering algorithm matter for experimental measurements, one does not expect sizeable effects for our NLO calculation. We have verified numerically that using the distance criteria of Ref. [47] with different exponents ($-1, 0, +1$) yields negligible differences both at integrated and at differential level for the process considered here. Note, however, that for resolved topologies, which we study in this work, large jet radii are not appropriate for high energies (above a TeV) as they artificially suppress the LO and induce large QCD corrections.

Note that a generalised version of the k_T algorithm has been used in the $t\bar{t}$ study in the fully-leptonic decay channel [23]. We use the following selection cuts, which are inspired by event selections applied in CLIC and FCC-ee studies [7–9]. In our calculation, the events are required to have:

- a minimum missing transverse-momentum $p_{T,\text{miss}} > 20 \text{ GeV}$, which is defined as the transverse momentum of the neutrino;
- a minimum transverse momentum $p_T > 20 \text{ GeV}$ for the antimuon, the light jets, and the two b-tagged jets;
- an angular acceptance of $10^\circ < \theta < 170^\circ$ for the antimuon, the light jets, and the two b-tagged jets;
- a minimum rapidity–azimuthal-angle distance between the antimuon and the jets, $\Delta R_{\ell j}, \Delta R_{\ell j b} > 0.4$;
- an invariant-mass cut on the system formed by the two hardest visible light jets, the charged lepton, and the neutrino of $M_{jj\mu^+\nu_\mu} > 130 \text{ GeV}$.

We require at least two light jets that fulfil all requirements on the transverse momentum, the angular acceptance, and the rapidity–azimuthal-angle distance to leptons (visible jets). Out of these jets, we select the two hardest ones (according to transverse momentum) for the distributions shown below. The condition on the invariant mass $M_{j_1 j_2 + \nu_\mu}$ selects a kinematic region that excludes the Higgs-boson decay into two leptons and two jets.

The two b jets present in the final state can be associated with the leptonically decaying top quark (j_b^{lep}) and the hadronically decaying antitop quark (j_b^{had}). This is achieved by finding the maximum of a likelihood function that is the product of two Breit–Wigner distributions (of the top and antitop quark), as done in Ref. [49]. The likelihood function mimics the top- and antitop-quark propagators, assuming three-body decays after recombination (both at LO and at NLO QCD), and reads

$$\mathcal{L}_{ij} = \frac{1}{\left(p_{\mu^+ \nu_\mu j_{b,i}}^2 - m_t^2\right)^2 + (m_t \Gamma_t)^2} \times \frac{1}{\left(p_{j_{b,i} j_{b,j}}^2 - m_t^2\right)^2 + (m_t \Gamma_t)^2}, \quad (6)$$

with $p_{abc} = p_a + p_b + p_c$. The combination of bottom jets $\{j_{b,i}, j_{b,j}\}$ that maximises \mathcal{L}_{ij} defines the two bottom jets originating from the leptonic and hadronic top quarks. Note that in Eq. (6) all possible combinations of light jets and b jets are considered. This includes also light jets with a minimum angle of 0.7721° that do not fulfil the transverse-momentum, angular, and rapidity–azimuthal-angle distance requirements for visible jets.

Note further that the neutrino momenta are extracted from Monte Carlo truth, assuming that the hard-scattering CM energy is exactly the one of the e^+e^- collision, i.e. neglecting ISR and beam-strahlung effects.

2.3 Implementation and validation

To carry out the present calculation, we have employed the Monte Carlo program MOCANLO. In the past, MOCANLO has been successfully used for several top-associated computations at NLO QCD and/or EW accuracy at hadron colliders [33, 49–55]. The present work is the first application of MOCANLO to a lepton-collider process. The program uses phase-space mappings similar to those of Refs. [40, 56, 57] and has shown to be particularly efficient for NLO calculations for high-multiplicity processes (up to $2 \rightarrow 8$). The tree-level and one-loop matrix elements are obtained from RECOLA [45, 58] using the integral library COLLIER [59]. For the subtraction of infrared divergences, the original code relies on the Catani–Seymour subtraction scheme [60–62].

For the present calculation, we have implemented the FKS scheme [34] following closely Refs. [63, 64].

To validate our implementation of the FKS scheme, we have compared our results against those obtained with the well-tested Catani–Seymour scheme for several NLO QCD calculations at lepton colliders including di-jet production, di-boson production in the semi-leptonic channel, off-shell top–antitop production in the fully leptonic channel, and the process considered in this work. In all cases, we have found perfect agreement within the Monte Carlo uncertainty at the level of both fiducial cross sections and differential distributions. For the $\sqrt{s} = 365$ GeV setup considered in this article, the fiducial cross section obtained at NLO QCD with the two subtraction schemes reads,

$$\sigma_{\text{NLO}}^{(\text{FKS})} = 21,419(14)_{-1.8\%}^{+2.2\%} \text{ fb}, \quad (7)$$

$$\sigma_{\text{NLO}}^{(\text{CS})} = 21,427(12)_{-1.8\%}^{+2.2\%} \text{ fb}, \quad (8)$$

respectively, showing perfect agreement. Agreement has also been found at the differential level, where the numerical differences between the NLO distributions obtained with the two schemes are well within integration uncertainties bin by bin. We provide in Fig. 2 a comparison of the differential results obtained with the two subtraction schemes for two observables, namely an angular one and a transverse-momentum one.

The integration errors displayed in the lower inset of the plots are the combined integration errors of both NLO calculations. We observe that the bin-wise agreement is within the integration uncertainties over the whole spectrum. An analogous picture has been found for all other observables that we have computed.

In addition to the comparison against the dipole formalism, our implementation of the FKS subtraction scheme has allowed for further tests:

- The cancellation of infrared poles in the n -body contribution to the NLO QCD cross section has been verified by evaluating the virtual contribution \mathcal{V} and the integrated FKS counterterm \mathcal{I} at different values of the infrared scale μ_{IR} (from 10^{-8} GeV to 10^8 GeV) and checking that the sum $\mathcal{V} + \mathcal{I}$ is independent of the μ_{IR} choice. This has been carried out for a large number of phase-space points finding agreement up to 12 digits.
- The cancellation of phase-space singularities between the real matrix element and the FKS subtraction counterterm has been verified by constructing real-phase-space points that approach the soft, collinear, and soft–collinear regions by means of a rescaling of the radiation variables.
- The FKS-subtraction parameters ξ_c and δ [34, 63, 64] which define the integration boundaries for the soft and collinear regimes, have been varied, confirming that the

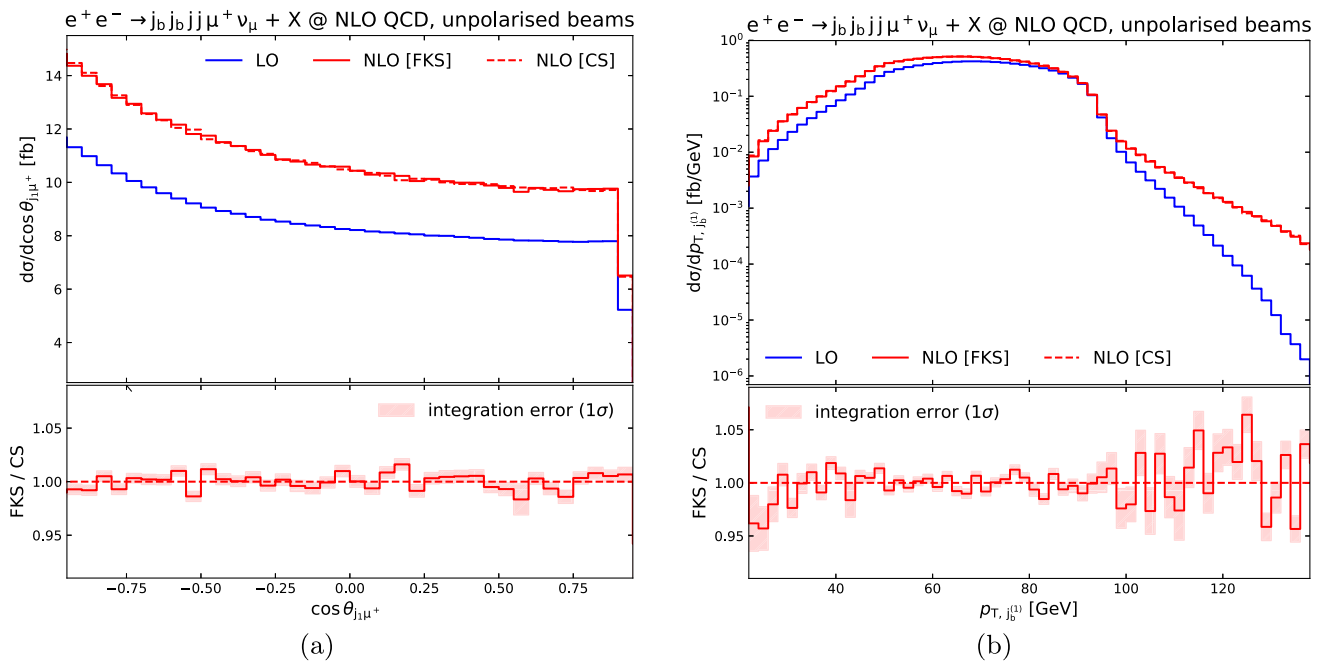


Fig. 2 Differential distributions in the cosine of the angular separation between the hardest light jet and the antimuon (left) and in the transverse momentum of the hardest b jet (right). The NLO QCD distributions obtained with the Catani–Seymour (dubbed CS) and FKS

subtraction schemes are plotted in the top panel besides the LO results. In the bottom panel the ratio between the FKS and CS results (solid line) and the corresponding Monte Carlo integration error (shaded band) are shown

Table 1 Comparison of subtracted virtual ($\mathcal{V}^{\text{subtr}}$) and real ($\mathcal{R}^{\text{subtr}}$) contributions to the NLO QCD correction to $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$ for different choices of FKS-subtraction soft (ξ_c) and collinear (δ) parameters

ξ_c	δ	$\mathcal{R}^{\text{subtr}}$	$\mathcal{V}^{\text{subtr}}$	$\mathcal{R}^{\text{subtr}} + \mathcal{V}^{\text{subtr}}$
0.01	0.01	37.497(4)	-32.94(1)	4.55(2)
0.4	0.01	-15.02(2)	19.51(7)	4.49(7)
0.01	0.4	6.14(2)	-1.620(9)	4.53(2)
0.4	0.4	-4.38(2)	8.86(5)	4.49(5)

sum of the subtraction counterterm and its integrated counterpart is independent of them. Selected results regarding this subtraction test are shown in Table 1, where the reader can observe a rather strong impact of the soft parameter ξ_c on the size and sign of subtracted real and virtual contributions, compared to a milder effect of the collinear one δ .

- The evaluation of FKS sector functions and their sum rules have been checked in the subtracted-real contribution by means of a variation of the exponents a, b that enter the sector functions, which are defined in Eq. (5.11) of Ref. [64]. For a fixed choice of the FKS parameters ($\xi_c = \delta = 0.01$) we have calculated the subtracted-real contributions for different choices of such exponents, e.g.

$$\begin{aligned}
 a, b = 1 : \quad \sigma_{\text{real}}^{\text{subtr}} &= 37.423(40) \text{ fb}, \\
 a, b = 4 : \quad \sigma_{\text{real}}^{\text{subtr}} &= 37.451(41) \text{ fb},
 \end{aligned}
 \tag{9}$$

finding perfect agreement within integration errors.

The default values used for the results presented here are $\xi_c = \delta = 0.01$ and $a = b = 1$.

3 Results

3.1 Fiducial cross sections

In this section, we report results for the fiducial cross section in the setup defined in Sect. 2. In Fig. 3, we provide the results at LO and NLO QCD accuracy for several choices of the CM energy, including regimes below and above the $t\bar{t}$ threshold. Since $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$ is a purely EW process at LO, there is no scale dependence at this order. At NLO, the QCD uncertainty comes from three-point variations of the renormalisation scale, $\mu_R/\mu_R^{(0)} = 1/2, 1, 2$.

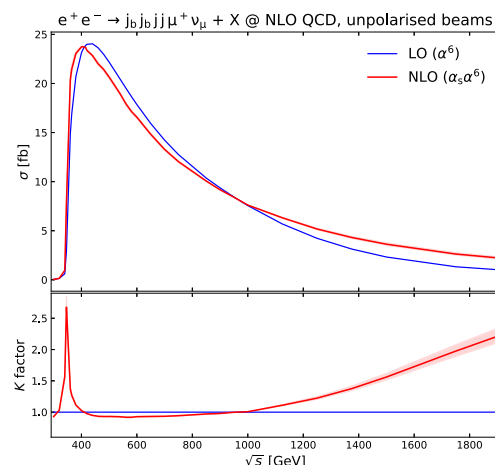
The NLO QCD corrections are strongly dependent on the CM energy as can be seen in Fig. 3, where the fiducial cross section is provided as a function of the CM energy in the range from 300 GeV to 2 TeV. The largest QCD correc-

\sqrt{s} [GeV]	σ_{LO} [fb]	$\sigma_{\text{NLO QCD}}$ [fb]	$\delta_{\text{NLO QCD}}[\%]$
320	0.14461(5)	0.1490(8) ^{+0.3%} _{-0.3%}	3.03
340	0.6153(2)	0.963(3) ^{+3.8%} _{-3.1%}	56.6
346	2.9127(8)	7.795(6) ^{+6.6%} _{-5.4%}	167.6
365	16.877(4)	21.42(2) ^{+2.2%} _{-1.8%}	26.9
405	23.437(7)	23.75(4) ^{+0.1%} _{-0.1%}	1.34
440	24.040(8)	22.84(9) ^{+0.5%} _{-0.6%}	-5.01
560	19.542(7)	17.95(4) ^{+0.8%} _{-0.9%}	-8.15
1125	5.683(1)	6.31(3) ^{+1.1%} _{-0.9%}	11.1
1500	2.3235(8)	3.627(9) ^{+3.8%} _{-3.1%}	56.1

Fig. 3 Fiducial cross sections for $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$ at LO and NLO QCD at various CM energies \sqrt{s} . Numerical values are shown in the table (left), where the digit in parentheses indicates the Monte Carlo statistical error, while the sub- and super-scripts in per cent indicate the renormalisation-scale uncertainties calculated with three-point scale

tions are observed slightly below the top–antitop threshold. In this regime, the presence of the Coulomb singularity renders the NLO QCD corrections divergent for on-shell top quarks [21,65], while the inclusion of decay effects makes them finite though still very large, reaching almost 170% of the LO cross section. In the case of the fully leptonic top-quark decays, a similar behaviour is observed at and around threshold [21,23]. Above threshold the corrections turn negative. For the semi-leptonic process, the NLO QCD corrections become positive for energies above 1 TeV. For example at 1.5 TeV, the corrections are very large of the order of 60%. This is in contrast with the fully leptonic case where the corrections stay negative at high energies. This difference is due to the specific event selection. At LO, the cross section is suppressed in the semi-leptonic channel by the jet clustering (with $R = 0.4$) which effectively forbids boosted W bosons decaying into two quarks. At NLO QCD, in the presence of real gluon radiation, this constraint is lifted for sufficiently hard gluons. This part of the phase space therefore opens up and leads to relatively large corrections. This effect is specific to the semi-leptonic final state as in the fully leptonic case there are no cuts preventing boosted W bosons. We have verified this explanation by running the calculation for different jet radii. Smaller relative QCD corrections are found for a smaller jet-clustering radius ($R = 0.1$, allowing configurations with more boosted W bosons). For instance, at $\sqrt{s} = 1$ TeV we find:

$$\begin{aligned}
 R = 0.4 : \quad & \sigma_{\text{LO}} = 2.3235(8) \text{ fb}, \quad \delta_{\text{QCD}} = +56.1\%, \\
 R = 0.1 : \quad & \sigma_{\text{LO}} = 4.1524(6) \text{ fb}, \quad \delta_{\text{QCD}} = +4.8\%. \quad (10)
 \end{aligned}$$



variations. The integrated cross section is shown in the figure (right) at LO (blue) and NLO QCD (red) as a function of the CM energy. The red-shaded band is obtained by means of three-point renormalisation-scale variations

The scale uncertainty increases from sub-percent to 5–7% when approaching the threshold ($\sqrt{s} \lesssim 346$ GeV) owing to the large QCD corrections in this region. For $400 \text{ GeV} \lesssim \sqrt{s} \lesssim 1$ TeV, it decreases down to sub-percent level, while for $\sqrt{s} \gtrsim 1$ TeV it increases up to $\mathcal{O}(5\text{--}10\%)$ level driven by large real-radiation corrections.

Finally, we mention that the off-shell calculation embeds $t\bar{t}$ contributions as well as irreducible-background contributions that become more and more important with increasing CM energy, as observed in ILC sensitivity studies at 500 GeV [29,31,32]. Analysing the Monte Carlo sampling in various phase-space integration channels, we have checked that background topologies become indeed relevant. The topologies with two resonant top quarks turn out to contribute roughly 90% (67%) to the off-shell calculation at 365 GeV (1.5 TeV), while the single-top ones (e.g. $e^+e^- \rightarrow \bar{b}W^-t, bW^+\bar{t}$, as shown in Fig. 1e) give the largest background contributions, accounting for the 5% (20%), respectively. Tri-boson topologies ($e^+e^- \rightarrow W^+W^-Z$, as shown in Fig. 1d) give a 1% (3%) contribution. We also checked that the above estimates of contributing resonant structures are in rough agreement with on-shell simulations performed at LO with MADGRAPH5_AMC@NLO [66]. It is therefore interesting to realise that at very high energy the final state under investigation is not only made of top–antitop topologies but also of many others, rendering the reconstruction of top–antitop pairs difficult.

3.2 Differential distributions at 365 GeV

In this section, several differential distributions are presented at LO and NLO QCD accuracy. While the upper panels of

the plots contain the absolute predictions, the lower ones show the corresponding K factors. In the following, when an observable refers to either the leptonically or the hadronically decaying top quark, their definition follows from the maximisation of the likelihood function in Eq. (6).

In Fig. 4, several transverse-momentum distributions are shown. The first two are for the reconstructed bottom quark from the leptonically and hadronically decaying top quarks, respectively. At LO, both distributions are almost identical with a pronounced drop around 95 GeV. This results from a hard cut for production of on-shell top-quark pairs decaying into on-shell W bosons that can be evaluated to

$$p_{T,j_b} < \frac{\sqrt{s}}{4} \left(1 - \frac{M_W^2}{m_t^2} \right) \left[1 + \sqrt{1 - \frac{4m_t^2}{s}} \right]. \quad (11)$$

For $\sqrt{s} = 365$ GeV, this amounts to $p_{T,j_b} \lesssim 94.4$ GeV. At NLO QCD, the picture changes as the hadronic top quark receives significantly more corrections at high transverse momenta. This is related to additional real-radiation jets that are mis-reconstructed as top-decay jets. It is worth mentioning that the reconstructed bottom-quark distributions are very close to those obtained with Monte Carlo truth. For what concerns the reconstructed top quarks (leptonic and hadronic), both at LO and NLO QCD accuracy the two distributions show very similar qualitative behaviours. On-shell production of top–antitop quarks is restricted to $p_{T,t} < (\sqrt{s}/2)\sqrt{1 - 4m_t^2/s} \approx 58$ GeV, leading to a sharp drop of the distribution above this value. In the off-shell region, the NLO QCD corrections are somewhat larger for the hadronically decaying top quark. For $p_{T,t} \gtrsim 127$ GeV the recoiling system cannot contain a resonant W boson anymore explaining the shoulder near 120 GeV. We note that in both cases, around 100 GeV and above, the top-transverse-momentum distributions become numerically unstable. Besides the lower statistics, this is simply due to the fact that at such energies, the process is not exclusively made of top–antitop topologies as explained in the previous section.

In Fig. 5, several invariant-mass distributions are displayed. The first two are for the invariant masses of the leptonically (Fig. 5a) and hadronically (Fig. 5b) decaying top quarks, respectively. It is interesting to observe that, as for the transverse-momentum distributions, the LO predictions are essentially identical while at NLO they significantly differ. This is due to the fact that the hadronically decaying top quark possesses three partons in the final state as opposed to only one for the leptonically decaying one, leading to more final-state radiations in the hadronic case. As a consequence, more events are moved from the resonance or above to below the resonance owing to final-state radiation that is not reconstructed with the decay products of the top/antitop quark forming hence a large radiative tail (see, for instance,

Refs. [33,67]). In the case of the hadronically decaying top quark, the effect is so large that the NLO cross section becomes negative above the resonance. Such a behaviour has already been observed for the same final state at a hadron collider [33] and requires the inclusion of higher-order corrections for a proper description of this observable.

A radiative tail also appears in the invariant-mass distribution of the two hardest light jets (Fig. 5c), which at LO reconstruct a W boson. Again, the effects are extremely large with K factors reaching ten below the resonance. The distribution in the invariant mass of the system formed by the reconstructed bottom quark from the leptonically decaying top quark and the antimuon (Fig. 5d) has been found to be very sensitive to the top-quark mass as it possesses an on-shell edge at $M_{\mu+j_b}^2 < m_t^2 - M_W^2 \approx (153 \text{ GeV})^2$ [67,68]. While the relative corrections are flat in the on-shell region, they strongly increase above the on-shell edge.

Finally, Fig. 5e, f show distributions in the cosine of the production angle of the reconstructed top quarks. Both distributions are relatively similar up to a reflection of the directions. Indeed, given that, as opposed to the LHC, the initial state is asymmetric, the top and antitop quarks have preferred directions while generally ending up in a back-to-back configuration. The NLO QCD corrections are flat and reproduce those of the fiducial cross section.

3.3 Differential distributions at 1.5 TeV

In Fig. 6 we show the results obtained for a few selected observables in e^+e^- collisions at 1.5 TeV CM energy. We stress that the technique used in the 365 GeV analysis to reconstruct the top and antitop quarks is not performing well at 1.5 TeV, owing to the presence of sizeable irreducible backgrounds not involving a top–antitop pair in the fiducial volume. Therefore, we do not show top-reconstructed observables in this section as their physical interpretation is unclear. The higher CM energy shifts the most populated transverse-momentum and invariant-mass regions to higher values than at 365 GeV and enhances irreducible backgrounds that are suppressed for CM energies around the top-pair threshold.

In Fig. 6a we show the differential results in the transverse momentum of the second-hardest light jet. At LO and for on-shell W bosons, this observable is characterised by a kinematic cut-off which, assuming small angles between the two jets, is given by $p_{T,j_2,\text{max}} \sim m_{jj,\text{max}}/\Delta R_{jj,\text{min}} \sim M_W/0.4 \sim 200$ GeV [33]. The NLO QCD corrections, rather flat in the most populated region ($p_{T,j_2} \lesssim 150$ GeV), fill the kinematic regime that is suppressed at LO, driven by real corrections with a third jet from gluon radiation that are tagged as the second-hardest jet.

The light-jet kinematics is strongly affected by QCD corrections at NLO, as can also be observed in Fig. 6b where the

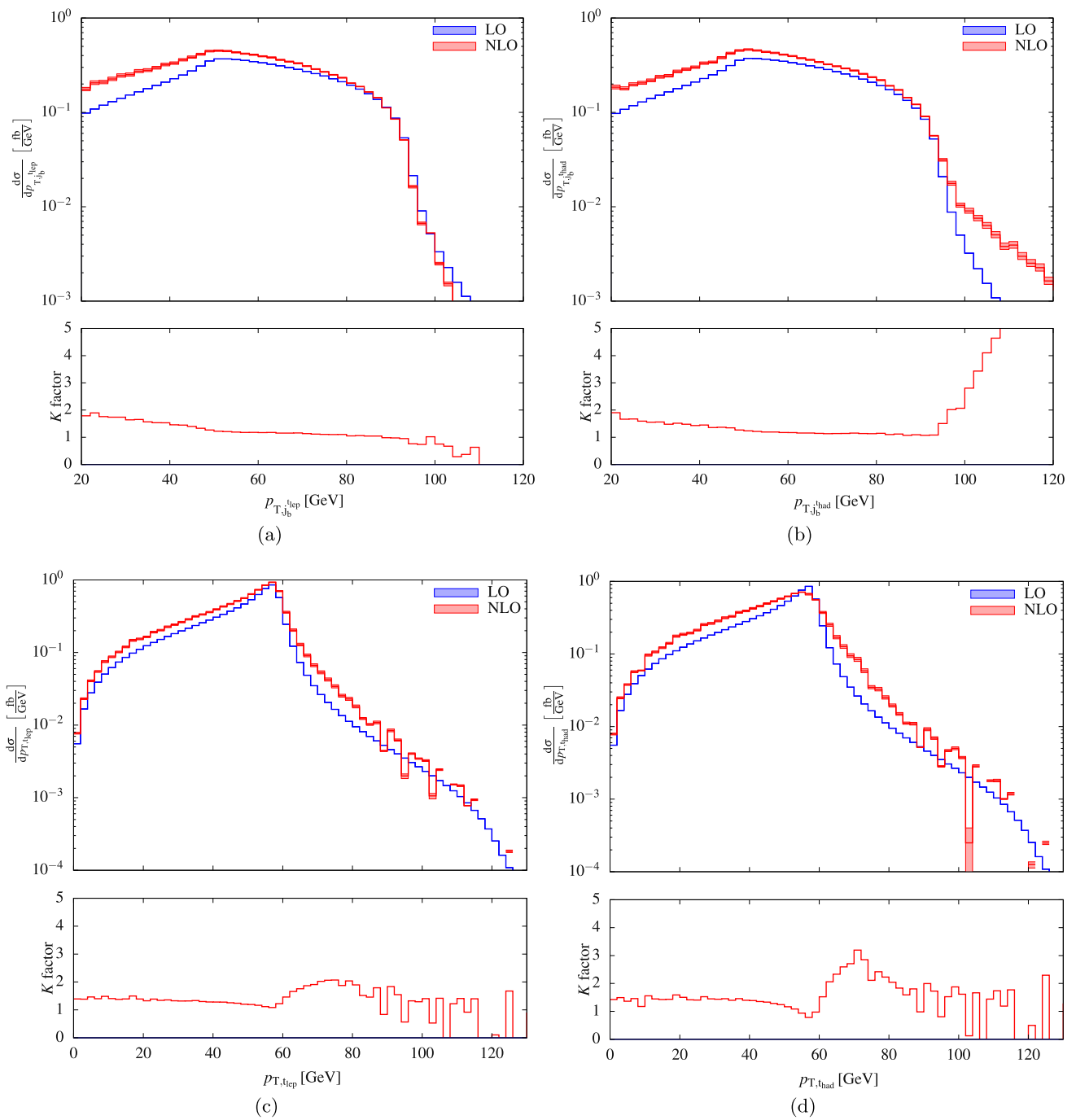


Fig. 4 Differential distributions for a 365 GeV CM energy in the transverse momentum of the reconstructed bottom quark from the leptonically decaying top quark (top left), the reconstructed bottom quark from the hadronically decaying top quark (top right), the reconstructed

leptonically decaying top quark (bottom left), and the reconstructed hadronically decaying top quark (bottom right). The red-shaded band is obtained by means of three-point renormalisation-scale variations and the lower panel displays the *K* factor

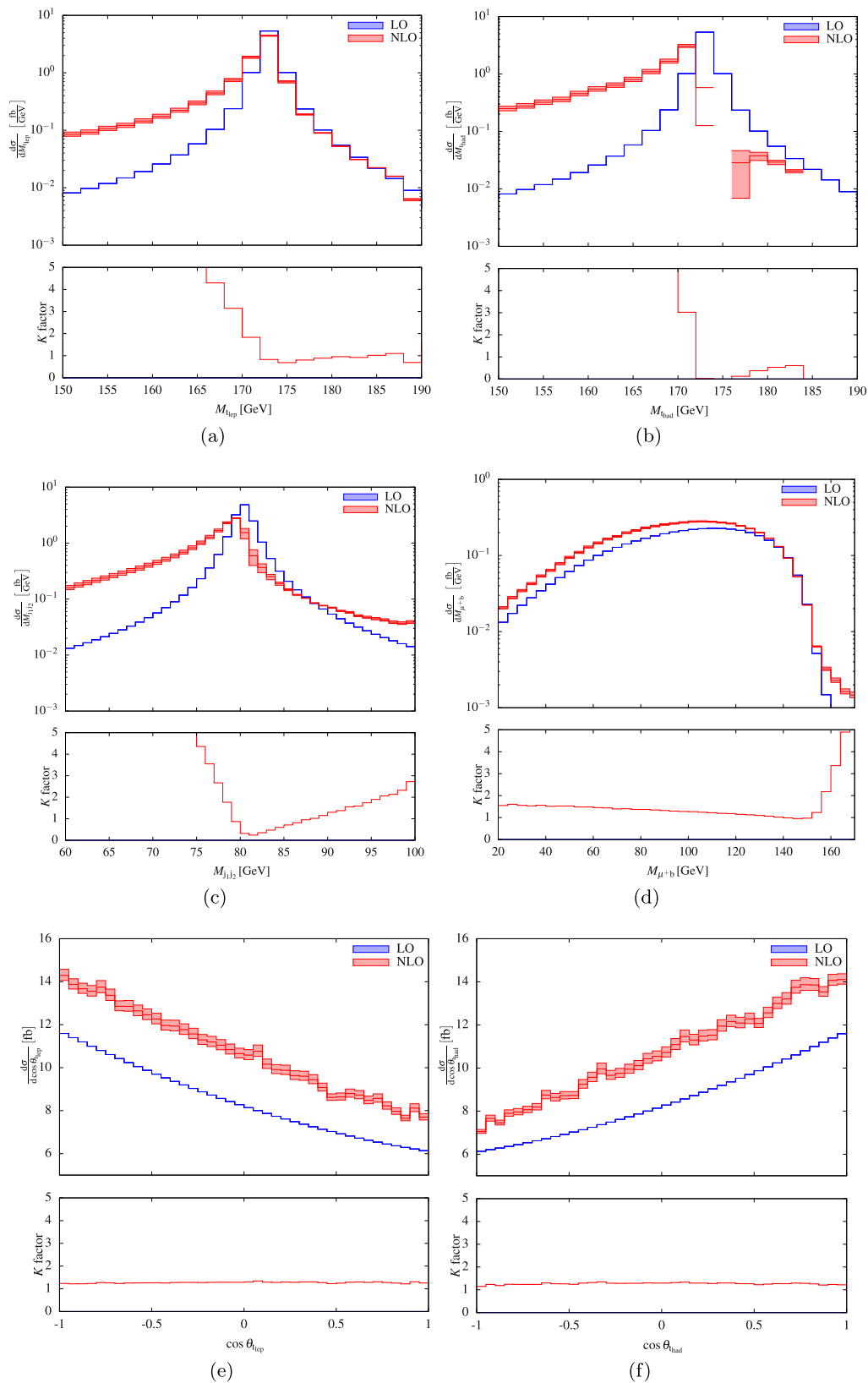


Fig. 5 Differential distributions for a 365 GeV CM energy in the invariant masses of the reconstructed leptonically decaying top quark (a), the reconstructed hadronically decaying top quark (b), the system of the two hardest light jets (c), and the bottom quark from the leptonically decaying top quark with the antimuon (d), as well as in the cosine of the

production angle of the reconstructed leptonically decaying top quark (e) and the reconstructed hadronically decaying top quark (f). Note that for Fig. 5d, the Monte Carlo truth momenta are used. Same structure as in Fig. 4

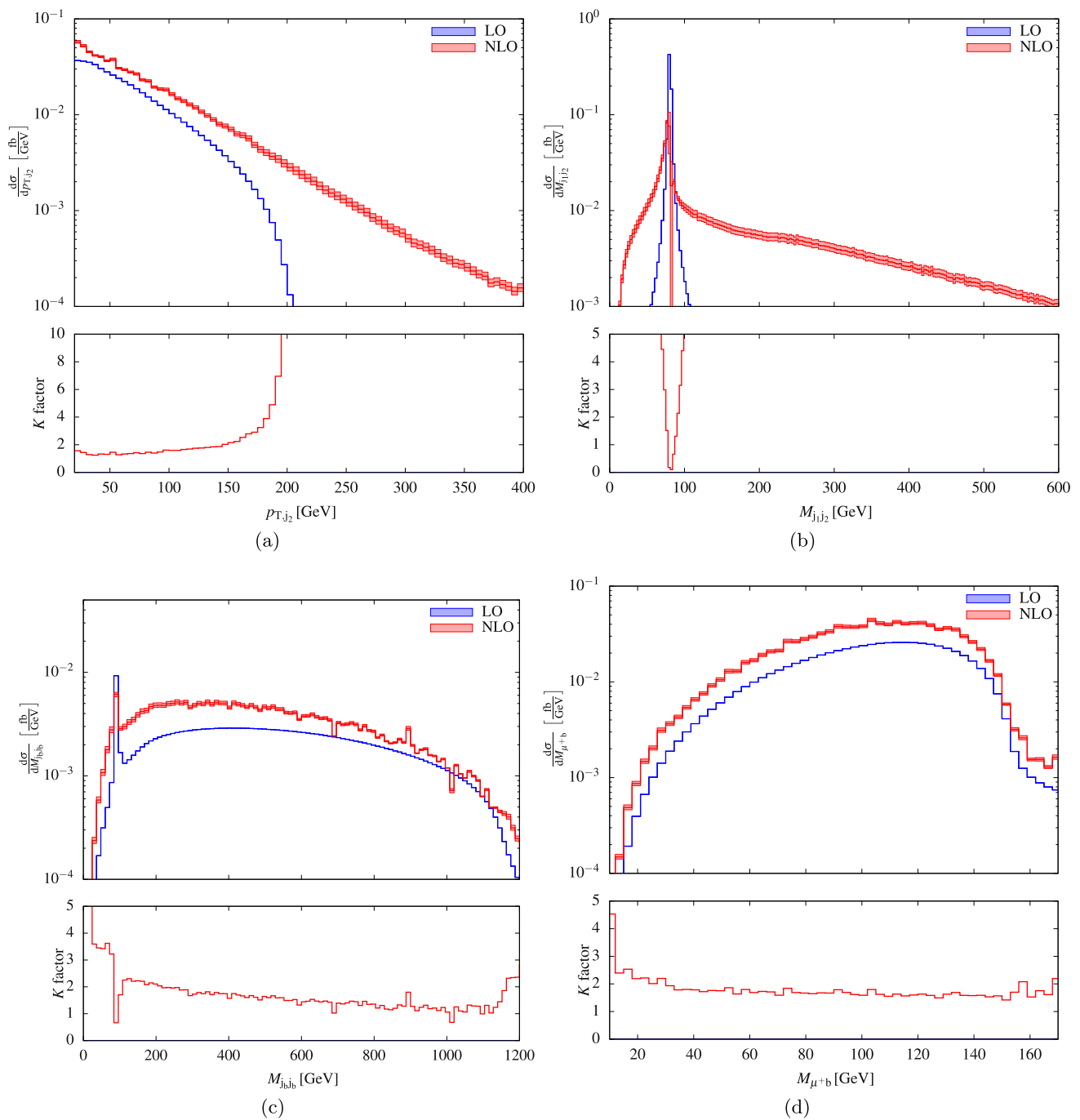


Fig. 6 Differential distributions for a 1.5 TeV CM energy: transverse momentum of the second hardest light jet **(a)**, invariant masses of the system formed by the two hardest light jets **(b)**, the two b jets **(c)**, and

the bottom and the antimuon **(d)**. The bottom momentum used in Fig. 6d is taken from Monte Carlo truth. Same structure as in Fig. 4

Table 2 Fiducial cross sections for $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$ at LO and NLO QCD for several beam polarisations and for CM energies 365 GeV and 1.5 TeV. The digit in parentheses indicates the Monte Carlo statis-

tical error, while the sub- and super-scripts in per cent indicate the scale variation

P_{e^+}	P_{e^-}	$\sqrt{s} = 365 \text{ GeV}$			$\sqrt{s} = 1.5 \text{ TeV}$		
		σ_{LO} [fb]	σ_{NLOQCD} [fb]	K factor	σ_{LO} [fb]	σ_{NLOQCD} [fb]	K factor
0	0	16.877(4)	21.42(1) $^{+2.23\%}_{-1.84\%}$	1.27	2.3235(8)	3.627(9) $^{+3.77\%}_{-3.11\%}$	1.56
+1	-1	48.013(9)	60.87(4) $^{+2.22\%}_{-1.83\%}$	1.27	7.427(3)	11.09(4) $^{+3.47\%}_{-2.86\%}$	1.49
-1	+1	19.501(2)	24.86(2) $^{+2.26\%}_{-1.86\%}$	1.28	1.866(1)	3.416(8) $^{+4.76\%}_{-3.92\%}$	1.83
0	-0.8	22.581(5)	28.63(2) $^{+2.22\%}_{-1.83\%}$	1.27	3.435(2)	5.16(2) $^{+3.51\%}_{-2.89\%}$	1.50
0	+0.8	11.176(3)	14.23(1) $^{+2.25\%}_{-1.86\%}$	1.27	1.211(1)	2.091(5) $^{+4.42\%}_{-3.64\%}$	1.73
-0.3	+0.8	13.088(3)	16.67(1) $^{+2.26\%}_{-1.86\%}$	1.27	1.352(1)	2.387(5) $^{+4.55\%}_{-3.75\%}$	1.77
+0.3	-0.8	28.770(5)	36.48(3) $^{+2.22\%}_{-1.83\%}$	1.27	4.410(2)	6.61(2) $^{+3.49\%}_{-2.88\%}$	1.50

invariant mass of the two hardest light jets is considered. The typical Breit–Wigner shape coming from the LO W-boson hadronic decay is distorted by QCD corrections both below and above the W-boson pole mass. The QCD corrections at the peak are negative and qualitatively similar to the ones at 365 GeV in Fig. 5b. The radiative tail below M_W is similar to the one observed at 365 GeV, while the enhanced K factor found for $M_{jj} \gtrsim 200 \text{ GeV}$ originates from hard gluon radiation.

The distribution in the invariant mass of the b-jet pair is considered in Fig. 6c. A clear peak at M_Z highlights the contributions with a resonant Z boson decaying into two b jets, produced in association with two W bosons. The Z-boson peak sits on top of contributions coming both from the $t\bar{t}$ -production process and other sizeable backgrounds as single-top production. The QCD corrections are large and negative at the Z-boson peak, similarly to what can be observed in Fig. 6b for the hadronic W-boson decay. Below M_Z , the QCD corrections are large and positive, due to the LO suppression and the presence of unclustered gluon radiation. Above M_Z the corrections are positive and diminish in size from 100% to a few percent around 1 TeV. For an invariant mass close to the maximal possible value for on-shell production of about 1170 GeV [twice the transverse momentum resulting from Eq. (11)] the leading order is suppressed and the K factor increases.

Rather flat relative QCD corrections are found for the distribution in the invariant mass of the bottom–antimuon system, shown in Fig. 6d. As in Fig. 5d, the bottom momentum is obtained from Monte Carlo truth. The edge at $\sqrt{m_t^2 - M_W^2}$ observed in the 365 GeV scenario is present also in the high-energy scenario, but the drop of the LO cross section around this threshold is less severe at TeV-scale energies due to the increased irreducible-background contributions which do not embed a resonant top quark. This leads to a QCD K factor that does not increase for $M_{\mu+b} > 153 \text{ GeV}$.

3.4 Polarised-beam effects

In the baseline FCC-ee scenarios, the beams are planned to be unpolarised [4, 69], while the CLIC and ILC facilities are envisioned to collide 80%-polarised electrons and possibly 30%-polarised positrons [1, 5]. It has been claimed [3] that polarised beams at lepton colliders are beneficial to enhance the sensitivity to EW parameters and possible new-physics effects, increase the signal-to-background ratio for several signatures, and keep systematics under control. Assuming a partial polarisation along the beam axis (often dubbed *longitudinal polarisation*) for both the positron (P_{e^+}) and the electron (P_{e^-}), the cross section for a given process reads,

$$\begin{aligned} \sigma(P_{e^+}, P_{e^-}) = & \frac{1}{4} \left[(1 + P_{e^+})(1 - P_{e^-}) \sigma_{\text{RL}} \right. \\ & + (1 - P_{e^+})(1 + P_{e^-}) \sigma_{\text{LR}} \\ & + (1 + P_{e^+})(1 + P_{e^-}) \sigma_{\text{RR}} \\ & \left. + (1 - P_{e^+})(1 - P_{e^-}) \sigma_{\text{LL}} \right], \end{aligned} \tag{12}$$

where σ_{XY} is the cross section for a positron with helicity X and an electron with helicity Y , and L(R) stands for left(right)-handed helicity. Note that in annihilation processes, like the one we consider in this work, the Standard-Model dynamics only allows for a combined angular momentum equal to 1. Therefore, the initial-state leptons cannot carry the same helicity, i.e. $\sigma_{\text{LL}} = \sigma_{\text{RR}} = 0$. In Table 2 we show the integrated cross sections for a number of beam-polarisation choices and for both energy scenarios considered in this work. Close to the threshold, the configuration with a right-handed positron and a left-handed electron (RL) gives a fiducial cross section approximately 2.5 times larger than the opposite helicity configuration (LR). The QCD corrections relative to the corresponding LO cross section are almost identical in all pure or mixed helicity configurations, in agreement with the results of Ref. [23] for undecayed top quarks. At high energy, the ratio between the RL and LR

cross sections sizeably increases at LO, being ≈ 4 at 1.5 TeV, while the relative NLO QCD corrections are larger for the LR state (+80%) than for the RL one (+50%). This difference between the two pure polarisation states is driven by the real-radiation contributions which open up new helicity configurations in the final state that are suppressed at LO. When irreducible-background effects become relevant, the trivial factorisation of QCD corrections from the initial-state helicity configuration does not hold anymore [23].

The effects of beam polarisation are maximal in differential distributions, mostly for angular observables. In Figs. 7, 8 and 9 we show differential results at fixed initial-state helicities (LR, RL) for both the 365 GeV and the 1.5 TeV energy scenarios. Differential results for partially polarised beams, though not considered here, can be estimated easily performing a bin-by-bin combination of the RL and LR distributions according to Eq. (12). At energies well above the $t\bar{t}$ threshold, selecting the RL helicity state is expected to enhance the ratio of single-top and non-resonant contributions over the top-antitop ones [30].

In Fig. 7 we consider the distribution in the cosine of the polar angle of the second hardest light jet.

The LR and RL shapes at 365 GeV are related by an almost perfect mirroring about $\cos \theta_{j_2} = 0$, up to the different overall normalisation. This holds both at LO and at NLO QCD. In the RL state, the antitop quark typically goes forward, as shown in Fig. 5f, and is mostly right handed [70]. According to the helicity structure of the tree-level on-shell top-decay amplitude, the W^- boson from the decay of a mostly right-handed antitop quark is produced backward with respect to the antitop direction (in the antitop rest frame), therefore giving light jets that are typically produced with $\cos \theta_j < 0$. In fact, this is the case for the second-hardest jet in Fig. 7a but also for the hardest jet, though with different distribution shapes. The same reasoning applies to the LR initial state, with a flip of sign in the left–right asymmetry in the decay matrix element, therefore motivating the almost exact mirror symmetry between the LR and RL states. The QCD effects are slightly different for the LR and RL states, but in both cases they are larger where the corresponding LO are more kinematically suppressed. The situation is significantly different at 1.5 TeV. The RL state features asymmetric peaks at the distribution endpoints, the LR one peaks in forward regions, while being suppressed in backward regions. This results from the fact that at high energies the antitop quarks have high energies, while still going mostly in the forward direction for the RL state. As a consequence, their decay products also often end up in forward direction. The QCD corrections mostly fill the region with negative $\cos \theta_{j_2}$ for both helicity states. Especially for the LR state, the QCD K factor is between 5 and 6 in the region that is mostly suppressed at LO.

Shape differences between the LR and RL helicity states are not observed for all angular observables. In Fig. 8 we

show distributions in the cosine of the polar angle of the hardest b jet.

For this variable, the LR and RL distributions are characterised by very similar LO shapes and QCD K factors at 365 GeV, with the most populated region being the central one. At 1.5 TeV the central region is suppressed, while the forward–backward ones are favoured by both the $t\bar{t}$ and the single-top contributions.² The inclusion of QCD corrections enhances mostly the suppressed central regions. In spite of a much larger K factor for the LR state, the NLO shapes are quite similar for the two pure helicity states. The rationale is that an approximate factorisation of QCD corrections w.r.t. the beam helicity is fulfilled in the fiducial volume for a beam energy around the top-mass threshold, while it is broken for TeV-scale CM energies mostly due to the LO suppression of some helicity configurations and the opening of new ones thanks to gluon emission.

The difference between different helicity selections can be appreciated not only in angular distributions, but also in the invariant mass of the b-jet pair considered in Fig. 9.

Note that we focus here on masses below 200 GeV for both CM energies, while the unpolarised distribution in Fig. 6c clearly shows that the relevant range for the 1.5 TeV scenario is much larger. The contributions involving the Z decay into two b jets give a peak at the Z mass that is sizeable only in the RL shape, while its presence is hardly visible for the LR state, since $e^+e^- \rightarrow W^+W^-Z$ contributes basically only for the former owing to the purely left-handed coupling of the W boson. While this holds for both energies, the tri-boson contribution to the cross section is small compared to the dominant $t\bar{t}$ -production process at 365 GeV, but its contribution is larger at 1.5 TeV. The difference between the LR and RL states at 1.5 TeV is propagated to the QCD corrections which are sizeable and negative at the Z peak for the RL state, while they are less pronounced for the LR one. On the contrary, at 365 GeV, the QCD K factors are almost identical for the two helicity states, apart from a narrow region around the Z peak and close to the kinematic boundary for on-shell top production $M_{j_b j_b} \lesssim 189$ GeV given by Eq. (11) and $M_{j_b j_b} < 2p_{T, j_b}$.

As shown above, the beam-polarisation effects are relevant for some observables but completely irrelevant for others. These effects are in general enhanced for higher CM energies, where the interplay among different subprocesses enhances spin configurations in the final state depending on the polarisation of the incoming beams. In particular, the

² This statement has been verified by investigating on-shell top amplitudes obtained from MADGRAPH5_AMC@NLO [66]. Qualitatively it can be understood from the fact that both top and antitop quarks are preferably produced in the backward and forward directions and thus at high energies also their decay products.

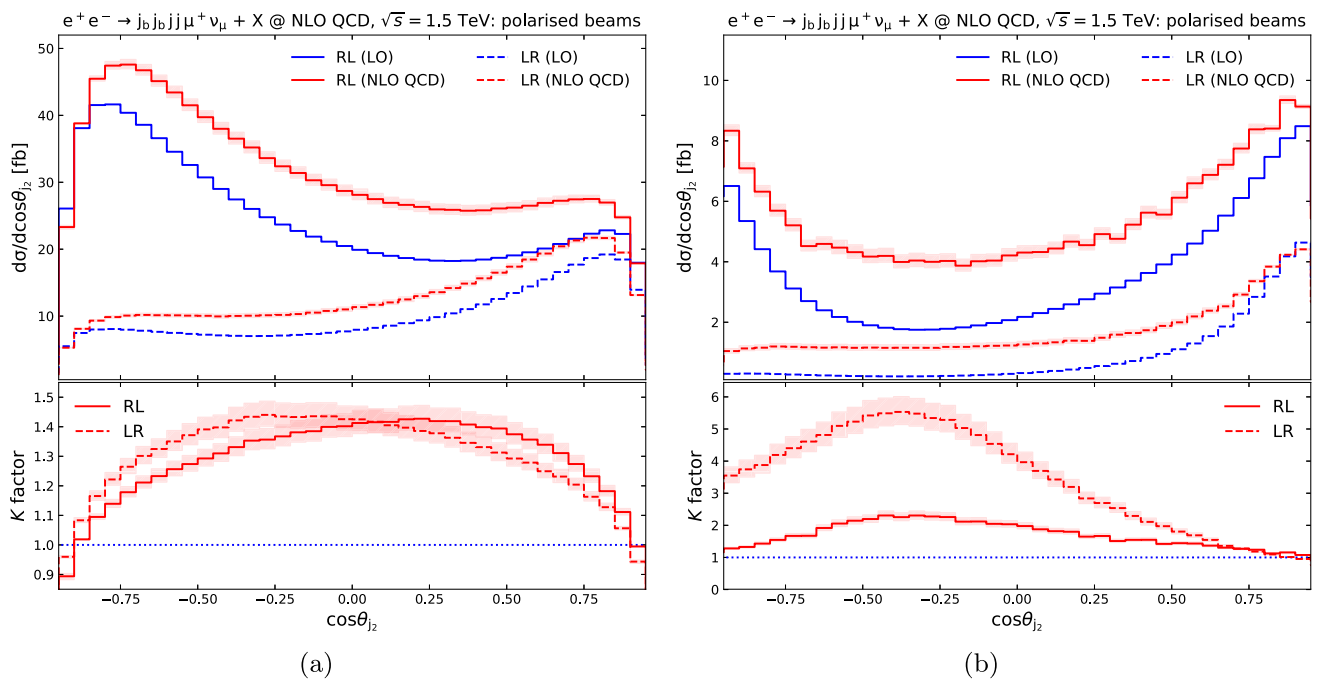


Fig. 7 Differential distributions in the cosine of the polar angle of the second hardest light jet for CM energies 365 GeV (left) and 1.5 TeV (right). The top panels show the absolute differential cross sections at LO (blue) and at NLO QCD (red) for the RL (solid) and LR (dashed)

helicity combinations of the initial-state leptons. The red-shaded band is obtained by means of three-point renormalisation-scale variations and the lower panel displays the K factor

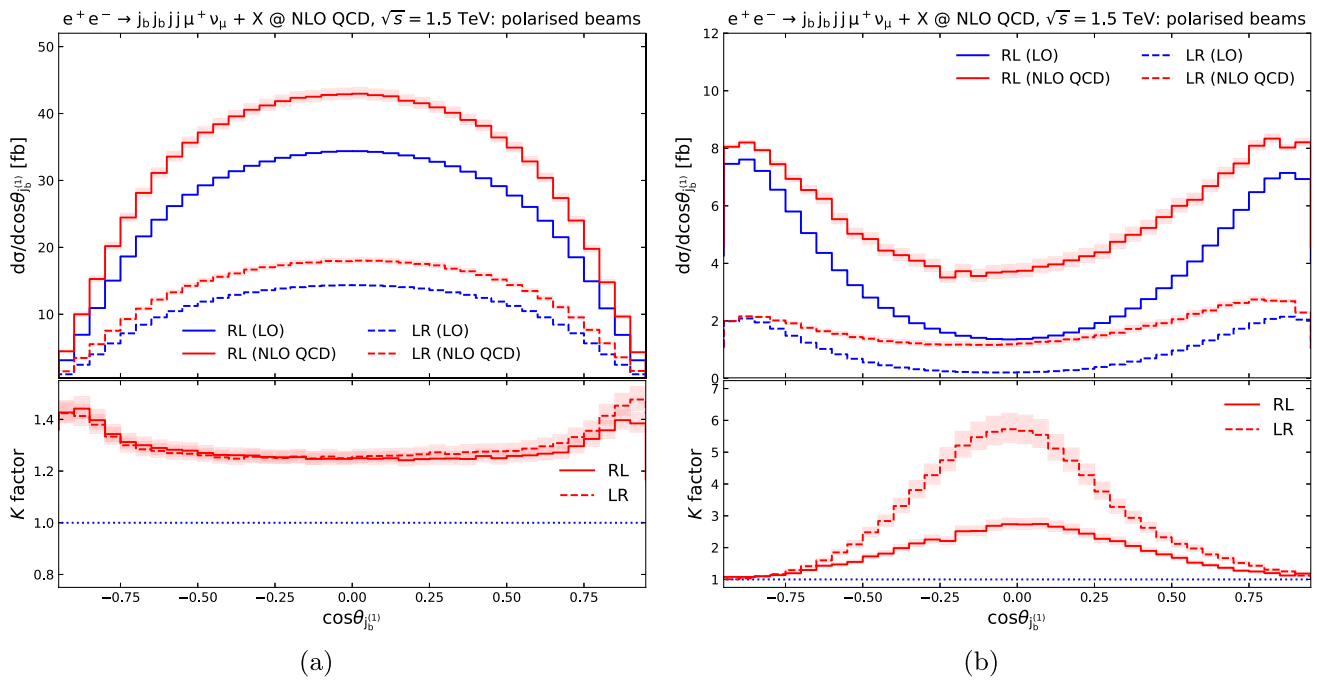


Fig. 8 Differential distributions in the cosine of the polar angle of the hardest b jet for CM energies 365 GeV (left) and 1.5 TeV (right). Same structure as in Fig. 7

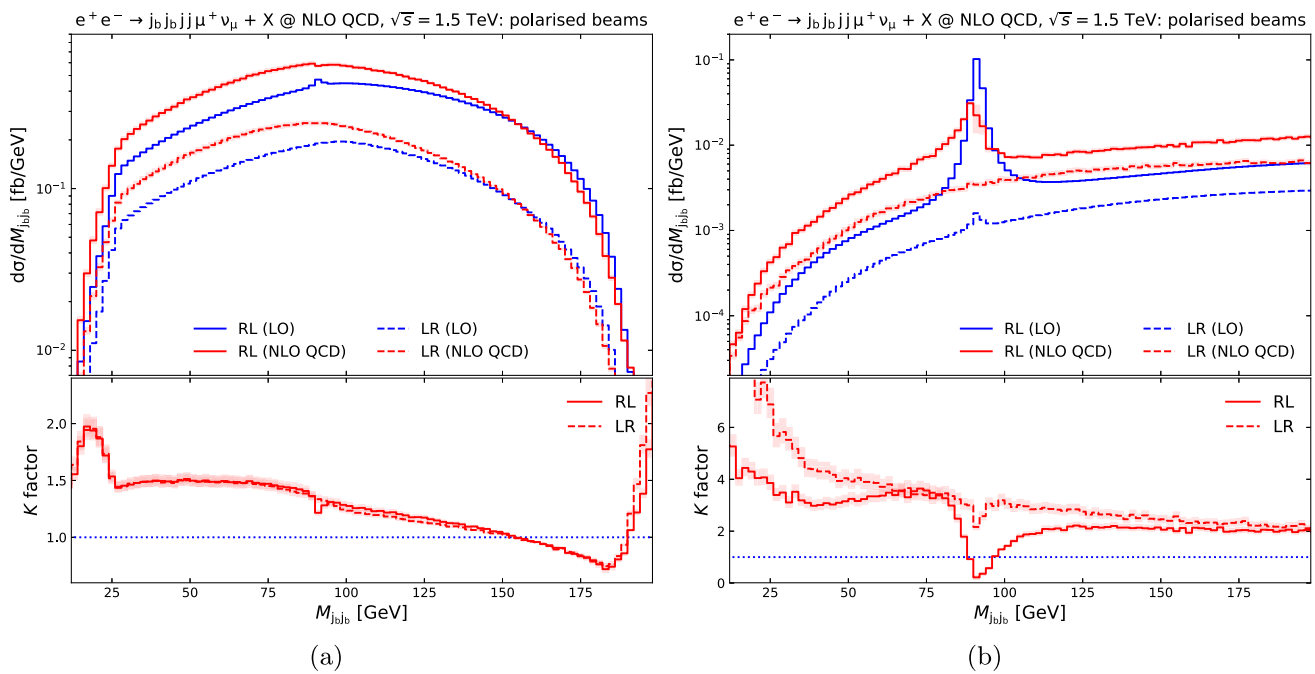


Fig. 9 Differential distributions in the invariant mass of the b-jet pair for CM energies 365 GeV (left) and 1.5 TeV (right). Same structure as in Fig. 7

appearance of hard-gluon radiation at NLO can allow for different spin configurations than at LO.

A due comment concerns the flavour of the final-state lepton. Since we have considered $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$, with different lepton flavours in the initial and final state no t -channel or boson-fusion topology appears. If a positron was considered in the final state, contributions from the RR and LL initial-state configurations would be non vanishing, and more resonant structures would be present making the spin structure of the process even more involved.

4 Conclusion

Top-quark physics will play a central role in any of the potential future lepton colliders. It is therefore very important to provide precise theoretical predictions. So far, only the off-shell production of a top–antitop pair with fully leptonic decays had been computed at NLO QCD.

Nonetheless, the semi-leptonic channel offers many advantages such as the larger cross section and the possibility to fully reconstruct the momenta of the top quarks. In this paper, we have provided the first calculation of the NLO QCD corrections to the full process $e^+e^- \rightarrow j_b j_b j j \mu^+ \nu_\mu$. To that end, we have implemented the FKS subtraction scheme in the Monte Carlo integrator MOCANLO. We have successfully validated this implementation against the already existing Catani–Seymour scheme in our Monte Carlo program.

At the level of fiducial cross sections, the QCD corrections strongly depend on the collision energy, ranging from huge positive values at the top-pair threshold to negative values above threshold but lower than 1 TeV and increasingly positive values beyond 1 TeV. This dependence results from the Coulomb singularity in the threshold region and the selection of the decay jets that suppresses specific kinematic regimes at high energies at LO but not at NLO because of hard gluon radiation.

The behaviour of QCD corrections becomes even more striking at differential level, with huge K factors in regions where the LO cross section is suppressed and the real corrections open up new kinematic topologies. This holds for both collision energies we have considered (365 GeV, 1.5 TeV). The radiative corrections become large especially for invariant-mass and transverse-momentum distributions. The most extreme K factors are found at a 1.5 TeV collision energy where the QCD effects are enhanced far from the top-pair threshold.

The structure of LO contributions and QCD radiative corrections can be understood in more detail when selecting pure helicity states for the electron and positron beams. The factorisation of higher-order QCD effects with respect to the spin state of the leptonic initial state is typically confirmed at collision energies around the top-pair threshold. It is clearly broken at 1.5 TeV, owing to enhanced irreducible background processes and the suppression of certain helicity configurations at high energy.

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