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Stability Estimates for Composite Identification–and–Control Maps Related to a Distributed Parameter System

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PARADIGM

Identification for Control

INVERSE (*identification*) PROBLEM *ill – posed*

data { $u\,,f$; extra information } → $(a u')' =_{\text{a.e.}} f \quad x \in (x_0, x_1) \equiv D$ → a, parameter

DIRECT (*control*) PROBLEM *well – posed*

parameter a → $(a r')' =_{\text{a.e.}} h \quad x \in (x_0, x_1) \equiv D$ → r, result

controls { h ; $r(x_0) = r_0$; $r(x_1) = r_1$ } →

"The solution of the ill–posed problem is only an intermediate construct intervening between the available data and the intended application"

(T. I. Seidman, 1990).

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PLAN

Consider the composite *data* – to – *result* map

1 Provide some *uniqueness* conditions for the *data* – to – *parameter* map

1.1 from *independent* data pairs =

= *regular* Cauchy pbm. w.r. to a in $(au')' =_{\text{a.e.}} f$

1.2 from a potential *stationary* at a point =

= *singular* Cauchy pbm. w.r. to a in $(au')' =_{\text{a.e.}} f$

2 Provide the corresponding *stability* estimates for the composite map.

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1.1 – UNIQUENESS FROM *INDEPENDENT* DATA PAIRS
(a *Regular* Cauchy Problem)

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$D := (x_0, x_1)$

$u\,,\, v \in C^1(\bar{D})\,;\, E_u := \{\, x \mid x \in \bar{D}\,,\, u'(x) = 0\,\}\,;\, E_v :=$ analogous

$f\,,\, g \in L^1(D)\,; f\,,\, g \not\equiv 0;\, F(x) := \int_{x_0}^{x} f\, ds\,;\, G(x) := \int_{x_0}^{x} g\, ds$

$A_{ad} := \{\, a \mid a \in C^0(\bar{D}),\, 0 < a_L \le a(x)\,,\, \forall\, x \in \bar{D}\,\}$

Hp. $\exists\, a \in A_{ad}\ \cdot\!\ni\!\cdot\ \{\, (au')' =_{\text{a.e.}} f\,,\, (av')' =_{\text{a.e.}} g\,\}$

$$E_u = E_v = \emptyset\,;\, \exists\, y_1, y_2 \in \bar{D}\ \cdot\!\ni\!\cdot\ \frac{1}{v'(y_1)\,u'(y_2)} - \frac{1}{v'(y_2)\,u'(y_1)} \ne 0$$

Th. $\exists!\ \hat{a} = \dfrac{F + c_1}{u'}$ *(reference solution)*

$$c_1 = \frac{(G_2 - G_1)\,u'_1 u'_2 - F_2\, u'_1 v'_2 + F_1 u'_2 v'_1}{u'_1 v'_2 - u'_2 v'_1}\,.$$

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1.2 – UNIQUENESS FROM A POTENTIAL *STATIONARY* AT A (*CRITICAL*) POINT (a *Singular* Cauchy Problem)

Hp. $\exists\ a \in A_{ad}\ \cdot\ni\cdot\ \{\ (au')' =_{\text{a.e.}} f\ \}$

$E_u = \{\ x_u\ \}$

Th. $\exists!\ \hat{a} = \dfrac{F - F(x_u)}{u'}$ (*reference solution*)

Rem.: special case of Kitamura – Nakagiri (1977) uniqueness prop.

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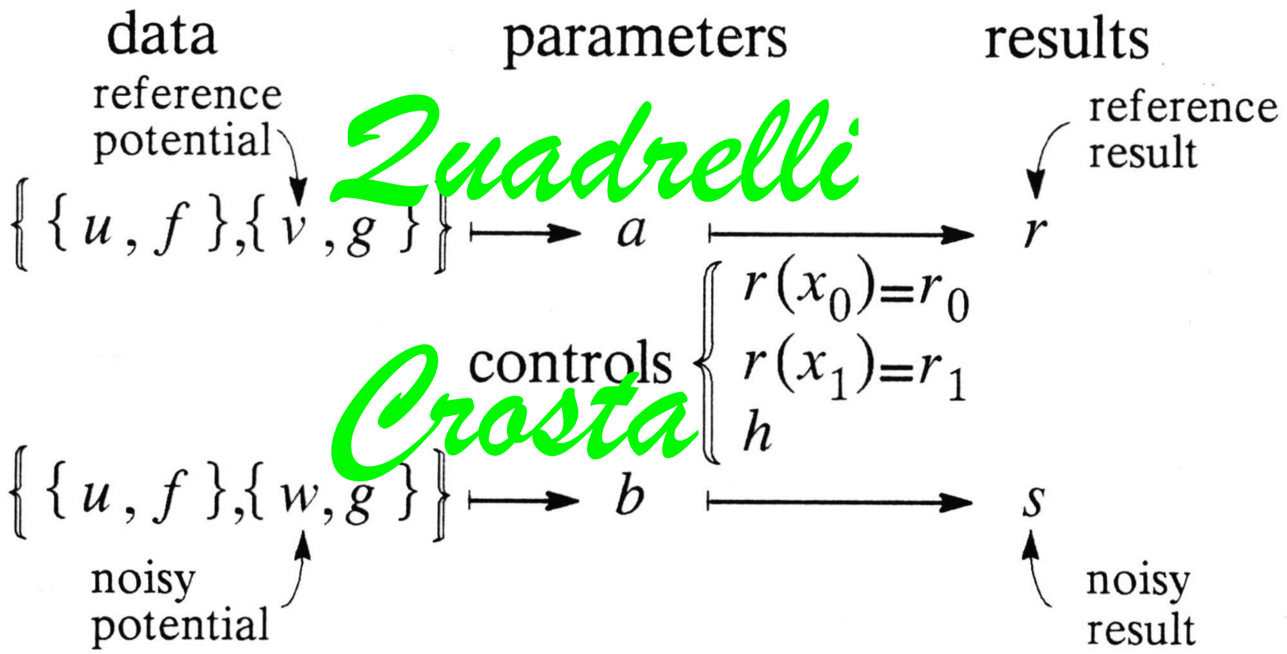
2.1 – STABILITY OF THE *DATA* – TO – *RESULT* MAP

Regular Cauchy Problem, begin

Target:



$$\| s - r \|_Z \le c_I(D, f, g, h, u, r_0, r_1, a_L, a_H)\| w - v \|_V$$

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2.1 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Regular Cauchy Problem, end

Hp. (additional, w.r. to uniqueness statement)

$\| f \|_1, \| g \|_1 \le c_s$;

$| u'_i | \quad | v'_i | \le c_P$, $i = 1, 2$ (noiseless potentials only)

$| u'_i | \ge c_M$ ⟵ # uniqueness from *singular* Cauchy problem

$| v'_2 u'_1 - v'_1 u'_2 |$, $| w'_2 u'_1 - w'_1 u'_2 | \ge \frac{1}{c_T}$;

$a \le a_H$ ⟵ No such constraint for b !

$\left| \frac{r_1 - r_0}{x_1 - x_0} \right| \le c_r$; $\| h \|_1 \le c_h$

Th. *i)* $Z = W^{1,\infty}(D)$; $V = \{ w'_i - v'_i \mid i = 1, 2 \}$

ii) (uniform estimate)

$$\| s' - r' \|_\infty \le C_0 (C_1 \Delta_{21} + C_2 \Delta_{21}^2)$$

$$\Delta_i := | w'_i - v'_i | , i = 1, 2 \quad ; \quad \Delta_{21} = 2 c_P^3 c_s c_T^2 (\Delta_2 + \Delta_1)$$

$$C_0 = \frac{a_H}{a_L^2 c_M} (c_r + \frac{c_h}{a_L}); \; C_1 = 1 + \frac{a_H}{a_L} \; ; \; C_2 = \frac{1}{a_L c_M}$$

iii) ($W^{1,\infty}$ – norm estimate)

$$\| r - s \|_{1,\infty} \le (1 + | x_1 - x_0 |) \| s' - r' \|_\infty$$

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2.2 – STABILITY OF THE *DATA* – TO – *RESULT* MAP: *Singular* Cauchy Problem, begin

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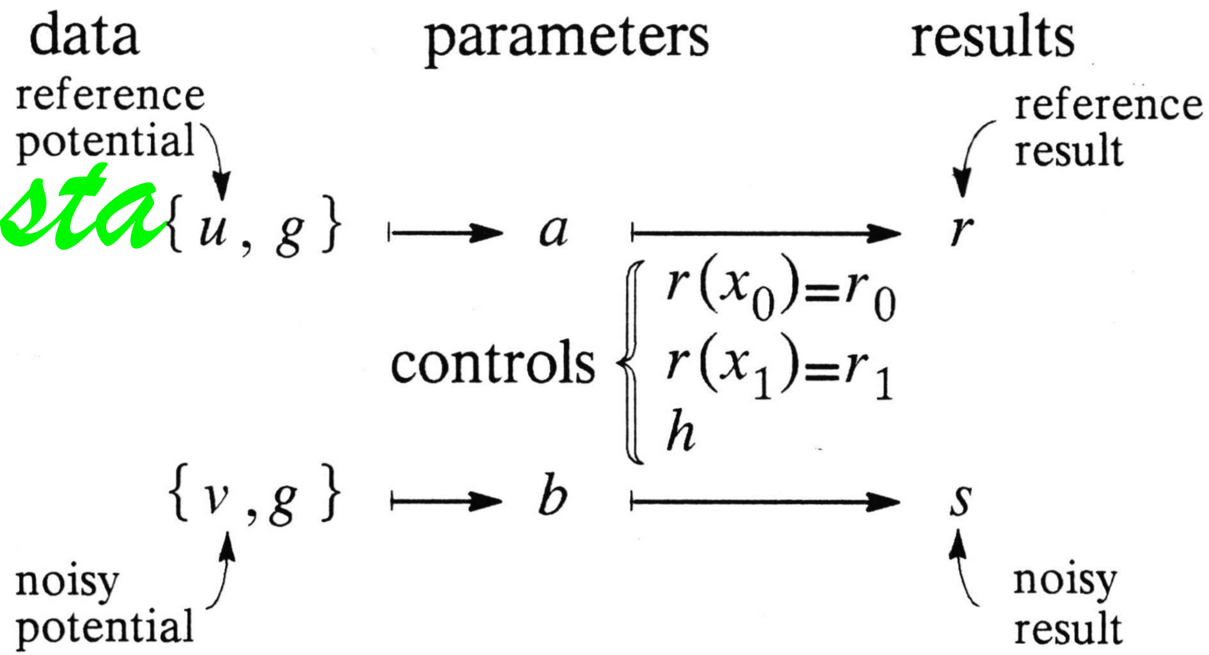

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2.2 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Singular Cauchy Problem, end

Hp. (additional, w.r. to uniqueness statement)

$\| \frac{1}{u'} \|_1 \, , \, \| \frac{1}{v'} \|_1 \leq c_v$ ⟵ major difference w.r. to *uniqueness* from *independence*)

Th. *i*) $Z = W^{1,1}(D)$; $V = W^{1,\infty}(D)$

ii) (estimate, $E_u = E_v = \{ x_u \}$)

$$\| s' - r' \|_1 \leq 4 \frac{a_H^2}{a_L^2} c_v (c_r + \frac{c_h}{a_L}) \| v' - u' \|_\infty$$

c_r , c_h , a_H , a_L : usual meaning.

Rem.

Given the two point BV *control* pbm, $\| s' - r' \|_1$ is equivalent to the natural norm of $W^{1,1}$.

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2.3 – STABILITY OF THE *DATA* – TO – *RESULT* MAP
Conclusion

Independent data => Biblioteca

=> *regular* Cauchy problem for $a \ni (au')' =_{\text{a.e.}} f$ =>

=> L^{∞}– estimate for the *data* – to – *parameter* map.

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Stationary potential e.g., $E_u = \{ x_u \}$ =>

=> *singular* Cauchy problem =>

=> L^1– estimate. Crosta

The *parameter* – to – *result* problem is well – posed.

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The stability of the composite map inherits the stability of the *data* – to – *parameter* map.