

### Planar Abelian mirror duals of $\mathcal{N} = 2$ SQCD<sub>3</sub>

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We propose an Abelian mirror dual for the  $\mathcal{N} = 2$  SQCD<sub>3</sub> that we obtain as real mass deformation of known  $\mathcal{N} = 4$  mirror pairs. We match the superconformal index and the  $S_b^3$  partition function, discuss the agreement of the moduli spaces, and provide a map of the gauge invariant operators and the global symmetries as evidence of this duality.

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*Introduction.* We propose an Abelian dual description for a family of  $\mathcal{N} = 2$   $SU(N)$  Chern-Simons (CS) SQCD<sub>3</sub> [1–3]. The dual theory is *planar*, in the sense that it is a quiver drawn on a plane, with a cubic superpotential term for each closed oriented loop, as in the 4D periodic quivers associated to the dimers of [4,5] and, in addition, contains linear monopole superpotentials. Our dualities are obtained by real mass deformations of 3d  $\mathcal{N} = 4$  mirror pairs of theories [6,7] and display the exchange of topological and flavor symmetries characteristic of 3D mirror symmetry.

In this letter, we focus on the  $SU(N)_k$  SQCD<sub>3</sub> with  $F \geq 2N$  fundamentals and CS level  $k = \frac{F}{2} - N$  and discuss extensions of the analysis to larger  $k$  via further real mass deformations. We can extend our results to theories with more general flavor content, CS levels, and quivers with unitary gauge groups as discussed in forthcoming papers [8,9]. As an example, we discuss a duality between chiral and planar quivers obtained from the self-mirror  $T[SU(N)]$  theory [10].

Our proposal is guided by the behavior under real mass deformations of the  $S_b^3$  partition function [11,12], which matches across our proposed duality, providing a nontrivial check of our claims. Details of this analysis will be given in

[8,9]. In this letter, we support our proposals by matching the superconformal index [13,14] (SCI), the global symmetries, and the chiral rings of the dual theories.

*A Planar Abelian Dual for  $SU(N)_{\frac{F}{2}-N}$  with  $F$  fundamentals.* We start from the  $\mathcal{N} = 4$  mirror duality relating  $SU(N)$  with  $F \geq 2N$  flavors to a quiver with  $F$  gauge nodes (Fig. 1). We deform the  $\mathcal{N} = 4$  electric theory, breaking the  $SU(2) \times SU(2)$  R symmetry to  $U(1)_R$  [15] by turning on a real mass  $m$  for a combination of the commutant of  $U(1)_R$  and the baryonic symmetry. Under this deformation, in the vacuum where the real scalar has no vacuum expectation value (VEV), only the  $F$  fundamental fields remain massless while the adjoint and the  $F$  antifundamental fields are massive and are integrated out generating a nonzero CS level and we obtain the  $\mathcal{N} = 2$   $SU(N)_{\frac{F}{2}-N}$  SQCD<sub>3</sub> with  $F$  fundamental fields [16].

In the magnetic theory, under the same deformation, something interesting happens—we propose that the magnetic vacuum corresponding to the electric theory is such that each gauge node  $U(k)$  is Higgsed to its maximal torus

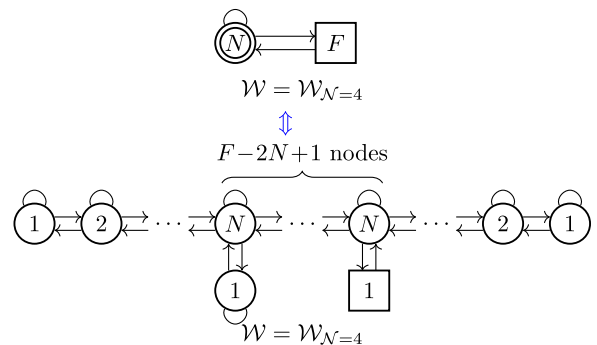


FIG. 1.  $\mathcal{N} = 4$  mirror duality for SQCD<sub>3</sub>. Single/double circles correspond to  $U/SU$  symmetry groups.

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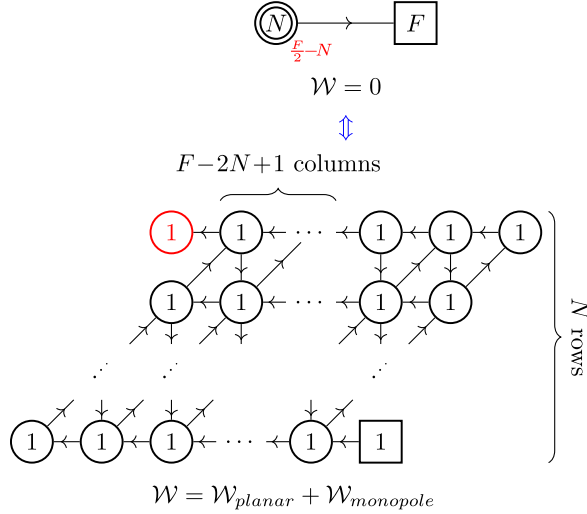
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 FIG. 2. Mirror-like duality for  $\mathcal{N} = 2$  SQCD<sub>3</sub>.

$U(1)^k$  producing a column of  $k$  black  $U(1)$  gauge nodes as in Fig. 2. For a given node, the Higgs mechanism is triggered by VEVs for the scalar  $\sigma$  in the corresponding  $\mathcal{N} = 2$  vector multiplet. To reach this vacuum we move along the Coulomb branch, where  $\langle \sigma \rangle = \text{diag}(\sigma_1, \sigma_2, \dots)$  with,

$$\sigma_i - \sigma_{i+1} = m \quad (1)$$

and the chiral fields depicted in Fig. 2 remain massless. We checked that this vacuum satisfies the F-term and D-term equations [3].

This claim is supported by the careful analysis of the effect of the real mass deformation on the  $\mathcal{S}_b^3$  partition function as in [17,18]. The large mass limit produces a highly oscillating phase corresponding to contributions from massive fields and is sensitive to the Higgsing [19]. Assuming the Higgsing pattern described above, we checked that the highly oscillating phases cancel between the electric and magnetic sides. This procedure implies the equality of the partition functions of the two theories in Fig. 2 and is a very nontrivial check of the duality. Details on these computations will appear in a forthcoming paper [8].

The mirror dual theory admits a Lagrangian description, all the gauge groups are Abelian and each arrow denotes a chiral field with charge  $(+1, -1)$  under the two nodes it connects. Importantly, the theory also contains (mixed) CS interactions and a nonzero superpotential:

- (i) each black (red) gauge node carries a  $-1$  ( $-\frac{1}{2}$ ) CS coupling. The CS interactions are a consequence of integrating out fermionic fields present in the  $\mathcal{N} = 4$  theory.
- (ii) For each vertical (nonvertical) arrow there is a  $-1$  ( $+1$ ) mixed CS coupling involving the two nodes connected by the arrow.

- (iii)  $\mathcal{W}_{\text{planar}}$  are cubic superpotential terms. There is one term with  $-1$  ( $+1$ ) coefficient for each clockwise (anticlockwise) closed triangle. The cubic terms in the superpotential are a remnant of the cubic  $\mathcal{N} = 4$  superpotential.
- (iv)  $\mathcal{W}_{\text{monopole}}$  are the monopole terms in the superpotential generated by the Polyakov mechanism [20] due to the Higgsing of a  $U(k)$  gauge symmetry to  $U(1)^k$ . For each vertical arrow, there is a monopole superpotential with Goddard-Nahm-Olive (GNO) flux  $+1$  and  $-1$  under the nodes connected by the arrow, from top to bottom.

*Checks of the duality.* As a preliminary check of the proposed duality, we count the rank of the global symmetries of the planar theory. All  $U(1)$  symmetries rotating the chiral fields are either broken by  $\mathcal{W}_{\text{planar}}$  or removed by gauge transformations. The monopole superpotential breaks the topological  $U(1)$  symmetries of the black nodes in each column to a diagonal combination. We thus have a single  $U(1)$  topological symmetry for each column and an extra one associated with the node indicated in red, resulting in a  $U(1)^F$  global symmetry. The duality predicts that the UV global symmetry enhances in the IR to  $U(F)$ , which is manifest in the UV in the electric SQCD<sub>3</sub>. This is inherited from the topological symmetry enhancement in the  $\mathcal{N} = 4$  quiver of Fig. 1, which is preserved by the real mass deformation we consider.

In addition to the already mentioned check of the matching of the  $\mathcal{S}_b^3$  partition functions, we also matched the refined SCIs for  $N < 5$  and  $F < 11$  up to  $\mathcal{O}(x^{12/5})$ , with trial R charge of the baryons set to  $R = \frac{3}{5}$ . In particular, we can detect the character of the enhanced  $U(F)$  current at order  $x^2$  in the SCI of the planar Abelian mirror.

*Structure of the moduli space.* We observe that the chiral ring generators in the electric  $SU(N)$  with  $F$  fundamentals theory are the  $\binom{F}{N}$  baryons  $B^{j_1, \dots, j_N} = \epsilon^{a_1 \dots a_N} Q_{a_1}^{j_1} \dots Q_{a_N}^{j_N}$  [21]. The full set of BPS baryons can be encoded in the following Hilbert series [22–24]:

$$\mathcal{H}_{\text{baryons}} = \sum_{k=0}^{\infty} [0^{N-1}, k, 0^{F-N-1}]_{SU(F)} t^k \xrightarrow{(u.r.)} \frac{\sum_{k=0}^A c_{N,F}^{(k)} t^k}{(1-t)^d}, \quad (2)$$

where  $[\lambda_1, \dots, \lambda_{F-1}]$  is the Dynkin label of the  $SU(F)$  representation. We have turned off the fugacities and resummed the unrefined Hilbert series in the second line of (2) (indicated by *u.r.*), wherein  $A = F(N-1) - N^2 + 1$ ,  $c_{N,F}^{(k)} = c_{N,F}^{(A-k)} > 0$ ,  $c_{N,F}^{(k)} = c_{F-N,F}^{(k)}$ , and  $d = FN - N^2 + 1$ . We deduce that the branch of the moduli space generated by the baryons is a  $d$ -dimensional complex cone.

In the planar theory, the chiral ring is generated by monopoles, as we will see explicitly in an example below. The proposed duality predicts that they satisfy quantum relations compatible with the Hilbert series (2).

*An example:  $SU(2)_{\frac{1}{2}}$  with five fundamentals.* Let us consider the example of  $SU(2)_{\frac{1}{2}}$  SQCD<sub>3</sub> with five fundamental flavors:

$$\begin{array}{c} \textcircled{2} \xrightarrow{\frac{1}{2}} \boxed{5} \bar{X} \\ \mathcal{W} = 0 \end{array} \quad (3)$$

where  $X_i$ ,  $i = 1, \dots, 5$  are the real masses for the Cartan of the flavor  $U(5)_{\bar{X}}$  symmetry and the chiral fields are assigned a trial R charge  $\frac{1}{2}$ . Our proposal for the mirror theory is given below,

$$\begin{array}{ccccccc} x_1 + x_2 - \frac{3}{4} & & x_3 - x_2 + \frac{1}{4} & & x_4 - x_3 + \frac{1}{4} & & x_5 - x_4 \\ \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \textcircled{1} \\ & \alpha_{1,1} & & \alpha_{1,2} & & \alpha_{1,3} & \\ \gamma_1 & & \beta_1 & & \gamma_2 & & \beta_2 & & \gamma_3 \\ & \alpha_{2,1} & & \alpha_{2,2} & & \alpha_{2,3} & \\ \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \boxed{1} \\ x_2 - x_1 & & x_3 - x_2 - \frac{1}{4} & & x_4 - x_3 - \frac{1}{4} & & \end{array} \quad (4)$$

$$\begin{aligned} \mathcal{W} &= \beta_1[\gamma_2\alpha_{1,2} - \alpha_{2,1}\gamma_1] + \beta_2[\gamma_3\alpha_{1,3} - \alpha_{2,2}\gamma_2] \\ &+ \mathfrak{M}\begin{pmatrix} 0 & + & 0 & 0 \\ 0 & & 0 & 0 \end{pmatrix} + \mathfrak{M}\begin{pmatrix} 0 & 0 & + & 0 \\ 0 & 0 & & 0 \end{pmatrix} \end{aligned}$$

with the same convention for the CS and mixed CS levels as in Fig. 2. The fields  $\beta_i$  have trial R charge 1 and the fields  $\alpha_{i,j}, \gamma_k$  have trial R charge  $\frac{1}{2}$ . The labels in blue are the Fayet–Iliopoulos (FI) parameters for the corresponding node, namely the real masses for the topological symmetries. These are written in terms of the  $X_i$ , reflecting the embedding in the enhanced symmetry,

$$U(1)_{\text{top}}^5 \times U(1)_R \rightarrow U(5)_{\bar{X}} \times U(1)_R \quad (5)$$

where  $U(1)_{\text{top}}^5$  is the subgroup of the seven topological symmetries unbroken by the monopole superpotential. Notice that the linear monopole superpotential induces a mixing between the topological and the trial R symmetry, encoded in the constant terms in the FI terms (4).

The chiral ring of the electric theory is generated by the  $\binom{5}{2} = 10$  baryons  $B$ . The mapping to the operators on the mirror side is,

$$\begin{aligned} B \leftrightarrow & \left\{ \mathfrak{M}\begin{pmatrix} - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & - & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \right. \\ & \mathfrak{M}\begin{pmatrix} - & - & - & - \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & 0 & 0 \\ - & 0 & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & - & 0 \\ - & 0 & 0 & 0 \end{pmatrix}, \\ & \mathfrak{M}\begin{pmatrix} - & - & - & - \\ - & 0 & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & - & 0 \\ - & - & 0 & 0 \end{pmatrix}, \mathfrak{M}\begin{pmatrix} - & - & - & - \\ - & - & 0 & 0 \end{pmatrix}, \\ & \left. \mathfrak{M}\begin{pmatrix} - & - & - & - \\ - & - & - & - \end{pmatrix} \right\}, \quad (6) \end{aligned}$$

which can be verified by computing the charges of monopole operators. The charges of a monopole with GNO fluxes  $m_i$ ,  $i = 1, \dots, n_g$  under the  $n_g$  Abelian gauge symmetries can be compactly encoded in the following polynomial of the fugacities [1,25]:

$$\begin{aligned} \mathcal{Q}(\vec{m}) &= -\frac{1}{2} \left( \sum_{b_{ij}} |m_i - m_j| ((R[b_{ij}] - 1) - u_i + u_j) \right. \\ &+ \sum_{f_i, \tilde{f}_i} |m_i| ((R[\tilde{f}_i, f_i] - 1) \pm u_i) \\ &+ \sum_{i=1}^{n_g} \lambda_i m_i - \frac{1}{2} \sum_{i \leq j} k_{ij} (m_i u_j + m_j u_i), \quad (7) \end{aligned}$$

where the first two lines are contributions from fermionic zero modes in the bifundamentals  $b_{ij}$  and (anti)fundamentals ( $\tilde{f}_i$ )  $f_i$  and the other terms are due to CS levels  $k_i$ , mixed CS levels  $k_{ij}$ , and FI terms  $\lambda_i$ . In our conventions, the CS and FI terms are given by (the SUSY completion of),

$$-\sum_{l,m} i \frac{k_{lm}}{4\pi} \int A_l \wedge dA_m + \sum_l \lambda_l \int D_l, \quad (8)$$

where  $A_j$  is the gauge field of the  $j$ th node, and with  $k_{jj} = k_j$  being the CS level for the  $j$ th node, while the FI terms  $\lambda_i$  come from mixed CS interactions between the gauge group  $U(1)_i$  and the associated topological symmetry  $U(1)_{T_i}$  and  $D_i$  is the auxiliary field in the vector superfield containing the gauge field  $A_i$ .

In Eq. (7), the charge under a gauge or flavor symmetry is encoded in the coefficient in front of the corresponding parameter,  $u_i$  are fugacities for the gauge symmetries, and the constant is the trial R charge. One can check that the charge of the monopoles in the superpotential and the chiral ring map do not depend on the  $u_i$  and they are, therefore, gauge invariant. In particular,

$$\mathcal{Q}\left[\mathfrak{M}\begin{pmatrix} 0 & + & 0 & 0 \\ 0 & - & 0 & 0 \end{pmatrix}\right] = \mathcal{Q}\left[\mathfrak{M}\begin{pmatrix} 0 & 0 & + & 0 \\ 0 & 0 & - & 0 \end{pmatrix}\right] = 2 \quad (9)$$

consistent with the monopole superpotential and,

$$\mathcal{Q} \left[ \begin{array}{c} \mathfrak{M} \begin{pmatrix} - & 0 & 0 & 0 \\ 0 & 0 & 0 & \end{pmatrix} \\ \vdots \\ \mathfrak{M} \begin{pmatrix} - & - & - & - \\ - & - & - & \end{pmatrix} \end{array} \right] = \mathcal{Q}[B^{1,2}] = 1 - X_1 - X_2$$

$$\mathcal{Q} \left[ \begin{array}{c} \mathfrak{M} \begin{pmatrix} - & - & - & - \\ - & - & - & \end{pmatrix} \end{array} \right] = \mathcal{Q}[B^{4,5}] = 1 - X_4 - X_5 \quad (10)$$

compatible with the chiral ring map.

We also report the superconformal index of both theories,

$$\mathcal{I} = 1 + \mathbf{10}f x^{3/5} + \mathbf{50}f^2 x^{6/5} + \mathbf{175}f^3 x^{9/5} - (\mathbf{24} + 1)x^2 + \mathcal{O}(x^{12/5}) \quad (11)$$

where bold numbers denote  $SU(5)$  representations and  $f$  is the fugacity for the baryonic  $U(1) \subset U(5)$ . Notice that to perform the SCI expansion we shifted the trial R-charge of the fundamentals in the SQCD<sub>3</sub> so that baryons have R charge  $\frac{3}{5}$ , which is closer to the superconformal value that can be computed via F-extremization [26] ( $R_{sc} = 0.67778\dots$ ). The shift can be performed by changing the mixing coefficient of the  $U(1)$  baryonic symmetry. In the mirror theory in (4), this corresponds to shifting only the FI terms of the red gauge node by a constant number. In the index in (11), we observe terms of the form  $[0, k, 0, 0]_{SU(5)} f^k x^{kR}$ , corresponding to the baryons on the electric side and to monopoles on the mirror side, and the negative term at order  $x^2$  corresponding to the  $U(5)$  conserved currents.

*More chiral  $\leftrightarrow$  planar duals.* Our analysis can be extended to obtain similar Abelian planar duals for unitary SQCD<sub>3</sub> with both fundamental and antifundamental matter and more general CS levels. As will be discussed in forthcoming papers [8,9], it is also possible to define an algorithmic procedure in the spirit of [27–29], which streamlines the study of unitary linear and circular chiral quiver gauge theories.

This Section discusses an example of an SQCD<sub>3</sub> with a more general CS level obtained via further real mass deformations and a linear quiver gauge theory.

Abelian mirror duals of  $3d \mathcal{N} = 2$  non-Abelian quivers appeared in [30–32]; a key difference in [30–32] from our proposal is that the non-Abelian side is nonchiral and the Abelian side is linear instead of planar.

*Real mass deformations.* The procedure outlined in the previous Section allows for the construction of Abelian planar duals for  $SU(N)$  SQCD with CS level  $k = \frac{F}{2} - N \geq 0$ . We can generalize to  $k > \frac{F}{2} - N$  by turning on further positive

real mass deformations for fundamental fields. As an example, we consider the duality for  $SU(2)$  with *five* fundamentals (3) studied in the previous section, and turn on a real positive mass associated to  $X_5$ , making one fundamental massive [33] and flowing to,

$$\begin{array}{c} \textcircled{2} \xrightarrow{\quad} \boxed{4} \vec{X} \\ \textcircled{1} \end{array} \quad (12)$$

$$\mathcal{W} = 0$$

We claim that the corresponding vacua on the mirror side (4) is such that the fields  $\alpha_{1,3}, \gamma_3, \alpha_{2,3}$  are massive. This can be followed on the  $\mathbf{S}_b^3$  partition function, where the matching of asymptotics in the large  $X_5$  limit provides a nontrivial check of the flow. Notice that all the fields charged under the rightmost gauge node are massive, leaving a  $U(1)_{-1}$  CS theory coupled to the rest of the quiver by BF terms. Furthermore, by a gauge field redefinition, we can render all chiral fields neutral under the gauge symmetry of the bottom right node. Then, the corresponding gauge field describes a decoupled  $U(1)_{-1}$  sector, and the bottom right node effectively becomes a flavor node.

The path integral over the two  $U(1)_{-1}$  gauge fields can be performed exactly [34,35], resulting in,

$$\begin{array}{ccccc} x_1 + x_2 - \frac{3}{4} & & x_3 - x_2 + \frac{1}{4} & & x_4 - x_3 \\ \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \textcircled{1} \\ & \alpha_{1,1} & & \alpha_{1,2} & \\ & \nearrow \gamma_1 & \downarrow \beta_1 & \nearrow \gamma_2 & \downarrow \beta_2 \\ \textcircled{1} & \leftarrow & \textcircled{1} & \leftarrow & \boxed{1} \\ x_2 - x_1 & & x_3 - x_2 - \frac{1}{4} & & \end{array} \quad (13)$$

$$\mathcal{W} = \beta_1 [\gamma_2 \alpha_{1,2} - \alpha_{2,1} \gamma_1] - \beta_2 \alpha_{2,2} \gamma_2 + \mathfrak{M} \begin{pmatrix} 0 & \dagger & 0 \\ 0 & & 0 \end{pmatrix}$$

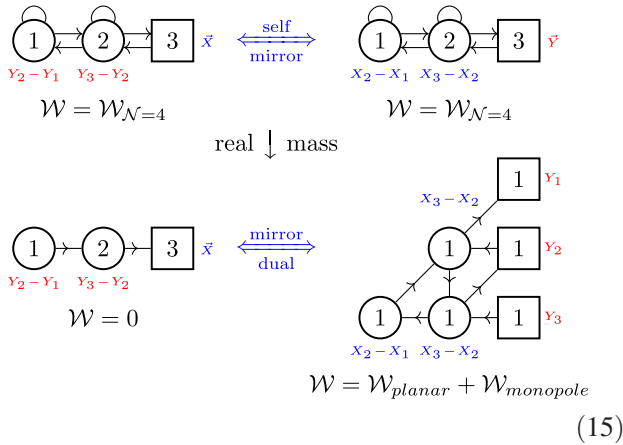
Notice that there are two gauge nodes at CS level  $-\frac{1}{2}$  (indicated in red). The duality map between the baryons on the electric side and the monopoles on the mirror side is now given by,

$$B \leftrightarrow \left\{ \begin{array}{l} \mathfrak{M} \begin{pmatrix} - & 0 & 0 \\ 0 & 0 & \end{pmatrix}, \mathfrak{M} \begin{pmatrix} - & - & 0 \\ 0 & 0 & \end{pmatrix}, \mathfrak{M} \begin{pmatrix} - & - & - \\ 0 & 0 & \end{pmatrix}, \\ \mathfrak{M} \begin{pmatrix} - & - & 0 \\ - & 0 & \end{pmatrix}, \mathfrak{M} \begin{pmatrix} - & - & - \\ - & 0 & \end{pmatrix}, \mathfrak{M} \begin{pmatrix} - & - & - \\ - & - & \end{pmatrix} \end{array} \right\}, \quad (14)$$

which can be checked by computing the charges of monopole operators as described in the previous Section.

*A linear quiver with chiral matter and its planar Abelian dual.* We can perform similar real mass deformations in mirror pairs of unitary quiver  $\mathcal{N} = 4$  theories. Here we

consider the example of the  $T[SU(N)]$  theory [10]. The global symmetry is  $SU(N)_{\bar{X}} \times SU(N)_{\bar{Y}}$  and mirror symmetry is a self-duality that exchanges the two  $SU(N)$  symmetries. We turn on a real mass deformation for the commutant of  $U(1)_R$  and the  $SU(N)_{\bar{Y}}$  symmetry that breaks SUSY to  $\mathcal{N} = 2$  and breaks  $SU(N)_{\bar{Y}} \rightarrow U(1)_{\bar{Y}_i}^{N-1}$ . We choose a vacuum in which the chiral adjoint multiplets and half of the chiral bifundamental fields are integrated. The dual vacuum in the mirror side Abelianizes all the gauge groups. For  $N = 3$ , we find the following duality:



In the chiral linear quiver, there is a CS term at level 1 for the diagonal  $U(1) \subset U(m)$  for each  $U(m)$  gauge group and there is a mixed CS term at level  $-1$  between adjacent nodes. The CS and mixed CS couplings of the mirror planar Abelian quiver follow from the prescription outlined in Sec. I.

The chiral rings of the two theories also match, with the  $N - 1$  dressed gauge invariant monopoles of the electric theory mapped to the  $N - 1$  mesonic operators constructed along the shortest path connecting two adjacent  $U(1)$  flavor nodes.

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*Data availability.* No data were created or analyzed in this study.

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